The Equivalence Of Gravitational Field And Time

Ilija Barukčić [1, 2]

[1] Horandstrasse, DE-26441 Jever, Germany.
[2] Corresponding author: Barukcic@t-online.de

Manuscript submitted to viXra.org on Saturday, June 20, 2015.

Abstract: A proper discussion of the various philosophical views of the nature of time and gravitational field and the different issues related to time as such would take us far beyond the scope of this article. For our purposes, time and gravitational field are related somehow. In any case, especially due to Einstein's relativity theory, there is a very close relationship between time the gravitational field and vice versa. The aim of this publication is to work out the interior logic between gravitational field and time. As we will see, the gravitational field is equivalent to time and vice versa, both are equivalent or identical.

Key words: Quantum theory, relativity theory, unified field theory, causality.

1. Introduction

Time has always featured prominently discussion in philosophy and is especially associated with the logic of change. A good deal of the philosophical work with respect to time has been especially important since the beginning of philosophy as such. Under which conditions are we allowed to assume a period of time during which nothing changes? A brief overview of this and some other of the main topics in the philosophy of time like Platonism with respect to time, Fatalism, Reductionism cannot be given. Further coverage, an historical overview and a general presentation of these and other topics related to time as such are available or can be found in literature. According to this line of thought, we will analyze the relationship between time and gravitational field from the standpoint of Einstein's theory of special relativity.
Questions about the nature of time appear to be closely connected to the issue of gravitational field itself. Under conditions of special theory of relativity, clocks which are moving with respect to an inertial system of observation are found to be running more slowly. A clock which is closer to the gravitational mass (i. e. deeper in its "gravity well") appears to go more slowly than a clock which is more distant from the same mass (energy). For example, clocks on the former Space Shuttle where found to ran slightly slower than reference clocks on the Earth. Thus far, the stronger the gravitational potential (i. e. the closer a clock is to the source of the gravitational field, the slower time passes). The gravitational time dilation has been repeatedly confirmed experimentally by several tests of general relativity. Altogether, time as such is linked to the gravitational field and vice versa even if we still don't know how. This publication will make the proof that the relationship between time and gravitational field is similar to the relationship between mass and energy. Finally, we must accept the gravitational field- time equivalence.

Manuscript submitted to ViXra.org (Saturday, June 20, 2015)

2. Definitions

2.1. **Thought Experiments**

The general importance, acceptance and enormous influence of properly constructed models or (real or) thought experiments (as devices of scientific investigation) is backgrounded by some important and common features. One main way to do this, is to develop a model, in which the different, fundamental terms of the quantum formalism find a correspondence. Such a model is able to produce new predictions and new explanations. Especially, under conditions where it is too expensive or too difficult to run a real experiment a model or a thought experiment can help us to prove how to deal with some basic properties of the nature in a mathematically and logically consistent and appropriate way. Again, it is necessary to highlight the possibility of a thought experiment to get the right against or in favor of a theory or a hypothesis. However, it is worth being mentioned that a thought experiment can draw out a contradiction in a theory and thereby refuting the same. Furthermore, models and thought experiments used for diverse reasons in a variety of areas are at the end no substitute for a real experiment.

2.2. **Definition. The Relativistic Energy Of A System** ${}_{R}E$

Let

$${}_{R}E$$

denote the total energy of a system.

Scholium.
Due to Einstein, matter/mass and energy are equivalent.
“Da Masse und Energie nach den Ergebnissen der speziellen Relativitätstheorie das Gleiche sind und die Energie formal durch den symmetrischen Energietensor ($T_{\mu\nu}$) beschrieben wird, so besagt dies, daß das G-Geld [gravitational field, author] durch den Energietensor der Materie bedingt und bestimmt ist.“ [1]

2.3. **Definition. The Co-ordinate or Relativistic Time And Proper Time**

Let

$${}_{R}t$$

denote the relativistic (i. e. co-ordinate time). Let

$${}_{0}t$$

denote the proper time.

Scholium.
An accurate **clock in motion slow down** with respect a stationary observer (observer at rest). The proper time ${}_{0}t$ of a clock moving at constant velocity v is related to a stationary observer's coordinate time ${}_{R}t$ by Einstein’s relativistic time dilation [2] and defined as

$$ {}_{0}t = {}_{R}t \times \sqrt[2]{1 - \frac{v^2}{c^2}} \tag{1} $$

where ${}_{0}t$ denotes the "proper" time, ${}_{R}t$ denotes the "relativistic" (i. e. stationary or coordinate) time, v denotes the relative velocity and c denotes the speed of light in vacuum. Equally, it is

$$ \frac{{}_{0}t}{{}_{R}t} = \sqrt[2]{1 - \frac{v^2}{c^2}} \tag{2} $$

or

$$ \frac{{}_{0}t}{c^2} \times \frac{c^2}{{}_{R}t} = \sqrt[2]{1 - \frac{v^2}{c^2}} . \tag{3} $$

Coordinate systems can be chosen freely, deepening upon circumstances. In many coordinate systems, an event can be specified by one time coordinate and three spatial coordinates. The time as specified by the time coordinate is denoted as coordinate time. Coordinate time is distinguished from proper time. The concept of proper time, introduced by Hermann Minkowski in 1908 and denoted as ${}_{0}t$, incorporates Einstein's *time dilation effect.* In principle, Einstein is defining time exclusively for every place where a watch, measuring this time, is located.

"... Definition ... der ... Zeit ... für den Ort, an welchem sich die Uhr … befindet ..." [3]

In general, a watch is treated as being at rest relative to the place, where the same watch is located.

"Es werde ferner mittels der **im ruhenden System** befindlichen **ruhend**en Uhren die Zeit t [i. e. ${}_{R}t$, author] des ruhenden Systems ... bestimmt, ebenso werde die Zeit τ [${}_{0}t$, author] des **bewegten System**s, in welchen sich relativ zu letzterem **ruhend**e Uhren befinden, bestimmt..." [4]

Only, the place where a watch at rest is located can move together with the watch itself. Therefore, due to Einstein, it is necessary to distinguish between clocks as such which are qualified to mark the time ${}_{R}t$ when at rest relatively to the stationary system R, and the time ${}_{0}t$ when at rest relatively to the moving system O.

"Wir denken uns ferner eine der Uhren, welche **relativ zum ruhenden System ruhend** die Zeit t [${}_{R}t$, author], **relativ zum bewegten System ruhend** die Zeit τ [${}_{0}t$, author] anzugeben befähigt sind ..." [5]

In other words, we have to take into account that both observers have at least one point in common, the stationary observer R and the moving observer O are at rest, but at rest relative to what? The stationary observer R is at rest relative to a stationary co-ordinate system R, the moving observer O is at rest relative to a moving co-ordinate system O. Both co-ordinate systems can but must not be at rest relative to each other. The time ${}_{R}t$ of the stationary system R is determined by clocks which are at rest relatively to that stationary system R. Similarly, the time $_0t$ of the moving system O is determined by clocks which are at rest relatively to that the moving system O. In last consequence, due to Einstein's theory of special relativity, an accelerated clock ($_0t$) will measure a smaller elapsed time between two events than that measured by a non-accelerated (inertial) clock ($_Rt$) between the same two events.

2.4. **Definition. The Normalized Relativistic Time Dilation Relation**

As defined above, due to Einstein's special relativity, it is

$$\frac{{}_0t}{{}_Rt} = \sqrt[2]{1-\frac{v^2}{c^2}} \,. \tag{4}$$

The normalized relativistic time dilation relation [6] follows as

$$\frac{{}_0t^2}{{}_Rt^2} + \frac{v^2}{c^2} = 1 \,. \tag{5}$$

2.5. **Definition. The Mathematical Identity Of A System** $_RS$

Let

$${}_RS \equiv {}_RE + {}_Rt \,. \tag{6}$$

denote the mathematical identity of energy and time of a system.

Scholium.
The notion $_RS$ can but must not be the mathematical equivalent of a very simple form of space. Following Aristotele's principle of the excluded middle, *tertium no datur*, it is important to stress out, that **all but energy** is denoted as time. Consequently, there is no third between energy and time, a third is not given.
Let

$${}_OS \equiv {}_O E + {}_O t \tag{7}$$

where $_0E$ denotes the 'rest' energy and $_0t$ denotes the 'proper' time. Consequently, due to special relativity it is

$${}_OS \equiv \sqrt[2]{1-\frac{v^2}{c^2}} \times {}_RS \,. \tag{8}$$

2.6. **Definition. The Matter** $_RM$

Let

$${}_RM \equiv \frac{{}_RE}{c^2} \equiv \frac{{}_R\hat{H}}{c^2} \,, \tag{9}$$

where $_RM$ is (the quantum mechanical operator of) matter (and not only of mass [7]), c is the speed of the light in vacuum and $_R\hat{H}$ is the Hamiltonian operator.

2.7. **Definition. The Gravitational Field** ${}_{R}G$

The wavefunction of the gravitational field ${}_{R}G \equiv {}_{R}\Gamma$ describes the gravitational field completely. In general, it is

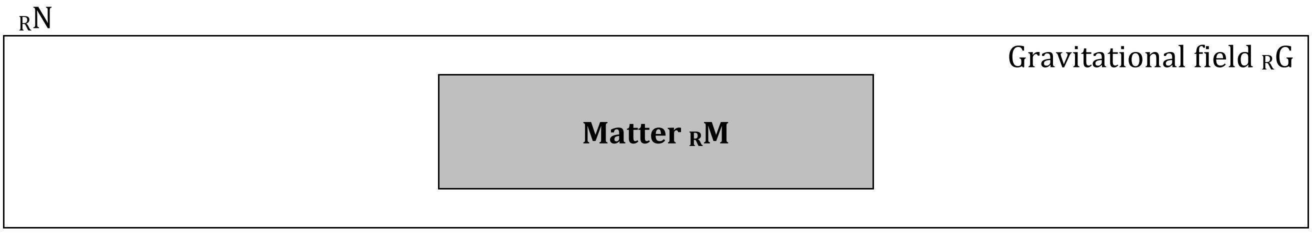
$$ {}_{R}G \equiv {}_{R}\Gamma \equiv {}_{R}N - {}_{R}M \; . \qquad (10) $$

where ${}_{R}M$ denotes matter (and not only of mass).

Scholium.
In our understanding, the relationship between matter and gravitational field is based on Einstein's definition of **matter** (i. e. not only mass) ex negativo. Einstein himself pointed out that everything but the gravitational field has to be treated as matter. Thus far, matter as such includes *matter in the ordinary sense* and the *electromagnetic field* as well. In other words, there is no third between matter and gravitational field. Einstein himself wrote:

"Wir unterscheiden im folgenden zwischen 'Gravitationsfeld' und 'Materie', in dem Sinne, daß alles außer dem Gravitationsfeld als 'Materie' bezeichnet wird, also nicht nur die 'Materie' im üblichen Sinne, sondern auch das elektro-magnetische Feld. " [8]

Einstein's writing translated into English:
>> We make a distinction hereafter between 'gravitational field' and 'matter' in this way, that we denote everything but the gravitational field as 'matter', the word matter therefore includes not only matter in the ordinary sense, but the electromagnetic field as well. <<
In terms of set theory we would obtain the following picture.



This approach to the relationship between matter and gravitational field is sometimes also referred to as the *de Broglie hypothesis* since all matter can exhibit wave-like behavior. Thus far, so called "matter waves", a concept having been proposed by Louis de Broglie in 1924, are a central part of the theory of quantum mechanics and a subject of ongoing debate too.

2.8. **Definition. The Mathematical Identity Of A System** ${}_{R}N$

From the standpoint of a stationary (fix) observer R let us define

$$ {}_{R}N \equiv {}_{R}M + {}_{R}G \qquad (11) $$

where ${}_{R}M$ denotes the matter and ${}_{R}G$ denotes the (wavefunction of the) gravitational field. From the standpoint of a moving observer O we define

$$ {}_O N \equiv {}_O M + {}_O G \tag{12} $$

where ${}_0M$ denotes the matter from the standpoint of a moving observer and ${}_0G$ denotes the gravitational field from the standpoint of a moving observer.

Scholium.
In the context of general relativity, Einstein himself demands that everything but the gravitational field has to be treated as matter. Thus far, matter as such includes matter in the ordinary sense and the electromagnetic field as well. In other words, there is no third between matter and gravitational field. In other words, matter and gravitational field are determining ${}_RN$.

2.9. **Definition. The Relationship Between ${}_R$S and ${}_R$N**

Let

$$ {}_R N \equiv \frac{{}_R S}{c^2} . \tag{13} $$

2.10. **Axioms.**

The following theory is based on the following axioms.

Axiom I.

$$ +1 = +1. \qquad \text{(Axiom I)} $$

3. Theorems

3.1. **Theorem. The Normalization Of The Relationship Between Energy And Time**

Claim.
The relationship between energy and time can be normalized as

$$1 \equiv \frac{{}_R E}{{}_R S} + \frac{{}_R t}{{}_R S}. \tag{14}$$

Proof.
Starting with Axiom I it is

$$+1 = +1. \tag{15}$$

Multiplying this equation by the wavefunction ${}_R t$ we obtain

$${}_R t = {}_R t. \tag{16}$$

Adding the ${}_R E$, the total energy of the system ${}_R E$ to this equation, it is

$${}_R E + {}_R t = {}_R E + {}_R t. \tag{17}$$

Due to our definition above, we obtain

$${}_R S \equiv {}_R E + {}_R t. \tag{18}$$

We divide this equation by ${}_R S$. The normalization of the relationship between energy and time follows as

$$1 \equiv \frac{{}_R E}{{}_R S} + \frac{{}_R t}{{}_R S}. \tag{19}$$

Quod erat demonstrandum.

Manuscript submitted to ViXra.org (Saturday, June 20, 2015)

3.2. **Theorem. The Normalization Of The Relationship Between The Matter And The Gravitational Field.**

Claim.
The relationship between matter and the gravitational field can be normalized as

$$1 = \frac{{}_{R}G}{{}_{R}N} + \frac{{}_{R}M}{{}_{R}N}. \tag{20}$$

Proof.
Starting with Axiom I it is

$$+1 = +1. \tag{21}$$

Multiplying this equation by ${}_{R}N$ we obtain

$${}_{R}N = {}_{R}N. \tag{22}$$

which is equivalent to

$${}_{R}N = {}_{R}N + 0. \tag{23}$$

In our understanding, matter ${}_{R}M$ is a determining part of ${}_{R}N$. In general it is

$${}_{R}N = {}_{R}N - {}_{R}M + {}_{R}M. \tag{24}$$

Following Einstein and de Broglie, **all but matter is defined as being equal to the gravitational field** or ${}_{R}G \equiv {}_{R}\Gamma \equiv {}_{R}N - {}_{R}M$. Consequently, there is no third between matter and gravitational field, a third is not given (**tertium non datur**). Thus far, we obtain

$${}_{R}N = {}_{R}G + {}_{R}M. \tag{25}$$

We divide this equation by ${}_{R}N$. The normalization of the relationship between matter and the gravitational field follows as

$$1 = \frac{{}_{R}G}{{}_{R}N} + \frac{{}_{R}M}{{}_{R}N}. \tag{26}$$

Quod erat demonstrandum.

Manuscript submitted to ViXra.org (Saturday, June 20, 2015)

3.3. **Theorem. The Equivalence Of Gravitational Field And Time.**

Claim.
The gravitational field and time are equivalent. In general, it is

$$ {}_{R}G = \frac{{}_{R}t}{c^{2}} . \tag{27} $$

Proof.
Starting with Axiom I it is

$$ +1 = +1 . \tag{28} $$

Due to our theorem above it is $1 = \frac{{}_{R}E}{{}_{R}S} + \frac{{}_{R}t}{{}_{R}S}$ and we obtain

$$ 1 = \frac{{}_{R}E}{{}_{R}S} + \frac{{}_{R}t}{{}_{R}S} . \tag{29} $$

Due to the other theorem above it is $1 = \frac{{}_{R}G}{{}_{R}N} + \frac{{}_{R}M}{{}_{R}N}$. The equation before changes to

$$ \frac{{}_{R}G}{{}_{R}N} + \frac{{}_{R}M}{{}_{R}N} = \frac{{}_{R}E}{{}_{R}S} + \frac{{}_{R}t}{{}_{R}S} . \tag{30} $$

Multiplying this equation by ${}_{R}N$, it is

$$ {}_{R}G + {}_{R}M = \frac{{}_{R}N}{{}_{R}S} \times {}_{R}E + \frac{{}_{R}N}{{}_{R}S} \times {}_{R}t . \tag{31} $$

According to our definition, it is ${}_{R}N \equiv \frac{{}_{R}S}{c^{2}}$ and thus far $\frac{{}_{R}N}{{}_{R}S} \equiv \frac{1}{c^{2}}$. We obtain

$$ {}_{R}G + {}_{R}M = \frac{1}{c^{2}} \times {}_{R}E + \frac{1}{c^{2}} \times {}_{R}t . \tag{32} $$

Due to Einstein's special theory of relativity, it is ${}_{R}M \equiv \frac{{}_{R}E}{c^{2}}$, we obtain

$$ {}_{R}G + {}_{R}M = {}_{R}M + \frac{1}{c^{2}} \times {}_{R}t . \tag{33} $$

After subtraction of ${}_{R}M$ on both sides of the equation, it is

$$ {}_{R}G = \frac{{}_{R}t}{c^{2}} . \tag{34} $$

Quod erat demonstrandum.

4. Discussion

The relationship between time and gravitational field is similar to the relationship between mass and energy. We must accept the gravitational field-time equivalance. This is of far reaching consequences especially for the mathematics of gravitational waves ${}_{w}G$. From the standpoint of a moving observer it makes sense to assume that ${}_{0}t$, the time as measured by a moving observer O, is equivalent to the eigenstate of the wavefunction ${}_{0}\psi$. (i. e. the situation after the collapse of the wavefunction). In other words, we assume that the gravitational field oG as determined by the moving observer O is ${}_{0}G = \frac{{}_{0}t}{c^2} \equiv \frac{{}_{0}\psi}{c^2}$. Under these conditions **the gravitational wave**, denoted as ${}_{w}G$, appears to be determined something like $\left|{}_{w}G\right| = \left|{}_{R}G\right| \times \sqrt[2]{\left(1 - \frac{{}_{0}G^2}{{}_{R}G^2}\right)} \equiv \left|\frac{{}_{R}\psi(t)}{c^2}\right| \times \sqrt[2]{\left(1 - \frac{{}_{0}\psi^2 \times c^2 \times c^2}{c^2 \times c^2 \times {}_{R}\psi(t)^2}\right)} \equiv \left|\frac{{}_{R}\psi(t)}{c^2}\right| \times \sqrt[2]{\left(1 - \frac{{}_{0}\psi^2}{{}_{R}\psi(t)^2}\right)}$ which can be tested by experiments. The straightforward question is, are there conditions, circumstances or experimental situations where $\sqrt[2]{\left(1 - \frac{v^2}{c^2}\right)} \equiv \sqrt[2]{\left(1 - \frac{{}_{0}\psi^2}{{}_{R}\psi(t)^2}\right)}$ can be accepted as being given?

5. Conclusion

The problem of the relationship between time and gravitational field is solved. In general, the gravitational field is equivalent to time and vice versa.

Acknowledgment

I am very happy to have the opportunity to express my deep gratitude to the Scientific Committee of the conference "Quantum Theory: from foundations to technologies (QTFT)". This paper was a part my presentation at the QFTF conference at Linnaeus University, Sweden, June 8-11, 2015.

Appendix

None.

References

[1] Albert Einstein, "Prinzipielles zur allgemeinen Relativitätstheorie", *Annalen der Physik*, vol. 55, no. 4, pp. 241-242, 1918.
[2] Albert Einstein, "Zur Elektrodynamik bewegter Körper," *Annalen der Physik*, vol. 322, Issue 10, p. 904, 1905.
[3] Albert Einstein, "Zur Elektrodynamik bewegter Körper," *Annalen der Physik*, vol. 322, Issue 10, p.893, 1905
[4] Albert Einstein, "Zur Elektrodynamik bewegter Körper," *Annalen der Physik*, vol. 322, Issue 10, p. 898, 1905.
[5] Albert Einstein, "Zur Elektrodynamik bewegter Körper," *Annalen der Physik*, vol. 322, Issue 10, p.903-904, 1905.
[6] Ilija Barukčić, "The Relativistic Wave Equation," *International Journal of Applied Physics and Mathematics,* vol. 3, no. 6, pp. 387-391, 2013.
[7] Albert Einstein, "Zur Elektrodynamik bewegter Körper," *Annalen der Physik*, vol. 322, Issue 10, pp.891-921, 1905.
[8] Albert Einstein, "Die Grundlage der allgemeinen Relativitätstheorie," *Annalen der Physik*, vol. 354, Issue 7, pp. 802-803, 1916.