The Topological Skyrme Model and the Missing Baryon Number in the Eight Fold Way Model

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Abstract

One of the most outstanding and puzzling problem of particle physics is, how come, no baryon number is needed to specify the spin-1/2 fermions of the Eight Fold Way model. Recently the author has shown that all the models proposed to solve this problem, at present, are fundamentally wrong. So what is the resolution of this conundrum? Here we show that the topological Skyrme model comes to our rescue. It is this model which fills the gap by providing a topologically generated baryon number for the spin-1/2 baryons in the Eight Fold Way model. The global nature of this baryon number complements perfectly well the global nature of the Eight Fold Way model baryons.

Keywords: Skyrme Model, Eightfold Way Model, Topological Baryon Number. SU(3) Symmetry, Non-Linear Sigma Model, Global Symmetry

The most amazing and puzzling property of the spin 1/2 baryons in the Eight Fold Way model is that it is bereft of a baryon number. But we know that the baryon number is an important quantum number which distinguishes it from others like the mesons and leptons etc. This problem had surfaced back in 1961 [1].

Much effort has been put in to understand this problem. All popular explanations of it may be classified as follows - Take the Lie algebra $u(3) = su(3) \oplus a0$ where $a0$ is a one dimensional Lie algebra representing the baryon number transformation. Now there are only five possibilities for the corresponding connected Lie group associated with the above Lie algebra. These are:

(a) $SU(3) \otimes R$. (b) $\frac{SU(3)}{Z_3} \otimes R$. (c) $SU(3) \otimes T$. (d) $\frac{SU(3)}{Z_3} \otimes T$. (e) U(3).

Here the irreducible representation of R are specified by a real number r which is an integer for the representations of T.

Now we demand that the baryon number and the electric charge are integer numbers for all the irreducible representations, then the possibilities (a), (b) and (c) are excluded and then only options of (d) and (e) are left as physically valid. Then (d) holds for the above Eightfold-Way model with the group T giving an external baryon number. The group U(3) has been very often tried by several persons to solve this problem.

Note that in all these standard way of understanding the role of the baryon number in the Eightfold-way model in (d) and (e) above, the baryon number arises fron outside the group SU(3). But we know that in the SU(3) model, the second diagonal generator of SU(3) defines hypercharge Y as Y= B + S ; so it is a composite of the baryon number B and the strangeness S. Thus in SU(3) the baryon number is arising from within SU(3) itself. Therefore clearly the above canonical models of understanding the Eightfold-way models are **fundamentally wrong**.

These issues were stressed by the author recently [1]. We pointed out the intrinsic global aspect of the issue in the Eightfold Way model. Thus given the fact that in the SU(3)-flavour model, with the u,d,s-quarks, the octet representation does arise as a microscopic model, and thus one is faced with a duality of these two equally fundamental representations of the same octet entity. One is global and the other is local [1].

As shown above, now we are left with no viable solution of the baryon-number-less-ness of the Eight Fold Way Model. Here we suggest a solution in terms of the Topological Skyrme Model [2-8].

First note that the essence of the Eight Fold Way model [1] is that the pseudoscalar octet meson may be represented as an eight-dimensional vector or as the traceless matrix

$$\frac{1}{\sqrt{2}}\lambda^a P^a = \begin{pmatrix} \frac{\pi^0}{\sqrt{2}} + \frac{\eta^0}{\sqrt{6}} & \pi^+ & K^+ \\ \pi^- & -\frac{\pi^0}{\sqrt{2}} + \frac{\eta^0}{\sqrt{6}} & K^0 \\ K^- & \bar{K}^0 & -\frac{2}{\sqrt{6}}\eta^0 \end{pmatrix} \tag{1}$$

In terms of the cartesian coordinates $P^a, a = 1, ..8$ the above states can be written as

$$P = \begin{pmatrix} \frac{P_8}{\sqrt{6}} + \frac{P_3}{\sqrt{2}} & \frac{P_1 - iP_2}{\sqrt{2}} & \frac{P_4 - iP_5}{\sqrt{2}} \\ \frac{P_1 + iP_2}{\sqrt{2}} & \frac{P_8}{\sqrt{6}} - \frac{P_3}{\sqrt{2}} & \frac{P_6 - iP_7}{\sqrt{2}} \\ \frac{P_4 + iP_5}{\sqrt{2}} & \frac{P_6 + iP_7}{\sqrt{2}} & -\frac{2}{\sqrt{6}}P_8 \end{pmatrix} = \frac{1}{\sqrt{2}}\lambda_j P_j \tag{2}$$

In the same manner, the $1/2^+$ baryons octet can be described either by an 8-dimensional vector or by a traceless matrix $\frac{1}{\sqrt{2}}\lambda_j \mathcal{B}^{\dashv}$. Here the components of $\mathcal{B}^{\dashv}$ are related to the physical baryons N, Λ, σ, Ξ in the same manner as for the above mesons as:

$$\frac{1}{\sqrt{2}}\lambda_j B^a = \begin{pmatrix} \frac{\Lambda^0}{\sqrt{6}} + \frac{\Sigma^0}{\sqrt{2}} & \Sigma^+ & p \\ \Sigma^- & \frac{\Lambda^0}{\sqrt{6}} - \frac{\Sigma^0}{\sqrt{2}} & n \\ \Xi^- & \Xi^0 & -\frac{2}{\sqrt{6}}\Lambda^0 \end{pmatrix} \tag{3}$$

So the similarity and paralellism between the 0^- meson octet in eqn. (1) and the spin-1/2 baryon octet as given in eqn. (3) is the basis of the Eightfold Way Model [1].

Next on to the Topological Skyrme Model. The Skyrme Lagrangian [2-8] is give as

$$L_S = \frac{{f_\pi}^2}{4} Tr(L_\mu L^\mu) + \frac{1}{32e^2} Tr[L_\mu, L_\nu]^2 \tag{4}$$

where $L_\mu = U^\dagger \partial_\mu U$. The U field for the three flavour case for example is

$$U(x) = exp[\frac{i\lambda^a \pi^a(x)}{f_\pi}] \tag{5}$$

with π^a the pseudoscalar octet of π, K and η mesons given in eqn. (1) as the 0^- meson octet of the Eightfold Way model.

Here the topological charge - baryon number, identified with the winding number is

$$B = \frac{i}{24\pi^2}\epsilon^{ijk}\int d^3x Tr[L_i L_j L_k] \tag{6}$$

Now the 3x3 matrix in eqn. (1) of the meson is exactly what the non-linear U of the Skyrme Lagrangian takes as its basic field. Out of this structure arises the baryon number, eqn. (6). We identitfy this baryon number of the Skyrme Model as the missing baryon number of the spin-1/2 fermions in the Eight Fold Way model.

Now the baryon 3x3 marix in the Eightfold Way model which did not have the baryon number, is getting a new topological baryon number deposited on it by its meson Eightfold Way partner. The whole thing is consistent and complete within itself.

For the 2-flavour case, it has been shown in Ref. [4, p. 11] that the baryon number B does not vanish only if the three pions π^+, π^-, π^0 are excited. We extrapolate to 3-flavours by stating that the baryon number B is generated only if all the meson fields of the Eight Fold Way are fully excited. Thus the 0^- Eight Fold Way mesons (eqn. (1)) are playing a fundamental role in the Skyrme model to generate a topological baryon number to fill the gap of the missing baryon number of its Eight Fold Way partner (eqn. (3)) - the spin 1/2 fermions.

What the spin 1/2 baryons in the Eight Way model are missing i.e. a baryon number, is being provided to it by the Skyrme model as a topological baryon number. The Eight Fold Way model and the Skyrme model are playing complemenatry roles with respect to each other. So to say, one is not complete without the other. Its is perfect global match!

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