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GENERALIZED NEUTRINO EQUATIONS BY THE SAKURAI-GERSTEN METHOD

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I discuss generalized spin-1/2 massless equations for neutrinos. They have been obtained by Gersten's method for derivation of arbitrary-spin equations. Possible physical consequences are discussed.

1. Introduction

Recently Gersten [1] proposed a method for derivations of massless equations of arbitrary-spin particles. In fact, his method is related to the van der Waerden-Sakurai [2] discussion of obtaining the massive Dirac equation. I commented the derivation of Maxwell equations [1a][1] in [3] and showed that the method is rather ambigious because instead of free-space Maxwell equations one can obtain *generalized* $S = 1$ equations, which connect the antisymmetric tensor field with additional scalar fields. The problem of physical significance of additional scalar chi-fields should be solved, of course, by experiment.

In the present article I apply the van der Waerden-Sakurai-Gersten procedure to the spin-1/2 fields. As a result one obtains equations which *generalize* the well-known Weyl equations. However, these equations are known for a long time [4]. Recently, Raspini [5, 6, 7, 8, 9] analized them again in detail. I add some comments on physical contents of the generalized spin-1/2 equations.

2. Derivation

I use the equation (4) of the Gersten paper [1a] for two-component spinor wave function:

$$(E^2 - c^2\vec{\mathbf{p}}^{\,2})I^{(2)}\psi = \left[EI^{(2)} - c\vec{\mathbf{p}}\cdot\vec{\sigma}\right]\cdot$$

$$\cdot\left[EI^{(2)} + c\vec{\mathbf{p}}\cdot\vec{\sigma}\right]\psi = 0 \quad (eq.(4) of\ [1a])\,. \qquad (1)$$

Actually this equation is a massless limit of the equation which has been presented (together with a corresponding method of derivation of the Dirac equation) in Sakurai book [2]; in the latter case one should substitute m^2c^4 into the right-hand side of eq. (1). However, instead of equation (3.25) of [2] one can define two-component 'right' wave function

$$\phi_R = \frac{1}{m_1 c}(i\hbar\frac{\partial}{\partial x_0} - i\hbar\sigma\cdot\nabla)\psi, \quad \phi_L = \psi \qquad (2)$$

with an additional mass parameter m_1. In such a way we come to the set of equations

$$(i\hbar\frac{\partial}{\partial x_0} + i\hbar\sigma\cdot\nabla)\phi_R = \frac{m_2^2 c}{m_1}\phi_L\,, \qquad (3)$$

$$(i\hbar\frac{\partial}{\partial x_0} - i\hbar\sigma\cdot\nabla)\phi_L = m_1 c\phi_R\,, \qquad (4)$$

which can be written in the 4-component form:

$$\begin{pmatrix} i\hbar(\partial/\partial x_0) & i\hbar\sigma\cdot\nabla \\ -i\hbar\sigma\cdot\nabla & -i\hbar(\partial/\partial x_0)\end{pmatrix}\begin{pmatrix}\psi_A\\ \psi_B\end{pmatrix} = \frac{c}{2}\cdot$$

$$\cdot\begin{pmatrix}(m_2^2/m_1 + m_1) & (-m_2^2/m_1 + m_1)\\ (-m_2^2/m_1 + m_1) & (m_2^2/m_1 + m_1)\end{pmatrix}\begin{pmatrix}\psi_A\\ \psi_B\end{pmatrix} \qquad (5)$$

for the function $\Psi = column(\phi_R + \phi_L \quad \phi_R - \phi_L)$. The equation (5) can be written in the covariant form (as one can see the standard representation of γ^μ matrices was used here):

$$\left[i\gamma^\mu\partial_\mu - \frac{m_2^2 c}{m_1\hbar}\frac{(1-\gamma^5)}{2} - \frac{m_1 c}{\hbar}\frac{(1+\gamma^5)}{2}\right]\Psi = 0\,. \qquad (6)$$

If $m_1 = m_2$ we can recover the standard Dirac equation. As noted in [4b] this procedure can be viewed as simply changing the representation of γ^μ matrices (unless $m_2 \neq 0$).

Furthermore, one can either repeat a similar procedure (the modified Sakurai procedure) starting from

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[1]In fact, the $S = 1$ equations.

the '*massless*' equation (4) of [1a] or put $m_2 = 0$ in eq. (6). The 'massless equations' is[2]

$$\left[i\gamma^\mu \partial_\mu - \frac{m_1 c}{\hbar}\frac{(1+\gamma^5)}{2}\right]\Psi = 0\,. \tag{7}$$

Then we may have *physical* difference with the Weyl equation (which is obtained as $m \to 0$ limit of the usual Dirac equation). The mathematical reason of such a possibility to have *different* massless limits is that the corresponding change of representation of γ^μ matrices involves mass parameters m_1 and m_2 themselves and in a certain limit the corresponding matrix may be non-existent (its elements tend to infinity).

It is interesting that we also can repeat this procedure for the definition (or even more general)

$$\phi_L = \frac{1}{m_3 c}(i\hbar\frac{\partial}{\partial x_0} + i\hbar\sigma\cdot\nabla)\psi, \quad \phi_R = \psi \tag{8}$$

since in the two-component equation the parity properties of the two-component spinor are undefined. The resulting equation is

$$\left[i\gamma^\mu \partial_\mu - \frac{m_4^2 c}{m_3\hbar}\frac{(1+\gamma^5)}{2} - \frac{m_3 c}{\hbar}\frac{(1-\gamma^5)}{2}\right]\tilde\Psi = 0\,, \tag{9}$$

which give us yet another equation in the massless limit ($m_4 \to 0$):

$$\left[i\gamma^\mu \partial_\mu - \frac{m_3 c}{\hbar}\frac{(1-\gamma^5)}{2}\right]\tilde\Psi = 0\,, \tag{10}$$

The above procedure can be generalized to *any* Lorentz group representations, *i. e., any* spins. In some sense the equations (7,10) are analogs of the '$S = 1$ equations' [3, (4-7,10-13)] which also contain additional parameters.

3. Physical Interpretation and the Conclusion

Is the physical content of the generalized $S = 1/2$ 'massless' equations the same as that of the Weyl equation? We can answer 'No'. The excellent discussion can be found in [4a,b]. The theory does *not* have chiral invariance. Those authors call the additional parameters as measures of the degree of chirality. Apart, Tokuoka introduced the concept of the gauge transformations (not to confuse with phase transformations) for the 4-spinor fields. He also found somewhat strange properties of the anti-commutation relations (see §3 in [4a] and cf. [11b]). And, the equation (7) describes *four* states, two of which answer for the positive energy $E = |\mathbf{p}|$, and two others answer for the negative energy $E = -|\mathbf{p}|$.

[2]It is necesary to stress that the term 'massless' is used in the sense that $p_\mu p^\mu = 0$.

We just want to add the following to the discussion. The operator of the *chiral-helicity* $\eta = (\alpha\cdot\hat{\mathbf{p}})$ (in the spinorial representation) used in [4b] (and re-discovered in [11a]) does *not* commute with the Hamiltonian of the equation (7):

$$[\mathcal{H}, \alpha\cdot\hat{\mathbf{p}}]_- = 2\frac{m_1 c}{\hbar}\frac{1-\gamma^5}{2}(\gamma\cdot\hat{\mathbf{p}})\,. \tag{11}$$

For the eigenstates of *chiral-helicity* the set of corresponding equations read ($\eta = \uparrow, \downarrow$)

$$i\gamma^\mu\partial_\mu\Psi_\eta - \frac{m_1 c}{\hbar}\frac{1+\gamma^5}{2}\Psi_{-\eta} = 0\,. \tag{12}$$

The conjugated eigenstates of the Hamiltonian $|\Psi_\uparrow + \Psi_\downarrow >$ and $|\Psi_\uparrow - \Psi_\downarrow >$ are connected, in fact, by γ^5 transformation $\Psi \to \gamma^5\Psi \sim (\alpha\cdot\hat{\mathbf{p}})\Psi$ (or $m_1 \to -m_1$). However, the γ^5 transformation is related to the PT ($t \to -t$ only) transformation [4b], which, in its turn, can be interpreted as $E \to -E$, if one accepts Stueckelberg ideas about antiparticles. We associate $|\Psi_\uparrow + \Psi_\downarrow >$ with the positive-energy eigenvalue of the Hamiltonian $E = |\mathbf{p}|$ and $|\Psi_\uparrow - \Psi_\downarrow >$, with the negative-energy eigenvalue of the Hamiltonian ($E = -|\mathbf{p}|$). Thus, the free chiral-helicity massless eigenstates may oscillate one to another with the frequency $\omega = E/\hbar$ (as the massive chiral-helicity eigenstates, see [10a] for details). Moreover, a special kind of interaction which is not symmetric with respect to the chiral-helicity states (for instance, if only left chiral-helicity eigenstates interact with the matter) may induce changes in the oscillation frequency.

It is not yet clear how can these frameworks be connected with the Ryder method of derivation of relativistic wave equations and with subsequent analysis of problems of the choice of normalization and of phase in the papers [10, 11, 12]. However, the conclusion may be similar to that achieved before: the dynamical properties of the massless particles (*e. g.*, neutrinos) may differ from those defined by well-known Weyl and Maxwell equations.

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