W.B.VASANTHA KANDASAMY
FLORENTIN SMARANDACHE

ALGEBRAIC STRUCTURES ON FINITE COMPLEX MODULO INTEGER INTERVAL C([0, n))

Algebraic Structures on Finite Complex Modulo Integer Interval C([0, n))

W. B. Vasantha Kandasamy
Florentin Smarandache

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PREFACE

In this book authors introduce the notion of finite complex modulo integer intervals. Finite complex modulo integers was introduced by the authors in 2011. Now using this finite complex modulo integer intervals several algebraic structures are built.

Further the concept of finite complex modulo integers itself happens to be new and innovative for in case of finite complex modulo integers the square value of the finite complex number varies with varying n of Z_n.

In case of finite complex modulo integer intervals also we can have only pseudo ring as the distributive law is not true, in general in C([0, n)). Finally the concepts of pseudo vector spaces and pseudo linear algebras are introduced. At every stage the neutrosophic analogue is also defined, developed and described.

Several interesting properties about these new structures built using $C([0, n))$ and $C(\langle[0, n) \cup I\rangle)$ are described. There are several open problems suggested.

We wish to acknowledge Dr. K Kandasamy for his sustained support and encouragement in the writing of this book.

W.B.VASANTHA KANDASAMY
FLORENTIN SMARANDACHE

Chapter One

INTRODUCTION

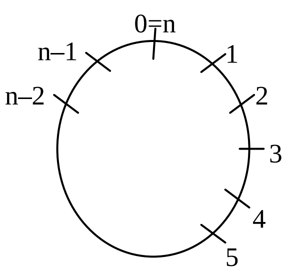
In this chapter we just introduce the notion of finite complex modulo integer interval

$$C([0, n)) = \{a + bi_F \mid a, b \in [0, n),\ i_F^2 = n-1\}.$$

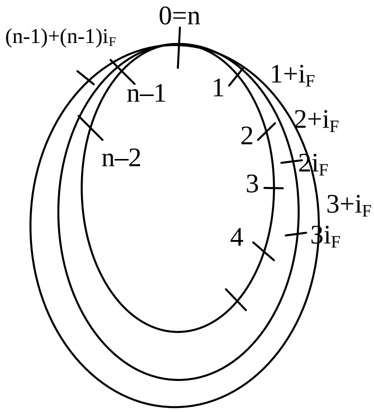
We know the finite modulo integer

$C(Z_n) = \{a + bi_F \mid a, b \in Z_n),\ i_F^2 = n-1\}$, i_F is the finite complex modulo integer which depends of the interger n. For more about finite complex modulo intergers refer [53].

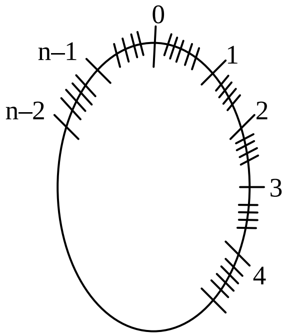
The diagrammatic representation of Z_n is as follows:



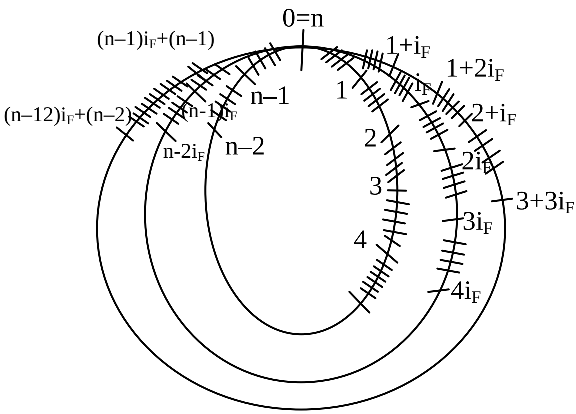
The diagrammatic representation of $C(Z_n)$ is given in the following :



The diagrammatic representation of the modulo interval [0, n) is as follows:



The diagrammatic representation of C([0, n))



$C([0, 3)) = \{a + bi_F \mid a, b \in [0, 3),\ i_F^2 = 2\}$.

$C([0, 19)) = \{a + bi_F \mid a, b \in [0, 3),\ i_F^2 = 18\}$
and so on.

Now B = C([0, n)) can be made into a pseudo ring for + and $\times$ are non distributive that is $a \times (b + c) \neq a \times b + b \times c$ in general for a, b, c $\in$ B.

However B is an abelian group under addition modulo n. B is an abelian semigroup under product modulo n.

Further B under min operation is a semigroup known as the finite complex modulo integer interval semigroup. Similarly {C([0, n)), max} also is a semigroup of the finite complex modulo integer interval.

However they exibit distinct properties under max and min operations. We see {C([0, n), min, max} is a semiring called the finite complex modulo integer interval semiring. In the ring no ideal can be a filter and vice versa. Both filters and ideals are of infinite cardinality. But $\{C([0, n)), \min, \times\}$ happens to be a pseudo semiring ring as

$a \times \min \{c, d\} \neq \min \{a \times c, a \times d\}$ in general for $a, c, d \in C([0, n))$. In this pseudo semiring we have filters to be ideals.

Further we see $D = C (\langle[0, n) \cup I\rangle) = \{a + bi_F + cI + dIi_F \mid a, b, c, d \in [0, n), \; i_F^2 = n–1 \; (Ii_F)^2 = (n–1)I$ and $I^2 = I\}$ is the neutrosophic finite complex modulo integer interval.

Now on this D also analogous study as in case of $C([0, n))$ has been made. This is a richer structure and of course $B \subseteq D$ as proper subsets. Certainly all these study can lead to several applications. For properties about the neutrosophic concepts please refer [53].

Finally we build vector spaces using $C([0, n))$ and $C (\langle[0, n) \cup I\rangle)$. We see they are vector spaces if n is a prime over Z_p.

If n is not a prime they can be vector spaces over S-rings Z_n or $C(Z_n)$ or $\langle Z_n \cup I\rangle$ or $C (\langle Z_n \cup I\rangle)$. However they fail to be linear algebras over Z_p or over S-rings mentioned above. They are only pseudo linear algebras for distributivity is not in general true for every triple.

But only when vector spaces or linear algebras are defined over $C([0, n))$ (or $C (\langle[0, n) \cup I\rangle)$ it can help us define inner product and linear functionals.

Hence this sort of study is carried out in the last chapter of this book. Several open problems are suggest as the field is very new hence can lead to several nice applications is various fields of research.

Chapter Two

FINITE COMPLEX MODULO INTEGER INTERVALS

In this chapter we for the first time define the notion the complex modulo integers on the intervals. Finite complex modulo integers have been defined in [53].

$C(Z_n) = \{a + bi_F \mid a, b \in Z_n \text{ and } i_F^2 = n-1\}$. Study on this collection was also made in [53].

However $|C(Z_n)| < \infty$. But these interval finite complex numbers are defined as follows.

DEFINITION 2.1: *Let $C([0, n)) = \{a + bi_F \mid a, b \in [0, n);\ i_F^2 = n-1\};\ n > 1$. $C([0, n))$ is defined as the finite complex number modulo integer interval.*

Clearly $C([0, n)) = \infty$. Further $C(Z_n) \underset{\neq}{\subset} C([0, n))$.

Note: We see n = 1 has no relevance to the situation as i_F^2 = 1 and 1 $\notin$ [0, 1). Hence complex finite modulo integer interval can be defined only when n > 1 (n = 1 it is not defined).

We will illustrate this situation by some simple examples.

Example 2.1: Let C([0, 5)) = {a + bi_F | a, b $\in$ [0, 5), i_F^2 = 4} be the finite complex modulo integer interval; clearly o(C([0, 5)) = ∞.

Example 2.2: Let C([0, 10)) = {a + bi_F | a, b $\in$ [0, 10), i_F^2 = 9} be the finite complex modulo integer interval.

Clearly o(C([0, 10)) = ∞.

Example 2.3: Let C([0, 45)) = {a + bi_F | a, b $\in$ [0, 45), i_F^2 = 44} be the finite complex modulo integer interval.

Example 2.4: Let C([0, 125)) = {a + bi_F | a, b $\in$ [0, 125), i_F^2 = 124} be the finite complex modulo integer interval.

Now having seen a few examples of them we now proceed onto define some operations on them.

We first define the plus operation modulo n on C([0, n)).

Example 2.5: Let us take
C([0, 6)) = {a + bi_F | a, b $\in$ [0, 6), i_F^2 = 5, +}.

Define + modulo 6 on C[0, 6).

Let x = 0.37 + 4.27 i_F and y = 5.63 + 1.2 i_F $\in$ C([0, 6)).

$$\begin{aligned} x + y &= (0.37 + 4.27i_F) + (5.63 + 1.2i_F) \\ &= (0.37 + 5.63) + (4.27 + 1.20)i_F \\ &= 6 + 5.47i_F \\ &= 0 + 5.47i_F \in C([0, 6)). \end{aligned}$$

This is the way the + modulo 6 operation is performed on C([0, 6)).

$0 + 0i_F = 0$ acts as the additive identity of C([0, 6)).

We see C([0, 6)) is closed under + and infact C([0, 6)) is a group.

For every $x \in$ C([0, 6)) we have a unique y such that $x + y = 0$.

Suppose $x = 3.2119 + 2.6075\, i_F \in$ C([0, 6)) is such that

$$\begin{aligned} x + y &= 3.2119 + 2.6075i_F + 2.7881 + 3.3925i_F \\ &= (3.2119 + 2.7881) + (2.6075 + 3.3925)i_F \\ &= 6 + 6i_F = 0 + 0i_F = 0 \text{ is the additive identity of C([0, 6)).} \end{aligned}$$

Thus C([0, 6)) is a finite complex modulo integer interval group of infinite order which is commutative. C([0, 6)) also has subgroups of finite order also.

Example 2.6: Let C[0, 8)) = $\{a + bi_F \mid a, b \in [[0, 8),\ i_F^2 = 7, +\}$ be the additive abelian group of infinite order.

Let $Z_8 = \{0, 1, 2, \ldots, 7\} \subseteq$ C([0, 8)) be a subgroup of finite order.

$C(Z_8) = \{a + bi_F \mid a, b \in Z_8,\ i_F^2 = 7, +\} \subseteq$ C([0, 8)) is a subgroup of finite order.

$L = \{0, 4, 4i_F, 4 + 4i_F\} \subseteq$ C([0, 8)) is a subgroup of order four.

$M = \{0, 4 + 4i_F, +\}$ is again a subgroup of finite order and $o(M) = 2$.

$N = \{0, 2i_F, 4i_F, 6i_F, +\} \subseteq C([0, 8))$ is a subgroup of order four under +.

Example 2.7: Let
$C([0, 19)) = \{a + bi_F \mid a, b \in [0, 19),\ i_F^2 = 18,\ +\}$ be the finite complex modulo integer group under + of infinite order.

$C([0, 19))$ has subgroups of both infinite and finite order. Z_{19} is a finite subgroup under +.

$C(Z_{19})$ is again a finite subgroup under + and so on; $\{[0, 19), +\} \subseteq C([0, 19))$ is a subgroup of infinite order. $S = \{ai_F \mid a \in [0, 19), +\}$ is a subgroup of $C([0, 19))$ under addition.

Thus $C([0, 19)$ has subgroups of both finite and infinite order.

Example 2.8: Let
$S = C([0, 24))) = \{a + bi_F \mid a, b \in [0, 24),\ i_F^2 = 23, +\}$ be the finite complex modulo integer group under +. S is an infinite group under +. S is of infinite order. S has subgroups both of finite and infinite order.

Infact this S has more number of finite subgroups as 24 is a composite number.

Inview of all these we have the following theorem.

THEOREM 2.1: *Let*
$S = \{C([0, n))\} = \{a + bi_F \mid a, b \in [0, n),\ i_F^2 = n–1, +\}$ be the finite complex modulo integer interval group of infinite order.

(i) S has both finite and infinite order subgroups.

(ii) If n is a composite number S has more number of finite subgroups.

(iii) *Z_n is a subgroup of order n.*

(iv) *$C(Z_n)$ is a finite complex modulo integer finite subgroup of S.*

(v) *$T = \{ai_F \mid a \in [0, n), +\}$ is an infinite complex number subgroup.*

(vi) *$M = \{a \mid a \in [0, n), +\}$ is an infinite real subgroup.*

The proof is direct hence left as an exercise to the reader.

Example 2.9: Let
$M = \{C([0, 28))\} = \{a + bi_F \mid a, b \in [0, 28);\ i_F^2 = 27\}$ be the complex finite modulo integer group of infinite order.

$W = \{a + bi_F \mid a, b \in Z_{28},\ i_F^2 = 27\} \subseteq M$ is a subgroup of finite order which is a complex finite modulo integer group of finite order.

$L = \{a \mid a \in Z_{28}, +\} \subseteq M$ is a finite subgroup of order 28.

$T = \{ai_F \mid a \in Z_{28}, +\} \subseteq M$ is a subgroup of finite order.

$V = \{a \mid a \in [0, 28), +\} \subseteq M$ is an infinite subgroup of real numbers in the semi open interval.

$B = \{ai_F \mid a \in [0, 28), +\} \subseteq M$ is also an infinite subgroup of finite complex modulo integers.

Example 2.10: Let
$M = \{C([0, 27)\} = \{a + bi_F \mid a, b \in [0, 27),\ i_F^2 = 26, +\}$ be a group of infinite order.

Now using C([0, n)) we can build groups under +. This is illustrated by the following examples.

Example 2.11: Let
$S = \{(a_1, a_2, \ldots, a_8) \mid a_i \in ([0, 9); 1 \leq i \leq 8, +\}$ be a group of infinite order; $(0, 0, \ldots, 0)$ acts as the additive identity.

Let $A = \{(a_1, 0, 0, \ldots, 0) \mid a_1 = a + bi_F \in C([0, 9)), +\} \subseteq S$ is a subgroup of infinite order. S has several subgroups of both finite and infinite order.

Example 2.12: Let
$M = \{(a_1, a_2, a_3)$ where $a_i \in C([0, 28)); 1 \leq i \leq 3, +\}$ be a group.

Let $x = (0.71 + 4.31i_F, 8.4 + 15.2i_F, 11.5 + 3.21i_F)$ and $y = (27.29 + 23.69i_F, 19.6 + 12.8i_F, 16.5 + 24.79i_F) \in M$.

We see $x + y = (0.71 + 4.31i_F, 8.4 + 15.2i_F, 11.5 + 3.2i_F) + (27.29 + 23.69i_F, 19.6 + 12.8i_F, 16.5 + 24.79i_F)$

$= (0.71 + 4.31i_F + 27.29 + 23.69i_F, 8.4 + 15.2i_F + 19.6 + 12.8i_F, 11.5 + 3.21i_F + 16.5 + 24.79i_F)$

$= (0.71 + 27.29 + 4.31i_F + 23.69i_F, 8.4 + 19.6 + (15.2 + 17.8)i_F, 11.5 + 16.5 + 3.21i_F + 24.79i_F)$

$= (0, 0, 0)$.

Thus x is the inverse of y and y is the inverse of x under +.

Example 2.13: Let

$$P = \left\{ \begin{bmatrix} a_1 \\ a_2 \\ a_3 \\ a_4 \\ a_5 \\ a_6 \end{bmatrix} \middle| \; a_i = a + bI \in C([0, 24)), 1 \leq i \leq 6, +\right\}$$

be the column matrix finite complex modulo integer interval group under +.

P has subgroups of finite order and infinite order.

Example 2.14: Let

$$V = \left\{ \begin{bmatrix} a_1 & a_2 & a_3 \\ a_4 & a_5 & a_6 \\ \vdots & \vdots & \vdots \\ a_{22} & a_{23} & a_{24} \end{bmatrix} \middle|\, a_i \in C\,([0, 48)),\ 1 \leq i \leq 24, +\right\}$$

be the group of infinite order.

V has several finite subgroups and infinite subgroups.

Example 2.15: Let

$$W = \left\{ \begin{bmatrix} a_1 & a_2 & a_3 \\ a_4 & a_5 & a_6 \\ a_7 & a_8 & a_9 \end{bmatrix} \middle|\, a_i \in C\,([0, 12)),\ 1 \leq i \leq 9, +\right\}$$

be the finite complex modulo integer interval group under + .

Example 2.16: Let

$$W = \left\{ \begin{bmatrix} a_1 & a_2 & a_3 & a_4 \\ a_5 & a_6 & a_7 & a_8 \\ \vdots & \vdots & \vdots & \vdots \\ a_{37} & a_{38} & a_{39} & a_{40} \end{bmatrix} \middle|\, a_i \in C\,([0, 28)),\ 1 \leq i \leq 40, +\right\}$$

be the group of finite complex modulo integer interval [0, 28). This S also has several subgroups of both finite and infinite order.

Example 2.17: Let

$$T = \{(a_1 \mid a_2\ a_3 \mid a_4\ a_5 \mid a_6) \mid a_i \in C([0, 27)), 1 \leq i \leq 6, +\}$$

be the super matrix finite complex modulo integer interval group.

Example 2.18: Let

$$M = \left\{ \left[\begin{array}{c|cccc|c} a_1 & a_2 & a_3 & a_4 & a_5 & a_6 \\ a_7 & \ldots & \ldots & \ldots & \ldots & a_{12} \\ \hline a_{13} & \ldots & \ldots & \ldots & \ldots & a_{18} \\ a_{19} & \ldots & \ldots & \ldots & \ldots & a_{24} \\ a_{25} & \ldots & \ldots & \ldots & \ldots & a_{30} \\ \hline a_{31} & \ldots & \ldots & \ldots & \ldots & a_{36} \\ \hline a_{37} & \ldots & \ldots & \ldots & \ldots & a_{42} \\ a_{43} & \ldots & \ldots & \ldots & \ldots & a_{49} \end{array} \right] \right| a_i \in C([0, 43)), \quad 1 \leq i \leq 49, +\}$$

be a super matrix group of finite complex modulo integer interval [0, 43). This M is of infinite order and has several subgroups of finite and infinite order.

Example 2.19: Let

$$S = \left\{ \left[\begin{array}{c|ccc|cc|cccc} a_1 & a_2 & a_3 & a_4 & a_5 & a_6 & a_7 & a_8 & a_9 & a_{10} \\ a_{11} & a_{12} & \ldots & \ldots & \ldots & \ldots & \ldots & \ldots & \ldots & a_{20} \\ a_{21} & a_{22} & \ldots & \ldots & \ldots & \ldots & \ldots & \ldots & \ldots & a_{30} \\ a_{31} & a_{32} & \ldots & \ldots & \ldots & \ldots & \ldots & \ldots & \ldots & a_{40} \end{array} \right] \right| a_i \in C([0, 248)), 1 \leq i \leq 40, i_F^2 = 247, +\}$$

be the super row matrix group of finite complex modulo integer intervals of infinite order. S has subgroups of finite as well as infinite order.

Infact S has atleast 40 subgroups which is isomorphic to the group G = {a + bi$_F$ | a, b $\in$ [0, 248), $i_F^2 = 247$, +} = {C ([0, 248), +}.

S has atleast 80 subgroups of order 248. S has atleast 80 subgroups of order $(248)^2$.

S has atleast 80 subgroups of infinite order equal to order of G and so on.

Example 2.20: Let

$$S = \left\{ \left[\begin{array}{c} a_1 \\ \hline a_2 \\ a_3 \\ \hline a_4 \\ a_5 \\ \hline a_6 \\ a_7 \\ a_8 \\ a_9 \\ \hline a_{10} \\ a_{11} \\ a_{12} \\ \hline a_{13} \end{array}\right] \middle| \; a_i \in C([0, 11)), 1 \le i \le 30, i_F^2 = 10, + \right\}$$

be the super column matrix of finite complex modulo integer interval group of infinite order.

S has 13 subgroups isomorphic to {C([0, 11)), +}

S has at least 3 $({}_{13}C_1 + {}_{13}C_2 + \ldots + {}_{13}C_{12})$ number of finite subgroups.

S has at least 3 $({}_{13}C_1 + {}_{13}C_2 + \ldots + {}_{13}C_{12})$ number of infinite subgroups.

Example 2.21: Let

$$S = \left\{\left[\begin{array}{cc|cc} a_1 & a_2 & a_3 & a_4 \\ \hline a_5 & a_6 & a_7 & a_8 \\ a_9 & a_{10} & a_{11} & a_{12} \\ a_{13} & a_{14} & a_{15} & a_{16} \end{array}\right] \middle|\; a_i \in C\,([0, 19)),\ 1 \leq i \leq 16,\ i_F^2 = 18\right\}$$

be the super square matrix of finite complex modulo integer interval group.

S has 16 subgroups isomorphic with G = {C([0, 19)), +}.

S has at least 3 $({}_{16}C_1 + {}_{16}C_2 + \ldots + {}_{16}C_{15})$ number of subgroups of finite order.

S has at least 3 $({}_{16}C_1 + {}_{16}C_2 + \ldots + {}_{16}C_{15})$ number of subgroups of infinite order.

Inview of all these we have the following theorem.

THEOREM 2.2: *Let* *S = {m × n matrices with entries from C([0, s)), $i_F^2 = s - 1$, +} be the group of finite complex modulo integer interval.*

(i) *S has at least m × n number of subgroups isomorphic to the group G = {C([0, s)), +}.*

(ii) *S has at least 3 $({}_{m\times n}C_1 + {}_{m\times n}C_2 + \ldots + {}_{m\times n}C_{m\times n-1})$ number of subgroups of finite order if s is a prime.*

(iii) *If s is not a prime say s has t number of subgroups of finite order then S has at least $(3 + 3t)\ ({}_{m\times n}C + {}_{m\times n}C_2 + \ldots + {}_{m\times n}C_{m\times n-1})$ number of subgroups of finite order.*

(iv) *S has at least 3 $({}_{m\times n}C_1 + \ldots + {}_{m\times n}C_{m\times n-1})$ subgroups of infinite order.*

The proof is direct and left as an exercise to the reader.

Now we proceed onto study finite neutrosophic complex modulo numbers interval.

Let $C(a + bI \mid a, b \in [0, n)) = \{a_1 + a_2I + a_3i_F + a_4i_FI \mid a_1, a_2, a_3, a_4 \in [0, n)$ with $I^2 = I$ and $i_F^2 = n-1$ and $(Ii_F)^2 = (n-1)I\}$ denote the finite neutrosophic complex modulo integer interval of infinite order. Clearly $n > 1$ we see on $C(a + bI \mid a, b \in [0, n))$ we can define the addition operation +. These are illustrated by some examples.

Example 2.22: Let $S = \{a_1 + a_2i_F + a_3I + a_4Ii_F \mid a_i \in [0, 5), 1 \leq i \leq 4;\ i_F^2 = 4, (Ii_F)^2 = 4I, I^2 = I\}$ be the finite neutrosophic complex modulo integer interval.

Example 2.23: Let $S = \{a_1 + a_2i_F + a_3I + a_4Ii_F \mid a_i \in [0, 12), 1 \leq i \leq 4,\ i_F^2 = 11, (Ii_F)^2 = 11I, I^2 = I\}$ be the finite neutrosophic complex modulo integer interval.

Example 2.24: Let $S = \{a_1 + a_2i_F + a_3I + a_4Ii_F \mid a_i \in [0, 29), 1 \leq i \leq 4,\ i_F^2 = 28, (Ii_F)^2 = 28I, I^2 = I\}$ be the finite neutrosophic complex modulo integer interval.

We can have infinite collection of such finite complex neutrosophic modulo integer intervals.

We will illustrate how they can be made into groups under '+' of infinite order.

Example 2.25: Let $S = \{a_1 + a_2i_F + a_3I + a_4Ii_F \mid a_i \in [0, 43), 1 \leq i \leq 4,\ i_F^2 = 42, (Ii_F)^2 = 42I, I^2 = I, +\}$ be the finite complex neutrosophic modulo integer interval group under "+" of infinite order. $0 + 0i_F + 0I + 0Ii_F = 0$ serves as the additive identity of S.

Let $x = 0.3 + 9.2i_F + 20I + 4.3i_FI$ and $y = 8.5 + 24.8I + 27.i_FI \in S$. We find x + y as follows:

$x + y = (0.3 + 9.2i_F + 20I + 4.3i_FI) + (8.5 + 24.8I + 2.7i_FI)$

$= (0.3 + 8.5) + (9.2 + 0)i_F + (20 + 24.8)I + (4.3 + 24.8)I + (4.3 + 2.7)i_FI$ [addition (mod 43)]

$= 8.8 + 9.2iF + 1.8I + 7i_FI \in S$.

This is the way addition + is performed on S.

0 is the additive identity in S.

To every $x \in S$ there exists a unique y in S with $x + y = 0$.

For if $x = 9.31 + 17.2i_F + 40.1I + 38i_FI \in S$ we have a unique y in S such that $x + y = 0$; we see
$y = 33.69 + 25.8i_F + 2.9I + 5i_FI$ is such that $x + y = 0$.

Thus y is the additive inverse of x and vice versa.

Thus S is an infinite commutative interval group under +.

S has both subgroups of finite and infinite order.

$P_1 = \{a \mid a \in [0, 43)\} \subseteq S$ is a subgroup of infinite order.

$P_2 = \{a + bI \mid a, b \in [0, 43)\} \subseteq S$ is a subgroup of infinite order.

$P_3 = \{a + bI + ci_F \mid a, b, c \in [0, 43)\} \subseteq S$ is a subgroup of infinite order.

$P_4 = \{a + bi_F \mid a, b \in [0, 43)\} \subseteq S$ is a subgroup of infinite order.

$P_5 = \{ai_F \mid a \in [0, 43)\} \subseteq S$ is a subgroup of infinite order.

$P_6 = \{aI \mid a \in [0, 43)\} \subseteq S$ is a subgroup of infinite order under + modulo 43.

$P_7 = \{aIi_F \mid a \in [0, 43)\} \subseteq S$ is a subgroup of infinite order under +.

$P_8 = \{aI + bi_FI \mid a, b \in [0, 43)\} \subseteq S$ is also a subgroup of infinite order.

$P_9 = \{ai_F + bi_FI \mid a, b \in [0, 43)\} \subseteq S$ is also a subgroup of infinite order. Thus we have several subgroups of infinite order.

Consider $R_1 = \{a \mid a \in Z_{43}, +\} \subseteq S$ is a subgroup of order 43.

$R_2 = \{ai_F \mid a \in Z_{43}, +\} \subseteq S$ is a subgroup of order 43.

$R_3 = \{aI \mid a \in Z_{43}, +\} \subseteq S$ is a subgroup of order 43.

$R_4 = \{ai_FI \mid a \in Z_{43}, +\} \subseteq S$ is a subgroup of order 43.

$R_5 = \{a + bI \mid a, b \in Z_{43}, +\} \subseteq S$ is a subgroup of order 43^2.

$R_6 = \{a + bi_F \mid a, b \in Z_{43}, +\} \subseteq S$ is a subgroup of order 43^2.

$R_t = \{a + bi_F + cI + dIi_F \mid a, b, c, d \in Z_{43}, +\} \subseteq S$ is a subgroup of order 43^4.

Thus S has both subgroups of finite order as well as infinite order.

Example 2.26: Let $S = \{a_1 + a_2i_F + a_3I + a_4Ii_F \mid a_i \in [0, 4)$ with $I^2 = I$, $i_F^2 = 3$, $(i_FI)^2 = 3I, +\}$ be the neutrosophic finite complex modulo integer group of infinite order.

S has both finite subgroups and subgroups of infinite order.

$T_1 = \{a_1 \mid a_1 \in \{0, 2\}, +\} \subseteq S$ is a subgroup of order two.

$T_2 = \{aI \mid a \in \{0, 2\}, +\} \subseteq S$ is a subgroup of order two.

$T_3 = \{a + bI \mid a, b \in \{0, 2\}, +\} \subseteq S$ is a subgroup of order 4.

$T_4 = \{ai_F \mid a \in \{0, 2\}, +\}$ is a subgroup of S of order two.

$T_5 = \{ai_F + bI \mid a, b \in \{0, 2\}, +\}$ is a subgroup of order four.

$T_6 = \{a_1 + ai_F + bI \mid a_1, b, a \in \{0, 2\}, +\} \subseteq S$ is a subgroup of order 8.

$T_7 = \{a_1 + a_2I + a_3i_F + a_4Ii_F \mid a_i \in \{0, 2\}, +, 1 \leq i \leq 4\} \subseteq S$ is a subgroup of order 16.

$T_8 = \{a_1 + a_2I \mid a_1, a_2 \in Z_4, +\} \subseteq S$ is a subgroup of order 16.

But $T_7 \cong T_8$.

$T_9 = \{a_1 + a_2I + a_3i_F \mid a_i \in Z_4, 1 \leq i \leq 3, +\}$ be a subgroup of order 64.

$T_{10} = \{a_1 + a_2I + a_3i_F + a_4Ii_F \mid a_i \in Z_4, 1 \leq i \leq 4, +\}$ is a subgroup of order 4^4.

$T_{11} = \{a_1 + a_2I + a_3i_F + a_4Ii_F \mid a_1, a_2 \in Z_4$ and $a_3, a_4 \in \{0, 2\}, +\} \subseteq S$ is a subgroup of order $16 \times 4 = 64$.

But T_{11} is not isomorphic with T_9 as subgroups we have also subgroups of S of infinite order.

$P_1 = \{a \mid a \in [0, 4), +\} \subseteq S$ is a subgroup of infinite order.

$P_2 = \{ai_F \mid a \in [0, 4), +\} \subseteq S$ is again a subgroup of infinite order.

$P_3 = \{aI \mid a \in [0, 4), +\} \subseteq S$ is a subgroup of infinite order.

$P_4 = \{a + bI \mid a, b \in [0, 4), +\}$ is a subgroup of infinite order.

$P_5 = \{a + bI + ci_F \mid a, b \in [0, 4), c \in Z_4, +\} \subseteq S$ is also a subgroup of infinite order.

Thus S has several subgroups both a finite and infinite order.

Example 2.27: Let M = $\{a_1 + a_2I + a_3i_F + a_4Ii_F \mid a_i \in [0, 23), 1 \leq i \leq 4, i_F^2 = 22, I^2 = I, (Ii_F)^2 = 22I, +\}$ be a neutrosophic finite complex modulo number interval group under addition.

M has several subgroups both of finite and infinite order.

Now using these neutrosophic finite complex modulo integer group we can build groups of matrices. All these we will only illustrate by examples.

Example 2.28: Let M = $\{a_1 + a_2i_F + a_3I + a_3Ii_F \mid a_i \in [0, 24), 1 \leq i \leq 4, i_F^2 = 23, I^2 = I, (i_FI)^2 = 23I, +\}$ be the group under +. M is an infinite group which is commutative.

M has subgroups of finite and infinite order.

Example 2.29: Let M = $\{(a_1, a_2, a_3, a_4, a_5) \mid a_i \in a + bi_F + cI + dIi_F; a, b, c, d \in [0, 14), 1 \leq i \leq 5, i_F^2 = 13, I^2 = I, (Ii_F)^2 = 13I, +\}$ be a group of infinite order.

This has both finite and infinite subgroups.

$W_1 = \{(a_1, a_2, a_3, a_4, a_5) \mid a_i = a + bi_F + ci_FI + dI$ with $a, b, c, d \in Z_{14}, 1 \leq i \leq 4, +\} \subseteq M$ is a subgroup of M of finite order. M has several subgroups of finite order. M has also subgroups of infinite order.

$P_1 = \{(a_1, 0, 0, 0, 0) \mid a_1 = a + bi_F + cI + dIi_F$ with $a, b, c, d \in [0, 14), +\} \subseteq M$ is a subgroup of infinite order.

$P_2 = \{(0, a_1, 0, 0, 0) \mid a_1 = a + bi_F + cI + dIi_F$ with $a, b, c, d \in [0, 14), +\} \subseteq M$ is a subgroup of infinite order.

$P_3 = \{(0, 0, a_1, 0, 0) \mid a_1 = a + bi_F + cI + dIi_F$ with $a, b, c, d \in [0, 14), +\} \subseteq M$ is a subgroup of infinite order.

We can have several subgroups of finite and infinite order.

$R_1 = \{(a, 0, 0, 0, 0) \mid a \in [0, 14), +\}$ is a subgroup of infinite order.

$R_2 = \{(a, 0, 0, 0, 0) \mid a \in [0, 7), +\}$ is a subgroup of infinite order two.

$R_3 = \{(a, 0, 0, 0, 0) \mid a = a + bI$ with $a, b \in [0, 7), +\}$ is a subgroup of finite order and so on.

Example 2.30: Let

$$S = \left\{ \left. \begin{bmatrix} a_1 \\ a_2 \\ a_3 \\ a_4 \\ a_5 \\ a_6 \\ a_7 \\ a_8 \end{bmatrix} \right| a_i = a + bi_F + cI + di_FI \text{ with } a, b, c, d \in [0, 6)), +, 1 \leq i \leq 8 \right\}$$

be the finite neutrosophic complex modulo integer group of infinite order. S has subgroups of order 2, 3, 4, 6 and so on.

Example 2.31: Let

$$S = \left\{ \left. \begin{bmatrix} a_1 & a_2 & a_3 \\ a_4 & a_5 & a_6 \\ a_7 & a_8 & a_9 \\ a_{10} & a_{11} & a_{12} \end{bmatrix} \right| a_i = a + bi_F + cI + di_FI \text{ with } a, b, c, d \in [0, 42), +, 1 \leq i \leq 8 \right\}$$

be the finite neutrosophic complex modulo integer interval group of infinite order.

$$P_1 = \left\{\left.\begin{bmatrix} a_1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}\right| a_1 \in [0, 42), +\right\}$$ is a subgroup of infinite order.

$$R_1 = \left\{\left.\begin{bmatrix} a_1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}\right| a_1 \in Z_{42}, +\right\} \subseteq S$$ is a subgroup of finite order.

$$T_1 = \left\{\left.\begin{bmatrix} a & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}\right| a \in C(Z_{42}), +\right\} \subseteq S$$

is a subgroup of finite order.

$$T_2 = \left\{\left.\begin{bmatrix} a & b & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}\right| a, b \in C(Z_{42}), +\right\} \subseteq S$$

is a subgroup of finite order.

$$T_1 \subseteq T_2.$$

Likewise we can construct several subgroups of finite and infinite order.

Example 2.32: Let

$$S = \left\{ \begin{bmatrix} a_1 & a_2 & a_3 \\ a_4 & a_5 & a_6 \\ \vdots & \vdots & \vdots \\ a_{28} & a_{29} & a_{30} \end{bmatrix} \middle| \; a_i = a + bi_F + cI + di_FI \text{ with } a, b, c, d \in [0, 15), +, 1 \leq i \leq 30 \right\}$$

be the neutrosophic complex finite modulo integer interval group of infinite order.

T has many subgroups of finite order and several subgroups of infinite order.

Now we give examples of super matrix finite neutrosophic complex modulo integer interval group of infinite order.

Example 2.33: Let $S = \{(a_1, a_2, a_3 \mid a_4 \mid a_5\ a_6) \mid a_i \in \{a + bi_F + cI + dIi_F \mid a, b, c, d \in [0, 18), 1 \leq i \leq 6, +\}$ be the super row matrix finite neutrosophic complex modulo integer interval group of infinite order.

S has subgroups of finite and infinite order.

$V_1 = (0\ 0\ 0 \mid a \mid 0\ 0) \mid a \in Z_{18}, +\}$ is a finite subgroup of S.

$V_2 = (0\ 0\ 0 \mid 0 \mid a\ 0) \mid a \in C(Z_{18}), +\}$ is again a finite subgroup of S.

But $|V_2| > |V_1|$.

$V_3 = \{(a\ 0\ 0 \mid 0 \mid 0\ 0) \mid a \in C(\langle Z_{18} \cup I\rangle), +\}$ be a finite subgroup of S.

$V_5 = (a_1\ a_2\ a_3 \mid a_4 \mid a_5\ a_6) \mid a_i \in C(Z_{18}), 1 \leq i \leq 6, +\}$ is a finite subgroup of S of finite order.

$W_1 = (a\ 0\ 0 \mid 0 \mid 0\ 0) \mid a \in [0, 18), +\}$ be a subgroup of infinite order.

$W_2 = (a\ b\ 0 \mid 0 \mid 0\ 0) \mid a, b \in \{a_1 + a_2 i_F$ where $a_1, a_2 \in [0, 18), +\}$ is a subgroup of infinite order.

$W_3 = (0\ 0\ 0 \mid a \mid 0\ 0) \mid a \in \{a_1 + a_2 i_F + a_3 I + a_4 i_F I \mid a_i \in [0, 18)\}, 1 \leq i \leq 4, +\} \subseteq S$ is a subgroup of infinite order.

The subgroups W_2 and W_3 are of different infinities.

Example 2.34: Let

$$S = \left\{ \left[\begin{array}{c} a_1 \\ a_2 \\ \hline a_3 \\ a_4 \\ a_5 \\ \hline a_6 \\ a_7 \\ a_8 \\ \hline a_9 \end{array} \right] \middle|\ a_i \in a + bi_F + cI + di_F I \mid a, b, c, d \in [0, 28)\}, \right.$$

$$I^2 = I,\ i_F^2 = 27,\ (Ii_F)^2 = 27I, +, 1 \leq i \leq 9\}$$

be the super column matrix finite neutrosophic complex modulo integer interval group of infinite order.

$$M_1 = \left\{ \left[\begin{array}{c} a \\ 0 \\ \hline 0 \\ 0 \\ 0 \\ \hline 0 \\ 0 \\ 0 \\ \hline 0 \end{array}\right] \middle| \; a \in Z_{28}, + \right\}$$

is a subgroup of finite order;

$$o(M_1) = 28.$$

$$M_2 = \left\{ \left[\begin{array}{c} 0 \\ a \\ \hline 0 \\ 0 \\ 0 \\ \hline 0 \\ 0 \\ 0 \\ \hline 0 \end{array}\right] \middle| \; a \in C(Z_{28}), + \right\}$$

is a subgroup of finite order.

$$M_3 = \left\{ \left. \begin{bmatrix} 0 \\ \underline{0} \\ 0 \\ 0 \\ \underline{0} \\ 0 \\ 0 \\ \underline{0} \\ a \end{bmatrix} \right| a \in \langle Z_{28} \cup I \rangle, + \right\}$$

is a subgroup of finite order.

$$M_4 = \left\{ \left. \begin{bmatrix} 0 \\ \underline{0} \\ 0 \\ 0 \\ \underline{a} \\ 0 \\ 0 \\ \underline{0} \\ 0 \end{bmatrix} \right| a \in \{a_1 + a_2 i_F + a_3 I + a_4 I i_F \mid a_i \in Z_8, 1 \le i \le 4\}, + \right\}$$

is a subgroup of finite order.

$$M_5 = \left\{ \left[\begin{array}{c} 0 \\ 0 \\ \hline 0 \\ 0 \\ 0 \\ \hline 0 \\ 0 \\ 0 \\ \hline a_1 \end{array} \right] \middle| \, a \in C(Z_8) \text{ and } a_1 \in \langle Z_8 \cup I \rangle \right\}$$

is a subgroup of finite order.

S has several subgroups of finite order and also subgroups of infinite order.

Example 2.35: Let

$$S = \left\{ \left(\begin{array}{c|ccc|c} a_1 & a_2 & a_3 & a_4 & a_5 \\ a_6 & a_7 & a_8 & a_9 & a_{10} \\ a_{11} & a_{12} & a_{13} & a_{14} & a_{15} \end{array} \right) \middle| \text{ where } a_i \in \{a + bI + ci_F + \right.$$

$$d i_F I \mid a, b, c, d \in [0, 40)\}, I^2 = I, \ i_F^2 = 39, (I i_F)^2 = 39I, +,$$

$$\left. 1 \le i \le 15, + \right\}$$

be a super row matrix finite neutrosophic complex modulo integer interval group of infinite order.

Example 2.36: Let

$$S = \left\{ \begin{bmatrix} a_1 & a_2 & a_3 & a_4 & a_5 & a_6 \\ a_7 & \cdots & \cdots & \cdots & \cdots & a_{12} \\ a_{13} & \cdots & \cdots & \cdots & \cdots & a_{18} \\ a_{19} & \cdots & \cdots & \cdots & \cdots & a_{24} \\ a_{25} & \cdots & \cdots & \cdots & \cdots & a_{30} \\ a_{31} & \cdots & \cdots & \cdots & \cdots & a_{36} \end{bmatrix} \right| \text{ where } a_i \in \{a + bI +$$

$$ci_F + di_F I \mid a, b, c, d \in [0, 6)\}, I^2 = I, i_F^2 = 5, (Ii_F)^2 = 5I, +,$$

$$1 \leq i \leq 36, +\}$$

be a super square matrix finite neutrosophic complex modulo integer interval group of infinite order. S has finite order subgroups and infinite order subgroups.

Now having seen groups built using the neutrosophic finite complex modulo integer intervals we now proceed onto define semigroups on C([0, n)) = $\{a + bi_F \mid a, b \in [0, n)\}$ under the three operations min, max and product.

min {a, b} = a or b, a, b $\in$ [0, n).

max {a, b} = a or b, a, b $\in$ [0, n).

min $\{ai_F, bi_F\}$ = min {a, b} i_F that is min a or b
so is max defined.

min $\{a + bi_F, c + di_F\}$ = min {a, c} + min $\{bi_F, di_F\}$.

For every x in C([0, n)) is only of the form $a + bi_F$ where a, b $\in$ [0, n).

We give examples of them in the following.

Let $\{0.3 + 5i_F, 8 + 3.1i_F\} \in \{C([0, 14))\}$.

We see min $\{0.3 + 5i_F, 8 + 3.1i_F\}$
$= \min \{0.3, 8\} + \min \{5i_F, 3.1i_F\}$
$= 0.3 + 3.1i_F$.

max $\{0.3 + 5i_F, 8 + 3.1i_F\}$
$= \max \{0.3, 8\} + \max \{5i_F, 3.1i_F\}$
$= 8 + 5i_F \in C([0, 14))$.

This is the way max or min operation is performed on $C([0, n))$.

Consider the following examples.

Example 2.37: Let $S = \{C([0, 7), \min\}$ be the semigroup.
Let $x = 0.7 + 2.5i_F$ and $y = 0.5 \in S$ min $\{0.7 + 2.5i_F, 0.5\}$
$= \min \{0.7, 0.5\} + \min \{2.5i_F, 0\}$
$= 0.5 + 0.i_F = 0.5 \in S$.

Let $x = 3.7 + 2i_F$, $3.16i_F = y \in S$;
$\min \{x, y\} = \min \{3.7 + 2i_F, 3.16i_F\}$
$= \min \{3.7, 0\} + \min \{2i_F, 3.16i_F\}$
$= 0 + 2i_F = 2i_F \in S$.

S is a semigroup of infinite order. Every singleton set is a subsemigroup of S.

Let $T = \{0.7i_F, 0.8\} \in S$;

T is clearly not closed under min operation as
$\min \{0.7i_F, 0.8\}$

$= \min \{0, 0.8\} + \min \{0.7i_F, 0\}$
$= 0 + 0i_F = 0$ and $0 \notin T$ so T can be completed as

$T_c = \{0, 0.7i_F, 0.8\} \subseteq S$ which is a subsemigroup of S under min operation.

Let P = $\{0.3 + 5.2i_F, 5.1 + 2i_F\} \in S$; P is only a subset and not a subsemigroup under min operation.

$$\begin{aligned}\text{For } &\min \{0.3 + 5.2i_F, 5.1 + 2i_F\} \\ &= \min \{0.3, 5.1\} + \min \{5.2i_F, 2i_F\} \\ &= 0.3 + 2i_F \notin P.\end{aligned}$$

Thus $P_c = \{0.3 + 5.2i_F, 5.1 + 2i_F, 0.3 + 2i_F\} \subseteq S$ is the completion of the subset P as a subsemigroup of S. $o(P_c) = 3$.

Let $M = \{0.7i_F, 4.1i_F, 2.3, 4.2\} \subseteq S$ be a subset of S to complete M into a subsemigroup under the min operation

$$\begin{aligned}&\min \{0.7i_F, 4.1i_F\} = 0.7i_F; \\ &\min \{0.7i_F, 2.3\} = 0, \\ &\min \{0.7i_F, 4.2\} = 0, \\ &\min \{4.1i_F, 2.3\} = 0 \text{ and} \\ &\min \{2.3, 4.2\} = 0.\end{aligned}$$

$M_c = \{0, 0.7i_F, 4.1i_F, 2.3, 4.2\}$ is a subsemigroup of order 5.

Let $A = \{0.3 + 5i_F, 4 + 0.4i_F, 0.7 + 0.2i_F\} \subseteq S$ be the subset of S.

We complete A into a subsemigroup as follows.

$$\begin{aligned}&\min \{0.3 + 5i_F, 4 + 0.4i_F\} \\ &= 0.3 + 0.4i_F \notin A.\end{aligned}$$

$$\begin{aligned}&\min \{0.3 + 5i_F, 0.7 + 0.2i_F\} \\ &= \min \{0.3, 0.7\} + \min \{5i_F, 0.2i_F\} \\ &= 0.3 + 0.2i_F \notin A.\end{aligned}$$

$$\begin{aligned}&\min \{4 + 0.4i_F, 0.7 + 0.2i_F\} \\ &= \min \{4, 0.7\} + \min \{0.4i_F, 0.2i_F\} \\ &= 0.7 + 0.2i_F \in A.\end{aligned}$$

Thus
$A_c = \{0.7 + 0.2i_F, 0.3 + 0.2i_F, 0.3 + 0.4i_F, 0.3 + 5i_F, 4 + 0.4i_F\} \subseteq$ S is a subsemigroup of S of order 5.

We see min {x, x} = x and min {x, 0} = 0 so 0 is the least element.

Example 2.38: Let S = {C([0, 12), min} be a finite complex modulo integer interval semigroup of infinite order.

Example 2.39: Let S = {C([0, 10), min} be the finite complex modulo integer semigroup of infinite order.

P = {x, 0} is a subsemigroup for every $x \in S$. So S has subsemigroups of order 2.

Clearly every singleton subset of S is a subsemigroup of S.

Consider $M = \{6.3 + 0.5i_F, 0.7 + 0.8i_F, 9 + 0.4i_F\} \subseteq S$ a subset of S. We complete M into a subsemigroup.

$$\min \{6.3 + 0.5i_F, 0.7 + 0.8i_F\} = 0.7 + 0.5i_F \notin M$$

$$\min \{6.3 + 0.5i_F, 9 + 0.4i_F\} = 6.3 + 0.4i_F \notin M.$$

$$\min \{0.7 + 0.8i_F, 9 + 0.4i_F\} = \{0.7 + 0.4i_F\} \notin M.$$

$M_c = \{6.3 + 0.5i_F, 0.7 + 0.8i_F, 9 + 0.4i_F, 0.7 + 0.5i_F, 6.3 + 0.4i_F, 0.7 + 0.4i_F\} \subseteq S$ is the completed subsemigroup of S.

Consider $W = \{0.4, 0.9i_F, 0.9 + 0.2i_F, 7i_F + 3.2, 0.5i_F + 2.1\} \subseteq S$; a subset of S.

We complete W as follows:

$\min \{0.4, 0.9i_F\} = 0 \notin W.$
$\min \{0.4, 0.9 + 2i_F\} = 0.4 \in W.$
$\min \{0.4, 7i_F + 3.2\} = 0.4 \in W.$
$\min \{0.4, 0.5i_F + 2.1\} = 0.4 \in W.$
$\min \{0.9i_F, 0.9 + 0.2i_F\} = 0.2i_F \notin W.$

$\min \{0.9i_F, 7i_F + 3.2\} = 0.9i_F \in W.$
$\min \{0.9i_F, 0.5i_F + 2.1\} = 0.5i_F \notin W.$

$\min \{0.9 + 0.2i_F, 7i_F + 3.2\} = 0.9 + 0.2i_F \in W.$
$\min \{0.9 + 0.2i_F, 0.5i_F + 2.1\} = 0.9 + 0.2i_F \in W.$

$\min \{7i_F + 3.2, 0.5i_F + 2.1\} = 0.5i_F + 2.1 \in W.$

Thus $W_c = \{0.4, 0.9i_F, 0, 0.9 + 0.2i_F, 7i_F, 3.2, 0.5i_F + 2.1, 0.2i_F, 0.5i_F\} \subseteq S$ is a subsemigroup of order 8.

Example 2.40: Let T = {C([0, 43), min} be the finite complex modulo integer semigroup under min.

W = {[0, 14), min} is a subsemigroup as well as ideal of T.

M = {[7, 43), min} is a subsemigroup and not an ideal of T.

P = {[6, 20), min} is a subsemigroup and not an ideal of T.

None of the finite subsemigroups of T are ideals.

Since $i_F^2 = 42$ we cannot get sub interval which has finite complex modulo integers, this problem does not arise as the operation on C([0, 43)) is under min and not under product.

Thus we take $A = \{a + bi_F \mid a, b \in [0,12)\} \subseteq T$ under min operation is an ideal.

However $B = \{a + bi_F \mid a, b \in [6, 15)\} \subseteq T$ is not an ideal.

C = {a + bi_F | a, b ∈ [20, 43)} ⊆ T is not an ideal only a subsemigroup.

Example 2.41: Let S = {C([0, 12), min} be the finite complex modulo integer semigroup of infinite order.

T = {C(Z_{12}), min} ⊆ S is a finite subsemigroup of S and is not an ideal of S.

W = {[0, 12), min} is a subsemigroup as well as an ideal of S.

M = {ai_F | a ∈ [0, 12), min} ⊆ S is a subsemigroup as well as an ideal of S.

Inview of all these we have the following theorem.

THEOREM 2.3: *Let S {C([0, n)), min, i_F^2 = n–1} be the finite complex modulo integer semigroup of infinite order.*

1. *S has infinite number of finite subsemigroups and none of them are ideals.*

2. *T = {[0, t), t < n–1, min} ⊆ S is a subsemigroup of infinite order, is also an ideal of S for t < n–1.*

3. *W = {[a, n) | a > 0, min} ⊆ S is a subsemigroup of infinite order which is not an ideal for all a > 0 and a < n–1.*

4. *N = {[a, b) | a > 0 and b < n–1, min} ⊆ S is only a subsemigroup for all a > 0 and b < n–1 and are not ideals of S.*

5. *Let V = {ai_F | a ∈ [0, t); t < n–1, min} ⊆ S be a subsemigroup which is an ideal of infinite order.*

6. *Let L = {ai_F | a ∈ [b, n), b > 0, min} ⊆ S be a subsemigroup of infinite order which is not an ideal of S.*

7. *R = {ai_F | a ∈ [m, t), m > 0, t < n–1} ⊆ S is again a subsemigroup of infinite order which is not an ideal of S.*

The proof is left as an exercise to the reader.

Now we build using the finite complex modulo integer interval semigroup under min operation; matrix finite complex modulo integer matrix semigroups under min operation which is also illustrated by examples.

Example 2.42: Let
S = {(a_1, a_2, a_3, a_4) | a_i ∈ ([0, 12), 1 ≤ i ≤ 4, min} be the row matrix finite complex modulo integer interval semigroup of infinite order.

S has subsemigroups of order one. For take A_1 = {(0.3i_F, 2, 4 + 2i_F, 0.7 + 0.5i_F) ⊆ S is a subsemigroup of order one.

S has finite subsemigroups of order two. This collection is also infinite in number.

B_1 = {(0, 0.7i_F, 8 + 0.4i_F, 4 + 6i_F), (0, 0, 0, 0)} ⊆ S is a subsemigroup of order two.

Thus B_1 = {x, (0, 0, 0, 0) | x ∈ S} is a subsemigroup of order two.

Now we can order any two elements in S under min operation as follows. This is not the usual order as S or for that matter C([0, n)) are not orderable but we can define a comparison relation.

Let x = 8.2 + 0.3i_F and y = 2.7 + 0.03i_F ∈ C([0, 10))

$y \leq_{min}$ x as min {x, y} = min $\{8.2 + 0.3i_F, 2.7 + 0.03i_F\}$
$= \min \{8.2, 2.7\} + \min \{0.3i_F, 0.03i_F\}$
$= 2.7 + 0z.03i_F = y.$

So we say $y \leq$ min x.

Likewise max {x, y} = max $\{8.2 + 0.3i_F, 2.7 + 0.03i_F\}$
$= \max \{8.2, 2.7\} + \max \{0.3i_F, 0.03i_F\}$
$= 8.2 + 0.3i_F = x.$

So max {x, y} = x and it is denoted by $x \geq_{max} y$.

Now we call this ordering as quasi special ordering as ordering is not possible in case of Z_n or $C(Z_n)$ or [0, n) or C([0, n)).

So if in example 2.42 we have $x = (6.3 + 7i_F, 4.2 + 0.9i_F, 10 + 2.9i_F, 11 + 11i_F)$ and $y = (2.5 + 3i_F, 0.6i_F, 7, 10 + 2i_F) \in S$ we see min {x, y} = max $\{(6.3 + 7i_F, 4.2 + 9i_F, 10 + 2.9i_F, 11 + 11i_F), (2.5 + 3i_F, 0.6i_F, 7, 10 + 2i_F)\} = (2.5 + 3i_F, 0.6i_F, 7, 10+2i_F) = y$.

Thus $y \leq_{min} x$, '$\leq_{min}$' is defined as a special quasi ordering.

We see for any $x, y \in S$ we cannot say $y \leq_{min} x$; we may or may not be in a position to put a special quasi ordering on them.

Example 2.43: Let

$$T = \left\{ \begin{bmatrix} a_1 \\ a_2 \\ a_3 \\ a_4 \\ a_5 \\ a_6 \\ a_7 \end{bmatrix} \middle| \; a_i \in C([0, 24)\}, 1 \leq i \leq 7, \min\} \right.$$

be a semigroup of infinite order.

T can have zero divisors.

$$B_1 = \left\{\begin{bmatrix} a_1 \\ a_2 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}\right\} \text{ and } A_1 = \left\{\begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ a_1 \\ a_2 \end{bmatrix}\right\} \in T;$$

$$\text{we see } \min\{A_1, B_1\} = \left\{\begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}\right\}.$$

Under min operation T has several zero divisors.

Infact T has infinite number of zero divisors.

However T has ideals and subsemigroups which are not ideals.

$$X = \left\{ \begin{bmatrix} 0.87 + 9i_F \\ 0 \\ 9 + 2i_F \\ 0 \\ 23 + 22i_F \\ 9 + 8.7i_F \\ 8 + 18i_F \end{bmatrix} \right\} \subseteq T$$

is a subsemigroup of T but is not as ideal.

Further no finite subsemigroup of T can be an ideal of T. We see T has infinite number of subsemigroups of order one and none of them are ideals of T.

$$\text{Let } A = \left\{ \begin{bmatrix} 9.2 + 11i_F \\ 13 + 5i_F \\ 17 + 10i_F \\ 7 + 13i_F \\ 14 + 12i_F \\ 15 + 9i_F \\ 10.7 + 20.3i_F \end{bmatrix}, \begin{bmatrix} 3 + 2i_F \\ 0.5 + 0.3i_F \\ 7 + 2.1i_F \\ 0 + 5i_F \\ 2 + 3.71i_F \\ 1 + 0.752i_F \\ 9.33 + 0.79i_F \end{bmatrix} \right\}$$

be a subsemigroup of T under min operation.

A is not an ideal of T.

Clearly T has infinite number of subsemigroups of order two.

Likewise T has order three subsemigroups which are not ideals.

Let

$$W = \left\{ \begin{bmatrix} a_1 \\ 0 \\ 0 \\ 0 \\ 0 \\ \vdots \\ 0 \end{bmatrix} \middle| \; a_1 \in C([0, 24)\}, \min\} \right.$$

be the subsemigroup of S. Clearly W is also an ideal of T of infinite order.

Let

$$V = \left\{ \begin{bmatrix} a_1 \\ a_2 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix} \middle| \; a_1, a_2 \in C(Z_{24}), \min\} \subseteq T \right.$$

be a subsemigroup of T.

Clearly V is not an ideal and V is of finite order.

$$\text{Let } M = \left\{ \left. \begin{bmatrix} a_1 \\ a_2 \\ a_3 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix} \right| a_1, a_2, a_3 \in C[0, 24), \min\} \right.$$

be a subsemigroup of T.

M is also an ideal of T. $o(M) = \infty$.

Thus T has several subsemigroups which are not ideals.

Let

$$B = \left\{ \left. \begin{bmatrix} a_1 \\ a_2 \\ 0 \\ 0 \\ 0 \\ \vdots \\ 0 \end{bmatrix} \right| a_1, a_2 \in C([0, 12)\}, \min\} \right.$$

be a subsemigroup B is an ideal for if

$$S = \begin{bmatrix} a + bi_F \\ c + di_F \\ 0 \\ 0 \\ 0 \\ \vdots \\ 0 \end{bmatrix} \in T;\ a, b, c, d \in C([0, 24))$$

then for any x ∈ B and we see min {x, s} ∈ B.

Let

$$D = \left\{ \begin{bmatrix} a+bi_F \\ c+di_F \\ e+gi_F \\ 0 \\ 0 \\ \vdots \\ 0 \end{bmatrix} \middle| \; a, b, c, d, e, f, g \in [0, 14), \min \right\} \subseteq T$$

be the subsemigroup.

Clearly D is an ideal of T of infinite order.

Let

$$E = \left\{ \begin{bmatrix} a+bi_F \\ c+di_F \\ 0 \\ 0 \\ 0 \\ \vdots \\ x+yi_F \end{bmatrix} \middle| \; a, b, c, d, x, y \in [7, 20), \min \right\} \subseteq T$$

be the subsemigroup of infinite order. E is not an ideal of T.

T has infinite number of infinite order subsemigroups which are not ideals of T. T has infinite number of zero divisors and every element of T is an idempotent and a subsemigroup of order one.

Example 2.44: Let

$$M = \left\{ \begin{bmatrix} a_1 & a_2 & a_3 \\ a_4 & a_5 & a_6 \\ \vdots & \vdots & \vdots \\ a_{13} & a_{14} & a_{15} \end{bmatrix} \middle| \; a_i \in C([0, 23)), 1 \leq i \leq 15, \min \right\}$$

be the finite complex modulo integer interval semigroup of infinite order.

Every element is an idempotent. Every singleton is a subsemigroup and is not an ideal.

Likewise we can have

$$P = \{x, \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}\} \subseteq M$$

to be a subsemigroup which is not an ideal of M.

Thus P is a subsemigroup for every $x \in M$.

Hence we have an infinite number of subsemigroups of order two which are not ideals of M.

However M has infinite number of zero divisors. Every element in M is an idempotent.

M has ideals of infinite order and no ideal in M can be of finite order.

Let

$$T = \left\{ \begin{bmatrix} a_1 & a_2 & a_3 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \middle| \; a_i \in [0, 10), 1 \leq i \leq 3, \min \right\} \subseteq M$$

be the subsemigroup of M.

Clearly T is an ideal of infinite order.

Let

$$S = \left\{ \begin{bmatrix} a_1 & 0 & 0 \\ a_2 & 0 & 0 \\ a_3 & 0 & 0 \\ a_4 & 0 & 0 \\ a_5 & 0 & 0 \end{bmatrix} \middle| \; a_i = c_i + d_iI_F \text{ where } c_i, d_i \in [0, 12), \right.$$

$$\left. 1 \leq i \leq 5, \min \right\} \subseteq M$$

be a subsemigroup of infinite order. S is an ideal.

Let

$$W = \left\{ \begin{bmatrix} a_1 & 0 & 0 \\ 0 & a_2 & 0 \\ 0 & 0 & a_3 \\ a_4 & 0 & 0 \\ 0 & a_5 & 0 \end{bmatrix} \middle| \; a_i \in C(Z_{23}), 1 \leq i \leq 5, \min \right\} \subseteq M$$

W is only subsemigroup of finite order and is not an ideal of M.

Let

$$V = \left\{ \begin{bmatrix} a_1 & 0 & 0 \\ 0 & a_2 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & a_3 \end{bmatrix} \middle| \; a_i = c_i + d_i i_F,\ c_i, d_i \in [7, 23),\ 1 \leq i \leq 3, \right.$$

$$\left. \min \right\} \subseteq M$$

be a subsemigroup of infinite order.

Clearly V is only a subsemigroup and not an ideal of M.

Let

$$B = \left\{ \begin{bmatrix} a_1 & a_2 & a_3 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & a_7 & a_8 \\ a_4 & a_5 & a_6 \end{bmatrix} \middle| \; a_i \in c_i + d_i i_F,\ c_i, d_i \in [20, 23), \right.$$

$$\left. 1 \leq i \leq 8,\ \min \right\}$$

be the subsemigroup of infinite order. B is not an ideal of M.

Thus M has subsemigroups of infinite order which are not ideals.

Example 2.45: Let

$$S = \left\{\begin{bmatrix} a_1 & a_2 & a_3 & a_4 \\ a_5 & a_6 & a_7 & a_8 \\ a_9 & a_{10} & a_{11} & a_{12} \\ a_{13} & a_{14} & a_{15} & a_{16} \end{bmatrix} \middle| \; a_i \in C([0, 20)), 1 \leq i \leq 16, \min\right\}$$

be the finite complex modulo integer semigroup of infinite order.

This S has subsemigroups of both finite and infinite order. S has ideals of infinite order.

S has infinite number of zero divisors and idempotents.

Example 2.46: Let

$$W = \left\{\begin{bmatrix} a_1 \\ \hline a_2 \\ a_3 \\ \hline a_4 \\ a_5 \\ \hline a_6 \\ a_7 \\ a_8 \\ \hline a_9 \end{bmatrix} \middle| \; a_i \in C([0, 13)), 1 \leq i \leq 9, \min\right\}$$

be the finite complex modulo integer interval column super matrix semigroup of infinite order.

W has the least element viz (0) = $\left[\begin{array}{c} 0 \\ \hline 0 \\ 0 \\ \hline 0 \\ 0 \\ \hline 0 \\ 0 \\ 0 \\ \hline 0 \end{array}\right]$

and W has infinite number of idempotents and zero divisors.

Example 2.47: Let

$$V = \left\{ \left[\begin{array}{c|cc|c} a_1 & a_2 & a_3 & a_4 \\ \hline a_5 & a_6 & a_7 & a_8 \\ a_9 & a_{10} & a_{11} & a_{12} \\ a_{13} & a_{14} & a_{15} & a_{16} \\ \hline a_{17} & a_{18} & a_{19} & a_{20} \\ a_{21} & a_{22} & a_{23} & a_{24} \end{array}\right] \middle| \; a_i \in C([0, 5)), 1 \leq i \leq 24, \min \right\}$$

be a super matrix finite complex modulo integer interval semigroup of infinite order.

V has subsemigroups of all finite orders which are infinite in number. None of them are ideals.

V also has infinite number of subsemigroups which are not ideals. V has ideals which are of infinite order.

Example 2.48: Let

$$S = \left\{ \left(\begin{array}{c|cc|cc|cc} a_1 & a_2 & a_3 & a_4 & a_5 & a_6 & a_7 \\ a_8 & \ldots & \ldots & \ldots & \ldots & \ldots & a_{14} \\ a_{15} & \ldots & \ldots & \ldots & \ldots & \ldots & a_{21} \end{array} \right) \right| a_i \in C([0, 7)),$$

$$1 \leq i \leq 21, \min\}$$

be the super row matrix of finite complex modulo integer interval semigroup of infinite order.

Every element is a subsemigroup of S. Every element of S is an idempotent. Ideals of S are of infinite order.

Example 2.49: Let

$$B = \left\{ \left[\begin{array}{cccc} a_1 & a_2 & a_3 & a_4 \\ a_5 & a_6 & a_7 & a_8 \\ \hline a_9 & \ldots & \ldots & a_{12} \\ a_{13} & \ldots & \ldots & a_{16} \\ a_{17} & \ldots & \ldots & a_{20} \\ a_{21} & \ldots & \ldots & a_{24} \\ \hline a_{25} & \ldots & \ldots & a_{28} \\ a_{29} & \ldots & \ldots & a_{32} \\ \hline a_{33} & \ldots & \ldots & a_{36} \\ a_{37} & \ldots & \ldots & a_{40} \\ a_{41} & \ldots & \ldots & a_{44} \\ \hline a_{45} & \ldots & \ldots & a_{48} \end{array} \right] \right| a_i \in C([0, 40)),$$

$$1 \leq i \leq 48,\ i_F^2 = 39, \min\}$$

be the neutrosophic finite complex modulo integer interval semigroup. B is of infinite order. B has both finite and infinite subsemigroups which are not ideals. B also has infinite subsemigroups which are ideals.

B has infinite number of zero divisors. Every element in B is a subsemigroup. Every element in B is an idempotent.

Example 2.50: Let

$$V = \left\{ \left[\begin{array}{c|cc} a_1 & a_2 & a_3 \\ \hline a_4 & a_5 & a_6 \\ a_7 & a_8 & a_9 \end{array} \right] \middle| \; a_i \in C([0, 4)), \right.$$

$$\left. 1 \leq i \leq 9, \min \right\}$$

be the finite complex modulo integer interval semigroup of infinit ordr. V has zero divisors.

$$P_1 = \left\{ \left[\begin{array}{c|cc} a & 0 & 0 \\ \hline 0 & 0 & 0 \\ 0 & 0 & 0 \end{array} \right] \middle| \; a_i \in C\,(Z_4), \min \right\}$$

is a finite subsemigroup of V.

$$P_2 = \left\{ \left[\begin{array}{c|cc} 0 & 0 & 0 \\ \hline 0 & a_1 & a_2 \\ 0 & a_3 & a_4 \end{array} \right] \middle| \; a_i \in Z_4, 1 \leq i \leq 4, \min \right\} \subseteq V$$

is again a subsemigroup.

Both P_1 and P_2 are not ideals of V.

$$P_3 = \left\{\left[\begin{array}{c|cc} a_1 & a_2 & 0 \\ \hline 0 & 0 & 0 \\ a_3 & 0 & a_4 \end{array}\right] \middle| a_i \in [0, 4), 1 \le i \le 4, \min\right\} \subseteq V$$

is an infinite subsemigroup as well as an ideal of V.

$$P_4 = \left\{\left[\begin{array}{c|cc} 0 & a_1 & a_2 \\ \hline a_3 & 0 & 0 \\ a_4 & 0 & 0 \end{array}\right] \middle| a_i \in [2, 4), 1 \le i \le 4, \min\right\} \subseteq V$$

is an infinite subsemigroup and is not an ideal of V.

V has infinite number of zero divisors.

For take

$$x = \left[\begin{array}{c|cc} a_1 & 0 & 0 \\ \hline a_2 & 0 & 0 \\ 0 & a_3 & a_4 \end{array}\right] \text{ and } y = \left[\begin{array}{c|cc} 0 & b_1 & b_2 \\ \hline 0 & b_4 & b_5 \\ b_3 & 0 & 0 \end{array}\right]$$

where $a_i, b_j \in C([0, 4))$ $1 \le i \le 4$ and $1 \le j \le 5$ are such that

$$\min\{x, y\} = \left[\begin{array}{c|cc} 0 & 0 & 0 \\ \hline 0 & 0 & 0 \\ 0 & 0 & 0 \end{array}\right] \text{ is a zero divisor in V.}$$

V has infinite number of zero divisors.

We now proceed on to illustrate complex finite modulo integer interval semigroups with max operation by a few examples.

Example 2.51: Let V = {C([0, 3), max} be the semigroup V is of infinite order.

Let $x = 0.21 + 0.4i_F$ and $y = 0.7 + 0.2i_F \in V$.

We see max $\{x, y\}$ = max $\{0.21 + 0.4i_F, 0.7 + 0.2i_F\}$ = max $\{0.21, 0.7\}$ + max $\{0.4i_F, 0.2i_F\} = 0.7 + 0.4i_F$. This is the way max operation is performed on V.

Let $x = 2.1 + 0.75I \in V$ we see max $\{x, x\} = x$ and max $\{x, 0\} = x$.

Example 2.52: Let S = {C([0, 20)), max} be a semigroup of finite complex modulo integer interval. S is a commutative semigroup of infinite order.

Every element in S is a subsemigroup as well as an idempotent.

S has no zero divisors.

This is the marked difference between semigroups under min operation. S under min operation has infinite number of zero divisors.

Example 2.53: Let S = {C([0, 43)), max} be a semigroup of finite complex modulo integer interval. S has subsemigroups of order one, two, three and so on.

Suppose $x = 8.31 + 9.7\,I$ and $y = 12.9 + 3.1I \in S$.

$P = \{x, y\}$ is only a subset of S and is not a subsemigroup.

For max $\{x, y\}$ = max $\{8.31 + 9.7I, 12.9 + 3.1I\}$
= max $\{8.31, 12.9\}$ + max $\{9.7I, 3.1I\}$
$= 12.9 + 9.7I \notin P$.

But $P_c = \{x, y\} \cup \{12.9 + 9.7I\}$
$= \{x, y, 12.9 + 9.7I\} \subseteq S$ is a subsemigroup of S called the completed subsemigroup of S.

Thus given any subset of S we can always completes to form a subsemigroup. However this completed subsemigroup in general is not an ideal of S.

Example 2.54: Let S = {C([0, 24)); max} be a finite complex modulo integer interval semigroup.

Let P = {[0, 12), max} $\subseteq$ S.

Clearly P is a subsemigroup of infinite order. However P is not an ideal for $14 + 3i_F \in S$ and for any $p \in P$; we see max $\{p, 14 + 3i_F\} = 14 + 3i_F \notin P$ hence the claim.

Consider W = {[6, 24), max} $\subseteq$ S, is a subsemigroup of S. W is not an ideal of S; for take $10i_F \in S$, max $\{10i_F, x\}$ for any $x \in W$ gives $x + 10i_F \notin W$; hence the claim.

Let M = $\{a + bi_F \mid a, b \in [8, 24), \max\} \subseteq S$; M is a subsemigroup as well as an ideal of M for any $s \in S$ and $m \in M$ we see max $\{s, m\} \in M$.

Let T = $\{a + bi_F \mid a, b \in Z_{24}, \max\} \subseteq S$, T is only a subsemigroup of finite order and it is not an ideal of S under max operation.

For any $x = 18.7 + 18.7i_F \in S$ and $y = 23 + 10i_F \in T$ we see max $\{x, y\} = 23 + 18.7i_F \notin T$. Hence the claim.

We see S has both finite and infinite subsemigroups which are not ideals. However every ideal of S is of infinite order.

Example 2.55: Let S = {C([0, 23); max} be a semigroup of the finite complex modulo integer integer interval. S has no zero divisors every element is an idempotent and every singleton is a subsemigroup of S.

S has subsemigroups of orders 1, 2, 3 and so on. None of the finite subsemigroups of S are ideals of S.

Now using this semigroup we can build matrix semigroups of finite complex modulo integer intervals.

Example 2.56: Let
M = {(a_1, a_2, a_3, a_4, a_5) | $a_i \in$ C([0, 42)); $1 \leq i \leq 5$, max} be a finite complex modulo integer interval row matrix semigroup. M is commutative and is of infinite order.

T_1 = {(a_1, 0, 0, 0, 0) | $a_1 \in$ C([0, 42)) max} $\subseteq$ M is a subsemigroup of infinite order and T_1 is not an ideal of M.

T_2 = {(0, a_2, 0, 0, 0) | $a_2 \in$ C([0, 42)) max} $\subseteq$ M is a subsemigroup and not an ideal.

For take x = (0.8, 9 + $3i_F$, 2 + $7.1i_F$, $0.9i_F$, 2.1 + i_F) in M and
y = (9.42 + $19.3i_F$, 0, 0, 0, 0) $\in T_1$.

max {x, y} = (9.42 + $19.3i_F$, 9 + $3i_F$, 2 + $7.1i_F$, $0.9i_F$, 2.1 + i_F) $\notin T_1$.

Thus T_1 is not an ideal of M.

Hence we have subsemigrosup of M which are nit ideals of M.

Let W_1 = {(a_1, a_2, a_3, a_4, a_5) | $a_i \in$ C([0, 42), $1 \leq i \leq 5$), max} $\subseteq$ M; W_1 is a subsemigroup of M also an ideal of M.

W_a = {(a_1, a_2, a_3, a_4, a_5) | $a_i \in$ C([a, 42), 0 < a < 41, $1 \leq i \leq 5$), max} $\subseteq$ M; to be a subsemigroup of M which is also an ideal of M. Thus we have an infinite collection of subsemigroups of infinite order which are also ideals of M.

Consider V_1 = {a + bi_F | a, b $\in$ [0, 1), i_F^2 = 41; max} $\subseteq$ M; V_1 is a subsemigroup of M but is not an ideal of M. However V_1 is of infinite order.

$V_{6.7} = \{a + bi_F \mid a, b \in [0, 6.7), i_F^2 = 41; \max\} \subseteq M$ is a subsemigroup of M but is not an ideal of M. $o(V_{6.7}) = \infty$.

In this way we have $V_\beta = \{a + bi_F \mid a, b \in [0, \beta), \beta < 41.999, i_F^2 = 41; \max\} \subseteq M$ to be a subsemigroup of M but is not an ideal of M.

Hence we have infinte number of subsemigroups of infinite order none of which are ideals of M.

Further M has also infinite collection of subsemigroups of order one, two, three and so on.

Example 2.57: Let

$$S = \left\{ \begin{bmatrix} a_1 & a_2 & a_3 \\ a_4 & a_5 & a_6 \\ \vdots & \vdots & \vdots \\ a_{28} & a_{29} & a_{30} \end{bmatrix} \middle| a_i \in C([0, 8)),\ i_F^2 = 7,\ 1 \leq i \leq 30, \min \right\}$$

be the finite complex modulo integer interval semigroup of infinite order.

Every $x \in S$ is an idempotent. Every singleton set is a subsemigroup and is not an ideal of S. S has ideals to be of infinite order only S has no ideals of finite order S has no zero divisors.

Example 2.58: Let

$$W = \left\{ \begin{bmatrix} a_1 \\ a_2 \\ a_3 \\ a_4 \\ a_5 \\ a_6 \\ a_7 \\ a_8 \end{bmatrix} \middle| \, a_i \in C([0, 15)), 1 \leq i \leq 8, \min \right\}$$

be the finite complex modulo integer interval semigroup of infinite order.

T has infinite number of finite ordered subsemigroups which are not ideals. T has infinite number of infinite ordered subsemigroups which are ideals.

Every element in T is an idempotent and a subsemigroup of order one. T has no units or zero divisors.

Example 2.59: Let

$$W = \left\{ \left[\begin{array}{c} a_1 \\ \hline a_2 \\ \hline a_3 \\ a_4 \\ a_5 \\ a_6 \\ \hline a_7 \\ \hline a_8 \\ a_9 \\ a_{10} \end{array} \right] \middle| \, a_i \in C([0, 24)), 1 \leq i \leq 10, \max \right\}$$

be the finite complex modulo integer interval super column matrix semigroup.

V has idempotents and are infinite in number. This semigroup has infinite number of subsemigroups which are ideals and also infinite number of subsemigroups which are not ideals.

Example 2.60: Let

$$S = \left\{ \left[\begin{array}{ccc} a_1 & a_2 & a_3 \\ \hline a_4 & a_5 & a_6 \\ a_7 & a_8 & a_9 \\ \hline a_{10} & a_{11} & a_{12} \\ a_{13} & a_{14} & a_{15} \\ a_{16} & a_{17} & a_{18} \\ \hline a_{19} & a_{20} & a_{21} \\ \hline a_{22} & a_{23} & a_{24} \\ a_{25} & a_{26} & a_{27} \end{array}\right] \middle| \; a_i \in C([0, 29)), 1 \leq i \leq 27, \max \right\}$$

be the finite complex modulo integer interval super column matrix semigroup of infinite order.

This S has subsemigroups of finite order as well as infinite order. Every element $x \in S$ is an idempotent under max operation.

Example 2.61: Let

$$M = \left\{ \left[\begin{array}{cc|ccc|c} a_1 & a_2 & a_3 & a_4 & a_5 & a_6 \\ a_7 & \dots & \dots & \dots & \dots & a_{12} \\ \hline a_{13} & \dots & \dots & \dots & \dots & a_{18} \\ a_{19} & \dots & \dots & \dots & \dots & a_{24} \\ a_{25} & \dots & \dots & \dots & \dots & a_{30} \\ \hline a_{31} & \dots & \dots & \dots & \dots & a_{36} \\ a_{37} & \dots & \dots & \dots & \dots & a_{42} \end{array}\right] \middle|\, a_i \in C([0, 14)), \right.$$

$$\left. 1 \leq i \leq 42, \max \right\}$$

be the finite complex modulo integer interval semigroup of super matrices. M has subsemigroups of order one, two, three and so on. However all ideals of M are of infinite order.

Now we proceed on to define semigroups on C([0, n)) using the product operation. Let x, y $\in$C([0, n)) then x $\times$ y (mod n) $\in$ C ([0, n)).

First we will describe this situation by some examples.

Example 2.62: Let S = {C([0, 6)), $\times$, $i_F^2 = 5$} be the finite complex modulo integer interval semigroup under the usual product.

$$\text{Let } 4, 5 \in C([0, 6))\ ; \quad 4 \times 5 = 20 \equiv 2 \pmod 6$$

Let $x = 2 + 5i_F$ and $y = 3 + 4i_F \in S$.

$$\begin{aligned} x \times y &= (2 + 5i_F)\,(3 \times 4i_F) \\ &= 6 + 15i_F + 8i_F + 20\,i_F^2 \\ &= 0 + 23i_F + 20 \times 5 \\ &= 5i_F + 4 \in S. \end{aligned}$$

This is the way product operation is performed on S. o(S) = ∞: S is a commutative semigroup. S contains zero divisors and idempotents which are finite in number.

$$\begin{aligned}
&\text{Let } x = 3 + 2i_F \text{ and } y = 4 + 3i_F \in S \\
&x \times y = (3 + 2i_F) \times (4 + 3i_F) \\
&\quad = 12 + 8i_F + 9i_F + 6 i_F^2 \\
&\quad = 0 + 2i_F + 3i_F + 0 \\
&\quad = 5i_F \in S.
\end{aligned}$$

It is easily verified $a \times (b \times c) = (a \times b) \times c$ for all $a, b, c \in S$.

Example 2.63: Let $S = \{C([0, 23)), i_F^2 = 22, \times\}$ be the finite complex modulo integer interval semigroup of infinite order.

$$\begin{aligned}
&\text{Let } x = 0.8 + 0.4i_F \text{ and } y = 0.3 + 0.2i_F \in S \\
&x \times y = (0.8 + 0.4i_F) \times (0.3 + 0.2i_F) \\
&= 0.24 + 0.12i_F + 0.16i_F + 0.08 \times 22 \;\; 0.24 + 0.38i_F + 1.76 \\
&= 2 + 0.38i_F \in S.
\end{aligned}$$

Example 2.64: Let $S = \{C([0, 5)), \times, i_F^2 = 4\}$ be the semigroup of infinite order. $P_1 = \{Z_5, \times\}$ is a subsemigroup of order 5. $P_2 = \{C(Z_5), \times, i_F^2 = 4\}$ a subsemigroup of S finite order.

$P_3 = \{[0, 5), \times\}$ is subsemigroup of infinite order.

However $P_4 = \{[0, 2.3), \times\}$ is only a subset and is not a subsemigroup.

For take $x = 2$ and $y = 2.1$ in P_4. $x \times y = 2 \times 2.1 = 4.2 \notin P_4$. Hence the claim.

Clearly none of the subsemigroup P_1, P_2 and P_3 are ideals of S.

Example 2.65: Let S = {C([0, 12)), ×, i_F^2 = 11} be the finite complex modulo integer interval semigroup of infinite order. S has zero divisors and finite number of units. S is of infinite order. All ideals in S are of infinite order.

Let x = $3i_F$ and y = 8 ∈ S
x × y = 0. x = $3i_F$ + 6 and y = 4 + $8i_F$ ∈ S we see

$$x \times y = (3i_F + 6) \times (4 + 8i_F)$$
$$= 12i_F + 24 + 24 \times 11 + 48i_F$$
$$= 0 \in S \text{ is a zero divisor in S.}$$

x = $6i_F$ + 6 and y = 4 ∈ S is such that x × y = $6i_F$ + 6 × 4 = 0 is a zero divisors.

Likewise x = i_F and y = $11i_F$ ∈ S is such that
x × y = $11i_F \times 11 i_F^2$ = 11 × 11 = 1 is a unit in C([0, 12)).

Also 11 × 11 = 1 for 11 ∈ C([0, 12)).

Thus S has units also S has subsemigroups of both finite and infinite order.

Finding ideals is a problem.

Now using this we build matrix semigroups, which is described by some examples.

Example 2.66: Let
S = {(a_1, a_2, a_3, a_4) | a_i ∈ C([0, 42)), 1 ≤ i ≤ 4, ×} be the complex modulo finite integer interval semigroup. S has zero divisors, ideals of infinite order. S also has subsemigroups of finite order.

P_1 = {(a_1, 0, 0, 0) | a_1 ∈ C([0, 42)), ×} ⊆ S is a subsemigroup as well as an ideal of S.

P_2 = {(0, a_1, 0, 0) | a_1 ∈ C([0, 42)), ×} ⊆ S is a subsemigroup as well as an ideal of S.

$P_3 = \{(0, 0, a_1, 0) \mid a_1 \in C([0, 42)), \times\} \subseteq S$ is a subsemigroup as well as an ideal of S.

$P_4 = \{(0, 0, 0, a_1) \mid a_1 \in C([0, 42)), \times\} \subseteq S$ is a subsemigroup as well as an ideal of S.

Let $R = S = \{(a_1, a_2, a_3, a_4) \mid a_i \in [0, 42), 1 \leq i \leq 4, \times\} \subseteq S$, is a subsemigroup and not an ideal of S.

$R_1 = \{(a_1, 0, 0, 0) \mid a_1 \in [0, 42), \times\} \subseteq S$ is a subsemigroup and not an ideal.

$R_2 = \{(0, a_1, 0, 0) \mid a_1 \in [0, 42), \times\} \subseteq S$ is a subsemigroup and not an ideal.

$R_3 = \{(0, 0, a_1, 0) \mid a_1 \in [0, 42), \times\} \subseteq S$ is a subsemigroup and not an ideal.

$R_4 = \{(0, 0, 0, a_1) \mid a_1 \in [0, 42), \times\}$ is only a subsemigroup. All the R_i's are of infinite order.

However we have finite order subsemigroups also.

$x = (0, a_1, 0, a_2)$ and $y = (a_1, 0, a_2, 0) \in S$ are such that

$$x \times y = (0, a_1, 0, a_2) \times (a_1, 0, a_2, 0)$$
$$= (0, 0, 0, 0) \text{ is a zero divisor.}$$

Infact we have infinite number of such zero divisors. $(1, 1, 1, 1) = I$ acts as the multiplicative identity $x = (41, 41, 41, 41) \in S$ is such that $x^2 = (1, 1, 1, 1)$.

Also $y = (1, 41, 1, 41) \in S$ is such that $y^2 = (1, 1, 1, 1)$. Thus S has only finite number of units.

If $P \subseteq S$ is an ideal certainly $(1, 1, 1, 1) \notin P$.

Example 2.67: Let

$$S = \left\{ \left[\begin{array}{c} a_1 \\ a_2 \\ a_3 \\ a_4 \\ a_5 \\ a_6 \\ a_7 \end{array}\right] \middle| \, a_i \in C([0, 4)), 1 \leq i \leq 7, \times_n \right\}$$

be the finite complex modulo integer interval semigroup of infinite order under the natural product.

$$\text{Let } x = \left[\begin{array}{c} a_1 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{array}\right] \text{ and } y = \left[\begin{array}{c} 0 \\ 0 \\ a_1 \\ a_2 \\ 0 \\ 0 \\ 0 \end{array}\right] \in S$$

$$\text{we see } x \times_n y = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

is a zero divisor.

$$P_1 = \left\{ \begin{bmatrix} a_1 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix} \middle| a_1 \in Z_4, \times_n \right\} \subseteq S$$

be a subsemigroup of order 4.

$$P_2 = \left\{ \begin{bmatrix} a_1 \\ a_2 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix} \middle| a_1, a_2 \in C(Z_4), \times_n \right\} \subseteq S$$

be a subsemigroup of finite order.

Both P_1 and P_2 are only subsemigroups and not ideals.

Consider

$$B_1 = \left\{\begin{bmatrix} a_1 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix} \middle| a_1 \in C([0, 4))\ \times_n\right\} \subseteq S$$

is a subsemigroup of infinite order which is also an ideal of S.

$$B = \left\{\begin{bmatrix} a_1 \\ a_2 \\ a_3 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix} \middle| a_i \in C([0, 4)), 1 \leq i \leq 3, \times_n\right\} \subseteq S$$

is only a subsemigroup of infinite order and is not an ideal of S.

Thus S has subsemigroups of both finite and infinite order which are not ideals.

Further S has infinite number of zero divisors only a finite number of units and idempotents.

Example 2.68: Let

$$W = \left\{ \left[\begin{array}{cccc} a_1 & a_2 & a_3 & a_4 \\ a_5 & \dots & \dots & a_8 \\ a_9 & \dots & \dots & a_{12} \\ a_{13} & \dots & \dots & a_{16} \end{array}\right] \middle| a_i \in C([0, 25)), 1 \leq i \leq 16, \times_n \right\}$$

be a finite complex modulo integer interval matrix semigroup of infinite order under the natural product $\times_n$.

$$\text{Let } x = \left[\begin{array}{cccc} 0 & 0 & 0 & 0 \\ a_1 & a_2 & a_3 & a_4 \\ 0 & 0 & 0 & 0 \\ a_5 & a_6 & a_7 & a_8 \end{array}\right] \text{ and}$$

$$y = \left[\begin{array}{cccc} a_1 & a_2 & a_3 & a_4 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{array}\right] \in W.$$

$$\text{We see } x \times_n y = \left[\begin{array}{cccc} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{array}\right].$$

$$\text{Clearly } \left[\begin{array}{cccc} 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 \end{array}\right] \text{ is the unit element of W.}$$

Infact W has infinite number of zero divisors but only finite number of units. W has subsemigroups of both finite and infinite order W also has ideals but all ideals of W are of infinite order. W is a commutative semigroup.

Example 2.69: Let

$$S = \left\{\left[\begin{bmatrix} a_1 & a_2 & a_3 & a_4 & a_5 \\ a_6 & a_7 & a_8 & a_9 & a_{10} \\ a_{11} & a_{12} & a_{13} & a_{14} & a_{15} \\ a_{16} & a_{17} & a_{18} & a_{19} & a_{20} \\ a_{21} & a_{22} & a_{23} & a_{24} & a_{25} \end{bmatrix}\right] \middle| a_i \in C([0, 6)), 1 \leq i \leq 25, \times\right\}$$

be the finite complex modulo integer interval semigroup under usual product of matrices. S is a non commutative semigroup of infinite order.

$$I_{5\times5} = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix}$$

acts as the multiplicative identity. S has units. S has ideals.

S has finite subsemigroups and infinite subsemigroups. This study and this example. Here we illustrate the situation by a collection of 2 × 2 matrices with entries from C([0, 6)).

$$\text{Let } A = \begin{bmatrix} 0.31 & 2.4 \\ 1.6 & 0.5 \end{bmatrix} \text{ and } B = \begin{bmatrix} 0.4 & 0.8 \\ 0.9 & 4.2 \end{bmatrix}$$

$$A \times B = \begin{bmatrix} 0.31 & 2.4 \\ 1.6 & 0.5 \end{bmatrix} \times \begin{bmatrix} 0.4 & 0.8 \\ 0.9 & 4.2 \end{bmatrix}$$

$$= \begin{bmatrix} 0.31\times 0.4+2.4\times 0.9 & 0.31\times 0.8+2.4\times 4.2 \\ 1.6\times 0.4+0.5\times 0.9 & 1.6\times 0.8+0.5\times 4.2 \end{bmatrix}$$

$$= \begin{bmatrix} 0.124+2.16 & 0.248+10.08 \\ 0.64+0.45 & 0.128+2.10 \end{bmatrix}$$

$$= \begin{bmatrix} 2.284 & 4.328 \\ 1.09 & 2.228 \end{bmatrix} \qquad \ldots \quad \mathrm{I}$$

Consider B × A = $\begin{bmatrix} 0.4 & 0.8 \\ 0.9 & 4.2 \end{bmatrix} \times \begin{bmatrix} 0.31 & 2.4 \\ 1.6 & 0.5 \end{bmatrix}$

$$= \begin{bmatrix} 0.4\times 0.31+0.8\times 1.6 & 0.4\times 2.4+0.8\times 0.5 \\ 0.9\times 0.3+4.2\times 1.6 & 0.9\times 2.4+4.2\times 0.5 \end{bmatrix}$$

$$= \begin{bmatrix} 0.124+1.28 & 0.96+0.40 \\ 0.279+6.70 & 2.16+2.10 \end{bmatrix}$$

$$= \begin{bmatrix} 1.404 & 1.36 \\ 0.979 & 4.26 \end{bmatrix} \qquad \ldots \quad \mathrm{II}$$

Clearly I and II distinct so A × B ≠ B × A.

Now A $\times_n$ B = $\begin{bmatrix} 0.31 & 2.4 \\ 1.6 & 0.5 \end{bmatrix} \times_n \begin{bmatrix} 0.4 & 0.8 \\ 0.9 & 4.2 \end{bmatrix}$

$$= \begin{bmatrix} 0.124 & 1.92 \\ 0.81 & 17.64 \end{bmatrix}$$

$$= \begin{bmatrix} 0.124 & 1.92 \\ 0.81 & 5.64 \end{bmatrix} = B \times_n A.$$

It is easily verified $A \times_n B = B \times_n A$ and $A \times B \neq B \times A$ further $A \times B \neq A \times_n B$.

Example 2.70: Let $S = \{(a_1\ a_2\ a_3 \mid a_4\ a_5\ a_6 \mid a_7 \mid a_8\ a_9 \mid a_{10}\ a_{11}\ a_{12} \mid a_{13}) \mid a_i \in C([0, 20)), 1 \leq i \leq 14, \times\}$ be the complex modulo finite integer interval semigroup of super row matrices.

S is of infinite order. S is commutative.

S has infinite number of zero divisors.

$I = \{(1\ 1\ 1 \mid 1\ 1\ 1 \mid 1 \mid 1\ 1\ 1 \mid 1)\}$ is the unit (multiplicative identity of S). Thus S is a commutative monoid.

S has both finite order subsemigroups as well as infinite order subsemigroups which are not ideals.

$P = \{(a\ 0\ 0 \mid 0\ 0\ 0 \mid 0 \mid 0\ 0 \mid 0\ 0\ 0 \mid 0) \mid a \in C([0, 20)), \times\}$ is a subsemigroup of infinite order which is also an ideal of P.

If in P C([0, 20)) is replaced by [0, 20); then that P is not an ideal only a subsemigroup.

This is so for if

$x = (3\ 0\ 0 \mid 0\ 0\ 0 \mid 0 \mid 0\ 0 \mid 0\ 0\ 0 \mid 0)$ is in P and

$y = (2 + i_F\ a_1\ a_2 \mid a_3\ a_4\ a_5 \mid a_6 \mid a_7\ a_8 \mid a_9\ a_{10}\ a_{11} \mid a_{12}) \in S$, $a_i \in C([0, 20))$; $1 \leq i \leq 12$.

$x \times y = (6 + 3i_F\ 0\ 0 \mid 0\ 0\ 0 \mid 0 \mid 0\ 0 \mid 0\ 0\ 0 \mid 0) \notin P$, hence the claim.

Thus S has infinite subsemigroups which are not ideals of S.

Example 2.71: Let

$$S = \left\{ \left[\begin{array}{c} a_1 \\ a_2 \\ \hline a_3 \\ \hline a_4 \\ a_5 \\ \hline a_6 \\ a_7 \\ a_8 \\ \hline a_9 \\ a_{10} \\ a_{11} \\ a_{12} \end{array} \right] \middle| \; a_i \in C([0, 19)), 1 \leq i \leq 12, \times_n \right\}$$

be the finite complex modulo integer interval semigroup super column matrix.

S is an infinite semigroup which is commutative.

S is infinite number of zero divisors and has ideals and subsemigroups which are not ideals.

Example 2.72: Let

$$S = \left\{ \left(\begin{array}{c|cc|ccc|cc} a_1 & a_2 & a_3 & a_4 & a_5 & a_6 & a_7 & a_8 \\ a_9 & \ldots & \ldots & \ldots & \ldots & \ldots & \ldots & a_{16} \\ a_{17} & \ldots & \ldots & \ldots & \ldots & \ldots & \ldots & a_{24} \end{array} \right) \middle| \; a_i \in C([0, 40)), \right.$$

$$\left. 1 \leq i \leq 23, \times_n \right\}$$

be the finite complex modulo integer interval semigroup super row matrices. S has infinite number of zero divisors.

We can have examples of other types of super matrix semigroups.

Now we proceed onto describe neutrosophic finite integer complex number interval semigroup using the operation min or max or product.

We shall denoted it $C(\langle[0, n) \cup I\rangle) = \{a + bi_F + cI + di_FI \mid a, b, c, d \in [0, n),\ i_F^2 = n–1,\ (i_FI)^2 = (n–1)I$ and $I^2 = I$ is the indeterminate or the neutrosophic number I}.

Example 2.73: Let $S = C(\langle[0, 9) \cup I\rangle),\ i_F^2 = 8$, min} be the neutrosophic finite complex modulo integer interval semigroup of infinite order.

Let $x = 0.8 + 3.9i_F + 2.6I + 7.1i_FI$ and $y = 8.2 + 4.3i_F + 1.1I + 0.92i_FI \in S$.

We find min {x, y} = min $\{0.8 + 3.9i_F + 2.6I + 7.14Ii_F, 8.2 + 4.3i_F + 1.1I + 0.92i_FI\}$

= min {0.8, 8.2} + min $\{3.9i_F, 4.3i_F\}$ + min {2.6I, 1.1I} + min $\{7.14i_FI, 0.92i_FI\} = 0.8 + 3.9i_F+ 1.1I + 0.92i_FI \in S$.

This is the way min operation is performed on S. Clearly min {x, x} = x. min {x, 0} = 0 for all $x \in S$.

S is an infinite semigroup in which every $x \in S$ is a subsemigroup as min {x, x} = x.

Let $x = 2i_F + 1.3i_FI$ and $y = 7.2 + 6I \in S$.

We find min {x, y} = min $\{2i_F + 1.3i_FI, 7.2 + 6I\}$
= min {0.7.2} + min {0, 6I} + min $\{1.3i_FI, 0\}$ + min $\{2i_F, 0\} = 0 + 0I + 0i_FI + 0i_F = 0$

Thus min {x, y} = 0. Hence S has infinite number of zero divisors. Every element x in S is an idempotent.

Let $X = \{3i_F + 2I, 2i_F + 3.7I\} \subseteq S$; X is only a subset of S as min $\{3i_F + 2I, 2i_F + 3.7I\}$

$$\min \{3i_F, 2i_F\} + \min \{2I, 3.7I\}$$
$$= 2i_F + 2I \notin X.$$

However

$X_c = \{3i_F + 2I, 2i_F + 3.7I, 2I + 2i_F\} = X \cup \{2I + 2i_F\}$ is a subsemigroup of S. We call X_c the completed subsemigroup of the set X. Thus it is always possible to complete a set into a subsemigroup. If X is a finite set the completed subsemigroup X_c of X will be finite. If on the other hand X is an infinite subset of S; X_c will be an infinite subsemigroup of S.

Example 2.74: Let $S = \{C(\langle[0, 7) \cup I\rangle) = a + bi_F + cI + di_FI$ where $a, b, c, d \in [0, 7)$ with $i_F^2 = 6$, $I^2 = I$, $(i_FI)^2 = 6I$, min$\}$ be a subsemigroup of infinite order.

Let $X = \{x = 1.7 + 2.5i_F + 3I + 1.79i_FI, y = 6 + 1.4i_F + I + 2.3i_FI, z = 6I + 2i_FI\} \subseteq S$.

Clearly X is not a subsemigroup only a subset.

$\min \{x, y\} = \min \{1.7 + 2.5i_F + 3I + 1.79i_FI, 6 + 1.4i_F + I + 2.3i_FI\}$

$$= \min \{1.7, 6\} + \min \{2.5i_F, 1.4i_F\} + \min \{3I, I\} + \min \{1.79i_FI, 2.3i_FI\}$$

$$= 1.7 + 1.4i_F + I + 1.79i_FI \notin X.$$

$\min \{x, z\} = \min \{1.7 + 2.5i_F + 3I + 1.79i_FI, 6I + 2i_F\}$

$$= \min \{1.7, 0\} + \min \{2.5i_F, 2i_F\} + \min \{3I, 6I\} + \min \{1.79i_FI, 0\}$$

$$= 0 + 2i_F + 3I + 0 \notin X.$$

$\min \{y, z\} = \min \{6 + 1.4i_F + I + 2.3i_FI, 6I + 2i_FI\}$

$= \min \{6, 0\} + \min \{1.4i_F, 0\} + \min \{I, 6I\} + \min \{2.3i_FI, 2i_FI\}$

$= 0 + 0 + I + 2i_FI$

$= I + 2i_FI \notin X.$

$X_c = \{x, y, z, I + 2i_F, 2i_F + 3I, 1.7 + 1.4i_F + I + 1.79i_FI\} \subseteq S$ is the completed subsemigroup of the subset X. Order of the subsemigroup X_c is six.

Thus given any subset in X we can complete X to form a subsemigroup of S.

Example 2.75: Let $M = \{C(\langle[0, 3) \cup I\rangle), \min\}$ be the finite complex neutrosophic modulo integer interval semigroup under min operation.

Let $X = \{0.2i_F, 2I, 0.7i_FI, 2.4, 2 + 0.4i_FI\} \subseteq M$.
Clearly X is not a subsemigroup of M only a subset of M.

Now we complete X into a subsemigroup.

$\min \{0.2i_F, 2I\} = 0.$
$\min \{0.2i_F, 0.7i_FI\} = 0.$
$\min \{0.2i_F, 2.4\} = 0.$
$\min \{0.2i_F, 2 + 0.4Ii_F\} = 0.$
$\min \{2I, 0.7i_FI\} = 0.$
$\min \{2I, 2.4\} = 0.$
$\min \{2I, 2 + 0.4Ii_F\} = 0.$
$\min \{0.7i_FI, 2.4\} = 0.$
$\min \{0.7i_FI, 2 + 0.4i_FI\}$
$= \min \{0.2\} + \min \{0.7i_FI, 0.4i_FI\}$
$= 0 + 0.4i_FI.$
$\min \{2.4 + 2 + 0.4i_FI\}$
$= \min \{2.4, 2\} + \min \{0, 0.4i_FI\}$
$= 2 \notin X.$

Thus $X_c = \{0.2i_F, 2I, 0.7i_FI, 2.4, 2 + 0.4i_FI, 0, 0.4i_F, 2\} \subseteq M$ is a subsemigroup of order 8.

However o (X) = 5.

Let $Y = \{x = 2 + 1.3i_F + 2.1I + 0.7i_FI, y = 0.2 + 2i_F, z = 0.4I + 1.2i_FI\} \subseteq M$. Y is only a subset of M.

$\min \{x, y\} = \{2 + 1.3i_F + 2.1I + 0.7i_FI, 0.2 + 2i_F\}$

$= \min \{2, 0.2\} + \min \{1.3i_F, 2i_F\} + \min \{2.1I, 0\} + \min \{0.7i_FI, 0\}$

$= 0.2 + 1.3i_F \notin Y$.

$\min \{x, z\} = \min \{2 + 1.3i_F + 2.1I + 0.7i_FI, 0.4I + 1.2i_FI\}$

$= \min \{2, 0\} + \min \{1.3i_F, 0\} + \min \{2.1I, 0.4I\} + \min \{0.7i_FI, 1.2i_FI\}$

$= 0 + 0 + 0.4I + 0.7i_FI \notin Y$.

$\min \{y, z\} = \{0.2 + 2i_F, 0.4I + 1.2i_FI\}$

$= \min \{0.2, 0\} + \min \{2i_F, 0\} + \min \{0.4I, 0\} + \min \{0, 1.2i_FI\}$

$= 0 + 0i_F + 0I + 0i_FI = 0 \notin Y$.

Thus $Y_c = \{0, 0.4I + 0.7i_FI, 0.2 + 1.3i_F, x, y, z\} \subseteq M$ is a subsemigroup of order 6.

We see $P_1 = \{[0, 3), \min\} \subseteq M$ is subsemigroup of infinite order which is an ideal of M.

$P_2 = \{C([0, 3), \min\} \subseteq M$ is an ideal of infinite order.

$P_3 = \{\{1, 3\}, \min\} \subseteq M$ is a subsemigroup which is not an ideal of M.

$P_4 = \{C(Z_3), \min\} \subseteq M$ is a subsemigroup and not an ideal of M.

$P_5 = \{\langle Z_3 \cup I\rangle, \min\}$ is only a subsemigroup of M and is not an ideal of M.

$P_6 = \{a + bI \mid a, b \in [0, 2.5), \min\}$ is a subsemigroup of M which is also an ideal of M.

Thus we see using $C(\langle[0, n) \cup I\rangle)$ we can build semigroups under min operation.

This will be illustrated by examples.

Example 2.76: Let $S = \{(a_1, a_2, a_3, a_4, a_5) \mid a_i \in C(\langle[0, 18) \cup I\rangle); 1 \leq i \leq 5, \min\}$ be the neutrosophic finite complex modulo integer interval semigroup. S is commutative and is of infinite order. S has several subsemigroups of all finite order.

Infact S has infinite number of subsemigroups of order one and of order two and so on. S also has infinite number of zero divisors. Every element in S is an idempotent.

$P_1 = \{(a_1, 0, 0, 0, 0) \mid a_1 \in C(\langle[0, 18) \cup I\rangle), \min\} \subseteq S$ is a subsemigroup and also an ideal of S.

$P_2 = \{(a, 0, 0, 0, 0) \mid a \in C(\langle[0, 18)), \min\} \subseteq S$ is a subsemigroup and also an ideal of S.

$P_3 = \{(a, 0, 0, 0, 0) \mid a \in [0, 18), \min\} \subseteq S$ is a subsemigroup and ideal of S.

$P_4 = \{(a, 0, 0, 0, 0) \mid a = b + cI$ where $b, c \in [0, 18)), \min\} \subseteq S$ is a subsemigroup as well as ideal of S.

$P_5 = \{(a, 0, 0, 0, 0) \mid a \in Z_{18}, \min\} \subseteq S$ is a subsemigroup and not an ideal of S.

$P_6 = \{(a, 0, 0, 0, 0) \mid a \in C(Z_{18}), \min\} \subseteq S$ is a subsemigroup and not an ideal of S.

$P_7 = \{(a, 0, 0, 0, 0) \mid a \in \langle Z_{18} \cup I\rangle, \min\} \subseteq S$ is a subsemigroup and not an ideal of S.

$P_8 = \{(a, 0, 0, 0, 0) \mid a \in [0, 9), \min\} \subseteq S$ is a subsemigroup and not an ideal of S.

$P_9 = \{(a, 0, 0, 0, 0) \mid a \in [0, 16), \min\} \subseteq S$ is a subsemigroup and not an ideal of S.

$P_{10} = \{(a, 0, 0, 0, 0) \mid a \in [3, 18), \min\}$ is a subsemigroup and not an ideal of S.

$P_{11} = \{(a, b, c, 0, 0) \mid a, b, c \in [10, 12), \min\}$ is a subsemigroup of S and is not an ideal of S.

Thus we have seen examples of ideals and subsemigroup. S has infinite number of idempotents and every singleton element in S is a subsemigroup of order one.

Example 2.77: Let

$$S = \left\{ \begin{bmatrix} a_1 \\ a_2 \\ a_3 \\ a_4 \\ a_5 \\ a_6 \\ a_7 \end{bmatrix} \middle| \, a_i \in C(\langle [0, 12) \cup I \rangle), 1 \leq i \leq 7, \min \right\}$$

be a subsemigroup of infinite order.

Every element is an idempotent as well as subsemigroup of order two.

We can study subsemigroups of finite order and infinite order.

Example 2.78: Let

$$S = \left\{ \left[\begin{array}{ccccc} a_1 & a_2 & a_3 & a_4 & a_5 \\ a_6 & a_7 & a_8 & a_9 & a_{10} \\ a_{11} & a_{12} & a_{13} & a_{14} & a_{15} \\ a_{16} & a_{17} & a_{18} & a_{19} & a_{20} \\ a_{21} & a_{22} & a_{23} & a_{24} & a_{25} \\ a_{26} & a_{27} & a_{28} & a_{29} & a_{30} \end{array}\right] \right| a_i \in C\langle([0, 14) \cup I); \quad 1 \le i \le 30, \min\}$$

be a semigroup of infinite order. S has subsemigroups of infinite and finite order.

S has infinite number of idempotents and no units but infinite number of zero divisors.

Example 2.79: Let

$$S = \left\{ \left[\begin{array}{cc|ccc} a_1 & a_2 & a_3 & a_4 & a_5 \\ a_6 & a_7 & a_8 & a_9 & a_{10} \\ \hline a_{11} & a_{12} & a_{13} & a_{14} & a_{15} \\ \hline a_{16} & a_{17} & a_{18} & a_{19} & a_{20} \\ a_{21} & a_{22} & a_{23} & a_{24} & a_{25} \\ a_{26} & a_{27} & a_{28} & a_{29} & a_{30} \\ \hline a_{31} & a_{32} & a_{33} & a_{34} & a_{35} \\ \hline a_{36} & a_{37} & a_{38} & a_{39} & a_{40} \\ a_{41} & a_{42} & a_{43} & a_{44} & a_{45} \end{array}\right] \right| a_i \in C\langle([0, 23) \cup I); \quad 1 \le i \le 45, \min\}$$

be the neutrosophic finite complex modulo integer interval super matrix semigroup. P has infinite number of zero divisors and idempotents.

Every singleton element of P is a subsemigroup of P. P has ideals and all ideals are of infinite order. P also has subsemigroups of order one, two, three and so on. none of which are ideals.

Next we study semigroups under max operation.

Example 2.80: Let S = $\{C\langle([0, 20) \cup I\rangle), \max\}$ be a finite complex modulo integer neutrosophic semigroup of infinite order under max operation. Let $x = 9.2 + 17.5I + 3.7i_F + 11i_FI$ and $y = 3.5 + 10I + 8.6i_F + 14i_FI \in S$.

$\max\{x, y\} = \max\{9.2 + 17.5I + 3.7i_F + 11i_FI, 3.5 + 10I + 8.6i_F + 14i_FI\}$

$= \max\{9.2, 3.5\} + \max\{17.5I, 10I\} + \max\{3.7i_F, 8.6i_F\} + \max\{11i_FI, 14i_FI\}$

$= 9.2 + 17.5I + 8.6i_F + 14i_FI.$

This is the way max operation is performed on S.

Let $x = 8.4 + 3.2i_F$ and $y = 9.4\,I + 3.7Ii_F \in S$

$\max\{x, y\} = \{8.4 + 3.2i_F, 9.4I + 3.7Ii_F\}$

$= \max\{8.4, 0\} + \max\{3.2i_F, 0\} + \max\{0, 9.4I\} + \max\{0, 3.7Ii_F\}$

$= 8.4 + 3.2i_F + 9.4I + 3.7Ii_F \in S$

This is the way max operation is performed on S.

We see $\max\{x, x\} = x$ and $\max\{x, 0\} = x$ for all $x \in S$.

Example 2.81: Let S = $C\langle([0, 20) \cup I\rangle), \max\}$ be the finite neutrosophic complex modulo integer interval semigroup under max.

Let $x = 3i_F + 12I$ and $y = 2.31 \in S$;
we see $\max\{x, y\} = 2.31 + 3i_F + 12I \in S$.

Let $x = 0.4i_F$ and $y = 0 \in S$.
$\max \{x, y\} = 0.4i_F$.

Let $x = 5.31$ and $y = 0 \in S$
$\max \{x, 0\} = 5.31 = x$.

Thus we see S under max operation has no zero divisors.

Infact S has every element to be an idempotent. Thus $\max \{x, x\} = x$. Hence every singleton element in S is a subsemigroup of order one.

S has subsemigroups of order two for $\max \{0, x\} = x$ for all $x \in S$. S has subsemigroups of order three and so on.

Let $x = 0.3I + 4.3i_F$ and $y = 8.1 + 9.1I + 3i_FI \in S$.

$\max \{x, y\} = \max \{0.l3I + 4.3i_F, 8.1 + 9.1I + 3i_FI\}$

$= \max \{0, 8.1\} + \max \{0.3I, 9.1I\} + \max \{4.3i_F, 0\} + \max \{0, 3i_FI\}$

$= 8.1 + 9.1I + 4.3i_F + 3i_FI \in S$.

So $P = \{x, y, 8.1 + 9.1I + 4.3i_F + 3i_FI\} \subseteq S$ is a subsemigroup of order there; infact P is the completion of the set $\{x, y\}$.

Thus we can always complete a finite or infinite set to form a subsemigroup but in general these subsemigroups need not be ideals. Infact none of the finite subsemigroups of S are ideals of S.

Inview of all these we have the following theorem.

THEOREM 2.4: *Let $S = \{C\langle([0, n) \cup I), \min\}$ be the neutrosophic fuzzy complex modulo integer interval semigroup.*

(i) $o(S) = \infty$.
(ii) *S has infinite number of zero divisors.*
(iii) *S has every element to be an idempotent.*
(iv) *S has subsemigroups of order one, two, three and so on.*
(v) *S has no finite ideals (Every ideal in S is of infinite order)*
(vi) *S has infinite number of subsemigroups of infinite order.*
(vii) *S has no units.*
(viii) *Every finite or infinite subset of S can be completed to get a subsemigroup.*

The proof is direct and hence left as an exercise to the reader.

THEOREM 2.5: *Let* $M = \{C\langle([0, n) \cup I), \max\}$ *be the neutrosophic finite complex modulo integer interval semigroup.*

(i) $o(S) = \infty$.
(ii) *S has no zero divisors.*
(iii) *S has subsemigroups of order one, two, … and so on.*
(iv) *S has no ideals of finite order.*
(v) *S has subsemigroups of infinite order.*
(vi) *Every* $x \in S$ *is an idempotent.*
(vii) *Every finite or infinite subset can be completed to form a subsemigroup.*

Proof is direct and hence left as an exercise to the reader.

Example 2.82: Let
$S = \{(a_1, a_2, \ldots, a_5) \mid a_i \in C\langle([0, 12) \cup I)\rangle$, max, $1 \leq i \leq 5\}$ be the neutrosophic finite complex modulo integer interval row matrix semigroup.

$x = (3.4 + 2I, 0\ 4.3i_F + 8Ii_F, 0, I, 3i_F\}$ and
$y = (2, 9.2I, 4 + 3i_F, 2i_F + 8, 4i_F + 7) \in S$.

We see max {x, y} = $(3.4 + 2I, 9.2I + 4.3i_F + 8Ii_F, 4 + 3i_F, I + 8 + 2i_F, 7 + 4i_F) \in S$.

This is the way operations are performed on S.

S has no zero divisor. S has finite subsemigroups of order one, two and so on. Every $x \in S$ is such that {x} is a subsemigroup of order one.

$P = \{x = (3i_F + 2I + 0.7Ii_F + 4, 0, 0, 0, 0)\} \subseteq S$ is a subsemigroup of order one.

$M = \{y = (0, 3i_F, 2 + 8I, 0, 0), x\} \subseteq S$ is only subset of S for

max $\{x, y\} = \{3i_F + 2I + 0.7Ii_F + 4, 3i_F, 2 + 8I, 0, 0) \not\in$ M or P.

$A = \{x, y, (3i_F + 2I + 0.7Ii_F + 4, 3i_F, 2 + 8I, 0, 0)\} \subseteq S$ is a subsemigroup of order three and A is not an ideal of S.

$P_1 = \{(a_1, 0, 0, 0, 0) \mid a_1 \in Z_{12}\}$ is a finite subsemigroup of order 12 and is not an ideal of S.

$P_2 = \{(0, a, 0, \ldots, 0) \mid a \in \langle Z_{12} \cup I\rangle\}$ is only a finite subsemigroup.

$P_3 = \{(a, 0, 0, 0, a) \mid a \in C(Z_{12})\}$ is only a finite subsemigroup.

$P_4 = \{(a, b, 0, 0, 0) \mid a, b \in C(\langle Z_{12} \cup I\rangle)\}$ is again a finite subsemigroup and not an ideal of S.

$P_5 = \{(a, b, c, d, 0) \mid a, b, c, d \in C(\langle[0, 12) \cup I\rangle), \max\} \subseteq S$ is only an infinite subsemigroup and not an ideal of S.

$P_6 = \{(a_1, a_2, \ldots, a_5) \mid a_i \in [0, 12),\ 1 \leq i \leq 5, \max\}$ is only a subsemigroup of infinite order and not an ideal of S.

Example 2.83: Let

$$T = \left\{ \begin{bmatrix} a_1 \\ a_2 \\ \vdots \\ a_{10} \end{bmatrix} \middle| \, a_i \in C\langle([0, 3) \cup I); 1 \leq i \leq 10, \max\} \right.$$

be a neutrosophic finite complex modulo integer semigroup of infinite order.

T has subsemigroups of finite and infinite order also T has ideals of infinite order.

Example 2.84: Let

$$T = \left\{ \begin{bmatrix} a_1 \\ a_2 \\ \vdots \\ a_{10} \end{bmatrix} \middle| \, a_i \in C\langle([0, 3) \cup I); 1 \leq i \leq 10, \max\} \right.$$

be a neutrosophic finite complex modulo integer semigroup of infinite order.

T has subsemigroups of finite and infinite order also T has ideals of infinite order.

Example 2.85: Let

$$S = \left\{ \begin{bmatrix} a_1 & a_2 & a_3 \\ a_4 & a_5 & a_6 \\ \vdots & \vdots & \vdots \\ a_{16} & a_{17} & a_{18} \end{bmatrix} \middle| \, a_i \in C\langle([0, 18) \cup I); 1 \leq i \leq 18, \max\} \right.$$

be a neutrosophic finite complex modulo integer semigroup of infinite order.

S has subsemigroups of order one, two etc and also subsemigroups of infinite order as well as ideals which are only a infinite order. S has no zero divisors.

Example 2.86: Let

$$L = \left\{ \left[\begin{array}{cccc} a_1 & a_2 & a_3 & a_4 \\ a_5 & a_6 & a_7 & a_8 \\ a_9 & a_{10} & a_{11} & a_{12} \\ a_{13} & a_{14} & a_{15} & a_{16} \end{array} \right] \middle| a_i \in C\langle([0, 10) \cup I); \right.$$

$$\left. 1 \leq i \leq 16, \max \right\}$$

neutrosophic complex finite modulo integer interval semigroup several ideals infinite number of subsemigroups of finite order and no zero divisors.

Example 2.87: Let M = $\{(a_1 \mid a_2 \mid a_3\ a_4\ a_5\ a_6 \mid a_7 \mid a_8\ a_9\ a_{10} \mid a_{11}\ a_{12} \mid a_{13}) \mid a_i \in C\langle([0, 7) \cup I); 1 \leq i \leq 13, \max\}$ be the neutrosophic finite complex modulo integer super row matrix semigroup of infinite order. M has subsemigroup, and ideals. M has no units or zero divisors.

Example 2.88: Let

$$S = \left\{ \left[\begin{array}{c} a_1 \\ a_2 \\ a_3 \\ \hline a_4 \\ a_5 \\ \hline a_6 \\ a_7 \\ a_8 \\ a_9 \\ \hline a_{10} \end{array} \right] \middle| a_i \in C(\langle[0, 15) \cup I\rangle), 1 \leq i \leq 10, \max \right\}$$

be the neutrosophic finite complex modulo integer interval semigroup. S has ideals of infinite order. Every element in S is an idempotent as well as the subsemigroup of S.

Example 2.89: Let

$$M = \left\{ \left[\begin{array}{cccc} a_1 & a_2 & a_3 & a_4 \\ a_5 & a_6 & a_7 & a_8 \\ \hline a_9 & \ldots & \ldots & a_{12} \\ a_{13} & \ldots & \ldots & a_{16} \\ a_{17} & \ldots & \ldots & a_{20} \\ a_{21} & \ldots & \ldots & a_{24} \\ \hline a_{25} & \ldots & \ldots & a_{28} \\ a_{29} & \ldots & \ldots & a_{32} \\ \hline a_{33} & \ldots & \ldots & a_{36} \\ a_{37} & \ldots & \ldots & a_{40} \\ a_{41} & \ldots & \ldots & a_{44} \\ \hline a_{45} & \ldots & \ldots & a_{48} \end{array} \right] \right| a_i \in C(\langle[0, 16) \cup I\rangle),$$

$$1 \leq i \leq 48, \max\}$$

be the finite complex modulo integer interval neutrosophic semigroup of infinite order.

M has ideals. Every singleton element is a subsemigroup. Every subsemigroup need not be an ideal of M.

Now we proceed onto give examples semigroups.

Example 2.90: Let $C(\langle[0, 3) \cup I\rangle), \times\}$ be the semigroup on the neutrosophic finite complex modulo integers interval.

Let $x = 0.2 + 1.5i_F + 2.1I + 0.4Ii_F$
and $y = 0.7 + 0.8i_F + 2I + 1.6i_F I \in S$

$x \times y = (0.2 + 1.5i_F + 2.1I + 0.4Ii_F) \times (0.7 + 0.8i_F + 2I + 1.61i_FI)$

$= 0.14 + 1.05i_F + 1.47I + 0.28\ Ii_F + 0.16i_F + 1.6 \times 2 + 1.68Ii_F + 0.32 \times 3I + 0.4I + 4.2I + 0.8Ii_F + 0.32Ii_F + 2.4 \times 2I + 3.36Ii_F + 0.64 \times 2I$

$= (0.14 + 0.2) + (1.05) + 0.16)\ i_F + (1.47 + 0.96 + 0.4 + 1.2 + 1.8 + 1.28)I + (0.28 + 1.68 + 0.8 + 0.32 + 0.44Ii_F.$

This is the way product is performed. Further we use the fact $i_F^2 = 2$ and $(Ii_F)^2 = 2I$ while finding the product.

S has several zero divisors finite number of units and idempotents.

S has subsemigroups of both finite and infinite order. o(S) $= \infty$ and S is a commutative subsemigroup.

Example 2.91: Let
$S = \{(C\langle[0, 4) \cup I\rangle), \times, i_F^2 = 3, I^2 = I\ (Ii_F)^2 = 3I\}$ is a neutrosophic complex finite modulo integer semigroup of infinite order which is commutative.

$P_1 = \{[0, 4), \times\}$ is a subsemigroup of infinite order but P_1 is not an ideal of S.

$P_2 = \{C([0, 4)), \times\}$ is a subsemigroup of infinite order but P_2 is not an ideal of S.

$P_3 = \{a + bI \mid a, b \in [0, 4), \times\}$ is again only a subsemigroup and not an ideal.

$P_4 = \{C(Z_4), \times\}$ is a subsemigroup of finite order.

$P_5 = \{\langle Z_4 \cup I\rangle, \times\}$ is a subsemigroup of finite order.

S has zero divisors for take x = 2 and y = $2i_FI$ we see xy = 0 and y = $2i_FI$ we see xy = 0 and y = 2I then also x × y = 0.

Let x = $2i_F$ and y = $3i_F$ then x × y = 2 × 3 × 3 = 2. So a complex number is transformed into a modulo integer.

Example 2.92: Let

M = {(C⟨[0, 23) ∪ I⟩), i_F^2 = 22, I^2 = I, $(Ii_F)^2$ = 22I, ×} be a neutrosophic complex modulo integer interval semigroup of infinite order x = $11.5i_F$ and y = 2 is such that x × y = 0. M has zero divisors.

M has finite subsemigroups like P_1 = {Z_{23}, ×{ ⊆ M, P_2 = {C(Z_{23}), ×} ⊆ M, P_3 = {⟨Z_{23} ∪ I⟩, ×} ⊆ M and P_4 = {C(⟨Z_{23} ∪ I⟩), ×} ⊆ M are subsemigroups of finite order.

T_1 = {0, 1, 22, ×} ⊆ M is also a subsemigroup of finite order. L = {0, 1, 22, i_F, $22i_F$} is also subgroup of order 5. L has zero divisors, finite number of units and I^2 = I is an idempotent.

Example 2.93: Let

$$W = \left\{ \begin{bmatrix} a_1 \\ a_2 \\ \vdots \\ a_{10} \end{bmatrix} \middle| \; a_i \in C\langle([0, 24) \cup I); \times_n, 1 \leq i \leq 10 \right\}$$

be a semigroup of the complex neutrosophic modulo integer interval.

W has infinite number of zero divisors.

$$\begin{bmatrix} 1 \\ 1 \\ 1 \\ 1 \\ 1 \\ 1 \\ 1 \\ 1 \\ 1 \\ 1 \end{bmatrix} \text{ is the unit element of W.}$$

$$\begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix} \text{ is the zero in W. W is a commutative monoid.}$$

$$P_1 = \left\{ \begin{bmatrix} a_1 \\ 0 \\ \vdots \\ 0 \end{bmatrix} \middle| a_1 \in C\langle([0, 24) \cup I); \times_n\} \subseteq W$$

is a subsemigroup as well as an ideal of W.

$$P_2 = \left\{ \begin{bmatrix} 0 \\ a_2 \\ \vdots \\ 0 \end{bmatrix} \middle| a_2 \in C\langle([0, 24) \cup I); \times_n\} \subseteq W$$

is also an ideal of W and so on.

$$P_{10} = \left\{ \begin{bmatrix} 0 \\ 0 \\ \vdots \\ a_{10} \end{bmatrix} \middle| a_{10} \in C\langle([0, 24) \cup I); \times_n\} \subseteq W$$

is again an ideal of W.

We see W has at least ${}_{10}C_1 + {}_{10}C_2 + \ldots + {}_{10}C_9$ number of ideals.

Further W has at least $20({}_{10}C_1 + {}_{10}C_2 + \ldots + {}_{10}C_9)$ number of finite subsemigroups.

Example 2.94: Let

$$M = \left\{ \begin{bmatrix} a_1 & a_2 & a_3 \\ a_4 & a_5 & a_6 \\ \vdots & \vdots & \vdots \\ a_{28} & a_{29} & a_{30} \end{bmatrix} \middle| a_i \in C\langle([0, 10) \cup I); 1 \leq i \leq 30, \times_n\} \right.$$

be the finite complex modulo integer interval column matrix semigroup of infinite order.

M has subsemigroups of finite and infinite order. All ideals of M are of infinite order.

Example 2.95: Let

$$M = \left\{\begin{bmatrix} a_1 & a_2 & a_3 & a_4 \\ a_5 & a_6 & a_7 & a_8 \\ a_9 & a_{10} & a_{11} & a_{12} \\ a_{13} & a_{14} & a_{15} & a_{16} \end{bmatrix} \middle|\, a_i \in C\langle([0, 40) \cup I); 1 \leq i \leq 16, \times\right\}$$

be the non commutative complex finite modulo integer interval. V is of infinite order. V has several finite subsemigroups as well as infinite subsemigroups. V has also ideals. V has zero divisors units and idempotents.

$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \text{ is the identity element of V.}$$

Example 2.96: Let

$$S = \{(a_1, a_2, \ldots, a_9) \mid a_i \in C\langle([0, 9) \cup I); \ 1 \leq i \leq 9, \times\}$$

be the neutrosophic finite modulo integer interval semigroup of infinite order. S has infinite number of zero divisors. (1 1 1 1 1 1 1 1 1) is the unit in S.

Example 2.97: Let

$$M = \left\{\left[\begin{array}{cc|ccc} a_1 & a_2 & a_3 & a_4 & a_5 \\ \hline a_6 & a_7 & a_8 & a_9 & a_{10} \\ a_{11} & a_{12} & a_{13} & a_{14} & a_{15} \\ a_{16} & a_{17} & a_{18} & a_{19} & a_{20} \\ \hline a_{21} & a_{22} & a_{23} & a_{24} & a_{25} \\ a_{26} & a_{27} & a_{28} & a_{29} & a_{30} \\ \hline a_{31} & a_{32} & a_{33} & a_{34} & a_{35} \end{array}\right] \middle|\, a_i \in C\langle([0, 12) \cup I);\right.$$

$1 \le i \le 35, \times_n\}$

be the neutrosophic finite complex modulo integer interval super matrix semigroup.

M has infinite number of zero divisors, finite number of units and idempotents of units and idempotents.

M has both finite and infinite subsemigroups. Has ideals which are always of infinite order.

Inview of all these we have the following theorem.

THEOREM 2.6: *Let $S = C\langle([0, n) \cup I\rangle), \times, i_F^2 = n–1, I^2 = 1, (Ii_F)^2 = (n–1)I, \times\}$ be the neutrosophic finite complex modulo integer interval semigroup.*

(i) $o(S) = \infty$.
(ii) *S has finite number of zero divisors.*
(iii) *S has finite number of units.*
(iv) *S has only finite number of idempotents.*
(v) $P_1 = \{C([0, n)), \times\} \subseteq S$ *is a subsemigroup and not an ideal of infinite order.*
(vi) $P_2 = \{[0, n), \times\} \subseteq S$ *is a subsemigroup and not an ideal of infinite order.*
(vii) $P_3 = \{a + bI \mid a, b \in [0, n), \times\} \subseteq S$ *is a subsemigroup and not an ideal of infinite order.*
(viii) $P_4 = \{Z_n, \times\}$ *is a finite subsemigroup and not an ideal.*
(ix) $P_5 = \{C(Z_n), \times\}$ *is a finite subsemigroup and not an ideal.*
(x) $P_6 = \{\langle Z_n \cup I\rangle, \times\}$ *is a finite subsemigroup and not an ideal.*
(xi) $P_7 = \{C\langle Z_n \cup I\rangle, \times\}$ *is a finite subsemigroup and not an ideal of S.*

The proof is direct and hence left as an exercise to the reader.

THEOREM 2.7: *Let S = {m × n matrices with entries from {C⟨[[0, n) ∪ I⟩), $I^2 = I$, $i_F^2 = n-1$, $(Ii_F)^2 = (n-1)I$, $\times_n$} be the neutrosophic finite complex modulo interval matrix semigroup.*

(i) *o(S) = ∞.*
(ii) *S has infinite number of zero divisors.*
(iii) *S has only finite number of units.*
(iv) *S has only finite number of idempotents.*
(v) *P_1 = {m × n matrices with entries from Z_n, $\times_n$} is a finite subsemigroup of S and is not an ideal of S.*
(vi) *P_2 = {m × n matrices with entries from ⟨Z_n ∪ I⟩, $\times_n$} is a finite subsemigroup of S and is not an ideal of S.*
(vii) *P_3 = {m × n matrices with entries from C(Z_n), $\times_n$} is a subsemigroup of finite order and not an ideal of S.*
(viii) *P_4 = {m × n matrices with entries from [0, n), $\times_n$} is an infinite subsemigroup and is not an ideal of S.*
(ix) *P_5 = {m × n matrices with entries from C([0, n), $\times_n$} is an infinite subsemigroup of S and is not an ideal of S.*
(x) *P_6 = {m × n matrices with elements of the form a + bI where a, b ∈ [0, n), ×} is a subsemigroup of S of infinite order and is not an ideal of S.*
(xi) *S has at least N = 4 $({}_{m\times n}C_1 + {}_{m\times n}C_2 + \ldots + {}_{m\times n}C_{m\times n-1})$ number of finite subsemigroups if n is a prime.*
(xii) *S has at least T = ${}_{m\times n}C_1 + {}_{m\times n}C_2 + \ldots + {}_{m\times n}C_{m\times n-1}$ number of ideals.*

Proof is direct and hence let as an exercise to the reader.

We give some more examples before we proceed onto suggest problems.

Example 2.98: Let

$$S = \left\{ \begin{bmatrix} a_1 & a_2 & a_3 \\ a_4 & a_5 & a_6 \\ a_7 & a_8 & a_9 \end{bmatrix} \middle| a_i \in C(\langle [0, 7) \cup I \rangle), 1 \leq i \leq 9, \times_n \right\}$$

be the neutrosophic finite complex modulo integer interval semigroup of infinite order. o(S) = ∞ and S is a commutative semigroup.

$$P_1 = \left\{ \begin{bmatrix} a_1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \middle| a_i \in Z_7, \times_n \right\}$$

is a subsemigroup of finite order and is not an ideal. We have 9 such subsemigroups.

$$P_2 = \left\{ \begin{bmatrix} a_1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \middle| a_1 \in C(Z_7), \times \right\}$$

is a subsemigroup of finite order and is not an ideal of S. We have 9 such subsemigroup.

Further if we all in more than one non zero entry in P_1 (or P_2) we have N= ${}_9C_1 + {}_9C_2 + \ldots + {}_9C_8$ number of subsemigroup.

$$P_3 = \left\{ \begin{bmatrix} a_1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \middle| a_1 \in \langle Z_7 \cup I \rangle, \times_n \right\}$$

is a finite subsemigroup and we have N+1 such subsemigroups.

$$P_4 = \left\{\left.\begin{bmatrix} a & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}\right| a \in (\langle Z_7 \cup I\rangle), \times_n\right\}$$

is a finite subsemigroups and we have N+1 such subsemigroups. Thus we have atleast 4N+4 subsemigroup of finite order.

$$P_5 = \left\{\left.\begin{bmatrix} a & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}\right| a \in [0, n), \times\right\}$$

is an infinite subsemigroup of S.

If we have more than one non zero entry then we can have N+1 number subsemigroups of infinite order which are not ideals.

$$P_6 = \left\{\left.\begin{bmatrix} a_1 & a_2 & a_3 \\ 0 & 0 & 0 \\ a_4 & a_5 & 0 \end{bmatrix}\right| a_i \in C([0, n)), 1 \leq i \leq 5, \times_n\right\}$$

is again an infinite subsemigroup we can vary i from 1 to 9 and thus we have N + 1 number of infinite subsemigroups which are not ideals.

$$P_7 = \left\{\left.\begin{bmatrix} a_1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}\right| a_1 = a + bI \text{ where } a, b \in [0, n), \times_n\right\}$$

is a subsemigroup of infinite order and is not an ideal.

We have N+1 such subsemigroups. Thus we have atleast 3N+3 subsemigroup which are not ideals but are of infinite order.

Apart from this we can have subsemigroups of infinite order which are ideals.

$$T_1 = \left\{\left.\begin{bmatrix} a_1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}\right| a \in (\langle[0, 7) \cup I\rangle), \times_n\right\}$$

is a subsemigroup of infinite order which is also an ideal of S.

$$T_2 = \left\{\left.\begin{bmatrix} 0 & a_2 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}\right| a_2 \in (\langle[0, 7) \cup I\rangle), \times_n\right\}$$

is a subsemigroup of infinite order which is an ideal of S and so on. Let

$$T_9 = \left\{\left.\begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & a_9 \end{bmatrix}\right| a_9 \in (\langle[0, 7) \cup I\rangle), \times_n\right\}$$

be a subsemigroup of infinite order which is an ideal of S.

Let

$$T_{1,2} = \left\{\left.\begin{bmatrix} a_1 & a_2 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}\right| a_1\ a_2 \in C\ (\langle[0, 7) \cup I\rangle), \times_n\right\}$$

be a subsemigroup of infinite order which is also an ideal of S.

$$T_{3,7} = \left\{\left.\begin{bmatrix} 0 & 0 & a_3 \\ 0 & 0 & 0 \\ a_7 & 0 & 0 \end{bmatrix}\right| a_3, a_7 \in C(\langle [0, 7) \cup I\rangle), \times_n\right\}$$

be a subsemigroup of infinite order which is also an ideal of S.

Thus we have ${}_9C_2$ number of such ideals.

Consider

$$T_{2,5,9} = \left\{\left.\begin{bmatrix} 0 & a_2 & 0 \\ 0 & a_5 & 0 \\ 0 & 0 & a_9 \end{bmatrix}\right| a_i \in C(\langle [0, 7) \cup I\rangle), I = 2, 5, 9, \times_n\right\} \subseteq S$$

is an infinite subsemigroup which is also an ideal of S. We have ${}_9C_3$ number of such ideals.

Consider

$$T_{3,4,5,8} = \left\{\left.\begin{bmatrix} 0 & 0 & a_3 \\ a_4 & a_5 & 0 \\ 0 & a_8 & 0 \end{bmatrix}\right| a_3, a_4, a_5, a_8 \in C(\langle [0, n) \cup I\rangle), \times_n\right\}$$

be the infinite subsemigroup of S which is also an ideal of S. We have ${}_9C_4$ such ideals in S.

Let

$$T_{1,3,5,6,7} = \left\{\left.\begin{bmatrix} a_1 & 0 & a_3 \\ 0 & a_5 & a_6 \\ a_7 & 0 & 0 \end{bmatrix}\right| a_1, a_3, a_5, a_6, a_7 \in C(\langle [0, n) \cup I\rangle), \times_n\right\}$$

be the infinite subsemigroup of S which is an ideal. S has ${}_9C_5$ such ideals.

Consider

$$T_{1,3,4,5,6,8} = \left\{\left.\begin{bmatrix} a_1 & 0 & a_3 \\ a_4 & a_5 & a_6 \\ 0 & a_8 & 0 \end{bmatrix}\right| a_1, a_3, a_4, a_5, a_6, a_8 \in\right.$$

$$C (\langle[0, 7) \cup I\rangle): I = 1, 2, 3, 4, 7, 8, 9, \times_n\}$$

be an infinite subsemigroup which is an ideal of S. We have ${}_9C_7$ such ideal in S.

Finally

$$T_{2,3,4,5,6,7,9,8} = \left\{\left.\begin{bmatrix} 0 & a_2 & a_3 \\ a_4 & a_5 & a_6 \\ a_7 & a_8 & a_9 \end{bmatrix}\right| a_2, a_3, a_4, a_5, a_6, a_7, a_8, a_9 \in\right.$$

$$C (\langle[0, n) \cup I\rangle), I = 2, 3, 4, 5, 6, 7, 8, 9, \times_n\}$$

be an ideal of S. We have ${}_9C_8 = 9$ number of such ideals.

Hence we have atleast ${}_9C_1 + {}_9C_2 + \ldots + {}_9C_8$ number of ideals in S.

However if in the example we have taken instead of 7 a composite number we will have more number of subsemigroups of finite order. More number of zero divisors and more number of units and idempotents. However we are not in a position to say more about the ideals.

Finally we leave it as open conjecture.

If $S = \{C(\langle[0, n) \cup I\rangle, \times_n\}$ be the finite neutrosophic complex modulo integer interval semigroup. Can S have ideals?

Now we proceed onto suggest problems some of which are really difficult research problems open conjectures.

Problems

1. Find some special features enjoyed by finite complex modulo integer interval semigroups under $\times$.

2. Study problem (1) under the operation min and compare them.

3. $S = \{C([0, 7)) = \{a + bi_F \mid a, b \in [0, 7);\ i_F^2 = 6, \times\}$ be the semigroup.

   (i) Prove $o(S) = \infty$.
   (ii) Can S have finite ideals?
   (iii) Is every ideal of S of infinite order?
   (iv) Can S have zero divisors?
   (v) Is S a Smarandache semigroup?
   (vi) Find subsemigroups of finite order in S.
   (vii) Can S have units?
   (viii) Can S have S-zero divisors?
   (ix) Can S have S-idempotents?

4. Let $S = \{C([0, 12)),\ i_F^2 = 11, \times\}$ be the finite complex modulo integer interval semigroup.

   Study questions (i) to (ix) of problem 3 for this S.

5. Let $M = \{C([0, 24),\ i_F^2 = 23, \times\}$ be the finite complex modulo integer interval semigroup.

   Study questions (i) to (ix) of problem 3 for this S.

6. Let $T = \{(a_1, a_2, a_3, a_4, a_5, a_6) \mid a_i \in C([0, 20));\ 1 \leq i \leq 6,\ i_F^2 = 19, \times\}$ be the finite complex modulo integer interval row matrix semigroup.

(i) Study questions (i) to (ix) of problem 3 for this S.

(ii) Prove T has atleast ${}_6C_1 + {}_6C_2 + \ldots + {}_6C_5$ number of ideals.

7. Let

$$T = \left\{\left[\begin{array}{ccccc} a_1 & a_8 & a_{15} & a_{22} & a_{29} \\ a_2 & a_9 & a_{16} & a_{23} & a_{30} \\ a_3 & a_{10} & a_{17} & a_{24} & a_{31} \\ a_4 & a_{11} & a_{18} & a_{25} & a_{32} \\ a_5 & a_{12} & a_{19} & a_{26} & a_{33} \\ a_6 & a_{13} & a_{20} & a_{27} & a_{34} \\ a_8 & a_{14} & a_{21} & a_{28} & a_{35} \end{array}\right] \right| a_i \in C([0, 24));$$

$1 \leq i \leq 35, \times_n\}$

is a semigroup in the finite complex modulo integer interval.

(i) Study questions (i) to (ix) of problem 3 for this S.

(ii) Prove L has atleast ${}_{35}C_1 + {}_{35}C_2 + \ldots + {}_{35}C_{34}$ number of ideals.

8. Let $S = \{(a_1 \mid a_2\ a_3\ a_4 \mid a_5\ a_6\ a_7\ a_8 \mid a_9) \mid a_i \in C([0, 27)),$ $1 \leq i \leq 9, \times\}$ be the complex finite modulo integer interval super row matrix semigroup.

Study questions (i) to (ix) of problem 3 for this S.

9. Let

$$W = \left\{ \left[\begin{array}{cc|ccc|cc} a_1 & a_2 & a_3 & a_4 & a_5 & a_6 & a_7 \\ a_8 & \cdots & \cdots & \cdots & \cdots & \cdots & a_{14} \\ \hline a_{15} & \cdots & \cdots & \cdots & \cdots & \cdots & a_{21} \\ a_{22} & \cdots & \cdots & \cdots & \cdots & \cdots & a_{28} \\ a_{29} & \cdots & \cdots & \cdots & \cdots & \cdots & a_{35} \\ \hline a_{36} & \cdots & \cdots & \cdots & \cdots & \cdots & a_{42} \\ a_{43} & \cdots & \cdots & \cdots & \cdots & \cdots & a_{49} \\ a_{50} & \cdots & \cdots & \cdots & \cdots & \cdots & a_{56} \\ \hline a_{57} & \cdots & \cdots & \cdots & \cdots & \cdots & a_{63} \end{array}\right] \right| a_i \in C([0, 18)),$$

$1 \leq i \leq 63, \times_n\}$ be the complex finite modulo integer interval super matrix semigroup.

Study questions (i) to (ix) of problem 3 for this W.

10. Let

$$M = \left\{ \left[\begin{array}{cccc} a_1 & a_2 & a_3 & a_4 \\ \hline a_5 & a_6 & a_7 & a_8 \\ a_9 & \cdots & \cdots & a_{12} \\ a_{13} & \cdots & \cdots & a_{16} \\ a_{17} & \cdots & \cdots & a_{20} \\ \hline a_{21} & \cdots & \cdots & a_{24} \\ a_{25} & \cdots & \cdots & a_{28} \\ \hline a_{29} & \cdots & \cdots & a_{32} \\ a_{33} & \cdots & \cdots & a_{36} \\ a_{37} & \cdots & \cdots & a_{40} \end{array}\right] \right| a_i \in C([0, 45)), 1 \leq i \leq 40, \times_n \right\}$$

be the finite complex modulo integer interval super column matrix semigroup.

Study questions (i) to (ix) of problem 3 for this M.

11. Let S = $\{C(\langle[0, 10) \cup I\rangle), \times,\ i_F^2 = 9,\ I^2 = I,\ (i_F I)^2 = 9I\}$ be the semigroup of finite neutrosophic complex modulo integer interval semigroup.

    (i) Prove o(S) = ∞.
    (ii) Can S have zero divisors?
    (iii) Find idempotents in S.
    (iv) Can S have units?
    (v) Can S have ideals of finite order?
    (vi) Can S have finite subsemigroup infinite in number?
    (vii) Show S have atleast 15 finite subsemigroups.

12. Obtain some special and important features enjoyed by S = $\{\{C(\langle[0, n) \cup I\rangle), \times,\ i_F^2 = n-1,\ I^2 = I,\ (i_F I)^2 = (n-1)I\}$ the neutrosophic finite complex modulo integer interval semigroup.

13. Let T = $\{C(\langle[0, 12) \cup I\rangle), \times,\ i_F^2 = 11,\ I^2 = I,\ (i_F I)^2 = 11I, \times\}$ be the neutrosophic complex modulo integer interval semigroup.

    Study questions (i) to (vii) of problem 11 for this T.

14. Let S = $\{C(\langle[0, 13) \cup I\rangle), \times,\ i_F^2 = 12,\ I^2 = I,\ (i_F I)^2 = 12I, \times\}$ be the neutrosophic finite complex modulo integer interval semigroup.

    Study questions (i) to (vii) of problem 11 for this S.

    Compare T in problem 13 with this S and show T has more number of finite subsemigroups.

15. Let S = $\{C(\langle[0, 24) \cup I\rangle), i_F^2 = 23, I^2 = I, (i_FI)^2 = 23I, \times\}$ be the neutrosophic finite complex number integer interval semigroup.

    (i) Study questions (i) to (vii) of problem 11 for this S.

    (ii) Prove this S has more number of finite subsemigroups than the T given in problem 13.

    (iii) Prove this S has more number of zero divisors and idempotents than the S given in problem 14

16. Obtain some special features enjoyed by the complex finite modulo integer interval semigroup B = $\{C([0, n), \times\}$.

17. Let D = $\{C([0, 23), \times\}$ the complex modulo finite integer semigroup.

    (i) Find ideals of D.

    (ii) Can D have finite ideals?

    (iii) Can D have finite subsemigroups?

    (iv) Can D have infinite number of zero divisors?

    (v) Can D have idempotents?

    (vi) Find in D units.

    (vii) Find any other special feature enjoyed by D.

18. Let M = $\{(a_1, a_2, a_3, a_4, a_5) \mid a_i \in C([0, 24)), 1 \leq i \leq 5, i_F^2 = 23, \times\}$ be the finite complex modulo integer interval semigroup.

(i) Study questions (i) to (vii) of problem 17 for this M.

(ii) Prove M has only finite number of idempotents.

19. Let T = $\left\{\left[\begin{matrix} a_1 & a_2 & a_3 & a_4 & a_5 & a_6 \\ a_7 & \cdots & \cdots & \cdots & \cdots & a_{12} \\ a_{13} & \cdots & \cdots & \cdots & \cdots & a_{18} \\ a_{19} & \cdots & \cdots & \cdots & \cdots & a_{24} \end{matrix}\right] \right| a_i \in$

$C(\langle[0, 4) \cup I\rangle)$, $i_F^2 = 3$, $I^2 = I$, $(i_F I)^2 = 3I$, $\times_n\}$ be the neutrosophic complex finite modulo integer interval semigroup.

(i) Study questions (i) to (vii) of problem (11) for this T.

(ii) Prove T has more number of finite subsemigroups than S in problem 11.

(iii) Prove T has more number of ideals than S in problem 11.

20. Let

L = $\left\{\left[\begin{matrix} a_1 & a_2 & a_3 & a_4 & a_5 & a_6 & a_7 \\ a_8 & \cdots & \cdots & \cdots & \cdots & \cdots & a_{14} \\ a_{15} & \cdots & \cdots & \cdots & \cdots & \cdots & a_{21} \\ a_{22} & \cdots & \cdots & \cdots & \cdots & \cdots & a_{28} \\ a_{29} & \cdots & \cdots & \cdots & \cdots & \cdots & a_{35} \\ a_{36} & \cdots & \cdots & \cdots & \cdots & \cdots & a_{42} \end{matrix}\right] \right| a_i \in$

$C(\langle[0, 11) \cup I\rangle)$, $1 \leq i \leq 42$, $i_F^2 = 10$, $I^2 = I$, $(i_F I)^2 = 10I$, $\times_n\}$ be the neutrosophic finite complex modulo integer interval matrix semigroup.

Study questions (i) to (vii) of problem (11) for this L.

21. Let

$$W = \left\{ \begin{bmatrix} a_1 & a_2 & \ldots & a_{10} \\ a_{11} & a_{12} & \ldots & a_{20} \\ a_{21} & a_{22} & \ldots & a_{30} \end{bmatrix} \middle| \; a_i \in C(\langle [0, 42) \cup I \rangle), \right.$$

$1 \leq i \leq 30$, $i_F^2 = 41$, $I^2 = I$, $(i_F I)^2 = 41I$, $\times_n\}$ be the neutrosophic complex finite modulo integer interval matrix semigroup.

Study questions (i) to (vii) of problem (11) for this W.

22. Let $S = \{C([0, 18)$, $i_F^2 = 17$, $\min\}$ be the finite complex modulo integer interval semigroup.

(i) Prove $o(S) = \infty$.

(ii) Prove S has order 1, 2, 3,… finite subsemigroups.

(iii) Prove S has zero divisors.

(iv) Can S have units?

(v) Prove all ideals of S are of infinite order.

(vi) Prove S has infinite order subsemigroups which are not ideals of S.

(vii) Is it possible to define S-units (or S idempotents or S zero divisors in S)?

(viii) Can S be a Smarandache semigroup?

23. Let

$$W = \left\{ \begin{bmatrix} a_1 & a_2 & \ldots & a_{10} \\ a_{11} & a_{12} & \ldots & a_{20} \\ a_{21} & a_{22} & \ldots & a_{30} \\ a_{31} & a_{32} & \ldots & a_{40} \\ a_{41} & a_{42} & \ldots & a_{50} \\ a_{51} & a_{52} & \ldots & a_{60} \\ a_{61} & a_{62} & \ldots & a_{70} \end{bmatrix} \right| a_i \in C(\langle[0, 12) \cup I\rangle),$$

$1 \leq i \leq 70$, min} be the finite complex modulo integer interval semigroup.

(i) Study questions (i) to (viii) of problem 22 for this W.

(ii) Prove W has more number of zero divisors (Infact infinite in number).

(iii) Prove W has more number ideals in comparison with S in problem 22.

(iv) Prove W has more number of finite subsemigroup of infinite order which are not ideals.

24. Let M = {C([0, 17)), $i_F^2 = 16$, min} be the finite complex modulo integer interval semigroup.

Study questions (i) to (viii) of problem (22) for this M.

25. Let
L = {(a_1, a_2, …, a_9) | $a_i \in$ C([0, 19)), $1 \leq i \leq 9$, $i_F^2 = 18$, min} be the finite complex modulo integer interval semigroup.

Study questions (i) to (viii) of problem (22) for this L.

26. Let

$$V = \left\{ \begin{bmatrix} a_1 & a_2 & \ldots & a_{12} \\ a_{13} & a_{14} & \ldots & a_{24} \\ a_{25} & a_{26} & \ldots & a_{36} \end{bmatrix} \middle| \; a_i \in C(\langle [0, 15) \cup I \rangle), \right.$$

$1 \leq i \leq 36$, min} be the complex finite modulo integer interval semigroup.

Study questions (i) to (viii) of problem (22) for this V.

27. Let

$$M = \left\{ \left[\begin{array}{ccc} a_1 & a_2 & a_3 \\ \hline a_4 & a_5 & a_6 \\ a_7 & a_8 & a_9 \\ \hline a_{10} & a_{11} & a_{12} \\ a_{13} & a_{14} & a_{15} \\ a_{16} & a_{17} & a_{18} \\ \hline a_{19} & a_{20} & a_{21} \\ \hline a_{22} & a_{23} & a_{24} \\ a_{25} & a_{26} & a_{27} \end{array} \right] \middle| \; a_i \in C([0, 6)),\ i_F^2 = 5, \right.$$

$1 \leq i \leq 27$, min} be the finite complex modulo integer interval super column matrix semigroup.

Study questions (i) to (viii) of problem (22) for this M.

28. Let

$$X = \left\{ \left[\begin{array}{cc|ccc|cc|c} a_1 & a_2 & a_3 & a_4 & a_5 & a_6 & a_7 & a_8 \\ a_9 & \ldots & \ldots & \ldots & \ldots & \ldots & \ldots & a_{16} \\ a_{17} & \ldots & \ldots & \ldots & \ldots & \ldots & \ldots & a_{24} \end{array} \right] \right| a_i \in$$

C([0, 252)), $i_F^2 = 251$, $1 \leq i \leq 24$, min}
be the finite complex modulo integer interval super row matrix semigroup.

Study questions (i) to (viii) of problem (22) for this X.

29. Let

$$Y = \left\{ \left[\begin{array}{cc|ccc|cc|c} a_1 & a_2 & a_3 & a_4 & a_5 & a_6 & a_7 & a_8 \\ a_9 & \ldots & \ldots & \ldots & \ldots & \ldots & \ldots & a_{16} \\ \hline a_{17} & \ldots & \ldots & \ldots & \ldots & \ldots & \ldots & a_{24} \\ a_{25} & \ldots & \ldots & \ldots & \ldots & \ldots & \ldots & a_{32} \\ a_{33} & \ldots & \ldots & \ldots & \ldots & \ldots & \ldots & a_{40} \\ \hline a_{41} & \ldots & \ldots & \ldots & \ldots & \ldots & \ldots & a_{48} \\ a_{49} & \ldots & \ldots & \ldots & \ldots & \ldots & \ldots & a_{56} \end{array} \right] \right| a_i \in$$

C([0, 2)), $i_F^2 = 1$, $1 \leq i \leq 56$, min}
be the finite complex modulo integer interval super matrix semigroup.

Study questions (i) to (viii) of problem (22) for this Y.

30. Let S = {C(⟨[0, 22) ∪ I⟩), $i_F^2 = 21$, $I^2 = I$, $(i_F I)^2 = 22I$, min}
be the neutrosophic finite complex modulo integer interval semigroup.

(i) Can S have zero divisors?
(ii) Prove every singleton is a subsemigroup.

(iii) Prove every element is an idempotent.
(iv) Prove S has finite and infinite subsemigroups which are not ideals.
(v) Prove all ideals of S are of infinite cardinality.
(vi) Find any other interesting features associated with S.

31. Obtain some special features enjoyed by neutrosophic finite complex modulo integer interval semigroups under min operation.

32. Let M = $\{C(\langle[0, 24) \cup I\rangle)$, $i_F^2 = 23$, $I^2 = I$, $(i_F I)^2 = 23I$, min$\}$ be the neutrosophic complex finite integer interval semigroup.

Study questions (i) to (vi) of problem (30) for this M.

33. Let

$$M_1 = \left\{\left[\begin{array}{c} a_1 \\ \hline a_2 \\ a_3 \\ \hline a_4 \\ a_5 \\ \hline a_6 \\ a_7 \\ a_8 \\ \hline a_9 \end{array}\right] \middle|\; a_i \in C(\langle[0, 10) \cup I\rangle),\ 1 \le i \le 9;\ i_F^2 = 9,\ I^2 = I,\right.$$

$(i_F I)^2 = 9I$, min$\}$ be the neutrosophic complex finite integer interval super column matrix semigroup of infinite order.

Study questions (i) to (viii) of problem 30 for this M_1.

34. Let $P_2 = C(\langle[0, 15) \cup I\rangle), i_F^2 = 14, \max\}$ be the neutrosophic complex modulo integer interval semigroup under max.

    (i) Study the special properties associated with P_2.
    (ii) Prove $o(P_2) = \infty$.
    (iii) Find all subsemigroups of finite order in P_2.
    (iv) Prove all ideals in P_2 are of infinite order.
    (iv) Find all subsemigroups of infinite order which are not ideals of P_2.
    (vi) Can P_2 have zero divisors?
    (vii) Prove P_2 has subsemigroups of order one, two, three… and so on.

35. Let $S = \{C(\langle[0, 29) \cup I\rangle), i_F^2 = 28, I^2 = I, Ii_F = 28I, \max\}$ be the neutrosophic complex finite modulo integer interval semigroup.

    Study questions (i) to (vii) of problem (34) for this S.

36. Let $S = \{C(\langle[0, 80) \cup I\rangle), i_F^2 = 79, I^2 = I, Ii_F = 79I, \max\}$ be the neutrosophic finite complex modulo integer interval semigroup.

    Study questions (i) to (vii) of problem (34) for this S.

37. Let $S = \{C(\langle[0, 79) \cup I\rangle), i_F^2 = 78, I^2 = I, Ii_F = 78I, \max\}$ be the neutrosophic finite complex modulo integer interval semigroup.

    Study questions (i) to (vii) of problem (34) for this P.

38. Let

$$D = \left\{\left[\begin{matrix} a_1 & a_9 \\ a_2 & a_{10} \\ a_3 & a_{11} \\ a_4 & a_{12} \\ a_5 & a_{13} \\ a_6 & a_{14} \\ a_7 & a_{15} \\ a_8 & a_{16} \end{matrix}\right] \middle|\ a_i \in C(\langle[0, 12) \cup I\rangle),\ 1 \leq i \leq 16;\ i_F^2 = 11,\right.$$

$I^2 = I$, $(i_F I)^2 = 11I$, max} be the neutrosophic finite modulo integer interval semigroup.

Study questions (i) to (vii) of problem (34) for this D.

39. Let

$$V = \left\{\left(\begin{array}{cccc|c|ccc|cc} a_1 & a_2 & a_3 & a_4 & a_5 & a_6 & a_7 & a_8 & a_9 & a_{10} \\ a_{11} & \dots & \dots & \dots & \dots & \dots & \dots & \dots & \dots & a_{20} \\ a_{21} & \dots & \dots & \dots & \dots & \dots & \dots & \dots & \dots & a_{30} \end{array}\right) \middle|\ a_i\right.$$

$\in C(\langle[0, 18) \cup I\rangle)$, $1 \leq i \leq 30$; $i_F^2 = 17$, $I^2 = I$, $(i_F I)^2 = 17I$, max} be the neutrosophic finite modulo integer interval super row matrix semigroup.

Study questions (i) to (vii) of problem (34) for this V.

40. Let {C([0, 43)), +} = G be the complex finite modulo integer interval group under +.

(i) Show $o(G) = \infty$.
(ii) Show G has finite subgroups.

(iii) Can G have infinite subgroup?

41. Let $G_1 = \{C([0, 45), i_F^2 = 44, +\}$ be the complex modulo integer interval group under +.

Study questions (i) to (iii) of problem (40) for this G_1.

Show G_1 has more number of finite subgroups than G in problem 39.

42. Let $H = \{C(Q[0, 28)), i_F^2 = 27, +\}$ be the finite complex modulo integer group.

Study questions (i) to (iii) of problem (40) for this H.

43. Let $K = \{(a_1, a_2, a_3, \ldots, a_{10}) \mid a_i \in ([0, 40), 1 \leq i \leq 10, i_F^2 = 39, +\}$ be the finite complex modulo integer interval group.

Study questions (i) to (iii) of problem (40) for this K.

44. Let

$$E = \left\{ \begin{bmatrix} a_1 \\ a_2 \\ a_3 \\ a_4 \\ \vdots \\ a_{12} \end{bmatrix} \middle| \; a_i \in C(\langle [0, 30) \cup I \rangle), 1 \leq i \leq 12; \; i_F^2 = 29, +\right\}$$

be the finite complex modulo integer group.

Study questions (i) to (iii) of problem (40) for this E.

45. Let

$$W = \left\{ \left. \begin{pmatrix} a_1 & a_2 & a_3 & \ldots & a_{15} \\ a_{16} & a_{17} & a_{18} & \ldots & a_{30} \\ a_{31} & a_{32} & a_{33} & \ldots & a_{45} \end{pmatrix} \right| a_i \in C(\langle [0, 48) \cup I \rangle), \right.$$

$1 \leq i \leq 45;\ i_F^2 = 47,\ +\}$ be the finite complex modulo integer group.

Study questions (i) to (iii) of problem (40) for this W.

46. Let

$$P = \left\{ \left. \left(\begin{array}{c|ccc|cc|c|cccc} a_1 & a_2 & a_3 & a_4 & a_5 & a_6 & a_7 & a_8 & a_9 & a_{10} & a_{11} \\ a_{12} & \ldots & \ldots & \ldots & \ldots & \ldots & \ldots & \ldots & \ldots & \ldots & a_{22} \\ a_{23} & \ldots & \ldots & \ldots & \ldots & \ldots & \ldots & \ldots & \ldots & \ldots & a_{33} \end{array} \right) \right| \right.$$

$a_i \in C(\langle [0, 4) \cup I \rangle)$, $1 \leq i \leq 33$; $i_F^2 = 3$, $+\}$ be the finite complex modulo integer group.

Study questions (i) to (iii) of problem (40) for this W.

47. Let

$$P = \left\{ \left. \left(\begin{array}{c|ccc|cc|ccc|cc} a_1 & a_2 & a_3 & a_4 & a_5 & a_6 & a_7 & a_8 & a_9 & a_{10} & a_{11} \\ a_{12} & \ldots & \ldots & \ldots & \ldots & \ldots & \ldots & \ldots & \ldots & \ldots & a_{22} \\ a_{23} & \ldots & \ldots & \ldots & \ldots & \ldots & \ldots & \ldots & \ldots & \ldots & a_{33} \\ a_{34} & \ldots & \ldots & \ldots & \ldots & \ldots & \ldots & \ldots & \ldots & \ldots & a_{44} \\ a_{45} & \ldots & \ldots & \ldots & \ldots & \ldots & \ldots & \ldots & \ldots & \ldots & a_{55} \\ a_{56} & \ldots & \ldots & \ldots & \ldots & \ldots & \ldots & \ldots & \ldots & \ldots & a_{66} \\ a_{67} & \ldots & \ldots & \ldots & \ldots & \ldots & \ldots & \ldots & \ldots & \ldots & a_{77} \end{array} \right) \right| \right.$$

$a_i \in C(\langle [0, 43) \cup I \rangle)$, $1 \leq i \leq 77$; $i_F^2 = 42$, $+\}$

be the finite complex modulo integer group.

Study questions (i) to (iii) of problem (40) for this P.

48. Let $\{C(\langle[0, 9) \cup I\rangle), +\} = B$ be the finite complex modulo integer neutrosophic interval group.

    (i) Find o(B).

    (ii) Show B has subgroups of finite order.

    (iii) How many subgroups of infinite order does B contain?

    (iv) Can B be written as a direct sum of subgroups?

49. Let $M = \{(a_1, a_2, \ldots, a_{15}) \mid a_i \in \{C(\langle[0, 23) \cup I\rangle), 1 \leq I \leq 15, i_F^2 = 22, (Ii_F)^2 = 22I, I^2 = I, +\}$ be the neutrosophic complex finite modulo integer interval group.

50. Let

$$S = \left\{ \begin{bmatrix} a_1 \\ a_2 \\ a_3 \\ a_4 \\ \vdots \\ a_{12} \end{bmatrix} \middle| \; a_i \in C(\langle[0, 7) \cup I\rangle), 1 \leq i \leq 12; \; i_F^2 = 6, \right.$$

$I^2 = I$, $(i_F I)^2 = 6I$, $+\}$ be the neutrosophic finite complex modulo integer group.

Study questions (i) to (iv) of problem (48) for this S.

51. Let

$$T = \left\{ \left[\begin{array}{ccccccc} a_1 & a_2 & a_3 & a_4 & a_5 & a_6 & a_7 \\ a_8 & \ldots & \ldots & \ldots & \ldots & \ldots & a_{14} \\ a_{15} & \ldots & \ldots & \ldots & \ldots & \ldots & a_{21} \\ a_{22} & \ldots & \ldots & \ldots & \ldots & \ldots & a_{28} \\ a_{29} & \ldots & \ldots & \ldots & \ldots & \ldots & a_{35} \\ a_{35} & \ldots & \ldots & \ldots & \ldots & \ldots & a_{42} \end{array} \right] \right| a_i \in C(\langle [0, 5) \cup I \rangle),$$

$1 \leq i \leq 42$; $i_F^2 = 4$, $I^2 = I$, $(i_F I)^2 = 4I$, $+\}$ be the neutrosophic finite complex modulo integer group.

Study questions (i) to (iv) of problem (48) for this T.

52. Let

$$T = \left\{ \left[\begin{array}{cccccccc} a_1 & a_2 & a_3 & a_4 & a_5 & a_6 & a_7 & a_8 \\ a_9 & \ldots & \ldots & \ldots & \ldots & \ldots & \ldots & a_{16} \end{array} \right] \right| a_i \in$$

$C(\langle [0, 20) \cup I \rangle)$, $1 \leq i \leq 16$; $i_F^2 = 19$, $I^2 = I$, $(i_F I)^2 = 19I$, $+\}$ be the neutrosophic finite complex modulo integer group.

Study questions (i) to (iv) of problem (48) for this M.

53. Let

$$P = \left\{ \left[\begin{array}{ccc} a_1 & a_2 & a_3 \\ a_4 & a_5 & a_6 \\ \hline a_7 & a_8 & a_9 \\ \hline a_{10} & a_{11} & a_{12} \\ a_{13} & a_{14} & a_{15} \\ \hline a_{16} & a_{17} & a_{18} \\ a_{19} & a_{20} & a_{21} \\ a_{22} & a_{23} & a_{24} \\ \hline a_{25} & a_{26} & a_{27} \\ a_{28} & a_{29} & a_{30} \\ a_{31} & a_{32} & a_{33} \\ a_{34} & a_{35} & a_{36} \\ \hline a_{37} & a_{38} & a_{39} \end{array} \right] \right| a_i \in C(\langle [0, 9) \cup I \rangle), 1 \leq i \leq 39;$$

$i_F^2 = 38$, $I^2 = I$, $(i_F I)^2 = 38I$, +} be the neutrosophic finite complex modulo integer interval super column matrix group. Study questions (i) to (iv) of problem (48) for this P.

54. Let

$$L = \left\{ \left(\begin{array}{cc|ccc|cccc|c} a_1 & a_2 & a_3 & a_4 & a_5 & a_6 & a_7 & a_8 & a_9 & a_{10} \\ a_{11} & \ldots & \ldots & \ldots & \ldots & \ldots & \ldots & \ldots & \ldots & a_{20} \\ a_{21} & \ldots & \ldots & \ldots & \ldots & \ldots & \ldots & \ldots & \ldots & a_{30} \\ a_{31} & \ldots & \ldots & \ldots & \ldots & \ldots & \ldots & \ldots & \ldots & a_{40} \\ \hline a_{41} & \ldots & \ldots & \ldots & \ldots & \ldots & \ldots & \ldots & \ldots & a_{50} \\ a_{51} & \ldots & \ldots & \ldots & \ldots & \ldots & \ldots & \ldots & \ldots & a_{60} \\ a_{61} & \ldots & \ldots & \ldots & \ldots & \ldots & \ldots & \ldots & \ldots & a_{70} \\ \hline a_{71} & \ldots & \ldots & \ldots & \ldots & \ldots & \ldots & \ldots & \ldots & a_{80} \\ a_{81} & \ldots & \ldots & \ldots & \ldots & \ldots & \ldots & \ldots & \ldots & a_{90} \end{array} \right) \right|$$

$a_i \in C(\langle[0, 8) \cup I\rangle)$, $1 \leq i \leq 90$; $i_F^2 = 7$, $I^2 = I$, $(i_F I)^2 = 7I$, max} be the neutrosophic finite complex modulo finite integer interval super matrix group.

Study questions (i) to (iv) of problem (48) for this L.

55. Let M = $\{(\langle[0, 3) \cup I\rangle) \times (\langle[0, 14) \cup I\rangle) \times (\langle[0, 23) \cup I\rangle)\}$ be the neutrosophic finite complex modulo integer interval group under addition ‘+’.

(i) Show $o(M) = \infty$.
(ii) Can M have subgroups of finite order?
(iii) If the operation + is replaced by $\times$ study M.
(iv) Prove $(M, \times)$ has infinite number of zero divisors.
(v) Find all ideals in $(M, \times)$.
(vi) Can $(M, \times)$ have finite subsemigroups?
(vii) Will $(M, \times)$ have ideals of finite order?
(viii) Find some special features enjoyed by $(M, \times)$ as a semigroup.

Chapter Three

PSEUDO RINGS AND SEMIRINGS BUILT USING FINITE COMPLEX MODULO INTEGER INTERVALS C([0, n))

In this chapter we build semirings of two types using $C([0, n)) = \{a + bi_F \mid a, b \in [0, n), i_F^2 = n-1\}$ which will be known as the finite complex modulo integer semiring.

We also build semirings using $C(\langle [0, n) \cup I \rangle) = \{a + bi_F + cI + dIi_F \mid a, b, c, d \in [0, n), I^2 = I, i_F^2 = n-1, (i_F I)^2 = (n-1)I\}$. Apart from this we construct pseudo rings using $C([0, n))$ or $C(\langle [0, n) \cup I \rangle)$ (or used in the mutually exclusive sense).

We will illustrate these situations by examples.

DEFINITION 3.1: *Let $C([0, n)) = \{a + bi_F \mid a, b \in [0, n), i_F^2 = n-1\}$ be the finite complex modulo integer interval. $C([0, n))$ under min operation is a semigroup. $C([0, n))$ under max operation is a semigroup. $\{C([0, n)), \min, \max\}$ is a semiring called the finite complex modulo integer interval semiring.*

We will illustrate this situation by some examples.

Example 3.1: Let M = {C([0, 5)), i_F^2 = 4, min, max} be the finite complex modulo integer interval semiring |M| = ∞. We see if

x = 0.7 + 2.4i_F and y = 2.1 + 0.5i_F ∈ M, then max {x, y} = 2.1 + 2.4i_F and min {x, y} = 0.7 + 0.5i_F ∈ M. This is the way min, max operations are defined on M. M has subsemirings.

Infact {0, x} for every x ∈ M is a subsemiring of order two.

Every proper subset of M in general is not a subsemiring.

For take P = {0, 2.1 + 0.4i_F = x, y = 0.7 + 2i_F, z = 3 + 0.02i_F} ⊆ M. P is only a subsemigroup.

For min {x, y} = 0.7 + 0.4i_F ∉ P.

max {x, y} = 2.1 + 2i_F ∉ P,
min {x, z} = {2.1 + 0.02i_F} ∉ P,
max {x, z} = {3 + 0.4i_F} ∉ P,
min {y, z} = {0.7 + 0.02i_F} ∉ P and
max {y, z} = {3 + 2i_F} ∉ P.

But T = P ∪ {0.7 + 0.4i_F, 2.1 + 2i_F, 2.1 + 0.02i_F, 3 + 0.4i_F, 0.7 + 0.02i_F, 3 + 2i_F} is a subsemiring known as the completion of P and denoted by P_c.

P_c is the completion of the subset into a subsemiring.

Example 3.2: Let S = {C([0, 12)), i_F^2 = 11, min, max} be the complex finite modulo integer semiring.

Consider P_1 = {Z_{12}, min, max} is a subsemiring of S of order 12.

P_2 = {$C(Z_{12})$, min, max} is a subsemiring of S of finite order.

P_3 = {{0, 2, 4, 6, 8, 10}, min, max} is also a subsemiring of order 6.

P_4 = {[0, 12), min, max} is a subsemiring of infinite order.

P_5 = {[0, 6), min, max} is a subsemiring of infinite order.

P_6 = {[0, 3), min, max} is a subsemiring of infinite order.

None of these finite subsemirings are ideals of S.

Clearly none of these subsemirings are also filters of S finite or infinite. However P_4, P_5 and P_6 are ideals of S.

Example 3.3: Let S = {C([0, 14)), i_F^2 = 13, min, max} be the complex finite modulo integer semiring. S has subsemirings of finite order.

We can build semirings using S = C([0, n)).

Example 3.4: Let
A = {(a_1, a_2, a_3) | $a_i \in$ C([0, 4)), i_F^2 = 3, $1 \leq i \leq 3$, min, max} be the finite complex modulo integer interval semiring.

Clearly o(A) = ∞.

P_1 = {$(a_1, 0, 0)$ | $a_1 \in$ C([0, 4)), min, max} $\subseteq$ A is a subsemiring as well as ideal of A.

P_2 ={$(0, a_2, 0)$ | $a_2 \in$ C([0, 4))} $\subseteq$ A is a subsemiring as well as an ideal of A.

P_3 = {$(0, 0, a_3)$ | $a_3 \in$ C([0, 4)) $\subseteq$ A is a subsemiring as well as an ideal of A.

$P_4 = \{(a_1, a_2, 0) \mid a_3 \in C([0, 4)) \subseteq A$ is a subsemiring as well as an ideal of A.

$P_5 = \{(0, a_2, a_3) \mid a_2, a_3 \in C([0, 4))\}$ is a subsemiring as well as an ideal of A.

$P_6 = \{(a_1, 0, a_3) \mid a_1, a_3 \in C([0, 4))\} \subseteq A$ is a subsemiring as well as an ideal of A.

$P_7 = \{(a_1, a_2, a_3) \mid a_i \in [0, 4); 1 \leq i \leq 3\}$ is an ideal of A.

However none of them are filters of A.

$M_1 = \{(a_1, 0, 0) \mid a_1 \in Z_4\}$ is only a subsemiring not an ideal or filter of A.

We have subsemirings which are neither ideals nor filters of A.

Let $x = (2.5 + 3i_F, 0, 1.4 + 0.8i_F)$
and $y = (1 + 3.5i_F, 0.7 + 0.5i_F, 2 + 0.06i_F) \in A$.

We see $T = \{x, y\}$ is only a subset of A and is not a subsemiring of A.

For consider min $\{x, y\}$ = min $\{(2.5 + 3i_F, 0, 1.4 + 0.8i_F)$, $(1 + 3.5i_F, 0.7 + 0.5i_F, 2 + 0.8i_F) \notin T$.

However $T_c = T \cup \{(1 + 3i_F, 0, 1.4 + 0.06i_F), (2.5 + 3.5i_F, 0.7 + 0.5i_F, 2 + 0.8i_F)\}$ is a subsemiring of order five.

Thus we can complete any finite set to get a subsemiring.

This procedure of getting a subsemiring from a subset is known as completing a subset or completion of a subset into a subsemiring.

Example 3.5: Let

$$S = \left\{\left.\begin{bmatrix} a_1 \\ a_2 \\ a_3 \\ a_4 \\ a_5 \\ a_6 \end{bmatrix}\right| a_i \in C\,([0, 2)),\ i_F^2 = 1;\ 1 \leq i \leq 6, \max, \min\right\}$$

be the semiring.

S has zero divisors which are infinite in number. S has ideals all of them are of infinite order.

$$P_1 = \left\{\left.\begin{bmatrix} a_1 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}\right| a_1 \in C\,([0, 2)), \max, \min\right\}$$

is a subsemiring of infinite order which is also an ideal of S.

$$P_2 = \left\{\left.\begin{bmatrix} 0 \\ a_2 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}\right| a_2 \in C\,([0, 2)), \max, \min\right\} \text{ is an ideal of S.}$$

$$P_3 = \left\{\left.\begin{bmatrix} 0 \\ 0 \\ a_3 \\ 0 \\ 0 \\ 0 \end{bmatrix}\right| a_3 \in C([0, 2)), \max, \min\right\}$$

is an ideal of the semiring S.

$$P_6 = \left\{\left.\begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ a_6 \end{bmatrix}\right| a_6 \in C([0, 2)), \max, \min\right\}$$

is an ideal of the semiring.

$$P_{1,2} = \left\{\left.\begin{bmatrix} a_1 \\ a_2 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}\right| a_1, a_2 \in C([0, 2)), \max, \min\right\}$$

is an ideal of S and so on.

None of them are filters of S. We have filters in S which are not ideals of S.

For take

$$M_1 = \left\{ \begin{bmatrix} a_1 \\ a_2 \\ a_3 \\ a_4 \\ a_5 \\ a_6 \end{bmatrix} \middle| a_i \in \{a + bI \mid a, b \in [0.5, 2), 1 \leq i \leq 6\} \right. \subseteq S.$$

M_1 is not an ideal but M_1 is a filter of S. S has several filters.

Example 3.6: Let

$$R = \left\{ \begin{bmatrix} a_1 & a_2 & a_3 & a_4 \\ a_5 & a_6 & a_7 & a_8 \\ a_9 & a_{10} & a_{11} & a_{12} \\ a_{13} & a_{14} & a_{15} & a_{16} \end{bmatrix} \middle| a_i \in C([0, 12)),\ i_F^2 = 11; \right.$$

$$1 \leq i \leq 16, \max, \min\}$$

be the complex finite modulo integer interval semiring o(R) is ∞.

R has atleast ${}_{16}C_1 + {}_{16}C_2 + \ldots + {}_{16}C_{15}$ number of ideals of infinite order and atleast 6 $({}_{16}C_1 + {}_{16}C_2 + \ldots + {}_{16}C_{15})$ number of finite subsemirings which are not ideals or filters of R apart from finite semirings of order 2, 3, 4, … which are infinite in number.

Example 3.7: Let T = $\{(a_1 \mid a_2\ a_3\ a_4\ a_5 \mid a_6 \mid a_7\ a_8 \mid a_9\ a_{10}\ a_{11} \mid a_{12}) \mid a_i \in C([0, 7)), 1 \leq i \leq 12, \max, \min\}$ be the finite complete modulo integer interval semiring of super row matrices.

T has subsemirings of finite order. T has ideals as well as filters.

Example 3.8: Let

$$W = \left\{ \left[\begin{array}{c} a_1 \\ \hline a_2 \\ a_3 \\ a_4 \\ \hline a_5 \\ a_6 \\ a_7 \\ a_8 \\ \hline a_9 \\ a_{10} \\ a_{11} \\ a_{12} \\ a_{13} \end{array} \right] \middle| \; a_i \in C([0, 11)), 1 \leq i \leq 13, \min, \max \right\}$$

be the finite complex modulo integer interval semiring of super column matrices. W has several subsemirings of finite order say order 2, order 3 and so on.

Example 3.9: Let

$$V = \left\{ \left[\begin{array}{cc|ccc} a_1 & a_2 & a_3 & a_4 & a_5 \\ a_6 & \cdots & \cdots & \cdots & a_{10} \\ a_{11} & \cdots & \cdots & \cdots & a_{15} \\ \hline a_{16} & \cdots & \cdots & \cdots & a_{20} \\ \hline a_{21} & \cdots & \cdots & \cdots & a_{25} \\ a_{26} & \cdots & \cdots & \cdots & a_{30} \\ a_{31} & \cdots & \cdots & \cdots & a_{35} \\ a_{36} & \cdots & \cdots & \cdots & a_{40} \end{array} \right] \middle| \; a_i \in C([0, 24)), 1 \leq i \leq 40, \right.$$

$i_F^2 = 23, n, \max\}$

be the finite complex modulo integer interval super matrix semiring.

V has subsemirings of order two which are infinite in number.

This is so for every $x \in V$ together with

$$(0) = \left[\begin{array}{cc|ccc} 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ \hline 0 & 0 & 0 & 0 & 0 \\ \hline 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{array}\right] \in V. \ \{(0), x\} = W$$

are all subsemirings of order two. We can as in case of semigroup with max or min define two element in V to be comparable. If x and y are comparable in V then $T = \{0, x, y\} \subseteq V$ are subsemirings of order three.

Such subsemirings are also infinite in number.

Now we study semiring built on the finite neutrosophic complex modulo integer intervals $S = \{C(\langle[0, n) \cup I\rangle) = \{a + bI + ci_F + dIi_F \mid a, b, c, d \in [0, n),\ i_F^2 = n-1,\ I^2 = I$ and $(Ii_F)^2 = (n-1)I\}$.

We see S under min and max operation is a finite neutrosophic complex modulo integer interval semiring. We will illustrate how we work with them in a line or two.

Let us take S = {C(⟨[0, 10) ∪ I⟩) = {a + bi_F + cI + dIi_F where a, b, c, d ∈ [0, 10), $i_F^2 = 9$, $I^2 = I$, and $(Ii_F)^2 = 9I$}.

Suppose x = 3 + $2.7i_F$ + 8.1I + $4.5i_FI$ and
y = 8.4 + $1.3i_F$ + 6.7I + $6.2i_FI$ ∈ S. We find max {x, y} and min {x, y}.

max {x, y} max {3 + $2.7i_F$ + 8.1I + $4.5i_FI$, 8.4 + $1.3i_F$ + 6.7I + $6.2i_FI$}

= max {3, 8, 4} + max {$2.7i_F$, $1.3i_F$} + max {8.1I, 6.7I} + max {$4.5i_F$, $6.2i_FI$}

= {8.4 + $2.7i_F$ + 8.1I + $6.2i_FI$} ∈ S.

This is the way max operation is performed.

Now min {x, y} = min {3 + $2.7i_F$ + 8.1I + $4.5Ii_F$, 8.4 + $1.3i_F$ + 6.7I + $6.2i_FI$}

= min {3, 8, 4} + min {$2.7i_F$, $1.3i_F$} + min {8.1I, 6.7I} + min {$4.5Ii_F$, $6.2Ii_F$}

= 3 + $1.3i_F$ + 6.7I + $4.5Ii_F$ ∈ S.

Thus S is closed under both min and max operation.

{S, min, max} forms the finite neutrosophic complex modulo integer interval semiring. o(S) = ∞. S has subsemirings of order two, three and so on.

Example 3.10: Let S = C(⟨[0, 5) ∪ I⟩); min, max} be the finite neutrosophic complex modulo integer interval semiring.

Let x = {2.3 + $3.7i_F$ + 2.1I + $0.8Ii_F$} ∈ S.

min {x, 0} = 0 and max {0, x} = x so that T = {0, x} ⊆ S is a subsemiring of order two and is not an ideal or filter of S.

We see S has infinite number of subsemirings of order two. Infact every $x \in S$ together with $0 \in S$ is a subsemiring of order two.

S has also zero divisors.

For take $x = 2.1 + 3i_F$ and $y = 2.5I + 0.38Ii_F$ we see $\min \{x, y\} = \min \{2.1 + 3i_F, 2.5I + 0.38i_FI\}$

$= \min \{2.1, 0\} + \min \{3i_F, 0\} + \min \{0, 2.5I\} + \min \{0, 0.38i_FI\}$

$= 0 + 0 + 0 + 0 = 0.$

Thus S has infinite number of zero divisor.

Now $\max \{x, y\} = \max \{2.1, 0\} + \max \{0, 3i_F\} + \max \{0, 2.5I\} + \max \{0, 0.38Ii_F\}$

$= 2.1 + 3i_F + 2.5I + 0.38Ii_F.$

This is the way min and min are found and min operation gives an infinite collection of zero divisor.

Suppose $x = 0.7$ and $y = 2i_F \in S$

$\min \{x, y\} = 0$ and $\max \{0.7, 2i_F\} = 0.7 + 2i_F$.

Thus $T = \{x, y, 0, 0.7 + 2i_F\}$ is a subsemiring of order four. We can complete proper subsets of S into subsemirings however they are not in general ideals or filters of S.

Example 3.11: Let $S = C(\langle[0, 6 \cup I\rangle), \min, \max\}$ be the neutrosophic complex finite modulo integer interval semiring.

$T_1 = \{Z_6, \min, \max\}$ is a subsemiring which is not an ideal and $o(T_1) = 6$.

$T_2 = \{\{0, 2, 4\}, \min, \max\}$ is a subsemiring of order 3 and not an ideal.

$T_3 = \{\{0, 5\}, \min, \max\}$ is a subsemiring of order 2 and not an ideal of S.

$T_4 = \{\{0, 3\}, \min, \max\}$ is a subsemiring of order two and not an ideal of S.

$T_5 = \{\{0, 1, 2, 4\}, \min, \max\}$ is a subsemiring of order four not an ideal

$T_6 = \{\langle Z_6 \cup I\rangle, \min, \max\}$ is a subsemiring and not an ideal of S.

$T_7 = \{3 + 4I, 2+ I, 0\} \subseteq S$ is a subsemiring of order three and not an ideal of S.

$T_8 = \{C(Z_6), \min, \max\}$ is a subsemiring and not ideal of S of finite order.

$T_9 = \{3 + 2i_F, 0\}$ is a subsemiring of S.

$T_{10} = \{0, 2, 4i_F, 2 + 4i_F\}$ is a subsemiring of S and is not an ideal of order four. Thus we have a class of subsemirings of finite order which are not ideals of S.

$R_1 = \{[0, 6), \min, \max\}$ is an ideal of S. But is not a filter for if $3i_F \in S$. $\max \{3i_F, 4\} = 3i_F + 4 \notin R_1$.

$R_2 = \{[0, 3), \min, \max\}$ is an subsemiring of S and is also an ideal of S.

R_2 is not a filter of S.

Thus we have infinite collections of subsemirings which are not filters or ideals and we have an infinite collection of ideals which are not filters and vice versa.

Example 3.12: Let
S = {C(⟨[0, 29) ∪ I⟩); $i_F^2 = 28$; $I^2 = I$, $(i_F I)^2 = 28I$, min, max} be the complex finite modulo integer interval semiring. S has infinite number of finite subsemirings. S has infinite subsemirings which are ideals and filters.

It is pertinent to note that in S = C(⟨[0, n) ∪ I⟩); n prime or composite is immaterial as only min max operations are performed.

Now we contruct matrices using S which will be illustrated by examples.

Example 3.13: Let S = {(a_1, a_2, …, a_6) | a_i ∈ C(⟨[0, 25) ∪ I⟩); 1 ≤ i ≤ 6, $i_F^2 = 24$, $(i_F I)^2 = 24I$, $I^2 = I$, min, max} be the neutrosophic finite complex modulo integer interval semiring.

S has ideals, filters and subsemirings which are neither ideals nor filters. S has infinite number of zero divisors. S has not units.

Let x = (0.7 + $3i_F$, 2, $3i_F$, 0, 0, 12.4 + $17i_F$) ∈ S; we have infinite number of elements in S such min {x, y} = (0 0 0 0 0 0). Thus A = {(0, 0, 0, a, b, 0) | a, b ∈ C(⟨[0, 25) ∪ I⟩) are such that min {x, y} = (0, 0, 0, 0, 0, 0) for all x ∈ A.

Example 3.14: Let

$$S = \left\{ \begin{bmatrix} a_1 & a_2 & a_3 \\ a_4 & a_5 & a_6 \\ a_7 & a_8 & a_9 \\ a_{10} & a_{11} & a_{12} \\ a_{13} & a_{14} & a_{15} \\ a_{16} & a_{17} & a_{18} \\ a_{19} & a_{20} & a_{21} \end{bmatrix} \middle|\, a_i \in C(\langle [0, 12) \cup I \rangle), \right.$$

$$1 \le i \le 21, \text{min, max}\}$$

is a neutrosophic complex modulo finite integer interval semiring of finite order. S has infinite number of zero divisors.

$$\text{For if } x = \begin{bmatrix} a & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \in S \text{ then we have}$$

$$B = \left\{ \begin{bmatrix} 0 & x_1 & x_2 \\ x_3 & x_4 & x_5 \\ \vdots & \vdots & \vdots \\ x_{18} & x_{19} & x_{20} \end{bmatrix} \middle| a_i \in C(\langle [0, 12) \cup I \rangle), \right.$$

$$\left. 1 \leq i \leq 20, \min, \max \right\}$$

$$\text{is such that } \min \{x, b\} = (0) = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ \vdots & \vdots & \vdots \\ 0 & 0 & 0 \end{bmatrix} \text{ for all } b \in B.$$

Thus for a single x we have are infinite collection of zero divisors such that min {x, b} = (0).

Every element in S is an idempotent with respect to max and min. S has infinite number of finite subsemiring and infinite number of infinite subsemirings which are ideals.

Example 3.15: Let

$$M = \left\{ \left(\begin{array}{c|cc|ccc|c} a_1 & a_2 & a_3 & a_4 & a_5 & a_6 & a_7 \\ a_8 & a_9 & a_{10} & a_{11} & a_{12} & a_{13} & a_{14} \\ a_{15} & a_{16} & a_{17} & a_{18} & a_{19} & a_{20} & a_{21} \end{array} \right) \middle| \; a_i \in \right.$$

$$\left. C(\langle [0, 12) \cup I \rangle), 1 \leq i \leq 21, \text{min, max}, \; i_F^2 = 11, I^2 = I \text{ and } (i_F I)^2 = 11I \right\}$$

be the neutrosophic finite complex modulo integer interval super row matrix semiring of infinite order.

M has infinite number of zero divisor has atleast ${}_{21}C_1 + {}_{21}C_2 + \ldots + {}_{21}C_{20}$ number of subsemirings which is ideals and not filters of M.

M has infinite number of subsemirings of order 2, 3, 4, and so on.

Example 3.16: Let

$$S = \left\{ \left[\begin{array}{cccc} a_1 & a_2 & a_3 & a_4 \\ \hline a_5 & \ldots & \ldots & a_8 \\ a_9 & \ldots & \ldots & a_{12} \\ a_{13} & \ldots & \ldots & a_{16} \\ \hline a_{17} & \ldots & \ldots & a_{20} \\ a_{21} & \ldots & \ldots & a_{24} \\ \hline a_{25} & \ldots & \ldots & a_{28} \\ a_{29} & \ldots & \ldots & a_{32} \\ \hline a_{33} & \ldots & \ldots & a_{36} \end{array} \right] \middle| \; a_i \in C(\langle [0, 3) \cup I \rangle), 1 \leq i \leq 36, \right.$$

$$\left. i_F^2 = 2, I^2 = I, \text{ and } (i_F I)^2 = 2I, \text{min, max} \right\}$$

be the neutrosophic complex finite modulo integer interval semiring of super column matrices. S has infinite number of finite subsemirings.

S has infinite number of zero divisors no units and every element is an idempotent.

Example 3.17: Let

$$S = \left\{ \left[\begin{array}{cc|cc} a_1 & a_2 & a_3 & a_4 \\ a_5 & \ldots & \ldots & a_8 \\ \hline a_9 & \ldots & \ldots & a_{12} \\ a_{13} & \ldots & \ldots & a_{16} \\ a_{17} & \ldots & \ldots & a_{20} \\ \hline a_{21} & \ldots & \ldots & a_{24} \end{array}\right] \middle| \, a_i \in C(\langle [0, 20) \cup I \rangle), 1 \leq i \leq 24, \right.$$

$$\left. i_F^2 = 19, I^2 = 19I, \text{ and } (i_F I)^2 = 19I, \min, \max \right\}$$

be the neutrosophic finite complex modulo integer interval super matrix semiring. o(S) = ∞. S has infinite number of zero divisors and no units. S has several atleast ${}_{24}C_1 + {}_{24}C_2 + \ldots + {}_{24}C_{23}$ number of ideals which are not filters.

Now we proceed onto describe the notion of pseudo semirings built on C([0, n)) and on C(⟨[0, n) ∪ I⟩).

Let C([0, n)) be the finite complex modulo integer interval. C([0, n)) is a semigroup under ×.

C(⟨[0, n) ∪ I⟩) is a semigroup under the min operation C([0, n)) is also semigroup under min operation S = {C([0, n), min, ×} is defined as the pseudo semiring of finite complex modulo integer interval.

We call this as a pseudo semiring as the distributive law is not true in S.

For the distributive law in S is $x \times \min \{y, z\} \neq \min \{x \times y, x \times z\}$ for $x, y, z \in S$.

Consider $x = 3.2$ $y = 6.5$ and $z = 7.1 \in C([0, 10))$.

$$x \times \min \{6.5, 7.1\} = 3.2 \times 6.5 = 0.80 \quad \ldots \text{ I}$$

$$\min \{x \times y, x \times z\} = \min \{6.5 \times 3.2, 7.1 \times 3.2\}$$
$$= \min \{0.80, 2.72\}$$
$$= 0.80 \quad \ldots \text{ II}$$

Here I and II are same.

Consider $0.7i_F$ and $z = 8.1 \in S$

$$x \times \min \{y, z\}$$
$$= x \times \min \{0.7i_F, 8.1\} = x \times 0 = 0 \quad \text{I}$$

Consider $\min \{x \times z, x \times y\}$

$$= \min \{1.3 + 2.7i_F \times 0.7i_F, 1.3 + 2.7i_F \times 8.1\}$$
$$= \min \{0.91i_F + 7.01, 0.53 + 1.87i_F\}$$
$$= \{0.53 + 0.9i_F\} \quad \text{II}$$

I and II are not equal so in general $x \times \min \{y, z\} \neq \min \{x \times y, x \times z\}$ so the distributive law is not true so only we call $\{S, \times, \min\}$ as a pseudo semiring.

Thus in a pseudo semiring we may not have for every triple the distributive law to be true.

Example 3.18: Let $S = \{C([0, 40)), \times, \min\}$ be the pseudo semiring. S has zero divisors. Every element under min is an idempotnets some elements under $\times$ are also idempotents.

$V = \{[0, 40), \times, +\} \subseteq S$ is a subsemiring of infinite order.

S has also subsemirings which are of finite order.

For P = $\{Z_{40}, \times, \min\}$ is a subsemiring of finite order o(P) = 40.

$P_1 = \{\{0, 2, 4, \ldots, 38\}, \times, \min\}$ is a subsemiring of finite order.

$P_2 = \{\{0, 4, 8, 12, 16, 20, 24, 28, 32, 36\}, \times, \min\}$ is a subsemiring of finite order.

None of these finite subsemirings are ideals or filters of S. However we wish to make clear how to define filter or ideal.

Let S be a pseudo semiring.

$P \subseteq S$ be a proper pseudo subsemiring.

If $\min \{p, s\} \in P$ for all $p \in P$ and $s \in S$ we call this the pseudo semi ideal.

If for the same P we have $p \times s \in P$ for all $p \in P$ and $s \in S$ we call P the pseudo semi semifilter of S.

We will give examples of them. Only in case of pseudo semirings; we see a subsemiring can be both a pseudo ideal as well as a pseudo filter.

We will give examples of them.

Example 3.19: Let S = $\{C([0, 10)), \times, \min\}$ be the pseudo semiring of filter complex modulo integer interval.

P = $\{[0, 10), \times, \min\}$ is a pseudo subsemiring of S.

However P is not a pseudo filter as if $x = 2 + 3.5i_F \in S$ and $y = 2.1 \in P$ then $x \times y = 4.2 + 7.35i_F \notin P$.

Let $W = \{a + bi_F \mid a, b \in Z_{10}\} \subseteq S$. W is only a pseudo subsemiring and not an ideal or filter of S.

Example 3.20: Let S = {C([0,17)), $\times$, min} be the pseudo semiring of finite complex modulo integer interval semiring. This has infinite number of zero divisors and finite number of units.

We see if x = $8.5i_F$ and y = 2 $\in$ S we see x $\times$ y = 0 also min {2, $8.5i_F$} = 0. S has several subsemigroups of infinite order.

Example 3.21: Let S = {C([0, 24), min, $\times$, i_F^2 = 23} be the finite complex modulo integer interval semiring. S has several finite subsemiring.

P_1 = {{0, 12}, $\times$, min} is a subsemiring of S has an ideal.

P_2 = {{0, 4, 8, 12, 16, 20}, $\times$, min} is again a subsemiring of S.

P_3 = {{0, 8, 16}, $\times$, min} is a subsemiring of finite order.

P_4 = {{0, 2, 4, 6, 8, …, 22}, min, $\times$} is a subsemiring of finite order.

P_5 = {{0, 6, 12, 18}, min, $\times$} is a subsemiring of finite order.

None of these subsemirings are ideals.

M_1 = {[0, 24), min, $\times$} is a subsemiring of infinite order is a pseudo ideal and not a pseudo filter.

Study in this direction is innovative and interesting. Now using these pseudo semirings using S = {C([0, n)), $\times$, min}.

Example 3.22: Let
A = {(a_1, a_2, …, a_8) | a_i $\in$ C([0, 20)), 1 $\leq$ i $\leq$ 8, $\times$, min} be the complex finite modulo integer interval row matrix pseudo semiring.

Clearly o(A) = ∞ and A is commutative.

A has infinite number of zero divisors finite number of units (1, 1, 1, 1, 1, 1, 1, 1) is the identity element of A. A has only finite number of idempotents with respect to ×.

Infact every element is an idempotent with respect to min operation. A has finite pseudo subsemiring of finite order.

$P_1 = \{(a_1, 0, 0, 0, 0, 0, 0, 0) \mid a_1 \in C([0, 20)), \times, \min\} \subseteq S$ is a pseudo subsemiring of infinite order which is also an pseudo ideal. Infact P_1 is a pseudo filter of S.

Thus we see P_1 is both a pseudo ideal and a pseudo filter of S.

Consider $P_2 = \{(0, a_2, 0, \ldots, 0) \mid a_2 \in C([0, 20)), \times, \min\} \subseteq$ S is a subsemiring which is both an ideal and filter of S. Thus pseudo semirings happens to be a special type of semirings in which ideals can be filters and filters can be ideals.

$P_3 = \{(0, 0, a_3, 0, \ldots, 0) \mid a_3 \in C([0, 20)), \times, \min\}$ is a subsemiring as well as a pseudo ideal and pseudo filter of S.

Infact S has atleast ${}_8C_1 + {}_8C_2 + \ldots. + {}_8C_7$ number of pseudo subsemirings which are ideals and filters of S.

Example 3.23: Let

$$S = \left\{ \begin{bmatrix} a_1 \\ a_2 \\ a_3 \\ a_4 \\ a_5 \\ a_6 \\ a_7 \end{bmatrix} \middle| a_i \in C([0, 4)), 1 \leq i \leq 7, i_F^2 = 3, \min, \times_n \right\}$$

be the finite complex modulo integer interval column matrix pseudo semigroup, o(S) = ∞.

S has finite pseudo subsemiring. Infact even the finite pseudo semirings are such that elements of them do not in general satisfy the distributive laws.

Let

$$P_1 = \left\{ \begin{bmatrix} a_1 \\ a_2 \\ \vdots \\ a_7 \end{bmatrix} \middle| a_i \in Z_4, 1 \leq i \leq 7, \min, \times_n \right\}$$

be a pseudo subsemiring of finite order.

Consider min {3 × (2, 1)}

$$x \times \min \{y, z\} \neq \min \{x \times y, x \times z\}$$
$$= 3 \times \min \{2, 1\}$$
$$= 3 \times 1 = 3 \qquad \ldots \quad \text{I}$$

$$\min \{x \times y, x \times z\}$$
$$= \min \{2, 3\}$$
$$= 2 \qquad \ldots \quad \text{II}$$

I and II are not equal.

$$\text{Let } P_2 = \left\{ \begin{bmatrix} a_1 \\ 0 \\ 0 \\ \vdots \\ 0 \end{bmatrix} \middle| a_1 \in C(Z_4),\ i_F^2 = 3, \min, \times_n \right\}$$

be the pseudo finite subsemiring of S. P_2 is only a pseudo subsemiring and does not satisfy the distributive laws.

Infact we have atleast $3({}_7C_1 + {}_7C_2 + {}_7C_3 + \ldots + {}_3C_6 + 1)$ number of pseudo subsemiring of finite order. None of them are ideals or filters of S.

Now $x = 2$ $y = 0$ and $z = 2 \in 2Z_4 = \{0, 2\}$
$x \times \min \{y, z\}$
$= 2 \times \min \{0, 2\} = 0$ … I

$\min \{0, 0\} = 0$ … II

I and II are equal. Thus we see

$$B_1 = \left\{ \begin{bmatrix} a_1 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix} \middle| a_1 \in \{0, 2\} \subseteq Z_4, \min, \times_n \right\}$$

is a subsemiring which satisfies the distributive law.

Infact we have atleast $2({}_7C_1 + {}_7C_2 + \ldots + {}_7C_6 + 1)$ number of subsemirings which are finite in order and satisfy the distributive law.

Let $L = \{0, 1, 3\} \subseteq Z_4$ is a subsemiring under $\times$ min.

$x = 3$, $y = 1$ and $z = 0 \in L$. $x \times \min \{1, 0\}$
$= 0$ … I

$\min \{3 \times 1, 3 \times 0\} = 0$ … II

I and II are equal.
$0 \times \min \{1, 3\}1$
$= 0 \times 1 = 0$ … I

min {0 × 1, 0 × 3} = 0 … II

I and II are equal.

3 × min {1, 1} = 3 × 1 = 3 … I

min {3 × 1, 3 × 1} = 3 … II

I and II are equal.

1 × min {3, 0} = 0 … I

min {3, 0} = 0 … II

Hence we see this is also a distributive set.

Hence we are justified in saying we that atleast $2({}_7C_1 + {}_7C_2 + \ldots + {}_7C_6 + 1)$ number of finite subsemirings which satisfy the distributive law.

Let

$$M_1 = \left\{ \left[\begin{array}{c} a_1 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{array}\right] \middle| \, a_1 \in C([0, 4), \min, \times\} \right.$$

be the pseudo subsemiring of S which is an ideal as well as a filter of S.

$$M_2 = \left\{ \begin{bmatrix} 0 \\ a_2 \\ 0 \\ \vdots \\ 0 \end{bmatrix} \middle| a_1 \in C([0, 4), \min, \times) \right\}$$

be the pseudo subsemiring of S which is an ideal as well as a filter of S.

$$M_3 = \left\{ \begin{bmatrix} 0 \\ 0 \\ a_3 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix} \middle| a_3 \in C([0, 4), \min, \times) \right\}$$

be the pseudo subsemiring of S which is an ideal as well as a filter of S. We see like wise M_4, M_5, M_6 and M_7 all of them are pseudo ideals as well as filters of S.

$$M_{1,2} = \left\{ \begin{bmatrix} a_1 \\ a_2 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix} \middle| a_1\ a_2 \in C([0, 4), \min, \times) \right\} \subseteq S$$

is again a pseudo subsemiring which is also a pseudo ideal and pseudo filter of S. We have atleast ${}_7C_1 + {}_7C_2 + \ldots + {}_7C_6$ number of pseudo subsemirings which are also pseudo filters of S.

Example 3.24: Let

$$S = \left\{\begin{bmatrix} a_1 & a_2 & a_3 & a_4 \\ a_5 & a_6 & a_7 & a_8 \\ a_9 & a_{10} & a_{11} & a_{12} \\ a_{13} & a_{14} & a_{15} & a_{16} \end{bmatrix} \middle| \, a_i \in C([0, 6)); 1 \leq i \leq 6, \min, \times\right\}$$

be the complex finite modulo integer interval pseudo semiring. S has atleast 4 (${}_{16}C_1 + {}_{16}C_2 + \ldots + {}_{16}C_{15} + 1$) number of pseudo subsemirings of finite order none of which is an ideal or filter of S.

Further S has atleast ${}_{16}C_1 + {}_{16}C_2 + \ldots + {}_{16}C_{15}$ number of pseudo subsemirings which are both ideals and filters of S.

S has infinite number of zero divisors and every element is an idempotents.

We can have pseudo subsemirings of infinite order which are not filters or ideals.

For take

$$M_1 = \left\{\begin{bmatrix} a_1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \middle| \, a_1 \in C([0, 6)); a_2 \in Z_6, \min, \times\right\}$$

is a pseudo subsemiring which is not an ideal or filter of S. $o(M_1) = \infty$.

Infact we have several such pseudo subsemirings of infinite order which are neither ideals nor filters of S.

Example 3.25: Let

$$S = \left\{ \left. \begin{bmatrix} a_1 & a_2 & a_3 & a_4 \\ a_5 & a_6 & a_7 & a_8 \\ \vdots & \vdots & \vdots & \vdots \\ a_{37} & a_{38} & a_{39} & a_{40} \end{bmatrix} \right| a_i \in C([0, 19)); 1 \leq i \leq 40, \min, \times \right\}$$

be the finite complex modulo integer interval pseudo semiring.

S has several finite pseudo subsemirings and also several infinite pseudo subsemirings which are pseudo filters and pseudo ideals of S.

Example 3.26: Let V = $\{(a_1 \mid a_2\ a_3\ a_4\ a_5 \mid a_6\ a_7 \mid a_8\ a_9\ a_{10} \mid a_{11}) \mid a_i \in C([0, 24)), 1 \leq i \leq 11, \times, \min\}$ be the finite complex modulo integer interval super row matrix pseudo semiring. V has infinite number of zero divisors and only finite number of units.

V has pseudo subsemirings of finite and infinite order.

Example 3.27: Let

$$M = \left\{ \left. \begin{bmatrix} a_1 \\ \hline a_2 \\ a_3 \\ \hline a_4 \\ a_5 \\ a_6 \\ \hline a_7 \\ a_8 \\ a_9 \\ a_{10} \\ \hline a_{11} \end{bmatrix} \right| a_i \in C([0, 15)); 1 \leq i \leq 11,\ i_F^2 = 14, \min, \times \right\}$$

be the finite complex modulo integer interval super column matrix pseudo semiring.

M has infinite number of zero divisors. Every element in M is an idempotents.

However M has pseudo subsemiring of order two. Hence M has pseudo subsemiring of order three order 5 order 15, order 9, order 27, order 25 and so on.

$$B_1 = \left\{ \left[\begin{array}{c} a_1 \\ \hline 0 \\ 0 \\ \hline 0 \\ 0 \\ 0 \\ \hline 0 \\ 0 \\ 0 \\ \hline 0 \end{array} \right] \middle|\, a_i \in \{0, 5, 10\}, \min, \times \right\} \subseteq M$$

is a pseudo subsemiring of order three.

This is the smallest order subsemiring of M.

$$B_2 = \left\{\left.\left[\begin{array}{c} 0 \\ \hline 0 \\ 0 \\ \hline 0 \\ 0 \\ 0 \\ \hline 0 \\ 0 \\ 0 \\ 0 \\ \hline a_1 \end{array}\right]\right| a_1 \in \{0, 1, 14\}, \min, \times\right\}$$

is a pseudo subsemiring of order three.

$$\text{Let } B_3 = \left\{\left.\left[\begin{array}{c} a_1 \\ \hline 0 \\ 0 \\ \hline 0 \\ 0 \\ 0 \\ \hline 0 \\ 0 \\ 0 \\ 0 \\ \hline a_2 \end{array}\right]\right| a_1, a_2 \in \{0, 1, 14\}, \min, \times\right\}$$

is the pseudo subsemiring of order nine which is not an ideal of S.

$$B_4 = \left\{ \left[\begin{array}{c} 0 \\ \hline 0 \\ 0 \\ \hline 0 \\ 0 \\ 0 \\ \hline 0 \\ 0 \\ 0 \\ a_1 \\ \hline 0 \end{array} \right] \right| a_1 \in \{0, 3, 6, 9, 12\}, \min, \times\}$$

is the pseudo subsemiring of order 5.

$$\text{Let } V_1 = \left\{ \left[\begin{array}{c} a_1 \\ \hline 0 \\ 0 \\ \hline 0 \\ 0 \\ 0 \\ \hline 0 \\ 0 \\ 0 \\ 0 \\ \hline 0 \end{array} \right] \right| a_1 \in \{0, 10\}, \min, \times\} \subseteq S$$

is a pseudo subsemiring of order two. ($\times$ is the natural product $\times_n$).

$$V_2 = \left\{ \left[\begin{array}{c} 0 \\ \hline a_1 \\ a_2 \\ \hline 0 \\ 0 \\ 0 \\ \hline 0 \\ 0 \\ 0 \\ 0 \\ \hline 0 \end{array}\right] \middle| \; a_1\, a_2 \in \{0, 10\}, \min, \times \right\}$$

is a pseudo subsemiring of order two.

$$V_2 = \left\{ \left[\begin{array}{c} 0 \\ \hline 0 \\ 0 \\ \hline 0 \\ 0 \\ 0 \\ \hline 0 \\ 0 \\ 0 \\ 0 \\ \hline 0 \end{array}\right], \left[\begin{array}{c} 0 \\ \hline 10 \\ 0 \\ \hline 0 \\ 0 \\ 0 \\ \hline 0 \\ 0 \\ 0 \\ 0 \\ \hline 0 \end{array}\right], \left[\begin{array}{c} 0 \\ \hline 0 \\ 10 \\ \hline 0 \\ 0 \\ 0 \\ \hline 0 \\ 0 \\ 0 \\ 0 \\ \hline 0 \end{array}\right], \left[\begin{array}{c} 0 \\ \hline 10 \\ 10 \\ \hline 0 \\ 0 \\ 0 \\ \hline 0 \\ 0 \\ 0 \\ 0 \\ \hline 0 \end{array}\right] \right\} \text{ is of order 4.}$$

$$V_3 = \left\{ \left[\begin{array}{c} a_1 \\ \hline a_2 \\ a_3 \\ \hline 0 \\ 0 \\ 0 \\ \hline 0 \\ 0 \\ 0 \\ 0 \\ \hline 0 \end{array} \right] \middle| \, a_1 \in \{0, 10\}, a_2, a_3 \in \{0, 5, 10\}, \min, \times \right\}$$

be the pseudo subsemiring. $o(V_3) = 25$.

Thus we can have several pseudo subsemirings of finite order.

Throughout this chapter authors have used in most cases $\times$ to denote the natural product $\times_n$.

However this is not difficult as from the situation of the problem the reader can easily know which product is defined.

Example 3.28: Let

$$M = \left\{ \left[\begin{array}{ccc} a_1 & a_2 & a_3 \\ a_4 & a_5 & a_6 \\ \hline a_7 & a_8 & a_9 \\ \hline a_{10} & a_{11} & a_{12} \\ a_{13} & a_{14} & a_{15} \\ a_{16} & a_{17} & a_{18} \\ \hline a_{19} & a_{20} & a_{21} \\ a_{22} & a_{23} & a_{24} \\ a_{25} & a_{26} & a_{27} \\ a_{28} & a_{29} & a_{30} \\ \hline a_{31} & a_{32} & a_{33} \end{array}\right] \middle| \; a_i \in C([0, 12)); 1 \leq i \leq 33, \min, \times \right\}$$

be the finite complex modulo integer super column matrix pseudo semiring. M has finite subsemirings of order two, three and so on.

$$W_1 = \left\{ \left[\begin{array}{ccc} a_1 & 0 & 0 \\ 0 & 0 & 0 \\ \hline 0 & 0 & 0 \\ \hline 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ \hline 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ \hline 0 & 0 & 0 \end{array}\right] \middle| \; a_i \in \{0, 6\}, \min, \times \right\} \subseteq M$$

is a subsemiring of order two.

We can have order three four, six and so on, pseudo subsemirings in M.

M has ideals, ideals which are filters and subsemirings which are neither ideal nor filters.

Example 3.29: Let

$$W = \left\{ \left[\begin{array}{cc|ccc} a_1 & a_2 & a_3 & a_4 & a_5 \\ a_6 & \dots & \dots & \dots & a_{10} \\ a_{11} & \dots & \dots & \dots & a_{15} \\ \hline a_{16} & \dots & \dots & \dots & a_{20} \\ a_{21} & \dots & \dots & \dots & a_{25} \\ \hline a_{26} & \dots & \dots & \dots & a_{30} \\ \hline a_{31} & \dots & \dots & \dots & a_{35} \end{array} \right] \middle| \, a_i \in C([0, 8)), \min, \times \right\}$$

be the finite complex modulo integer interval super matrix pseudo semiring. W has finite subsemirings of order two, four three and so on.

We see W has atleast 35 subsemirings of order two 35 subsemiring of order three and so on.

Example 3.30: Let

$$S = \left\{ \left[\begin{array}{cc|c|cc} a_1 & a_2 & a_3 & a_4 & a_5 \\ a_6 & \dots & \dots & \dots & a_{10} \\ a_{11} & \dots & \dots & \dots & a_{15} \end{array} \right] \middle| \, a_i \in C([0, 3)), 1 \leq i \leq 15, \right.$$

$$\left. i_F^2 = 2, \min, \times \right\}$$

be the finite complex modulo integer interval pseudo semiring. S has pseudo subsemirings of order two and order three.

We see S has atleast 15 pseudo subsemirings of order two and 15 pseudo subsemirings of order 15 and so on.

Now we proceed onto build pseudo semirings using $C(\langle[0, n) \cup I\rangle) = \{a + bI + ci_F + dIi_F \mid a, b, c, d \in [0, n)\}$.

Example 3.31: Let
$S = C(\langle[0, 4 \cup I\rangle)\ i_F^2 = 3, I^2 = I, (i_FI) = 3I, \times, \min\}$ be the pseudo semiring of neutrosophic finite complex modulo integer intervals.

If $x = 3.2 + i_F + 0.3I + 2.1i_FI$
and $y = 0.8 + 0.9i_F + 2.2I + 0.5i_FI \in S$ then

$\min\{x, y\} = \min\{3.2 + i_F + 0.3I + 2.1i_FI, 0.8 + 0.9i_F + 2.2I + 0.5i_FI\}$

$= 0.8 + 0.9i_F + 0.3I + 0.5i_FI$

$= \min\{3.2, 0.8\} + \min\{i_F, 0.9i_F\} + \min\{0.3I, 2.2I\} + \min\{2.1i_FI, 0.5i_FI\}$ is in S.

Now $x \times y = (3.2 + i_F + 0.3I + 2.1i_FI) \times (0.8 + 0.9i_F + 2.2I + 0.5i_FI)$

$= 2.56 + 0.8i_F + 0.24I + 1.68i_FI + 2.88i_F + 0.9 \times (i_F^2 = 3) + 0.27i_FI + 1.89I \times 3 + 3.04I + 2.2Ii_F + 0.66I + 0.62i_FI + 1.6i_FI + 0.5 \times 3I + 0.15Ii_F + 1.05 \times 3I$

$= (2.56 + 2.7) + (0.8 + 2.88)\, i_F + (0.24 + 1.67 + 3.04 + 0.66 + 1.5 + 3.15)\, I + (1.68 + 0.27 + 2.2 + 0.62 + 1.6 + 0.15)Ii_F$

$= 1.26 + 2.96i_F + 2.26I + 2.52Ii_F \in S.$

This is the way the min and × operations are performed on S. S is a only pseudo subsemiring as the distributive law is not true.

For if $x = 3.1 + 0.5i_F$ and $y = 0.9I$ and $z = 2.7 + 0.5I \in S$

$x \times \min(y, z) \neq \min\{x \times y, x \times z\}$ for $x \times \min(0.9I, 2.7 + 0.5I)$

$= (3.1 + 0.5i_F) \times 0.5I$
$= 1.55I + 0.25i_FI$ I

Consider
$\min\{x \times y, x \times z\} = \{3.1 + 0.5i_F \times 0.9I, (2.7 + 0.5I) \times 0.9I\}$
$= \min(2.79I + 0.45i_FI + 1.35I + 0.45I)$
$= 0.45I$ II

I and II are distinct so $x \times \min(y, z) \neq \min(x \times y, x \times z\}$ in general for $x, y, z \in S$.

Thus S is only a pseudo semiring of infinite order.

Example 3.32: Let $S = C(\langle[0, 7) \cup I\rangle)$ $i_F^2 = 6$, $I^2 = I$, $(i_FI)^2 = 6I$, $\times$, min} be a neutrosophic finite complex modulo integer interval semiring.

S has zero divisors for if $x = 2$ and $y = 3.5I$ then $x \times y = 0$ and $\min\{x, y\} = 0$

However if $x = 2$ and $y = 3.5$ only $x \times y = 0$ and $\min\{x, y\} = 2 \neq 0$.

Thus $x \times y = 0$ does not imply $\min\{x, y\} = 0$

Further if $x = 3Ii_F$ and $y = 2I$ then $\min\{x, y\} = 0$ but
$x \times y = 3Ii_F \times 2I = 6IiF \neq 0$.

Thus S has zero divisors. Inview of this result the following theorem.

THEOREM 3.1: *Let $S = \{C([0, n), \min, \times\}$ (or $S = \{C(\langle[0, n) \cup I\rangle), \times, \min\}$ be the pseudo semiring of finite complex modulo* *integer interval or (neutrosophic finite complex modulo integer interval pseudo semiring).*

If x, y $\in$ S such that x $\times$ y = 0 then min {x, y} $\neq$ 0 in general and if min {x, y} = 0 then in general x $\times$ y $\neq$ 0.

However S also contains elements like min {x, y} = 0 and x $\times$ y = 0.

The proof is direct and hence left as an exercise to the reader.

Example 3.33: Let
A = {C(⟨[0, 20) ∪ I⟩) i_F^2 = 2, I^2 = I, $(i_F I)^2$ = 19I, ×, min} be the neutrosophic finite complex modulo integer interval pseudo semiring.

P_1 = {Z_{20}, ×, min} is a pseudo semiring.
For take x = 10, y = 0 and z = 11 ∈ P_1.

Is x × min {y, z} = min {xy, xz}
Consider x × min {y, z}
= 10 × min {9, 11}
= 10 × 9 = 10 … I

min {x × y, x × z} = min {10 × 9, 10 × 11}
= {10, 10}
= 10 … II

I and II are equal for this triple.

Let x = 3 y = 7 and z = 19 ∈ P_1.

x × min {y, z} = 3 × min {7, 19}
= 3 × 7
= 21 = 1 … I

$\min \{x \times y, x \times z\} = \min \{3 \times 7, 57\}$
$= \min \{1, 17\}$
$= 1$ … II

I and II are equal.

Let $x = 8$ and $y = 6$ and $z = 2 \in P_1$
$x \times \min \{y, z\} = x \times 2 = 16$ I

$\min \{x \times y, x \times z\} = \min \{48, 16\}$
$= \{8, 16\}$
$= 8$ … II

I and II are distinct so P_1 is only a pseudo subsemiring as the distributive law in general is not true.

Let $P_2 = \{C(Z_{20}), \times, \min\}$ be the pseudo subsemiring of finite order.

Let $P_3 = \{\langle Z_{20} \cup I\rangle, \times, \min\}$ be the pseudo subsemiring of finite order.

Let $P_4 = \{C(\langle Z_{20} \cup I\rangle), \times, \min\}$ be the pseudo subsemiring of finite order.

$P_5 = \{\{0, 5, 10, 15\} \min, \times\}$ is a pseudo subsemiring.

$P_6 = \{\{0, 10\}, \min, \times\}$ is a pseudo subsemiring of order two.

$P_7 = \{\{0, 4, 8, 12, 16\}, \times, \min\}$ is a pseudo subsemiring of order 5 and so on.

We can have several pseudo subsemirings of finite order.

$P_8 = \{0, 10I, 10, 10I + 10, \times, \min\}$ is also a pseudo subsemiring of finite order.

$M_1 = \{[0, 20), \times, \min\}$ is a pseudo subsemiring of infinite order. Clearly M_1 is not an ideal of A.

$M_2 = \{aI \mid a \in [0, 20), \min, \times\}$ is a pseudo subsemiring of infinite order in A.

Clearly M_2 is not an ideal of A.

$M_3 = \{C([0, 20)), \times, \min\}$ is a pseudo subsemiring of infinite order and is not an ideal of A.

Example 3.34: Let $M = \{C(\langle[0, 5), \cup I\rangle, \times, \min, i_F^2 = 4, I2 = I, (i_F I)^2 = 4I\}$ be the pseudo semiring of infinite order.

$P_1 = \{0, 5), \times, \min\}$is only a pseudo subsemiring of infinite order and is not an ideal of M.

$P_2 = \{C([0, 5)), \times, \min\}$ is only a pseudo subsemiring of infinite order and is not an ideal of M.

$P_3 = \{a + bI \mid a, b \in [0, 5), \times, \min\}$ is only a pseudo subsemiring and is not an ideal of M.

$P_4 = \{Z_5, \min, \times\}$ is only a pseudo subsemiring and not an ideal of order 5.

Infact M has infinite number of zero divisors. Only finite number of idempotents. We now proceed onto build pseudo semirings using $C(\langle[0, n), \cup I\rangle)$.

This is illustrated by the following examples.

Example 3.35: Let
$M = \{(a_1, a_2, \ldots, a_7)$ where $a_i \in C(\langle[0, 9) \cup I\rangle)$, $1 \leq i \leq 7$, $\times$, $\min\}$ be the pseudo row matrix semiring built in the neutrosophic finite complex modulo integer interval.

M has pseudo subsemirings of both finite and infinite order.

Infact $P_1 = \{(a_1, 0, 0, 0, 0, 0, 0) \mid a_1 \in C(\langle[0, 9), \cup I\rangle); \times, \min\} \subseteq M$ is a pseudo subsemiring which is also an ideal of M.

P_1 is also a filter.

$P_2 = \{(0, a_2, 0, \ldots, 0) \mid a_2 \in C(\langle[0, 9), \cup I\rangle); \times, \min\} \subseteq M$ is a pseudo subsemiring which is a filter as well as an ideal of M; and so on.

$P_7 = \{(0, 0, \ldots, a_7) \mid a_7 \in C(\langle[0, 9), \cup I\rangle, \times, \min\} \subseteq M$ is a pseudo subsemiring as well as a pseudo filter and pseudo ideal of M.

Infact M has atleast ${}_7C_1 + {}_7C_2 + \ldots + {}_7C_6$ number of pseudo subsemirings which are both pseudo ideals and pseudo filters of M.

M has infact atleast $6({}_7C_1 + {}_7C_2 + \ldots + {}_7C_6)$ number of pseudo subsemirings of finite order.

Example 3.36: Let

$$T = \left\{\left.\begin{bmatrix} a_1 \\ a_2 \\ a_3 \\ \vdots \\ a_{15} \end{bmatrix}\right| a_i \in C(\langle[0, 15), \cup I\rangle), 1 \leq i \leq 15, \min, \times\right\}$$

be the finite complex modulo integer neutrosophic interval column matrix pseudo semigroup.

$$P_1 = \left\{\left.\begin{bmatrix} a_1 \\ 0 \\ \vdots \\ 0 \end{bmatrix}\right| a_i \in C(\langle[0, 15), \cup I\rangle), \max, \times\right\} \subseteq T$$

is a pseudo subsemiring which is both a pseudo filter as well as pseudo ideal of T.

$$P_8 = \left\{\left.\begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ a_8 \\ 0 \\ \vdots \\ 0 \end{bmatrix}\right| a_i \in C(\langle[0, 15), \cup I\rangle), \min, \times\right\} \subseteq T$$

is again a pseudo subsemiring which is both a pseudo filter as well as pseudo ideal of T and so on we have

$$P_{15} = \left\{\left.\begin{bmatrix} 0 \\ 0 \\ \vdots \\ 0 \\ a_{15} \end{bmatrix}\right| a_i \in C(\langle[0, 15), \cup I\rangle), \min, \times\right\} \subseteq T$$

is a pseudo subsemiring as well as a pseudo ideal and pseudo filter of T.

$$P_{1,2} = \left\{ \begin{bmatrix} a_1 \\ a_2 \\ 0 \\ \vdots \\ 0 \end{bmatrix} \middle| a_1\ a_2 \in C(\langle[0, 15), \cup I\rangle), \min, \times \right\}$$

is a pseudo subsemiring as well as a pseudo ideal of pseudo filter of T.

$$P_{1,3} = \left\{ \begin{bmatrix} a_1 \\ 0 \\ a_3 \\ 0 \\ \vdots \\ 0 \end{bmatrix} \middle| a_1, a_3 \in C(\langle[0, 15), \cup I\rangle), \min, \times \right\}$$

is a pseudo subsemiring as well as a pseudo ideal and pseudo filter of T and so on.

$$P_{1,5} = \left\{ \begin{bmatrix} a_1 \\ 0 \\ 0 \\ \vdots \\ 0 \\ a_{15} \end{bmatrix} \middle| a_1, a_{15} \in C(\langle[0, 15), \cup I\rangle), \min, \times \right\}$$

is a pseudo subsemiring as well as a pseudo ideal pseudo filter.

$$P_{2,3} = \left\{ \left[\begin{array}{c} 0 \\ a_2 \\ a_3 \\ 0 \\ \vdots \\ 0 \end{array}\right] \middle| a_2, a_3 \in C(\langle [0, 15), \cup I \rangle), \min, \times \right\}$$

is a pseudo subsemiring as well as a pseudo ideal and pseudo filter and so on.

$$P_{7,10} = \left\{ \left[\begin{array}{c} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ a_7 \\ 0 \\ 0 \\ a_{10} \\ 0 \\ \vdots \\ 0 \end{array}\right] \middle| a_7, a_{10} \in C(\langle [0, 15), \cup I \rangle), \min, \times \right\}$$

is a pseudo subsemiring as well as an ideal and filter of T.

$$P_{14,15} = \left\{ \begin{bmatrix} 0 \\ 0 \\ \vdots \\ 0 \\ a_{14} \\ a_{15} \end{bmatrix} \middle| a_{14}, a_{15} \in C(\langle [0, 15), \cup I \rangle), \min, \times \right\}$$

is a pseudo subsemiring as well as a pseudo filter and pseudo ideal of T.

$$P_{1,2,3} = \left\{ \begin{bmatrix} a_1 \\ a_2 \\ a_3 \\ 0 \\ \vdots \\ 0 \end{bmatrix} \middle| a_1, a_2, a_3 \in C(\langle [0, 15), \cup I \rangle), \min, \times \right\}$$

be the pseudo subsemiring as well as pseudo filter and pseudo ideal of T.

$$P_{13,14,15} = \left\{ \begin{bmatrix} 0 \\ \vdots \\ 0 \\ a_{13} \\ a_{14} \\ a_{15} \end{bmatrix} \middle| a_{13}, a_{14}, a_{15} \in C(\langle [0, 15), \cup I \rangle), \min, \times \right\}$$

be the pseudo subsemiring as well as pseudo ideal and pseudo filter and of T.

Thus we have atleast ${}_{15}C_1 + {}_{15}C_2 + {}_{15}C_3 + \ldots + {}_{15}C_{14}$ number are pseudo filters and pseudo ideals of T.

Example 3.37: Let

$$M = \left\{ \begin{bmatrix} a_1 & a_2 & a_3 & a_4 \\ a_5 & \ldots & \ldots & a_8 \\ a_9 & \ldots & \ldots & a_{12} \\ a_{13} & \ldots & \ldots & a_{16} \\ a_{17} & \ldots & \ldots & a_{20} \\ a_{21} & \ldots & \ldots & a_{24} \end{bmatrix} \middle| \, a_i \in C(\langle [0, 15), \cup I \rangle), \right.$$

$$\left. 1 \leq i \leq 24, \min, \times \right\}$$

is a finite complex modulo integer neutrosophic interval pseudo semiring.

M has atleast ${}_{24}C_1 + {}_{24}C_2 + \ldots + {}_{24}C_{23}+1$ number of pseudo subsemirings of finite order none of which are ideals or filters of M.

Example 3.38: Let $S = \{(a_1, a_2, a_3, a_4 \mid a_5\ a_6 \mid a_7\ a_8\ a_9 \mid a_{10}) \mid a_i \in C(\langle [0, 3), \cup I \rangle), 1 \leq i \leq 10, i_F^2 = 2, I^2 = I, Ii_F = 2I, \min, \times\}$ be the finite complex modulo integer neutrosophic interval pseudo super row matrix semiring. S has infinite number of zero divisors.

S has atleast ${}_{10}C_1 + {}_{10}C_2 + \ldots + {}_{10}C_9$ number of subsemirings of infinite order which are both ideals and filters of S. S has atleast $4({}_{10}C_1 + {}_{10}C_2 + \ldots + {}_{10}C_9)$ number of finite pseudo subsemirings which are neither ideals nor filters of S.

S has only finite number of idempotents with respect to ×. Infact every element is an idempotent with respect to min operation.

Example 3.39: Let

$$T = \left\{ \left(\begin{array}{cccccc} a_1 & a_2 & a_3 & a_4 & a_5 & a_6 \\ a_7 & a_8 & a_9 & a_{10} & a_{11} & a_{12} \\ a_{13} & a_{14} & a_{15} & a_{16} & a_{17} & a_{18} \end{array} \right) \middle| \, a_i \in C(\langle [0, 17), \cup I \rangle), \right.$$

$$\left. 1 \leq i \leq 19, \min, \times \right\}$$

be the neutrosophic finite complex modulo integer interval pseudo semiring of super row matrix. T has infinite number of zero divisors with respect to × and min.

T has atleast ${}_{18}C_1 + {}_{18}C_2 + \ldots + {}_{18}C_{17}$ number of pseudo subsemiring which are both filters and ideals all of which are of infinite order. T has $4({}_{18}C_1 + {}_{18}C_2 + \ldots + {}_{18}C_{17})$ number of pseudo subsemiring of finite order none of them is an ideal or filter.

T has only finite number of idempotents with respect to ×.

Example 3.40: Let

$$M = \left\{ \left[\begin{array}{c} a_1 \\ a_2 \\ a_3 \\ \hline a_4 \\ \hline a_5 \\ a_6 \\ \hline a_7 \\ a_8 \\ a_9 \\ \hline a_{10} \\ a_{11} \\ a_{12} \\ a_{13} \end{array} \right] \middle| \, a_i \in C(\langle [0, 19), \cup I \rangle), 1 \leq i \leq 13, \max, \times \right\}$$

be the neutrosophic finite complex modulo integer interval super column matrix pseudo semiring. M has infinite number of pseudo subsemiring which are not filters or ideals.

M has atleast ${}_{13}C_1 + {}_{13}C_2 + \ldots + {}_{13}C_{12}$ number of pseudo subsemirings. M has atleast $3({}_{13}C_1 + {}_{13}C_2 + \ldots + {}_{13}C_{12})$ number of finite pseudo subsemiring which are not ideals or filters. M has infinite number of zero divisors and only finite number of units with respect to ×.

Example 3.41: Let

$$T = \left\{ \left[\begin{array}{cccc} a_1 & a_2 & a_3 & a_4 \\ a_5 & a_6 & a_7 & a_8 \\ \hline a_9 & \ldots & \ldots & a_{12} \\ a_{13} & \ldots & \ldots & a_{16} \\ \hline a_{17} & \ldots & \ldots & a_{20} \\ \hline a_{21} & \ldots & \ldots & a_{24} \\ a_{25} & \ldots & \ldots & a_{28} \\ a_{29} & \ldots & \ldots & a_{32} \\ a_{33} & \ldots & \ldots & a_{36} \\ \hline a_{37} & \ldots & \ldots & a_{40} \\ a_{41} & \ldots & \ldots & a_{44} \\ a_{45} & \ldots & \ldots & a_{48} \\ \hline a_{49} & \ldots & \ldots & a_{52} \\ a_{53} & \ldots & \ldots & a_{56} \end{array}\right] \right| a_i \in C(\langle[0, 5), \cup I\rangle), 1 \leq i \leq 56, \text{min}, \times \right\}$$

be the neutrosophic complex finite modulo integer interval super column matrix pseudo semiring.

T has infinite number of zero divisors.

Only finite number of units and idempotents with respect to $\times$.

Example 3.42: Let

$$W = \left\{ \left[\begin{array}{cc|c|cc} a_1 & a_2 & a_3 & a_4 & a_5 \\ a_6 & a_7 & a_8 & a_9 & a_{10} \\ \hline a_{11} & a_{12} & a_{13} & a_{14} & a_{15} \\ a_{16} & a_{17} & a_{18} & a_{19} & a_{20} \\ a_{21} & a_{22} & a_{23} & a_{24} & a_{25} \\ \hline a_{26} & a_{27} & a_{28} & a_{29} & a_{30} \\ a_{31} & a_{32} & a_{33} & a_{34} & a_{35} \\ \hline a_{36} & a_{37} & a_{38} & a_{39} & a_{40} \\ a_{41} & a_{42} & a_{43} & a_{44} & a_{45} \\ a_{46} & a_{47} & a_{48} & a_{49} & a_{50} \end{array} \right] \middle| \; a_i \in C(\langle [0, 12), \cup I \rangle), \; 1 \leq i \leq 50, \min, \times \right\}$$

be the neutrosophic finite complex modulo integer super matrix pseudo semiring. W has several pseudo subsemirings which are ideals and filters.

W has several pseudo subsemirings which are not ideals or filters.

Now we proceed onto describe pseudo rings built using the neutrosophic complex finite modulo integer interval.

Example 3.43: Let $M = \{C(\langle [0, 3), \cup I \rangle); +, \times\}$ be the neutrosophic modulo finite complex integer interval pseudo ring. Clearly M under the operation of '+' is an abelian group. M under the operation $\times$ is a semigroup.

However $a \times (b + c) \neq a \times b + a \times c$ for $a, b, c \in M$. Since the distributive law is not true in M we call M to be the pseudo ring.

Let a = 0.3 b = 2.1 and c = $0.7i_F \in M$.

$$a \times (b + c) = 0.3\times (2.1 + 0.7i_F)$$
$$= 0.63 + 0.21\, i_F \qquad \text{I}$$

Consider $a \times b + a \times c = 0.3 \times 2.1 + 0.3 \times 0.7i_F$
$$= 0.63 + 0.21i_F \qquad \text{II}$$

I and II are identical hence the distributive law is true for this triple.

Now take a = 0.8, b = 2.3 and c = $1.2 \in M$
$$a \times b + a \times c = 0.8 \times 2.3 + 0.8 \times 1.2$$
$$= 1.84 + 0.96$$
$$= 2.80 \qquad \text{I}$$

Now $a \times (b + c) = 0.8 \times (2.3 + 1.2)$
$$= 0.8\ (3.5)$$
$$= 0.8 \times 0.5$$
$$= 0.4 \qquad \text{II}$$

Clearly I and II are distinct for this triple so the distributive law is not true for this triple.

Let $x = 0.3 + 2I + 0.7i_F + 0.4i_FI$ and
$y = 2.1 + I + 0.2i_F + 0.6i_FI \in M$.

$$x + y = 0.3 + 2I + 0.7i_F + 0.4i_FI + 2.1 + I + 0.2i_F + 0.6i_FI$$
$$= 2.4 + 0 + 0.9i_F + Ii_F$$
$$= 1.5 + I + 1.1i_F + 1.6i_FI$$

$$x \times y = (0.3 + 2I + 0.7i_F + 0.4i_FI) \times (2.1 + I + 0.2i_F + 0.6i_FI)$$

$= 0.63 + 0.3I + 0.06i_F + 0.18i_FI + 1.2I + 2I + 0.4i_FI + 1.2i_FI + 1.47i_F + 0.7i_FI + 0.14 \times 2 + 0.42 \times 2I + 0.84Ii_F + 0.4Ii_F + 1.2i_FI + 1.47i_F + 0.7i_FI + 0.14 \times 2 + 0.42 \times 2I + 0.84Ii_F + 0.4Ii_F + 0.08 \times 2 \times I + 0.24 \times 2I$

= (0.63 + 0.28) + (0.3 + 1.2 + 2 + 0.84 + 0.16 + 0.48) I + (0.06 + 1.47)i_F + (0.18 + 0.4 + 1.2 + 0.7 + 0.84 + 0.4) Ii_F

= 0.91 + 1.98I + 1.53i_F + 0.72Ii_F

This is the way product is performed. It is easily verified that the distributive laws are not true in case of these rings.

Example 3.44: Let S = {C([0, 20), ×, +} be a pseudo ring of finite complex modulo integer interval. S has only finite number of zero and idempotents. S has only finite number of units. S has subrings which satisfy distributive law. S has subrings which do not satisfy distributive law that is pseudo subrings.

P = {[0, 20), ×, +} is a pseudo subring of infinite order which is not an ideal.

Example 3.45: Let B = {C([0, 41)), i_F^2 = 40, +, ×} is a finite complex modulo integer interval pseudo ring of infinite order. B has zero and divisors units. We define a pseudo ring. B to be a Smarandache special pseudo ring if B has a subring T which is not a pseudo ring.

We do not demand T to contain a field.

Study in this direction is interesting.

T = {Z_{40}, +, ×} is a subring.

P_1 = {2Z_{40}, ×, +} is also a subring which is not pseudo.

P_2 = {{10, 20, 30, 0}, ×, +} is again a subring of order four.

P_3 = {{5, 10, 15, …, 30, 35, 0}, ×, +} is again a subring of finite order.

Example 3.46: Let V = {C([0, 24)), ×, +, i_F^2 = 23} be a pseudo ring of finite complex modulo integer interval.

V has several finite subrings pseudo as well as non pseudo.

Example 3.47: Let S = $\{(a_1, a_2, a_3, a_4, a_5) \mid a_i \in C([0, 15)), 1 \leq i \leq 5, +, \times\}$ is a pseudo ring of finite complex modulo integer interval.

$P_1 = \{(a_1, 0, 0, 0, 0) \mid a_1 \in C([0, 5)), +, \times\} \subseteq S$,

$P_2 = \{(0, a_2, 0, 0, 0) \mid a_2 \in C([0,15)), +, \times\} \subseteq S$,

$P_3 = \{(0, 0, a_3, 0, 0) \mid a_3 \in C([0, 5)), +, \times\} \subseteq S$,

$P_5 = \{(0, 0, 0, 0, a_5) \mid a_5 \in C([0, 5)), +, \times\} \subseteq S$ and so on.

$P_{1,2,3} = \{(a_1, a_2, a_3, 0, 0) \mid a_1, a_2, a_3 \in C([0, 15)), +, \times\}$ is also pseudo subring as well as a pseudo ideal.

S has infinite number of zero divisors, only finite number of units and idempotents.

S has atleast ${}_5C_1 + {}_5C_2 + {}_5C_3 + {}_5C_4$ number of pseudo subrings which are also pseudo ideals of S.

S has atleast $4({}_5C_1 + {}_5C_2 + {}_5C_3 + {}_5C_4 + 1)$ number of subrings of finite order which are not pseudo.

Example 3.48: Let

$$B = \left\{ \begin{bmatrix} a_1 \\ a_2 \\ a_3 \\ \vdots \\ a_{12} \end{bmatrix} \middle| \; a_i \in C([0, 7)), 1 \leq i \leq 12, +, \times \right\}$$

be a finite complex modulo integer interval pseudo ring. B is of infinite order.

B has infinite number of zero divisors but only a finite number of units and idempotents.

B has atleast ${}_{12}C_1 + {}_{12}C_2 + \ldots + {}_{12}C_{11}$ number of infinite pseudo subrings which are pseudo ideals of B.

B has atleast 2 $({}_{12}C_1 + {}_{12}C_2 + \ldots + {}_{12}C_{11} + 1)$ number of subrings of finite order none of which are ideals of B.

Example 3.49: Let

$$S = \left\{\left.\begin{bmatrix} a_1 & a_2 & a_3 & a_4 \\ a_5 & \ldots & \ldots & a_8 \\ \vdots & \vdots & \vdots & \vdots \\ a_{37} & a_{38} & a_{39} & a_{40} \end{bmatrix}\right| a_i \in C([0, 19)), 1 \leq i \leq 40, +, \times\right\}$$

be a finite complex modulo integer interval pseudo ring.

L has atleast ${}_{40}C_1 + \ldots + {}_{40}C_{39}$ number of pseudo ideals and $2({}_{40}C_1 + \ldots + {}_{40}C_{39} + 1)$ number finite subrings which are not pseudo.

Example 3.50 Let

$$M = \{(a_1 \mid a_2\ a_3\ a_4\ a_5 \mid a_6\ a_7 \mid a_8\ a_9\ a_{10} \mid a_{11}) \mid a_i \in C([0, 14)), \quad 1 \leq i \leq 11, +, \times\}$$

be a finite complex modulo integer interval super row matrix pseudo ring.

M has infinite number of zero divisors. M has only finite number of units and idempotents.

M has both finite and infinite pseudo subrings.

Example 3.51 Let

$$L = \left\{ \left[\begin{array}{c} a_1 \\ \hline a_2 \\ \hline a_3 \\ a_4 \\ \hline a_5 \\ a_6 \\ a_7 \\ \hline a_8 \\ a_9 \\ a_{10} \\ a_{11} \\ \hline a_{12} \end{array} \right] \middle| \; a_i \in C([0, 23)), 1 \leq i \leq 12, +, \times \right\}$$

be the finite complex modulo integer interval super column matrix pseudo ring of infinite order.

L has atleast ${}_{12}C_1 + {}_{12}C_2 + \ldots + {}_{12}C_{11}$ number of pseudo subring of infinite order which are ideals of L.

L has atleast $2({}_{12}C_1 + {}_{12}C_2 + \ldots + {}_{12}C_{11} + 1)$ number of finite subrings which are not pseudo.

L has infinite number of zero divisors only a finite number of units and idempotents.

Example 3.52: Let

$$V = \left\{ \left[\begin{array}{cccc} a_1 & a_2 & a_3 & a_4 \\ \hline a_5 & a_6 & a_7 & a_8 \\ a_9 & \cdots & \cdots & a_{12} \\ a_{13} & \cdots & \cdots & a_{16} \\ a_{17} & \cdots & \cdots & a_{20} \\ \hline a_{21} & \cdots & \cdots & a_{24} \\ a_{25} & \cdots & \cdots & a_{28} \\ \hline a_{29} & \cdots & \cdots & a_{32} \\ a_{33} & \cdots & \cdots & a_{36} \\ a_{37} & \cdots & \cdots & a_{40} \\ \hline a_{41} & \cdots & \cdots & a_{44} \\ a_{45} & \cdots & \cdots & a_{48} \\ \hline a_{49} & \cdots & \cdots & a_{52} \end{array} \right] \middle| \; a_i \in C([0, 41)), 1 \leq i \leq 52, +, \times \right\}$$

be a super column matrix pseudo ring. V has subrings of finite order which are pseudo subring.

V has pseudo subrings of infinite order which are ideals.

Example 3.53: Let

$$B = \left\{ \left[\begin{array}{cc|ccc} a_1 & a_2 & a_3 & a_4 & a_5 \\ a_6 & \cdots & \cdots & \cdots & a_{10} \\ a_{11} & \cdots & \cdots & \cdots & a_{15} \\ \hline a_{16} & \cdots & \cdots & \cdots & a_{20} \\ a_{21} & \cdots & \cdots & \cdots & a_{25} \\ \hline a_{26} & \cdots & \cdots & \cdots & a_{30} \\ \hline a_{31} & \cdots & \cdots & \cdots & a_{35} \end{array} \right] \middle| \; a_i \in C([0, 23)), 1 \leq i \leq 35, +, \times \right\}$$

be the super matrix pseudo ring of finite complex modulo integers.

B has infinite number of zero divisors and finite number units and idempotents.

We can on similar lines build pseudo rings using neutrosophic finite complex modulo integers.

Example 3.54: Let V = $\{(a_1, a_2, a_3, \ldots, a_8) \mid a_i \in C(\langle[0, 23) \cup I\rangle), 1 \leq i \leq 8, +, \times\}$ be the neutrosophic complex modulo integer interval pseudo ring. V has infinite number of zero divisor.

V has atleast $4({}_8C_1 + {}_8C_2 + \ldots + {}_8C_7+1)$ number of finite subrings which are not pseudo.

V has atleast ${}_8C_1 + {}_8C_2 + \ldots + {}_8C_7$ number of pseudo subrings of infinite order which are pseudo ideals of V.

Example 3.55: Let

$$W = \left\{\begin{bmatrix} a_1 & a_2 \\ a_3 & a_4 \\ a_5 & a_6 \\ a_7 & a_8 \\ a_9 & a_{10} \\ a_{11} & a_{12} \\ a_{13} & a_{14} \\ a_{15} & a_{16} \end{bmatrix} \middle| \; a_i \in C([0, 13)), 1 \leq i \leq 16, +, \times\right\}$$

be the neutrosophic finite complex modulo integer matrix pseudo ring of infinite order.

W has infinite number of zero divisors and finite number of units and idempotents.

Example 3.56: Let

$$W = \left\{\begin{bmatrix} a_1 & a_2 & a_3 \\ a_4 & a_5 & a_6 \\ a_7 & a_8 & a_9 \end{bmatrix} \middle| \; a_i \in C([0, 21)), 1 \leq i \leq 9, +, \times\right\}$$

be the neutrosophic finite complex modulo integer interval matrix pseudo ring. W has ${}_9C_1 + {}_9C_2 + \ldots + {}_9C_8$ number of infinite pseudo subrings which are pseudo ideals.

W has $6({}_9C_1 + {}_9C_2 + \ldots + {}_9C_8+1)$ number finite subrings which are not pseudo. W has infinite number of zero divisor but only a finite number of units and idempotents.

Example 3.57: Let B = $(a_1\ a_2 \mid a_3 \mid a_4\ a_5\ a_6 \mid a_7\ a_8\ a_9\ a_{10} \mid a_{11}) \mid a_i \in C(\langle[0, 3) \cup I\rangle); 1 \leq i \leq 11, +, \times\}$ be the finite neutrosophic complex modulo integer interval super row matrix pseudo ring.

B has infinite number of zero divisors only finite number of idempotents and units.

$(1\ 1 \mid 1 \mid 1\ 1\ 1 \mid 1\ 1\ 1\ 1 \mid 1)$ is the unit in B. B has atleast ${}_{11}C_1 + {}_{11}C_2 + \ldots + {}_{11}C_{10}$ number of pseudo ideals.

B has atleast $4({}_{11}C_1 + {}_{11}C_2 + \ldots + {}_{11}C_{10} + 1)$ number of finite subrings which are not pseudo.

Let
$S = \{(a_1\ a_2 \mid 0 \mid 0\ 0\ 0 \mid 0\ 0\ 0\ 0 \mid a_3) \mid a_1\ a_2 \in [0, 3), a_3 \in \langle Z_3 \cup I\rangle\}$ be the infinite pseudo subring which is not a pseudo ideal of B.

S has several such pseudo subrings of infinite order which are not pseudo ideals. Infact B is a special Smarandache pseudo ring.

Example 3.58: Let

$$S = \left\{ \left[\begin{array}{c} a_1 \\ \hline a_2 \\ a_3 \\ a_4 \\ a_5 \\ \hline a_6 \\ a_7 \\ a_8 \\ \hline a_9 \\ a_{10} \\ \hline a_{11} \\ a_{12} \\ a_{13} \end{array} \right] \middle| \; a_i \in C(\langle [0, 2), \cup I \rangle), 1 \leq i \leq 13, +, \times \right\}$$

be the pseudo ring of super column matrices.

S has infinite number of zero divisors. Only finite number idempotents.

Further S has subrings of finite order say of order three, nine and so on. All pseudo ideals of S are of define order. None of the finite subrings are ideals.

S is a pseudo ring with multiplicative identity $I = \begin{bmatrix} \frac{1}{1} \\ 1 \\ 1 \\ \frac{1}{1} \\ 1 \\ \frac{1}{1} \\ \frac{1}{1} \\ 1 \\ 1 \end{bmatrix}$.

Take $x = \begin{bmatrix} \frac{i_F}{1} \\ 1 \\ 1 \\ \frac{1}{i_F} \\ 1 \\ \frac{i_F}{1} \\ \frac{1}{i_F} \\ 1 \\ 1 \end{bmatrix}$ and $y = \begin{bmatrix} \frac{i_F}{i_F} \\ 1 \\ 1 \\ \frac{i_F}{i_F} \\ 1 \\ \frac{i_F}{i_F} \\ \frac{1}{i_F} \\ 1 \\ i_F \end{bmatrix}$ $\in$ S are such that $x^2 = I$ and $y^2 = I$.

S has only units of this form.

$$a = \left[\begin{array}{c} 1 \\ \hline 0 \\ 1 \\ 1 \\ 1 \\ \hline 1 \\ 0 \\ 0 \\ \hline 0 \\ 0 \\ \hline 0 \\ 1 \\ 0 \end{array}\right] \in S \text{ such that } a^2 = a.$$

But S has only finite number of units and idempotents and they take entries from 1, i_F in case units and 1 and 0 in case of idempotents. Study in this direction is interesting.

Example 3.59: Let

$$V = \left\{\left[\begin{array}{cc|ccc} a_1 & a_2 & a_3 & a_4 & a_5 \\ a_6 & \dots & \dots & \dots & a_{10} \\ a_{11} & \dots & \dots & \dots & a_{15} \\ \hline a_{16} & \dots & \dots & \dots & a_{20} \\ a_{21} & \dots & \dots & \dots & a_{25} \\ \hline a_{26} & \dots & \dots & \dots & a_{30} \\ \hline a_{31} & \dots & \dots & \dots & a_{35} \end{array}\right] \right| a_i \in C(\langle[0, 5) \cup I\rangle), \quad 1 \leq i \leq 35, +, \times\}$$

be the finite neutrosophic complex modulo integer interval super matrix pseudo ring. V has finite number of idempotents and units but has infinite number of zero divisors. V has atleast $4({}_{35}C_1 + {}_{35}C_2 + \ldots + {}_{35}C_{34})$ number of finite subrings which are not pseudo.

Here we suggest a few problems some of which are at research level.

Problems

1. Obtain any special features enjoyed by the finite complex modulo integer interval semiring
   $S = \{C([0, n)),\ i_F^2 = n–1, \min, \max\}$.

2. Compare this S in problem 1 with $M = \{[0, n), \min, \max\}$.

3. Let $S = \{C([0, 10)),\ i_F^2 = 9, \min, \max\}$ be the finite complex modulo integer semiring.

   (i) Prove $o(S) = \infty$.
   (ii) Find ideals in S.
   (iii) Can S have filters?
   (iv) Prove S can have subsemirings of order 2, 3 and so on.

4. Let $R = \{C([0, 23)),\ i_F^2 = 22, \min, \max\}$ be the finite complex modulo integer interval semiring.

   Study questions (i) to (v) of problem 3 for this R.

5. Let $S = \{C([0, 24)),\ i_F^2 = 23, \max, \min\}$ be the semiring of finite complex modulo integer interval.

   Study questions (i) to (v) of problem 3 for this R.

6. Let $V = \{(a_1, a_2, \ldots, a_9) \mid a_i \in C([0, 4)), 1 \leq I \leq 9, \max, \min,$ $i_F^2 = 3\}$ be the complex finite modulo integer interval row matrix semiring.

Study questions (i) to (v) of problem 3 for this V.

7. Let

$$M = \left\{ \begin{bmatrix} a_1 \\ a_2 \\ \vdots \\ a_{18} \end{bmatrix} \middle| \; a_i \in C([0, 11)),\ 1 \leq i \leq 18,\ i_F^2 = 10, \min, \max \right\}$$

be the complex finite modulo integer interval column matrix semiring.

Study questions (i) to (v) of problem 3 for this M.

8. Let

$$T = \left\{ \begin{bmatrix} a_1 & a_2 & \dots & a_{12} \\ a_{13} & a_{14} & \dots & a_{24} \\ a_{25} & a_{26} & \dots & a_{36} \\ a_{37} & a_{37} & \dots & a_{48} \\ a_{49} & a_{50} & \dots & a_{60} \end{bmatrix} \middle| \; a_i \in C([0, 15)),\ 1 \leq i \leq 60, \right.$$

$i_F^2 = 14$, min, max}
be the finite complex modulo integer interval super row matrix semiring.

Study questions (i) to (v) of problem 3 for this T.

9. Let V = $\{(a_1\ a_2 \mid a_3\ a_4 \mid a_5\ a_6\ a_7 \mid a_8) \mid a_i \in C([0, 7)),\ 1 \leq I \leq 8,$ $i_F^2 = 6$, max, min} be the finite complex modulo integer interval super row matrix semiring.

Study questions (i) to (v) of problem 4 for this V.

10. Let

$$M = \left\{ \left(\begin{array}{cc|ccc|c|cccc} a_1 & a_2 & a_3 & a_4 & a_5 & a_6 & a_7 & a_8 & a_9 & a_{10} \\ a_{11} & \ldots & \ldots & \ldots & \ldots & \ldots & \ldots & \ldots & \ldots & a_{20} \\ a_{21} & \ldots & \ldots & \ldots & \ldots & \ldots & \ldots & \ldots & \ldots & a_{30} \end{array} \right) \right| a_i \in$$

C([0, 3)), $1 \leq i \leq 30$, min, max}
be the finite complex modulo integer interval super row matrix semiring.

Study questions (i) to (v) of problem 3 for this M.

11. Let

$$T = \left\{ \left[\begin{array}{c} a_1 \\ a_2 \\ \hline a_3 \\ a_4 \\ \hline a_5 \\ a_6 \\ \hline a_7 \\ a_8 \\ \hline a_9 \\ a_{10} \\ \hline a_{11} \\ a_{12} \end{array} \right] \right| a_i \in C([0, 24)), 1 \leq i \leq 12, \min, \max\}$$

be the finite complex modulo integer interval super column matrix semiring.

Study questions (i) to (v) of problem 3 for this T.

12. Let
$M = \{C(\langle[0, 20), \cup I\rangle), i_F^2 = 19, (i_F I)^2 = 19I, I^2 = I, \min, \max\}$ be the finite neutrosophic complex modulo integer interval semiring.

    (i) Prove $o(M) = \infty$
    (ii) Find all ideal of M.
    (iii) Can ideal of M be a filter?
    (iv) Find all filters of M.
    (v) Can ideals of M be of finite order?
    (vi) Can filters of M be a finite order?
    (vii) Show M has subsemiring of orders 2, 3, 4, …, n (n any finite integer)
    (viii) Can M have zero divisors?
    (ix) Find infinite subsemiring of M which are neither filter nor an ideal of M.

13. Let $M = \{C(\langle[0, 24), \cup I\rangle), i_F^2 = 23, (i_F I)^2 = 23I, I^2 = I, \min, \max\}$ be the finite neutrosophic complex modulo integer interval semiring.

    Study questions (i) to (ix) of problem 12 for this M.

14. Let
$T = \{C(\langle[0, 15), \cup I\rangle), i_F^2 = 14, (i_F I)^2 = 14I, I^2 = I, \min, \max\}$ be the finite neutrosophic complex modulo integer interval semiring.

    Study questions (i) to (ix) of problem 12 for this T.

15. Let $S = \{(a_1, a_2, a_3, \ldots, a_{15}) \mid a_i \in \{C(\langle[0, 29), \cup I\rangle), i_F^2 = 28, (i_F I)^2 = 28I, I^2 = I, \min, \max\}$ be the finite neutrosophic complex modulo integer interval semiring.

    Study questions (i) to (ix) of problem 12 for this S.

16. Let

$$B = \left\{ \begin{bmatrix} a_1 & a_2 & a_3 & a_4 & a_5 \\ a_6 & \ldots & \ldots & \ldots & a_{10} \\ a_{11} & \ldots & \ldots & \ldots & a_{15} \\ a_{16} & \ldots & \ldots & \ldots & a_{20} \\ a_{21} & \ldots & \ldots & \ldots & a_{25} \\ a_{26} & \ldots & \ldots & \ldots & a_{30} \end{bmatrix} \right| a_i \in \{C(\langle[0, 29) \cup I\rangle),$$

$i_F^2 = 29$, $(i_F I)^2 = 29I$, $I^2 = I$, min, max}
be the neutrosophic finite complex modulo integer interval semiring.

Study questions (i) to (ix) of problem 12 for this B.

17. Let

$$M = \left\{ \begin{bmatrix} a_1 & a_2 & \ldots & a_9 \\ a_{10} & a_{11} & \ldots & a_{18} \\ a_{19} & a_{20} & \ldots & a_{27} \end{bmatrix} \right| a_i \in \{C(\langle[0, 7) \cup I\rangle), 1 \leq i \leq 27,$$

min, max}
be the neutrosophic finite complex modulo integer interval semiring.

Study questions (i) to (ix) of problem 12 for this M.

18. Let $W = \{(a_1\ a_2 \mid a_3\ a_4 \mid a_5\ a_6\ a_7 \mid a_8) \mid a_i \in \{C(\langle[0, 10) \cup I\rangle)$, $1 \leq i \leq 8$, min, max} be the neutrosophic complex modulo integer interval super row matrix semiring.

Study questions (i) to (ix) of problem 12 for this W.

19. Let

$$S = \left\{ \left[\begin{array}{c} a_1 \\ \hline a_2 \\ a_3 \\ \hline a_4 \\ a_5 \\ \hline a_6 \\ a_7 \\ a_8 \end{array} \right] \middle| \; a_i \in \{C(\langle [0, 11) \cup I \rangle), 1 \leq i \leq 8, \min, \max\} \right.$$

be the neutrosophic complex modulo integer interval super row matrix semiring.

Study questions (i) to (ix) of problem 12 for this S.

20. Let

$$S = \left\{ \left[\begin{array}{cc|ccc|c} a_1 & a_2 & a_3 & a_4 & a_5 & a_6 \\ a_7 & \cdots & \cdots & \cdots & \cdots & a_{12} \\ \hline a_{13} & \cdots & \cdots & \cdots & \cdots & a_{18} \\ a_{19} & \cdots & \cdots & \cdots & \cdots & a_{24} \\ \hline a_{25} & \cdots & \cdots & \cdots & \cdots & a_{30} \end{array} \right] \middle| \; a_i \in \{C(\langle [0, 23) \cup I \rangle), \right.$$

$1 \leq i \leq 30$, min, max} be the neutrosophic complex modulo integer interval super row matrix semiring.

Study questions (i) to (ix) of problem 12 for this W.

21. Let S = {C([0, 14)), min, ×} be the finite complex modulo integer interval pseudo semiring.

(i) Find o(S).
(ii) Can S have finite ideals?
(iii) Can S have finite filters?

(iv) Can a subsemiring be both a filter and ideals?
(v) Prove S has zero divisors.
(vi) Prove S can have only finite number of idempotents with respect t to $\times$.
(vii) Prove S has have only finite number of idempotents with respect to $\times$.
(viii) Prove S have finite pseudo subsemirings.

22. Let T = {C([0, 23)), min, $\times$} be a finite complex modulo integer integer pseudo semiring.

    Study questions (i) to (viii) of problem 21 for this T.

23. Let B = {C([0, 42)), min, $\times$} be a finite complex modulo integer integer pseudo semiring.

    Study questions (i) to (viii) of problem 21 for this B.

24. Let L = {C([0, 251)), min, $\times$} be a finite complex modulo integer integer pseudo semiring.

    Study questions (i) to (viii) of problem 21 for this L.

25. Let M = {(a_1, a_2, …, a_8) | C([0, 43)), $1 \leq i \leq 8$, min, $\times$} be a finite complex modulo integer integer pseudo semiring.

    Study questions (i) to (viii) of problem 21 for this M.

26. Let

$$S = \left\{ \begin{bmatrix} a_1 \\ a_2 \\ \vdots \\ a_{18} \end{bmatrix} \middle| \; C([0, 49)), 1 \leq i \leq 18, \min, \times \right\}$$

    be a finite complex modulo integer integer pseudo semiring. Study questions (i) to (viii) of problem 21 for this S.

27. Let

$$S = \left\{\left[\begin{array}{cccc} a_1 & a_2 & \ldots & a_{10} \\ a_{11} & a_{12} & \ldots & a_{20} \\ a_{21} & a_{22} & \ldots & a_{30} \\ a_{31} & a_{32} & \ldots & a_{40} \\ a_{41} & a_{42} & \ldots & a_{50} \end{array}\right] \right| C([0, 5)), 1 \leq i \leq 50, \min, \times\}$$

be a finite complex modulo integer integer pseudo semiring.

Study questions (i) to (viii) of problem 21 for this V.

28. Let M = $\{(a_1\ a_2 \mid a_3 \mid a_4\ a_5 \mid a_6\ a_7\ a_8 \mid a_9) \mid C([0, 7)), 1 \leq i \leq 9, \min, \times\}$ be a finite complex modulo integer integer pseudo semiring.

Study questions (i) to (viii) of problem 21 for this M.

29. Let

$$M = \left\{\left[\begin{array}{c} a_1 \\ \hline a_2 \\ \hline a_3 \\ a_4 \\ \hline a_5 \\ a_6 \\ a_7 \\ a_8 \\ \hline a_9 \\ a_{10} \\ a_{11} \\ \hline a_{12} \end{array}\right] \right| a_i \in C([0, 41)), 1 \leq i \leq 12, \min, \times\}$$

be a finite complex modulo integer interval super column matrix pseudo semiring.

Study questions (i) to (viii) of problem 21 for this M.

30. Let

$$P = \left\{ \left[\begin{array}{cc|ccc|cc} a_1 & a_2 & a_3 & a_4 & a_5 & a_6 & a_7 \\ a_8 & \cdots & \cdots & \cdots & \cdots & \cdots & a_{14} \\ a_{15} & \cdots & \cdots & \cdots & \cdots & \cdots & a_{21} \\ \hline a_{22} & \cdots & \cdots & \cdots & \cdots & \cdots & a_{28} \\ a_{29} & \cdots & \cdots & \cdots & \cdots & \cdots & a_{35} \\ \hline a_{36} & \cdots & \cdots & \cdots & \cdots & \cdots & a_{42} \\ a_{43} & \cdots & \cdots & \cdots & \cdots & \cdots & a_{49} \\ a_{50} & \cdots & \cdots & \cdots & \cdots & \cdots & a_{56} \\ \hline a_{57} & \cdots & \cdots & \cdots & \cdots & \cdots & a_{63} \end{array} \right] \right| a_i \in C([0, 41)),$$

$1 \leq i \leq 63$, min, $\times\}$ be a finite complex modulo integer interval super column matrix pseudo semiring.

Study questions (i) to (viii) of problem 21 for this M.

31. Let S = $C(\langle[0, 11) \cup I\rangle)$, min, $\times\}$ be the neutrosophic finite complex modulo integer interval pseudo semiring.

(i) Show S has finite number of zero divisor with respect to $\times$.
(ii) Can S have finite pseudo ideals which are not filters?
(iii) Can S have finite pseudo filters which are pseudo ideals?
(iv) Can S have finite pseudo subsemiring which is both an ideal and filter of S.
(v) Is every pseudo ideal has infinite number of elements in it?
(vi) Is every pseudo filter is of infinite order?
(vii) Obtain any other special feature enjoyed by S.

32. Let $S_1 = C(\langle[0, 24) \cup I\rangle)$, min, $\times\}$ be the neutrosophic finite complex modulo integer interval pseudo semiring.

    Study questions (i) to (vii) of problem 31 for this S_1.

33. Let $W = C(\langle[0, 34) \cup I\rangle)$, min, $\times\}$ be the neutrosophic finite complex modulo integer interval pseudo semiring.

    Study questions (i) to (vii) of problem 31 for this W.

34. Let $V = \{(a_1, a_2, a_3, a_4, a_5, a_6) \mid a_i \in C(\langle[0, 27) \cup I\rangle), 1 \leq i \leq 6$, min, $\times\}$ be the neutrosophic finite complex modulo integer interval pseudo semiring.

    Study questions (i) to (vii) of problem 31 for this W.

35. Let

$$P = \left\{ \left. \begin{bmatrix} a_1 \\ a_2 \\ \vdots \\ a_{18} \end{bmatrix} \right| a_i \in C([0, 18)), 1 \leq i \leq 18, \text{min}, \times \right\}$$

    be a finite complex modulo integer interval pseudo semiring.

    Study questions (i) to (vii) of problem 31 for this P.

36. Let

$$W = \left\{ \left. \left[\begin{array}{cc|cc} a_1 & a_2 & a_3 & a_4 \\ a_5 & a_6 & a_7 & a_8 \\ \hline a_9 & a_{10} & a_{11} & a_{12} \\ a_{13} & a_{14} & a_{15} & a_{16} \\ a_{17} & a_{18} & a_{19} & a_{20} \end{array}\right] \right| a_i \in C(\langle[0, 24) \cup I\rangle), \right.$$

$1 \leq i \leq 20$, min, $\times\}$ be a finite complex modulo integer interval pseudo semiring.

Study questions (i) to (vii) of problem 31 for this W.

37. Let T = {C([0, 47)), $\times$, +} be the finite complex modulo integer pseudo ring.

    (i) Find all finite pseudo subrings of T.
    (ii) Can T have zero divisors?
    (iii) Can T have idempotents.
    (iv) Can T have units?
    (v) Can T have pseudo ideals of finite order.
    (vi) Can T have infinite order pseudo subrings which are not ideals?

    (vii) Obtain any other special property associated with T.
    (viii) Find all finite subrings of T which are pseudo subsemiring.

38. Let B = {C($\langle [0, 27) \cup I \rangle$), $\times$, +} be the neutrosophic finite complex modulo integer interval pseudo ring.

    (i) Compare problem 37 of T with this B.

    (ii) Study questions (i) to (viii) of problem 37 for this B.

39. Let S = {C([0, 17)), +, $\times$} be the finite complex modulo integer interval pseudo ring.

    Study questions (i) to (viii) of problem 37 for this S.

40. Let M = $\{(a_1, a_2, a_3, \ldots, a_{19}) \mid a_i \in C([0, 3)), 1 \leq i \leq 10, +, \times\}$ be the finite complex modulo integer interval pseudo row matrix ring.

    Study questions (i) to (viii) of problem 37 for this M.

41. Let

$$L = \left\{ \begin{bmatrix} a_1 & a_2 & \ldots & a_9 \\ a_{10} & a_{11} & \ldots & a_{18} \\ a_{19} & a_{20} & \ldots & a_{27} \\ a_{28} & a_{29} & \ldots & a_{36} \\ a_{37} & a_{38} & \ldots & a_{45} \\ a_{46} & a_{47} & \ldots & a_{54} \\ a_{55} & a_{56} & \ldots & a_{60} \end{bmatrix} \middle| \; a_i \in C([0, 11), 1 \leq i \leq 60+, \times \right\}$$

be the finite complex modulo integer interval pseudo row matrix ring.

Study questions (i) to (viii) of problem 37 for this L.

42. Let

$$W = \left\{ \begin{bmatrix} a_1 \\ a_2 \\ \vdots \\ a_{36} \end{bmatrix} \middle| \; a_i \in C([0, 4)), 1 \leq i \leq 36, +, \times_n \right\}$$

be a finite complex modulo integer interval pseudo semiring.

Study questions (i) to (viii) of problem 37 for this W.

43. Let B = $\{(a_1 \mid a_2\ a_3\ a_4 \mid a_5\ a_6 \mid a_7\ a_8\ a_9) \mid a_i \in C([0, 7)), 1 \leq i \leq 9, +, \times_n\}$ be a finite complex modulo integer interval pseudo semiring.

Study questions (i) to (viii) of problem 37 for this B.

44. Let

$$M = \left\{ \left[\begin{array}{c} a_1 \\ \hline a_2 \\ a_3 \\ \hline a_4 \\ a_5 \\ \hline a_6 \\ a_7 \\ a_8 \\ \hline a_9 \end{array} \right] \middle| \; a_i \in C([0, 11)), 1 \le i \le 9, +, \times_n \right\}$$

be a finite complex modulo integer interval pseudo semiring.

Study questions (i) to (viii) of problem 37 for this M.

45. Let W = $\{(a_1, a_2, \ldots, a_{19}) \mid a_i \in C(\langle [0, 12) \cup I \rangle), 1 \le i \le 19, +, \times\}$ be a finite complex modulo integer interval pseudo semiring.

Study questions (i) to (viii) of problem 37 for this W.

46. Let

$$T = \left\{ \begin{bmatrix} a_1 & a_2 & \ldots & a_8 \\ a_9 & a_{10} & \ldots & a_{16} \\ a_{17} & a_{18} & \ldots & a_{24} \end{bmatrix} \middle| \; a_i \in C([0, 7) \cup I\rangle), 1 \le i \le 24, +, \right.$$

$\times\}$ be the finite complex modulo integer interval pseudo row matrix ring.

Study questions (i) to (viii) of problem 37 for this T.

47. Let

$$W = \left\{ \left[\begin{array}{cc|ccc|cc} a_1 & a_2 & a_3 & a_4 & a_5 & a_6 & a_7 \\ a_8 & \cdots & \cdots & \cdots & \cdots & \cdots & a_{14} \\ a_{15} & \cdots & \cdots & \cdots & \cdots & \cdots & a_{21} \\ \hline a_{22} & \cdots & \cdots & \cdots & \cdots & \cdots & a_{28} \\ \hline a_{29} & \cdots & \cdots & \cdots & \cdots & \cdots & a_{35} \\ a_{36} & \cdots & \cdots & \cdots & \cdots & \cdots & a_{42} \end{array} \right] \right| a_i \in C([0, 5)$$

$\cup$ I⟩), $1 \leq i \leq 42$, +, $\times_n$} be the finite complex modulo integer interval pseudo super matrix ring.

Study questions (i) to (viii) of problem 37 for this W.

48. Let

$$P = \left\{ \left[\begin{array}{c|ccc|c|cc} a_1 & a_2 & a_3 & a_4 & a_5 & a_6 & a_7 \\ a_8 & \cdots & \cdots & \cdots & \cdots & \cdots & a_{14} \\ a_{15} & \cdots & \cdots & \cdots & \cdots & \cdots & a_{21} \end{array} \right] \right| a_i \in C([0, 24) \cup$$

I⟩), $1 \leq i \leq 21$, +, $\times_n$} be the finite complex modulo integer interval pseudo super matrix ring.

Study questions (i) to (viii) of problem 37 for this P.
If in P; C(⟨[0, 24) $\cup$ I⟩) replaced by C([0, 24) study and compare them.

49. Let

$$S = \left\{ \left[\begin{array}{ccc} a_1 & a_2 & a_3 \\ a_4 & a_5 & a_6 \\ \vdots & \vdots & \vdots \\ a_{31} & a_{32} & a_{33} \end{array} \right] \right| a_i \in C([0, 11) \cup I\rangle), 1 \leq i \leq 11, +,$$

$\times_n$} be the finite complex modulo integer interval pseudo super matrix ring.

Study questions (i) to (viii) of problem 37 for this S.

Chapter Four

PSEUDO VECTOR SPACES OVER C([0, n))

In this chapter authors for the first time define, develop and describe several types of pseudo vector spaces and pseudo neutrosophic vector spaces over C([0, n)) and C(⟨[0, n) ∪ I⟩) respectively. They are illustrated by examples.

$C([0, n)) = \{a + bi_F \mid a, b \in [0, n),\ i_F^2 = n-1\}$ and $C(\langle[0, n) \cup I\rangle) = \{a + bi_F + cI + dIi_F \mid a, b, c, d \in [0, n),\ i_F^2 = n-1,\ I^2 = I,\ (Ii_F)^2 = (n-1)I$.

DEFINITION 4.1: *Let V = {C([0, p)), +} be a group under +. V is a vector space over Z_p where p is a prime defined as the complex finite modulo integer interval vector space over Z_p.*

Example 4.1: Let S = {C([0, 7)), +} be a complex finite modulo integer interval vector space over Z_7.

Example 4.2: Let V = {C([0, 19), +} be the complex finite modulo integer interval vector space over the field Z_{19}.

Example 4.3: Let V = {C([0, 23), +} be the finite complex modulo integer interval vector space over the field Z_{23}.

We see only pseudo linear finite complex modulo integer interval algebra can be defined, as the × and + do not satisfy the distributive law in general.

We will first illustrate this situation by some example.

Example 4.4: Let V = {C([0, 29), +, ×} be the finite complex modulo integer interval pseudo linear algebra over the field Z_{29}.

Example 4.5: Let V = {C([0, 2), +, ×} be the finite complex modulo integer interval pseudo linear algebra over Z_2.

All these spaces and linear algebras are infinite dimensional.

Now using the set C([0, n)) we can build matrix vector spaces.

Example 4.6: Let M = $\{(a_1, a_2, a_3) \mid a_i \in ([0, 7)), 1 \leq i \leq 3, +\}$ be a finite complex modulo integer interval vector space over Z_7.

M is infinite dimensional over Z_7.

$P_1 = \{(a_1, 0, 0) \mid a_1 \in C([0, 7), +\} \subseteq M$,

$P_2 = \{(0, a_2, 0, 0) \mid a_2 \in C([0, 7), +\} \subseteq M$ and

$P_3 = \{(0, 0, a_3) \mid a_3 \in C([0, 7), +\} \subseteq M$.

We see P_1, P_2, P_3 are subspaces of M of infinite order. $M = P_1 + P_2 + P_3$ and $P_i \cap P_j = \{(0, 0, 0)\}$ $i \neq j$, $1 \leq i, j \leq 3$.

M is infinite dimensional vector spaces and M is the direct sum of P_1, P_2 and P_3.

$L_1 = \{(a_1, 0, 0) \mid a_1 \in Z_7, +\}$ is a finite dimensional subspace of M over Z_7.

M has atleast 2 $({}_3C_1 + {}_3C_2 + 1)$ number of finite dimensional subspaces.

Example 4.7: Let

$$P = \left\{ \begin{bmatrix} a_1 & a_2 & a_3 \\ a_4 & a_5 & a_6 \\ a_7 & a_8 & a_9 \end{bmatrix} \middle| a_i \in ([0, 11)), 1 \leq i \leq 9, + \right\}$$

be the vector space over Z_{11}.

$$K_1 = \left\{ \begin{bmatrix} a_1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \middle| a_1 \in ([0, 11)), + \right\} \subseteq P,$$

$$K_2 = \left\{ \begin{bmatrix} 0 & a_2 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \middle| a_2 \in ([0, 11)), + \right\} \subseteq P \text{ and so on.}$$

$$K_9 = \left\{ \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & a_9 \end{bmatrix} \middle| a_9 \in ([0, 11)), + \right\} \subseteq P$$

are all subspaces of P and are of infinite dimension over Z_{11}.

$$P = K_1 + K_2 + \ldots + K_9 \text{ and}$$

$$K_i \cap K_j = \left\{\begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}\right\}, i \neq j \; 1 \leq i, j \leq 9.$$

P has atleast ${}_9C_1 + {}_9C_2 + \ldots + {}_9C_8$ number of subspaces of infinite dimension over Z_{11} and $2({}_9C_1 + {}_9C_2 + \ldots + {}_9C_8)$ number of subspaces of infinite dimension over Z_{11} and $2\; {}_9C_1 + {}_9C_2 + \ldots + {}_9C_8 + 1)$ number of subspaces of finite dimension over Z_{11}.

Example 4.8: Let

$$T = \left\{\begin{bmatrix} a_1 \\ a_2 \\ a_3 \\ \vdots \\ a_{10} \end{bmatrix} \middle| \; a_i \in C([0, 13)), 1 \leq i \leq 10, +\right\}$$

be the vector space of finite complex modulo integer interval over the field Z_{13}.

T has subspaces of both finite dimension as well as of infinite dimension.

$$W_{1,2} = \left\{\begin{bmatrix} a_1 \\ a_2 \\ 0 \\ \vdots \\ 0 \end{bmatrix} \middle| \; a_1, a_2 \in C([0, 13)), +\right\}$$

is a subspace of T over Z_{13} of infinite dimension.

$$M_{3,7} = \left\{ \left[\begin{array}{c} 0 \\ 0 \\ a_3 \\ 0 \\ 0 \\ 0 \\ a_7 \\ 0 \\ 0 \\ 0 \end{array} \right] \middle| \; a_3, a_7 \in Z_{13}, +\right\}$$

is a subspace of T of finite dimension over Z_{13}. We have atleast ${}_{10}C_1 + {}_{10}C_2 + \ldots + {}_{10}C_9$ number of subspaces of infinite dimension over Z_{13}.

T has atleast $10({}_{10}C_1 + {}_{10}C_2 + \ldots + {}_{10}C_9 + 1)$ number of subspaces of finite dimension over Z_{13}.

Example 4.9: Let

$$W = \left\{ \left[\begin{array}{cccc} a_1 & a_2 & a_3 & a_4 \\ a_5 & a_6 & a_7 & a_8 \\ \vdots & \vdots & \vdots & \vdots \\ a_{45} & a_{46} & a_{47} & a_{48} \end{array} \right] \middle| \; a_i \in C([0, 29)), 1 \leq i \leq 48, +\right\}$$

be the finite complex modulo integer vector space over the field Z_{19}.

W has infinite dimensional subspaces as well as finite dimensional subspaces over Z_{29}.

We will give one or two examples of super matrix vector spaces built using C([0, p)).

Example 4.10: Let

$$W = \left\{ \left[\begin{array}{cc|c|ccc} a_1 & a_2 & a_3 & a_4 & a_5 & a_6 \\ a_7 & \dots & \dots & \dots & \dots & a_{12} \\ a_{13} & \dots & \dots & \dots & \dots & a_{18} \\ \hline a_{19} & \dots & \dots & \dots & \dots & a_{24} \\ a_{25} & \dots & \dots & \dots & \dots & a_{30} \\ \hline a_{31} & \dots & \dots & \dots & \dots & a_{36} \end{array} \right] \middle| \, a_i \in C([0, 43)), \right.$$

$$1 \leq i \leq 36, +\}$$

be the finite complex modulo integer super matrix vector space over the field Z_{43}.

All properties of these spaces can be derived.

Example 4.11: Let

$$W = \left\{ \left[\begin{array}{c} a_1 \\ \hline a_2 \\ a_3 \\ \hline a_4 \\ a_5 \\ \hline a_6 \\ a_7 \\ a_8 \\ a_9 \\ \hline a_{10} \\ a_{11} \\ a_{12} \\ \hline a_{13} \end{array} \right] \middle| \, a_i \in C([0, 31)), 1 \leq i \leq 13, +\right\}$$

be the finite complex modulo integer super column matrix vector space over the field Z_{31}. S has both finite and infinite dimensional subspaces over Z_{31}.

We can as in case of usual vector space define both the concept of linear transformation (provided both spaces are defined over the same field) and linear operator.

However we are not in a position to define the notion of inner product or linear functional on these spaces.

Example 4.12: Let

$$V = \left\{ \begin{bmatrix} a_1 & a_2 \\ \vdots & \vdots \\ a_{11} & a_{12} \end{bmatrix} \middle| a_i \in C([0, 43)), 1 \leq i \leq 12, + \right\}$$

and

$$W = \left\{ \begin{bmatrix} a_1 & a_2 & a_3 & a_4 \\ a_5 & a_6 & a_7 & a_8 \\ a_9 & a_{10} & a_{11} & a_{12} \end{bmatrix} \middle| a_i \in C([0, 43)), 1 \leq i \leq 12, + \right\}$$

be two finite complex modulo integer interval matrix vector space over the field Z_{43}.

We can define a map $T : V \to W$ such that

$$T = \left\{ \begin{bmatrix} a_1 & a_2 \\ a_3 & a_4 \\ a_5 & a_6 \\ a_7 & a_8 \\ a_9 & a_{10} \\ a_{11} & a_{12} \end{bmatrix} \right\} = \begin{bmatrix} a_1 & a_2 & a_3 & a_4 \\ a_5 & a_6 & a_7 & a_8 \\ a_9 & a_{10} & a_{11} & a_{12} \end{bmatrix};$$

it is easily verified T is a linear transformation from V to W.

Example 4.13: Let

$$V = \left\{ \begin{bmatrix} a_1 \\ a_2 \\ a_3 \\ \vdots \\ a_7 \end{bmatrix} \middle| \; a_i \in C([0, 13)), 1 \le i \le 7 \right\}$$

and W = $\{(a_1, a_2, \ldots, a_{10}) \mid a_i \in C([0, 13)), 1 \le i \le 10\}$ be any two infinite dimensional finite complex modulo integer interval vector space over the field Z_{13}.

Define $T : V \to W$ by

$$T \{ \begin{bmatrix} a_1 \\ a_2 \\ a_3 \\ a_4 \\ a_5 \\ a_6 \\ a_7 \end{bmatrix} \} = (a_1, a_2, 0, a_4, a_5, 0, a_7, a_2, 0, a_4)$$

It is easily verified T is a linear transformation from V to W.

All properties associated with linear transformation can be derived in this case also. It is a matter of routine hence left as an exercise to the reader.

Now we proceed onto describe linear operations on finite complex modulo integer interval vector spaces defined over a field Z_p.

Example 4.14: Let

$$M = \left\{ \begin{bmatrix} a_1 & a_2 & a_3 \\ a_4 & a_5 & a_6 \\ a_7 & a_8 & a_9 \\ a_{10} & a_{11} & a_{12} \\ a_{13} & a_{14} & a_{15} \\ a_{16} & a_{17} & a_{18} \end{bmatrix} \middle| \; a_i \in C([0, 23)), 1 \leq i \leq 18, + \right\}$$

be the finite complex modulo integer interval vector space defined over the field Z_{23}.

Define $T : M \to M$ by

$$T \left\{ \begin{bmatrix} a_1 & a_2 & a_3 \\ a_4 & a_5 & a_6 \\ a_7 & a_8 & a_9 \\ a_{10} & a_{11} & a_{12} \\ a_{13} & a_{14} & a_{15} \\ a_{16} & a_{17} & a_{18} \end{bmatrix} \right\} = \begin{bmatrix} a_1 & a_2 & a_3 \\ 0 & 0 & 0 \\ a_7 & a_8 & a_9 \\ 0 & 0 & 0 \\ a_{13} & a_{14} & a_{15} \\ 0 & 0 & 0 \end{bmatrix}.$$

It is easily verified T is a linear operator on M.

Example 4.15: Let

$$V = \left\{ \begin{bmatrix} a_1 & a_2 & a_3 \\ a_4 & a_5 & a_6 \\ a_7 & a_8 & a_9 \end{bmatrix} \middle| \; a_i \in C([0, 5)), 1 \leq i \leq 9, + \right\}$$

be the finite complex modulo integer interval vector space over Z_5.

Define T : S → S by

$$T\left\{\begin{bmatrix} a_1 & a_2 & a_3 \\ a_4 & a_5 & a_6 \\ a_7 & a_8 & a_9 \end{bmatrix}\right\} = \begin{bmatrix} a_1 & a_2 & a_3 \\ 0 & a_4 & a_5 \\ 0 & 0 & a_6 \end{bmatrix}$$

It is easily verified T is a linear operator on S.

In this way linear operators are defined all properties of linear operators can be easily extended and defined for these spaces also.

We in case of finite complex modulo integer vector spaces defined over a finite field Z_p, cannot define inner product or linear functionals.

Example 4.16: Let $S = \{(a_1, a_2, \ldots, a_{10}) \mid a_i \in C([0, 11)), 1 \leq i \leq 10, +, \times\}$ be a pseudo linear algebra over Z_{11}. We can define $T : S \to S$ by $T((a_1, a_2, \ldots, a_{10})) = (a_1\ 0\ a_3\ 0\ a_5\ 0\ a_7\ 0\ a_9\ 0)$.

T is a linear operator on S. So in case of pseudo linear algebras also we can define the notion of pseudo linear transformation and pseudo linear operators on them. The work of constructing pseudo linear algebra of finite complex modulo integers is a matter of routine so is left as an exercise to the reader.

Next we define the notion of S - vector spaces over the S-ring Z_n or $C(Z_p)$.

Let $V = \{C(Z_n), +\}$ be a vector space over the S-ring $C(Z_n)$ or the S-ring Z_n then we define V to be a S-vector space of the finite complex modulo integer over the S-ring Z_n or $C(Z_n)$.

We will illustrate this situation by some examples.

Example 4.17: Let V = {C([0, 11)), +} be the S-vector space over the S-ring $C(Z_{11})$.

Example 4.18: Let M = {C([0, 12)) be the S-vector space over the S-ring Z_{12}.

Example 4.19: Let W = {C([0, 35)), +} be the S-vector space over the S-ring Z_{35}.

Example 4.20: Let W = {C([0, 16), +} be a S-vector space over the S-ring $C(Z_6)$.

We can use matrices and have many such examples.

Example 4.21: Let
M = $\{(a_1, a_2, \ldots, a_9) \mid a_i \in C[0, 43); 1 \leq i \leq 9, +\}$ be the finite complex modulo integer interval S-vector space over the S-ring $C(Z_{43})$. M has finite dimensional S-vector subspaces as well as infinite dimensional S-vector subspaces.

$P_1 = \{(a_1, 0, 0, 0, \ldots, 0) \mid a_1 \in C([0, 43)), +\}$ is an infinite dimensional vector subspace where as
$B_1 = \{(a_1, 0, \ldots, 0) \mid a_1 \in C(Z_{43})\}$ is a finite dimensional vector subspace of M over the S-ring $C(Z_{43})$.

Example 4.22: Let

$$B = \left\{\begin{bmatrix} a_1 \\ a_2 \\ \vdots \\ a_{20} \end{bmatrix} \middle| a_i \in C([0, 29)), 1 \leq i \leq 20\right\}$$

be the finite complex modulo integer interval column matrix S-vector space over the S-ring $C(Z_{29})$.

B has S-subspaces of both finite and infinite dimension over $C(Z_{29})$.

Example 4.23: Let

$$M = \left\{ \left[\begin{matrix} a_1 & a_2 & \cdots & a_6 \\ \vdots & \vdots & & \vdots \\ a_{31} & a_{32} & \cdots & a_{36} \end{matrix} \right] \middle| \; a_i \in C([0, 14)), 1 \leq i \leq 36, + \right\}$$

be a finite complex modulo integer interval matrix S-vector space over the S-ring Z_{14}.

M has several S-subvector spaces of both finite and infinite dimension over Z_{14}.

Example 4.24: Let

$$L = ((a_1\ a_2 \mid a_3\ a_4\ a_5 \mid a_6) \mid a_i \in C([0, 42)), 1 \leq i \leq 6, +\}$$

be a finite complex modulo integer interval row super matrix S-vector space over the S-ring $C(Z_{42})$.

L has S-subspaces of both finite and infinite dimension over the S-ring $C(Z_{42})$.

Example 4.25: Let

$$B = \left\{ \left[\begin{array}{cc} a_1 & a_2 \\ \hline a_3 & a_4 \\ a_5 & a_6 \\ a_7 & a_8 \\ \hline a_9 & a_{10} \\ a_{11} & a_{12} \\ \hline a_{13} & a_{14} \\ \hline a_{15} & a_{16} \\ a_{17} & a_{18} \\ a_{19} & a_{20} \end{array}\right] \right| a_i \in C([0, 15)), 1 \leq i \leq 20, +\}$$

be the finite complex modulo integer interval column super matrix S-vector space defined over the S-ring $C(Z_{15})$.

B has both finite and infinite dimensional over $C(Z_{15})$.

Example 4.26: Let

$$S = \left\{ \left(\begin{array}{ccc|cccc|cc|c} a_1 & a_2 & a_3 & a_4 & a_5 & a_6 & a_7 & a_8 & a_9 & a_{10} \\ a_{11} & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & a_{20} \\ a_{21} & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & a_{30} \\ \hline a_{31} & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & a_{40} \\ a_{41} & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & a_{50} \\ a_{51} & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & a_{60} \\ a_{61} & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & a_{70} \\ \hline a_{71} & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & a_{80} \end{array}\right) \right| a_i \in$$

$$C([0, 43)), 1 \leq i \leq 48, +\}$$

be the finite complex modulo integer interval super matrix S-vector space over the S-ring R = $C(Z_{43})$.

S has atleast ${}_{80}C_1 + {}_{80}C_2 + \ldots + {}_{80}C_{79}$ number of S-subspaces of infinite dimension and ${}_{80}C_1 + {}_{80}C_2 + \ldots + {}_{80}C_{79} + 1$ number of finite dimensional S-subspaces over $C(Z_{43})$.

Now we give some examples of neutrosophic finite dimensional complex modulo integer vector spaces over a field and over a S-ring.

Example 4.27: Let T = $\{C(\langle[0, 7) \cup I\rangle), +\}$ be the neutrosophic finite complex modulo integer interval vector space over the field R = Z_7.

T has subspace of both finite and infinite dimension over R = Z_7.

Example 4.28: Let B = $\{C(\langle[0, 43) \cup I\rangle), +\}$ be the neutrosophic finite complex modulo integer interval vector space over the field F = Z_{43}.

B has $T_1 = \{[0, 43)\} \subseteq B$ is an infinite dimensional subspace over F.

$T_2 = \{C([0, 43)\} \subseteq B$ is also an infinite dimensional subspace over F.

$T_3 = \{a + bI \mid a, b \in [0, 43)\} \subseteq B$ is an infinite dimensional subspace of B over F.

$T_4 = \{Z_{43}\}$ is a finite dimensional subspace of B over F.

$T_5 = \{\langle Z_{43} \cup I\rangle\} \subseteq B$ is a subspace of finite dimension over F = Z_{43}.

$T_6 = \{C(Z_{43})\} \subseteq B$ is a subspace of finite dimensional over the field F = Z_{43}.

Example 4.29: Let
M = {(a_1, a_2, …, a_9) where $a_i \in C(\langle[0, 23) \cup I\rangle)$, $1 \le i \le 9$, +} be the neutrosophic finite complex modulo integer interval vector space over the field F = Z_{23}. M has several subspaces of finite dimension as well as some subspaces of infinite dimension.

Example 4.30: Let

$$V = \left\{ \begin{bmatrix} a_1 & a_2 & a_3 \\ \vdots & \vdots & \vdots \\ a_{28} & a_{29} & a_{30} \end{bmatrix} \middle| \; a_i \in C(\langle[0, 5) \cup I\rangle), 1 \le i \le 30, + \right\}$$

be the finite complex modulo integer neutrosophic interval column matrix vector space over the field F = Z_5. V has several subspaces of finite and infinite dimension.

V has atleast $3({}_{30}C_1 + {}_{30}C_2 + \ldots + {}_{30}C_{29})$ subspaces of infinite dimension over Z_5. V has $4\,({}_{30}C_1 + {}_{30}C_2 + \ldots + {}_{30}C_{29} + 1)$ number of finite dimensional vector subspaces over Z_5.

Example 4.31: Let

$$V = \left\{ \begin{bmatrix} a_1 & a_2 & \ldots & a_8 \\ a_9 & a_{10} & \ldots & a_{16} \\ a_{17} & a_{18} & \ldots & a_{24} \\ a_{25} & a_{26} & \ldots & a_{32} \\ a_{33} & a_{34} & \ldots & a_{40} \\ a_{41} & a_{42} & \ldots & a_{48} \end{bmatrix} \middle| \; a_i \in C(\langle[0, 17) \cup I\rangle), 1 \le i \le 48, + \right\}$$

be the neutrosophic complex modulo finite integer interval vector space over the field Z_{17}. M has atleast $4({}_{48}C_1 + \ldots + {}_{48}C_{47})$ number of finite integer interval vector subspace of infinite dimension over Z_{17}.

Also M has atleast $4({}_{48}C_1 + \ldots + {}_{48}C_{47} + 1)$ number finite dimensional vector subspaces.

THEOREM 4.1: *Let V = {m × n matrices with entries from C([0, p)) (or C(⟨[0, p) ∪ I⟩)} be the finite complex modulo integer interval vector space (or neutrosophic finite complex modulo integer interval vector space) over the field* Z_p.

(i) *V has atleast 2* $({}_{m\times n}C_1 + {}_{m\times n}C_2 + \ldots + {}_{m\times n}C_{m\times n-1})$ *number of subspaces of infinite dimension over* Z_p. *(or 4* $({}_{m\times n}C_1 + {}_{m\times n}C_2 + \ldots + {}_{m\times n}C_{m\times n-1})$ *number of infinite dimensional subspaces over* Z_p *in case of neutrosophic finite complex modulo integer intervals).*

(ii) *V has atleast* $2({}_{m\times n}C_1 + {}_{m\times n}C_2 + \ldots + {}_{m\times n}C_{m\times n-1}+1)$ *number of finite dimensional vector subspaces over* Z_p *(or in case of neutrosophic entries V has atleast* $4({}_{m\times n}C_1 + {}_{m\times n}C_2 + \ldots + {}_{m\times n}C_{m\times n-1}+1)$ *number of subspaces over* Z_p.

The proof is direct and hence left as an exercise to the reader.

Example 4.32: Let M = $\{(a_1\ a_2\ a_3 \mid a_4 \mid a_5\ a_6 \mid a_7\ a_8\ a_9\ a_{10}\ a_{11}) \mid a_i \in C(\langle[0, 7 \cup I\rangle), 1 \leq i \leq 11, +\}$ be the finite complex modulo integer interval neutrosophic super row matrix vector spaces over the field Z_7. M has both infinite and finite dimensional subspaces over Z_7.

We can build super matrix neutrosophic complex modulo integer interval spaces over Z_p; p a prime.

Such study is a matter of routine and hence left as an exercise to the reader.

Finally we can as in case of finite complex modulo integer interval vector spaces define for the neutrosophic finite complex modulo integer intervals the notion of S-vector spaces which is left as a matter of routine to the reader.

However we give some examples of this situation.

Example 4.33: Let T = $\{C(\langle[0, 17) \cup I\rangle), +\}$ be the neutrosophic finite complex modulo integer interval S-vector space over the S-ring $(Z_{17} \cup I)$ (or $C(Z_{17})$ or $(C\langle Z_{17} \cup I\rangle)$).

In all cases we get S-vector spaces and study in this direction is a matter of routine.

Example 4.34: Let M = $\{C(\langle[0, 12) \cup I\rangle), +\}$ be the neutrosophic finite modulo integer interval S-vector space over the S-ring Z_{12} (or $C(Z_{12})$ or $\langle Z_{12} \cup I\rangle$ or $C(\langle Z_{12} \cup I\rangle)$.

M has both infinite and finite dimensional S-subspaces over the S-ring Z_{12} (or C(12) or $\langle Z_{12} \cup I\rangle$ or $C(\langle Z_{12} \cup I\rangle)$.

Example 4.35: Let

$$B = \left\{\left[\begin{array}{cc} a_1 & a_2 \\ a_3 & a_4 \\ a_5 & a_6 \\ a_7 & a_8 \\ a_9 & a_{10} \\ a_{11} & a_{12} \\ a_{13} & a_{14} \end{array}\right] \middle| \; a_i \in C(\langle[0, 15) \cup I\rangle), 1 \leq i \leq 14\right\}$$

be the neutrosophic finite complex modulo integer interval S-vector space over the S-ring Z_{15} (or $C(Z_{15})$ or $\langle Z_{15} \cup I\rangle$ or $C(\langle Z_{15} \cup I\rangle)$.

B has several S-vector subspaces of both finite and infinite dimension over the S-ring Z_{15} (or $C(Z_{15})$ or $\langle Z_{15} \cup I\rangle$ or $C(\langle Z_{15} \cup I\rangle)$.

Example 4.36: Let

$$V = \left\{\left.\begin{bmatrix} a_1 & a_2 & a_3 \\ a_4 & a_5 & a_6 \\ a_7 & a_8 & a_9 \end{bmatrix}\right| a_i \in C(\langle[0, 5) \cup I\rangle), 1 \leq i \leq 9, +\right\}$$

be the finite neutrosophic complex modulo integer interval S-vector space over the S-ring $C(Z_5)$ (or $\langle Z_5 \cup I\rangle$ or $C(\langle Z_5 \cup I\rangle)$.

T is of infinite dimension. T has finite number of infinite dimensional S-vector subspaces as well as finite dimensional S-vector subspaces.

This study is again a matter of routine.

Now we build pseudo S-vector spaces and pseudo strong S-vector spaces.

Let V = {C([0, n))} be finite complex modulo integer interval vector space over the pseudo S-ring [0, n), then we define V to be S-pseudo vector space or a pseudo S-vector space over the pseudo S-ring [0, n).

If V = {m × n matrix with entries from C([0, n))} be the vector space over C([0, n)) then V is defined as the S strong pseudo vector space or strong Smarandache pseudo vector space over the finite complex modulo integer interval pseudo ring C([0, n)).

The main advantage of defining this is that only on these spaces we can define the concept of inner product pseudo strong vector spaces and linear functions.

Thus the dire need to define and study these concepts arises.

Example 4.37: Let

$$M = \left\{ \left[\begin{array}{cc|ccc} a_1 & a_2 & a_3 & a_4 & a_5 \\ a_6 & \ldots & \ldots & \ldots & a_{10} \\ \hline a_{11} & \ldots & \ldots & \ldots & a_{15} \\ a_{16} & \ldots & \ldots & \ldots & a_{20} \\ a_{21} & \ldots & \ldots & \ldots & a_{25} \\ \hline a_{26} & \ldots & \ldots & \ldots & a_{30} \end{array} \right] \right| a_i \in C(\langle [0, 23) \cup I \rangle), \; 1 \leq i \leq 30, + \}$$

be the neutrosophic complex finite modulo integer pseudo vector space over the pseudo ring R = {[0, 23), +, ×}.

Example 4.38: Let

$$L = \left\{ \left[\begin{array}{ccc} a_1 & a_2 & a_3 \\ \hline a_4 & a_5 & a_6 \\ a_7 & a_8 & a_9 \\ a_{10} & a_{11} & a_{12} \\ \hline a_{13} & a_{14} & a_{15} \\ a_{16} & a_{17} & a_{18} \\ \hline a_{19} & a_{20} & a_{21} \\ a_{22} & a_{23} & a_{24} \\ a_{25} & a_{26} & a_{27} \\ a_{28} & a_{29} & a_{30} \\ \hline a_{31} & a_{32} & a_{33} \end{array} \right] \right| a_i \in C(\langle [0, 23) \cup I \rangle), \; 1 \leq i \leq 33, + \}$$

be the neutrosophic complex finite modulo super column matrix vector space over the pseudo ring R = {[0, 23), +, ×}
(or R_1 = {C([0, 23)). +, ×} or R_2 = {a + bI | a, b ∈ [0, 23), +, ×}.

Example 4.39: Let

$$M = \left\{ \begin{bmatrix} a_1 & a_2 & a_3 & a_4 \\ \vdots & \vdots & \vdots & \vdots \\ a_{41} & a_{42} & a_{43} & a_{44} \end{bmatrix} \middle| \; a_i \in C(\langle [0, 46) \cup I \rangle), \; 1 \leq i \leq 44, + \right\}$$

be the neutrosophic complex finite modulo integer interval pseudo vector space over R = {[0, 46), +, ×} (or {C([0, 46), +, ×} or {a + bI | a, b ∈ [0, 46), +, ×}.

Several properties related with them can be derived as in case of usual vector spaces.

Example 4.40: Let

$$S = \left\{ \left[\begin{array}{c|cccc} a_1 & a_2 & a_3 & a_4 & a_5 \\ \hline a_6 & \cdots & \cdots & \cdots & a_{10} \\ a_{11} & \cdots & \cdots & \cdots & a_{15} \\ a_{16} & \cdots & \cdots & \cdots & a_{20} \\ a_{21} & \cdots & \cdots & \cdots & a_{25} \\ a_{26} & \cdots & \cdots & \cdots & a_{30} \\ \hline a_{31} & \cdots & \cdots & \cdots & a_{35} \\ a_{36} & \cdots & \cdots & \cdots & a_{40} \\ a_{41} & \cdots & \cdots & \cdots & a_{45} \\ \hline a_{46} & \cdots & \cdots & \cdots & a_{50} \end{array} \right] \middle| \; a_i \in C(\langle [0, 17) \cup I \rangle), \; 1 \leq i \leq 50, + \right\}$$

be the neutrosophic finite complex modulo integer interval super matrix pseudo vector space over the pseudo ring R = {[0, 17), +, ×} (or R_1 = {([0, 17), +, ×} or R_2 = {(⟨[0, 17) ∪ I⟩), +, ×} or R_3 = {a + bI | a, b ∈ [0, 17), +, ×}).

Now using only these strong pseudo rings over the pseudo ring $R_2 = \{C(\langle[0, 17) \cup I\rangle), +, \times\}$ we can define inner product and linear functionals.

For if we take $M = \{(a_1, a_2, a_3, a_4)$ where $a_i \in C(\langle[0, 11) \cup I\rangle); 1 \leq i \leq 4, +, \times\}$ be the neutrosophic complex modulo integer interval strong pseudo linear algebras over the pseudo neutrosophic interval ring $R = \{C(\langle[0, 11) \cup I\rangle), +, \times\}$.

Let $x = (0.7 + 10I + 4i_F + 3.2i_FI, 9 + 0.2I + 7.5i_F + 10i_FI, 10i_FI + 0.3i_F, 0.8i_F + 6.2i_FI + I + 1)$ and

$y = (0.4 + 0.2I + 2i_F + 10i_FI, 0, 8i_F, 0.7 + I, 0) \in M.$

The inner product $\langle x, y\rangle = \sum_{i=1}^{4} x_i y_i = (0.7 + 10I + 4i_F + 3.2Ii_F) \times (0.4 + 0.2I + 2i_F + 10i_FI) + 0 + (10i_FI + 0.3i_F) \times (8i_F + 0.7 + I) + 0$

$$= 0.28 + 4I + 1.6i_F + 1.28Ii_F + 0.14I + 2I + 0.8i_FI + 0.64Ii_F + 1.4i_F + 9i_FI + 8 \times 10 + 6.4I \times 10 + 7i_FI + i_FI + 7 \times I \times 10 + 7i_FI + 0.21i_F + 10Ii_F + 0.3i_FI$$

$$= (0.28 + 3 + 2) + (4I + 0.14I + 2I + I + 4I + I + 8I) + (1.6i_F + 0.21i_F) + (1.28 + 0.8 + 0.64 + 9 + 7 + 1 + 7 + 10 + 0.3)i_FI$$

$$= 5.28 + 9.14I + 1.81i_F + 6.02i_FI \text{ is in } C(\langle[0, 11) \cup I\rangle).$$

Thus we see only on strong pseudo neutrosophic vector spaces inner product can be defined.

Likewise we see linear functionals can be defined only on strong neutrosophic complex finite modulo integer vector spaces over $C(\langle[0, n) \cup I\rangle)$.

A linear functional will be a linear transformation from V to $C(\langle[0, n) \cup I\rangle)$ where V is defined over the pseudo neutrosophic complex modulo integer interval ring $C(\langle[0, n) \cup I\rangle)$.

We will give a few examples to illustrate of these situations.

Example 4.41: Let

$$V = \left\{ \begin{bmatrix} a_1 \\ a_2 \\ a_3 \\ a_4 \\ a_5 \\ a_6 \\ a_7 \end{bmatrix} \middle| a_i \in C(\langle[0, 3) \cup I\rangle), 1 \leq i \leq 7, +, \times_n \right\}$$

be the neutrosophic finite complex modulo integer interval column matrix pseudo vector space over
$R = C(\langle[0, 3) \cup I\rangle), +, \times\}$.

We define inner product on V by $x, y \in V$

$$\langle x, y\rangle = \sum_{i=1}^{7} x_i y_i$$

$$\text{where } x = \begin{bmatrix} x_1 \\ x_2 \\ \vdots \\ x_7 \end{bmatrix} \text{ and } y = \begin{bmatrix} y_1 \\ y_2 \\ \vdots \\ y_7 \end{bmatrix}; \text{ then}$$

$$\langle x, y\rangle = \sum x \times_n y = x_1y_1 + \ldots + x_7y_7$$

Clearly $\langle x, y\rangle \in \{C(\langle[0, 3) \cup I\rangle), +, \times\} = R$.

Now we can define linear functionals on V as follows

$f : V \to R$ is given by for any $x \in V$;

$$f(x) = f\left(\begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \\ x_5 \\ x_6 \\ x_7 \end{bmatrix}\right) = \sum_{i=1}^{7} x_i y_i \in R.$$

We can define this in any other way also.

Example 4.42: Let

$$M = \left\{ \begin{bmatrix} a_1 & a_2 & a_3 & a_4 \\ a_5 & a_6 & a_7 & a_8 \\ a_9 & a_{10} & a_{11} & a_{12} \\ a_{13} & a_{14} & a_{15} & a_{16} \end{bmatrix} \middle| \; a_i \in C(\langle[0, 17) \cup I\rangle), \right.$$

$$\left. 1 \leq i \leq 16, +, \times_n \right\}$$

be the neutrosophic complex modulo integer interval strong pseudo matrix vector space over the strong pseudo S-ring $R = \{C(\langle[0, 17) \cup I\rangle), +, \times_n\}$.

We can have several linear functionals defined on M. However we can make M into a inner pseudo product space.

Example 4.43: Let $L = \{(a_1 \mid a_2\ a_3\ a_4 \mid a_5\ a_6\ a_7\ a_8 \mid a_9\ a_{10}\ a_{11}\ a_{12}\ a_{13} \mid a_{14}) \mid a_i \in C(\langle[0, 5) \cup I\rangle), 1 \leq i \leq 15, +, \times_n\}$ be the neutrosophic finite complex modulo integer interval strong pseudo vector space over the strong pseudo neutrosophic S-ring $R = \{C(\langle[0, 5) \cup I\rangle), +, \times\}$.

V is a strong pseudo inner product linear algebra. Several linear functionals can be defined on V.

Example 4.44: Let

$$M = \left\{ \begin{bmatrix} a_1 & a_2 & a_3 \\ \vdots & \vdots & \vdots \\ a_{28} & a_{29} & a_{30} \end{bmatrix} \middle| \; a_i \in C(\langle [0, 29) \cup I \rangle),\ 1 \leq i \leq 30, +, \times_n \right\}$$

be the strong neutrosophic finite complex modulo integer interval strong pseudo vector space over the pseudo S-ring $R = \{C(\langle [0, 29) \cup I \rangle), +, \times\}$.

We have several pseudo subvector spaces of infinite order and using them. We can as in case of usual vector spaces define the notion of projections.

This is also considered as a matter of routine and left as an exercise to the reader.

We can on M define an inner product so that M becomes an inner product spaces.

We can define several linear functionals on V.

Example 4.45: Let

$$M = \left\{ \left[\begin{array}{ccc|c} a_1 & a_2 & a_3 & a_4 \\ a_5 & a_6 & a_7 & a_8 \\ a_9 & a_{10} & a_{11} & a_{12} \\ \hline a_{13} & a_{14} & a_{15} & a_{16} \end{array} \right] \middle| \; a_i \in C(\langle [0, 43) \cup I \rangle), \right.$$

$$\left. 1 \leq i \leq 16, +, \times_n \right\}$$

be the neutrosophic finite complex modulo integer interval pseudo strong vector space over the pseudo ring
$R = \{C(\langle [0, 43) \cup I \rangle), +, \times\}$.

$$\text{Let } B_1 = \left\{ \left[\begin{array}{ccc|c} a_1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ \hline 0 & 0 & 0 & 0 \end{array} \right] \middle| \; a_i \in C(\langle [0, 43) \cup I \rangle) \right\} \subseteq M$$

is a pseudo vector subspace of M and P_1 the linear operator on M.

$$P_1 \left(\left[\begin{array}{ccc|c} a_1 & a_2 & a_3 & a_4 \\ a_5 & a_6 & a_7 & a_8 \\ a_9 & a_{10} & a_{11} & a_{12} \\ \hline a_{13} & a_{14} & a_{15} & a_{16} \end{array} \right] \right) = \left[\begin{array}{ccc|c} a_1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ \hline 0 & 0 & 0 & 0 \end{array} \right].$$

It is easily verified p_1 is a projection on M.

We can define several subspaces of only of infinite dimension over R. However we do not have the concept of finite dimensional subspaces over the pseudo ring $R = \{C(\langle [0, n) \cup I \rangle) +, \times\}$.

Example 4.46: Let

$$V = \left\{ \left[\begin{array}{cc} a_1 & a_2 \\ a_3 & a_4 \\ a_5 & a_6 \\ \hline a_7 & a_8 \\ \hline a_9 & a_{10} \\ a_{11} & a_{12} \\ \hline a_{13} & a_{14} \\ a_{15} & a_{16} \\ a_{17} & a_{18} \end{array} \right] \middle| \; a_i \in C(\langle [0, 29) \cup I \rangle), 1 \leq i \leq 18, +, \times_n \right\}$$

be the neutrosophic finite complex modulo integer interval pseudo strong vector space over the pseudo ring
R = {C(⟨[0, 29) ∪ I⟩), +, ×}.

We see V is a inner product space we can define on V linear functionals and it is considered as a matter of routine we have atleast ${}_{18}C_1 + {}_{18}C_2 + \ldots + {}_{18}C_1$ number of nontrivial strong pseudo subspaces of infinite dimension over R.

Using these spaces we can define projections appropriately.

Example 4.47: Let

$$M=\left\{\left(\begin{array}{c|ccc|cccccc|cc} a_1 & a_2 & a_3 & a_4 & a_5 & a_6 & a_7 & a_8 & a_9 & a_{10} & a_{11} & a_{12} \\ a_{13} & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & a_{24} \\ a_{25} & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & a_{36} \end{array}\right)\right|$$

$$a_i \in C(\langle[0, 7 \cup I\rangle), 1 \leq i \leq 36+, \times_n\}$$

be the neutrosophic finite complex modulo integer interval strong pseudo linear algebra over the pseudo ring
R = {(⟨[0, 7 ∪ I⟩), +, ×}.

We can define on M inner product so that M is a pseudo inner product space and we can define using M the notion of linear functionals and M* the pseudo strong dual space over R. Such study can also be carried out as a matter of routine.

We suggest a few problems some of which are open conjectures.

Problems:

1. Let V = {C([0, p)), p a prime, +} be a finite complex modulo integer vector space over the field Z_p.

   (i) Is V infinite dimensional over Z_p?
   (ii) Find a basis of V over Z_p.

(iii) How many finite dimensional subspaces of V over Z_p exist?
(iv) Find any other special feature enjoyed by V.

2. Let M = {C([0, 23))} be the finite complex modulo integer vector space over the field Z_{23}.

   Study questions (i) to (iv) of problem 1 for this M.

3. Let L = {C([0, 3)), +} be a finite complex modulo integer vector space over the field Z_3.

   Study questions (i) to (iv) of problem 1 for this L.

4. Let V = {C([0, 7)), +, ×} be the pseudo linear algebra over the field Z_7.

   (i) Obtain the special properties enjoyed by V.
   (ii) Find all subalgebras of V.
   (iii) Prove V is infinite dimensional.
   (iv) Can V have sublinear algebra of finite dimension which are not pseudo?
   (iv) Find Hom(V,V), what is the algebraic structure enjoyed by it.

5. Let M = {$(a_1, a_2, \ldots, a_9)$ | $a_i \in$ C([0, 3)), $1 \leq i \leq 9$, +} be the vector space over the field Z_3.

   (i) Find all subspaces of finite dimension over Z_3.
   (ii) Find all subspaces of infinite dimension over Z_3.
   (iii) Find Hom (m, m).
   (iv) Write M as a direct sum of subspaces.
   (v) Find a basis of M over Z_3.
   (vi) Prove M is infinite dimensional over Z_3.

6. Let

$$T = \left\{ \left[\begin{array}{c} a_1 \\ a_2 \\ \vdots \\ a_{12} \end{array} \right] \middle| \; a_i \in C([0, 19)), 1 \leq i \leq 12, + \right\}$$

be a vector space over Z_{19}.

Study questions (i) to (vi) of problem 5 for this T.

7. Let

$$S = \left\{ \left[\begin{array}{cc|c|ccccc|c|c} a_1 & a_2 & a_3 & a_4 & a_5 & a_6 & a_7 & a_8 & a_9 \\ a_9 & a_{10} & \ldots & \ldots & \ldots & \ldots & \ldots & \ldots & a_{18} \\ \hline a_{19} & a_{20} & \ldots & \ldots & \ldots & \ldots & \ldots & \ldots & a_{27} \\ a_{28} & a_{29} & \ldots & \ldots & \ldots & \ldots & \ldots & \ldots & a_{36} \\ a_{37} & a_{38} & \ldots & \ldots & \ldots & \ldots & \ldots & \ldots & a_{45} \\ \hline a_{46} & a_{47} & \ldots & \ldots & \ldots & \ldots & \ldots & \ldots & a_{54} \end{array} \right] \middle| \; a_i \in C([0, 13)), 1 \leq i \leq 54, + \right\}$$

be a vector space over Z_{19}.

Study questions (i) to (vi) of problem 5 for this S.

8. Let $A = \{(a_1, a_2, \ldots, a_{15}) \mid a_i \in C([0, 190), +\}$ be the S-vector space over the S-ring Z_{190} (or $C(Z_{190})$).

Study questions (i) to (vi) of problem 5 for this A.

9. Let

$$M = \left\{ \left[\begin{array}{c|c|ccc|cc} a_1 & a_2 & a_3 & a_4 & a_5 & a_6 & a_7 \\ a_8 & a_9 & \cdots & \cdots & \cdots & \cdots & a_{14} \\ a_{15} & a_{16} & \cdots & \cdots & \cdots & \cdots & a_{21} \\ \hline a_{22} & a_{23} & \cdots & \cdots & \cdots & \cdots & a_{28} \\ \hline a_{29} & a_{30} & \cdots & \cdots & \cdots & \cdots & a_{35} \\ a_{36} & a_{37} & \cdots & \cdots & \cdots & \cdots & a_{42} \end{array} \right] \right| a_i \in C([0, 15)),$$

$1 \leq i \leq 42$, +} be the finite complex modulo integer vector space over the S-ring Z_{15}. (or $C(Z_{15})$).

Study questions (i) to (vi) of problem 5 for this M.

10. Let

$$T = \left\{ \left[\begin{array}{c} a_1 \\ a_2 \\ \vdots \\ a_8 \end{array} \right] \right| a_i \in C([0, 17)), 1 \leq i \leq 8, +\}$$

be the finite complex modulo integer S-vector space over the S-ring $C(Z_{17})$.

Study questions (i) to (vi) of problem 5 for this W.

11. Let P = $\{C(\langle[0, 23) \cup I\rangle), +\}$ be the vector space of neutrosophic complex finite modulo integer interval over the field F = Z_{23}.

(i) Find a basis of P over Z_{23} = F.
(ii) Find Hom (V, V).
(iii) Find all subspace of finite dimension over F.
(iv) Find all subspaces of infinite dimension over F.

12. Obtain some special features enjoyed by pseudo neutrosophic complex finite modulo integer interval vector space over a strong pseudo ring $R = \{C(\langle[0, n) \cup I\rangle), +, \times\}$.

13. Let $W = \{ C(\langle[0, 23) \cup I\rangle), +\}$ be a vector space over Z_{23}.

    (i) Study questions (i) to (vi) of problem 5 for this W.
    (ii) Find a basis of W over Z_{23}.

14. Let $W = \{(a_1, a_2, \ldots, a_{10})$ where $a_i \in C(\langle[0, 13) \cup I\rangle)$, $1 \leq i \leq 10, +\}$ be the neutrosophic finite complex modulo integer interval vector space over the field $F = Z_{13}$.

    (i) Find a basis of W over Z_{13}.
    (ii) Find all subspaces of W which are finite dimensional over Z_{13}.
    (iii) Find all subspaces of W, which are infinite dimensional over Z_{13}.
    (iv) Can W be written as a direct sum?
    (v) Find the algebraic structure enjoyed by Hom(W,W).
    (vi) Find projections of V.
    (vii) If Z_{13} is replaced by $\langle Z_{13} \cup I\rangle$ so that W is a S-vector space for that changed W study questions (i) to (vi).

15. Let

$$S = \left\{ \begin{bmatrix} a_1 & a_2 & a_3 & a_4 & a_5 \\ a_6 & \cdots & \cdots & \cdots & a_{10} \\ a_{11} & \cdots & \cdots & \cdots & a_{15} \\ a_{16} & \cdots & \cdots & \cdots & a_{20} \\ a_{21} & \cdots & \cdots & \cdots & a_{25} \\ a_{26} & \cdots & \cdots & \cdots & a_{30} \\ a_{31} & \cdots & \cdots & \cdots & a_{35} \\ a_{36} & \cdots & \cdots & \cdots & a_{40} \end{bmatrix} \right| a_i \in C(\langle[0, 19) \cup I\rangle), 1 \leq i \leq 40,$$

+} be the neutrosophic finite complex modulo integer vector space over the field F = Z_{19}.

Study questions (i) to (vii) of problem 14 for this S.

16. Let

$$B = \left\{ \left[\begin{array}{cc|ccc|c|ccc} a_1 & a_2 & a_3 & a_4 & a_5 & a_6 & a_7 & a_8 & a_9 \\ a_9 & a_{10} & \ldots & \ldots & \ldots & \ldots & \ldots & \ldots & a_{18} \\ a_{19} & a_{20} & \ldots & \ldots & \ldots & \ldots & \ldots & \ldots & a_{27} \\ a_{28} & a_{29} & \ldots & \ldots & \ldots & \ldots & \ldots & \ldots & a_{36} \end{array} \right] \right| a_i \in$$

C(⟨[0, 23) ∪ I⟩), 1 ≤ i ≤ 36, +} be the neutrosophic finite complex modulo integer super row matrix vector space over the field F = Z_{23}.

Study questions (i) to (viii) of problem 14 for this B.

17. Let

$$T = \left\{ \left[\begin{array}{c|c|ccc|cc} a_1 & a_2 & a_3 & a_4 & a_5 & a_6 & a_7 \\ a_8 & a_9 & \ldots & \ldots & \ldots & \ldots & a_{14} \\ a_{15} & a_{16} & \ldots & \ldots & \ldots & \ldots & a_{21} \\ \hline a_{22} & a_{23} & \ldots & \ldots & \ldots & \ldots & a_{28} \\ \hline a_{29} & a_{30} & \ldots & \ldots & \ldots & \ldots & a_{35} \\ a_{36} & a_{37} & \ldots & \ldots & \ldots & \ldots & a_{42} \end{array} \right] \right| a_i \in C([0, 53)),$$

1 ≤ i ≤ 42, +} be the neutrosophic finite complex modulo integer super matrix vector space over the field Z_{53}.

Study questions (i) to (vii) of problem 14 for this T.

18. Let V = {(a_1, a_2, …, a_9) | a_i ∈ C(⟨[0, 23) ∪ I⟩), 1 ≤ i ≤ 9, +} be the neutrosophic finite complex modulo integer S-vector

space over the S-ring $R = (C(Z_{23})$ (or $R_1 = \langle Z_{23} \cup I\rangle$ or $R_2 = (\langle Z_{23} \cup I\rangle)$.

(i) Find dimension of V over R (or R_1 or R_2)
(ii) Can V have finite dimensional vector subspaces over R (or R_1 or R_2).
(iii) Find all infinite dimensional vector subspaces of V over R (or R_1 or R_2)
(iv) Find S = Hom (V, V); what is the algebraic structure enjoyed by S.
(v) Write V as a direct sum.
(vi) Find projection operators on V.

19. Let

$$P = \left\{ \left[\begin{array}{cc|c|ccc|cc} a_1 & a_2 & a_3 & a_4 & a_5 & a_6 & a_7 & a_8 \\ a_9 & a_{10} & \ldots & \ldots & \ldots & \ldots & \ldots & a_{16} \\ a_{17} & a_{18} & \ldots & \ldots & \ldots & \ldots & \ldots & a_{24} \\ a_{25} & a_{26} & \ldots & \ldots & \ldots & \ldots & \ldots & a_{32} \end{array} \right] \right| a_i \in$$

$C([0, 43))$, $1 \leq i \leq 32$, +} be the finite complex modulo integer interval neutrosophic S-vector space over the S-ring $C(Z_{43}) = R$ (or $R_1 = \langle Z_{43} \cup I\rangle$ or $R_2 = (\langle Z_{43} \cup I\rangle)$.

Study questions (i) to (vi) of problem 18 for this P.

20. Let

$$B = \left\{ \left[\begin{array}{c} a_1 \\ a_2 \\ \vdots \\ a_{20} \end{array} \right] \right| a_i \in C(\langle [0, 31) \cup I\rangle), 1 \leq i \leq 20, +\}$$

be the complex modulo integer interval neutrosophic S-vector space over the S-ring $R = C(Z_{31})$ (or $R_1 = \langle Z_{31} \cup I\rangle$ or $R_2 = \langle Z_{31} \cup I\rangle$.

Study questions (i) to (vi) of problem 18 for this B.

21. Let M = $\{(a_1, a_2, \ldots, a_9) \mid a_i \in C(\langle[0, 23) \cup I\rangle), 1 \leq i \leq 9, +\}$ be a pseudo neutrosophic complex modulo finite integer vector space over the S-ring R = $\{[0, 23), +, \times\}$ (or R_1 = $\{C([0, 23), +, \times\}$ or $R_2 = C(\langle[0, 23) \cup I\rangle), +, \times\}$ or R_3 = $\{\langle[0, 23) \cup I\rangle), +, \times\}$.

    (i) Study questions (i) to (vi) of problem 18 for this M.
    (ii) What is dimension of M over R (or R_1 or R_2 or R_3)?
    (iii) Will dimension of M over R_2 be finite?

22. Let

$$P = \left\{\left(\begin{array}{cccc} a_1 & a_2 & \ldots & a_{10} \\ a_{11} & a_{12} & \ldots & a_{20} \\ a_{21} & a_{22} & \ldots & a_{30} \\ a_{31} & a_{32} & \ldots & a_{40} \end{array}\right) \middle| a_i \in C(\langle[0, 43) \cup I\rangle), 1 \leq i \leq 40,\right.$$

$+, \times_n\}$ be the neutrosophic finite complex modulo integer pseudo linear algebra over the pseudo ring R = $\{[0, 43), +, \times\}$ (or $R_1 = \{C([0, 43)), +, \times\}$ or $R_2 = \{C(\langle[0, 43) \cup I\rangle, +, \times\}$ or $R_3 = \{\langle[0, 43) \cup I\rangle, +, \times\})$.

    (i) Study questions (i) to (vi) of problem 18 for this P.
    (ii) Define inner product operation on P.

23. Let

$$M = \left\{\left[\begin{array}{c|cc|cc} a_1 & a_2 & a_3 & a_4 & a_5 \\ \hline a_6 & \ldots & \ldots & \ldots & a_{10} \\ a_{11} & \ldots & \ldots & \ldots & a_{15} \\ \hline a_{16} & \ldots & \ldots & \ldots & a_{20} \\ a_{21} & \ldots & \ldots & \ldots & a_{25} \end{array}\right] \middle| a_i \in C(\langle[0, 11) \cup I\rangle),\right.$$

$1 \leq i \leq 25, +, \times\}$ be a neutrosophic finite complex modulo integer interval pseudo super matrix vector space over R = $\{[0, 11), +, \times\}$ (or $R_1 = \{C([0, 11)), +, \times\}$ or $R_3 = \{\langle[0, 11) \cup I, +, \times\}$ or $R_2 = \{C(\langle[0, 11) \cup I\rangle), +, \times\}$.

(i) Study questions (i) to (vi) of problem 21 for this M.
(ii) Define inner product on M.
(iii) Give two linear functionals on M which are distinct.
(iv) What is the algebraic structure enjoyed by $L(M, R_2)$ = {Collection of all linear functionals from M to R_2}?
(v) Find M* of M.

24. Let

$$V = \left\{\begin{pmatrix} a_1 & a_2 & \dots & a_7 \\ a_8 & a_9 & \dots & a_{14} \end{pmatrix} \middle| \, a_i \in C([0, 43)), 1 \leq i \leq 14\right\}$$

and

$$W = \left\{\begin{bmatrix} a_1 & a_2 & a_3 & a_4 \\ a_5 & a_6 & a_7 & a_8 \\ a_9 & a_{10} & a_{11} & a_{12} \end{bmatrix} \middle| \, a_i \in C(\langle[0, 43) \cup I\rangle), 1 \leq i \leq 12\right\}$$

be two finite complex modulo integer interval vector spaces over the field $F = Z_{43}$.

(i) Find Hom(V,W), Hom (V, V) and Hom (W, W) and describe the algebraic structure enjoyed by them.

25. Let $P = \left\{\begin{bmatrix} a_1 & a_2 & \dots & a_{12} \\ a_{13} & a_{14} & \dots & a_{24} \\ a_{25} & a_{26} & \dots & a_{36} \end{bmatrix} \middle| \, a_i \in C([0, 5)), 1 \leq i \leq 36\right\}$

and $M = \left\{\left.\begin{bmatrix} a_1 & a_2 & \ldots & a_6 \\ a_7 & a_8 & \ldots & a_{12} \\ \vdots & \vdots & & \vdots \\ a_{31} & a_{32} & \ldots & a_{36} \end{bmatrix}\right| a_i \in C([0, 15)), 1 \leq i \leq 36\right\}$

be two finite complex modulo integer interval S-vector space over the S-ring Z_{15}.

(i) Study questions (i) of problem 24 for this P and M.

26. Let $V = \left\{\left.\begin{bmatrix} a_1 & a_2 & \ldots & a_{10} \\ a_{11} & a_{12} & \ldots & a_{20} \\ a_{21} & a_{22} & \ldots & a_{30} \\ a_{31} & a_{32} & \ldots & a_{40} \end{bmatrix}\right| a_i \in C([0, 7)), 1 \leq i \leq 40\right\}$

and $W = \left\{\left.\begin{bmatrix} a_1 & a_2 & a_3 & a_4 \\ a_5 & a_6 & a_7 & a_8 \\ a_9 & a_{10} & a_{11} & a_{12} \end{bmatrix}\right| a_i \in C(\langle [0, 7) \cup I\rangle),\right.$

$1 \leq i \leq 12\}$ be two finite complex modulo integer interval S-pseudo vector space over the pseudo ring $R = \{[0, 7), +, \times\}$.

(i) Study questions (i) of problem 24 for this V and W.

27. Let $T_1 = \{(a_1\ a_2 \mid a_3\ a_4\ a_5 \mid a_6 \mid a_7\ a_8) \mid a_i \in C([0, 11)), +, \times,$

$1 \leq i \leq 8\}$ and $S_1 = \left\{\left.\left[\begin{array}{c|cc|cccc|c} a_1 & a_2 & a_3 & a_4 & a_5 & a_6 & a_7 & a_8 \\ a_9 & a_{10} & \ldots & \ldots & \ldots & \ldots & \ldots & a_{16} \end{array}\right]\right|\right.$

$a_i \in C(\langle [0, 11) \cup I\rangle), 1 \leq i \leq 16\}$ be two finite complex modulo integer interval strong pseudo S-vector spaces over the complex finite modulo integer S-ring
$R = \{C([0, 11), +, \times\}$.

(i) Study questions (i) of problem 24.
(ii) Find T_1^*.
(iii) Find S_1^*.
(iv) Define inner product on S_1 and T_1.

28. Give some special features enjoyed by pseudo S-vector spaces built over R = {[0, p), +, ×}.

29. Study the special properties enjoyed by pseudo Strong S-vector spaces built over R = {C([0, p), +, ×}

30. Compare the structures in problems (28) and (29).

31. What are the special features associated with neutrosophic S-pseudo vector space built using C(⟨[0, n) ∪ I⟩) defined over R = {⟨[0, n) ∪ I⟩, +, ×}?

32. Study the algebraic structure enjoyed by the strong S-pseudo neutrosophic finite complex modulo integer interval vector over F = {C(⟨[0, n) ∪ I⟩), +, ×}.

33. Let M = {(a_1, a_2, a_3, a_4, a_5, a_6, a_7) | a_i ∈ C(⟨[0, 37) ∪ I⟩)} be the strong pseudo neutrosophic finite complete S-linear algebra over the S-pseudo ring P = { C(⟨[0, 37) ∪ I⟩), +, ×}.

    (i) Find a basis of M over P.
    (ii) Is M finite dimensional over P?
    (iii) Find Hom (M, M)
    (iv) Find the dual space M*.
    (v) Define an inner product on M so that m is an inner product space.
    (vi) Can M have subspaces with finite cardinality?
    (vii) Can M subspaces of dimension one, four etc.?

34. Let T = $\left\{ \begin{bmatrix} a_1 \\ a_2 \\ \vdots \\ a_9 \end{bmatrix} \right|$ $a_i \in C(\langle[0, 23) \cup I\rangle)1 \leq i \leq 12, +, \times_n\}$ and

S = $\left\{ \begin{bmatrix} a_1 & a_2 & a_3 & a_4 & a_5 \\ a_6 & \ldots & \ldots & \ldots & a_{10} \\ a_{11} & \ldots & \ldots & \ldots & a_{15} \end{bmatrix} \right|$ $a_i \in C(\langle[0, 23) \cup I\rangle)$, $1 \leq i \leq 15, +, \times_n\}$ be two pseudo strong neutrosophic finite complex modulo integer interval S-linear algebra over the S-pseudo neutrosophic finite complex number integer interval ring R = $\{C(\langle[0, 23) \cup I\rangle), +, \times\}$.

(i) Study questions (i) to (vii) of problem 33 for this T and S.

35. Derive some special results regarding dual spaces V* of V built over the pseudo neutrosophic finite complex modulo integer interval rings R = $\{C(\langle[0, n) \cup I\rangle), +, \times\}$.

36. Derive special theorems on these special pseudo linear algebras by overcoming the lack of distributivity on + and ×.

37. Find pseudo special strong linear algebras of dimension 2 over S pseudo ring R = $\{C(\langle[0, n) \cup I\rangle), +, \times\}$.

38. Let M = $\{(a, b) \mid a, b \in \{C(\langle[0, 2) \cup I\rangle), +, \times\}$ be the pseudo strong neutrosophic finite complex modulo integer linear algebra over R = $\{C(\langle[0, 2) \cup I\rangle), +, \times\}$ the S-pseudo ring.

(i) Find dimension of M over R.
(ii) Can W be a S-pseudo sublinear algebra such that o(W) $< \infty$ by o(W) we mean the number of distinct elements in W?
(iii) Find Hom(M,M).

(iv) What is dimension of M*, the dual S-pseudo space of M?
(v) If R is replaced by Z_2 show M has infinite basis.
(vi) If R is replaced by P = {[0, 2), +, ×} will M have infinite basis?
(vii) Can M be a inner product space?
(viii) If R is replaced by B = {C([0, 2)), +, ×} will M be infinite dimensional?
(ix) Study M when R is replaced by D = {a + bI | a, b ∈ [0, 2), +, ×}.

FURTHER READING

1. Albert, A.A., *Non-associative algebra I, II,* Ann. Math. (2), **43**, 685-707, (1942).

2. Birkhoff, G. and Bartee, T.C. *Modern Applied Algebra*, Mc-Graw Hill, New York, (1970).

3. Bruck, R. H., *A survey of binary systems,* Springer-Verlag, (1958).

4. Bruck, R.H, *Some theorems on Moufang loops,* Math. Z., **73**, 59-78 (1960).

5. Castillo J., *The Smarandache Semigroup*, International Conference on Combinatorial Methods in Mathematics, II Meeting of the project 'Algebra, Geometria e Combinatoria', Faculdade de Ciencias da Universidade do Porto, Portugal, 9-11 July 1998.

6. Chang Quan, Zhang, *Inner commutative rings*, Sictiuan Dascue Xuebao (Special issue), **26**, 95-97 (1989).

7. Chein, Orin and Goodaire, Edgar G., *Loops whose loop rings in characteristic 2 are alternative*, Comm. Algebra, **18**, 659-668 (1990).

8. Chein, Orin, and Goodaire, Edgar G., *Moufang loops with unique identity commutator (associator, square)*, J. Algebra, **130**, 369-384 (1990).

9. Chein, Orin, and Pflugfelder, H.O., *The smallest Moufang loop*, Arch. Math., **22**, 573-576 (1971).

10. Chein.O, Pflugfelder.H.O and Smith.J.D.H, (eds), *Quasigroups and loops: Theory and applications*, Sigma Series in Pure Maths, Vol. 8, Heldermann Verlag, (1990).

11. Chein, Orin, Kinyon, Michael. K., Rajah, Andrew and Vojlechovsky, Peter, *Loops and the Lagrange property*, (2002). http://lanl.arxiv.org/pdf/math.GR/0205141

12. Fenyves.F, *Extra loops II: On loops with identities of Bol-Moufang types*, Publ. Math. Debrecen, Vol.16, 187-192 (1969).

13. Goodaire, E.G., and Parmenter, M.M., *Semisimplicity of alternative loop rings*, Acta. Math. Hung, **50**. 241-247 (1987).

14. Hall, Marshall, *Theory of Groups*. The Macmillan Company, New York, (1961).

15. Hashiguchi. K, Ichihara, S. and Jimbo, S., *Formal languages over binoids*, J. Autom Lang Comb, **5**, 219-234 (2000).

16. Herstein., I.N., *Topics in Algebra*, Wiley Eastern Limited, (1975).

17. Ivan, Nivan and Zukerman. H. S., *Introduction to number theory*, Wiley Eastern Limited, (1984).

18. Kepka.T, *A construction of commutative Moufang loops and quasi modulus*, Comment Math. Univ. Carolin. Vol.27, No.3, 499-518 (1986).

19. Lang, S., *Algebra*, Addison Wesley, (1967).

20. Maggu, P.L., *On introduction of Bigroup concept with its applications in industry*, Pure App. Math Sci., **39**, 171-173 (1994).

21. Maggu, P.L., and Rajeev Kumar, *On sub-bigroup and its applications*, Pure Appl. Math Sci., **43**, 85-88 (1996).

22. Michael.K.Kinyon and Phillips.J.D, *Commutants of Bol loops of odd order*, (2002). http://lanl.arxiv.org/pdf/math.GR/0207119

23. Michael.K.Kinyon and Oliver Jones, *Loops and semidirect products*, (2000). http://lanl.arxiv.org/pdf/math.GR/9907085 (To appear in Communications in Algebra)

24. Pflugfelder.H.O, *A special class of Moufang loops*, Proc. Amer. Math. Soc., Vol. 26, 583-586 (1971).

25. Pflugfelder.H.O, *Quasigroups and loops: Introduction*, Sigma Series in Pure Mathematics, Vol. 7, Heldermann Verlag, (1990).

26. Raul, Padilla, *Smarandache Algebraic Structures*, Smarandache Notions Journal, **9**, 36-38 (1998).

27. Singh, S.V., On a new class of loops and loop rings, Ph.D. thesis IIT (Madras), guided by Vasantha. W.B., (1994).

28. Smarandache, Florentin, (editor), Proceedings of the First International Conference on Neutrosophy, Neutrosophic Set, Neutrosophic Probability and Statistics, University of New Mexico, (2001).

29. Smarandache, Florentin, *A Unifying Field in Logics: Neutrosophic Logic*, Preface by Charles Le, American Research Press, Rehoboth, 1999, 2000. Second edition of the Proceedings of the First International Conference on *Neutrosophy, Neutrosophic Logic, Neutrosophic Set, Neutrosophic Probability and Statistics*, University of New Mexico, Gallup, (2001).

30. Smarandache, Florentin, *Special Algebraic Structures*, in Collected Papers, Abaddaba, Oradea, **3**, 78-81 (2000).

31. Smarandache Florentin, Multi structures and Multi spaces, (1969) www.gallup.unm.edu/~smarandache/transdis.txt

32. Smarandache, Florentin, *Definitions Derived from Neutrosophics*, In Proceedings of the First International Conference on Neutrosophy, Neutrosophic Logic, Neutrosophic Set, Neutrosophic Probability and Statistics, University of New Mexico, Gallup, 1-3 December (2001).

33. Smarandache, Florentin, *Neutrosophic Logic—Generalization of the Intuitionistic Fuzzy Logic*, Special Session on Intuitionistic Fuzzy Sets and Related Concepts, International EUSFLAT Conference, Zittau, Germany, 10-12 September 2003.

34. Solarin, A.R.T., and Sharma B.L., *On the identities of Bol-Moufang type*, Kyungpook Math. J., **28**, 51-62 (1988).

35. Tim Hsu, *Class 2 Moufang loops small Frattini Moufang loops and code loops*, (1996). http://lanl.arxiv.org/pdf/math.GR/9611214

36. Vasantha Kandasamy, W. B., *Fuzzy subloops of some special loops*, Proc. 26th Iranian Math. Conf., 33-37 (1995).

37. Vasantha Kandasamy, W. B., *On ordered groupoids and groupoid rings*, J. Math. Comp. Sci., **9**, 145-147 (1996).

38. Vasantha Kandasamy, W. B. and Meiyappan, D., *Bigroup and Fuzzy bigroup*, Bol. Soc. Paran Mat, **18**, 59-63 (1998).

39. Vasantha Kandasamy, W. B., *On Quasi loops*, Octogon, **6**, 63-65 (1998).

40. Vasantha Kandasamy, W. B., *On a new class of Jordan loops and their loop rings*, J. Bihar Math. Soc., **19**, 71-75 (1999).

41. Vasantha Kandasamy, W. B. and Singh S.V., *Loops and their applications to proper edge colouring of the graph K_{2n}*, Algebra and its applications, edited by Tariq et al., Narosa Pub., 273-284 (2001).

42. Vasantha Kandasamy, W. B., *Biloops*, U. Sci. Phy. Sci., **14**, 127-130 (2002).

43. Vasantha Kandasamy, W. B., *Groupoids and Smarandache groupoids*, American Research Press, Rehoboth, (2002). http://www.gallup.unm.edu/~smarandache/Vasantha-Book2.pdf

44. Vasantha Kandasamy, W. B., *On Smarandache Cosets*, (2002). http://www.gallup.unm.edu/~smaranandache/pseudo ideals.pdf

45. Vasantha Kandasamy, W. B., *Smarandache groupoids*, (2002). http://www.gallup.unm.edu/~smarandache/Groupoids.pdf

46. Vasantha Kandasamy, W. B., *Smarandache loops*, Smarandache Notions Journal, **13**, 252-258 (2002). http://www.gallup.unm.edu/~smarandache/Loops.pdf

47. Vasantha Kandasamy, W. B., *Smarandache Loops*, American Research Press, Rehoboth, NM, (2002). http://www.gallup.unm.edu/~smarandache/Vasantha-Book4.pdf

48. Vasantha Kandasamy, W. B., *Bialgebraic Structures and Smarandache Bialgebraic Structures*, American Research Press, Rehoboth, NM, (2002). http://www.gallup.unm.edu/~smarandache/NearRings.pdf

49. Vasantha Kandasamy, W. B., *Smarandache Semigroups*, American Research Press, Rehoboth, NM, (2002). http://www.gallup.unm.edu/~smarandache/Vasantha-Book1.pdf

50. Vasantha Kandasamy, W. B. and Smarandache, F., *N-algebraic structures and S-N-algebraic structures*, Hexis, Phoenix, Arizona, (2005).

51. Vasantha Kandasamy, W. B. and Smarandache, F., *Neutrosophic algebraic structures and neutrosophic N-algebraic structures*, Hexis, Phoenix, Arizona, (2006).

52. Vasantha Kandasamy, W. B. and Smarandache, F., *Smarandache Neutrosophic algebraic structures*, Hexis, Phoenix, Arizona, (2006).

53. Vasantha Kandasamy, W. B. and Smarandache, F., *Finite Neutrosophic Complex Number*, Zip Publishing, Ohio, (2011).

54. Vasantha Kandasamy, W.B. and Florentin Smarandache, *Algebraic Structures using [0, n),* Educational Publisher Inc, Ohio, (2013).

55. Vasantha Kandasamy, W.B. and Florentin Smarandache, *Algebraic Structures on the fuzzy interval [0, 1),* Educational Publisher Inc, Ohio, (2014).

56. Vasantha Kandasamy, W.B. and Florentin Smarandache, *Algebraic Structures on the fuzzy unit square and neutrosophic unit square*, Educational Publisher Inc, Ohio, (2014).

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ABOUT THE AUTHORS

Dr.W.B.Vasantha Kandasamy is a Professor in the Department of Mathematics, Indian Institute of Technology Madras, Chennai. In the past decade she has guided 13 Ph.D. scholars in the different fields of non-associative algebras, algebraic coding theory, transportation theory, fuzzy groups, and applications of fuzzy theory of the problems faced in chemical industries and cement industries. She has to her credit 653 research papers. She has guided over 100 M.Sc. and M.Tech. projects. She has worked in collaboration projects with the Indian Space Research Organization and with the Tamil Nadu State AIDS Control Society. She is presently working on a research project funded by the Board of Research in Nuclear Sciences, Government of India. This is her 96th book.

On India's 60th Independence Day, Dr.Vasantha was conferred the Kalpana Chawla Award for Courage and Daring Enterprise by the State Government of Tamil Nadu in recognition of her sustained fight for social justice in the Indian Institute of Technology (IIT) Madras and for her contribution to mathematics. The award, instituted in the memory of Indian-American astronaut Kalpana Chawla who died aboard Space Shuttle Columbia, carried a cash prize of five lakh rupees (the highest prize-money for any Indian award) and a gold medal.

She can be contacted at vasanthakandasamy@gmail.com
Web Site: http://mat.iitm.ac.in/home/wbv/public_html/
or http://www.vasantha.in

Dr. Florentin Smarandache is a Professor of Mathematics at the University of New Mexico in USA. He published over 75 books and 200 articles and notes in mathematics, physics, philosophy, psychology, rebus, literature. In mathematics his research is in number theory, non-Euclidean geometry, synthetic geometry, algebraic structures, statistics, neutrosophic logic and set (generalizations of fuzzy logic and set respectively), neutrosophic probability (generalization of classical and imprecise probability). Also, small contributions to nuclear and particle physics, information fusion, neutrosophy (a generalization of dialectics), law of sensations and stimuli, etc. He got the 2010 Telesio-Galilei Academy of Science Gold Medal, Adjunct Professor (equivalent to Doctor Honoris Causa) of Beijing Jiaotong University in 2011, and 2011 Romanian Academy Award for Technical Science (the highest in the country). Dr. W. B. Vasantha Kandasamy and Dr. Florentin Smarandache got the 2012 New Mexico-Arizona and 2011 New Mexico Book Award for Algebraic Structures. He can be contacted at smarand@unm.edu

In this book authors introduce the notion of finite complex modulo integer intervals. Finite complex modulo integers were introduced by the authors in 2011. Now using this finite complex modulo integer intervals several algebraic structures are built.
Open problems are suggested.

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