# Particle Solution to the Klein-Gordon-Dirac Equation in the Context of a Big Bang Universe

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## Abstract

The purpose of this paper is to develop a single solution to both the Klein-Gordon & Dirac equations that expresses both the QM and the classical aspects of particles. It is found that this can be done, but only in the context of a system that has an initial event (T = 0) and is expanding at c, thus it is consistent with a big bang representation of the universe. The equations are defined in *geometric algebraic*, and the KGD equation will be considered a single equation factorable into linear products of the two linear Dirac expressions, with a solution defined analogously to path integrals. The solution has botha Gaussian shaped amplitude, (classical), and phase, (QM) components satisfying the quadratic KG equation, and the linear Dirac expression. The equation differentials are not restricted to representing the normal QM operator replacement of p and E, applicable to the linear equation, but have a broader context in operating on the more complex function with amplitude and phase factors. The solutions represent the particle at a single event, thus the standard view of the solution being a probability amplitude field over spacetime is not applicable, but an alternate observational field is illustrated that demonstrates the connection of the solutions to the observed wave characteristics. The phase factors are as usual cyclic, but the amplitude factors exist only in the context of the entire interval. The amplitude factor of the solution is proportional to mass and thus should offer insight into particle mass ratios.

Keywords: Klein-Gordon-Dirac, particles

## INTRODUCTION

The purpose of this work is to develop a single point particle solution the KGD that defines the phase, quantum mechanical as well as the classical particle-particle dynamics and the electromagnetic interactions of particles. Instead of regarding the Klein-Gordon, and Dirac equations as analogs of the classical equations with the momentum and energy replaced by differential operators, the equations are treated as differentials operating on complex particle function.

$$\not{p} = m\not{v} \neq i\hbar\not{\partial} \tag{1}$$

The coordinates in this development are the end event coordinates of a particle solution, and not field variables of the function.

The particle solution will be formulated as the end point of a propagator, with function exponents that are the square of the sum of the action of the canonical momentum, over a classical fourspace interval from the initial, (Big Bang), to the current event. The end point function will be designated as the Systemfunction.

Section I, forms an analogy with the path integral approach, and proposes a particle solution that is the propagator, with an action that is the square of the fourspace bivector action of a particle from the Big Bang to its current position. The square of the complex bivector has both first and second order terms which are both real and imaginary. The gauge field is evaluated at the action endpoint making the solution a point function, and an eigensolution for the particle.

Section II separates a point Systemfunction into amplitude and phase functions. The complete Systemfunction is a solution to the KG expression, and also a solution to the Dirac expression.

Section III Shows that the phase factor of the point Systemfunction is equivalent to the Dirac solution for the first order equation, and develops the function properties.

Section IV defines an observational field associated with a point particle, having features similar to the Dirac probability amplitude field. The observational field illustrates planar deBroglie waves, and spherical Compton waves with a phase velocity of c.

Section V illustrates the point Systemfunction is a solutions to the classical KG equation, showing the relativistic mechanical properties, and the classical electromagnetic particle-particle interactions.

# Preliminaries

## A. Reviewing the standard QM relations

Free KG:

$$\left(-\frac{\partial^2}{\partial x^2}-\frac{\partial^2}{\partial y^2}-\frac{\partial^2}{\partial z^2}+\frac{\partial^2}{\partial t^2}\right)\psi=-m_0^2\,\psi \qquad (2)$$

or in Feynman slash notation:

$$\left(\not{\partial}^2 + m_0^2\right)\psi = 0 \qquad \not{\partial} = \gamma^\mu \partial_\mu \tag{3}$$

The mass $m_0$ is the invariant rest mass, and $\psi$ is the one component KG field.

Dirac:

$$\left(i\not{\partial} - m_0\right)\psi = 0 \tag{4}$$

Dirac with potential operator:

$$\left(i\not{\partial} \;-\; Q\not{A} - m_0\right)\psi = 0 \tag{5}$$

The wavefunction, $\psi$ in this case is the Dirac spinor field, and $\not{A}$ is the electromagnetic gauge field in which the charge is immersed.

Schrödinger free particle:

$$\left(\hbar^2 \partial^k \partial_k + 2im_0\partial_0\right)\psi = 0 \tag{6}$$

The notation is such that the rest mass is the reciprocal of the Compton radius $1/\lambdabar_0 = m_0 c/\hbar \to m_0$, and he units are the *natural units* $\hbar = c = 1$, except for clarification at section ends. (For general conventions, and notation, see **Appendix I**)

The point solutions in this paper will be designated as single component scalar, *Systemfunction* $\tilde{\Theta}$, to distinguish from standard wavefunctions, and it is presumed to be a solution to:

$$\left(\not{\partial}^2 + m_0^2\right)\tilde{\Theta} = 0 \tag{7}$$

If the Systemfunction is factored into a real amplitude $\left(\tilde{\Theta}_R\right)$ and phase $\left(\tilde{\Theta}_I\right)$ functions such that:

$$\tilde{\Theta} = \tilde{\Theta}_R \tilde{\Theta}_I \tag{8}$$

*Separation*

Using the standard geometrical algebra factoring of the KG operator using the Clifford-Dirac gamma matrix Eq.(7), is:

$$\left(i\cancel{\partial} + m_0\right)\left(i\cancel{\partial} - m_0\right)\tilde{\Theta} = 0 \quad , \tag{9}$$

It is easy to show by the chain rule that $\tilde{\Theta}_I$ cannot be a solution of:

$$\left(i\cancel{\partial} - m_0\right)\tilde{\Theta}_I \neq 0 \tag{10}$$

Unless any amplitude factor is a solution of:

$$\cancel{\partial}\tilde{\Theta}_R = 0 \tag{11}$$

See **Note 2.**

This implies that the gradient of the amplitude function is a constant, or is a null vector, thus having a phase velocity of c. This restriction on $\tilde{\Theta}_R$ can be met in the context of coordinate system originating at the initial event at $x = 0$, and $t = 0$, and traveling away from that event at the speed of c (see later development).

The linear expression for the phase, excluding the amplitude, is obtained directly from Eq.(9) with the condition of Eq.(11):

$$\left(i\cancel{\partial} + m_0\right)\left[\left(i\cancel{\partial} - m_0\right)\tilde{\Theta}_I\right]\tilde{\Theta}_R = 0 \tag{12}$$

Since $\mathrm{Det}\ |AB| \ = \ \mathrm{Det}\ |A| \ \times \mathrm{Det}\ |B|$, the square brackets can be picked from Eq.(12), and set as:

$$\mathrm{Det}\left|\left(i\cancel{\partial} - m_0\right)\tilde{\Theta}_I\right| = 0 \tag{13}$$

The eigenvector equation for this expression is thus:

$$i\cancel{\partial}\ \mathbf{a}\tilde{\Theta}_I = m_0\ \mathbf{a}\tilde{\Theta}_I \tag{14}$$

The complete Systemfunction is a solution to equation Eq.(7), the amplitude is a solution to Eq.(11), and the phase part of the Systemfunction is a solution to Eq.(14),

The factorization has also induced an expansion of the number of solutions to the equation Eq.(7), by four. These extra solutions are for spin ½ particles, and the expression is identical in form to the Dirac expression if $\psi = \mathbf{a}\tilde{\Theta}_I$

## B. Action & Wavefunctions

In this section a particle based solution, to Eq.(7), and Eq.(14), will be developed based of the square of a classical path of action, over a Minkowski fourspace interval defined over the life of the system of particles.

In the Path Integral formulation of QM, the amplitude of the probability for the m particle to transition from one state to another is the integral over a scalar action between the two states over all possible paths. The path integral depends on the final coordinate and time in such a way that it obeys the Schrödinger equation, [2], thus heuristically for a Schrödinger wavefunction:

$$\psi(x_f, t_f) = \int dx_i \left[ \int Dx(t) e^{i\int \frac{L}{\hbar} dt} \right] \psi(x_i, t_i) = \int dx_i \left[ \int Dx(t) e^{i\frac{S}{\hbar}} \right] \psi(x_i, t_i) \tag{15}$$

Where the propagator for the wavefunction going from an initial state *i* to a final state *f* is:

$$K\left(z_f t_f ; x_i t_i\right) = \int Dx(t) e^{i\frac{S}{\hbar}} \tag{16}$$

And S is the particle action.

Analogously this development will propose a point solution to the expressions for Eq.(7), and Eq. (14), that represents a single particle, starting in the system at the Big Bang, (T = 0) and transitioning to the current event.

Observation of the wavefunction for a system of particles in the Schrodinger picture from Eq.(15), suggests that for a system of particles coming into existence the initial state at the big bang should be a constant $(x_i, T_i = 0,0)$.

$$\psi(x_{1i}, t_{1i} \cdots x_{ni}, t_{ni}) = \text{cons}\tan\text{t} \tag{17}$$

Thus the primary constituent of the wavefunction going from initial to the current state is just the propagator. The path for the propagator considered here will be the classical path which is sufficient to illustrate the concepts. Path integral summation would induce refinements, but as in standard methodology the general picture would not be changed.

The Systemfunction would thus become:

$$\Theta = \int dx_i \left[ \int Dx(t) e^{i\frac{S}{\hbar}} \right] \psi(x_i, t_i) \rightarrow A e^{i\frac{S}{\hbar}} \tag{18}$$

## C. Defining the Lagrangian

The action of Eq.(16), and, Eq.(18), for a particle is in general the time integral of the path, and in the classical view the path taken minimizes this functional.

For the relativistic action of a particle it is presumed that a particle follows a path through 4-space that minimizes the space-time interval. In curved space-time, this path would be geodesic, but for our purposes, spacetime will be considered as flat, and the Minkowski action is:

$$S = \int_{t_i}^{t_f} L dt d^3x \tag{19}$$

L is an invariant, t & x are the spacetime coordinates, and S is an invariant action integrated over the spacetime interval. (Henceforth, $\hbar = 1$).

For defining the Lagrangian, the spacetime vector four potential generated by a charged particle is.

$$\not{A} = \pm\alpha \left( \frac{\not{v}}{r} \right) \tag{20}$$

Where $\not{v}$ the four-velocity, r is the distance to the particle, $\pm$ is the charge of the particle, and $\alpha$ is the fine structure constant [1]. (This is slightly a nonstandard notation, since it contains $Q^2$, but it is helpful this development.)

From experimental observations of charged particles, there is a photon scattering sphere at the classical electron radius, $\alpha\lambda\!\!\!^{-}$ which is also the radius at which the integral of the electric energy equals the mass.

For a charged particle, it is proposed that the potential is actually a function of the distance to the charge radius and not the central point. Instead of the potential prescribed by Eq.(20), the gauge potential for charged particles is proposed to be:

$$\not{A} = \pm\alpha\left(\frac{\not{\varphi}}{r-\alpha\lambda\!\!\!^{-}}\right) \tag{21}$$

The $\pm_j$ is the charge sign of the particle, and $\lambda\!\!\!^{-}_j = \hbar / mc$, is the Compton radius.
Since the photon cross section of a particle decreases as the velocity is increased [8], the mass in $\lambda\!\!\!^{-}$ is the relativistic mass, and the function is shown in figure 1.

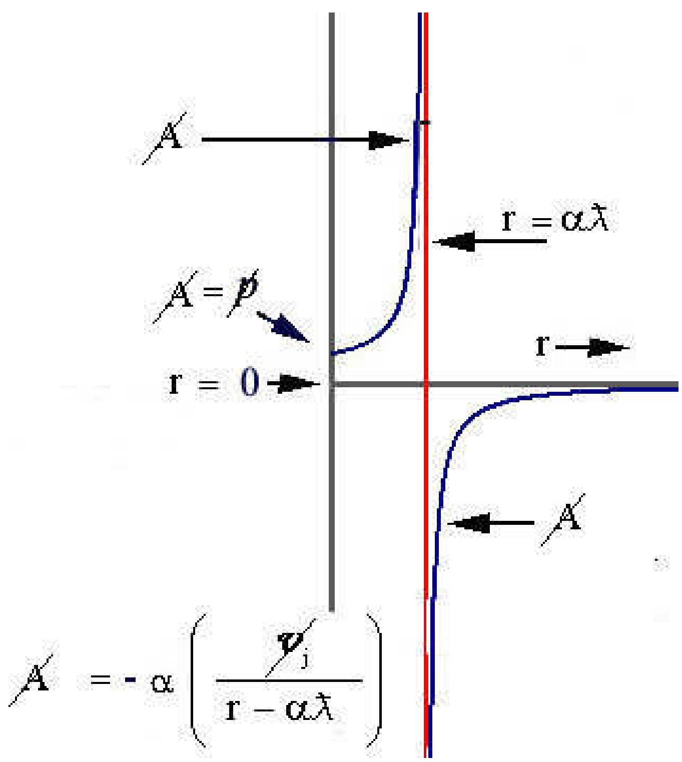


**Figure 1.**T Plot of modified charged particle Potential

The $\alpha\lambda$ term in Eq.(21), is the classical electron radius, and it can be noted that at the center of the particle $r=0$, the value of $\not{A}$ is not infinite, but the negative of the four-momentum for that particle, or:

$$-\not{A}_n\Big|_{r_n=0} \rightarrow m\not{v}_n = \not{p}_n \tag{22}$$

Thus, the sum of the gauge potentials of a collection of particles, evaluated at a single n particle:

$$\left(c\,m\not{v}_n - \sum_j \not{A}_j\right)\Bigg|_{r_n=0} = \left(c\not{P}_n - \sum_j \not{A}_j\right)\Bigg|_{r_n=0} \quad , \tag{23}$$

This is the canonical momentum for the n particle. The magnitude has the appearance of a Lagrangian for a particle in a collection of other charged particles. Both terms are covariant, and thus the sum is covariant. [6]

Noting that the standard classical Lagrangian for a particle in an electromagnetic field is:

$$L = -m_0c^2\sqrt{1-\frac{v^2}{c^2}} + Q(\phi - A\bullet v) = -m_0c^2\sqrt{1-\frac{v^2}{c^2}} + \not{A}\bullet\not{v} \quad , \tag{24}$$

This is not a covariant function but is shown to yield a relativistic action.[9]

It is proposed that instead of Eq.(24), an acceptable relativistic Lagrangian for a particle in a collection of charged particles, which is covariant, is the magnitude of Eq.(23)

$$L_n = \left| c\not{p}_n - \sum_j \not{A}_j \right| \tag{25}$$

The vector function is just the canonical momentum, and the n particle selection can be the Lagrangian for any of the particles in the system. (See **Note 1,** for discussion of this selection)

More explicitly in velocity terms Eq.(25), is:

$$L_n = \left| \underset{n}{\pm} c\,m_n\not{\boldsymbol{v}}_n - \sum \underset{m}{\pm}\alpha\left(\frac{\not{\boldsymbol{v}}_j}{r_j - \alpha\lambda_j}\right)\right| \quad , \tag{26}$$

The velocities are geometrical fourspace vectors. (This should not be confused with an integral of a Lagrange field density.)

## D. Total Lifetime Bivector Action

This section will define the total spacetime action of a particle from its initial existence in the universe to its present event.

Inserting the invariant Lagrangian of Eq.(25), into the spacetime action of Eq.(19), gives for the action:

$$\left(S_n\right) = -\int_{t_i}^{t_f} \left| c\,\cancel{p}_n - \sum_j \cancel{A}_j \right| dt\, d^3x \tag{27}$$

As noted in Eq.(22), the center of the n particle is the endpoint.

As was noted for Eq.(17), the action for the particle is to be summed over the interval from the initiation of the system, (Big Bang) to its current event. This interval is expanding at c and thus must be a null vector, consistent with our solution for the amplitude function, EQ.(11). Since the particle is moving at the velocity of light with respect to the initial event the time and space integrals are on coordinate time and the sum of the amplitude action is therefore, zero.

$$\cancel{\eta}T = \;= \left(\gamma^0 + \vec{\eta}\cdot\vec{\gamma}\right)T \;\; = \left(\gamma^0 T + \gamma^k X_k\right) \tag{28}$$

Where T is the interval time and X is the observed radius of the universe, now 13.8 billion years. Then:

$$\left(S_n\right) = \left| \left(\gamma^0 T + \gamma^k X_k\right)\left(\cancel{p}_n - \sum_j \cancel{A}_j\right) - S_0 \right| \tag{29}$$

S is the magnitude of the lifetime action of a particle from inception to the current event, and $S_0$ is the initial value of the action.

It has been presumed that the particle has followed a path through 4-space that minimizes the space-time interval from the initial event to the current event and that the classical vector integral over a flat Minkowski geodesic is that path.

The frame of the action is the frame in which the Big Bang is the zero point, at rest in space, and the particle is traveling at c. If the action over the life of the particle is to be calculated, the initial frame has to be the proper starting point. The particle is moving along the light cone, a null vector, in a direction away from the BB point. Relativistically the particle is moving at c respect to the BB frame, and proper time and relative motion is stopped, thus there is no contribution of the local relative velocity to the action integral. The action can thus be trivially integrated.

*Initial value*

Without orientation provided by the pseudoscalar of the fourspace, the action of a particle can have no spin. It is therefore asserted that the initial action at T = 0 must contain the unit fourspace orientation vector $I$ . The pseudoscalar is represented in fourspace geometrical algebra by $I = \gamma^1\gamma^2\gamma^3$ which serves the role of i in complex bivector spaces. In this case it is a unit orientation vector and not the spin per se. This can be noted from its role in vector rotation.

$$e^{I\varphi} = \cos(\varphi) + I\sin(\varphi) \tag{30}$$

The initial action for a half spin particle should be the spin or $1/2\hbar$ so $S_0$ will be set as 1/2 the unit bivector,

$$S_0 = \frac{1}{2} I\gamma^k \tag{0.31}$$

$\gamma^k$ is the initial direction.

The total lifetime particle action, S’, then is the complex bivector.

$$S_n = \left| \left(\gamma^0 T + \gamma^k X_k\right)\left(c\,\not{p}_n - \sum_j \not{A}_j\right) - \frac{1}{2} I\gamma^k \right| \tag{32}$$

This is the lifetime fourspace action of a charged particle existing at the current time.

In terms of the standard view of action, this would be:

$$S \to \left| S - \frac{1}{2} I \right| \tag{33}$$

For the purposes of the next section it is noted that the invariant magnitude of a velocity-position bivector AB is:

$$|AB|^2 = ABBA\,, \tag{34}$$

For simplicity it will be designated as the square, [7] thus:

$$(iS_n)^2 = \left[ \left(\gamma^0 T + \gamma^k X_k\right)\left(c\,\cancel{p}_n - \sum_j \cancel{A}_j\right) + \frac{1}{2} I \gamma^k \right]^2 \;, \tag{35}$$

or heuristically can be expressed as:

$$(S')^2 = \left(S^2 + iS - \frac{1}{4}\right) \tag{36}$$

The square of the magnitude defined in Eq.(36), is scalar but not just the square of the action between two events, but also includes a linear component representing the phase difference between the initial and final event.

In particle velocity terms the square of the lifetime action of the n particle in the potential of all the other charged particles, Eq.(35), is:

$$(iS'_n)^2 = -\left[ \left(\gamma^0 T + \gamma^k X_k\right)\left(cm_n\,\cancel{v} - \sum_m \pm \frac{\alpha\,\cancel{v}_j}{r_j}\right) + \frac{1}{2} I \gamma^k \right]^2 \quad , \tag{37}$$

## I. PROPOSED COMPLETE SOLUTION, THE SYSTEMFUNCTION

This section constructs a point solution that properly models both the phase and amplitude of a particle existing in a collection of other particles originating in an initial single event, and satisfying both the Klein-Gordon and Dirac equations.

Focusing on the action path solutions to the Schrödinger equations, Eq.(15), and noting that a particle can be considered as an energy level or state of the system. It is presumed that the initial state for a particle in the Big Bang is a constant. $(x, T = 0, 0)$.

$$\psi\left(x_{1i}, t_{1i} \cdots x_{ni}, t_{ni}\right) = \text{constant} \tag{38}$$

Thus the primary constituent of the wavefunction must be the propagator that evolves the state to the final state. For the classical path Eq.(15), would be:

$$\psi\left(x_f, t_f\right) = \int Dx(t)\, e^{i\frac{S}{\hbar}} \rightarrow e^{i\frac{S}{\hbar}} \tag{39}$$

An observation of Eq.(36), shows an action square which has both the linear phase relation of the action, and the real square of the action which must be related to the relativistic mass. It is proposed that the square of this is the basis of a propagator action that includes more than just the standard first order terms; there are also real terms that set the value of the amplitude factors of the solutions to the KG equation.

It is proposed that the Systemfunction point solution, $\tilde{\Theta}$, for a particle satisfying Eq.(7), and Eq.(14), is:

$$\tilde{\Theta}_n = Ae^{(S')^2\big|_{r_n = 0}} = Ae^{\left(S^2 + iS - \frac{1}{4}\right)\big|_{r_n = 0}}, \tag{40}$$

Note that the phase part of the function is scalar, linear, and somewhat conventional, but the $S^2$, is real, and constitutes the amplitude of the particle function. The $r_n = 0$, indicates endpoint action evaluated at the center of the n particle.

The Systemfunction then from Eq.(37), for the single n particle is:

$$\tilde{\Theta} = Ae^{\left(\left(T^2 - X^2\right) \times \left(c\cancel{p}_n - \sum_j \cancel{A}_j\right) \times \left(c\cancel{p}_n - \sum_k \cancel{A}_k\right) + I\gamma^k T \cancel{\eta} \bullet \left(c\cancel{p}_n - \sum_j \cancel{A}_j\right)\right)} \tag{41}$$

The function will be shown to be a solution of the KG Eq.(7), and the phase factor is a solution to Eq.(14), and the amplitude is a solution to Eq. (11).

Note that the first term, which is the square of the real action, is zero $\not{\eta}\not{\eta} = 0$, Making the real term a ratio of the space and time functions a constant equal to one.

$$\tilde{\Theta} = A\ \frac{e^{\left(T^2\left(c\not{p}_n - \sum_j \not{A}_j\right)\times\left(c\not{p}_n - \sum_k \not{A}_k\right)\right)}}{e^{\left(X^2\left(c\not{p}_n - \sum_j \not{A}_j\right)\times\left(c\not{p}_n - \sum_k \not{A}_k\right)\right)}} \times e^{+I\left(\gamma^0 T + \gamma^k X_k\right)\bullet\left(c\not{p}_n - \sum_j \not{A}_j\right)} \tag{0.42}$$

Note, also that at the center of the function $T^2 = X^2$, the value of the real function is A, and at a distance $\Delta x$ from the center, with the velocity equal to zero the observed function is:

$$\tilde{\Theta}_{\Delta x} = Ae^{-\left(\frac{Mc}{\hbar}\Delta x\right)^2} \tag{0.43}$$

This function describes a real particle having a spherical Gaussian shape with the proper dimensions for a classical particle such an electron.

The phase function of Eq.(41), comes from:

$$\frac{I\gamma^k}{2}\left(\gamma^0 T + \gamma^k X_k\right)\left(c\not{p}_n - \sum_j \not{A}_j\right) + \left(c\not{p}_n - \sum_k \not{A}_k\right)\left(\gamma^0 T + \gamma^k X_k\right)\frac{I\gamma^k}{2}$$
$$= I\gamma^k\left(\gamma^0 T + \gamma^k X_k\right)\bullet\left(c\not{p}_n - \sum_j \not{A}_j\right)$$

(44)

The first term in the exponent of Eq.(41), is real, scalar, and related to the classical properties including particle mass ratios. The product $\not{\eta}\not{\eta}$ is the square of a null vector and always zero, and thus the exponential does not run away. The Second term is imaginary, scalar, and generates the phase or QM properties of the function.

It is strange that the KG differentials operating on the first term are not also zero, but since the differentials are a difference between the time and space coordinates, rather than a sum, it has a value.

The $\not\eta$ vector is not a field variable as would be the case for a Dirac particle wavefunction, but is the null location vector of the particle with respect to the initial event.

## II. SEPARATING THE SYSTEMFUNCTION

The Systemfunction $\tilde{\Theta}$ for the n particle defined in Eq.(40), as shown earlier, Eq.(8), can be separated into a product of amplitude and phase functions, and the single equation can be separated into amplitude and phase equations.

$$\tilde{\Theta} = \Theta_R \Theta_I \quad , \tag{45}$$

From Eq.(41), the amplitude scalar Systemfunction is:

$$\tilde{\Theta}_R = Ae^{\left(\gamma^0 T + \gamma^k X_k\right)^2 \times \left(c\not p_n - \sum_j \not A_j\right) \times \left(c\not p_n - \sum_k \not A_k\right)} , \tag{46}$$

The scalar phase Systemfunction for the n particle and solution to Eq.(14), is:

$$\tilde{\Theta}_I = Ae^{IT\not\eta \bullet \left(c\not p_n - \sum_j \not A_j\right)} \tag{47}$$

## III THE FIRST ORDER COMPLEX FUNCTION PROPERTIES

This section shows the phase term is a solution to the first order equation.

Letting $I = \mathrm{i}$ for familiarity, and $\mathrm{T} = \mathrm{t}$, be the local time, since the phase is cyclic, the scalar complex portion of the function of Eq.(47), is then:

$$\tilde{\Theta}_I = A e^{\mathrm{i t} \not{\eta} \bullet \left( c \not{p}_n - \sum_j \not{A}_j \right)} \tag{48}$$

Noting that:

$$\mathrm{t}\not{\eta} = \left( ct\gamma^0 + \gamma^1 x + \gamma^2 y + \gamma^3 z \right) \qquad (ct)^2 = |r|^2 \tag{49}$$

$$\mathrm{t}\not{\eta} \bullet \not{p} = m \not{\eta} \bullet \not{v} = Et + p \bullet \vec{r} \tag{50}$$

The free particle part of the exponent in Eq.(48), becomes identical to the Dirac free particle wavefunction:

$$\tilde{\Theta}_I = A e^{\mathrm{i} \left( \left( Et + p \bullet \vec{r} \right) - \sum \left( t \not{\eta} \bullet \sum_j \not{A}_j \right) \right)} \tag{51}$$

If the potential term is ignored the form of the phase Systemfunction is identical to the Dirac wavefunction for a massive free particle. The difference is that the Systemfunction is the value of the function only at an event in spacetime, at the center of a particle, whereas the Dirac solution is the probability amplitude throughout spacetime. In the Dirac case the solution is a function of r, whereas the r in the Systemfunction is the particle location.

### A. Gauge Potential Equivalence

The notable difference in The Systemfunction, Eq.(51), and the Dirac wavefunction is the presence of the potential in the function. This can be shown to be equivalent to the Dirac solution of an equation with the potential operator included.

Defining the phase function of Eq., as $\not{\partial}\tilde{\Theta}_I = \not{\partial}\tilde{\Theta}_a \tilde{\Theta}_b$ , where the potential function for the j particle is a separate function:

$$\tilde{\Theta}_b = \exp\left[ \mp \mathrm{i} \sum \left( t \not{\eta} \bullet \not{A}_j \right) \right] , \tag{52}$$

Noting the explicit form of the null vector $t\not{\eta}$ Eq.(49), multiplying and taking the derivative the result is:

$$\not{\partial}\left[\mp it\not{\eta}\bullet\not{A}_j\right] = \pm i\not{A}_j \tag{53}$$

(The more conventional form of $\not{A}$ would have Q $\not{A}$ .)

Applying the chain rule to the scalar Systemfunction:

$$\not{\partial}\tilde{\Theta}_I = \not{\partial}\tilde{\Theta}_a\tilde{\Theta}_b = \tilde{\Theta}_b\left(\not{\partial}\tilde{\Theta}_a\right)+\tilde{\Theta}_a\left(\not{\partial}\tilde{\Theta}_b\right) = \tilde{\Theta}_b\left(\not{\partial}\tilde{\Theta}_a\right)+\tilde{\Theta}_a\tilde{\Theta}_b\left(iQ\not{A}\right) \tag{54}$$

Thus:

$$\tilde{\Theta}_b\left[-i\not{\partial}+\not{A}_j\right]\tilde{\Theta}_a = m_0\tilde{\Theta}_a\tilde{\Theta}_b, \tag{55}$$

or:

$$\left[-i\not{\partial}+\not{A}_j\right]\tilde{\Theta}_a = m_0\tilde{\Theta}_a \tag{56}$$

The phase Systemfunction with the potential included in the solution has equivalence with the Dirac wavefunction, as a solution to the Dirac operator, with a potential operator.

The phase Systemfunction is thus a solution to the first order Dirac equation.

## IV OBSERVATION FIELD

It is undisputable that the wave structure associated with the Dirac and Schrodinger first order equations are associated with probability amplitude, and it is well known that some of the associated negative wave structures are not related to probability amplitudes. The following is a presentation of a wave structure associated with the massive point particle defined here that exhibits the known features and interaction properties

The function value of Eq.(47), at the $r_n = 0$ position is can be observed in the future along the direction of the same null vector locating the particle in the initial event coordinate system. The direction to the initial event is ubiquitous, meaning that the Big Bang is at 13.8B light years distant, but the direction is arbitrary over 4 pi steradians. If an arbitrary null interval is added to the null location vector the, value of the

Systemfunction event at the particle can be observed at a distant point at a future time, which is at the time of arrival of light from the event.

$$\not{n}T \rightarrow \not{n}\left(T + r_o\right) \ , \tag{57}$$

The value depends on the angle, distance, and time from the particle, and can be referred to as the "observation field" of the point function

Figure 4 shows the n particle with the $\kappa$ event located at the center $r_n = 0$, as observed at another event. The value of the Systemfunction, which is a point function, is viewed along the null observation vector $\not{n}r_o$, and has a different value depending on distance and direction from the event.

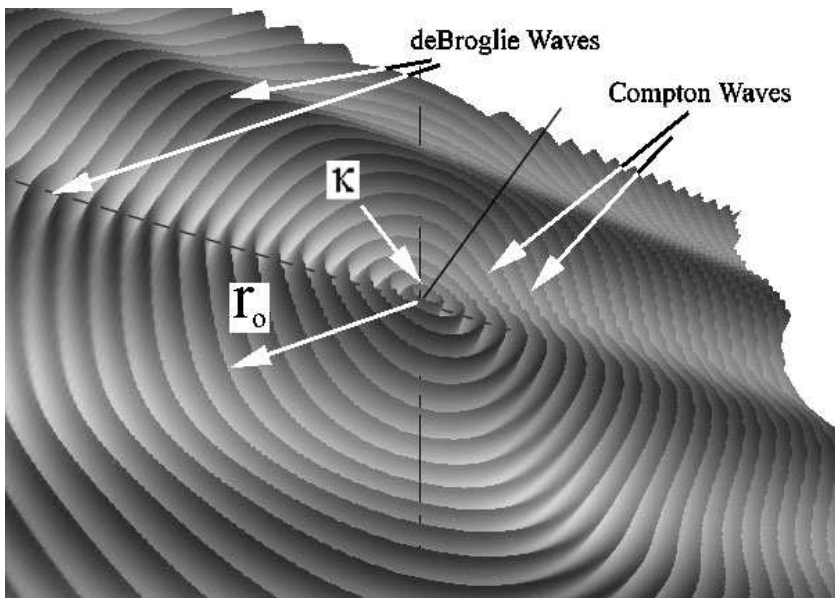


**Figure 4**. Observed value of $\Theta_I$ function evaluated at n particle as viewed at $r_o$

This is the observation of the value of the function at the $\kappa$ event as transmitted by a photon. There is no physical field, or any substance defined, at that point. $r_o$. This observation field would represent the interaction of the point particle with another particle, as communicated by a photon from the point particle.

## V THE CLASSICAL KG EQUATION

Putting the Systemfunction, $\tilde{\Theta}$ into the KG equation, results in the standard coordinate free equations of motion for a charged particle in the presence of the electromagnetic field of other particles, and illustrates that the same Systemfunction is not only a solution to the Dirac Equation but also the Klein Gordon equation.

Putting the complete Systemfunction of Eq.(41):

$$\tilde{\Theta} = Ae^{\left. \left( \left(T^2 - X^2\right) \times \sum_{j=1}^{N} \left( c\not{p}_n - \sum_j \not{A}_j \right) \times \sum_{k=1}^{N} \left( c\not{p}_n - \sum_k \not{A}_k \right) + i \sum \left( \gamma^0 T + \gamma^k X_k \right) \bullet \not{A}_j \right) \right|_{r_{n=0}}}, \tag{58}$$

into Eq(7),

$$\left( \not{\partial}^2 + m_0^2 \right) \tilde{\Theta} = 0, \tag{59}$$

and executing the differential, gives:

$$-\left( c\not{p}_n - \sum_j \not{A}_j \right)\left( c\not{p}_n - \sum_k \not{A}_k \right) = -m_0^2 \tag{60}$$

Expanding Eq.(60), and using notation in Eq.(35),

$$-\left( \left( cm_n \not{v} \right)^2 + \left( cm_n \not{v}_n \right) \sum_j \not{A}_j + \sum_k \not{A}_k \vec{S}_n \left( cm_n \not{v}_n \right) \bullet\bullet\bullet \right) = -m_0^2 \tag{61}$$

Explicitly putting in the potential from Eq.(21), for the m particles this is:

$$-\left( cm_n \not{v}_n \right)^2 + \sum_j \pm_n \pm_j 2m_n \frac{\alpha}{r_j} \not{v}_j \bullet \not{v}_n = -m_0^2 \tag{62}$$

Or in more elementary terminology:

$$\left(\frac{m_{n0}c}{\hbar}\right)^2 = \left(\frac{m_n c}{\hbar}\right)^2\left[\left(1-\frac{\vec{v}_n^{\,2}}{c^2}\right)-\sum \pm_n \pm_j \frac{2}{m_n c^2}\frac{Q^2}{r_j}\left(1-\frac{\vec{v}_j}{c}\bullet\frac{\vec{v}_n}{c}\right)\right], \tag{63}$$

Taking the square root gives the familiar classical linear Lagrangian expression for a particle the presence of other electrically charged particles. Note that the proper electromagnetic interaction between the particles could not have been possible without the square of the action of Eq.(35).

The square root of Eq.(63), is:

$$m_{n0}c^2 \approx +\left[m_n c^2\left(1-\frac{1}{2}\vec{v}^{\,2}{}_n\right)\mp\sum_j \frac{Q^2}{r_j}\left(1-\vec{v}_j\bullet\vec{v}_n\right)\right] \tag{64}$$

Note this equation is the particle-particle electromagnetic interaction Lagrangian, and the classical coordinate free equation of motion for the particle. (See Appendix II) The amplitude of the function is therefore just the classical mass of a particle in an electromagnetic field. This equation is the core of the Bohr-Sommerfeld model of the atom, thus showing that the Systemfunction contains both the Quantum and Classical descriptions of particles.

The amplitude factor of the solution thus has the proper classical values of the electromagnetic interactions, but the phase factor of the Systemfunction demonstrated earlier, sets the quantization rules and values.

It is of interest to note that the next term from the expansion of Eq.(21), in the bracket of Eq.(64), is:

$$\frac{Q^2}{r}\left(\frac{\alpha\lambda_j}{r}\right)\left(\frac{\cancel{\boldsymbol{v}_n}\bullet\cancel{\boldsymbol{v}_j}}{c^2}\right), \tag{65}$$

This is the classical value of the coefficient of the Larmor spin orbit interaction energy interaction which is:

$$\Delta\boldsymbol{H}_L = \frac{Q^2}{r}\left(\alpha\frac{\lambda\!\!\!^{-}_e}{r}\right)\left(\frac{1}{n}\frac{\vec{v}\times\vec{r}\bullet\boldsymbol{S}}{vr\hbar}\right) \tag{66}$$

This illustrates that the classical amplitudes do not contain the QM detail available in the multiple phase solutions of the linear form.

From the foregoing, it has been shown that the real amplitude factor of the Systemfunction is a solution to the KG equation, and thus the Systemfunction is a solution to both the quadratic and linear forms of the KGD equation.

## CONCLUSION

An alternate point particle solution has been presented that is a solution to both the KG and Dirac expressions. It is only viable in the context of the particle system as a whole, with the initial event with T = 0 (Big Bang). The solution defines both classical (amplitude), and quantum mechanical, (phase) properties of charged particles originating in a system of similar particles heretofore not connected. This is a straightforward arithmetical approach in geometric algebra to a solution of the equation in a universe that is expanding at c. The constructs and observables are outside the normal QM parameters, and thus do not conflict with standard QM methods or results, but adds another perspective. It is presumed that the defining the connection between the real amplitude and the imaginary phase factors of a complete wavefunction can be used in defining particle mass ratios.

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# Appendix I

## *Definitions, Notation, and Conventions*

Feynman slash notation:

$$\not{A} = \gamma^{\mu} A_{\mu} \qquad \not{a}\,\not{a} = a^{\mu} a_{\mu} = a^{2} \qquad \not{a}\,\not{b} + \not{b}\,\not{a} = 2a \cdot b$$

The radius of particle system $\Re = cT = \Re_0 + ct$

Four velocity $\gamma^{\mu} v_{\mu} = \not{v}$

Three velocity $\gamma^{k} \left(v\right)_{k} = \vec{v} = \gamma^{k} \cdot \vec{v}$

Null unit vector $\not{\eta} = {}^{\mu}\gamma\eta_{\mu} = \left(\gamma^{0} + \vec{\eta} \cdot \vec{\gamma}\right) \qquad \vec{\eta} \cdot \vec{\eta} = -1$

Mass in this paper $m = \dfrac{mc}{\hbar} = \dfrac{1}{\lambdabar}$

Rest mass $m_0 = \dfrac{m_0 c}{\hbar}$

Compton radius $\lambdabar = \dfrac{\hbar}{mc}$

Vector 4 potential $\not{A}$

Most equations $\hbar = c = 1$

Charge sign of j particle $\underset{j}{\pm}$

Derivatives in slash notation $\gamma^{\mu} \dfrac{\partial}{\partial\left(x\right)_{\mu}} = \gamma^{\mu} \partial_{\mu} = \not{\partial}$

Local reference coordinates x

Initial event coordinates X

The Weyl representation of the Dirac gamma matrix:

$$\gamma^1 = \begin{bmatrix} & & & +1 \\ & & +1 & \\ & -1 & & \\ -1 & & & \end{bmatrix} \quad \gamma^2 = \begin{bmatrix} & & & -i \\ & & i & \\ & i & & \\ -i & & & \end{bmatrix} \quad \gamma^3 = \begin{bmatrix} & & 1 & \\ & & & -1 \\ -1 & & & \\ & 1 & & \end{bmatrix} \quad \gamma^0 = \begin{bmatrix} & & 1 & \\ & & & 1 \\ 1 & & & \\ & 1 & & \end{bmatrix}$$

$$\gamma^1\gamma^1 = -1, \quad \gamma^2\gamma^2 = -1, \quad \gamma^3\gamma^3 = -1, \quad \gamma^0\gamma^0 = +1.$$

Four velocity $\quad \not{\boldsymbol{v}} = \left(\gamma^1 v_x + \gamma^2 v_y + \gamma^3 v_z + \gamma^0 c\right)$

The product of two unitless four velocities:

$$\not{\boldsymbol{v}}_n \not{\boldsymbol{v}}_m = \left(\gamma^1 v_{xn} + \gamma^2 v_{yn} + \gamma^3 v_{zn} + \gamma^0 c\right)\left(\gamma^1 v_{xm} + \gamma^2 v_{ym} + \gamma^3 v_{zm} + \gamma^0 c\right)$$

or

$$\not{\boldsymbol{v}}_n \not{\boldsymbol{v}}_m = \left[\vec{v}_n \bullet \vec{v}_m + \bar{\sigma}\left(\vec{v}_n \times \vec{v}_m\right) + \gamma_4 c\left(\vec{v}_m - \vec{v}_n\right)\right] \qquad \bar{\sigma} = \gamma^1\gamma^2\gamma^3$$

The inner product:

$$\not{\boldsymbol{v}}_n \not{\boldsymbol{v}}_m + \not{\boldsymbol{v}}_m \not{\boldsymbol{v}}_n = 2\boldsymbol{v}_n \bullet \boldsymbol{v}_m$$

The outer product:

$$\not{\boldsymbol{v}}_n \not{\boldsymbol{v}}_m - \not{\boldsymbol{v}}_m \not{\boldsymbol{v}}_n = \left[2\bar{\sigma}\left(\vec{v}_n \times \vec{v}_m\right) + 2\gamma_4 c\left(\vec{v}_m - \vec{v}_n\right)\right]$$

## **Appendix II** (Standard units)

### ***Particle Interaction Lagrangian***

From Eq.(64), the interaction of a charged particle in an electromagnetic field of a charged particle j is:

$$m_{n0}c^2 = +\left[ m_n c^2 - \frac{1}{2} m_n \vec{v}^2{}_n \quad \underset{n}{\pm}\underset{J}{\pm} \left( \frac{Q^2}{r_J} \left( 1 - \frac{\vec{v}_J}{c} \bullet \frac{\vec{v}_n}{c} \right) \right) \right] \tag{1.1}$$

So the interaction Lagrangian is:

$$L_{Int} = \underset{n}{\overset{C}{\pm}}\underset{J}{\overset{C}{\pm}} \left( \frac{Q^2}{r_J} \left( 1 - \frac{\vec{v}_J}{c} \bullet \frac{\vec{v}_n}{c} \right) \right) \tag{1.2}$$

From Jackson p407 [6], the interaction Lagrangian for two particles is:

$$L_{int} = \frac{Q}{m_0 c} \left( p \bullet A \right) = Q \left( \cancel{v} \bullet \cancel{A} \right) \tag{1.3}$$

With:

$$m_0 = m \sqrt{1 - \left( \frac{v}{c} \right)^2} \tag{1.4}$$

From Jackson, [6], the Potentials for the classical electrodynamics interactions are: (*not* the quantum electrodynamics interactions ).

$$A = \frac{Q \cancel{v}}{r} \tag{1.5}$$

Thus

$$L_{int} = Q \frac{\vec{v}_n}{c} \bullet A_J \; = \; Q \left[ \gamma^0 + \gamma^k \left( \frac{v_n}{c} \right)_k \right] \bullet \left[ \frac{Q}{r_J} \left( \gamma^0 + \gamma^k \left( \frac{v_J}{c} \right)_k \right) \right] \; = \; \frac{Q^2}{r_J} \left( 1 - \left( \frac{v_J}{c} \right) \bullet \left( \frac{v_n}{c} \right) \right), \tag{1.6}$$

and is the same as our interaction Lagrangian in the classical particle Eq.(1.2).

## Note 1

*Alternate Electromagnetic Lagrangian*

The Standard classical non-covariant Lagrangian for a particle in an electromagnetic field is Eq.(24) , and using our defined notation Eq.(21), this is:

$$L = -m_0 c^2 \sqrt{1-\frac{v^2}{c^2}} + Q\left(\phi - A \bullet v\right) \quad = \quad -m_0 c^2 \sqrt{1-\frac{v^2}{c^2}} + \not{A} \bullet \not{v} \tag{2.1}$$

Letting $m_0 / m = 1/\gamma \rightarrow m_0 = m/\gamma$, and expanding:

$$L = -mc^2 + mv^2 + \not{A} \bullet \not{v} \tag{2.2}$$

In this proposal, the Lagrangian is the magnitude of the invariant gauge potential evaluated at the particle.

$$L_n = -\left| c\,\not{p}_n - \sum_j \not{A}_j \right| \tag{2.3}$$

The first order terms of this are:

$$L_n{}^2 = \left| c\,\not{p}_n - \sum_j \not{A}_j \right|^2 \quad = \quad c^2\,\not{p}_n{}^2 - 2c\,\not{p}_n \bullet \left(\not{A}_j\right) + \left(A\right)^2 \tag{2.4}$$

Taking square root of product;

$$L_n = -\sqrt{\left(mc^2\right)^2 \left(1-\frac{v^2}{c^2}\right) \pm 2mc^2\,\not{v} \bullet \not{A} + \left(A\right)^2} \tag{2.5}$$

$$L_n = -\left(mc^2\right)\sqrt{\left(1-\frac{v^2}{c^2}\right) \pm \frac{2}{mc^2}\,\not{A} \bullet \not{v}_n + \frac{\left(A\right)^2}{\left(mc^2\right)^2}} \tag{2.6}$$

Since most of the terms are small the square root is:

$$L_n = -\left(mc^2\right)\left(1 - \frac{1}{2}\frac{v^2}{c^2}\right) \mp \cancel{A} \bullet \cancel{v}_n + \frac{1}{2}\frac{\left(A\right)^2}{mc^2} \tag{2.7}$$

$$L_n = -mc^2 + \frac{1}{2}mv^2 \pm \cancel{A} \bullet \cancel{v}_n - \frac{\left(A\right)^2}{2mc^2} \tag{2.8}$$

Comparing the classical version Eq.(2.2), with the proposed relation Eq.(2.8), we see that the first order difference in the proposed version is that the kinetic energy enters with the classical value, whereas in the classical Lagrangian the enters with twice the classical value of the kinetic energy.

Since multiplying the Lagrangian by a constant or adding a constant, has no effect on the equations of motion, the difference between the classical and the proposed Lagrangian should have no effect on the predicted equations of motion.

It is straight forward to show the Lagrange equations of motion resulting from the proposed Lagrangian produce acceptable equations of motion for a free particle.

Momentum:

$$\frac{\partial L}{\partial \dot{x}^\mu} = 2p \tag{2.9}$$

Acceleration:

$$\frac{d}{dt}\left(\frac{\partial L}{\partial \dot{x}^\mu}\right) = 0 \tag{2.10}$$

Coordinate dependence:

$$\frac{\partial L}{\partial x^\mu} = 0 \tag{2.11}$$

**Note 2**

*Constraint on Amplitude Function*

Starting with the factored expression:

$$\left(i\cancel{\partial} + m_0\right)\left(i\cancel{\partial} - m_0\right)\tilde{\Theta} = 0 \quad , \tag{3.1}$$

Or:

$$\left(i\not{\partial} - m_0\right)\tilde{\Theta}_R\tilde{\Theta}_I = 0 \tag{3.2}$$

Applying the chain rule:

$$\left(i\not{\partial} - m_0\right)\tilde{\Theta}_R\tilde{\Theta}_I = \tilde{\Theta}_R i\not{\partial}\tilde{\Theta}_I + \tilde{\Theta}_I i\not{\partial}\tilde{\Theta}_R - m_0\tilde{\Theta}_R\tilde{\Theta}_I = 0 \tag{3.3}$$

Dividing by $\tilde{\Theta}_R$ this is:

$$\left(\frac{\tilde{\Theta}_I}{\tilde{\Theta}_R} i\not{\partial}\tilde{\Theta}_R\right) + \left(i\not{\partial} - m_0\right)\tilde{\Theta}_I = 0 \tag{3.4}$$

Thus:

$$\left(i\not{\partial} - m_0\right)\tilde{\Theta}_I = 0 \tag{3.5}$$

Only if:

$$\not{\partial}\tilde{\Theta}_R = 0 \tag{3.6}$$