Neutrosophic Sets and Systems

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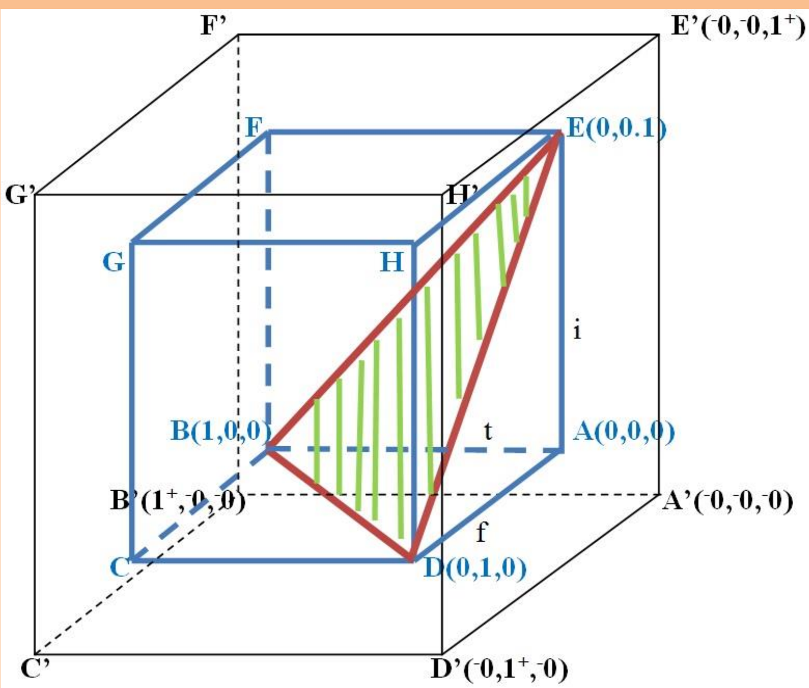

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Neutrosophic Sets and Systems

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Contents

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Neutrosophic Sets and Systems has been created for publications on advanced studies in neutrosophy, neutrosophic set, neutrosophic logic, neutrosophic probability, neutrosophic statistics that started in 1995 and their applications in any field, such as the neutrosophic structures developed in algebra, geometry, topology, etc.

The submitted papers should be professional, in good English, containing a brief review of a problem and obtained results.

Neutrosophy is a new branch of philosophy that studies the origin, nature, and scope of neutralities, as well as their interactions with different ideational spectra.

This theory considers every notion or idea <A> together with its opposite or negation <antiA> and with their spectrum of neutralities <neutA> in between them (i.e. notions or ideas supporting neither <A> nor <antiA>). The <neutA> and <antiA> ideas together are referred to as <nonA>.

Neutrosophy is a generalization of Hegel's dialectics (the last one is based on <A> and <antiA> only).

According to this theory every idea <A> tends to be neutralized and balanced by <antiA> and <nonA> ideas - as a state of equilibrium.

In a classical way <A>, <neutA>, <antiA> are disjoint two by two. But, since in many cases the borders between notions are vague, imprecise, Sorites, it is possible that <A>, <neutA>, <antiA> (and <nonA> of course) have common parts two by two, or even all three of them as well.

Neutrosophic Set and *Neutrosophic Logic* are generalizations of the fuzzy set and respectively fuzzy logic (especially of intuitionistic fuzzy set and respectively intuitionistic fuzzy logic). In neutrosophic logic a proposition has a degree of truth (T), a degree of indeterminacy (I), and a degree of falsity (F), where T, I, F are standard or non-standard subsets of $]^-0, 1^+[$.

Neutrosophic Probability is a generalization of the classical probability and imprecise probability.

Neutrosophic Statistics is a generalization of the classical statistics.

What distinguishes the neutrosophics from other fields is the <neutA>, which means neither <A> nor <antiA>.

<neutA>, which of course depends on <A>, can be indeterminacy, neutrality, tie game, unknown, contradiction, ignorance, imprecision, etc.

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NSS

Expanding Newton Mechanics with Neutrosophy and Quad-stage Method —New Newton Mechanics Taking Law of Conservation of Energy as Unique Source Law

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Abstract. Neutrosophy is a new branch of philosophy, and "Quad-stage" (Four stages) is the expansion of Hegel's triad thesis, antithesis, synthesis of development. Applying Neutrosophy and "Quad-stage" method, the purposes of this paper are expanding Newton Mechanics and making it become New Newton Mechanics (NNW) taking law of conservation of energy as unique source law. In this paper the examples show that in some cases other laws may be contradicted with the law of conservation of energy. The original Newton's three laws and the law of gravity, in principle can be derived by the law of conservation of energy. Through the example of free falling body, this paper derives the original Newton's second law by using the law of conservation of energy, and proves that there is not the contradiction between the original law of gravity and the law of conservation of energy; and through the example of a small ball rolls along the inclined plane (belonging to the problem cannot be solved by general relativity that a body is forced to move in flat space), derives improved Newton's second law and improved law of gravity by using law of conservation of energy. Whether or not other conservation laws (such as the law of conservation of momentum and the law of conservation of angular momentum) can be utilized, should be tested by law of conservation of energy. When the original Newton's second law is not correct, then the laws of conservation of momentum and angular momentum are no longer correct; therefore the general forms of improved law of conservation of momentum and improved law of conservation of angular momentum are presented. In the cases that law of conservation of energy cannot be used effectively, New Newton Mechanics will not exclude that according to other theories or accurate experiments to derive the laws or formulas to solve some specific problems. For example, with the help of the result of general relativity, the improved Newton's formula of universal gravitation can be derived, which can be used to solve the problem of advance of planetary perihelion and the problem of deflection of photon around the Sun. Again, according to accurate experimental result, the synthesized gravitational formula (including the effects of other celestial bodies and sunlight pressure) for the problem of deflection of photon around the Sun is presented. Unlike the original Newton Mechanics, in New Newton Mechanics, for different problems, may have different laws of motion, different formulas of gravity, as well as different expressions of energy. For example, for the problem of a small ball rolls along the inclined plane, and the problem of advance of planetary perihelion, the two formulas of gravity are completely different.

Keywords: Neutrosophy, "Quad-stage" (Four stages), law of conservation of energy, unique source law, New Newton Mechanics

1 Introduction

As a new branch of philosophy, Neutrosophy studies the origin, nature, and scope of neutralities, as well as their interactions with different ideational spectra. According to Neutrosophy that there is a 3D Neutrosophic Space, where each dimension of the space represents respectively the truth (T), the falsehood (F), and the indeterminacy (I) of the statement under consideration. More information about Neutrosophy may be found in references [1,2]. Quad-stage is introduced in reference [3], it is the expansion of Hegel's triad-stage (triad thesis, antithesis, synthesis of development). The four stages are "general theses", "general antitheses", "the most important and the most complicated universal relations", and "general syntheses". In quad-stage method, "general theses" may be considered as the notion or idea <A> in neutrosophy; "general antitheses" may be considered as the notion or idea <Anti-A> in neutrosophy; "the most important and the most complicated universal relations" may be considered as the notion or idea <Neut-A> in neutrosophy; and "general syntheses" are the final results. The different kinds of results in the above mentioned four stages can also be classified and induced with the viewpoints of neutrosophy. Thus, the theory and achievement of neutrosophy can be applied as many as possible, and the method of quad-stage will be more effective. The combination of Neutrosophy and quad-stage will be a powerful method to realize many innovations in areas of science, technology, literature and art. Therefore, this paper expands Newton Mechanics with Neutrosophy and Quad-stage Method and creates New Newton Mechanics (NNW) taking law of conservation of energy as unique source law.

One of the development trends of natural science is using fewer laws to solve increasing problems. In this process, according to the viewpoint of neutrosophy, some laws will play the increasingly great roles; some laws will play the smaller roles, or even disappear from the ranks of laws; and the middle ones will be improved and expanded to play the greater roles.

As expanding Newton mechanics with neutrosophy and quad-stage, the whole process can be divided into the following four stages.

The first stage (stage of "general theses"), for the beginning of development, the thesis (namely Newton mechanics) should be widely, deeply, carefully and repeatedly contacted, explored, analyzed, perfected and so on.

Regarding the advantages of Newton mechanics, that will not be repeated here, while we should stress the deficiencies of Newton mechanics.

As well-known, Newton mechanics cannot be used to solve the problem of advance of planetary perihelion and the problem of deflection of photon around the Sun. For other perspectives on Newton mechanics, we will discuss in detail below, in order to avoid duplication.

The second stage, for the appearance of opposite (antithesis), the antithesis should be also widely, deeply, carefully and repeatedly contacted, explored, analyzed, perfected and so on.

There are many opposites (antitheses) to Newton mechanics. For example: special and general theory of relativity, "theory of everything", law of conservation of energy, and so on, this paper focuses on the problems related to law of conservation of energy.

The third stage is the one that the most important and the most complicated universal relations. The purpose of this provision stage is to establish the universal relations in the widest scope.

To link and combine Newton mechanics with law of conservation of energy, as well as the brilliant achievements of modern science and technology, then Newton mechanics can be expanded and developed effectively and successfully in the maximum area.

The fourth stage, to carry on the unification and synthesis regarding various opposites and the suitable pieces of information, factors, and so on; and reach one or more results to expand Newton mechanics which are the best or agreed with some conditions; this is the stage of "general syntheses".

Now we discuss the law of conservation of energy. Its main contents are as follows: In a closed system, the total energy of this system remains unchanged.

Because the law of conservation of energy is the most important one in natural sciences, it should play an increasingly great role. For this reason and according to the principle of the uniqueness of truth, this paper presents the New Newton Mechanics (NNM) taking law of conservation of energy as unique source law with Neutrosophy and Quad-stage Method.

In the area of Newton Mechanics, there should be one truth only. Other so-called truth, either it can be derived by the unique truth, or we can prove that in certain cases it is not true. As well-known, when Newton founded the classical mechanics, four laws were proposed, they were Newton's three laws and the law of gravity. If the law of conservation of energy is choosing as the unique source law, that in principle, all the Newton's four laws can be derived according to the law of conservation of energy; after studying carefully we found that this may indeed be the real case. In addition, in the areas such as physics, mechanics, engineering and so on, there are three very important laws: the law of conservation of energy, the law of conservation of momentum and the law of conservation of angular momentum. If we believe that the law of conservation of energy is the truth, then for the law of conservation of momentum and the law of conservation of angular momentum, either they can be derived by the law of conservation of energy, or we can prove that in certain cases they are not true. We believe that the true situation is the latter, namely, the law of conservation of momentum and the law of conservation of angular momentum are not true in some cases (or their results are contradicted to the law of conservation of energy). Of course, we can also find that in some cases, these two laws still can be used. Taking the example that a man walks along the car located on the horizontal smooth rail, we can see that at present in the area of Newton mechanics, some people do not notice the case of the contradiction between the law of conservation of energy and the law of conservation of momentum.

2 New three laws of motion and new law of gravity (formula) created by law of conservation of energy for New Newton Mechanics

The original Newton's three laws of motion (partial theses) are as follows.

Newton's First Law of Motion: Every object in a state of uniform motion (or at rest) tends to remain in that state of motion (or at rest) unless an external force is applied to it. For short: rest remains rest, and moving remains moving.

Newton's Second Law of Motion: The relationship between an object's mass m, its acceleration a, and the applied force F is F = ma. The direction of the force is the same as the direction of the acceleration.

Newton's Third Law of Motion: For every action there is an equal and opposite reaction.

The original Newton's law of gravity (partial theses): The attractive force between two objects is as follows

$$F = -\frac{GMm}{r^2} \qquad (1)$$

While through the stage of "general antitheses" and the stage of "the most important and the most complicated universal relations", for NNM, taking law of conservation of energy as unique source law, then we have the following NNM's three laws of motion and law of gravity.

NNM's First Law of Motion: Every object in a state of uniform motion (or in a state of uniform rotation, or at rest) tends to remain in that state of motion (or in a state of uniform rotation, or at rest) unless an external force is applied to it; otherwise the law of conservation of energy will be destroyed. For short: rest remains rest, moving remains moving, and rotating remains rotating.

NNM's Second Law of Motion: The relationship between an object's mass m, its acceleration a, and the applied force F is a function that should be derived by law of conservation of energy. The direction of the force is the same as the direction of the acceleration. In general, the function can be written as the form of variable dimension fractal: $F = ma^{1+\varepsilon}$, where: ε is a constant or a variable. For different problems, the forms of second law may be different.

NNM's Third Law of Motion: In general, for every action there is an equal and opposite reaction. In special case, the function relationship between action and reaction should be derived by law of conservation of energy. The improved form of the original Newton's third law ($F_{AB} = -F_{BA}$) is as follows: $F_{AB} = -F_{BA}^{1+\lambda}$, where: λ is a constant or a variable. For different problems, the forms of third law may be different.

NNM's law (formula) of gravity: The attractive force between two objects is a function that should be derived by law of conservation of energy, or experimental data; or derived with the help of other theories. For different problems, the forms of law (formula) of gravity may be different. The results of original Newton's law of gravity are only accurate in the cases that two objects are relative static or running the straight line between one center and another center, and the like; for other cases its results are all approximate. In general, NNM's law (formula) of gravity may be taken as the form that adding the amending term to original Newton's law of gravity, or the following form of variable dimension fractal:

$$F = -\frac{GMm}{r^{2-\delta}} \qquad (2)$$

where: δ is a constant or a variable.

Now for an example, a NNM's law (formula) of gravity (an improved Newton's law of gravity) and a NNM's second law of motion (an improved Newton's second law of motion), they are suitable for this example only, are derived simultaneously by law of conservation of energy.

Firstly, through "universal relations", the variational principles established by the law of conservation of energy can be given with least squares method (LSM).

Supposing that the initial total energy of a closed system equals $W(0)$, and for time t the total energy equals $W(t)$, then according to the law of conservation of energy:

$$W(0) = W(t) \qquad (3)$$

This can be written as:

$$R_W = \frac{W(t)}{W(0)} - 1 = 0 \qquad (4)$$

According to LSM, for the interval [t_1, t_2], we can write the following variational principle:

$$\Pi = \int_{t_1}^{t_2} R_W^2 dt = \min{}_0 \qquad (5)$$

where: $\min_0$ denotes the minimum value of functional Π and it should be equal to zero.

It should be noted that, in many cases $W(t)$ is approximate, and R_W is not identically equal to zero, therefore Eq.(5) can be used to solve the problem. Besides the time coordinate, another one can also be used. For example, for interval [x_1, x_2], the following variational principle can be given according to the law of conservation of energy:

$$\Pi = \int_{x_1}^{x_2} R_W^2 dx = \min{}_0 \qquad (6)$$

The above-mentioned principles are established by using the law of conservation of energy directly. Sometimes, a certain principle should be established by using the law of conservation of energy indirectly. For example, a special physical quantity Q may be interested, not only it can be calculated by using the law of conservation of energy, but also can be calculated by using other laws (for this paper they are the law of gravity, and Newton's second law). For distinguishing the values, let's denote the value given by other laws as Q, while denote the value given by the law of conservation of energy as Q', then the value of R_W can be redefined as follows:

$$R_W = \frac{Q}{Q'} - 1 = 0 \qquad (7)$$

Substituting Eq.（7）into Eqs.（5）and（6），as Q' is the result calculated with the law of conservation of energy, it gives the variational principle established by using the law of conservation of energy indirectly. Otherwise, it is clear that the extent of the value of Q accords with Q'. Substituting the related quantities into Eq.（5）or Eq.（6），the equations derived by the condition of an extremum can be written as follows:

$$\frac{\partial \Pi}{\partial a_i} = \frac{\partial \Pi}{\partial k_i} = 0 \qquad (8)$$

After solving these equations, the improved law of gravity, and Newton's second law can be reached at once.

According to the value of Π, the effect of the solution can be judged. The nearer the value of Π is to zero, the better the effect of the solution. It should be noted that besides of solving equations, optimum-seeking methods could also be used for finding the minimum and the constants to be determined. In fact, the optimum seeking method will be used in this paper.

Now we solve an example. As shown in Fig.1, supposing that the small ball rolls along a long incline from A to B. Its initial velocity is zero and the friction and the rotational energy of small ball are neglected.

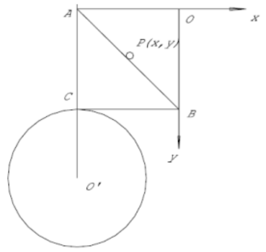

Fig.1 A small ball rolls from *A* to *B*

Supposing that circle O' denotes the Earth, M denotes its mass; m denotes the mass of the small ball (treated as a mass point P), O'A is a plumb line, coordinate x is orthogonal to O'A, coordinate y is orthogonal to coordinate x (parallel to O'A), BC is orthogonal to O'A. The lengths of OA, OB, BC, and AC are all equal to H, and O'C equals the radius R of the Earth. In this example, the value of v_P^2 which is the square of the velocity for the ball located at point P is investigated. To distinguish the quantities, denote the value given by the improved law of gravity and improved Newton's second law as v_P^2, while v'^2_P denotes the value given by the law of conservation of energy, then Eq.（6）can be written as

$$\Pi = \int_{-H}^{0} \left(\frac{v_P^2}{v'^2_P} - 1\right)^2 dx = \min{}_0 \qquad (9)$$

Supposing that the improved law of gravity and improved Newton's second law can be written as the following constant dimension fractal forms

$$F = -\frac{GMm}{r^D} \qquad (10)$$

$$F = ma^{1+\varepsilon} \qquad (11)$$

where: D and ε are constants.

Now we calculate the related quantities according to the law of conservation of energy. From Eq.(10), the potential energy of the small ball located at point P is

$$V = -\frac{GMm}{(D-1)r_{O'P}^{D-1}} \qquad (12)$$

According to the law of conservation of energy, we can get

$$-\frac{GMm}{(D-1)r_{O'A}^{D-1}} = \frac{1}{2}mv'^2_P - \frac{GMm}{(D-1)r_{O'P}^{D-1}} \qquad (13)$$

And therefore

$$v'^2_P = \frac{2GM}{D-1}\left[\frac{1}{r_{O'P}^{D-1}} - \frac{1}{(R+H)^{D-1}}\right] \qquad (14)$$

Now we calculate the related quantities according to the improved law of gravity and improved Newton's second law. Supposing that the equation of rolling line is

$$y = x + H \qquad (15)$$

For the ball located at point P,

$$dv/dt = a \qquad (16)$$

because

$$dt = \frac{ds}{v} = \frac{\sqrt{2}dx}{v}$$

Therefore

$$vdv = a\sqrt{2}dx \qquad (17)$$

According to the improved law of gravity, the force along to the tangent is

$$F_a = \frac{GMm}{r_{O'P}^D}\frac{1}{\sqrt{2}} \qquad (18)$$

According to the improved Newton's second law, for point P, the acceleration along to the tangent is

$$a = \left(\frac{F_a}{m}\right)^{1/1+\varepsilon} = \left(\frac{GM}{r_{O'P}^D\sqrt{2}}\right)^{1/1+\varepsilon} \qquad (19)$$

From Eq.（17）, it gives

$$vdv = \left\{\frac{GM}{[(H+x)^2+(R+H-y)^2]^{D/2}\sqrt{2}}\right\}^{1/1+\varepsilon}\sqrt{2}dx \qquad (20)$$

Substituting Eq.(15) into Eq.(20), and for the two sides, we run the integral operation from A to P, it gives

$$v_P^2 = 2\int_{-H}^{x_P}\left\{\frac{GM}{[(H+x)^2+(R-x)^2]^{D/2}}\right\}^{1/1+\varepsilon}(\sqrt{2})^{\varepsilon/1+\varepsilon}dx \qquad (21)$$

then the value can be calculated by a method of numerical integral.

The given data are assumed to be: for Earth, GM=3.99×10^{14}m^3/s^2; the radius of the Earth R=6.37×10^6m, H=R/10, try to solve the problem shown in Fig. 1, find the solution for the value of v_B^2, and derive the improved law of gravity and the improved Newton's second law. Firstly, according to the original law of gravity, the original Newton's second law (i.e., let D =2 in Eq.(10), ε =0 in Eq.(11)) and the law of conservation of energy, all the related quantities can be calculated, then substitute them into Eq.(9), it gives

$$\Pi_0 = 571.4215$$

Here, according to the law of conservation of energy, it gives v'^2_B=1.0767×10^7, while according to the original law of gravity, and the original Newton's second law, it gives v_B^2=1.1351×10^7, the difference is about 5.4 %. For the reason that the value of Π_0 is not equal to zero, then the values of D and ε can be decided by the optimum seeking method. At present all the optimum seeking methods can be divided into two types, one type may not depend on the initial values which program may be complicated, and another type requires the better initial values which program is simple. One method of the second type, namely the searching method will be used in this paper.

Firstly, the value of D is fixed so let D=2, then search the value of ε, as ε=0.0146, the value of Π reaches the minimum 139.3429; then the value of ε is fixed, and search the value of D, as D=1.99989, the value of Π reaches the minimum 137.3238; then the value of D is fixed, and search the value of ε, as ε=0.01458, the value of Π reaches minimum 137.3231. Because the last two results are highly close, the searching can be stopped, and the final results are as follows

D=1.99989, ε=0.01458, Π=137.3231

Here the value of Π is only 24% of Π_0. While according to the law of conservation of energy, it gives v'^2_B=1.0785×10^7, according to the improved law of gravity and the improved Newton's second law, it gives v^2_B=1.1073×10^7, the difference is about 2.7 % only.

The results suitable for this example with the constant dimension fractal form are as follows

The improved law of gravity reads

$$F = -\frac{GMm}{r^{1.99989}} \tag{22}$$

The improved Newton's second law reads

$$F = ma^{1.01458} \tag{23}$$

The above mentioned results have been published on reference [1].

According to the results for the example shown in Fig.1, it can be said that we could not rely on any experimental data, only apply the law of conservation of energy to derive the improved law of gravity, and improved Newton's second law; and demonstrate that the original Newton's law of gravity and Newton's second law are all tenable approximately for this example. So, can only apply the law of conservation of energy to derive that these two original laws or demonstrate they are tenable accurately in some cases? The answer is that in some cases we can indeed derive the original Newton's second law and prove the original Newton's law of gravity is tenable accurately.

Now, in the case that a small ball free falls (equivalent to free fall from A to C in Fig. 1), we derive the original Newton's second law and prove the original Newton's law of gravity is tenable accurately. Assuming that for the original law of gravity and Newton's second law, the related exponents are unknown, only know the forms of these two formulas are as follows: $F = -\frac{GMm}{r^D}$,

$F = ma^{D'}$; where: D and D' are undetermined constants. As shown in Fig.1, supposing that a small ball free falls from point A to point C. Similar to the above derivation, when the small ball falls to point P (point P is not shown in Fig.1), the value of v_P^2 calculated by the undetermined Newton's second law and the law of gravity, as well as the value of v'^2_P calculated by the law of conservation of energy are as follows:

$$v'^2_P = \frac{2GM}{D-1}\left[\frac{1}{r_{O'P}^{D-1}} - \frac{1}{(R+H)^{D-1}}\right]$$

$$v_P^2 = 2(GM)^{1/D'} \int_0^{y_p} (R+H-y)^{-D/D'} dy$$

$$v_P^2 = 2(GM)^{1/D'} \left\{-\frac{1}{1-D/D'}\left[(R+H-y)^{1-D/D'}\right]\Big|_0^{y_p}\right\}$$

$$v_P^2 = \frac{2(GM)^{1/D'}}{(D/D')-1}\left[\frac{1}{r_{O'P}^{(D/D')-1}} - \frac{1}{(R+H)^{(D/D')-1}}\right]$$

Let $v_P^2 = v'^2_P$, then we should have: $1 = 1/D'$, and $D-1 = (D/D')-1$; these two equations all give: $D' = 1$, this means that for free fall problem, by using the law of conservation of energy, we strictly derive the original Newton's second law $F = ma$.

Here, although the original law of gravity cannot be derived (the value of D may be any constant, certainly including the case that D=2), we already prove that the original law of gravity is not contradicted to the law of conservation of energy, or the original law of gravity is tenable accurately. For the example shown in Fig.1 that a small ball rolls along the inclined plane, in order to obtain the better results, we discuss the variable dimension fractal solution with Eq.(4) that is established by the law of conservation of energy directly.

Supposing that the improved Newton's second law and the improved law of gravity with the form of variable dimension fractal can be written as follows: $F = ma^{1+\varepsilon}$, $\varepsilon = k_1 u$; $F = -GMm/r^{2-\delta}$, $\delta = k_2 u$; where: u is the horizon distance that the small ball rolls ($u = x + H$).

With the similar searching method, the values of k_1, k_2 can be determined, and the results are as follows

$$\varepsilon = 8.85\times10^{-8}u\text{,}\quad \delta = 2.71\times10^{-13}u$$

The results of variable dimension fractal are much better than that of constant dimension fractal. For example, the final $\Pi = 5.8662\times10^{-4}$, it is only 0.019% of Π_0 (3.1207). While according to the law of conservation of energy, it gives v'^2_B =1.0767×10^7, according to the improved law of gravity and the improved Newton's second law, it gives v^2_B =1.0777×10^7, the difference is about 0.093 % only. The results suitable for this example with the variable dimension fractal form are as follows

The improved law of gravity reads

$$F = -\frac{GMm}{r^{2-2.71\times10^{-13}u}} \tag{24}$$

The improved Newton's second law reads

$$F = ma^{1+8.85\times10^{-8}u} \tag{25}$$

where: u is the horizon distance that the small ball rolls （$u = x + H$）

There is another problem should also be discussed. That is the improved kinetic energy formula. As well-known, the kinetic energy formula has been modified in the theory of relativity, now we improve the kinetic energy formula with the law of conservation of energy.

Supposing that the improved kinetic energy formula is $E_d = \frac{1}{2}mv^{2-\lambda}$, $\lambda = k_3 u$; where: u is the horizon distance that the small ball rolls （$u = x + H$）.

With the similar searching method, we can get: $k_3 = 9.95\times10^{-13}$, then the improved kinetic energy formula with variable dimension fractal form reads

$$E_d = \frac{1}{2}mv^{2-9.95\times10^{-13}u}$$

Because the effect of improvement is very small (the value of Π is only improved from 5.8662×10^{-4} into 5.8634×10^{-4}), therefore these results should be for reference only.

3 With the help of general relativity and accurate experimental data to derive the improved Newton's formula of universal gravitation

We already point out that, according to Neutrosophy and Quad-stage Method, various results can be reached. Prof. Hu Ning derived an equation according to general relativity, with the help of Hu's equation and Binet's formula, we get the following improved Newton's formula of universal gravitation[2]

$$F = -\frac{GMm}{r^2} - \frac{3G^2M^2mp}{c^2r^4} \tag{26}$$

where: G is gravitational constant, M and m are the masses of the two objects, r is the distance between the two objects, c is the speed of light, p is the half normal chord for the object m moving around the object M along with a curve, and the value of p is given by: p = a(1-e^2) (for ellipse), p = a (e^2-1) (for hyperbola), p = y^2/2x (for parabola).

It should be noted that, this improved Newton's formula of universal gravitation can also be written as the form of variable dimension fractal.

Suppose

$$-\frac{GMm}{r^D} = -\frac{GMm}{r^2} - \frac{3G^2M^2mp}{c^2r^4}$$

It gives $D = -\ln\left(\frac{1}{r^2} + \frac{3GMp}{c^2r^4}\right)/\ln r$

For the problem of gravitational defection of a photon orbit around the Sun, M=1.99×10^{30}kg, r_0=6.96×10^8m, c=2.9979×10^8m/s, then we have: 1.954997≤D≤2.

The improved Newton's universal gravitation formula (Eq.(26)) can give the same results as given by general relativity for the problem of planetary advance of perihelion and the problem of gravitational defection of a photon orbit around the Sun.

For the problem of planetary advance of perihelion, the improved Newton's universal gravitation formula reads

$$F = -\frac{GMm}{r^2} - \frac{3G^2M^2ma(1-e^2)}{c^2r^4} \quad (27)$$

For the problem of gravitational defection of a photon orbit around the Sun, the improved Newton's universal gravitation formula reads

$$F = -\frac{GMm}{r^2} - \frac{1.5GMmr_0^2}{r^4} \quad (28)$$

where : r_0 is the shortest distance between the light and the Sun, if the light and the Sun is tangent, it is equal to the radius of the Sun.

The funny thing is that, for this problem, the maximum gravitational force given by the improved Newton's universal gravitation formula is 2.5 times of that given by the original Newton's law of gravity.

Although the deflection angles given by Eq.(26) and Eq.(28) are all exactly the same as given by general relativity, they have still slight deviations with the precise astronomical observations. What are the reasons? According to "universal relations", the answer is that the deflection angle not only is depended on the gravitational effect of the Sun, but also depended on the gravitational effects of other celestial bodies, as well as the influences of sunlight pressure and so on. If all factors are taken into account, not only general relativity can do nothing for this problem, but also for a long time it could not be solved by theoretical method. Therefore, at present the only way to solve this problem is based on the precise observations to derive the synthesized gravitational formula (including the effects of other celestial bodies and sunlight pressure) for the problem of deflection of photon around the Sun.

As well-known, the deflection angle ϕ_0 given by general relativity or the improved Newton's formula of universal gravitation is as follows

$$\phi_0 = 1.75''$$

Adding an additional term to Eq.(28), it gives the synthesized gravitational formula between the photon and the Sun as follows

$$F = -\frac{GMm}{r^2}\left(1 + \frac{3GMp}{c^2r^2} + \frac{wG^2M^2p^2}{c^4r^4}\right) \quad (29)$$

where: w is a constant to be determined.

Now we determine the value of w according to accurate experimental data. Firstly the problem of deflection of photon around the Sun as shown in Fig.2 will be solved with Eq.(29). The method to be used is the same as presented in references [2] and [3].

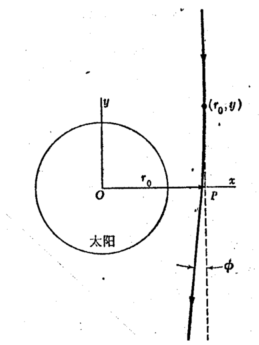

Fig. 2 Deflection of photon around the Sun

Supposing that m represents the mass of photon. Because the deflection angle is very small, we can assume that $x=r_0$; thus on point (x, y), its coordinate can be written as (r_0, y), then the force acted on photon reads

$$F_x = \frac{Fr_0}{(r_0^2 + y^2)^{1/2}} \quad (30)$$

whrer : The value of F is given by Eq.(29).

Because

$$mv_x = \int F_x dt = \int F_x \frac{dy}{v_y} \approx \frac{1}{c}\int F_x dy \quad (31)$$

Hence

$$v_x \approx -\frac{2GMr_0}{c}\int_0^\infty \frac{dy}{(r_0^2 + y^2)^{3/2}} - \frac{6G^2M^2pr_0}{c^3}\int_0^\infty \frac{dy}{(r_0^2 + y^2)^{5/2}}$$

$$-\frac{2wG^3M^3p^2r_0}{c^5}\int_0^\infty \frac{dy}{(r_0^2 + y^2)^{7/2}} \quad (32)$$

Because

$$\int_0^\infty \frac{dy}{(r_0^2+y^2)^{3/2}} = \frac{1}{r_0^2}, \quad \int_0^\infty \frac{dy}{(r_0^2+y^2)^{5/2}} = \frac{2}{3r_0^4},$$

$$\int_0^\infty \frac{dy}{(r_0^2+y^2)^{7/2}} = \frac{8}{15r_0^6}$$

Therefore

$$v_x \approx -\frac{2GM}{cr_0} - \frac{4G^2M^2p}{c^3r_0^3} - \frac{16wG^3M^3p^2}{15c^5r_0^5}$$

Because

$$\phi \approx tg\phi \approx \frac{|v_x|}{c}$$

By using the half normal chord given in reference [2], it gives

$$p = \frac{c^2r_0^2}{2GM}$$

Then the deflection angle is as follows

$$\phi = \frac{4GM}{c^2r_0}\left|1+\frac{w}{15}\right| \quad (33)$$

where: r_0 is the radius of Sun. Because

$$\phi_0 = \frac{4GM}{c^2r_0} \quad (34)$$

Then, it gives

$$\phi = \phi_0(1+\frac{w}{15}) \quad (35)$$

Thus the value of w can be solved as follows

$$w = 15(\frac{\phi}{\phi_0} - 1) \quad (36)$$

Now we can determine the value of w according to the experimental data.

Table 1 shows the experimental data of radio astronomy for the deflection angle of photon around the Sun (taken from reference [4]).

Table 1. The experimental data of radio astronomy for the deflection angle of photon around the Sun

Year	Observer	Observed value / ”
1969	G.A.Seielstud et al	1.77±0.20
1969	D.O.Muhleman et al	$1.82^{+0.24}_{-0.17}$
1969	I.I.Shapiro	1.80±0.2
1970	R.A.Sramak	1.57±0.08
1970	J.M.Hill	1.87±0.3
1972		1.82±0.14
1974		1.73±0.05
1975		1.78±0.02

Now we choose the experimental data in 1975, it gives

$$1.76 \le \varphi \le 1.80$$

Then, we have

$$0.08571 \le w \le 0.42857$$

Taking the average value, it gives

$$w=0.25714$$

Thus, according to the experimental data, the synthesized gravitational formula can be decided.

4 Contradiction between the law of conservation of energy and the law of conservation of momentum as well as the law of conservation of angular momentum

According to Neutrosophy, any law may be in three states: correct, wrong, and it is correct under certain conditions.

As well-known, unlike the law of conservation of energy, the law of conservation of momentum and the law of conservation of angular momentum are only correct under certain conditions. For example, considering friction force and the like, these two laws will not be correct.

Now we point out further that for NNM the law of conservation of momentum as well as the law of conservation of angular momentum will be not correct under certain conditions (or their results contradict with the law of conservation of energy). As well-known, in order to prove the law of conservation of momentum as well as the law of conservation of angular momentum, the original Newton's second law should be applied. However, as we have made clear, the original Newton's second law will not be correct under certain conditions, for such cases, these two laws also will not correct.

Here we find another problem, if the original three conservation laws are all correct, therefore for certain issues, the law of conservation of energy and the other two conservation laws could be combined to apply. While for NNM, if the other two conservation laws cannot be applied, how to complement the new formulas to replace these two conservation laws? The solution is very simple: according to the law of conservation of energy, for any time, the derivatives of total energy $W(t)$ should be all equal to zero, then we have

$$\frac{d^n W(t)}{dt^n} = 0 \qquad n = 1,2,3,\cdots \tag{37}$$

In addition, running the integral operations to the both sides of Eq.(3), it gives

$$W(0)t = \int_0^t W(t)dt \tag{38}$$

Now we illustrate that, because there is one truth only, even within the scope of original classical mechanics, the contradiction could also appear between the law of conservation of energy and the law of conservation of momentum.

As shown in Fig.3, a man walks along the car located on the horizontal smooth rail, the length of the car equals L, the mass of the man is m_1 and the car is m_2. At beginning the man and the car are all at rest, then the man walks from one end to the other end of the car, try to decide the moving distances of the man and the car. This example is taken from references [5].

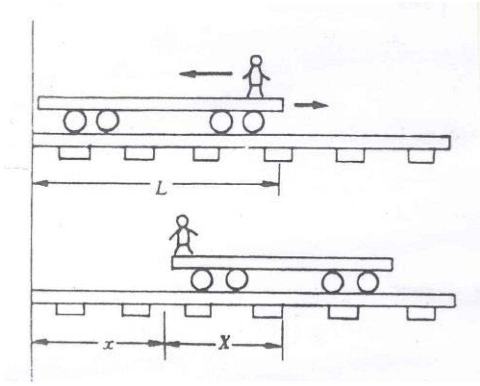

Fig.3 A man walks along the car located on the horizontal smooth rail.

As solving this problem by using the original classical mechanics, the law of conservation of momentum will be used, it gives

$$m_1 v_1 + m_2 v_2 = 0$$

However, at beginning the man and the car are all at rest, the total energy of the system is equal to zero; while once they are moving, they will have speeds, and the total energy of the system is not equal to zero; thus the law of conservation of energy will be destroyed. For this paradox, the original classical mechanics looks without seeing. In fact, considering the lost energy of the man and applying the law of conservation of energy, the completely different result will be reached.

As the original law of conservation of momentum ($P_t = P_0 = Const$) and the law of conservation of angular momentum ($L_t = L_0 = Const$) are not correct, we can propose their improved forms of variable dimension fractal. The improved law of conservation of momentum: $P_t = P_0^{1+\delta}$ (δ is a constant or a variable), and the improved law of conservation of angular momentum: $L_t = L_0^{1+\varepsilon}$ (ε is a constant or a variable).

References

[1] Fu Yuhua, Deriving Improved Newton's Second Law and the Law of Gravity at One Time with Form of Fractal Formula, Engineering Science. 2003, Vol.5, No.6, 55-58

[2] Fu Yuhua, Improved Newton's formula of universal gravitation, Ziranzazhi (Nature Journal), 2001(1), 58-59

[3] C. Kittel et al, Translated into Chinese by Chen Bingqian et al, Mechanics, Beijing: Science Press, 1979, 535-537

[4] Liu Liao, General relativity, Beijing: Higher education press, 1987,202

[5] Xu Hexing, Mechanics (revised edition), Shanghai: East China Normal University Press, 1998, 175-176

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The Characteristic Function of a Neutrosophic Set

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Abstract. The purpose of this paper is to introduce and study the characteristic function of a neutrosophic set. After given the fundamental definitions of neutrosophic set operations generated by the characteristic function of a neutrosophic set (for short), we obtain several properties, and discussed the relationship between neutrosophic sets generated by Ng and others. Finally, we introduce the neutrosophic topological spaces generated by . Possible application to GIS topology rules are touched upon.

Keywords: Neutrosophic Set; Neutrosophic Topology; Characteristic Function.

1 Introduction

Neutrosophy has laid the foundation for a whole family of new mathematical theories generalizing both their classical and fuzzy counterparts, such as a neutrosophic set theory. After the introduction of the neutrosophic set concepts in [2-13]. In this paper we introduce definitions of neutrosophic sets by characteristic function. After given the fundamental definitions of neutrosophic set operations by , we obtain several properties, and discussed the relationship between neutrosophic sets and others. Added to, we introduce the neutrosophic topological spaces generated by Ng .

2 Terminologies

We recollect some relevant basic preliminaries, and in particular, the work of Smarandache in [7- 9], Hanafy, Salama et al. [2- 13] and Demirci in [1].

3 Neutrosophic Sets generated by Ng

We shall now consider some possible definitions for basic concepts of the neutrosophic sets generated by and its operations.

3.1 Definition

Let X is a non-empty fixed set. A neutrosophic set (NS for short) A is an object having the form $A=\langle x,\mu_A(x),\sigma_A(x),\nu_A(x)\rangle$ where $\mu_A(x),\sigma_A(x)$ and $\gamma_A(x)$ which represent the degree of member ship function (namely $\mu_A(x)$), the degree of indeterminacy (namely $\sigma_A(x)$), and the degree of non-member ship (namely $\gamma_A(x)$) respectively of each element $x\in X$ to the set A .and let $g_A : X\times[0,1]\to[0,1]=I$ be reality function, then $Ng_A(\lambda)=Ng_A(\langle x,\lambda_1,\lambda_2,\lambda_3\rangle)$is said to be the characteristic function of a neutrosophic set on X if

$$Ng_A(\lambda)=\begin{cases}1 & \text{if } \mu_A(x)=\lambda_1,\sigma_{A(x)}=\lambda_2,\nu_A(x)=\lambda_3\\ 0 & \text{otherwise}\end{cases}$$

Where $\lambda=(\langle x,\lambda_1,\lambda_2,\lambda_3\rangle)$. Then the object $G(A)=\langle x,\mu_{G(A)}(x),\sigma_{G(A)}(x),\nu_{G(A)}(x)\rangle$ is a neutrosophic set generated by where

$$\mu_{G(A)}=\sup_{\lambda_1}\{Ng_A(\lambda)\wedge\lambda\}$$

$$\sigma_{G(A)}=\sup_{\lambda_2}\{Ng_A(\lambda)\wedge\lambda\}$$

$$\nu_{G(A)}=\sup_{\lambda_3}\{Ng_A(\lambda)\wedge\lambda\}$$

3.1 Proposition

1) $A\subseteq^{Ng} B\Leftrightarrow G(A)\subseteq G(B)$.

2) $A =^{Ng} B \Leftrightarrow G(A) = G(B)$

3.2 Definition

Let A be neutrosophic set of X. Then the neutrosophic complement of A generated by denoted by A^{Ngc} iff $[G(A)]^c$ may be defined as the following:

(Ng^{c1}) $\langle x, \mu^c{}_A(x), \sigma^c{}_A(x), \nu^c{}_A(x)\rangle$

(Ng^{c2}) $\langle x, \nu_A(x), \sigma\ {}_A(x), \mu_A(x)\rangle$

(Ng^{c3}) $\langle x, \nu_A(x), \sigma^c{}_A(x), \mu_A(x)\rangle$

3.1 Example. Let $X = \{x\}$, $A = \langle x, 0.5, 0.7, 0.6\rangle$, $Ng_A = 1$, $Ng_A = 0$. Then $G(A) = (\langle x, 0.5, 0.7, 0.6\rangle)$
Since our main purpose is to construct the tools for developing neutrosophic set and neutrosophic topology, we must introduce the $G(0_N)$ and $G(1_N)$ as follows $G(0_N)$ may be defined as:

i) $G(0_N) = \langle x, 0, 0, 1\rangle$

ii) $G(0_N) = \langle x, 0, 1, 1\rangle$

iii) $G(0_N) = \langle x, 0, 1, 0\rangle$

iv) $G(0_N) = \langle x, 0, 0, 0\rangle$

$G(1_N)$ may be defined as:

i) $G(1_N) = \langle x, 1, 0, 0\rangle$

ii) $G(1_N) = \langle x, 1, 0, 1\rangle$

iii) $G(1_N) = \langle x, 1, 1, 0\rangle$

iv) $G(1_N) = \langle x, 1, 1, 1\rangle$

We will define the following operations intersection and union for neutrosophic sets generated by Ng denoted by $\cap^{Ng}$ and $\cup^{Ng}$ respectively.

3.3 Definition. Let two neutrosophic sets $A = \langle x, \mu_A(x), \sigma_A(x), \nu_A(x)\rangle$ and

$B = \langle x, \mu_B(x), \sigma_B(x), \nu_B(x)\rangle$ on X, and

$G(A) = \langle x, \mu_{G(A)}(x), \sigma_{G(A)}(x), \nu_{G(A)}(x)\rangle$,

$G(B) = \langle x, \mu_{G(B)}(x), \sigma_{G(B)}(x), \nu_{G(B)}(x)\rangle$. Then $A \cap^{Ng} B$ may be defined as three types:

i) Type $G(A \cap B) =$

$\langle \mu_{G(A)}(x) \wedge \mu_{G(B)}, \sigma_{G(A)}(x) \wedge \sigma_{G(B)}(x), \nu_{G(A)}(x) \vee \nu_{G(B)}(x)\rangle$

ii) Type II:

$G(A \cap B) =$

$\langle \mu_{G(A)}(x) \wedge \mu_{G(B)}, \sigma_{G(A)}(x) \vee \sigma_{G(B)}(x), \nu_{G(A)}(x) \vee \nu_{G(B)}(x)\rangle$.

ii) Type III:

$G(A \cap B) =$

$\langle \mu_{G(A)}(x) \times \mu_{G(B)}, \sigma_{G(A)}(x) \times \sigma_{G(B)}(x), \nu_{G(A)}(x) \times \nu_{G(B)}(x)\rangle$

$A \cup^{Ng} B$ may be defined as two types:

Type I :

$G(A \cup B) =$

$\langle \mu_{G(A)}(x) \vee \mu_{G(B)}, \sigma_{G(A)}(x) \wedge \sigma_{G(B)}(x), \nu_{G(A)}(x) \wedge \nu_{G(B)}(x)\rangle$ ii) Type II:

$G(A \cup B) =$

$\langle \mu_{G(A)}(x) \vee \mu_{G(B)}, \sigma_{G(A)}(x) \vee \sigma_{G(B)}(x), \nu_{G(A)}(x) \wedge \nu_{G(B)}(x)\rangle$

. **3.4 Definition**

Let a neutrosophic set $A = \langle x, \mu_A(x), \sigma_A(x), \nu_A(x)\rangle$ and $G(A) = \langle x, \mu_{G(A)}(x), \sigma_{G(A)}(x), \nu_{G(A)}(x)\rangle$. Then

(1) $[\ \ ]^{Ng} A = \langle x : \mu_{G(A)}(x), \sigma_{G(A)}(x), 1 - \nu_{G(A)}(x)\rangle$

(2) $<>^{Ng} A =$
$\langle x : 1 - \mu_{G(A)}(x), \sigma_{G(A)}(x), \nu_{G(A)}(x)\rangle$

3.2 Proposition

For all two neutrosophic sets A and B on X generated by Ng, then the following are true

1) $(A \cap B)^{cNg} = A^{cNg} \cup B^{cNg}$.

2) $(A \cup B)^{cNg} = A^{cNg} \cap B^{cNg}$.

We can easily generalize the operations of intersection and union in definition 3.2 to arbitrary family of neutrosophic subsets generated by Ng as follows:

3.3 Proposition.

Let $\{A_j : j \in J\}$ be arbitrary family of neutrosophic subsets in X generated by two types , then

a) $\cap^{Ng} A_j$ may be defined as :

1) Type I :
$G(\cap A_j) = \langle \wedge \mu_{G(A_j)}(x), \wedge \sigma_{G(A_j)}(x), \vee \nu_{G(A_j)}(x)\rangle$,

2) Type II:
$G(\cap A_j) = \langle \wedge \mu_{G(A_j)}(x), \vee \sigma_{G(A_j)}(x), \vee \nu_{G(A_j)}(x)\rangle$,

b) $\cup^{Ng} A_j$ may be defined as :

1) $G(\cup A_j) = \langle \vee \mu_{G(A_j)}(x), \wedge \sigma_{G(A_j)}(x), \wedge \nu_{G(A_j)}(x)\rangle$ or

2) $G(\cup A_j) = \left\langle \vee \mu_{G(A_j)}(x), \vee \sigma_{G(A_j)}(x), \wedge \nu_{G(A_j)}(x) \right\rangle$.

3.4 Definition

Let f. X → Y be a mapping .

(i) The image of a neutrosophic set A generated by on X under f is a neutrosophic set B on Y generated by , denoted by f (A) whose reality function g_B : Y x I → I=[0, 1] satisfies the property

$$\mu_{G(B)} = \sup_{\lambda_1} \{Ng_A(\lambda) \wedge \lambda\}$$

$$\sigma_{G(B)} = \sup_{\lambda_2} \{Ng_A(\lambda) \wedge \lambda\}$$

$$\nu_{G(B)} = \sup_{\lambda_3} \{Ng_A(\lambda) \wedge \lambda\}$$

(ii) The preimage of a neutrosophic set B on Y generated by under f is a neutrosophic set A on X generated by , denoted by f^{-1} (B) , whose reality function g_A : X x [0, 1] →[0, 1] satisfies the property G(A) = G (B) o f

3.4 Proposition

Let $\{A_j : j \in J\}$ and $\{B_j : j \in J\}$ be families of neutrosophic sets on X and Y generated by , respectively. Then for a function f: X → Y, the following properties hold:

(i) If $A_j \subseteq^{Ng} A_k$;i , j∈J, then $f(A_j) \subseteq^{Ng} f(A_k)$

(ii) If $B_j \subseteq^{Ng} B_k$, for j , K ∈ J, then

$f^{-1}(B_j) \subseteq^{Ng} f^{-1}(B_K)$

(iii) $f^{-1}(\underset{jEJ}{\cup}^{Ng} B_j) =^{Ng} \underset{jEJ}{\cup}^{Ng} f^{-1}(B_j)$

3.5 Proposition

Let A and B be neutrosophic sets on X and Y generated by , respectively. Then, for a mappings f : X → Y , we have :

(i) $A \subseteq^{Ng} f^{-1}(f(A))$ (if f is injective the equality holds) .

(ii) $f(f^{-1}(B)) \subseteq^{Ng} B$ (if f is surjective the equality holds) .

(iii) $[f^{-1}(B)]^{Ngc} \subseteq^{Ng} f^{-1}(B^{N gc})$.

3.5 Definition . Let X be a nonempty set, Ψ a family of neutrosophic sets generated by and let us use the notation

$$G(\Psi) = \{G(A) : A \in \Psi\} .$$

If (X , G (Ψ)= $N\tau$) is a neutrosophic topological space on X is Salama's sense [3] , then we say that Ψ is a neutrosophic topology on X generated by and the pair (X , Ψ) is said to be a neutrosophic topological space generated by (ngts , for short). The elements in Ψ are called genuine neutrosophic open sets. also , we define the family

$$G(\Psi^c) = \{1 - G(A) : A \in \Psi\} .$$

3.6 Definition

Let (X , Ψ) be a ngts . A neutrosophic set C in X generated by is said to be a neutrosophic closed set generated by , if 1- G(C) ∈ G (Ψ) = $N\tau$.

3.7 Definition

Let (X , Ψ) be a ngts and A a neutrosophic set on X generated by . Then the neutrosophic interior of A generated by , denoted by, ngintA, is a set characterized by G(intA) = $\underset{G(\Psi)}{\text{int}}$ G(A) , where $\underset{G(\Psi)}{\text{int}}$ denotes the interior operation in neutrosophic topological spaces generated by .Similarly, the neutrosophic closure of A generated by , denoted by ngclA , is a neutrosophic set characterized by G(ngclA)= $\underset{G(\psi)}{cl}$ G(A) , where $\underset{G(\psi)}{cl}$ denotes the closure operation in neutrosophic topological spaces generated by .

The neutrosophic interior gnint(A) and the genuine neutrosophic closure gnclA generated by can be characterized by :

gnintA $=^{Ng} \cup^{Ng}$ { U : U ∈ Ψ and $U \subseteq^{Ng} A$ }

gnclA $=^{Ng} \cap^{Ng}$ { C : C is neutrosophic closed generated by and $A \subseteq^{Ng} C$ }

Since : G (gnint A) = ∪ { G (U) : G (U) ∈ G (Ψ) , G (U) ⊆ G (A) }

G (gncl A) = ∩ { G (C) : G (C) ∈ G (Ψ^c) , G (A) ⊆ G(C) }.

3.6 Proposition . For any neutrosophic set A generated by on a NTS (X , Ψ) , we have

(i) $\mathrm{cl}\ A^{Ngc} = {}^{Ng}(\mathrm{int}\ A)^{Ngc}$

(ii) $\mathrm{Int}\ A^{Ngc} = {}^{Ng}(\mathrm{cl}\ A)^{Ngc}$

References

[1] D. Demirci, Genuine sets, Fuzzy sets and Systems, 100 (1999), 377-384.

[2] A.A. Salama and S.A. Alblowi, "Generalized Neutrosophic Set and Generalized Neutrousophic Topological Spaces ", Journal computer Sci. Engineering, Vol. (2) No. (7), (2012), pp129-132.

[3] A.A. Salama and S.A. Alblowi,"Neutrosophic set and neutrosophic topological space" ISOR J.Math,Vol.(3), Issue(4), .(2012), pp-31-35.

[4] A.A. Salama and S.A. Alblowi, Intuitionistic Fuzzy Ideals Topological Spaces, Advances in Fuzzy Mathematics , Vol.(7), Number 1, (2012), pp 51- 60.

[5] A.A.Salama, and H.Elagamy, "Neutrosophic Filters" International Journal of Computer Science Engineering and Information Technology Reseearch (IJCSEITR), Vol.3, Issue(1), (2013)pp 307-312.

[6] S. A. Alblowi, A. A. Salama & Mohmed Eisa, New Concepts of Neutrosophic Sets, International Journal of Mathematics and Computer Applications Research (IJMCAR),Vol.3, Issue 4, Oct (2013), 95-102.

[7] Florentin Smarandache , Neutrosophy and Neutrosophic Logic , First International Conference on Neutrosophy , Neutrosophic Logic , Set, Probability, and Statistics University of New Mexico, Gallup, NM 87301, USA(2002) .

[8] Florentin Smarandache , An introduction to the Neutrosophy probability applid in Quntum Physics , International Conference on introducation Neutrosoph Physics , Neutrosophic Logic, Set, Probability, and Statistics University of New Mexico, Gallup, NM 87301, USA2-4 December (2011) .

[9] F. Smarandache. A Unifying Field in Logics: Neutrosophic Logic. Neutrosophy, Neutrosophic Set, Neutrosophic Probability. American Research Press, Rehoboth, NM, 1999.

[10] I. Hanafy, A.A. Salama and K. Mahfouz, "Correlation of neutrosophic Data" International Refereed Journal of Engineering and Science (IRJES), Vol.(1), Issue 2,(2012), pp.39-43.

[11] I.M. Hanafy, A.A. Salama and K.M. Mahfouz," Neutrosophic Classical Events and Its Probability" International Journal of Mathematics and Computer Applications Research(IJMCAR) Vol.(3),Issue 1,(2013), pp171-178.

[12] A. A. Salama,"Neutrosophic Crisp Points & Neutrosophic Crisp Ideals", Neutrosophic Sets and Systems, Vol.1, No. 1,(2013) pp50-54.

[13] A. A. Salama and F. Smarandache, " Filters via Neutrosophic Crisp Sets", Neutrosophic Sets and Systems, Vol.1, No. 1,(2013) pp 34-38.

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Neutrosophic Left Almost Semigroup

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Abstract. In this paper we extend the theory of neutrosophy to study left almost semigroup shortly LA-semigroup. We generalize the concepts of LA-semigroup to form that for neutrosophic LA-semigroup. We also extend the ideal theory of LA-semigroup to neutrosophy and discuss different kinds of neutrosophic ideals. We also find some new type of neutrosophic ideal which is related to the strong or pure part of neutrosophy. We have given many examples to illustrate the theory of neutrosophic LA-semigroup and display many properties of neutrosophic LA-semigroup in this paper.

Keywords: LA-semigroup,sub LA-semigroup, ideal, neutrosophic LA-semigroup, neutrosophic sub LA- semigroup, neutrosophic ideal.

1 Introduction

Neutrosophy is a new branch of philosophy which studies the origin and features of neutralities in the nature. Florentin Smarandache in 1980 firstly introduced the concept of neutrosophic logic where each proposition in neutrosophic logic is approximated to have the percentage of truth in a subset T, the percentage of indeterminacy in a subset I, and the percentage of falsity in a subset F so that this neutrosophic logic is called an extension of fuzzy logic. In fact neutrosophic set is the generalization of classical sets, conventional fuzzy set $[1]$, intuitionistic fuzzy set $[2]$ and interval valued fuzzy set $[3]$. This mathematical tool is used to handle problems like imprecise, indeterminacy and inconsistent data etc. By utilizing neutrosophic theory, Vasantha Kandasamy and Florentin Smarandache dig out neutrosophic algebraic structures in $[11]$. Some of them are neutrosophic fields, neutrosophic vector spaces, neutrosophic groups, neutrosophic bigroups, neutrosophic N-groups, neutrosophic semigroups, neutrosophic bisemigroups, neutrosophic N-semigroup, neutrosophic loops, neutrosophic biloops, neutrosophic N-loop, neutrosophic groupoids, and neutrosophic bigroupoids and so on.

A left almost semigroup abbreviated as LA-semigroup is an algebraic structure which was introduced by M .A. Kazim and M. Naseeruddin $[3]$ in 1972. This structure is basically a midway structure between a groupoid and a commutative semigroup. This structure is also termed as Able-Grassmann's groupoid abbreviated as AG-groupoid $[6]$. This is a non associative and non commutative algebraic structure which closely resemble to commutative semigroup. The generalization of semigroup theory is an LA-semigroup and this structure has wide applications in collaboration with semigroup. We have tried to develop the ideal theory of LA-semigroups in a logical manner. Firstly, preliminaries and basic concepts are given for LA-semigroups. Section 3 presents the newly defined notions and results in neutrosophic LA-semigroups. Various types of ideals are defined and elaborated with the help of examples. Furthermore, the homomorphisms of neutrosophic LA-semigroups are discussed at the end.

2 Preliminaries

Definition 1. A groupiod $(S,*)$ is called a left almost semigroup abbreviated as LA-semigroup if the left invertive law holds, i.e.

$(a*b)*c=(c*b)*a$ for all $a,b,c\in S$.

Similarly $(S,*)$ is called right almost semigroup denoted as RA-semigroup if the right invertive law holds, i.e.

$a*(b*c)=c*(b*a)$ for all $a,b,c\in S$.

Proposition 1. In an LA-semigroup S, the medial law holds. That is

$(ab)(cd)=(ac)(bd)$ for all $a,b,c,d\in S$.

Proposition 2. In an LA-semigrup S, the following statements are equivalent:

1) $(ab)c=b(ca)$
2) $(ab)c=b(ac)$. For all $a,b,c\in S$.

Theorem 1. An LA-semigroup S is a semigroup if and only if $a(bc)=(cb)a$, for all $a,b,c\in S$.

Theorem 2. An LA-semigroup with left identity satisfies the following Law,

$$(ab)(cd)=(db)(ca) \text{ for all } a,b,c,d\in S.$$

Theorem 3. In an LA-semigroup S, the following holds, $a(bc)=b(ac)$ for all $a,b,c\in S$.

Theorem 4. If an LA-semigroup S has a right identity, then S is a commutative semigroup.

Definition 2. Let S be an LA-semigroup and H be a proper subset of S. Then H is called sub LA-semigroup of S if $H.H\subseteq H$.

Definition 3. Let S be an LA-semigroup and K be a subset of S. Then K is called Left (right) ideal of S if $SK\subseteq K$, $(KS\subseteq K)$.

If K is both left and right ideal, then K is called a two sided ideal or simply an ideal of S.

Lemma 1. If K is a left ideal of an LA-semigroup S with left identity e, then aK is a left ideal of S for all $a\in S$.

Definition 4. An ideal P of an LA-semigroup S with left identity e is called prime ideal if $AB\subseteq P$ implies either

$A\subseteq P$ or $B\subseteq P$, where A,B are ideals of S.

Definition 5. An LA-semigroup S is called fully prime LA-semigroup if all of its ideals are prime ideals.

Definition 6 An ideal P is called semiprime ideal if

$T.T\subseteq P$ implies $T\subseteq P$ for any ideal T of S.

Definition 7. An LA-semigroup S is called fully semiprime LA-semigroup if every ideal of S is semiprime ideal.

Definition 8. An ideal R of an LA-semigroup S is called strongly irreducible ideal if for any ideals H,K of S, $H\cap K\subseteq R$ implies $H\subseteq R$ or $K\subseteq R$.

Proposition 3. An ideal K of an LA-semigroup S is prime ideal if and only if it is semiprime and strongly irreducible ideal of S.

Definition 9. Let S be an LA-semigroup and Q be a non-empty subset of S. Then Q is called Quasi ideal of S if $QS\cap SQ\subseteq Q$.

Theorem 5. Every left $(right)$ ideal of an LA-semigroup S is a quasi-ideal of S.

Theorem 6. Intersection of two quasi ideals of an LA-semigroup is again a quasi ideal.

Definition 10. A sub LA-semigroup B of an LA-semigroup is called bi-ideal of S if $(BS)B \subseteq B$.

Definition 11. A non-empty subset A of an LA-semigroup S is termed as generalized bi-ideal of S if $(AS)A \subseteq A$.

Definition 12. A non-empty subset L of an LA-semigroup S is called interior ideal of S if $(SL)S \subseteq L$.

Theorem 7. Every ideal of an LA-semigroup S is an interior ideal.

3 Neutrosophic LA-semigroup

Definition 13. Let $(S,*)$ be an LA-semigroup and let $\langle S \cup I \rangle = \{a + bI : a, b \in S\}$. The neutrosophic LA-semigroup is generated by S and I under $*$ denoted as $N(S) = \{\langle S \cup I \rangle, *\}$, where I is called the neutrosophic element with property $I^2 = I$. For an integer n, $n + I$ and nI are neutrosophic elements and $0.I = 0$. I^{-1}, the inverse of I is not defined and hence does not exist.

Example 1. Let $S = \{1,2,3\}$ be an LA-semigroup with the following table

*	1	2	3
1	1	1	1
2	3	3	3
3	1	1	1

Then the neutrosophic LA-semigroup $N(S) = \langle S \cup I \rangle = \{1,2,3,1I,2I,3I\}$ with the following table

*	1	2	3	1I	2I	3I
1	1	1	1	1I	1I	1I
2	3	3	3	3I	3I	3I
3	1	1	1	1I	1I	1I
1I	1I	1I	1I	1I	1I	1I
2I	3I	3I	3I	3I	3I	3I
3I	1I	1I	1I	1I	1I	1I

Similarly we can define neutrosophic RA-semigroup on the same lines.

Theorem 9. All neutrosophic LA-semigroups contains corresponding LA-semigroups.

Proof : straight forward.

Proposition 4. In a neutrosophic LA-semigroup $N(S)$, the medial law holds. That is

$$(ab)(cd) = (ac)(bd) \text{ for all } a,b,c,d \in N(S).$$

Proposition 5. In a neutrosophic LA-semigrup $N(S)$, the following statements are equivalent.

1) $(ab)c = b(ca)$
2) $(ab)c = b(ac)$. For all $a,b,c \in N(S)$.

Theorem 9. A neutrosophic LA-semigroup $N(S)$ is a neutrosophic semigroup if and only if $a(bc) = (cb)a$, for all $a,b,c \in N(S)$.

Theorem 10. Let $N(S_1)$ and $N(S_2)$ be two neutrosophic LA-semigroups. Then their cartesian product $N(S_1) \times N(S_2)$ is also a neutrosophic LA-semigroups.

Proof : The proof is obvious.

Theorem 11. Let S_1 and S_2 be two LA-semigroups. If $S_1 \times S_2$ is an LA-semigroup, then $N(S_1)\times N(S_2)$ is also a neutosophic LA-semigroup.

Proof : The proof is straight forward.

Definition 14. Let $N(S)$ be a neutrosophic LA-semigroup. An element $e \in N(S)$ is said to be left identity if $e*s = s$ for all $s \in N(S)$. Similarly e is called right identity if $s*e = s$.

e is called two sided identity or simply identity if e is left as well as right identity.

Example 2. Let $N(S) = \langle S \cup I \rangle = \{1,2,3,4,5,1I,2I,3I,4I,5I\}$

with left identity 4, defined by the following multiplication table.

.	1	2	3	4	5	1I	2I	3I	4I	5I
1	4	5	1	2	3	4I	5I	1I	2I	3I
2	3	4	5	1	2	3I	4I	5I	1I	2I
3	2	3	4	5	1	2I	3I	4I	5I	1I
4	1	2	3	4	5	1I	2I	3I	4I	5I
5	5	1	2	3	4	5I	1I	2I	3I	4I
1I	4I	5I	1I	2I	3I	4I	5I	1I	2I	3I
2I	3I	4I	5I	1I	2I	3I	4I	5I	1I	2I
3I	2I	3I	4I	5I	1I	2I	3I	4I	5I	1I
4I	1I	2I	3I	4I	5I	1I	2I	3I	4I	5I
5I	5I	1I	2I	3I	4I	5I	1I	2I	3I	4I

Proposition 6. If $N(S)$ is a neutrosophic LA-semigroup with left identity e, then it is unique.

Proof : Obvious.

Theorem10. A neutrosophic LA-semigroup with left identity satisfies the following Law,

$(ab)(cd) = (db)(ca)$ for all $a,b,c,d \in N(S)$.

Theorem 11. In a neutrosophic LA-semigroup $N(S)$, the following holds,

$a(bc) = b(ac)$ for all $a,b,c \in N(S)$.

Theorem 12. If a neutrosophic LA-semigroup $N(S)$ has a right identity, then $N(S)$ is a commutative semigroup.

Proof. Suppose that e be the right identity of $N(S)$. By definition $ae = a$ for all $a \in N(S)$.So $ea = (e.e)a = (a.e)e = a$ for all $a \in N(S)$. Therefore e is the two sided identity. Now let $a,b \in N(S)$, then $ab = (ea)b = (ba)e = ba$ and hence $N(S)$ is commutative. Again let $a,b,c \in N(S)$, So $(ab)c = (cb)a = (bc)a = a(bc)$ and hence $N(S)$ is commutative semigroup.

Definition 15. Let $N(S)$ be a neutrosophic LA-semigroup and $N(H)$ be a proper subset of $N(S)$. Then $N(H)$ is called a neutrosophic sub LA-semigroup if $N(H)$ itself is a neutrosophic LA-semigroup under the operation of $N(S)$.

Example 3. Let $N(S) = \langle S \cup I \rangle = \{1,2,3,1I,2I,3I\}$

be a neutrosophic LA-semigroup as in example (1). Then $\{1\}, \{1,3\}, \{1,1I\}, \{1,3,1I,3I\}$ etc are neutrosophic sub LA-semigroups but $\{2,3,2I,3I\}$ is not neutrosophic sub LA-semigroup of $N(S)$.

Theorem 13 Let $N(S)$ be a neutrosophic LA-semigroup and $N(H)$ be a proper subset of $N(S)$. Then $N(H)$ is a neutrosophic sub LA-semigroup of $N(S)$ if $N(H).N(H) \subseteq N(H)$.

Theorem 14 Let H be a sub LA-semigroup of an LA-semigroup S, then $N(H)$ is aneutrosophic sub LA-semigroup of the neutrosophic LA-semigroup $N(S)$, where $N(H) = \langle H \cup I \rangle$.

Definition 16. A neutrosophic sub LA-semigroup $N(H)$ is called strong neutrosophic sub LA-semigroup or pure neutrosophic sub LA-semigroup if all the elements of $N(H)$ are neutrosophic elements.

Example 4. Let $N(S) = \langle S \cup I \rangle = \{1,2,3,1I,2I,3I\}$ be a neutrosophic LA-semigroup as in example (1). Then $\{1I,3I\}$ is a strong neutrosophic sub LA-semigroup or pure neutrosophicsub LA-semigroup of $N(S)$.

Theorem 15. All strong neutrosophic sub LA-semigroups or pure neutrosophic sub LA-semigroups are trivially neutrosophic sub LA-semigroup but the converse is not true.

Example 5. Let $N(S) = \langle S \cup I \rangle = \{1,2,3,1I,2I,3I\}$ be a neutrosophic LA-semigroup as in example (1). Then $\{1\}, \{1,3\}$ are neutrosophic sub LA-semigroups but not strong neutrosophic sub LA-semigroups or pure neutrosophic sub LA-semigroups of $N(S)$.

Definition 17 Let $N(S)$ be a neutrosophic LA-semigroup and $N(K)$ be a subset of $N(S)$. Then $N(K)$ is called Left (right) neutrosophic ideal of $N(S)$ if $N(S)N(K) \subseteq N(K)$, $\{N(K)N(S) \subseteq N(K)\}$.

If $N(K)$ is both left and right neutrosophic ideal, then $N(K)$ is called a two sided neutrosophic ideal or simply a neutrosophic ideal.

Example 6. Let $S = \{1,2,3\}$ be an LA-semigroup with the following table.

*	1	2	3
1	3	3	3
2	3	3	3
3	1	3	3

Then the neutrosophic LA-semigroup $N(S) = \langle S \cup I \rangle = \{1,2,3,1I,2I,3I\}$ with the following table.

*	1	2	3	1I	2I	3I
1	3	3	3	3I	3I	3I
2	3	3	3	3I	3I	3I
3	1	3	3	1I	3I	3I
1I	3I	3I	3I	3I	3I	3I
2I	3I	3I	3I	3I	3I	3I
3I	1I	3I	3I	1I	3I	3I

Then clearly $N(K_1)=\{3,3I\}$ is a neutrosophic left ideal and $N(K_2)=\{1,3,1I,3I\}$ is a neutrosophic left as well as right ideal.

Lemma 2. If $N(K)$ be a neutrosophic left ideal of a neutrosophic LA-semigroup $N(S)$ with left identity e, then $aN(K)$ is a neutrosophic left ideal of $N(S)$ for all $a\in N(S)$.

Proof : The proof is satraight forward.

Theorem 16. $N(K)$ is a neutrosophic ideal of a neutrosophic LA-semigroup $N(S)$ if K is an ideal of an LA-semigroup S, where $N(K)=\langle K\cup I\rangle$.

Definition 18 A neutrosophic ideal $N(K)$ is called strong neutrosophic ideal or pure neutrosophic ideal if all of its elements are neutrosophic elements.

Fxample 7. Let $N(S)$ be a neutrosophic LA-semigroup as in example (5), Then $\{1I,3I\}$ and $\{1I,2I,3I\}$ are strong neutrosophic ideals or pure neutrosophic ideals of $N(S)$.

Theorem 17. All strong neutrosophic ideals or pure neutrosophic ideals are neutrosophic ideals but the converse is not true.

To see the converse part of above theorem, let us take an example.

Example 7 Let $N(S)=\langle S\cup I\rangle=\{1,2,3,1I,2I,3I\}$ be as in example (5). Then $N(K_1)=\{2,3,2I,3I\}$ and $N(K_2)=\{1,3,1I,3I\}$ are neutrosophic ideals of $N(S)$ but clearly these are not strong neutrosophic ideals or pure neutrosophic ideals.

Definition 19 : A neutorophic ideal $N(P)$ of a neutrosophic LA-semigroup $N(S)$ with left identity e is called prime neutrosophic ideal if $N(A)N(B)\subseteq N(P)$ implies either $N(A)\subseteq N(P)$ or $N(B)\subseteq N(P)$, where $N(A),N(B)$ are neutrosophic ideals of $N(S)$.

Example 8. Let $N(S)=\langle S\cup I\rangle=\{1,2,3,1I,2I,3I\}$ be as in example (5) and let $N(A)=\{2,3,2I,3I\}$ and $N(B)=\{1,3,1I,3I\}$ and $N(P)=\{1,3,1I,3I\}$ are neutrosophic ideals of $N(S)$. Then clearly $N(A)N(B)\subseteq N(P)$ implies $N(A)\subseteq N(P)$ but $N(B)$ is not contained in $N(P)$. Hence $N(P)$ is a prime neutrosophic ideal of $N(S)$.

Theorem 18. Every prime neutrosophic ideal is a neutrosophic ideal but the converse is not true.

Theorem19. If P is a prime ideal of an LA-semigoup S, Then $N(P)$ is prime neutrosophic ideal of $N(S)$ where $N(P)=\langle P\cup I\rangle$.

Definition 20. A neutrosophic LA-semigroup $N(S)$ is called fully prime neutrosophic LA-semigroup if all of its neutrosophic ideals are prime neutrosophic ideals.

Definition 21. A prime neutrosophic ideal $N(P)$ is called strong prime neutrosophic ideal or pure neutrosophic ideal if x is neutrosophic element for all $x\in N(P)$.

Example 9. Let $N(S)=\langle S\cup I\rangle=\{1,2,3,1I,2I,3I\}$ be as in example (5) and let $N(A)=\{2I,3I\}$ and

$N(B)=\{1I,3I\}$ and $N(P)=\{1I,3I\}$ are neutrosophic ideals of $N(S)$. Then clearly $N(A)N(B)\subseteq N(P)$ implies $N(A)\subseteq N(P)$ but $N(B)$ is not contained in $N(P)$. Hence $N(P)$ is a strong prime neutrosophic ideal or pure neutrosophic ideal of $N(S)$.

Theorem 20. Every prime strong neutrosophic ideal or pure neutrosophic ideal is neutrosophic ideal but the converse is not true.

Theorem 21. Every prime strong neutrosophic ideal or pure neutrosophic ideal is a prime neutrosophic ideal but the converse is not true.

For converse, we take the following example.

Example 10. In example (6), $N(P)=\{1,3,1I,3I\}$ is a prime neutrosophic ideal but it is not strong neutrosophic ideal or pure neutrosophic ideal.

Definition 22. A neutrosophic ideal $N(P)$ is called semiprime neutrosophic ideal if $N(T).N(T)\subseteq N(P)$ implies $N(T)\subseteq N(P)$ for any neutrosophic ideal $N(T)$ of $N(S)$.

Example 11. Let $N(S)$ be the neutrosophic LA-semigroup of example (1) and let $N(T)=\{1,1I\}$ and $N(P)=\{1,3,1I,3I\}$ are neutrosophic ideals of $N(S)$. Then clearly $N(P)$ is a semiprime neutrosophic ideal of $N(S)$.

Theorem 22. Every semiprime neutrosophic ideal is a neutrosophic ideal but the converse is not true.

Definition 23. A neutrosophic semiprime ideal $N(P)$ is said to be strong semiprime neutrosophic ideal or pure semiprime neutrosophic ideal if every element of $N(P)$ is neutrosophic element.

Example 12. Let $N(S)$ be the neutrosophic LA-semigroup of example (1) and let $N(T)=\{1I,3I\}$ and $N(P)=\{1I,2I,3I\}$ are neutrosophic ideals of $N(S)$. Then clearly $N(P)$ is a strong semiprime neutrosophic ideal or pure semiprime neutrosophic ideal of $N(S)$.

Theorem 23. All strong semiprime neutrosophic ideals or pure semiprime neutrosophic ideals are trivially neutrosophic ideals but the converse is not true.

Theorem 24. All strong semiprime neutrosophic ideals or pure semiprime neutrosophic ideals are semiprime neutrosophic ideals but the converse is not true.

Definition 24. A neutrosophic LA-semigroup $N(S)$ is called fully semiprime neutrosophic LA-semigroup if every neutrosophic ideal of $N(S)$ is semiprime neutrosophic ideal.

Definition 25. A neutrosophic ideal $N(R)$ of a neutrosophic LA-semigroup $N(S)$ is called strongly irreducible neutrosophic ideal if for any neutrosophic ideals $N(H),N(K)$ of $N(S)$, $N(H)\cap N(K)\subseteq N(R)$ implies $N(H)\subseteq N(R)$ or $N(K)\subseteq N(R)$.

Example 13. Let $N(S)=\langle S\cup I\rangle=\{1,2,3,1I,2I,3I\}$ be as in example (5) and let $N(H)=\{2,3,2I,3I\}$, $N(K)=\{1I,3I\}$ and $N(R)=\{1,3,1I,3I\}$ are neutrosophic ideals of $N(S)$. Then clearly $N(H)\cap N(K)\subseteq N(R)$ implies $N(K)\subseteq N(R)$ but $N(H)$ is not contained in $N(R)$. Hence $N(R)$ is a strong irreducible neutrosophic ideal of $N(S)$.

Theorem 25. Every strongly irreducible neutrosohic ideal is a neutrosophic ideal but the converse is not true.

Theorem 26. If R is a strong irreducible neutrosophic ideal of an LA-semigoup S, Then $N(I)$ is a strong irreducible neutrosophic ideal of $N(S)$ where $N(R)=\langle R\cup I\rangle$.

Proposition 7. A neutrosophic ideal $N(I)$ of a neutrosophic LA-semigroup $N(S)$ is prime neutrosophic ideal if and only if it is semiprime and strongly irreducible neutrosophic ideal of $N(S)$.

Definition 26. Let $N(S)$ be a neutrosophic ideal and

$N(Q)$ be a non-empty subset of $N(S)$. Then $N(Q)$ is called quasi neutrosophic ideal of $N(S)$ if

$$N(Q)N(S)\cap N(S)N(Q)\subseteq N(Q).$$

Example 14. Let $N(S)=\langle S\cup I\rangle=\{1,2,3,1I,2I,3I\}$ be as in example (5). Then $N(K)=\{3,3I\}$ be a non-empty subset of $N(S)$ and $N(S)N(K)=\{3,3I\}$, $N(K)N(S)=\{1,3,1I,3I\}$ and their intersection is $\{3,3I\}\subseteq N(K)$. Thus clearly $N(K)$ is quasi neutrosophic ideal of $N(S)$.

Theorem27. Every left $(right)$ neutrosophic ideal of a neutrosophic LA-semigroup $N(S)$ is a quasi neutrosophic ideal of $N(S)$.

Proof : Let $N(Q)$ be a left neutrosophic ideal of a neutrosophic LA-semigroup $N(S)$, then $N(S)N(Q)\subseteq N(Q)$ and so

$$N(S)N(Q)\cap N(Q)N(S)\subseteq N(Q)\cap N(Q)\subseteq N(Q)$$

which proves the theorem.

Theorem 28. Intersection of two quasi neutrosophic ideals of a neutrosophic LA-semigroup is again a quasi neutrosophic ideal.

Proof : The proof is straight forward.

Definition 27. A quasi-neutrosophic ideal $N(Q)$ of a neutrosophic LA-semigroup $N(Q)$ is called quasi-strong neutrosophic ideal or quasi-pure neutosophic ideal if all the elements of $N(Q)$ are neutrosophic elements.

Example 15. Let $N(S)=\langle S\cup I\rangle=\{1,2,3,1I,2I,3I\}$ be as in example (5). Then $N(K)=\{1I,3I\}$ be a quasi-neutrosophic ideal of $N(S)$. Thus clearly $N(K)$ is quasi-strong neutrosophic ideal or quasi-pure neutrosophic ideal of $N(S)$.

Theorem 29. Every quasi-strong neutrosophic ideal or quasi-pure neutrosophic ideal is qausi-neutrosophic ideal but the converse is not true.

Definition 28. A neutrosophic sub LA-semigroup $N(B)$ of a neutrosophic LA-semigroup is called bi-neutrosophic ideal of $N(S)$ if

$$\big(N(B)N(S)\big)N(B)\subseteq N(B).$$

Example 16. Let $N(S)=\langle S\cup I\rangle=\{1,2,3,1I,2I,3I\}$ be a neutrosophic LA-semigroup as in example (1) and $N(B)=\{1,3,1I,3I\}$ is a neutrosophic sub LA-semigroup of $N(S)$. Then Clearly $N(B)$ is a bi-neutrosophic ideal of $N(S)$.

Theorem 30. Let B be a bi-ideal of an LA-semigroup S, then $N(B)$ is bi-neutrosophic ideal of $N(S)$ where $N(B)=\langle B\cup I\rangle$.

Proof : The proof is straight forward.

Definition 29. A bi-neutrosophic ideal $N(B)$ of a neutrosophic LA-semigroup $N(S)$ is called bi-strong neutrosophic ideal or bi-pure neutrosophic ideal if every element of $N(B)$ is a neutrosophic element.

Example 17. Let $N(S)=\langle S\cup I\rangle=\{1,2,3,1I,2I,3I\}$ be a neutrosophic LA-semigroup as in example (1) and $N(B)=\{1I,3I\}$ is a bi-neutrosophic ideal of $N(S)$. Then Clearly $N(B)$ is a bi-strong neutrosophic ideal or bi-pure neutosophic ideal of $N(S)$.

Theorem 31. All bi-strong neutrosophic ideals or bi-pure neutrosophic ideals are bi-neutrosophic ideals but the converse is not true.

Definition 30. A non-empty subset $N(A)$ of a neutrosophic LA-semigroup $N(S)$ is termed as generalized bi-neutrosophic ideal of $N(S)$ if $(N(A)N(S))N(A)\subseteq N(A)$.

Example 18. Let $N(S)=\langle S\cup I\rangle=\{1,2,3,1I,2I,3I\}$ be a neutrosophic LA-semigroup as in example (1) and $N(A)=\{1,1I\}$ is a non-empty subset of $N(S)$. Then Clearly $N(A)$ is a generalized bi-neutrosophic ideal of $N(S)$.

Theorem 32. Every bi-neutrosophic ideal of a neutrosophic LA-semigroup is generalized bi-ideal but the converse is not true.

Definition 31. A generalized bi-neutrosophic ideal $N(A)$ of a neutrosophic LA-semigroup $N(S)$ is called generalized bi-strong neutrosophic ideal or generalized bi-pure neutrosophic ideal of $N(S)$ if all the elements of $N(A)$ are neutrosophic elements.

Example 19. Let

$N(S)=\langle S\cup I\rangle=\{1,2,3,1I,2I,3I\}$ be

a neutrosophic LA-semigroup as in example (1) and

$N(A)=\{1I,3I\}$ is a generalized bi-neutrosophic ideal of $N(S)$. Then clearly $N(A)$ is a generalized bi-strong neutrosophic ideal or generalized bi-pure neutrosophic ideal of $N(S)$.

Theorem 33. All generalized bi-strong neutrosophic ideals or generalized bi-pure neutrosophic ideals are generalized bi-neutrosophic ideals but the converse is not true.

Theorem 34. Every bi-strong neutrosophic ideal or bi-pureneutrosophic ideal of a neutrosophic LA-semigroup is generalized bi-strong neutrosophic ideal or generalized bi-pure neutrosophic ideal but the converse is not true.

Definition 32. A non-empty subset $N(L)$ of a neutrosophic LA-semigroup $N(S)$ is called interior neutrosophic ideal of $N(S)$ if $(N(S)N(L))N(S) \subseteq N(L)$.

Example 20. Let $N(S) = \langle S \cup I \rangle = \{1,2,3,1I,2I,3I\}$ be a neutrosophic LA-semigroup as in example (1) and $N(L) = \{1,1I\}$ is a non-empty subset of $N(S)$. Then Clearly $N(L)$ is an interior neutrosophic ideal of $N(S)$.

Theorem 35. Every neutrosophic ideal of a neutrosophic LA-semigroup $N(S)$ is an interior neutrosophic ideal.

Proof : Let $N(L)$ be a neutrosophic ideal of a neutrosophic LA-semigroup $N(S)$, then by definition $N(L)N(S) \subseteq N(L)$ and $N(S)N(L) \subseteq N(L)$. So clearly $(N(S)N(L))N(S) \subseteq N(L)$ and hence $N(L)$ is an interior neutrosophic ideal of $N(S)$.

Definition 33. An interior neutrosophic ideal $N(L)$ of a neutrosophic LA-semigroup $N(S)$ is called interior strong neutrosophic ideal or interior pure neutrosophic ideal if every element of $N(L)$ is a neutrosophic element.

Example 21. Let

$N(S) = \langle S \cup I \rangle = \{1,2,3,1I,2I,3I\}$ be a neutrosophic LA-semigroup as in example (1) and

$N(L) = \{1I,3I\}$ is a non-empty subset of $N(S)$. Then Clearly $N(L)$ is an interior strong neutrosophic ideal or interior pure neutrosophic ideal of $N(S)$.

Theorem 36. All interior strong neutrosophic ideals or interior pure neutrosophic ideals are trivially interior neutrosophic ideals of a neutrosophic LA-semigroup $N(S)$ but the converse is not true.

Theorem 37. Every strong neutrosophic ideal or pure neutosophic ideal of a neutrosophic LA-semigroup $N(S)$ is an interior strong neutrosophic ideal or interior pure neutrosophic ideal.

Neutrosophic homomorphism

Definition 34. Let S,T be two LA-semigroups and $\phi : S \to T$ be a mapping from S to T. Let $N(S)$ and $N(T)$ be the corresponding neutrosophic LA-semigroups of S and T respectively. Let $\theta : N(S) \to N(T)$ be another mapping from $N(S)$ to $N(T)$. Then θ is called neutrosophic homomorphis if ϕ is homomorphism from S to T.

Example 22. Let Z be an LA-semigroup under the operation $a*b = b-a$ for all $a,b \in Z$. Let Q be another LA-semigroup under the same operation defined above. For some fixed non-zero rational number x, we define $\phi : Z \to Q$ by $\phi(a) = a/x$ where $a \in Z$. Then ϕ is a homomorphism from Z to Q. Let $N(Z)$ and $N(Q)$ be the corresponding neutrosophic LA-semigroups of Z and Q respectively. Now Let $\theta : N(Z) \to N(Q)$ be a map from neutrosophic LA-semigroup $N(Z)$ to the neutrosophic LA-semigroup $N(Q)$. Then clearly θ is the corresponding neutrosophic homomorphism of $N(Z)$ to $N(Q)$ as ϕ is homomorphism.

Theorem 38. If ϕ is an isomorphism, then θ will be the neutrosophic isomorphism.

Proof : It's easy.

Conclusion

In this paper we extend the neutrosophic group and subgroup,pseudo neutrosophic group and subgroup to soft neutrosophic group and soft neutrosophic subgroup and respectively soft pseudo neutrosophic group and soft pseudo neutrosophic subgroup. The normal neutrosophic subgroup is extended to soft normal neutrosophic subgroup.

We showed all these by giving various examples in order to illustrate the soft part of the neutrosophic notions used.

References

[1] A.A.A. Agboola, A.D. Akwu, Y.T.Oyebo, Neutrosophic groups and subgroups, Int. J. Math. Comb. 3(2012) 1-9.

[2] Aktas and N. Cagman, Soft sets and soft group, Inf. Sci 177(2007) 2726-2735.

[3] M.I. Ali, F. Feng, X.Y. Liu, W.K. Min and M. Shabir, On some new operationsin soft set theory, Comput. Math. Appl. (2008) 2621-2628.

[4] K. Atanassov, Intuitionistic fuzzy sets, Fuzzy Sets Syst. 64(2) (1986)87-96.

[5] S.Broumi, F. Smarandache, Intuitionistic Neutrosophic Soft Set, J. Inf. & Comput. Sc. 8(2013) 130-140.

[6] D. Chen, E.C.C. Tsang, D.S. Yeung, X. Wang, The parameterization reduction of soft sets and its applications,Comput. Math. Appl. 49(2005) 757-763.

[7] M.B. Gorzalzany, A method of inference in approximate reasoning based on interval-valuedfuzzy sets, Fuzzy Sets and Systems 21 (1987) 1-17.

[8] P. K. Maji, Neutrosophic Soft Set, Annals of Fuzzy Mathematics and Informatics, 5 (1) (2013), 2093-9310.

[9] P.K. Maji, R. Biswas and R. Roy, Soft set theory, Comput. Math. Appl. 45(2003) 555-562.

[10] P.K. Maji, A.R. Roy and R. Biswas, An application of soft sets in a decision making problem, Comput. Math. Appl. 44(2002) 1007-1083.

[11] D. Molodtsov, Soft set theory first results, Comput. Math. Appl. 37(1999) 19-31.

[12] Z. Pawlak, Rough sets, Int. J. Inf. Compu. Sci. 11(1982) 341-356.

[13] Florentin Smarandache,A Unifying Field in Logics. Neutrosophy: Neutrosophic Probability, Set and Logic. Rehoboth: American Research Press, (1999).

[14] W. B. Vasantha Kandasamy & Florentin Smarandache, Some Neutrosophic Algebraic Structures and Neutrosophic N-Algebraic Structures, 219 p., Hexis, 2006.

[15] W. B. Vasantha Kandasamy & Florentin Smarandache, N-Algebraic Structures and S-N-Algebraic Structures, 209 pp., Hexis, Phoenix, 2006.

[16] W. B. Vasantha Kandasamy & Florentin Smarandache, Basic Neutrosophic Algebraic Structures and their Applications to Fuzzy and Neutrosophic Models, Hexis, 149 pp., 2004.

[17] A. Sezgin & A.O. Atagun, Soft Groups and Normalistic Soft Groups, Comput. Math. Appl. 62(2011) 685-698.

[18] L.A. Zadeh, Fuzzy sets, Inform. Control 8(1965) 338 -353.

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Neutrosophic Hypercompositional Structures defined by Binary Relations

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Abstract: The objective of this paper is to study *neutrosophic* hypercompositional structures $H(I)_\tau$ arising from the hypercompositions derived from the binary relations τ on a *neutrosophic* set $H(I)$. We give the characterizations of τ that make $H(I)_\tau$ hypergroupoids,quasihypergroups, semihypergroups, *neutrosophic* hypergroupoids, *neutrosophic* quasihypergroups, *neutrosophic* semihypergroups and *neutrosophic* hypergroups.

Keywords: hypergroup, neutrosophic hypergroup, binary relations.

1 Introduction

The concept of hyperstructure together with the concept of hypergroup was introduced by F. Marty at the 8th Congress of Scandinavian Mathematicians held in 1934. A comprehensive review of the concept can be found in [5, 6, 12]. The concept of neutrosophy was introduced by F. Smarandache in 1995 and the concept of *neutrosophic* algebraic structures was introduced by F. Smarandache and W.B. Vasantha Kandasamy in 2006. A comprehensive review of *neutrosophy* and *neutrosophic* algebraic structures can be found in [1, 2, 3, 4, 15, 24 , 25].

One of the techniques of constructing hypergoupoids, quasi hypergroups, semihypergroups and hypergroups is to endow a nonempty set H with a hypercomposition derived from the binary relation ρ on H that give rise to a hypercompositional structure H_ρ. In this paper, we consider binary relations τ on a neutrosophic set H(I) that define hypercompositional structures $H(I)_\tau$ .Hypercompositions in H(I) considered in this paper are in the sense of Rosenberg [22], Massouros and Tsitouras [16, 17], Corsini [8, 9], and De Salvo and Lo Maro [13, 14]. We give the characterizations of τ that make $H(I)_\tau$ hypergroupoids, quasihypergroups, semihypergroups, *neutrosophic* hypergroupoids, *neutrosophic* quasihypergroups, *neutrosophic* semihypergroups, and *neutrosophic* hypergroups.

2 Preliminaries

Definition 2.1. Let H be a non-empty set, and $\circ : H\times H \rightarrow P^*(H)$ be a hyperoperation.

(1) The couple $(H,\circ)$ is called a hypergroupoid. For any two non-empty subsets A and B of H and $x\in H$, we define

$$A\circ B=\bigcup_{a\in A, b\in B} a\circ b,\ A\circ x = A\circ\{x\} \text{ and}$$

$$x\circ B=\{x\}\circ B$$

(2) A hypergroupoid $(H,\circ)$ is called a semihypergroup if for all a,b,c of H we have $(a\circ b)\circ c = a\circ(b\circ c)$, which means that

$$\bigcup_{u\in a\circ b} u\circ c=\bigcup_{v\in b\circ c} a\circ v.$$

A hypergroupoid $(H,\circ)$ is called a quasihypergroup if for all a of H we have $a\circ H = H\circ a = H$. This condition is also called the reproduction axiom.

(3) A hypergroupoid $(H,\circ)$ which is both a semihypergroup and a quasihypergroup is called a hypergroup.

Definition 2.2. Let $(G,*)$ be any group and let $G(I)=\langle G\cup I\rangle$. The couple $(G(I),*)$ is called a neutrosophic group generated by G and I under the binary operation $*$. The indeterminancy factor I is such that $I*I=I$. If $*$ is ordinary multiplication, then $IastI*...*I=I^n=I$ and if $*$ is ordinary addition, then $I*I*I*...*I=nI$ for $n\in\mathbb{N}$.

If $a*b=b*a$ for all $a,b\in G(I)$, we say that G(I) is commutative. Otherwise, G(I) is called a non-commutative *neutrosophic* group.

Theorem 2.3. *[24] Let G(I) be a neutrosophic group. Then,*

(1) G(I) in general is not a group;

(2) G(I) always contain a group.

Example 1. [3] Let G(I)={e, a, b, c, I, aI, bI, cI} be a set, where $a^2=b^2=c^2=e$, bc=cb=a, ac=ca=b, ab=ba=c. Then (G(I),.) is a commutative *neutrosophic* group.

Definition 2.4. [4] Let $(H,\circ)$ be any hypergroup and let $H(I)=\langle H\cup I\rangle=\{(a,bI):a,b\in H\}$. The couple $(H(I),\circ)$ is called a *neutrosophic hypergroup* generated by H and I under the hyperoperation $\circ$.

For all (a,bI),(c,dI) $\in H(I)$, the composition of elements of H(I) is defined by

$$(a,bI)\circ(c,dI)=\{(x,yI):x\in a\circ c,$$
$$y\in a\circ d\cup b\circ c\cup b\circ d\}.$$

Example 2. [4] Let H(I)={a,b,(a,aI),(a,bI),(b,aI), (b,bI)} be a set and let $\circ$ be a hyperoperation on H defined in the table below.

$\circ$	a	b	(a,aI)	(a,bI)	(b,aI)	(b,bI)
a	a	b	(a,aI)	(a,bI)	(b,aI)	(b,bI)
b	b	a b	(b,bI)	(b,aI) (b,bI)	(a,bI) (b,bI)	(a,aI) (a,bI) (b,aI) (b,bI)
(a,aI)	(a,aI)	(b,bI)	(a,aI)	(a,aI) (b,bI)	(b,aI) (b,bI)	(b,bI)
(a,bI)	(a,bI)	(b,aI) (b,bI)	(a,aI) (a,bI)	(a,aI) (a,bI)	(b,aI) (b,bI)	(b,aI) (b,bI)
(b,aI)	(b,aI)	(b,bI) (a,bI)	(b,aI) (b,bI)	(b,aI) (b,bI)	(a,aI) (a,bI) (b,aI) (b,bI)	(a,aI) (a,bI) (b,aI) (b,bI)
(b,bI)	(b,bI)	(a,aI) (a,bI) (b,aI) (b,bI)	(b,bI)	(b,aI) (b,bI)	(a,aI) (a,bI) (b,aI) (b,bI)	(a,aI) (a,bI) (b,aI) (b,bI)

Then $(H(I),\circ)$ is a *neutrosophic* hypergroup.

Definition 2.5. Let H be a nonempty set and let ρ be a binary relation on H.

(1) $\rho\circ\rho=\rho^2=\{(x,y):(x,z),(z,y)\in\rho,$ for some $z\in H\}.$

(2) An element $x\in H$ is called an outer element of ρ if $(z,x)\notin\rho^2$ for some $z\in H.$ Otherwise, x is called an inner element.

(3) The domain of ρ is the set $D(\rho)=\{x\in H:(x,z)\in\rho,$ for some $z\in H\}.$

(4) The range of ρ is the set

$R(\rho) = \{x \in H : (z, x) \in \rho,$ for some $z \in H\}.$

In [22], Rosenberg introduced in H the hypercomposition

$x \circ x = \{z \in H : (x, z) \in \rho\}$ and

$x \circ y = x \circ x \cup y \circ y$ (1)

and proved the following:

Proposition 2.6. *[22] $H_\rho = (H, \circ)$ is a hypergroupoid if and only if $H = D(\rho)$.*

Proposition 2.7. *[22] H_ρ is a quasihypergroup if and only if*
(1) $H = D(\rho)$.
(2) $H = R(\rho)$.

Proposition 2.8. *[22] H_ρ is a semihypergroup if and only if*
(1) $H = D(\rho)$.
(2) $\rho \subseteq \rho^2$.
(3) $(a, x) \in \rho^2$ implies that $(a, x) \in \rho$ whenever x is an outer element of ρ.

Proposition 2.9. *[22] H_ρ is a hypergroup if and only if*
(1) $H = D(\rho)$.
(2) $H = R(\rho)$.
(3) $\rho \subseteq \rho^2$.
(4) $(a, x) \in \rho^2$ implies that $(a, x) \in \rho$ whenever x is an outer element of ρ.

In [17], Massouros and Tsitouras noted that whenever x is an outer element of ρ, then it can be deduced from condition (2) and (3) (conditions (3) and (4)) of Proposition 2.8 (Proposition 2.9) that $(a, x) \in \rho$ if and only if $(a, x) \in \rho^2$ for some $a \in H_\rho$. Hence, they restated Propositions 2.8 and 2.9 in the following equivalent forms:

Proposition 2.10. *[17] H_ρ is a semihypergroup if and only if*
(1) $H = D(\rho)$.
(2) $(a, x) \in \rho^2$ if and only if $(a, x) \in \rho$ for all $a \in H$ whenever x is an outer element of ρ.

Proposition 2.11. *[17] H_ρ is a semihypergroup if and only if*
(1) $H = D(\rho)$.
(2) $H = R(\rho)$.
(3) $(a, x) \in \rho^2$ if and only if $(a, x) \in \rho$ for all $a \in H$ whenever x is an outer element of ρ.

If H is a nonempty set and ρ is a binary on H, Massouros and Tsitouras [17] defined hypercomposition • on H as follows:

$x \bullet x = \{z \in H : (z, x) \in \rho\}$ and

$x \bullet y = x \bullet x \cup y \bullet y$ (2)

and stated that:

Proposition 2.12. *[17] If ρ is symmetric, then the hypercompositional structures $(H, \circ)$ and $(H, \bullet)$ coincide.*

Following Rosenberg's terminology in [22], Massouros and Tsitouras established the following:

Definition 2.13. [17]
(1) For $(a, b) \in \rho$, a is called a predecessor of b and b a successor of a.
(2) An element x of H is called a predecessor outer element of ρ if $(x, z) \notin \rho^2$ for some $z \in H$.
Using hypercomposition •, Massouros and Tsitouras established the following:

Proposition 2.14. [17] $H_\rho = (H, \bullet)$ is hypergroupoid if and only if $H = R(\rho)$.

Proposition 2.15. [17] $H_\rho = (H, \bullet)$ is quasihypergroup if and only if
(1) $H = D(\rho)$.
(2) $H = R(\rho)$.

Proposition 2.16. [17] $H_\rho = (H, \bullet)$ is semihypergroup if and only if
(1) $H = R(\rho)$.
(2) $(x, y) \in \rho^2$ if and only if $(x, y) \in \rho$ for all $y \in H$ whenever x is a predecessor outer element of ρ.

Proposition 2.17. [17] $H_\rho = (H, \bullet)$ is hypergroup if and only if
(1) $H = D(\rho)$.
(2) $H = R(\rho)$.
(3) $(x, y) \in \rho^2$ if and only if $(x, y) \in \rho$ for all $y \in H$ whenever x is a predecessor outer element of ρ.

If H is a nonempty set and ρ is a binary relation on H, Corsini [8, 9] introduced in H the hypercomposition:

$x * y = \{z \in H : (x, z) \in \rho$ and

$(z,y)\in\rho$ for some $z\in H\}$. (3)

It is clear that $(H,*)$ is a partial hypergroupoid and it is a hypergroupoid if for each pair of elements $x,y\in H$, there exists $z\in H$ such that $(x,z)\in\rho$ and $(z,y)\in\rho$. Equivalently, $(H,*)$ is a hypergroupoid if and only if $\rho^2=H^2$.

If H_ρ is the hypercompositional structure defined by equation (3) , Massouros and Tsitouras [16] proved the following:

Proposition 2.18. [16] H_ρ is a quasihypergroup if and only if $(x,y)\in\rho$ for all $x,y\in H_\rho$.

Lemma 2.19. *[16] If H_ρ is a semihypergroup and $(z,z)\notin\rho$ for some $z\in H_\rho$, then $(s,z)\in\rho$ implies that $(z,s)\notin\rho$.*

Corrolary 2.20. [16] If H_ρ is a semihypergroup and ρ is not reflexive, then ρ is not symmetric.

Lemma 2.21. *If H_ρ is a semihypergroup then ρ is reflexive.*

Proposition 2.22. [16] *H_ρ is a semihypergroup if and only if $(x,y)\in\rho$ for all $x,y\in H_\rho$.*

Definition 2.23. A hyperoperation $*$ defined through ρ is said to be a total hypercomposition if and only if $(x,y)\in\rho$ for all $x,y\in H_\rho$. In other words, $*$ is said to be a total hypercomposition if $x*y=H_\rho$ for all $x,y\in H_\rho$.

Remark 1. If a hypercompositional structure H_ρ is endowed with the total hypercomposition $*$, then $(H_\rho,*)$ is a hypergroup.

Theorem 2.24. *[16] The only semihypergroup and the only quasihypergroup defined by the binary relation ρ is the total hypergroup.*

If H is a nonempty set and ρ is a binary relation on H, De Salvo and Lo Faro [13, 14] introduced in H the hypercomposition:

$$x\lozenge y=\{z\in H:(x,z)\in\rho$$
$$(x,y)\in\rho \text{ for some } z\in H\}.$$

They characterized the relations ρ which give quasihypergoups, semihypergroups and hypergroups.

3 Neutrosophic Hypercompositional Structures

3.1 Neutrosophic Hypercompositional Structures of Rosenberg Type

Let τ be a binary relation on H(I) and let $\rho=\tau\big|_H$. For all $(a,bI),(c,dI)\in H(I)$, define hypercomposition on H(I) as follows:

$$\begin{aligned}(a,bI)\circ(a,bI)&=\{(x,yI)\in H(I):x\in a\circ a,\\ &\qquad y\in a\circ a\cup b\circ b\}\\ &=\{(x,yI)\in H(I):(a,x)\in\rho,\\ &\qquad (a,y)\in\rho \text{ or } (b,y)\in\rho\}.\end{aligned}$$

(5)

$$\begin{aligned}(a,bI)\circ(c,dI)&=\{(x,yI)\in H(I):x\in a\circ a\cup c\circ c,\\ &\qquad y\in a\circ a\cup b\circ b\cup c\circ c\cup d\circ d\}\\ &=\{(x,yI)\in H(I):(a,x)\in\rho,\\ &\qquad \text{or } (c,x)\in\rho,(a,y)\in\rho\\ &\qquad \text{or } (b,y)\in\rho \text{ or } (c,y)\in\rho \text{ or } (d,y)\in\rho\}.\end{aligned} \quad (6)$$

Let $H(I)_\tau=(H(I),\circ)$ be a hypercompositional structure arising from the hypercomposition defined by equation (6).

Proposition 3.1.1. $H(I)_\tau$ is a hypergroupoid if and only if H_ρ is a hypergroupoid.

Proof. Suppose that H_ρ is a hypergroupoid. Then $H=D(\rho)$ and from equation (6) we have $(a,bI)\circ(c,dI)\subseteq H(I)_\tau$ for all $(a,bI),(c,dI)\in H(I)$. Hence $H(I)_\tau$ is a hypergroupoid. The converse is obvious.

Proposition 3.1.2. $H(I)_\tau$ is a quasihypergroup if and only if H_ρ is a quasihypergroup.

Proof. Suppose that H_ρ is a quasihypergroup. Then $H=D(\rho)=R(\rho)$. Let $(x,yI)\in(a,bI)\circ(c,dI)$ for an arbitrary $(c,dI)\in H(I)$. Then

$$(a,bI)\circ H(I)_\tau=\bigcup\{(a,bI)\circ(c,dI)\}$$

$$=\bigcup\{(x,yI)\in H(I):(a,x)\in\rho,$$
$$\text{or } (c,x)\in\rho,(a,y)\in\rho$$
$$\text{or } (b,y)\in\rho \text{ or } (c,y)\in\rho \text{ or } (d,y)\in\rho\}.$$
$$=H(I)_\tau$$

Similarly, it can be shown that $H(I)_\tau\circ(a,bI)=H(I)_\tau$ for all $(a,bI)\in H(I)$.

Hence $(H(I)_\tau,\circ)$ is a quasihypergroup. The converse is obvious.

Lemma 3.1.1. If ρ is not reflexive, then $(a,bI)\notin(a,bI)\circ(a,bI)$ for all $(a,bI)\in H(I)$.

Proof. Suppose that ρ is not reflexive and suppose that $(a,bI)\notin(a,bI)\circ(a,bI)$ for all $(a,bI)\in H(I)$. Assuming that $(a,b)\in\rho$, we have from equation (5):

$$(a,bI)\circ(a,bI)=\{(a,bI)\in H(I):(a,a)\in\rho,$$
$$(a,b)\in\rho \text{ or } (b,b)\in\rho\}$$
$$=\varnothing$$

a contradiction. Hence $(a,bI)\notin(a,bI)\circ(a,bI)$.

Proposition 3.1.3. $H(I)_\tau$ is a semihypergroup if ρ is reflexive and symmetric.

Proof. Suppose that ρ is reflexive and symmetric. Let $(a,bI),(b,aI)\in H(I)$ be arbitrary and let $(x,a)\in\rho$, $(x,b)\in\rho$ and $(y,a)\in\rho$. Then $(b,aI)\in(a,bI)\circ((b,aI)\circ(a,bI))$ implies that

$$(a,bI)\circ((b,aI)\circ(a,bI))=\{(b,aI)\in H(I):(a,b)\in\rho$$
$$\text{or } (x,b)\in\rho,\ (a,a)\in\rho,(b,a)\in\rho \text{ or}$$
$$(x,a)\in\rho \text{ or } (y,a)\in\rho\}$$
$$=((a,bI)\circ(b,aI))\circ(a,bI).$$

This shows that $(b,aI)\in((a,bI)\circ(b,aI))\circ(a,bI)$. Since (a,bI) and (b,aI) are arbitrary, it follows that $H(I)_\tau$ is a semihypergroup.

The following results are immediate from the hypercomposition defined by equation (6):

Proposition 3.1.4. (1) $H(I)_\tau$ is a *neutrosophic* hypergroupoid if and only if H_ρ is a hypergroupoid.

(2) $H(I)_\tau$ is a *neutrosophic* semihypergroup if and only if H_ρ is a semihypergroup.

(3) $H(I)_\tau$ is a *neutrosophic* hypergroup if and only if H_ρ is a hypergroup.

3.2 Neutrosophic Hypercompositional Structures of Massouros and Tsitouras Type

Let τ be a binary relation on H(I) and let $\rho=\tau|_H$. For all $(a,bI),(c,dI)\in H(I)$, define hypercomposition on H(I) as follows:

$$(a,bI)\bullet(a,bI)=\{(x,yI):x\in a\bullet a,$$
$$y\in a\bullet a\cup b\bullet b\}$$
$$=\{(x,yI):(x,a)\in\rho,$$
$$(y,a)\in\rho \text{ or } (y,b)\in\rho\} \qquad (7)$$

$$(a,bI)\bullet(c,dI)=\{(x,yI):x\in a\bullet a\cup c\bullet c,$$
$$y\in a\bullet a\cup b\bullet b\cup c\bullet c\cup d\bullet d\}$$
$$=\{(x,yI):(x,a)\in\rho,$$
$$\text{or } (x,c)\in\rho,\ (y,a)\in\rho \text{ or}$$
$$(y,b)\in\rho \text{ or } (y,c)\in\rho \text{ or } (y,d)\in\rho\} \qquad (8)$$

$H(I)_\tau=(H(I),\bullet)$ be a hypercompositional structure arising from the hypercomposition defined by equation (8).

Proposition 3.2.1. If ρ is symmetric, then hypercompositional structure $(H(I),\bullet)$ coincide with hypercompositional structure $(H(I),\circ)$.

Proof. This follows directly from equations (6) and (8).

Proposition 3.2.2. $H(I)_\tau$ is a hypergroupoid if and only if H_ρ is a hypergroupoid.

Proof. Suppose that H_ρ is a hypergroupoid. Then $H=R(\rho)$ and from equation (8) we have $(a,bI)\bullet(c,dI)\subseteq H(I)_\tau$ for all $(a,bI),(c,dI)\in H(I)$. Hence $H(I)_\tau$ is a hypergroupoid. The converse is obvious.

Proposition 3.2.3. $H(I)_\tau$ is a quasihypergroup if and only if H_ρ is aquasi hypergroup.

Proof. Suppose that H_ρ is a quasihypergroup. Then $H=D(\rho)=R(\rho)$. Let $(x,yI)\in(a,bI)\bullet(c,dI)$ for an arbitrary $(c,dI)\in H(I)$. Then

$$(a,bI)\bullet H(I)_\tau=\bigcup\{(a,bI)\bullet(c,dI)\}$$
$$=\bigcup\{(x,yI)\in H(I):(x,a)\in\rho$$
$$\text{or } (x,c)\in\rho,\ (y,a)\in\rho \text{ or}$$
$$(y,b)\in\rho \text{ or } (y,c)\in\rho \text{ or } (y,d)\in\rho\}$$
$$=H(I)_\tau$$

Similarly, it can be shown that $H(I)_\tau\bullet(a,bI)=H(I)_\tau$ for all $(a,bI)\in H(I)$. Hence $H(I)_\tau$ is a quasihypergroup. The converse is obvious.

Lemma 3.2.1. If ρ is not reflexive, then $(a,bI)\notin(a,bI)\bullet(a,bI)$ for all $(a,bI)\in H(I)$.

Proof. The same as the proof of Lemma 3.1.1.

Proposition 3.2.4. $H(I)_\tau$ is a semihypergroup if ρ is reflexive and symmetric.

Proof. This follows from Proposition 3.1.3 and Proposition 3.2.1.

Proposition 3.2.5. (1) $H(I)_\tau$ is a *neutrosophic* hypergroupoid if and only if H_ρ is a hypergroupoid.

(2) $H(I)_\tau$ is a *neutrosophic* semihypergroup if and only if H_ρ is a semihypergroup.

(3) $H(I)_\tau$ is a *neutrosophic* hypergroup if and only if H_ρ is a hypergroup.

3.3 Neutrosophic Hypercompositional Structures of Corsini Type

Let τ be a binary relation on H(I) and let $\rho = \tau|_H$. For all $(a,bI),(c,dI) \in H(I)$, define hypercomposition on H(I) as follows:

$$(a,bI)*(c,dI) = \{(x,yI) \in H(I) : x \in a*a, y \in a*d \cup b*c \cup b*d\}$$
$$= \{(x,yI) \in H(I) : (a,x) \in \rho, \text{ and } (x,c) \in \rho, [(a,y) \in \rho \text{ and } (y,d) \in \rho] \text{ or } [(b,y) \in \rho \text{ and } (y,c) \in \rho] \text{ or } [(b,y) \in \rho \text{ and } (y,d) \in \rho]\}. \quad (9)$$

Let $H(I)_\tau = (H(I),*)$ be a hypercompositional structure arising from the hypercomposition defined by equation (9).

Proposition 3.3.1. $H(I)_\tau$ is a hypergroupoid if and only if H_ρ is a hypergroupoid.

Proof. Suppose that H_ρ is a hypergroupoid. Then $H^2=\rho^2$. Since $(a,c),(a,d),(b,c),(b,d) \in \rho^2$ from equation (9), it follows that $(a,bI)*(c,dI) \subseteq H(I)_\tau$ for all $(a,bI),(c,dI) \in H(I)$. Hence $H(I)_\tau$ is a hypergroupoid. The converse is obvious.

Proposition 3.3.2. $H(I)_\tau$ is a quasihypergroup if and only if H_ρ is a quasihypergroup.

Proof. Suppose that H_ρ is a quasihypergroup. Then $(x,y) \in \rho$ for all $x,y \in H$. Let $(x,yI) \in (a,bI)*(c,dI)$ for an arbitrary $(c,dI) \in H(I)$. Then

$$(a,bI)*H(I)_\tau = \bigcup\{(a,bI)*(c,dI)\}$$
$$= \{(x,yI) \in H(I) : (a,x) \in \rho, \text{ and } (x,c) \in \rho, [(a,y) \in \rho \text{ and } (y,d) \in \rho] \text{ or } [(b,y) \in \rho \text{ and } (y,c) \in \rho] \text{ or } [(b,y) \in \rho \text{ and } (y,d) \in \rho]\}.$$
$$= H(I)_\tau$$

Similarly, it can be shown that $H(I)_\tau * (a,bI) = H(I)_\tau$ for all $(a,bI) \in H(I)$. Hence $H(I)_\tau$ is a quasihypergroup. The converse is obvious.

Proposition 3.3.3. $H(I)_\tau$ is a *neutrosophic* quasihypergroup if and only if H_ρ is aquasihypergroup.

Proof. Follows directly from equation (9).

Lemma 3.3.1. If ρ is not reflexive and symmetric, then

(1) $(a,bI) \notin (a,bI)*(a,bI)$ for all $(a,bI) \in H(I)$.

(2) $(b,aI) \notin (a,bI)*(a,bI)$ for all $(a,bI),(b,aI) \in H(I)$.

(3) $(a,aI) \notin (a,bI)*(a,bI)$ for all $(a,aI),(a,bI) \in H(I)$.

(4) $(a,bI) \notin (a,bI)*(a,bI)$ for all $(a,bI),(b,aI) \in H(I)$.

(5) $(b,aI) \notin (a,bI)*(b,aI)$ for all $(a,bI),(b,aI) \in H(I)$.

(6) $(a,aI) \notin (a,bI)*(b,aI)$ for all $(a,aI),(a,bI),(b,aI) \in H(I)$.

Proof. (1) Suppose that ρ is not reflexive and symmetric and suppose that $(a,bI) \notin (a,bI)*(a,bI)$. Then

$$(a,bI)*(a,bI) = \{(a,bI) \in H(I) : (a,a) \in \rho, (b,b) \in \rho \text{ or } [(a,b) \in \rho \text{ and } (b,b) \in \rho] \text{ or } [(b,b) \in \rho \text{ and } (a,b) \in \rho]$$
$$= \varnothing$$

a contradiction. Hence $(a,bI) \notin (a,bI)*(a,bI)$. Using similar argument, (2), (3), (4), (5) and (6) can be established.

Proposition 3.3.4. $H(I)_\tau$ is a semihypergroup if ρ is reflexive and symmetric.

Proof. Suppose that ρ is reflexive and symmetric. Let $(a,bI),(b,aI) \in H(I)$ be arbitrary and let $(x,a) \in \rho$, $(x,b) \in \rho$, $(y,b) \in \rho$ and $(b,a) \in \rho$. Then $(a,bI) \in (a,bI)*((b,aI)*(a,bI))$ implies that

$$(a,bI)*((b,aI)*(a,bI)) = \{(a,bI) \in H(I) : (x,a) \in \rho \text{ and } (a,a) \in \rho, [(x,b) \in \rho \text{ and } (b,b) \in \rho] \text{ or } [(y,a) \in \rho \text{ and } (b,a) \in \rho] \text{ or } [(y,b) \in \rho \text{ and } (b,b) \in \rho]\}$$
$$= ((a,bI)*(b,aI))*(a,bI).$$

This shows that $(b,aI) \in ((a,bI)*(b,aI))*(a,bI)$. Since (a,bI) and (b,aI) are arbitrary, it follows that $H(I)_\tau$ is a semihypergroup.

Corollary 3.3.1. $H(I)_\tau$ is a semihypergroup if and only if H_ρ is a semihypergroup.

Proposition 3.3.5. If any pair of elements of H_ρ does not belong to ρ, then $H(I)_\tau$ is not a semihypergroup.

3.1 Neutrosophic Hypercompositional Structures of De Salvo and Lo Faro Type

Let τ be a binary relation on H(I) and let $\rho=\tau\big|_H$. For all $(a,bI),(c,dI)\in H(I)$, define hypercomposition on H(I) as follows:

$$(a,bI)\Diamond(c,dI)=\{(x,yI)\in H(I): x\in a\Diamond c,$$
$$y\in a\Diamond d\cup b\Diamond c\cup b\Diamond d\}$$
$$=\{(x,yI)\in H(I):(a,x)\in\rho,$$
$$\text{or } (x,c)\in\rho,(a,y)\in\rho$$
$$\text{or } (b,y)\in\rho \text{ or } (y,c)\in\rho \text{ or } (y,d)\in\rho\}. \quad (10)$$

Let $H(I)_\tau=(H(I),\Diamond)$ be a hypercompositional structure arising from the hypercomposition defined by equation (10).

Proposition 3.4.1. If ρ is symmetric, then hypercompositional structures $(H(I),\Diamond)$, $(H(I),\circ)$ and $(H(I),\bullet)$ coincide.

Proof. Follows directly from equations (6), (8) and (10).

Proposition 3.4.2. $H(I)_\tau$ is a hypergroupoid if and only if H_ρ is a hypergroupoid.

Proof. Suppose that H_ρ is a hypergroupoid. Then H=D(ρ) or H=R(ρ) and from equation (10) we have $(a,bI)\Diamond(c,dI)\subseteq H(I)_\tau$ for all $(a,bI),(c,dI)\in H(I)$. Hence $H(I)_\tau$ is a hypergroupoid. The converse is obvious.

Proposition 3.4.3. $H(I)_\tau$ is a quasihypergroup if and only if H_ρ is a quasihypergroup.

Proof. The same as the proof of Proposition 3.2.3.

Lemma 3.4.1. If ρ is not reflexive and symmetric, then

(1) $(a,bI)\notin(a,bI)\Diamond(a,bI)$ for all $(a,bI)\in H(I)$.
(2) $(b,aI)\notin(a,bI)\Diamond(a,bI)$ for all $(a,bI),(b,aI)\in H(I)$.
(3) $(a,aI)\notin(a,bI)\Diamond(a,bI)$ for all $(a,aI),(a,bI)\in H(I)$.
(4) $(a,bI)\notin(a,bI)\Diamond(a,bI)$ for all $(a,bI),(b,aI)\in H(I)$.
(5) $(b,aI)\notin(a,bI)\Diamond(b,aI)$ for all $(a,bI),(b,aI)\in H(I)$.
(6) $(a,aI)\notin(a,bI)\Diamond(b,aI)$ for all $(a,aI),(a,bI),(b,aI)\in H(I)$.

Proof. (1) Suppose that ρ is not reflexive and symmetric and suppose that $(a,bI)\notin(a,bI)\Diamond(a,bI)$. Then

$$(a,bI)\Diamond(a,bI)=\{(a,bI)\in H(I):(a,a)\in\rho,$$
$$(a,b)\in\rho \text{ or } (b,b)\in\rho \text{ or } (b,a)\in\rho\}$$
$$=\varnothing$$

a contradiction. Hence $(a,bI)\notin(a,bI)\Diamond(a,bI)$. Using similar argument, (2), (3), (4), (5) and (6) can be established.

Proposition 3.4.4. $H(I)_\tau$ is a semihypergroup if ρ is reflexive and symmetric.

Proof. Suppose that ρ is reflexive and symmetric. Let $(a,bI),(b,aI)\in H(I)$ be arbitrary and let $(a,x)\in\rho$, $(b,x)\in\rho$, $(b,y)\in\rho$ and $(a,b)\in\rho$. Then $(a,bI)\in(a,bI)\Diamond\ ((b,aI)\Diamond(a,bI))$ implies that

$$(a,bI)\Diamond((b,aI)\Diamond(a,bI))=\{(a,bI)\in H(I):$$
$$(a,a)\in\rho \text{ or } (a,x)\in\rho,(a,b)\in\rho \text{ or}$$
$$(b,y)\in\rho \text{ or } (b,b)\in\rho \text{ or } (b,x)\in\rho\}$$
$$=((a,bI)\Diamond(b,aI))\Diamond(a,bI).$$

This shows that $(a,bI)\in((a,bI)\Diamond(b,aI))\ \Diamond(a,bI)$. Since (a,bI) and (b,aI) are arbitrary, it follows that $H(I)_\tau$ is a semihypergroup.

The following results are immediate from the hypercomposition defined by equation (10):

Proposition 3.4.5. (1) $H(I)_\tau$ is a *neutrosophic* hypergroupoid if and only if H_ρ is a hypergroupoid.
(2) $H(I)_\tau$ is a *neutrosophic* semihypergroup if and only if H_ρ is a semihypergroup.
(3) $H(I)_\tau$ is a *neutrosophic* hypergroup if and only if H_ρ is a hypergroup.

References

[1] A. A. A. Agboola, A. D. Akinola, and O. Y. Oyebola. Neutrosophic Rings I, Int. J. Math. Comb. 4 (2011), 1-14.

[2] A. A. A. Agboola, E. O. Adeleke, and S. A. Akinleye. Neutrosophic Rings II, Int. J. Math. Comb. 2 (2012), 1-8.

[3] A. A. A. Agboola, A. O. Akwu, and Y. T. Oyebo. Neutrosophic Groups and Neutrosophic Subgroups, Int. J. Math. Comb. 3 (2012), 1-9.

[4] A. A. A. Agboola and B. Davvaz. Introduction to Neutrosophic Hypergroups (To appear in ROMAI Journal of Mathematics).

[5] P. Corsini. Prolegomena of Hypergroup Theory. Second edition, Aviain Editore, 1993.

[6] P. Corsini and V. Leoreanu. Applications of Hyperstructure Theory. Advances in Mathematics. Kluwer Academic Publisher, Dordrecht, 2003.

[7] P. Corsini and V. Loereanu, Hypergroups and Binary Relations. Algebra Universalis 43 (2000), 321-330.

[8] P. Corsini, On the Hypergroups associated with Binary Relations. Multiple Valued Logic, 5 (2000), 407-419.

[9] P. Corsini, Binary Relations and Hypergroupoids. Italian J. Pure and Appl. Math 7 (2000), 11-18.

[10] L. Cristea and M. Stefanescu, Binary Relations and Reduced Hypergroups. Discrete Math. 308 (2008), 3537-44.

[11] L. Cristea, M. Stefanescu, and C. Angheluta, About the Fundamental Relations defined on the Hypergroupoids associated with Binary Relations. Electron. J. Combin. 32 (2011), 72-81.

[12]B. Davvaz and V. Leoreanu-Fotea. Hyperring Theory and Applications. International Academic Press, USA, 2007.

[13]M. De Salvo and G. Lo Faro. Hypergroups and Binary Relations, Multi-Val. Logic 8 (2002), 645-657.

[14]M. De Salvo and G. Lo Faro. A New class of Hypergroupoids Associated with Binary Relations, Multi-Val. Logic Soft Comput., 9 (2003), 361-375.

[15]F. Smarandache. A Unifying Field in Logics: Neutrosophic Logic, Neutrosophy, Neutrosophic Set, Neutrosophic Probability (3rd Ed.). American Research Press, Rehoboth, 2003. URL: http://fs.gallup.unm.edu/eBook-Neutrosophic4.pdf.

[16]Ch.G. Massouros and Ch. Tsitouras, Enumeration of Hypercompositional Structures defined by Binary Relations, Italian J. Pure and App. Math. 28 (2011), 43-54.

[17]Ch.G. Massouros and Ch. Tsitouras, Enumeration of Rosenberg-Type Hypercompositional Structures defined by Binary Relations, Euro J. Comb. 33 (2012), 1777-1786.

[18]F. Marty, Sur une generalization de la notion de groupe, 8th Congress Math. Scandinaves, Stockholm, Sweden, (1934), 45-49.

[19]S. Mirvakili, S.M. Anvariyeh and B. Davvaz, On α-relation and transitivity conditions of α, Comm. Algebra, 36 (2008), 1695-1703.

[20]S. Mirvakili and B. Davvaz, Applications of the α*-relation to Krasner hyperrings, J. Algebra, 362 (2012), 145-146.

[21]J. Mittas, Hypergroups canoniques, Math. Balkanica 2 (1972), 165-179.

[22]I.G. Rosenberg, Hypergroups and Join Spaces determined by relations, Italian J. Pure and App. Math. 4 (1998), 93-101.

[23]S.I. Spartalis and C. Mamaloukas, Hyperstructures Associated with Binary Relations, Comp. Math. Appl. 51 (2006), 41-50.

[24]W.B. Vasantha Kandasamy and F. Smarandache. Some Neutrosophic Algebraic Structures and Neutrosophic N-Algebraic Structures. Hexis, Phoenix, Arizona, 2006. URL: http://fs.gallup.unm.edu/NeutrosophicN-AlgebraicStructures.pdf.

[25] W.B. Vasantha Kandasamy and F. Smarandache. Neutrosophic Rings, Hexis, Phoenix, Arizona, 2006. URL: http://fs.gallup.unm.edu/NeutrosophicRings.pdf.

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A Note on Square Neutrosophic Fuzzy Matrices

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Abstract. In this article, we shall define the addition and multiplication of two neutrosophic fuzzy matrices. Thereafter, some properties of addition and multiplication of these matrices are also put forward.

Keywords: Neutrosophic fuzzy matrice, Neutrosophic Set.

1 Introduction

Neutrosophic sets theory was proposed by Florentin Smarandache [1] in 1999, where each element had three associated defining functions, namely the membership function (T), the non-membership (F) function and the indeterminacy function (I) defined on the universe of discourse X, the three functions are completely independent. The theory has been found extensive application in various field [2,3,4,5,6,7,8,9,10,11] for dealing with indeterminate and inconsistent information in real world.Neutrosophic set is a part of neutrosophy which studied the origin, nature and scope of neutralities, as well as their interactions with ideational spectra. The neutrosophic set generalized the concept of classical fuzzy set [12, 13], interval-valued fuzzy set, intuitionistic fuzzy set [14, 15], and so on.

Also as we know, matrices play an important role in science and technology. However, the classical matrix theory sometimes fails to solve the problems involving uncertainties, occurring in an imprecise environment. In [17] Thomason, introduced the fuzzy matrices to represent fuzzy relation in a system based on fuzzy set theory and discussed about the convergence of powers of fuzzy matrix. In 2004, W. B. V. Kandasamy and F. Smarandache introduced fuzzy relational maps and neutrosophic relational maps.

Our aim ,In this paper is to propose another type of fuzzy neutrosophic matrices ,called “square neutrosophic fuzzy matrices”, whose entries is of the form a+Ib (neutrosophic number) , where a,b are the elements of [0,1] and I is an indeterminate such that I^n=I, n being a positive integer. In this study we will focus on square neutrosophic fuzzy matrices.The paper unfolds as follows. The next section briefly introduces some definitions related to neutrosophic set, neutrosophic matrices, Fuzzy integral neutrosophic matrices and fuzzy matrix. Section 3 presents a new type of fuzzy neutrosophic matrices and investigated some properties such as addition and multiplication. Conclusions appear in the last section.

2 Preliminaries

In this section we recall some concept such as , neutrosophic set, neutrosophic matrices and fuzzy neutrosophic matrices proposed by W. B. V. Kandasamy and F. Smarandache in their books [16] , and also the concept of fuzzy matrix .

Definition 2.1 (Neutrosophic Sets).[1]

Let U be an universe of discourse then the neutrosophic set A is an object having the form

A = {< x: $T_A(x)$, $I_A(x)$, $F_A(x)$>,x $\in$ U}, where the functions T, I, F : U→ $]^-0, 1^+[$ define respectively the degree of membership (or Truth) , the degree of indeterminacy, and the degree of non-membership (or

Falsehood) of the element x ∈ U to the set A with the condition.

$$^{-}0 \le T_A(x) + I_A(x) + F_A(x) \le 3^{+}.$$

From philosophical point of view, the neutrosophic set takes the value from real standard or non-standard subsets of $]^{-}0, 1^{+}[$. So instead of $]^{-}0, 1^{+}[$ we need to take the interval [0, 1] for technical applications, because $]^{-}0, 1^{+}[$will be difficult to apply in the real applications such as in scientific and engineering problems.

Definition 2.2 (Neutrosophic matrix) [16].

Let $\boldsymbol{M}_{m\,x\,n}= \{(\boldsymbol{a}_{ij}) \,/\, \boldsymbol{a}_{ij} \in K(I)\}$, where K (I), is a neutrosophic field. We call $\boldsymbol{M}_{m\,x\,n}$ to be the neutrosophic matrix.

Example 1: Let Q(I) = ⟨Q ∪ I ⟩be the neutrosophic field

$$\boldsymbol{M}_{3\,x\,3}=\begin{pmatrix} 0 & 1 & 0 \\ -2 & 4I & 2 \\ 3I & 1 & -I \end{pmatrix}$$

$M_{3\,x\,3}$ denotes the neutrosophic matrix, with entries from rationals and the indeterminacy.

Definition 2.3 (Fuzzy integral neutrosophic matrices)

Let N = [0, 1] ∪ I where I is the indeterminacy. The m ×n matrices $\mathbf{M}_{m\,x\,n}= \{(\boldsymbol{a}_{ij}) \,/\, \boldsymbol{a}_{ij} \in [0, 1] \cup I\}$ is called the fuzzy integral neutrosophic matrices. Clearly the class of m ×n matrices is contained in the class of fuzzy integral neutrosophic matrices.

An integral fuzzy neutrosophic row vector is a **1 × n** integral fuzzy neutrosophic matrix, Similarly an integral fuzzy neutrosophic column vector is a m ×1 integral fuzzy neutrosophic matrix.

Example 2 : Let $A_{3\,x\,3}=\begin{pmatrix} 0 & 1 & 0.3 \\ 0.9 & I & 0.2 \\ I & 1 & I \end{pmatrix}$

A is a 3 ×3 integral fuzzy neutrosophic matrix.

Definition 2.5 (Fuzzy neutrosophic matrix) [16]

Let $\mathbf{N_s}$= [0, 1] ∪ nI / n ∈ (0, 1]}; we call the set $\mathbf{N_s}$ to be the fuzzy neutrosophic set. Let $\mathbf{N_s}$ be the fuzzy neutrosophic set. $\mathbf{M}_{m\,x\,n}= \{(\boldsymbol{a}_{ij}) \,/\, \boldsymbol{a}_{ij} \in \mathbf{N_s}\}$ we call the matrices with entries from $\mathbf{N_s}$ to be the fuzzy neutrosophic matrices.

Example 3: Let $\mathbf{N_s}$= [0,1] ∪{nI/ n ∈ (0,1]} be the set

$$P = \begin{pmatrix} 0 & 0.2I & I \\ I & 0.01I & 0 \\ 0.31I & 0.53I & 0.1 \end{pmatrix}$$

is a 3 ×3 fuzzy neutrosophic matrix

Definition 2.6 (Fuzzy matrix) [17]

A fuzzy matrix is a matrix which has its elements from the interval [0, 1], called the unit fuzzy interval. A m x n fuzzy matrix for which m = n (i.e the number of rows is equal to the number of columns) and whose elements belong to the unit interval [0, 1] is called a fuzzy square matrix of order n. A fuzzy square matrix of order two is expressed in the following way

A= $\begin{pmatrix} a & b \\ c & d \end{pmatrix}$, where the entries a,b,c,d all belongs to the interval [0,1].

3 Some Properties of Square Neutrosophic Fuzzy Matrices

In this section ,we define a new type of fuzzy neutrosophic set and define some operations on this neutrosophic fuzzy matrice.

3.1 .Definition (Neutrosophic Fuzzy Matrices)

Let A be a neutrosophic fuzzy matrices, whose entries is of the form a+Ib (neutrosophic number) , where a,b are the elements of [0,1] and I is an indeterminate such that I^n=I, n being a positive integer.

$$A=\begin{pmatrix} a_1 + I\,b_1 & a_2 + I\,b_2 \\ a_3 + I\,b_3 & a_4 + I\,b_4 \end{pmatrix}$$

3.2.Arithmetic with Square Neutrosophic Fuzzy Matrices

In this section we shall define the addition and multiplication of neutrosophic fuzzy matrices along with some properties associated with such matrices.

3.2.1. Addition Operation of two Neutrosophic Fuzzy Matrices

Let us consider two neutrosophic fuzzy matrices as

A=$\begin{pmatrix} a_1 + I\, b_1 & a_2 + I\, b_2 \\ a_3 + I\, b_3 & a_4 + I\, b_4 \end{pmatrix}$and B=$\begin{pmatrix} c_1 + I\, d_1 & c_2 + I\, d_2 \\ c_3 + I\, d_3 & c_4 + I\, d_4 \end{pmatrix}$

Then we would like to define the addition of these two matrices as

$$A + B = [C_{ij}]$$

Where

$C_{11} = \max(a_{1,} c_1) + I\max(b_1, d_1)$

$C_{12} = \max(a_{2,} c_2) + I\max(b_2, d_2)$

$C_{21} = \max(a_{3,} c_3) + I\max(b_3, d_3)$

$\mathbf{C_{22} = \max(a_{4,} c_4) + I\max(b_4, d_4)}$

It is noted that the matrices defined by our way is reduced to fuzzy neutrosophic matrix when a =

Properties 1

The following properties can be found to hold in cases of neutrosophic fuzzy matrix multiplication

(i) A+B = B+A

(ii) (A+B)+C = A+(B+C)

3.2.2 Multiplication Operation of Neutrosophic Fuzzy Matrices

Let us consider two neutrosophic fuzzy matrices as $A = [a_{ij} + Ib_{ij}]$ and $B = [c_{ij} + Id_{ij}]$. Then we shall define the multiplication of these two neutrosophic fuzzy matrices as

$AB = [\max\min(a_{ij}, c_{ji}) + I\max\min(b_{ij}, d_{ji})]$. It can be defined in the following way:

If the above mentioned neutrosophic fuzzy matrices are considered then we can define the product of the above matrices as

A B = $[D_{ij}]$, where

$D_{11} = [\max\min\{(a_{1,}c_1), (a_{2,}c_3)\} + I\max\min\{(b_1, d_1), (b_2, d_3)\}]$

$D_{12} = [\max\min\{(a_{1,}c_2), (a_{2,}c_4)\} + I\max\min\{(b_1, d_2), (b_2, d_4)\}]$

$D_{21} = [\max\min\{(a_{3,}c_1), (a_{4,}c_3)\} + I\max\min\{(b_3, d_1), (b_4, d_3)\}]$

$D_{22} = [\max\min\{(a_{3,}c_2), (a_{4,}c_4)\} + I\max\min\{(b_3, d_2), (b_4, d_4)\}]$

It is important to mention here that if the multiplication of two neutrosophic fuzzy matrices is defined in the above way then the following properties can be observed to hold:

Properties

(i) $AB \neq BA$

(ii) A(B+C)=AB+AC

2.4.1 Numerical Example

Let us consider three neutrosophic fuzzy matrices as

A=$\begin{pmatrix} 0.1 + I\, 0.3 & 0.4 + I\, 0.1 \\ 0.2 + I\, 0.4 & 0.1 + I\, 0.7 \end{pmatrix}$

B=$\begin{pmatrix} 0.2 + I\, 0.3 & 0.5 + I\, 0.4 \\ 0.3 + I\, 0.8 & 0.9 + I\, 0.1 \end{pmatrix}$

C= $\begin{pmatrix} 0.4 + I\, 0.3 & 0.4 + I\, 0.3 \\ 0.2 + I\, 0.7 & 0.2 + I\, 0.4 \end{pmatrix}$

B+C = $\begin{pmatrix} 0.4 + I\, 0.6 & 0.5 + I\, 0.4 \\ 0.6 + I\, 0.8 & 0.9 + I\, 0.2 \end{pmatrix}$

A (B +C)= $\begin{pmatrix} 0.1 + I\, 0.3 & 0.4 + I\, 0.1 \\ 0.2 + I\, 0.4 & 0.1 + I\, 0.7 \end{pmatrix} \begin{pmatrix} 0.4 + I\, 0.6 & 0.5 + I\, 0.4 \\ 0.6 + I\, 0.8 & 0.9 + I\, 0.2 \end{pmatrix}$

Let us take

A (B +C) = $\begin{pmatrix} A_{11} & A_{12} \\ A_{21} & A_{22} \end{pmatrix}$, where

A_{11} =max{min(0.1, 0.4), min(0.4,0.6)}+I max{min(0.3, 0.6), min(0.1, 0.8)}

= max(0.1, 0.4) + I max (0.3, 0.1)

= 0.4 + I 0.3

A_{12} = max{min(0.1, 0.5), min(0.4,0.9)}+I max{min(0.3, 0.4), min(0.1, 0.2)}

= max(0.1, 0.4) + I max (0.3, 0.1)

= 0.4 + I 0.3

A_{21} = max{min(0.2, 0.4), min(0.1,0.6)}+I max{min(0.4, 0.6), min(0.7, 0.8)}

= max(0.2, 0.1) + I max (0.4, 0.7)

= 0.2 + I 0.7

A_{22} = max{min(0.2, 0.5), min(0.1,0.9)}+I max{min(0.4, 0.4), min(0.7, 0.2)}

= max(0.2, 0.1) + I max (0.4, 0.2)

= 0.2 + I 0.4

Therefore we have

A (B + C)= $\begin{pmatrix} 0.4 + I\,0.3 & 0.4 + I\,0.3 \\ 0.2 + I\,0.7 & 0.2 + I\,0.4 \end{pmatrix}$

Now we shall see what happens to AB+BC

Then let us calculate AB

A B = $\begin{pmatrix} 0.1 + I\,0.3 & 0.4 + I\,0.1 \\ 0.2 + I\,0.4 & 0.1 + I\,0.7 \end{pmatrix} \begin{pmatrix} 0.2 + I\,0.3 & 0.5 + I\,0.4 \\ 0.3 + I\,0.8 & 0.9 + I\,0.1 \end{pmatrix}$

Let is now consider

A B= $\begin{pmatrix} C_{11} & C_{12} \\ C_{21} & C_{22} \end{pmatrix}$, where

C_{11} = max{min(0.1, 0.2), min(0.4,0.3)}+I max{min(0.3, 0.3), min(0.1, 0.8)}

= max(0.1, 0.3) + I max (0.3, 0.1)

= 0.3 + I 0.3

C_{12} = max{min(0.1, 0.5), min(0.4,0.9)}+I max{min(0.3, 0.4), min(0.1, 0.1)}

= max(0.1, 0.4) + I max (0.3, 0.1)

= 0.4 + I 0.3

C_{21} = max{min(0.2, 0.2), min(0.1,0.3)}+I max{min(0.4, 0.3), min(0.7, 0.8)}

= max(0.2, 0.1) + I max (0.3, 0.7)

= 0.2 + I 0.7

C_{22} = max{min(0.2, 0.5), min(0.1,0.9)}+I max{min(0.4, 0.4), min(0.7, 0.1)}

= max(0.2, 0.1) + I max (0.4, 0.1)

= 0.2 + I 0.4

Let us consider A C= $\begin{pmatrix} E_{11} & E_{12} \\ E_{21} & E_{22} \end{pmatrix}$, where

E_{11} = max{min(0.1, 0.4), min(0.4,0.6)}+I max{min(0.3, 0.6), min(0.1, 0.2)}

= max(0.1, 0.4) + I max (0.3, 0.1)

= 0.4 + I 0.3

E_{12} = max{min(0.1, 0.5), min(0.4,0.3)}+I max{min(0.3, 0.3), min(0.1, 0.2)}

= max(0.1, 0.3) + I max (0.3, 0.1)

= 0.3 + I 0.3

E_{21} = max{min(0.2, 0.4), min(0.1,0.6)}+I max{min(0.4, 0.6), min(0.7, 0.2)}

= max(0.2, 0.1) + I max (0.4, 0.2)

= 0.2 + I 0.2

E_{22} = max{min(0.2, 0.5), min(0.1,0.3)}+I max{min(0.4, 0.3), min(0.7, 0.2)}

= max(0.2, 0.1) + I max (0.3, 0.2)

= 0.2 + I 0.3

Thus we have

$C_{11} + E_{11}$= (0.3 + I 0.3) + (0.4 + I 0.3)

= 0.4 + I 0.3

$C_{12} + E_{12}$= (0.4+ I 0.3) + (0.3 + I 0.3)

= 0.4 + I 0.3

$C_{21} + E_{21}$= (0.2+ I 0.7) + (0.2 + I 0.2)

= 0.2+ I 0.7

$C_{22} + E_{22}$= (0.2+ I 0.4) + (0.2 + I 0.3)

=0.2+ I 0.4

Thus, we get, A B + A C= $\begin{pmatrix} 0.4+I\,0.3 & 0.3+I\,0.3 \\ 0.2+I\,0.7 & 0.2+I\,0.4 \end{pmatrix}$

From the above results, it can be established that

A (B+C) = AB + AC

4. Conclusions

According the newly defined addition and multiplication operation of neutrosophic fuzzy matrices, it can be seen that some of the properties of arithmetic operation of these matrices are analogous to the classical matrices. Further some future works are necessary to deal with some more properties and operations of such kind of matrices.

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References

[1] F. Smarandache, "A Unifying Field in Logics. Neutrosophy : Neutrosophic Probability, Set and Logic". Rehoboth: American Research Press,1999.

[2] Ansari, Biswas, Aggarwal," Proposal for Applicability of Neutrosophic Set Theory in Medical AI", International Journal of Computer Applications (0975 – 8887),Vo 27– No.5, (2011), pp.5-11.

[3] M. Arora, R. Biswas, U.S. Pandy, "Neutrosophic Relational Database Decomposition", International Journal of Advanced Computer Science and Applications, Vol. 2, No. 8, (2011), pp.121-125.

[4] M. Arora and R. Biswas," Deployment of Neutrosophic Technology to Retrieve Answers for Queries Posed in Natural Language", in 3rd International Conference on Computer Science and Information Technology ICCSIT, IEEE catalog Number CFP1057E-art, Vol No. 3, ISBN: 978-1-4244-5540-9, (2010), pp.435-439.

[5] F.G. Lupiáñez ,"On Neutrosophic Topology", Kybernetes, Vol. 37 Iss: 6, (2008), pp.797 – 800.

[6] H. D. Cheng, & Y Guo. "A New Neutrosophic Approach to Image Thresholding". New Mathematics and Natural Computation, 4(3), (2008), pp. 291–308.

[7] Y. Guo, &, H. D. Cheng "New Neutrosophic Approach to Image Segmentation". Pattern Recognition, 42, (2009), pp.587–595.

[8] M. Zhang, L. Zhang, and H.D. Cheng. "A Neutrosophic Approach to Image Segmentation based on Watershed Method". Signal Processing 5, 90 , (2010), pp.1510-1517.

[9] A. Kharal, "A Neutrosophic Multicriteria Decision Making Method", New Mathematics &Natural Computation, to appear in Nov 2013.

[10] J. Ye,"Similarity measures between interval neutrosophic sets and their multicriteria decision-making method "Journal of Intelligent & Fuzzy Systems, DOI: 10.3233/IFS-120724 ,(2013),.

[11] S. Broumi, F. Smarandache , "Correlation Coefficient of Interval Neutrosophic Set", Applied Mechanics and Materials Vol. 436 (2013) pp 511-517.

[12] L. A. Zadeh, "Fuzzy sets". Information and Control, 8,(1965), pp. 338-353.

[13] L. A. Zadeh, "Probability Measures of Fuzzy Events", Journal of Mathematical Analysis and Applications, 23,(1968), pp. 421-427.

[14] K. Atanassov, "Intuitionistic Fuzzy Sets". Proc. of Polish Symp. On Interval and Fuzzy Mathematics, Poznan ,(1983), pp.23-26.

[15] K. Atanassov, Intuitionistic Fuzzy Sets". Fuzzy Sets and Systems, 20,(1986), pp. 87-96.

[16] W. B. V. Kandasamy and F. Smarandache," Fuzzy Relational Maps and Neutrosophic Relational Maps" , HEXIS Church Rock ,2004, book, 302 pages.

[17] M.G.Thomas. Convergence of powers of a fuzzy matrix. J.Math. Annal. Appl. 57 (1977), pp 476-480.

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A New Methodology for Neutrosophic Multi-Attribute Decision-Making with Unknown Weight Information

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Abstract. In this paper, we present multi-attribute decision-making problem with neutrosophic assessment. We assume that the information about attribute weights is incompletely known or completely unknown. The ratings of alternatives with respect to each attributes are considered as single-valued neutrosophic set to catch up imprecise or vague information. Neutrosophic set is characterized by three independent degrees namely truth-membership degree (T), indeterminacy-membership degree (I), and falsity-membership degree (F). The modified grey relational analysis method is proposed to find out the best alternative for multi-attribute decision-making problem under neutrosophic environment. We establish a deviation based optimization model based on the ideal alternative to determine attribute weight in which the information about attribute weights is incompletely known. Again, we solve an optimization model with the help of Lagrange functions to find out the completely unknown attributes weight. By using these attributes weight we calculate the grey relational coefficient of each alternative from ideal alternative for ranking the alternatives. Finally, an illustrative example is provided in order to demonstrate its applicability and effectiveness of the proposed approach.

Keywords: Neutrosophic set; Single-valued neutrosophic set; Grey relational analysis; Multi-attribute decision making; Unknown weight information.

1 Introduction

In the real world problem, we often encounter different type of uncertainties that cannot be handled with classical mathematics. In order to deal differen types of uncertainty, Fuzzy set due to Zadeh [1] is very useful and effective. It deals with a kind of uncertainty known as "fuzziness". Each real value of [0, 1] represents the membership degree of an element of a fuzzy set i.e partial belongingness is considered. If $\mu_A(x) \in [0,1]$ is the membership degree of an element x of a fuzzy set *A*, then (1-$\mu_A(x)$) is assumed as the non-membership degree of that element. This is not generally hold for an element with incomplete information. In 1986, Atanassov [2] developed the idea of intuitionistic fuzzy set (IFS). An element of intuitionistic fuzzy set *A* characterized by the membership degree $\mu_A(x) \in [0,1]$ as well as non-membership degree $\nu_A(x) \in [0,1]$ with some restriction $0 \le \mu_A(x) + \nu_A(x) \le 1$. Therefoe certain amount of indeterminacy or incomplete information $1-(\mu_A(x)+\nu_A(x))$ remains by default. However, one may also consider the possibility $\mu_A(x)+\nu_A(x)>1$, so that inconsistent beliefs are also allowed. In this case, an element may be regarded as both member and non-member at the same time. A set connected with this features is called Para-consistent Set [3]. Smarandache [3-5] introduced the concept of neutrosophic set (NS) which is actually generalization of different type of FSs and IFSs. Consider an example, if $\mu_A(x) \in [0,1]$ is a membership degree, $\nu_A(x) \in [0,1]$ is a non-membership degree of an element x of a set *A*, then fuzzy set can be expressed as *A*= $\langle x/(\mu_A(x), 0, 1-\mu_A(x))\rangle$ and IFS can be represented as *A* $=\langle x/(\mu_A(x), 1-\mu_A(x)-\nu_A(x), \nu_A(x))\rangle$

with $0 \le \mu_A(x) + \nu_A(x) \le 1$. The main feature of neutrosophic set is that every element of the universe has not only a certain degree of truth (T) but also a falsity degree (F) and indeterminacy degree (I). These three degrees have to consider independently from each other. Another interesting feature of neutrosophic set is that we do not even assume that the incompleteness or indeterminacy degree is always given by $1-(\mu_A(x)+\nu_A(x))$.

Multiple attribute decision-making (MADM) problem in the area of operation research, management science, economics, systemic optimization, urban planning and many other fields has gained very much attention to the researchers during the last several decades. These problems generally consist of choosing the most desirable alternative that has the highest degree of satisfaction from a set of alternatives with respect to their attributes. In this approach the decision makers have to provide qualitative and/or quantitative assessments for determining the performance of each alternative with respect to each attribute, and the relative importance of evaluation attribute.

There are many MADM methods available in the literature in crisp environment such as TOPSIS (Hwang & Yoon [6]), PROMETHEE (Brans et al. [7]), VIKOR (Opricovic [8-9]), and ELECTRE (Roy [10]) etc. However it is not always possible to evaluate the importance of attributes weights and the ratings of alternatives by using crisp numbers due to un-availability of complete information about attribute values. Chen [11] extended the classical TOPSIS by developing a methodology for solving multi-criteria decision-making problems in fuzzy environment. Zeng [12] solved fuzzy MADM problem with known attribute weight by using expected value operator of fuzzy variables. However, fuzzy set can only focus on the membership grade of vague parameters or events. It fails to handle non-membership degree and indeterminacy degree of imprecise parameters. Boran et al. [13] extended the TOPSIS method for multi-criteria intuitionistic decision-making problem. Xu and Hu [14] developed two projection models for MADM problems with intuitionistic fuzzy information. Xu [15] studied MADM problem with interval-valued intuitionistic fuzzy decision-making by using distance measure.

In IFS the sum of membership degree and non-membership degree of a vague parameter is less than unity. Therefore, a certain amount of incomplete information or indeterminacy arises in an intuitionistic fuzzy set. It cannot handle all types of uncertainties successfully in different real physical problems. Hence further generalizations of fuzzy set as well as intuitionistic fuzzy sets are required.

Neutrosophic set information is helpful to handling MADM for the most general ambiguity cases, including paradox. The assessment of attribute values by the decision maker takes the form of single-valued neutrosophic set (SVNS) which is defined by Wang et al. [16]. Ye [17] studied multi-criteria decision-making problem under SVNS environment. He proposed a method for ranking of alternatives by using weighted correlation coefficient. Ye [18] also discussed single-valued neutrosophic cross entropy for multi-criteria decision-making problems. He used similarity measure for interval valued neutrosophic set for solving multi-criteria decision-making problems. Grey relational analysis (GRA) is widely used for MADM problems. Deng [19-20] developed the GRA method that is applied in various areas, such as economics, marketing, personal selection and agriculture. Zhang et al. [21] discussed GRA method for multi attribute decision-making with interval numbers. An improved GRA method proposed by Rao & Singh [22] is applied for making a decision in manufacturing situations. Wei [23] studied the GRA method for intuitionistic fuzzy multi-criteria decision-making. Biswas et al. [24] developed an entropy based grey relational analysis method for multi-attribute decision-making problem under single valued neutrosophic assessments.

The objective of this paper is to study neutrosophic MADM with unknown weight information using GRA. The rest of the paper is organized as follows. Section 2 briefly presents some preliminaries relating to neutrosophic set and single-valued neutrosophic set. In Section 3, Hamming distance between two single-valued neutrosophic sets is defined. Section 4 is devoted to represent the new model of MADM with SVNSs based on modified GRA. In section 5, an illustrative example is provided to show the effectiveness of the proposed model. Finally, section 6 presents the concluding remarks.

2 Preliminaries of Neutrosophic sets and Single valued neutrosophic set

In this section, we provide some basic definition about neutrosophic set due to Smrandache [3], which will be used to develop the paper.

Definition 1 Let X be a space of points (objects) with generic element in X denoted by x. Then a neutrosophic set A in X is characterized by a truth membership function T_A, an indeterminacy membership function I_A and a falsity membership function F_A. The functions T_A, I_A and F_A are real standard or non-standard subsets of$]0^-, 1^+[$ that is $T_A : X \to]0^-, 1^+[$; $I_A : X \to]0^-, 1^+[$; $F_A : X \to]0^-, 1^+[$

It should be noted that there is no restriction on the sum of $T_A(x)$, $I_A(x)$, $F_A(x)$ i.e. $0^- \le T_A(x) + I_A(x) + F_A(x) \le 3^+$.

Definition 2 The complement of a neutrosophic set *A* is denoted by A^c and is defined by

$$T_{A^c}(x) = \{1^+\} - T_A(x)\,;\; I_{A^c}(x) = \{1^+\} - I_A(x)\,;$$
$$F_{A^c}(x) = \{1^+\} - F_A(x)$$

Definition 3 A neutrosophic set *A* is contained in the other neutrosophic set *B*, $A \subseteq B$ if and only if the following result holds.

$$\inf T_A(x) \le \inf T_B(x)\,,\; \sup T_A(x) \le \sup T_B(x) \qquad (1)$$

$$\inf I_A(x) \ge \inf I_B(x)\,,\; \sup I_A(x) \ge \sup I_B(x) \qquad (2)$$

$$\inf F_A(x) \ge \inf F_B(x)\,,\; \sup F_A(x) \ge \sup F_B(x) \qquad (3)$$

for all x in *X*.

3 Some basics of single valued neutrosophic sets (SVNSs)

In this section we provide some definitions, operations and properties about single valued neutrosophic sets due to Wang et al. [17]. It will be required to develop the rest of the paper.

Definition 4 (Single-valued neutrosophic set). Let *X* be a universal space of points (objects), with a generic element of X denoted by x. A single-valued neutrosophic set $\tilde{\mathcal{N}} \subset X$ is characterized by a true membership function $T_{\tilde{\mathcal{N}}}(x)$, a falsity membership function $F_{\tilde{\mathcal{N}}}(x)$ and an indeterminacy function $I_{\tilde{\mathcal{N}}}(x)$ with $T_{\tilde{\mathcal{N}}}(x)$, $I_{\tilde{\mathcal{N}}}(x)$, $F_{\tilde{\mathcal{N}}}(x) \in [0, 1]$ for all x in *X*.

When X is continuous a SVNSs, $\tilde{\mathcal{N}}$ can be written as

$$\tilde{\mathcal{N}} = \int_x \left\langle T_{\tilde{\mathcal{N}}}(x), I_{\tilde{\mathcal{N}}}(x), F_{\tilde{\mathcal{N}}}(x) \right\rangle \Big/ x,\;\; \forall x \in X.$$

and when X is discrete a SVNSs $\tilde{\mathcal{N}}$ can be written as

$$\tilde{\mathcal{N}} = \sum_{i=1}^{m} \left\langle T_{\tilde{\mathcal{N}}}(x), I_{\tilde{\mathcal{N}}}(x), F_{\tilde{\mathcal{N}}}(x) \right\rangle / x\,,\;\; \forall x \in X.$$

Actually, SVNS is an instance of neutrosophic set that can be used in real life situations like decision making, scientific and engineering applications. In case of SVNS, the degree of the truth membership $T_{\tilde{\mathcal{N}}}(x)$, the indeterminacy membership $I_{\tilde{\mathcal{N}}}(x)$ and the falsity membership $F_{\tilde{\mathcal{N}}}(x)$ values belong to [0, 1] instead of non-standard unit interval$]0^-, 1^+[$as in the case of ordinary neutrosophic sets.

It should be noted that for a SVNS $\tilde{\mathcal{N}}$,

$$0 \le \sup T_{\tilde{\mathcal{N}}}(x) + \sup I_{\tilde{\mathcal{N}}}(x) + \sup F_{\tilde{\mathcal{N}}}(x) \le 3\,,\; \forall x \in X. \qquad (4)$$

and for a neutrosophic set, the following relation holds

$$0^- \le \sup T_{\tilde{\mathcal{N}}}(x) + \sup I_{\tilde{\mathcal{N}}}(x) + \sup F_{\tilde{\mathcal{N}}}(x) \le 3^+,\; \forall x \in X. \qquad (5)$$

Definition 5 The complement of a neutrosophic set $\tilde{\mathcal{N}}$ is denoted by $\tilde{\mathcal{N}}^c$ and is defined by

$$T_{\tilde{\mathcal{N}}^c}(x) = F_{\tilde{\mathcal{N}}}(x)\,;\; I_{\tilde{\mathcal{N}}^c}(x) = 1 - I_{\tilde{\mathcal{N}}}(x)\,;\; F_{\tilde{\mathcal{N}}^c}(x) = T_{\tilde{\mathcal{N}}}(x)$$

Definition 6 A SVNS $\tilde{\mathcal{N}}_A$ is contained in the other SVNS $\tilde{\mathcal{N}}_B$, denoted as $\tilde{\mathcal{N}}_A \subseteq \tilde{\mathcal{N}}_B$, if and only if $T_{\tilde{\mathcal{N}}_A}(x) \le T_{\tilde{\mathcal{N}}_B}(x)$; $I_{\tilde{\mathcal{N}}_A}(x) \ge I_{\tilde{\mathcal{N}}_B}(x)$; $F_{\tilde{\mathcal{N}}_A}(x) \ge F_{\tilde{\mathcal{N}}_B}(x)$ $\forall x \in X$.

Definition 7 Two SVNSs $\tilde{\mathcal{N}}_A$ and $\tilde{\mathcal{N}}_B$ are equal, i.e. $\tilde{\mathcal{N}}_A = \tilde{\mathcal{N}}_B$, if and only if $\tilde{\mathcal{N}}_A \subseteq \tilde{\mathcal{N}}_B$ and $\tilde{\mathcal{N}}_A \supseteq \tilde{\mathcal{N}}_B$.

Definition 8 (Union) The union of two SVNSs $\tilde{\mathcal{N}}_A$ and $\tilde{\mathcal{N}}_B$ is a SVNS $\tilde{\mathcal{N}}_C$, written as $\tilde{\mathcal{N}}_C = \tilde{\mathcal{N}}_A \cup \tilde{\mathcal{N}}_B$. Its truth membership, indeterminacy-membership and falsity membership functions are related to those of $\tilde{\mathcal{N}}_A$ and $\tilde{\mathcal{N}}_B$ by

$T_{\tilde{\mathcal{N}}_C}(x) = \max(T_{\tilde{\mathcal{N}}_A}(x), T_{\tilde{\mathcal{N}}_B}(x))$;
$I_{\tilde{\mathcal{N}}_C}(x) = \max(I_{\tilde{\mathcal{N}}_A}(x), I_{\tilde{\mathcal{N}}_B}(x))$;
$F_{\tilde{\mathcal{N}}_C}(x) = \min(F_{\tilde{\mathcal{N}}_A}(x), F_{\tilde{\mathcal{N}}_B}(x))$ for all x in X.

Definition 9 (Intersection) The intersection of two SVNSs $\tilde{\mathcal{N}}_A$ and $\tilde{\mathcal{N}}_B$ is a SVNS $\tilde{\mathcal{N}}_C$, written as $\tilde{\mathcal{N}}_C = \tilde{\mathcal{N}}_A \cap \tilde{\mathcal{N}}_B$, whose truth membership, indeterminacy-membership and falsity membership functions are related to those of $\tilde{\mathcal{N}}_A$ and $\tilde{\mathcal{N}}_B$ by $T_{\tilde{\mathcal{N}}_C}(x) = \min(T_{\tilde{\mathcal{N}}_A}(x), T_{\tilde{\mathcal{N}}_B}(x))$;
$I_{\tilde{\mathcal{N}}_C}(x) = \min(I_{\tilde{\mathcal{N}}_A}(x), I_{\tilde{\mathcal{N}}_B}(x))$;
$F_{\tilde{\mathcal{N}}_C}(x) = \max(F_{\tilde{\mathcal{N}}_A}(x), F_{\tilde{\mathcal{N}}_B}(x))$ for all x in X.

4 Distance between two neutrosophic sets.

Similar to fuzzy or intuitionistic fuzzy set, the general SVNS having the following pattern

$\tilde{\mathcal{N}} = \{(x/(T_{\tilde{\mathcal{N}}}(x), I_{\tilde{\mathcal{N}}}(x), F_{\tilde{\mathcal{N}}}(x)) : x \in X\}$. For finite SVNSs can be represented by the ordered tetrads:

$$\tilde{\mathcal{N}} = \left\{ \begin{array}{l} (x_1/(T_{\tilde{\mathcal{N}}}(x_1), I_{\tilde{\mathcal{N}}}(x_1), F_{\tilde{\mathcal{N}}}(x_1)), \\ \quad ..., x_m/(T_{\tilde{\mathcal{N}}}(x_m), I_{\tilde{\mathcal{N}}}(x_m), F_{\tilde{\mathcal{N}}}(x_m)) \end{array} \right\}, \quad \forall x \in X$$

Definition 10 Let

$$\tilde{\mathcal{N}}_A = \{(x_1/(T_{\tilde{\mathcal{N}}_A}(x_1), I_{\tilde{\mathcal{N}}_A}(x_1), F_{\tilde{\mathcal{N}}_A}(x_1)), ..., x_n/(T_{\tilde{\mathcal{N}}_A}(x_n), I_{\tilde{\mathcal{N}}_A}(x_n), F_{\tilde{\mathcal{N}}_A}(x_n))\}$$

and

$$\tilde{\mathcal{N}}_B = \{(x_1/(T_{\tilde{\mathcal{N}}_B}(x_1), I_{\tilde{\mathcal{N}}_B}(x_1), F_{\tilde{\mathcal{N}}_B}(x_1)), ..., x_n/(T_{\tilde{\mathcal{N}}_B}(x_n), I_{\tilde{\mathcal{N}}_B}(x_n), F_{\tilde{\mathcal{N}}_B}(x_n))\} \quad (6)$$

be two SVNSs in $X = \{x_1, x_2, \ldots, x_n\}$.

Then the Hamming distance between two SVNSs $\tilde{\mathcal{N}}_A$ and $\tilde{\mathcal{N}}_B$ is defined as follows:

$$d_{\tilde{\mathcal{N}}}\left(\tilde{\mathcal{N}}_A, \tilde{\mathcal{N}}_B\right) = \sum_{i=1}^{n} \left\{ \begin{array}{l} \left|T_{\tilde{\mathcal{N}}_A}(x_1) - T_{\tilde{\mathcal{N}}_B}(x_1)\right| + \left|I_{\tilde{\mathcal{N}}_A}(x_1) - I_{\tilde{\mathcal{N}}_B}(x_1)\right| \\ \quad + \left|F_{\tilde{\mathcal{N}}_A}(x_1) - F_{\tilde{\mathcal{N}}_B}(x_1)\right| \end{array} \right\} \quad (7)$$

and normalized Hamming distance between two SVNSs $\tilde{\mathcal{N}}_A$ and $\tilde{\mathcal{N}}_B$ is defined as follows:

$$^N d_{\tilde{\mathcal{N}}}\left(\tilde{\mathcal{N}}_A, \tilde{\mathcal{N}}_B\right) = \frac{1}{3n} \sum_{i=1}^{n} \left\{ \left|T_{\tilde{\mathcal{N}}_A}(x_1) - T_{\tilde{\mathcal{N}}_B}(x_1)\right| + \left|I_{\tilde{\mathcal{N}}_A}(x_1) - I_{\tilde{\mathcal{N}}_B}(x_1)\right| + \left|F_{\tilde{\mathcal{N}}_A}(x_1) - F_{\tilde{\mathcal{N}}_B}(x_1)\right| \right\}$$

(8) with the following two properties

1. $0 \le d_{\tilde{\mathcal{N}}}\left(\tilde{\mathcal{N}}_A, \tilde{\mathcal{N}}_B\right) \le 3n$ (9)
2. $0 \le {}^N d_{\tilde{\mathcal{N}}}\left(\tilde{\mathcal{N}}_A, \tilde{\mathcal{N}}_B\right) \le 1$ (10)

5 GRA based single valued neutrosophic multiple attribute decision-making problems with incomplete weight information.

Consider a multi-attribute decision-making problem with *m* alternatives and *n* attributes. Let $A_1, A_2, ..., A_m$ be a discrete set of alternatives, and $C_1, C_2, ..., C_n$ be the set of attributes. The rating provided by the decision maker, describes the performance of alternative A_i against attribute C_j. The values associated with the alternatives for MADM problems can be presented in the following decision matrix

Table 1. Decision matrix of attribute values

$$D = \left\langle d_{ij} \right\rangle_{m\times n} = \begin{array}{c} \\ A_1 \\ A_2 \\ . \\ . \\ A_m \end{array} \begin{array}{c} \begin{array}{cccc} C_1 & C_2 & ... & C_n \end{array} \\ \begin{bmatrix} d_{11} & d_{12} & ... & d_{1n} \\ d_{21} & d_{22} & ... & d_{2n} \\ ... & ... & ... & ... \\ ... & ... & ... & ... \\ d_{m1} & d_{m2} & ... & d_{mn} \end{bmatrix} \end{array} \quad (11)$$

The weight $w_j \in [0,1]$ (j = 1, 2, ..., n) reflects the relative importance of attribute C_j (j = 1, 2, ..., m) to the decision-making process such that $\sum_{j=1}^{n} w_j = 1$. S is a set of known weight information that can be represented by the following forms due to Park et al. [25], Park and Kim [26], Kim et al. [27], Kim and Ahn [28], and Park [29].

Form 1. A weak ranking: $w_i \ge w_j$, for i $\neq$ j;

Form 2. A strict ranking: $w_i - w_j \ge \varphi_i$, $\varphi_i > 0$, for i $\neq$ j;

Form 3. A ranking of differences: $w_i - w_j \geq w_k - w_l$, for $j \neq k \neq l$;

Form 4. A ranking with multiples: $w_i \geq \beta_j w_j$, $\beta_j \in [0,1]$, for $i \neq j$;

Form 5. An interval form: $\alpha_i \leq w_i \leq \alpha_i + \varepsilon_i$, $0 \leq \alpha_i < \alpha_i + \varepsilon_i \leq 1$.

GRA is one of the derived evaluation methods for MADM based on the concept of grey relational space. The first step of GRA method is to create a comparable sequence of the performance of all alternatives. This step is known as data pre-processing. A reference sequence (ideal target sequence) is defined from these sequences. Then, the grey relational coefficient between all comparability sequences and the reference sequence for different values of distinguishing coefficient are calculated. Finally, based on these grey relational coefficients, the grey relational degree between the reference sequence and every comparability sequences is calculated. If an alternative gets the highest grey relational grade with the reference sequence, it means that the comparability sequence is most similar to the reference sequence and that alternative would be the best choice (Fung [30]). The steps of improved GRA method under SVNS are described below.

Step 1. Determine the most important criteria.

Generally, there are many criteria or attributes in decision-making problems, where some of them are important and others may not be so important. So it is crucial to select the proper criteria or attributes for decision-making situation. The most important attributes may be chosen with the help of experts' opinions or by some others method that are technically sound.

Step 2. Construct the decision matrix with SVNSs

Assume that a multiple attribute decision making problem have *m* alternatives and *n* attributes. The general form of decision matrix as shown in Table1 can be presented after data pre-processing. The original GRA method can effectively deal with quantitative attributes. However, there exist some difficulties in the case of qualitative attributes. In the case of a qualitative attribute, an assessment value may be taken as SVNSs. In this paper we assume that the ratings of alternatives A_i (i = 1, 2,…m) with respect to attributes C_j (j = 1, 2,…n) are SVNSs. Thus the neutrosophic values associated with the alternatives for MADM problems can be represented in the following decision matrix:

Table 2. Decision matrix with SVNS

$$D_{\tilde{\mathcal{N}}} = \langle T_{ij}, I_{ij}, F_{ij} \rangle_{m \times n}$$

$$= \begin{array}{c} \\ A_1 \\ A_2 \\ . \\ . \\ A_m \end{array} \begin{array}{c} \begin{array}{cccc} C_1 & C_2 & & C_n \end{array} \\ \begin{bmatrix} \langle T_{11}, I_{11}, F_{11} \rangle & \langle T_{12}, I_{12}, F_{12} \rangle & \ldots & \langle T_{1n}, I_{1n}, F_{1n} \rangle \\ \langle T_{21}, I_{21}, F_{21} \rangle & \langle T_{22}, I_{22}, F_{22} \rangle & \ldots & \langle T_{2n}, I_{2n}, F_{2n} \rangle \\ \ldots & \ldots & \ldots & \ldots \\ \ldots & \ldots & \ldots & \ldots \\ \langle T_{m1}, I_{m1}, F_{m1} \rangle & \langle T_{m2}, I_{m2}, F_{m2} \rangle & \ldots & \langle T_{mn}, I_{mn}, F_{mn} \rangle \end{bmatrix} \end{array} \quad (12)$$

In this matrix $D_{\tilde{\mathcal{N}}} = \langle T_{ij}, I_{ij}, F_{ij} \rangle_{m \times n}$, T_{ij}, I_{ij} and F_{ij} denote the degrees of truth membership, degree of indeterminacy and degree of falsity membership of the alternative A_i with respect to attribute C_j. These three degrees for SVNS satisfying the following properties:

1. $0 \leq T_{ij} \leq 1$, $0 \leq I_{ij} \leq 1$, $0 \leq F_{ij} \leq 1$
2. $0 \leq T_{ij} + I_{ij} + F_{ij} \leq 3$.

Step 3. Determine the ideal neutrosophic estimates reliability solution (INERS) and the ideal neutrosophic estimates un-reliability solution (INEURS) for neutrosophic decision matrix.

The ideal reliability estimation can be easily determined due to Biswas et al. [24].

Definition 11 The ideal neutrosophic estimates reliability solution (INERS) $Q^+_{\tilde{\mathcal{N}}} = [q^+_{\tilde{\mathcal{N}}_1}, q^+_{\tilde{\mathcal{N}}_2}, \ldots, q^+_{\tilde{\mathcal{N}}_n}]$ is a solution in which every component $q^+_{\tilde{\mathcal{N}}_j} = \langle T^+_j, I^+_j, F^+_j \rangle$, where $T^+_j = \max_i \{T_{ij}\}$, $I^+_j = \min_i \{I_{ij}\}$ and $F^+_j = \min_i \{F_{ij}\}$ in the neutrosophic decision matrix $D_{\tilde{\mathcal{N}}} = \langle T_{ij}, I_{ij}, F_{ij} \rangle_{m \times n}$.

Definition 12 The ideal neutrosophic estimates un-reliability solution (INEURS) $Q^{-}_{\tilde{\mathcal{N}}} = [q^{-}_{\tilde{\mathcal{N}}_1}, q^{-}_{\tilde{\mathcal{N}}_2}, ..., q^{-}_{\tilde{\mathcal{N}}_n}]$ can be taken as a solution in the form $q^{-}_{\tilde{\mathcal{N}}j} = \langle T^{-}_j, I^{-}_j, F^{-}_j \rangle$, where

$T^{-}_j = \min_i \{T_{ij}\}$, $I^{-}_j = \max_i \{I_{ij}\}$ and $F^{-}_j = \max_i \{F_{ij}\}$ in the neutrosophic decision matrix $D_{\tilde{\mathcal{N}}} = \langle T_{ij}, I_{ij}, F_{ij} \rangle_{m\times n}$.

Step 4. Calculate the neutrosophic grey relational coefficient of each alternative from INERS and INEURS.

Grey relational coefficient of each alternative from INERS is defined as:

$$\chi^{+}_{ij} = \frac{\min_i \min_j \Delta^{+}{}_{ij} + \rho \max_i \max_j \Delta^{+}{}_{ij}}{\Delta^{+}{}_{ij} + \rho \max_i \max_j \Delta^{+}{}_{ij}}, \text{ where}$$

$$\Delta^{+}{}_{ij} = d\left(q^{+}_{\tilde{\mathcal{N}}j}, q_{\tilde{\mathcal{N}}ij}\right), \text{ i= 1, 2,…,m. and j=1, 2,…,n.} \quad (13)$$

Grey relational coefficient of each alternative from INEURS is defined as:

$$\chi^{-}_{ij} = \frac{\min_i \min_j \Delta^{-}{}_{ij} + \rho \max_i \max_j \Delta^{-}{}_{ij}}{\Delta^{-}{}_{ij} + \rho \max_i \max_j \Delta^{-}{}_{ij}}, \text{ where}$$

$$\Delta^{-}{}_{ij} = d\left(q_{\tilde{\mathcal{N}}ij}, q^{-}_{\tilde{\mathcal{N}}j}\right), \text{ i = 1, 2,…,m. and j = 1, 2,…,n.} \quad (14)$$

ρ is the distinguishing coefficient or the identification coefficient, $\rho \in [0,1]$. Smaller value of distinguishing coefficient will yield in large range of grey relational coefficient. Generally, $\rho = 0.5$ is considered for decision- making situation.

Step 5. Determine the weights of criteria.

In the decision-making process, decision maker may often feel that the importance of the attributes is not same. Due to the complexity and uncertainty of real world decision-making problems, the information about attribute weights is usually incomplete. The estimation of the attribute weights plays an important role in MADM. Therefore, we need to determine reasonable attribute weight for making a reasonable decision. Many methods are available to determine the unknown attribute weight in the literature such as maximizing deviation method (Wu and Chen [31]), entropy method (Wei and Tang [32]; Xu and Hui [33]), optimization method (Wang and Zhang [34-35]) etc. In this paper, we use optimization method to determine unknown attribute weights for neutrosophic MADM.

The basic principle of the GRA method is that the chosen alternative should have the largest degree of grey relation from the INERS. Thus, the larger grey relational coefficient determines the best alternative for the given weight vector. To obtain the grey relational coefficient, we have to calculate weight vector of attributes if the information about attribute weights is incompletely known. The grey relational coefficient between INERS and itself is (1, 1, …, 1), similarly, coefficient between INEURS and itself is also (1, 1, …, 1). So the corresponding comprehensive deviations are

$$d^{+}_i(W) = \sum_{j=1}^{n} \left(1 - \chi^{+}_{ij}\right) w_j \quad (15)$$

$$d^{-}_i(W) = \sum_{j=1}^{n} \left(1 - \chi^{-}_{ij}\right) w_j \quad (16)$$

Smaller value of (15) as well as (16) indicates the better alternative A_i. A satisfactory weight vector W= (w_1, w_2,…, w_n) is determined by making smaller all the distances $d^{+}_i(W) = \sum_{j=1}^{n} \left(1 - \chi^{+}_{ij}\right) w_j$ and $d^{-}_i(W) = \sum_{j=1}^{n} \left(1 - \chi^{-}_{ij}\right) w_j$. We utilize the max-min operator developed by Zimmermann and Zysco [36] to integrate all the distances $d^{+}_i(W) = \sum_{j=1}^{n} \left(1 - \chi^{+}_{ij}\right) w_j$ for i = 1, 2, …, m and $d^{-}_i(W) = \sum_{j=1}^{n} \left(1 - \chi^{-}_{ij}\right) w_j$ for i = 1, 2, …, m separately. Therefore, we can formulate the following programming model:

$$\text{(M-1a)} \begin{cases} \text{Min } \xi^{+} \\ \text{subject to} : \sum_{j=1}^{n} \left(1 - \chi^{+}_{ij}\right) w_j \le \xi^{+} \text{for } i = 1, 2, ...m \\ W \in S \end{cases} \quad (17)$$

$$\text{(M-1b)} \begin{cases} \text{Min } \xi^- \\ \text{subject to}: \sum_{j=1}^{n}\left(1-\chi_{ij}^-\right)w_j \le \xi^- \text{for } i=1,2,...m \\ W \in S \end{cases} \quad (18)$$

Here $\xi^+ = \max_i \left\{ \sum_{j=1}^{n}\left(1-\chi_{ij}^+\right)w_j \right\}$ (19)

and $\xi^- = \max_i \left\{ \sum_{j=1}^{n}\left(1-\chi_{ij}^-\right)w_j \right\}$ for 1, 2, …, m. (20)

Solving these two model (M-1a) and (M-1b), we obtain the optimal solutions $W^+ = (w_1^+, w_2^+, ...w_n^+)$ and $W^- = (w_1^-, w_2^-, ...w_n^-)$ respectively. Combinations of these two optimal solutions will give us the weight vector of the attributes i.e. $W = \gamma W^+ + (1-\gamma)W^-$ for $\gamma \in [0,1]$. (21)

If the information about attribute weights is completely unknown, we can establish another multiple objective programming:

$$\text{(M-2)} \begin{cases} \min\ d_i^+(W) = \sum_{j=1}^{n}\left[\left(1-\chi_{ij}^+\right)w_j\right]^2, \quad i=1,...,m. \\ \text{subject to}: \sum_{j=1}^{n} w_j = 1 \end{cases} \quad (22)$$

Since each alternative is non-inferior, so there exists no preference relation between the alternatives. Then, we can aggregate the above multiple objective optimization models with equal weights in to the following single objective optimization model:

(M-3)

$$\begin{cases} \min\ d^+(W) = \sum_{i=1}^{m} d_i^+(W) = \sum_{i=1}^{m}\sum_{j=1}^{n}\left[\left(1-\chi_{ij}^+\right)w_j\right]^2, \quad i=1,...,m. \\ \text{subject to}: \sum_{j=1}^{n} w_j = 1 \end{cases} \quad (23)$$

To solve this model, we construct the Lagrange function:

$$L(W,\lambda) = \sum_{i=1}^{m}\sum_{j=1}^{n}\left[\left(1-\chi_{ij}^+\right)w_j\right]^2 + 2\lambda\left(\sum_{j=1}^{n} w_j - 1\right) \quad (24)$$

Where λ is the Lagrange multiplier. Differentiating equation (24) with respect to w_j (j = 1, 2,…, n) and λ, and putting these partial derivatives equal to zero, we have the following set of equations:

$$\frac{\partial L(w_j,\lambda)}{\partial w_j} = 2\sum_{i=1}^{m} w_j (1-\chi_{ij}^+)^2 + 2\lambda = 0 \quad (25)$$

$$\frac{\partial L(w_j,\lambda)}{\partial \lambda} = \sum_{j=1}^{n} w_j - 1 = 0 \quad (26)$$

Solving equations (25) and (26), we obtain the following relation

$$w_j^+ = \left[\sum_{j=1}^{n}\left(\sum_{i=1}^{m}\left(1-\chi_{ij}^+\right)^2\right)^{-1}\right]^{-1} \Bigg/ \sum_{i=1}^{m}\left(1-\chi_{ij}^+\right)^2 \quad (27)$$

Then we get χ_i^+ for i = 1, 2,…, m.

Similarly, we can find out the attribute weight w_j^- taking into consideration of INERUS as:

$$w_j^- = \left[\sum_{j=1}^{n}\left(\sum_{i=1}^{m}\left(1-\chi_{ij}^-\right)^2\right)^{-1}\right]^{-1} \Bigg/ \sum_{i=1}^{m}\left(1-\chi_{ij}^-\right)^2 \quad (28)$$

Combining (27) and (28), we can determine the j-th attribute weight with the help of (21).

Step 6. Calculate of neutrosophic grey relational coefficient (NGRC).

The degree of neutrosophic grey relational coefficient of each alternative from INERS and INEURS are calculated by using the following equations:

$$\chi_i^+ = \sum_{j=1}^{n} w_j \chi_{ij}^+$$

(29)

and $\chi_i^- = \sum_{j=1}^{n} w_j \chi_{ij}^-$ for i= 1, 2,…,m. (30)

Step 7. Calculate the neutrosophic relative relational degree (NRD).

We calculate the neutrosophic relative relational degree of each alternative from INERS by employing the following equation:

$$R_i = \frac{\chi_i^+}{\chi_i^+ + \chi_i^-}, \text{ for } i = 1, 2,…, m. \quad (31)$$

Step 8. Rank the alternatives.

Based on the neutrosophic relative relational degree, the ranking order of all alternatives can be determined. The highest value of R_i presents the most desired alternatives.

5 . Illustrative Examples

In this section, neutrosophic MADM is considered to demonstrate the application and the effectiveness of the proposed approach. Let us consider the decision-making problem adapted from Ye [37]. Suppose there is an investment company, which wants to invest a sum of money in the best option. There is a panel with four possible alternatives to invest the money: (1) A_1 is a car company; (2) A_2 is a food company; (3) A_3 is a computer company; and (4) A_4 is an arms company. The investment company must take a decision based on the following three criteria: (1) C_1 is the risk analysis; (2) C_2 is the growth analysis; and (3) C_3 is the environmental impact analysis. We obtain the following single-valued neutrosophic decision matrix based on the experts' assessment:

Table3. Decision matrix with SVNS

$$D_{\tilde{\mathcal{N}}} = \left\langle T_{ij}, I_{ij}, F_{ij} \right\rangle_{4\times 3} =$$

$$\begin{array}{c} \\ A_1 \\ A_2 \\ A_3 \\ A_4 \end{array} \begin{array}{c} \begin{array}{ccc} C_1 & C_2 & C_3 \end{array} \\ \begin{bmatrix} \langle 0.4, 0.2, 0.3\rangle & \langle 0.4, 0.2, 0.3\rangle & \langle 0.2, 0.2, 0.5\rangle \\ \langle 0.6, 0.1, 0.2\rangle & \langle 0.6, 0.1, 0.2\rangle & \langle 0.5, 0.2, 0.2\rangle \\ \langle 0.3, 0.2, 0.3\rangle & \langle 0.5, 0.2, 0.3\rangle & \langle 0.5, 0.3, 0.2\rangle \\ \langle 0.7, 0.0, 0.1\rangle & \langle 0.6, 0.1, 0.2\rangle & \langle 0.4, 0.3, 0.2\rangle \end{bmatrix} \end{array} \qquad (32)$$

Information about the attribute weights is partially known. The known weight information is given as follows: $S = \{.30 \le w_1 \le .35, .36 \le w_2 \le .48, .26 \le w_3 \le .30\}$ such that $w_j \ge 0$ for j = 1, 2, 3and $\sum_{j=1}^{3} w_j = 1$.

Step 1. Determine the ideal neutrosophic estimates reliability solution (INERS) from the given decision matrix (see Table 3) as:

$$Q^+_{\tilde{\mathcal{N}}} = [q^+_{\tilde{\mathcal{N}}_1}, q^+_{\tilde{\mathcal{N}}_2}, q^+_{\tilde{\mathcal{N}}_3}] =$$

$$\left[\left\langle \max_i\{T_{i1}\}, \min_i\{I_{i1}\}, \min_i\{F_{i1}\} \right\rangle, \left\langle \max_i\{T_{i2}\}, \min_i\{I_{i2}\}, \min_i\{F_{i2}\} \right\rangle, \left\langle \max_i\{T_{i3}\}, \min_i\{I_{i3}\}, \min_i\{F_{i3}\} \right\rangle \right]$$

$$= \left[\langle 0.7, 0.0, 0.1\rangle, \langle 0.6, 0.1, 0.2\rangle, \langle 0.5, 0.2, 0.2\rangle \right]$$

Step 2. Similarly, determine the ideal neutrosophic estimates un-reliability solution (INEURS) as:

$$Q^+_{\tilde{\mathcal{N}}} = [q^-_{\tilde{\mathcal{N}}_1}, q^-_{\tilde{\mathcal{N}}_2}, q^-_{\tilde{\mathcal{N}}_3}] =$$

$$\left[\left\langle \min_i\{T_{i1}\}, \max_i\{I_{i1}\}, \max_i\{F_{i1}\} \right\rangle, \left\langle \min_i\{T_{i2}\}, \max_i\{I_{i2}\}, \max_i\{F_{i2}\} \right\rangle, \left\langle \min_i\{T_{i3}\}, \max_i\{I_{i3}\}, \max_i\{F_{i3}\} \right\rangle \right]$$

$$= \left[\langle 0.4, 0.2, 0.3\rangle, \langle 0.4, 0.2, 0.3\rangle, \langle 0.2, 0.3, 0.5\rangle \right]$$

Step 3. Calculation of the neutrosophic grey relational coefficient of each alternative from INERS and INEURS

By using equation (13) the neutrosophic grey relational coefficient of each alternative from INERS can be obtained as:

$$\left[\chi^+_{ij}\right]_{4\times 3} = \begin{bmatrix} 0.3636 & 0.5000 & 0.4000 \\ 0.5714 & 1.0000 & 1.0000 \\ 0.3333 & 0.5714 & 0.8000 \\ 1.0000 & 1.0000 & 0.6666 \end{bmatrix}$$

(33)

and from equation (14), the neutrosophic grey relational coefficient of each alternative from INEUS is

$$\left[\chi^-_{ij}\right]_{4\times 3} = \begin{bmatrix} 1.0000 & 1.0000 & 0.7778 \\ 0.4667 & 0.4667 & 0.3333 \\ 0.7778 & 0.7778 & 0.3684 \\ 0.3333 & 0.4667 & 0.4111 \end{bmatrix} \qquad (34)$$

Step 4. Determine the weights of attribute.

Case 1. Utilizing the model (M-1a) and (M-2b), we establish the single objective programming model:

Case 1a. Min ξ^+

subject to: $0.6364\, w_1 + 0.5000\, w_2 + 0.6000 w_3 \le \xi^+$;

$0.4286\, w_1 \le \xi^+$;

$$0.6667w_1 + 0.4286\,w_2 + 0.2000w_3 \le \xi^+;$$

$$0.3334w_3 \le \xi^+;$$

$$30 \le w_1 \le 35;\ 36 \le w_2 \le 48;\ 26 \le w_3 \le 30;$$

$$w_1 + w_2 + w_3 = 1;\ w_j \ge 0,\ j = 1, 2, 3.$$

Case 1b.

Similarly, $\text{Min}\ \xi^-$

subject to: $$0.2222w_3 \le \xi^-;$$

$$0.5353w_1 + 0.5353\,w_2 + 0.6667\,w_3 \le \xi^-;$$

$$0.2222w_1 + 0.2222\,w_2 + 0.6316w_3 \le \xi^-;$$

$$0.6667\,w_1 + 0.5353w_2 + 0.5889w_3 \le \xi^-;$$

$$30 \le w_1 \le 35;\ 36 \le w_2 \le 48;\ 26 \le w_3 \le 30;$$

$$w_1 + w_2 + w_3 = 1;\ w_j \ge 0,\ j = 1, 2, 3.$$

We obtain the same solution set $W^+ = W^- = (0.30, 0.44, 0.26)$ after solving *Case 1a* and *Case 1b* separately. Therefore, the obtained weight vector of attributes is W = (0.30, 0.44, 0.26).

Case 2. If the information about the attribute weights is completely unknown, we can use another proposed formula given in (27) and (28) to determine the weight vector of attributes. The weight vector W^+= (0.1851, 0.4408, 0.3740) is determined from equation (27) and W^-= (0.3464, 0.4361, 0.2174) from equation (28). Therefore, the resulting weight vector of attribute with the help of equation (21) (taking $\gamma = 0.5$) is $W' = (0.2657, 0.4384, 0.2957)$. After normalizing, we obtain the final weight vector of the attribute as W= (0.2657, 0.4385, 0.2958).

Step 5. Determine the degree of neutrosophic grey relational co-efficient (NGRC) of each alternative from INERS and INEUS.

The required neutrosophic grey relational co-efficient of each alternative from INERS is determined by using equations (29) with the corresponding obtained weight vector W for Case-1 and Case-2 are presented in Table 4.

Similarly, the neutrosophic grey relational co-efficient of each alternative from INEURS is obtained with the help of equation (30) for all two cases are listed in Table 4.

Step 6. Neutrosophic relative degree (NRD) of each alternative from INERS can be obtained with the help of equation (31) and these are shown in Table 4

Table 4. Calculation of NGRC and NRD of each alternative from neutrosophic estimates reliability solution

Proposed method	Weight Vector	NGRC from INERS	NGRC from INEURS	NRD from INERS	Ranking Result	Selection
		0.4331	0.9422	0.3149		
		0.8714	0.4320	0.6686	$R_4>R_2>R_3>R_1$	A_4
Case-1	(0.30, 0.44, 0.26)	0.5594	0.6714	0.4545		
		0.9133	0.4122	**0.6890**		
		0.4342	0.9343	0.3173		
		0.8861	0.4272	0.6747	$R_4>R_2>R_3>R_1$	A_4
Case-2	(0.2657, 0.4385, 0.2958)	0.5758	0.6567	0.4672		
		0.9014	0.4149	**0.6847**		

Step 7. From Table 4, we can easily determine the ranking order of all alternatives according to the values of neutrosophic relational degrees. For case-1, we see that A_4 i.e. Arms company is the best alternative for investment purpose. Similarly, for case-2 A_4 i.e. Arms company also is the best alternative for investment purpose.

6 Conclusion

In this paper, we introduce single-valued neutrosophic multiple attribute decision-making problem with incompletely known or completely unknown attribute weight information based on modified GRA. In order to determine the incompletely known attribute weight minimizing deviation based optimization method is used. On the other hand, we solve an optimization model to find out the completely unknown attributes weight by using Lagrange functions. Finally, an illustrative example is provided to show the feasibility of the proposed approach and to demonstrate its practicality and effectiveness. However, we hope that the concept presented here will create new avenue of research in current neutrosophic decision-making arena. The main thrust of the paper will be in the field of practical decision-making, pattern recognition, medical diagnosis and clustering analysis.

References

[1] L. A. Zadeh. Fuzzy Sets, Information and Control, 8(1965), 338-353.
[2] K. T. Atanassov. Intuitionistic fuzzy sets, Fuzzy Sets and Systems, 20 (1986), 87–96.
[3] F. Smarandache. A unifying field in logics. Neutrosophy: Neutrosophic probability, set and logic. Rehoboth: American Research Press (1999).
[4] F. Smarandache. Neutrosophic set-a generalization of intuitionistic fuzzy set. Journal of Defense Resources Management, 1(1) (2010), 107-116.
[5] F. Smarandache. Neutrosophic set- a generalization of intuitionistic fuzzy sets. International Journal of Pure and Applied Mathematics, 24 (3) (2005), 287-297.
[6] C. L. Hwang, and K. Yoon. Multiple attribute decision making: methods and applications: a state-of-the-art survey, Springer, London (1981).
[7] J.P. Brans, P. Vinvke, and B. Mareschal. How to select and how to rank projects: The PROMETHEE method, European Journal of Operation Research, 24(1986), 228–238.
[8] S. Opricovic. Multicriteria optimization of civil engineering systems, Faculty of Civil Engineering, Belgrade (1998).
[9] S. Opricovic, and G. H. Tzeng. Compromise solution by MCDM methods: a comparative analysis of VIKOR and TOPSIS. European Journal of Operation Research, 156 (2004), 445–455.
[10] B. Roy. The outranking approach and the foundations of ELECTRE methods. Theory Decision, 31(1991), 49–73.
[11] C. T. Chen. Extensions of the TOPSIS for group decision making under fuzzy environment, Fuzzy Sets and Systems, 114(2000), 1-9.
[12] L. Zeng. Expected value method for fuzzy multiple attribute decision making, Tsinghua Science and Technology, 11 (2006) 102–106.
[13] F. E. Boran, S. Genc, M. Kurt, and D. Akay. A multi-criteria intuitionistic fuzzy group decision making for supplier selection with TOPSIS method, Expert Systems with Applications, 36(8) (2009), 11363–11368.
[14] Z. S. Xu. And H. Hu. Projection models for intuitionistic fuzzy multiple attribute decision making. International Journal of Information Technology & Decision Making. 9 (2) (2010), 285-297.
[15] Z. Xu. A method based on distance measure for interval-valued intuitionistic fuzzy group decision making, Information Sciences, 180(2010), 181-190.
[16] H. Wang, F. Smarandache, Y. Q. Zhang, and R. Sunderraman. Single valued neutrosophic sets. Multispace and Multistructure, 4(2010), 410–413.
[17] J. Ye. Multicriteria decision-making method using the correlation coefficient under single-valued neutrosophic environment, International Journal of General Systems, 42(4) (2013), 386-394.
[18] J. Ye. Single valued neutrosophic cross entropy for multicriteria decision making problems, Applied Mathematical Modeling (2013), doi: 10.1016/j.apm.2013.07.020.
[19] J. L. Deng. Introduction to grey system theory, The Journal of Grey System (UK), 1(1) (1989), 1–24.
[20] J. L. Deng. The primary methods of grey system theory, Huazhong University of Science and Technology Press, Wuhan (2005).
[21] J. J. Zhang, D. S. Wu, and D. L. Olson. The method of grey related analysis to multiple attribute decision making problems with interval numbers. Mathematical and Computer Modelling, 42(2005), 991–998.
[22] R. V. Rao, and D. Singh. An improved grey relational analysis as a decision making method for manufacturing situations, International Journal of Decision Science, Risk and Management, 2(2010), 1–23.
[23] G. W. Wei. Grey relational analysis method for intuitionistic fuzzy multiple attribute decision making. Expert systems with Applications, 38(2011), 11671-11677.
[24] P. Biswas, S. Pramanik, and B. C. Giri. Entropy based grey relational analysis method for multi-attribute decision-making under single valued neutrosophic assessments. Neutrosophic Sets and Systems, 2 (2014). (In Press).

[25] K.S. Park, S.H. Kim, and W.C. Yoon. Establishing strict dominance between alternatives with special type of incomplete information, European Journal of Operational Research, 96 (1996), 398-406.
[26] K.S. Park, S.H. Kim. Tools for interactive multi-attribute decision-making with incompletely identified information, European Journal of Operational Research, 98 (1997), 111-123.
[27] S.H. Kim, S.H. Choi, and J.K. Kim. An interactive procedure for multiple attribute group decision-making with incomplete information: range based approach, European Journal of Operational Research, 118 (1999), 139-152.
[28] S.H. Kim, B.S. Ahn. Interactive group decision-making procedure under incomplete information, European Journal of Operational Research, 116 (1999), 498-507.
[29] K.S. Park. Mathematical programming models for charactering dominance and potential optimality when multicriteria alternative values and weights are simultaneously incomplete, IEEE transactions on systems, man, and cybernetics-part A, Systems and Humans, 34 (2004),601-614.

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Introduction to Develop Some Software Programs for Dealing with Neutrosophic Sets

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Abstract. In this paper, we have developed an Excel package to be utilized for calculating neutrosophic data and analyze them. The use of object oriented programming techniques and concepts as they may apply to the design and development a new framework to implement neutrosophic data operations, the c# programming language, NET Framework and Microsoft Visual Studio are used to implement the neutrosophic classes. We have used Excel as it is a powerful tool that is widely accepted and used for statistical analysis. Figure 1 shows Class Diagram of the implemented package. Figure 2 presents a working example of the package interface calculating the complement. Our implemented Neutrosophic package can calculate Intersection, Union, and Complement of the nuetrosophic set. Figure 3 presents our neutrosphic package capability to draw figures of presented neutrosphic set. Figure 4 presents charting of Union operation calculation, and figure 5 Intersection Operation. nuetrosophic set are characterized by its efficiency as it takes into consideration the three data items: True, Intermediate, and False.

Keywords: Neutrosophic Data; Software Programs.

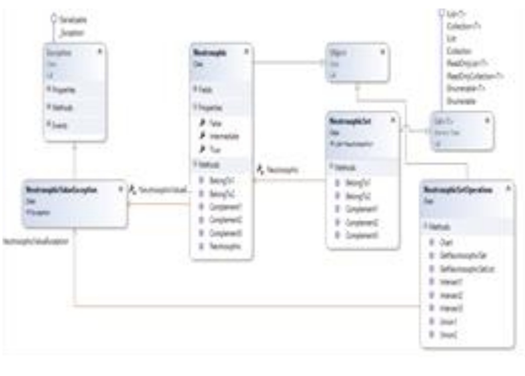
1 Introduction

The fundamental concepts of neutrosophic set, introduced by Smarandache in [8, 9] and Salama at el. in [1, 2, 3, 4, 5, 6, 7], provides a natural foundation for treating mathematically the neutrosophic phenomena which exist pervasively in our real world and for building new branches of neutrosophic mathematics. In this paper, we have developed an Excel package to be utilized for calculating neutrosophic data and analyze them. We have used Excel as it is a powerful tool that is widely accepted and used for statistical analysis. In this paper, we have developed an Excel package to be utilized for calculating neutrosophic data and analyze them. The use of object oriented programming techniques and concepts as they may apply to the design and development a new framework to implement neutrosophic data operations, the c# programming language, NET Framework and Microsoft Visual Studio are used to implement the neutrosophic classes.

2 Related Works

We recollect some relevant basic preliminaries, and in particular, the work of Smarandache in [8, 9], and Salama at el. [1, 2, 3, 4, 5, 6, 7]. The c# programming language, NET Framework and Microsoft Visual Studio are used to implement the neutrosophic classes.

3 Proposed frameworks



We introduce the neutrosophic package class diagram :

Figure 1: Neutrosophic Package Class Diagram.

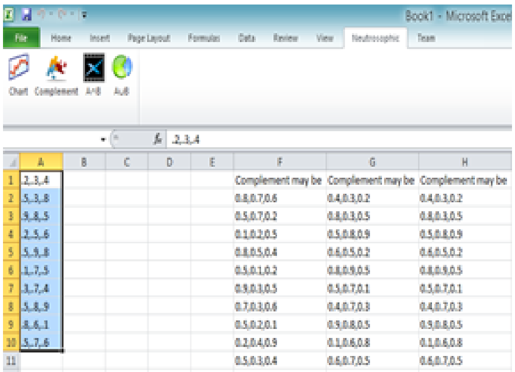


Figure 2: Neutrosophic Package Interface and Calculating Complement.

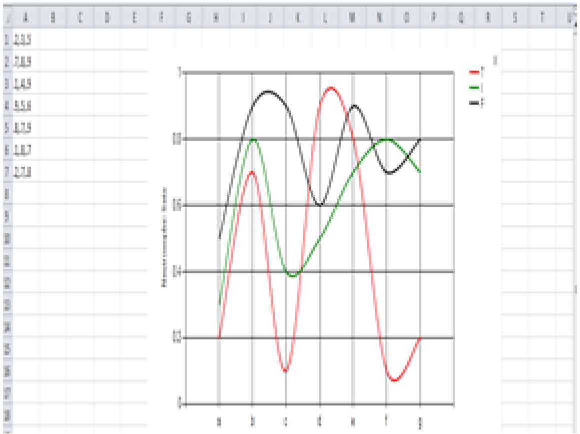


Figure 3: Neutrosophic Chart

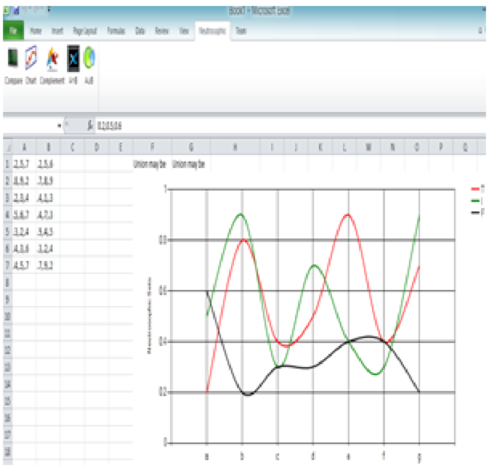
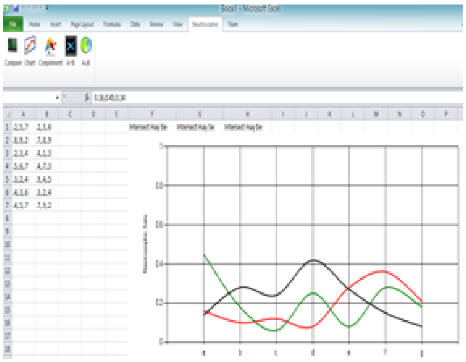


Figure 4: Neutrosophic Packege Union Chart

Figure 5: Neutrosophic Packege Intersection Chart



4 Conclusions and Future Work

In future studies we will develop some software programs to deal with the statistical analysis of the neutrosophic data.

References

[1] S. A. Alblowi, A. A. Salama & Mohmed Eisa, New Concepts of Neutrosophic Sets, International Journal of Mathematics and Computer Applications Research (IJMCAR),Vol. 3, Issue 4, Oct 2013, .(2013) pp 95-102.

[2] I. M. Hanafy, A.A. Salama and K. Mahfouz, Correlation of neutrosophic Data, International Refereed Journal of Engineering and Science (IRJES) , Vol.(1), Issue 2.(2012) PP.39-43

[3] I.M. Hanafy, A.A. Salama and K.M. Mahfouz,," Neutrosophic Classical Events and Its Probability" International Journal of Mathematics and Computer Applications Research(IJMCAR) Vol.(3),Issue 1,Mar (2013), pp171-178.

[4] A.A. Salama and S.A. Alblowi, "Generalized Neutrosophic Set and Generalized Neutrousophic Topological Spaces ", Journal computer Sci. Engineering, Vol. (2) No. (7) (2012)pp129-132.

[5] A.A.Salama and S.A.Albolwi, Neutrosophic set and neutrosophic topological space, ISORJ. Mathematics,Vol.(3), Issue(4), .(2012) pp-31-35.

[6] A.A.Salama, and H.Elagamy, "Neutrosophic Filters" International Journal of Computer Science Engineering and Information Technology Reseearch (IJCSEITR), Vol.3, Issue(1),Mar 2013, .(2013), pp 307-312

[7] Florentin Smarandache, Neutrosophy and Neutrosophic Logic , First International Conference on Neutrosophy , Neutrosophic Logic , Set, Probability, and Statistics University of New Mexico, Gallup, NM 87301, USA(2002).

[8] F.Smarandache. A Unifying Field in Logics: Neutrosophic Logic. Neutrosophy, Neutrosophic Set, Neutrosophic Probability. American Research Press, Rehoboth, NM, (1999).

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Soft Nuetrosophic Ring and Soft Neutrosophic Field

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Abstract. In this paper we extend the theory of neutrosophic rings and neutrosophic fields to soft sets and construct soft neutrosophic rings and soft neutrosophic fields. We also extend neutrosophic ideal theory to form soft neutrosophic ideal over a neutrosophic ring and soft neutrosophic ideal of a soft neutrosophic ring . We have given many examples to illustrate the theory of soft neutrosophic rings and soft neutrosophic fields and display many properties of of these. At the end of this paper we gave soft neutrosophic ring homomorphism.

Keywords: Neutrosophic ring, neutrosophic field,neutrosophic ring homomorphism, soft neutrosophic

1 Introduction

Neutrosophy is a new branch of philosophy which studies the origin and features of neutralities in the nature. Florentin Smarandache in 1980 firstly introduced the concept of neutrosophic logic where each proposition in neutrosophic logic is approximated to have the percentage of truth in a subset T, the percentage of indeterminacy in a subset I, and the percentage of falsity in a subset F so that this neutrosophic logic is called an extension of fuzzy logic. In fact neutrosophic set is the generalization of classical sets, conventional fuzzy set, intuitionistic fuzzy set and interval valued fuzzy set. This mathematical tool is used to handle problems like imprecise, indeterminacy and inconsistent data etc. By utilizing neutrosophic theory, Vasantha Kandasamy and Florentin Smarandache dig out neutrosophic algebraic structures. Some of them are neutrosophic fields, neutrosophic vector spaces, neutrosophic groups, neutrosophic bigroups, neutrosophic N-groups, neutrosophic semigroups, neutrosophic bisemigroups, neutrosophic N-semigroup, neutrosophic loops, neutrosophic biloops, neutrosophic N-loop, neutrosophic groupoids, and neutrosophic bigroupoids and so on.

Molodtsov in $[11]$ laid down the stone foundation of a richer structure called soft set theory which is free from t**he** parameterization inadequacy, syndrome of fuzzy se theory, rough set theory, probability theory and so on. In many areas it has been successfully applied such as smoothness of functions, game theory, operations research, Riemann integration, Perron integration, and probability. Recently soft set theory has attained much attention since its appearance and the work based on several operations of soft sets introduced in $[2,9,10]$. Some more exciting properties and algebra may be found in $[1]$. Feng et al. introduced the soft semirings $[5]$. By means of level soft sets an adjustable approach to fuzzy soft sets based decision making can be seen in $[6]$. Some other new concept combined with fuzzy sets and rough sets was presented in $[7,8]$. AygÄunoglu et al. introduced the Fuzzy soft groups $[4]$.

Firstly, fundamental and basic concepts are given for neutrosophic rings neutrosohic fields and soft rings. In the next section we presents the newly defined notions and results in soft neutrosophic rings and neutrosophic fields. Various types of soft neutrosophic ideals of rings are defined and elaborated with the help of examples. Furthermore, the homomorphisms of soft neutrosophic rings are discussed at the end.

2 Fundamental Concepts

Neutrosophic Rings and Neutrosophic Fields

Definition 1. Let R be any ring. The neutrosophic ring $\langle R\cup I\rangle$ is also a ring generated by R and I under the operations of R. I is called the neutrosophic element with the property $I^2 = I$. For an integer n, $n+I$ and nI are neutrosophic elements and $0.I = 0$. I^{-1}, the inverse of I is not defined and hence does not exist.

Definition 2. Let $\langle R\cup I\rangle$ be a neutrosophic ring. A proper subset P of $\langle R\cup I\rangle$ is said to be a neutrosophic subring if P itself is a neutrosophic ring under the operations of $\langle R\cup I\rangle$.

Definition 2. Let $\langle R\cup I\rangle$ be any neutrosophic ring, a non empty subset P of $\langle R\cup I\rangle$ is defined to be a neutrosophic ideal of $\langle R\cup I\rangle$ if the following conditions are satisfied;

1. P is a neutrosophic subring of $\langle R\cup I\rangle$.
2. For every $p\in P$ and $r\in\langle R\cup I\rangle$, rp and $pr\in P$.

Definition 4. Let K be a field . We call the field generated by $K\cup I$ to be the neutrosophic field for it involves the indeterminacy factor in it. We define $I^2 = I$, $I+I=2I$ i.e., $I+I+,...,+I=nI$, and if $k\in K$ then $k.I=kI, 0\mathrm{I}=0$. We denote the neutrosophic field by $K(I)$ which is generated by $K\cup I$ that is $K(I)=\langle K\cup I\rangle$. $\langle K\cup I\rangle$ denotes the field generated by K and I.

Definition 5. Let $K(I)$ be a neutrosophic field, $P\subset K(I)$ is a neutrosophic subfield of P if P itself is a neutrosophic field.

Soft Sets

Throughout this subsection U refers to an initial universe, E is a set of parameters, $P(U)$ is the power set of U, and $A\subset E$. Molodtsov [12] defined the soft set in the following manner:

Definition 6. *A pair (F,A) is called a soft set over U where F is a mapping given by* $F: A\longrightarrow P(U)$.In other words, a soft set over U is a parameterized family of subsets of the universe U. For $a\in A$, $F(a)$ may be considered as the set of a -elements of the soft set (F,A), or as the set of a -approximate elements of the soft set.

Definition 7. For two soft sets (F,A) and (H,B) over U, (F,A) is called a soft subset of (H,B) if

1) $A\subseteq B$ and
2) $F(a)\subseteq H(a)$, for all $a\in A$.

This relationship is denoted by $(F,A)\subset(H,B)$. Similarly (F,A) is called a soft superset of (H,B) if (H,B) is a soft subset of (F,A) which is denoted by $(F,A)\supset(H,B)$.

Definition 8. Two soft sets (F,A) and (H,B) over U are called soft equal if (F,A) is a soft subset of (H,B) and (H,B) is a soft subset of (F,A).

Definition 9. Let (F,A) and (K,B) be two soft sets over a common universe U such that $A\cap B\neq\phi$. Then their restricted intersection is denoted by $(F,A)\cap_R(\mathrm{K},B)=(H,C)$ where (H,C) is defined as $H(c)=F(c)\cap\mathrm{K}(c)$ for all $c\in C=A\cap B$.

Definition 10. The extended intersection of two soft sets (F,A) and (K,B) over a common universe U is the soft set (H,C), where $C=A\cup B$, and for all $c \in C$, $H \quad c$ is defined as

$$H(\mathrm{c}) = \begin{cases} F(\mathrm{c}) & \text{if } \mathrm{c} \in A - B \\ \mathrm{K}(\mathrm{c}) & \text{if } \mathrm{c} \in B - A \\ F(\mathrm{c}) \cap \mathrm{K}(\mathrm{c}) & \text{if } \mathrm{c} \in A \cap B. \end{cases}$$

We write $(F, A) \cap_{\varepsilon} (\mathrm{K}, B) = (H, C)$.

Definition 11. The restricted union of two soft sets (F, A) and (K, B) over a common universe U is the soft set (H, C) , where $C = A \cup B$, and for all $c \in C$, $H \quad c$ is defined as the soft set $(H, C) = (F, A) \cup_{R} (K, B)$ where $C = A \cap B$ and $H(\mathrm{c}) = F(c) \cup \mathrm{K}(c)$ for all $c \in C$.

Definition 12. The extended union of two soft sets (F, A) and (K, B) over a common universe U is the soft set (H, C) , where $C = A \cup B$, and for all $c \in C$, $H(c)$ is defined as

$$H(\mathrm{c}) = \begin{cases} F(\mathrm{c}) & \text{if } \mathrm{c} \in A - B \\ \mathrm{K}(\mathrm{c}) & \text{if } \mathrm{c} \in B - A \\ F(\mathrm{c}) \cup K(\mathrm{c}) & \text{if } \mathrm{c} \in A \cap B. \end{cases}$$

We write $(F, A) \cup_{\varepsilon} (\mathrm{K}, B) = (H, C)$.

Soft Rings

Definition 13. Let R be a ring and let (F, A) be a non-null soft set over R. Then (F, A) is called a soft ring over R if $F(a)$ is a subring of R , for all $a \in A$.

Definition 14. Let (F, A) and (K, B) be soft rings over R. Then (K, B) is called a soft sub ring of (F, A), If it satisfies the following;

1. $B \subset A$
2. $K(a)$ is a sub ring of $F(a)$, for all $a \in A$.

Definition 15. Let (F, A) and (K, B) be soft rings over R. Then (K, B) is called a soft ideal of F, A , If it satisfies the following;

1. $B \subset A$
2. $K(a)$ is an idela of $F(a)$, for all $a \in A$.

3 Soft Nuetrosophic Ring

Definition. Let $\langle R \cup I \rangle$ be a neutrosophic ring and (F, A) be a soft set over $\langle R \cup I \rangle$. Then (F, A) is called soft neutrosophic ring if and only if $F(a)$ is a neutrosophic subring of $\langle R \cup I \rangle$ for all $a \in A$.

Example. Let $\langle Z \cup I \rangle$ be a neutrosophic ring of integers and let (F, A) be a soft set over $\langle Z \cup I \rangle$. Let $A = \{a_1, a_2, a_3, a_4\}$ be a set of parameters. Then clearly (F, A) is a soft neutrosophic ring over $\langle Z \cup I \rangle$, where

$$F(a_1) = \langle 2Z \cup I \rangle, F(a_2) = \langle 3Z \cup I \rangle$$

$$F(a_3) = \langle 5Z \cup I \rangle, F(a_4) = \langle 6Z \cup I \rangle.$$

Theorem . Let (F, A) and (H, A) be two soft neutrosophic rings over $\langle R \cup I \rangle$. Then their intersection $(F, A) \cap (H, A)$ is again a soft neutrosophic ring over $\langle R \cup I \rangle$.

Proof. The proof is straightforward.

Theorem. Let (F, A) and (H, B) be two soft neutrosophic rings over $\langle R \cup I \rangle$. If $A \cap B = \phi$, then $(F, A) \cup (H, B)$ is a soft neutrosophic ring over $\langle R \cup I \rangle$.

Proof. This is straightforward.

Remark. The extended union of two soft neutrosophic rings (F, A) and (K, B) over $\langle R \cup I \rangle$ is not a soft neutrosophic ring over $\langle R \cup I \rangle$.

We check this by the help of following Example.

Example. Let $\langle Z \cup I \rangle$ be a neutrosophic ring of integers. Let (F, A) and (K, B) be two soft neutrosophic rings over $\langle Z \cup I \rangle$, where

$$F(a_1) = \langle 2Z \cup I \rangle, F(a_2) = \langle 3Z \cup I \rangle, F(a_3) = \langle 4Z \cup I \rangle,$$

And

$$K(a_1) = \langle 5Z \cup I \rangle, K(a_3) = \langle 7Z \cup I \rangle.$$

Their extended union

$(F, A) \cup_E (K, B) = (H, C)$, where

$$H(a_1) = \langle 2Z \cup I \rangle \cup \langle 5Z \cup I \rangle,$$

$$H(a_2) = \langle 3Z \cup I \rangle,$$

$$H(a_3) = \langle 5Z \cup I \rangle \cup \langle 7Z \cup I \rangle.$$

Thus clearly $H(a_1) = \langle 2Z \cup I \rangle \cup \langle 5Z \cup I \rangle$, $H(a_3) = \langle 5Z \cup I \rangle \cup \langle 7Z \cup I \rangle$ is not a neutrosophic rings.

Remark. The restricted union of two soft neutrosophic rings (F, A) and (K, B) over $\langle R \cup I \rangle$ is not a soft neutrosophic ring over $\langle R \cup I \rangle$.

Theorem. The OR operation of two soft neutrosophic rings over $\langle R \cup I \rangle$ may not be a soft neutrosophic ring over $\langle R \cup I \rangle$.

One can easily check these remarks with the help of Examples.

Theorem. The extended intersection of two soft neutrosophic rings over $\langle R \cup I \rangle$ is soft neutrosophic ring over $\langle R \cup I \rangle$.

Proof. The proof is straightforward.

Theorem. The restricted intersection of two soft neutrosophic rings over $\langle R \cup I \rangle$ is soft neutrosophic ring over $\langle R \cup I \rangle$.

Proof. It is obvious.

Theorem. The AND operation of two soft neutrosophic rings over $\langle R \cup I \rangle$ is soft neutrosophic ring over $\langle R \cup I \rangle$.

Proof. Easy.

Definition. Let (F, A) be a soft set over a neutrosophic ring $\langle R \cup I \rangle$. Then (F, A) is called an absolute soft neutrosophic ring if $F(a) = \langle R \cup I \rangle$ for all $a \in A$.

Definition. Let (F, A) be a soft set over a neutrosophic ring $\langle R \cup I \rangle$. Then (F, A) is called soft neutrosophic ideal over $\langle R \cup I \rangle$ if and only if $F(a)$ is a neutrosophic ideal over $\langle R \cup I \rangle$.

Example. Let $\langle Z_{12} \cup I \rangle$ be a neutrosophic ring. Let $A = \{a_1, a_2\}$ be a set of parameters and (F, A) be a soft set over $\langle Z_{12} \cup I \rangle$. Then clearly (F, A) is a soft neutrosophic ideal over $\langle R \cup I \rangle$, where

$$F(a_1) = \{0, 6, 2I, 4I, 6I, 8I, 10I, 6 + 2I, ..., 6 + 10I\},$$

$$F(a_2) = \{0, 6, 6I, 6 + 6I\}.$$

Theorem. Every soft neutrosophic ideal (F, A) over a neutrosophic ring $\langle R \cup I \rangle$ is trivially a soft neutrosophic ring.

Proof. Let (F, A) be a soft neutrosophic ideal over a neutrosophic ring $\langle R \cup I \rangle$. Then by definition $F(a)$ is a neutrosophic ideal for all $a \in A$. Since we know that every neutrosophic ideal is a neutrosophic subring. It follows that $F(a)$ is a neutrosophic subring of $\langle R \cup I \rangle$. Thus by definition of soft neutrosophic ring, this implies that (F, A) is a soft neutrosophic ring.

Remark. The converse of the above theorem is not true.

To check the converse, we take the following Example.

Example. Let $\langle Z_{10} \cup I \rangle$ be a neutrosophic ring. Let $A = \{a_1, a_2\}$ be a set of parameters and (F, A) be a soft neutrosophic ring over $\langle Z_{10} \cup I \rangle$, where

$$F(a_1) = \{0, 2, 4, 6, 8, 2I, 4I, 6I, 8I\},$$

$$\mathrm{F}(a_2) = \{0, 2I, 4I, 6I, 8I\}.$$

Then obviously (F, A) is not a soft neutrosophic ideal over $\langle Z_{10} \cup I \rangle$.

Proposition. Let (F, A) and (K, B) be two soft neutosophic ideals over a neutrosophic ring $\langle R \cup I \rangle$. Then

1. Their extended union $(F, A) \cup_E (K, B)$ is again a soft neutrosophic ideal over $\langle R \cup I \rangle$.
2. Their extended intersection $(F, A) \cap_E (K, B)$ is again a soft neutrosophic ideal over $\langle R \cup I \rangle$.
3. Their restricted union $(F, A) \cup_R (K, B)$ is again a soft neutrosophic ideal over $\langle R \cup I \rangle$.
4. Their restricted intersection $(F, A) \cap_R (K, B)$ is again a soft neutrosophic ideal over $\langle R \cup I \rangle$.
5. Their OR operation $(F, A) \vee (K, B)$ is again a soft neutrosophic ideal over $\langle R \cup I \rangle$.
6. Their AND operation $(F, A) \vee (K, B)$ is again a soft neutrosophic ideal over $\langle R \cup I \rangle$.

Proof. Supoose (F, A) and (K, B) be two soft neutrosophic ideals over $\langle R \cup I \rangle$. Let $C = A \cup B$. Then for all $c \in C$, The extended union is $(F, A) \cup_E (K, B) = (H, C)$, where

$$H(\mathrm{c}) = \begin{cases} F(c), \text{If c} \in A - B, \\ K(c), \text{If c} \in B - A, \\ F(c) \cup K(c), \text{If c} \in A \cap B. \end{cases}$$

As union of two neutrosophic ideals is again a neutrosophic ideal of $\langle R \cup I \rangle$. Hence the extended union H, C is a soft neutrosophic ideal over $\langle R \cup I \rangle$.

Similarly $(2), (3), (4), (5),$ and (6) can be proved respectively.

Definition. Let (F, A) and (K, B) be two soft neutrosophic rings over $\langle R \cup I \rangle$. Then (K, B) is called soft neutrosophic subring of (F, A), if

1. $B \subseteq A$, and
2. $K(a)$ is a neutrosophic subring of $F(a)$ for all $a \in A$.

Example. Let $\langle C \cup I \rangle$ be the neutrosophic ring of complex numbers. Let $A = \{a_1, a_2, a_3\}$ be a set of parameters. Then (F, A) be a soft neutrosophic ring over $\langle C \cup I \rangle$, where

$$F(a_1) = \langle Z \cup I \rangle, F(a_2) = \langle Q \cup I \rangle,$$

$$F(a_3) = \langle R \cup I \rangle.$$

Where $\langle Z \cup I \rangle, \langle Q \cup I \rangle$ and $\langle R \cup I \rangle$ are neutrosophic rings of integers, rational numbers, and real numbers respectively.

Let $B = \{a_2, a_3\}$ be a set of parmeters . Let (K, B) be the neutrosophic subring of (F, A) over $\langle C \cup I \rangle$, where

$$\mathrm{K}(a_2) = \langle Z \cup I \rangle, \mathrm{K}(a_3) = \langle Q \cup I \rangle.$$

Theorem. Every soft ring (H, B) over a ring R is a soft neutrosophic subring of a soft neutrosophic ring (F, A) over the corresponding neutrosophic ring $\langle R \cup I \rangle$ if $B \subseteq A$.

Proof. Straightforward.

Definition. Let (F, A) and (K, B) be two soft neutrosophic rings over $\langle R \cup I \rangle$. Then (K, B) is called soft neutrosophic ideal of (F, A), if

1. $B \subseteq A$, and

2. $K(a)$ is a neutrosophic ideal of $F(a)$ for all $a \in A$.

Example. Let $\langle Z_{12} \cup I \rangle$ be a neutrosophic ring. Let $A = \{a_1, a_2\}$ be a set of parameters and (F, A) be a soft set over $\langle Z_{12} \cup I \rangle$. Then clearly (F, A) is a soft neutrosophic ring over $\langle Z_{12} \cup I \rangle$,where

$F(a_1) = \{0, 6, 2I, 4I, 6I, 8I, 10I, 6 + 2I, ..., 6 + 10I\}$,

$F(a_2) = \{0, 2, 4, 6, 8, 2\mathrm{I}, 4\mathrm{I}, 6\mathrm{I}, 8\mathrm{I}\}$.

Let $B = \{a_1, a_2\}$ be a set of parameters. Then clearly (H, B) is a soft neutrosophic ideal of (F, A) over $\langle Z_{12} \cup I \rangle$, where

$$H(a_1) = \{0, 6, 6 + 6I\},$$

$$H(a_2) = \{0, 2, 4, 6, 8\}.$$

Proposition. All soft neutrosophic ideals are trivially soft neutrosophic subrings.

Proof. Straightforward.

4 Soft Neutrosophic Field

Defintion. Let $K(I) = \langle K \cup I \rangle$ be a neutrosophic field and let (F, A) be a soft set over $K(I)$. Then (F, A) is said to be soft neutrosophic field if and only if $F(a)$ is a neutrosophic subfield of $K(I)$ for all $a \in A$.

Example. Let $\langle C \cup I \rangle$ be a neutrosophic field of complex numbers. Let $A = \{a_1, a_2\}$ be a set of parameters and let (F, A) be a soft set of $\langle C \cup I \rangle$. Then (F, A) is called soft neutrosophic field over $\langle C \cup I \rangle$, where

$$F(a_1) = \langle R \cup I \rangle, F(a_2) = \langle Q \cup I \rangle.$$

Where $\langle R \cup I \rangle$ and $\langle Q \cup I \rangle$ are the neutosophic fields of real numbers and rational numbers.

Proposition. Every soft neutrosophic field is trivially a soft neutrosophic ring.

Proof. The proof is trivial.

Remark. The converse of above proposition is not true.

To see the converse, lets take a look to the following example.

Example. Let $\langle Z \cup I \rangle$ be a neutrosophic ring of integers. Let $A = \{a_1, a_2, a_3, a_4\}$ be a set of parameters and let (F, A) be a soft set over $\langle Z \cup I \rangle$. Then (F, A) is a soft neutrosophic ring over $\langle Z \cup I \rangle$, where

$$F(a_1) = \langle 2Z \cup I \rangle, F(a_2) = \langle 3Z \cup I \rangle$$

$$F(a_3) = \langle 5Z \cup I \rangle, F(a_4) = \langle 6Z \cup I \rangle.$$

Clearly (F, A) is not a soft neutrosophic field.

Definition. Let (F, A) be a soft neutrosophic field over a neutrosophic field $\langle K \cup I \rangle$. Then (F, A) is called an absolute soft neutrosophic field if $F(a) = \langle K \cup I \rangle$, for all $a \in A$.

5 Soft Neutrosophic Ring Homomorphism

Definition. Let (F, A) and (K, B) be the soft neutrosophic rings over $\langle R \cup I \rangle$ and $\langle R' \cup I \rangle$ respectively. Let $f : \langle R \cup I \rangle \to \langle R' \cup I \rangle$ and $g : A \to B$ be mappings. Let $(f, g) : (\mathrm{F}, \mathrm{A}) \to (\mathrm{K}, \mathrm{B})$ be another mapping. Then (f, g) is called a soft neutrosophic ring homomorphism if the following conditions are hold.

1. f is a neutrosophic ring homomorphism from $\left\langle R \cup I \right\rangle$ to $\left\langle R^{'} \cup I \right\rangle$.
2. g is onto mapping from A to B, and
3. $f(F(a)) = K(g(a))$ for all $a \in A$.

If f is an isomorphicm and g is a bijective mapping. Then (f, g) is called soft neutrosophic ring isomorphism.

Conclusions

In this paper we extend the neutrosophic ring, neutrosophic field and neutrosophic subring to soft neutrosophic ring, soft neutrosophic field and soft neutrosophic subring respectively. The neutrosophic ideal of a ring is extended to soft neutrosophic ideal. We showed all these by giving various examples in order to illustrate the soft part of the neutrosophic notions used.

References

1. Aktas and N. Cagman, Soft sets and soft group, Inf. Sci 177(2007) 2726-2735.
2. M.I. Ali, F. Feng, X.Y. Liu, W.K. Min and M. Shabir, On some new operationsin soft set theory, Comput. Math. Appl. (2008) 2621-2628.
3. D. Singh, Onyeozili, I.A, On the Ring Structure of Soft Set Theory, Int. J of Scientific and Technology Research. 2(3) (2013).
4. M.B. Gorzalzany, A method of inference in approximate reasoning based on interval-valuedfuzzy sets, Fuzzy Sets and Systems 21 (1987) 1-17.
5. P. K. Maji, Neutrosophic Soft Set, Annals of Fuzzy Mathematics and Informatics, 5(1)(2013), 2093-9310.
6. P.K. Maji, R. Biswas and R. Roy, Soft set theory, Comput. Math. Appl. 45(2003) 555-562.
7. P.K. Maji, A.R. Roy and R. Biswas, An application of soft sets in a decision making problem, Comput. Math. Appl. 44(2002) 1007-1083.
8. D. Molodtsov, Soft set theory first results, Comput. Math. Appl. 37(1999) 19-31.
9. Z. Pawlak, Rough sets, Int. J. Inf. Compu. Sci. 11(1982) 341-356.
10. Florentin Smarandache,A Unifying Field in Logics. Neutrosophy: Neutrosophic Probability, Set and Logic. Rehoboth: American Research Press, (1999).
11. W. B. Vasantha Kandasamy & Florentin Smarandache, Some Neutrosophic Algebraic Structures and Neutrosophic N-Algebraic Structures, 219 p., Hexis, 2006.
12. W. B. Vasantha Kandasamy & Florentin Smarandache, N-Algebraic Structures and S-N-Algebraic Structures, 209 pp., Hexis, Phoenix, 2006.
13. W. B. Vasantha Kandasamy & Florentin Smarandache, Basic Neutrosophic Algebraic Structures and their Applications to Fuzzy and Neutrosophic Models, Hexis, 149 pp., 2004.
14. A. Sezgin & A.O. Atagun, Soft Groups and Normalistic Soft Groups, Comput. Math. Appl. 62(2011) 685-698.
15. L.A. Zadeh, Fuzzy sets, Inform. Control 8(1965) 338 -353.

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Rough Neutrosophic Sets

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Abstract. Both neutrosophic sets theory and rough sets theory are emerging as powerful tool for managing uncertainty, indeterminate, incomplete and imprecise information .In this paper we develop an hybrid structure called " rough neutrosophic sets" and studied their properties.

Keywords: Rough set, rough neutrosophic set.

1 Introduction

In 1982, Pawlak [1] introduced the concept of rough set (RS), as a formal tool for modeling and processing incomplete information in information systems. There are two basic elements in rough set theory, crisp set and equivalence relation ,which constitute the mathematical basis of RSs. The basic idea of rough set is based upon the approximation of sets by a pair of sets known as the lower approximation and the upper approximation of a set .Here, the lower and upper approximation operators are based on equivalence relation. After Pawlak, there has been many models built upon different aspect, i.e, universe, relations, object, operators by many scholars [2, 3,4,5,6,7]. Various notions that combine rough sets and fuzzy sets, vague set and intuitionistic fuzzy sets are introduced ,such as rough fuzzy sets, fuzzy rough sets, generalize fuzzy rough, intuitionistic fuzzy rough sets, rough intuitionistic fuzzy sets, rough vagues sets. The theory of rough sets is based upon the classification mechanism, from which the classification can be viewed as an equivalence relation and knowledge blocks induced by it be a partition on universe.

One of the interesting generalizations of the theory of fuzzy sets and intuitionistic fuzzy sets is the theory of neutrosophic sets introduced by F.Smarandache [8,9]. Neutrosophic sets described by three functions : a membership function indeterminacy function and a non-membership function that are independently related. The theorie of neutrosophic set have achieved great success in various areas such as medical diagnosis [10], database [11,12], topology[13],image processing [14,15,16], and decision making problem[17]. While the neutrosophic set is a powerful tool to deal with indeterminate and inconsistent data, the theory of rough sets is a powerful mathematical tool to deal with incompleteness.

Neutrosophic sets and rough sets are two different topics ,none conflicts the other. Recently many researchers applied the notion of neutrosophic sets to relations, group theory, ring theory,Soft set theory [23,24,25,26,27,28,29,30,31,32] and so on.The main objective of this study was to introduce a new hybrid intelligent structure called " rough neutrosophic sets". The significance of introducing hybrid set structures is that the computational techniques based on any one of theses structures alone will not always yield the best results but a fusion of two or more of them can often give better results.

The rest of this paper is organized as follows. Some preliminary concepts required in our work are briefly recalled in section 2. In section 3 , the concept of rough neutrosophic sets is investigated. Section 4 concludes the paper.

2 .Preliminaries

In this section we present some preliminaries which will be useful to our work in the next section . For more details the reader may refer to [1, 8,9,]

Definition 2.1[8]. Let X be an universe of discourse, with a generic element in X denoted by x, the neutrosophic (NS) set is an object having the form

A = {< x: $\mu_A(x)$, $\nu_A(x)$, $\omega_A(x)$>,x $\in$X}, where the functions μ, ν, ω : X→ $]^-0, 1^+[$ define respectively the degree of membership (or Truth) , the degree of indeterminacy, and the degree of non-membership (or Falsehood) of the element x $\in$X to the set A with the condition.

$$^-0 \le \mu_A(x) + \nu_A(x) + \omega_A(x) \le 3^+. \qquad (1)$$

From philosophical point of view, the neutrosophic set takes the value from real standard or non-standard subsets of $]^-0, 1^+[$. So instead of $]-0, 1^+[$ we need to take the interval [0, 1] for technical applications, because $]^-0, 1^+[$ will be difficult to apply in the real applications such as in scientific and engineering problems. For two NS ,

A ={ <x , $\mu_A(x)$, $\nu_A(x)$, $\omega_A(x)$> | x $\in$ X }

And B={<x , $\mu_B(x)$, $\nu_B(x)$, $\omega_B(x)$> | x $\in$ X } the relations are defined as follows:

i. $A \subseteq B$ if and only if $\mu_A(x) \le \mu_B(x)$,$\nu_A(x) \ge \nu_B(x)$,$\omega_A(x) \ge \omega_B(x)$
ii. $A = B$ if and only if , $\mu_A(x) = \mu_B(x)$,$\nu_A(x) = \nu_B(x)$ and $\omega_A(x) = \omega_B(x)$
iii. $A \cap B$={ <x ,min($\mu_A(x)$, $\mu_B(x)$), max($\nu_A(x)$, $\nu_B(x)$),max($\omega_A(x)$, $\omega_B(x)$)> | x $\in$ X }
iv. $A \cup B$={ <x ,max($\mu_A(x)$, $\mu_B(x)$), min($\nu_A(x)$, $\nu_B(x)$),min($\omega_A(x)$, $\omega_B(x)$)> | x $\in$ X }
v. A^C=={ <x , $\omega_A(x)$,1- $\nu_A(x)$, $\mu_A(x)$> | x $\in$ X }
vi. 0_n =(0 , 1, 1) and 1_n =(1 , 0, 0)

As an illustration ,let us consider the following example.

Example 2.2 Assume that the universe of discourse U={x1,x2,x3},where x_1 characterizes the capability, x_2 characterizes the trustworthiness and x_3 indicates the prices of the objects. It may be further assumed that the values of x_1, x_2 and x_3 are in [0,1] and they are obtained from some questionnaires of some experts. The experts may impose their opinion in three components viz. the degree of goodness, the degree of indeterminacy and that of poorness to explain the characteristics of the objects. Suppose A is a neutrosophic set (NS) of U, such that,

A = {< x_1, (0.3, 0.5 ,0.6) >,< x_2, (0.3, 0.2 ,0.3)>,< x_3, (0.3, 0.5 ,0.6)>}, where the degree of goodness of capability is 0.3, degree of indeterminacy of capability is 0.5 and degree of falsity of capability is 0.6 etc.

Definition 2.3 [1]

Let U be any non-empty set . Suppose R is an equivalence relation over U. For any non-null subset X of U, the sets A_1(x)={x: $[x]_R \subseteq$ X} and A_2(x)={x: $[x]_R \cap X \neq \phi$}

are called the lower approximation and upper approximation, respectively of X, where the pair S = (U,R) is called an approximation space. This equivalent relation R is called indiscernibility relation. The pair A (X)= (A_1(x), A_2(x)) is called the rough set of X in S. Here $[x]_R$ denotes the equivalence class of R containing x.

Definition 2.4 [1]. Let A = (A_1, A_2) and B = (B_1, B_2) be two rough sets in the approximation space S = (U, R) . Then,

A ∪ B = ($A_1 \cup B_1$, $A_2 \cup B_2$),

A ∩ B = ($A_1 \cap B_1$, $A_2 \cap B_2$),

A ⊂ B if A ∩B = A ,

~A = { U – A_2 , U – A_1}.

3 Rough Neutrosophic Sets

In this section we introduce the notion of rough neutrosophic sets by combining both rough sets and nuetrosophic sets. and some operations viz. union, intersection , inclusion and equalities over them. Rough neutrosophic set are the generalization of rough fuzzy sets [2] and rough intuitionistic fuzzy sets [22].

Definition 3.1. Let U be a non-null set and R be an equivalence relation on U. Let F be neutrosophic set in U with the membership function μ_F ,indeterminacy function ν_F and non-membership function ω_F . The lower and the upper approximations of F in the approximation (U ,R) denoted by $\underline{N}$ (F) and $\overline{N}$(F) are respectively defined as follows:

$\underline{N}$(F)= { <x, $\mu_{\underline{N}(F)}$ (x) , $\nu_{\underline{N}(F)}$ (x) , $\omega_{\underline{N}(F)}$ (x)> |

$y \in [x]_R$,x∈ U},

$\overline{N}$(F) ={ <x, $\mu_{\overline{N}(F)}$ (x) , $\nu_{\overline{N}(F)}$ (x) , $\omega_{\overline{N}(F)}$ (x)>

| $y \in [x]_R$, x∈ U}, where:

$\mu_{\underline{N}(F)}(x)=\bigwedge_{y \in [x]_R} \mu_F(y)$

, $\nu_{\underline{N}(F)}(x)=\bigvee_{y \in [x]_R} \nu_F(y)$, $\omega_{\underline{N}(F)} =\bigvee_{y \in [x]_R} \omega_F(y)$,

$\mu_{\overline{N}(F)}(x)=\bigvee_{y \in [x]_R} \mu_F(y)$, $\nu_{\overline{N}(F)}(x)=\bigwedge_{y \in [x]_R} \nu_F(y)$

, $\omega_{\overline{N}(F)} =\bigwedge_{y \in [x]_R} \omega_F(y)$,

So $0 \leq \mu_{\overline{N}(F)}(x) + \nu_{\overline{N}(F)}(x) + \omega_{\overline{N}(F)}(x) \leq 3$

And $0 \leq \mu_{\underline{N}(F)}(x) + \nu_{\underline{N}(F)}(x) + \omega_{\underline{N}(F)}(x) \leq 3$

Where " ∨ " and " ∧ " mean "max" and "min " operators respectively, $\mu_F(y), \nu_F(y)$ and $\omega_F(y)$ are the membership ,indeterminacy and non-membership of y with respect to F. It is easy to see that $\overline{N}$(F) and $\underline{N}$(F) are two neutrosophic sets in U ,thus NS mapping

$\overline{N}$, $\underline{N}$:N(U) → N(U) are, respectively, referred to as the upper and lower rough NS approximation operators, and the pair ($\underline{N}$(F), $\overline{N}$(F)) is called the rough neutrosophic set in (U, R).

From the above definition , we can see that $\underline{N}$(F) and $\overline{N}$(F) have constant membership on the equivalence clases of U . if $\underline{N}$(F) =$\overline{N}$(F) ;i .e $\mu_{\underline{N}(F)} =\mu_{\overline{N}(F)}$

, $\nu_{\underline{N}(F)} =\nu_{\overline{N}(F)}$ and $\omega_{\underline{N}(F)} = \omega_{\overline{N}(F)}$. For any x∈ U. we call F a definable neutrosophic set in the approximation (U ,R). it is easily to be proved that Zero 0_N neutrosophic set and unite neutrosophic sets 1_N are definable neutrosophic sets. Let us consider a simple example in the following

Example 3.2 . Let U = $\{p_1, p_2, p_3, p_4, p_5, p_6, p_7, p_8\}$ be the universe of discourse. Let R be an equivalence relation its partition of U is given by

U/R =$\{\{p_1, p_4\}, \{p_2, p_3, p_6\}, \{p_5\}, \{p_7, p_8\}\}$.

Let N(F) ={(p_1, (0.2, 0.3, 0.4), (p_4, (0.3, 0.5, 0.4)), (p_5, (0.4, 0.6, 0.2)),(p_7, (0.1, 0.3, 0.5)) be a neutrosophic set of U .By the definition 3.1, we obtain:

$\underline{N}$(F) ={(p_1, (0.2, 0.5, 0.4)), (p_4, (0.2, 0.5, 0.4)), (p_5, (0.4, 0.6, 0.2))};

$\overline{N}$(F) ={(p_1, (0.2, 0.3, 0.4)), (p_4, (0.2, 0.3, 0.4)), (p_5, (0.4, 0.6, 0.2)),(p_7, (0.1, 0.3, 0.5)), (p_8, (0.1, 0.3, 0.5))}:

For another neutrosophic sets

N(G) ={(p_1, (0.2, 0.3, 0.4), (p_4, (0.2, 0.3, 0.4)), (p_5, (0.4, 0.6, 0.2)}

The lower approximation and upper approximation of N(G) are calculated as

$\underline{N}$(G) ={(p_1, (0.2, 0.3, 0.4)), (p_4, (0.2, 0.3, 0.4)), (p_5, (0.4, 0.6, 0.2))};

$\overline{N}$(G) ={(p_1, (0.2, 0.3, 0.4)), (p_4, (0.2, 0.3, 0.4)), (p_5, (0.4, 0.6, 0.2))

Obviously $\underline{N}$(G) = $\overline{N}$(G) is a definable neutrosophic set in the approximation space (U, R)

Definition 3.3. If N(F)= ($\underline{N}$(F), $\overline{N}$(F) is a rough neutrosophic set in (U,R) ,the rough complement of N(F) is the rough neutrosophic set denoted ~ N(F)=($\underline{N}(F)^c$, $\overline{N}(F)^c$) ,where $\underline{N}(F)^c$, $\overline{N}(F)^c$ are the complements of neutrosophic sets $\underline{N}$(F) and $\overline{N}$(F) respectively.

$\underline{N}(F)^c$ ={ <x, $\omega_{\underline{N}(F)}$,1- $\nu_{\underline{N}(F)}$ (x) , $\mu_{\underline{N}(F)}$ (x) > | x∈ U},And

$\overline{N}(F) = \{ <x, \omega_{\overline{N}(F)}(x), 1-\nu_{\overline{N}(F)}(x), \mu_{\overline{N}(F)}(x)> \mid x \in U\}$,

Definition 3.4. If $N(F_1)$ and $N(F_2)$ are two rough neutrosophic set of the neutrosophic sets F_1 and F_2 respectively in U ,then we define the following:

i. $N(F_1) = N(F_2)$ iff $\underline{N}(F_1) = \underline{N}(F_2)$ and $\overline{N}(F_1) = \overline{N}(F_2)$

ii. $N(F_1) \subseteq N(F_2)$ iff $\underline{N}(F_1) \subseteq \underline{N}(F_2)$ and $\overline{N}(F_1) \subseteq \overline{N}(F_2)$

iii. $N(F_1) \cup N(F_2) = <\underline{N}(F_1) \cup \underline{N}(F_2), \overline{N}(F_1) \cup \overline{N}(F_2)>$

iv. $N(F_1) \cap N(F_2) = <\underline{N}(F_1) \cap \underline{N}(F_2), \overline{N}(F_1) \cap \overline{N}(F_2)>$

v. $N(F_1) + N(F_2) = <\underline{N}(F_1) + \underline{N}(F_2), \overline{N}(F_1) + \overline{N}(F_2)>$

vi. $N(F_1) . N(F_2) = <\underline{N}(F_1) . (F_2), \overline{N}(F_1) . \overline{N}(F_2)>$

If N , M, L are rough neutrosophic set in (U,R) ,then the results in the following proposition are straightforward from definitions

Proposition 3.5:

i. $\sim N(\sim N) = N$

ii. $N \cup M = M \cup N$, $N \cap M = M \cap N$

iii. $(N \cup M) \cup L = N \cup (M \cup L)$ and $(N \cap M) \cap L = N \cap (M \cap L)$

iv. $(N \cup M) \cap L = (N \cup M) \cap (N \cup L)$ and $(N \cap M) \cup L = (N \cap M) \cup (N \cap L)$

De Morgan ‘s Laws are satisfied for neutrosophic sets :

Proposition 3.6

i. $\sim (N(F_1) \cup N(F_2)) = (\sim N(F_1)) \cap (\sim N(F_2))$

ii. $\sim (N(F_1) \cap N(F_2)) = (\sim N(F_1)) \cup (\sim N(F_2))$

Proof :

i. $(N(F_1) \cup N(F_2))$

$= \sim (\{\underline{N}(F_1) \cup \underline{N}(F_2)\}, \{\overline{N}(F_1) \cup \overline{N}(F_2)\})$

$= (\sim\{\underline{N}(F_1) \cup \underline{N}(F_2)\}\}, \sim \{\{\overline{N}(F_1) \cup \overline{N}(F_2)\})$

$= (\{\underline{N}(F_1) \cup \underline{N}(F_2)\}^c, \{\overline{N}(F_1) \cup \overline{N}(F_2)\}^c)$

$= (\sim \{\underline{N}(F_1) \cap \underline{N}(F_2)\}, \sim \{\overline{N}(F_1) \cap \overline{N}(F_2)\})$

$= \sim N(F_1)) \cap (\sim N(F_2))$

ii. Similar to the proof of (i)

Proposition 3.7:

If F_1 and F_2 are two neutrosophic sets in U such that $F_1 \subseteq F_2$, then $N(F_1) \subseteq N(F_2)$

i. $N(F_1 \cup F_2) \supseteq N(F_1) \cup N(F_2)$

ii. $N(F_1 \cap F_2) \subseteq N(F_1) \cap N(F_2)$

Proof:

$\mu_{\underline{N}(F_1 \cup F_2)}(x) = \inf\{ \mu_{(F_1 \cup F_2)}(x) \mid x \in X_i\}$

$= \inf(\max\{\mu_{F_1}(x), \mu_{F_2}(x) \mid x \in X_i)$

$\geq \max\{\inf\{\mu_{F_1}(x) \mid x \in X_i\}, \inf\{\mu_{F_2}(x) \mid x \in X_i\}\}$

$= \max\{\mu_{\underline{N}(F_1)}(x_i), \mu_{\underline{N}(F_2)}(x_i)\}$

$= (\mu_{\underline{N}(F_1)} \cup \mu_{\underline{N}(F_2)})(x_i)$

Similarly ,

$\nu_{\underline{N}(F_1 \cup F_2)}(x_i) \leq (\nu_{\underline{N}(F_1)} \cup \nu_{\underline{N}(F_2)})(x_i)$

$\omega_{\underline{N}(F_1 \cup F_2)}(x_i) \leq (\omega_{\underline{N}(F_1)} \cup \omega_{\underline{N}(F_2)})(x_i)$

Thus, $\underline{N}(F_1 \cup F_2) \supseteq \underline{N}(F_1) \cup \underline{N}(F_2)$

We can also see that

$\overline{N}(F_1 \cup F_2) = \overline{N}(F_1) \cup \overline{N}(F_2)$

Hence,

$N(F_1 \cup F_2) \supseteq N(F_1) \cup N(F_2)$

(ii) proof is similar to the proof of (i)

Proposition 3.8:

i. $\underline{N}(F) = \sim \overline{N}(\sim F)$
ii. $\overline{N}(F) = \sim \underline{N}(\sim F)$
iii. $\underline{N}(F) \subseteq \overline{N}(F)$

Proof , according to definition 3.1, we can obtain

i) $F = \{ <x, \mu_F(x), \nu_F(x), \omega_F(x)> \mid x \in X \}$
$\sim F = \{ <x, \omega_F(x), 1- \nu_F(x), \mu_F(x)> \mid x \in X \}$

$\overline{N}(\sim F) = \{ <x, \omega_{\overline{N}(\sim F)}(x), 1- \nu_{\overline{N}(\sim F)}(x), \mu_{\overline{N}(\sim F)}(x)> \mid y \in [x]_R, x \in U\}$

$\sim \overline{N}(\sim F) = \{ <x, \mu_{\overline{N}(\sim F)}(x), 1-(1- \nu_{\overline{N}(\sim F)}(x)), \omega_{\overline{N}(\sim F)}(x)> \mid y \in [x]_R, x \in U\}$

$= \{ <x, \mu_{\overline{N}(\sim F)}(x), \nu_{\overline{N}(\sim F)}(x), \omega_{\overline{N}(\sim F)}(x)>$

$\mid y \in [x]_R, x \in U\}$, where

$\mu_{\overline{N}(\sim F)} = \bigwedge_{y \in [x]_R} \mu_F(y)$, $\nu_{\overline{N}(\sim F)}(x) = \bigvee_{y \in [x]_R} \nu_F(y)$ and $\omega_{\overline{N}(\sim F)}(x) = \bigvee_{y \in [x]_R} \omega_F(y)$

Hence $\underline{N}(F) = \sim \overline{N}(\sim F)$

ii) The proof is similar to i
iii) For any $y \in \underline{N}(F)$, we can have

$\mu_{\underline{N}(F)}(x) = \bigwedge_{y \in [x]_R} \mu_F(y) \leq \bigvee_{y \in [x]_R} \mu_F(y)$

$\nu_{\underline{N}(F)}(x) = \bigvee_{y \in [x]_R} \nu_F(y) \geq \bigwedge_{y \in [x]_R} \nu_F(y)$

And $\omega_{\underline{N}(F)} = \bigvee_{y \in [x]_R} \omega_F(y) \geq \bigwedge_{y \in [x]_R} \omega_F(y)$

Hence, $\underline{N}(F) \subseteq \overline{N}(F)$

4 Conclusions

In this paper we have defined the notion of rough neutrosophic sets. We have also studied some properties on them and proved some propositions. The concept combines two different theories which are rough sets theory and neutrosophic theory. While neutrosophic set theory is mainly concerned with ,indeterminate and inconsistent information, rough set theory is with incompleteness; but both the theories deal with imprecision. Consequently , by the way they are defined, it is clear that rough neutrosophic sets can be utilized for dealing with both of indeterminacy and incompleteness.

References

[1] Z. Pawlak , Rough Sets , Int. J. Comput. Inform. Sci. 11 (1982),341-356.

[2] D. Dubios and H. Prade, “Rough fuzzy sets and fuzzy rough sets,” Int. J. Gen. Syst., vol. 17, pp. 191-208, 1990

[3] Z. Gong, B. Sun, and D. Chen, “Rough set theory for the interval-valued fuzzy information systems,” Inf. Sci., vol. 178, pp. 1968-1985, 2008.

[4] B. Sun, Z. Gong, and D. Chen, “Fuzzy rough set theory for the interval-valued fuzzy information systems,” Inf. Sci., vol. 178, pp. 2794-2815, 2008.

[5] W.-Z. Wu, J.-S. Mi, and W.-X. Zhang, “Generalized Fuzzy Rough Sets,” Inf. Sci., vol. 151, pp. 263-282, 2003.

[6] Z. Zhang, “On interval type-2 rough fuzzy sets,” Knowledge-Based Syst., vol. 35, pp. 1-13, 2012.

[7] J.-S. Mi, Y. Leung, H.-Y. Zhao, and T. Feng, “Generalized Fuzzy Rough Sets determined by a triangular norm,” Inf. Sci., vol. 178, pp. 3203-3213, 2008.

[8] F.Smarandache, “A Unifying Field in Logics. Neutrosophy: Neutrosophic Probability, Set and Logic”. Rehoboth: American Research Press.1999.

[9] F. Smarandache, “ Linguistic Paradoxists and Tautologies”, Libertas Mathematica, University of Texas at Arlington, Vol. XIX (1999),pp.143-154.

[10] Ansari, Biswas, Aggarwal, ” Proposal for Applicability of Neutrosophic Set Theory in Medical AI”,

International Journal of Computer Applications (0975 – 8887),Vo 27– No.5, (2011), pp.5-11.

[11] M. Arora, R. Biswas, U.S.Pandy, "Neutrosophic Relational Database Decomposition", International Journal of Advanced Computer Science and Applications, Vol. 2, No. 8, (2011), pp.121-125.

[12] M. Arora and R. Biswas, " Deployment of Neutrosophic technology to retrieve answers for queries posed in natural language", in 3rd International Conference on Computer Science and Information Technology ICCSIT, IEEE catalog Number CFP1057E-art, Vol. No. 3, (2010), pp.435-439.

[13] F.G. Lupiáñez "On neutrosophic topology", Kybernetes, Vol. 37 Iss: 6, (2008), pp.797 - 800,Doi:10.1108/03684920810876990.

[14] H. D. Cheng,& Y Guo. "A new neutrosophic approach to image thresholding". New Mathematics and Natural Computation, 4(3), (2008), pp. 291–308.

[15] Y.Guo & H. D. Cheng "New neutrosophic approach to image segmentation". Pattern Recognition, 42, (2009), pp.587–595.

[16] M.Zhang, L.Zhang, and H.D.Cheng. "A neutrosophic approach to image segmentation based on watershed method". Signal Processing 5, 90, (2010), pp.1510-1517.

[17] A. Kharal, "A Neutrosophic Multicriteria Decision Making Method", New Mathematics & Natural Computation, to appear in Nov 2013.

[20] A. Nakamura, Fuzzy rough sets, Note on Multiple-Valued Logic in Japan 9,1988,p. 1–8.

[21] S. Nanda, S. Majumdar, Fuzzy rough sets, Fuzzy Sets and Systems 45,1992, p. 157–160.

[22] K. V. Thomas, L. S. Nair, Rough intutionistic fuzzy sets in a lattice, Int. Math. Forum 6(27) ,(2011) 1327–1335

[23] S.Broumi and F. Smarandache, "Intuitionistic Neutrosophic Soft Set", Journal of Information and Computing Science, England, UK ,ISSN 1746-7659,Vol. 8, No. 2, (2013) 130-140.

[24] S.Broumi, "Generalized Neutrosophic Soft Set", International Journal of Computer Science, Engineering and Information Technology (IJCSEIT), ISSN: 2231-3605, E-ISSN : 2231-3117, Vol.3, No.2, (2013) 17-30

[25] S.Broumi and F.Smarandache," More on Intuitionistic Neutrosophic Soft Sets", Computer Science and Information Technology 1(4): 257-268, 2013 ;DOI: 10.13189/csit.2013.010404.

[26] S. Broumi,I. Deli and F.Smarandache," Relations on Interval Valued Neutrosophic Soft Sets",Journal of New Results in Science,ISSN:1304-798,;2014,accepted.

[27] S.Broumi, F.Smarandache, P. K. Maj, Intuitionistic Neutrosphic Soft Set over Rings, Mathematics and Statistics 2(3): 120-126, 2014, DOI: 10.13189/ms.2014.020303

[28] Said Broumi, and Florentin Smarandache, Several Similarity Measures of Neutrosophic Sets" ,Neutrosophic Sets and Systems,VOL1 ,2013,54-62.

[29] S. Broumi, F. Smarandache , "Correlation Coefficient of Interval Neutrosophic set", Periodical of Applied Mechanics and Materials, Vol. 436, 2013, with the title Engineering Decisions and Scientific Research in Aerospace, Robotics, Biomechanics, Mechanical Engineering and Manufacturing; Proceedings of the International Conference ICMERA, Bucharest, October 2013.

[30] I. Deli, S.Broumi," Neutrosophic soft sets and neutrosophic soft matrices based on decision making",2014,(submitted)

[31] S. Broumi,F.Smarandache," More on Intuitionistic Neutrosophic Soft Sets", Computer Science and Information Technology 1(4): 257-268, 2013 ;DOI: 10.13189/csit.2013.010404.

[32] Deli, I.Interval-valued neutrosophic soft sets and its decision making http://arxiv.org/abs/1402.3130.

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NSS

New Type of Fuzzy Relational Equations and Neutrosophic Relational Equations – To analyse Customers Preference to Street Shops

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Abstract. In this paper authors study the customer's preference of street shops to other eateries using Fuzzy Relation Equations (FREs) and Neutrosophic Relation Equations (NREs). We have constructed a new type of FRE and NRE called the new average FRE and new average NRE. This study is based on interviews /discussions taken from 32 tuck shops in and around Tambaram. This paper is organized into five sections. In section one we just recall the working of FRE and NRE. We define the new notion of average FREs and average NREs and use this new model to study the problem which forms section two of this paper. Section three describes the attributes related with the customers and the types of customers based on the pilot survey made by us. The new FRE and NRE models constructed in section two of this paper is used in analysing the problem in section four. The final section gives the conclusions and suggestions made from this study.

Keywords: Fuzzy Relation Equation(FRE), Neutrosophic Relation Equation(NRE), New Average Fuzzy Relation Equation (NAFRE), New Average Neutrosophic Relation Equation(NANRE).

1 Basic Concepts

Here we just recall the basic definitions and describe the functioning of Fuzzy Relation Equations (FRE) and Neutrosophic Relation Equations (NRE). We have taken the basic definitions from [1,6]. The notion of Fuzzy Relation Equations (FREs) is associated with the concept of composition of binary relations. The FREs are based upon the max-min composition. Considering the three binary relations P(X,Y), Q(Y,Z) and R(X,Z) which are defined on the sets, $X = \{x_i \mid i \in I\}$, $Y = \{y_j \mid j \in J\}$ and $Z = \{z_k \mid k \in K\}$ where we assume that $I = N_n$, $J = N_m$ and $K = N_s$. Let the membership matrices of P, Q and R be denoted by $P = [p_{ij}]$, $Q = [q_{jk}]$ and $R = [r_{ik}]$ respectively where $p_{ij} = P(x_i, y_i)$, $q_{jk} = Q(y_j, z_k)$ and $r_{ik} = R(x_i, z_k)$ for all $i \in I(= N_n), j \in J(= N_m)$ and $k \in K(= N_s)$ .This means all the entries in the matrices P, Q and R are real numbers in the unit interval [0, 1].

Assume now that the three relations constrain each other in such a way that

$$P \circ Q = R \quad (1)$$

where '$\circ$' denotes the max-min composition. This means that

$$\max_{j \in J} \min(p_{ij,} q_{jk}) = r_{ik} \quad (2)$$

for all $i \in I$ and $k \in K$. That is, the matrix equation (1) encompasses $n \times s$ simultaneous equations of the form (2). When the two of the components in each of the equations are given and one is unknown, these equations are referred to as FREs.

It is pertinent to mention that in general the equation P o Q = R need not give a solution. In case when we do not have a solution to equation (1) we use neural networks to find the solution [1,3-6].

We just recall the definition of Neutrosophic Relation Equations (NREs). To this new notion we need the concept of the indeterminate I, where $I^2 = I$ and $I + I + \ldots + I = nI$, for more about these neutrosophic concept please refer [2, 5]. We denote by $N_I = \{a + bI \;/\; a, b \in [0,1]\}$ and N_I is defined as Fuzzy neutrosophic values.

To construct Neutrosophic Relation Equations we make use of N_I clearly $[0,1] \subseteq N_I$; this is the case when b = 0. The Neutrosophic Relation Equations are based upon the max-min composition. Considering the three binary relations N(X,Y) Q(Y,Z) and B(X,Z) which are defined on the sets, $X = \{x_i \mid i \in I\}, Y = \{y_j \mid j \in J\}$ and $Z = \{z_k \mid k \in K\}$ where we assume that $I = N_n, J = N_m$ and $K = N_s$. Let the membership matrices of N, Q and B be denoted by $N = [n_{ij}]$, $Q = [q_{jk}]$ and $B = [b_{jk}]$ respectively where $n_{ij} = N(x_i, y_j)$, $q_{jk} = Q(y_j, z_k)$ and $b_{ik} = B(x_i, z_k)$ for all $i \in I(= N_n)$, $j \in J(= N_m)$ and $k \in K(= N_s)$. This means all the entries in the neutrosophic matrices N, Q and B are fuzzy neutrosophic values from N_I .

Assume now that the three relations constrain each other in such the way that

$$N \circ Q = B \quad (3)$$

where '$\circ$' denotes the max-min composition. This means that

$$\max_{j \in J} \min(p_{ij}, q_{jk}) = r_{ik} \qquad (4)$$

for all i ∈ I and k ∈ K. That is, the matrix equation (3) encompasses n × s simultaneous equations of the form (4). However if an expert wishes to work in a different way he/she can choose min{a, bI} = a even if a < b or min{a, bI} = b even if a > b. This flexibility alone makes the system more agile for any researcher. For more refer[1-6].

2 New Average Fuzzy Relation Equations (NAFRE) and New Average Neutrosophic Relation Equations (NANRE)

The main motivation for construction of these new models Average Fuzzy Relation Equations and Average Neutrosophic Relation Equations arises from following factors. These models functions on the wishes of all the experts who work with the problem. If for any problem we use more than one experts opinion we may have problem of choosing the experts opinion for preference of one expert over the other may not give satisfaction to the other experts as they may feel their suggestions are ignored and this may lead to an unpleasant situation and bias in the choice.

To overcome this problem we have defined two new models called the New Average Fuzzy Relation Equations (NAFRE) and New Average Neutrosophic Relation Equations (NANRE). Here we define and describe the New Average Fuzzy Relation Equations model (NAFRE) and New Average Neutrosophic Relation Equations model (NANRE).

Suppose $P_1(X,Y)$, $P_2(X,Y),...,P_n(X,Y)$ be the Fuzzy Relation of X on Y given by n-distinct experts, where all the n-experts agree to work with the same set of attributes from the range and domain spaces.

Let $R_1, R_2, ..., R_n$ denote the related matrices of the FRE of the n-experts associated with $P_1(X, Y)$, $P_2(X, Y)$, ..., $P_n(X, Y)$ the fuzzy relation of X on Y respectively.

We define

$$P(X,Y) = \frac{P_1(X,Y) + ... + P_n(X,Y)}{n}$$

that in terms of the Fuzzy Relation Equations, that is if R is the matrix related with P(X, Y) then R is got from

$$\frac{1}{n}\sum_{i=1}^{n} P_i$$

using FRE based on max-min composition is again a matrix which gives the fuzzy relation of X with Y.

The merit of using this model is that every expert is given the same preference so the experts have no disappointment when forming the final result and further this saves time and economy for we can work with one model instead of n-models. The advantages of using the average FRE model is as follows. Just like other fuzzy models the extreme values do not cancel out as the values of all the FRE matrices R related with the respective $P_i(X,Y)$ has its entries in [0, 1]; $1 \le i \le n$. Hence at the outset we are justified in using this specially constructed New Average Fuzzy Relation Equations(NAFREs) model. This model also caters to the law of large numbers. So the results become more and more sensitive by increasing the number of experts and further only a single matrix represents the opinion of all these n-experts. Hence time and economy are not affected by using this new model.

Next we proceed to define the New Average NRE model. Suppose $N_1(X,Y)$, $N_2(X,Y),...,N_n(X,Y)$ be the neutrosophic relation of X on Y given by n-distinct experts, where all the n-experts agree to work with the same set of attributes from the range and domain spaces.

Let $B_1, B_2, ..., B_n$ denote the related matrices of the NREs of the n-experts associated with $N_1(X,Y), N_2(X,Y)$, …, $N_n(X,Y)$ the neutrosophic relation of X on Y respectively. We define

$$N(X,Y) = \frac{N_1(X,Y) + ... + N_n(X,Y)}{n}$$

that in terms of the Neutrosophic Relation Equations, that is if B is the matrix related with N(X, Y) then B is got from $\frac{1}{n}\sum_{i=1}^{n} N_i$ using NRE based on max-min composition is again a matrix which gives the neutrosophic relation of X with Y. There is no dependency between the average taken for real and indeterminacy; since as per the experts who have deterministic opinion the average of their opinion is taken separately and the experts who have indeterminacy opinion is dealt with separately. However we prefer to use NRE models mainly as certain experts express their inability to give opinion had forced us to deploy neutrosophic models.

In NRE models the extreme values do not cancel out as the values of all the NRE matrices B related with the respective $N_i(X,Y)$ has neutrosophic values {a + bI | a, b ∈ [0,1]}. Hence at the outset we are justified in using this specially constructed New Average Neutrosophic Relation Equations (NANREs) model. This model also caters to the law of large numbers. So the results become more and more sensitive by increasing the number of experts and further only a single matrix represents the opinion of all these n-experts. Hence time and economy are not affected by using this new model.

3. Description of the attributes related with the preference of the customers to road side eateries

In this section we keep on record that we have taken a pilot survey from different types of customers and for their preferences to these road side eateries from 32 number of customers. After analysing the collected data the experts felt the following attributes can be given preference in the study of the problem. Accordingly X denotes the attributes related with the preferences of the customers which is taken as the 'domain' space of the Fuzzy Relation Equations. Y correspond to attributes related with the types of the customers.

We briefly describe in a line (or) two the attributes of X and Y in this section.

Let $S = \{S_1, S_2, ..., S_7\}$ denote the domain space.

S_1: "Cost" – The cost is reasonably fair because the street shop owners do not charge VAT, no tips for the servers and they do not charge for even hygienic water.

S_2: "Quality is good" – The view of the experts (customers) felt that the phrase "Quality is good" means that the food they get from the street shops is less in adulteration with chemical for taste and smell. They also claimed that the food is just like home made food so they prefer the tuck shops to that of big restaurant or multicusine hotels.

S_3: "Quantity is more" – The quantity is more in comparison since for the same amount we spend on street shops, we get more and substantial amount of food which is really fulfilling the customers.

S_4: "Better Hygiene" – Since the food are instantly made we do not get left out foods. They keep the surroundings clean because they are always watched keenly by all the customers and public. They give us clean can water and they serve the food in paper plates and cups which is used only once. Added to this even sometimes they serve in fresh green banana leaves.

S_5: "Service is good" - Most of the street shop owners are themselves servers. So they take care of each customer. They are friendlier. They serve the food immediately and the customers need not wait.

S_6: "Prepared in our presence" – Since the food is prepared in our presence we can give instruction to prepare for our taste. Food are just made so hot and hygienic.

S_7: "Waiting Time" – Comparatively since the owners are themselves servers they give importance to each customer and they serve the food very quickly. The customers need not wait for long time in a long queue which very often happens in big multicusine restaurant even to pay bills and for parcelling the food and for every service many hours are wasted.

The attributes related with the types of customers R = $\{C_1, C_2, ..., C_7\}$ is taken as range space. We briefly describe in a word or two the attributes $C_1, C_2, ..., C_7$;

C_1: Bachelors :Most of the bachelors take food from road side shops because of so many factors like the quantity of food is large for what they pay.

C_2: Students: Both day scholars and hostellers like to have food due to the less price they charge.

C_3:I.T and Call centre Employees: These type of customers give importance for hygiene food and less waiting time.

C_4: House Wives; These type of customers give much importance to better hygiene and for more quantity.

C_5: Daily Wage Labours: These labour in tambaram used to go road side eateries for various reason like more quantity of food they get for what they pay, less cost which is affordable by them and for good hospitality.

C_6: Local Employees: These type of customers mainly prefer these shops for better hygiene and for better service.

C_7: Children above 10-Years: Children prefer for some special food which is not always prepared in their home and for less cost charged for the food.

The collected data was analysed and the following limit sets are derived using the questionnaire.

$S_1 \geq 0.6$ (The cost is reasonably fair so we are forced to give just 60%, $S_1 < 0.6$ means the cost in not reasonably fair to their expectation).

$S_2 \geq 0.5$ (The quality is preferred by those who have experience in eating quality food so we are forced to give just 50% $S_2 < 0.5$ means the quality is not as good to their expectation).

$S_3 \geq 0.6$ (Several like school students, daily wage people, etc, prefer quantity with so the expert feel after pilot survey $S_3 < 0.6$ is not as good as to their expectation).

$S_4 \geq 0.5$ (Most of the customers like I.T employees, house wives, etc, prefer better hygiene $S_4 < 0.5$ means the better hygiene is not as good as to their expectation).

$S_5 \geq 0.6$ (The service is preferred by those who have experienced the better service when compared with the multi cuisine hotels. So we are forced to give 60% $S_5 < 0.6$ means the service is not good as to their expectation).

$S_6 \geq 0.4$ (The expert feels that they prefer the food which is prepared in their presence so we are forced to give 40% $S_6 < 0.4$ means they do not give much importance for the food which is prepared in their presence).

$S_7 \geq 0.6$ (The waiting time is much important and they have less waiting time compared to multiCuisine hotels $S_7 < 0.6$ means the waiting time is comparatively more).

In the next section we analyse the collected data using FRE and NRE.

4 Use of FRE and NRE models to analyse the problem

Here we have collected the data from 32 tuck shops. We have used five experts to work with FRE and NRE model. The FRE matrices of 5 experts P_1, P_2, P_3, P_4 and P_5 are given as follows. Now we work with first expert. Let P_1 be the membership matrix given by the first expert which is as follows:

$$P_1 = \begin{array}{c} S_1 \\ S_2 \\ S_3 \\ S_4 \\ S_5 \\ S_6 \\ S_7 \end{array} \begin{bmatrix} 0 & 0.6 & 0 & 0.7 & 0.8 & 0.6 & 0.8 \\ 0.3 & 0.1 & 0.6 & 0.6 & 0 & 0.5 & 0.7 \\ 0.6 & 0.8 & 0 & 0.7 & 0.7 & 0.5 & 0.6 \\ 0 & 0 & 0.7 & 0.6 & 0 & 0 & 0 \\ 0 & 0 & 0.6 & 0.3 & 0 & 0.2 & 0 \\ 0.1 & 0 & 0 & 0.2 & 0 & 0.1 & 0 \\ 0.2 & 0 & 0.7 & 0.6 & 0 & 0.5 & 0.7 \end{bmatrix}$$

and the expert wishes to work with this

$$Q_1^t = [0.7\ \ 0.5\ \ 0.7\ \ 0.4\ \ 0.8\ \ 0.6\ \ 0.3].$$

Now we find the solution to the following fuzzy relation equations. That is $P_1 \circ Q_1^t = R_1^t = \text{MaxMin}(p_{ij}q_{jk})$; which gives

$$R_1^t = [0.8\ \ 0.6\ \ 0.7\ \ 0.7\ \ 0.6\ \ 0.2\ \ 0.7].$$

As analysed from resultant R_1^t, the first expert feels that least preference for the food prepared in their presence and the much preference is given for all the remaining constrains.

Suppose the expert wishes to work with

$$Q_{11}^t = [0.9\ \ 0.8\ \ 0.9\ \ 0.8\ \ 0.7\ \ 0.7\ \ 0.6].$$

Then we find solution for the fuzzy relation equation as

$$P_1 \circ Q_{11}^t = \text{MaxMin}(p_{ij}, q_{jk}) = R_{11}^t$$

which gives $R_{11}^t = [0.7\ \ 0.6\ \ 0.8\ \ 0.7\ \ 0.6\ \ 0.2\ \ 0.7]$.

As analysed from resultant R_{11}^t the expert feels the least preference is given for the food prepared in their presence and much importance is given for all the remaining constrains.

Suppose the expert wishes to work with

$$Q_{12}^t = [0.4\ \ 0.2\ \ 0.3\ \ 0.4\ \ 0.5\ \ 0.4\ \ 0.3]$$

then we find solution for the fuzzy relation equation as

$$P_1 \circ Q_{12}^t = \text{MaxMin}(p_{ij}, q_{jk}) = R_{12}^t$$

which gives $R_{12}^t = [0.5\ \ 0.4\ \ 0.5\ \ 0.4\ \ 0.3\ \ 0.2\ \ 0.4]$.

As analysed from resultant R_{12}^t the expert feels least preference is given for all the constrains.

Next we work with second expert. Let P_2 be the membership matrix given by the second expert which is as follows

$$P_2 = \begin{array}{c} S_1 \\ S_2 \\ S_3 \\ S_4 \\ S_5 \\ S_6 \\ S_7 \end{array} \begin{bmatrix} 0.1 & 0.7 & 0.1 & 0.8 & 0.9 & 0.7 & 0.8 \\ 0.4 & 0.2 & 0.7 & 0.7 & 0.1 & 0.4 & 0.6 \\ 0.7 & 0.8 & 0.1 & 0.7 & 0.1 & 0.6 & 0.6 \\ 0.2 & 0.1 & 0.8 & 0.8 & 0 & 0.1 & 0 \\ 0.5 & 0 & 0.6 & 0.4 & 0 & 0.2 & 0.3 \\ 0.1 & 0.1 & 0.2 & 0.3 & 0.1 & 0.1 & 0.1 \\ 0.3 & 0 & 0.8 & 0.4 & 0 & 0.4 & 0.8 \end{bmatrix}.$$

Now using

$$Q_1^t = [0.7\ \ 0.5\ \ 0.7\ \ 0.4\ \ 0.8\ \ 0.6\ \ 0.3]$$

in the equation $P_2 \circ Q_1^t = \text{MaxMin}(p_{ij}, q_{jk}) = R_2^t$ we get

$$R_2^t = [0.8\ \ 0.7\ \ 0.7\ \ 0.7\ \ 0.6\ \ 0.3\ \ 0.7].$$

As analysed from R_2^t the expert feels least preference is given for the food prepared in their presence and much preference is given for all the remaining constrains.

Now using

$$Q_{11}^t = [0.9\ \ 0.8\ \ 0.9\ \ 0.8\ \ 0.7\ \ 0.7\ \ 0.6]$$

in the equation $P_2 \circ Q_{11}^t = \text{MaxMin}(p_{ij}, q_{jk}) = R_{21}^t$ we get

$$R_{21}^t = [0.8\ \ 0.7\ \ 0.8\ \ 0.8\ \ 0.6\ \ 0.3\ \ 0.8].$$

As analysed from R_{21}^t the expert feels least preference is given for the food prepared in their presence and much preferences is given for all the remaining constrains.

Now using

$$Q_{12}^t = [0.4\ \ 0.2\ \ 0.3\ \ 0.4\ \ 0.5\ \ 0.4\ \ 0.3]$$

in the equation $P_2 \circ Q_{12}^t = \text{MaxMin}(p_{ij}, q_{jk}) = R_{22}^t$ we get

$$R^t{}_{22} = [0.5\ \ 0.4\ \ 0.4\ \ 0.4\ \ 0.4\ \ 0.3\ \ 0.4].$$

As analysed from resultant R_{22}^t the expert feels much preference is given for the food prepared in their presence and gives least preference for all the other constrains.

Next we work with third expert. Let P_3 be the membership matrix given by third expert which is as follows

$$P_3 = \begin{array}{c} S_1 \\ S_2 \\ S_3 \\ S_4 \\ S_5 \\ S_6 \\ S_7 \end{array} \begin{bmatrix} 0.1 & 0.6 & 0 & 0.6 & 0.8 & 0.7 & 0.8 \\ 0.2 & 0.1 & 0.5 & 0.6 & 0.1 & 0.5 & 0.7 \\ 0.6 & 0.9 & 0.2 & 0.8 & 0.7 & 0.8 & 0.7 \\ 0.4 & 0.5 & 0.7 & 0.6 & 0.1 & 0.2 & 0.1 \\ 0.6 & 0.5 & 0.6 & 0.5 & 0.1 & 0.2 & 0.5 \\ 0.5 & 0.4 & 0.1 & 0.4 & 0.1 & 0.2 & 0.4 \\ 0.2 & 0.6 & 0.4 & 0.6 & 0.1 & 0.6 & 0.7 \end{bmatrix}.$$

Now using

$$Q_1^t = [0.7\ \ 0.5\ \ 0.7\ \ 0.4\ \ 0.8\ \ 0.6\ \ 0.3]$$

in the equation $P_3 \circ Q_1^t = \text{MaxMin}(p_{ij}, q_{jk}) = R_3^t$ we get

$$R_3^t = [0.8\ \ 0.5\ \ 0.7\ \ 0.7\ \ 0.6\ \ 0.5\ \ 0.6].$$

As analysed from resultant R_3^t the expert feels much preference is given for all the constrains.

Now using

$$Q_{11}^t = [0.9\ \ 0.8\ \ 0.9\ \ 0.8\ \ 0.7\ \ 0.7\ \ 0.6]$$

in the equation $P_3 \circ Q_{11}^t = \text{MaxMin}(p_{ij}, q_{jk}) = R_{31}^t$ we get

$$R_{31}^t = [0.7\ \ 0.6\ \ 0.8\ \ 0.7\ \ 0.6\ \ 0.5\ \ 0.6].$$

As analysed from resultant R_{31}^t the expert feels much preference is given for all the remaining constrains.

Now using

$$Q_{12}^t = [0.4\ \ 0.2\ \ 0.3\ \ 0.4\ \ 0.5\ \ 0.4\ \ 0.3]$$

in the equation $P_3 \circ Q_{12}^t = \text{MaxMin}(p_{ij}, q_{jk}) = R_{32}^t$ we get

$R^t_{32} = [0.5 \quad 0.4 \quad 0.4 \quad 0.4 \quad 0.4 \quad 0.4 \quad 0.4]$.

As analysed from resultant R^t_{32} the expert feels much preference is given for the food prepared in their presence and given least importance for all the remaining constrains. Next we work with fourth expert. Let P_4 be the membership matrix given by fourth expert which is as follows

$$P_4 = \begin{array}{c} S_1 \\ S_2 \\ S_3 \\ S_4 \\ S_5 \\ S_6 \\ S_7 \end{array} \begin{bmatrix} 0.1 & 0.7 & 0 & 0.7 & 0.8 & 0.8 & 0.2 \\ 0.5 & 0.5 & 0.6 & 0.7 & 0.4 & 0.6 & 0.7 \\ 0.7 & 0.7 & 0.5 & 0.4 & 0.7 & 0.6 & 0.5 \\ 0.6 & 0.5 & 0.5 & 0.6 & 0.1 & 0.2 & 0.4 \\ 0.6 & 0.4 & 0.6 & 0.7 & 0.4 & 0.3 & 0.5 \\ 0.4 & 0.2 & 0.4 & 0.5 & 0.2 & 0.1 & 0 \\ 0.2 & 0.1 & 0.7 & 0.5 & 0 & 0.5 & 0.5 \end{bmatrix}.$$

Now using

$$Q^t_1 = [0.7 \ 0.5 \ 0.7 \ 0.4 \ 0.8 \ 0.6 \ 0.3]$$

in the equation $P_4 \circ Q^t_1 = \text{MaxMin}(p_{ij}, q_{jk}) = R^t_4$ we get

$$R^t_4 = [0.8 \quad 0.6 \quad 0.7 \quad 0.6 \quad 0.6 \quad 0.4 \quad 0.7].$$

As analysed from resultant R^t_4 the expert feels much preference is given for all the constrains .

Now using

$$Q^t_{11} = [0.9 \quad 0.8 \quad 0.9 \quad 0.8 \quad 0.7 \quad 0.7 \quad 0.6]$$

in the equation $P_4 \circ Q^t_{11} = \text{MaxMin}(p_{ij}, q_{jk}) = R^t_{41}$ we get

$$R^t_{41} = [0.7 \quad 0.7 \quad 0.7 \quad 0.6 \quad 0.7 \quad 0.5 \quad 0.7].$$

As analysed from resultant R^t_{41} the expert feels much preferences is given for all the constrains.

Now using

$$Q^t_{12} = [0.4 \quad 0.2 \quad 0.3 \quad 0.4 \quad 0.5 \quad 0.4 \quad 0.3]$$

in the equation $P_4 \circ Q^t_{12} = \text{MaxMin}(p_{ij}, q_{jk}) = R^t_{42}$ we get

$$R^t_{42} = [0.5 \quad 0.4 \quad 0.5 \quad 0.4 \quad 0.4 \quad 0.4 \quad 0.4].$$

As analysed from resultant R^t_{42} the expert feels much preference is given for the food prepared in their presence and least preference is given for all the remaining constrains.

Next we work with fifth expert. Let P_5 be the membership matrix given by fifth expert which is as follows

$$P_5 = \begin{array}{c} S_1 \\ S_2 \\ S_3 \\ S_4 \\ S_5 \\ S_6 \\ S_7 \end{array} \begin{bmatrix} 0 & 0.6 & 0 & 0.6 & 0.7 & 0.7 & 0.8 \\ 0.3 & 0.1 & 0.5 & 0.5 & 0 & 0.5 & 0.7 \\ 0.5 & 0.7 & 0.1 & 0.6 & 0.6 & 0.5 & 0.6 \\ 0.1 & 0 & 0.7 & 0.6 & 0.1 & 0.1 & 0.1 \\ 0.1 & 0 & 0.6 & 0.3 & 0 & 0.2 & 0.1 \\ 0.1 & 0.1 & 0.1 & 0.2 & 0.1 & 0.1 & 0.2 \\ 0.3 & 0.1 & 0.7 & 0.6 & 0.1 & 0.5 & 0.6 \end{bmatrix}.$$

Now using

$$Q^t_1 = [0.7 \ 0.5 \ 0.7 \ 0.4 \ 0.8 \ 0.6 \ 0.3]$$

in the equation $P_5 \circ Q^t_1 = \text{MaxMin}(p_{ij}, q_{jk}) = R^t_5$ we get

$$R^t_5 = [0.7 \quad 0.5 \quad 0.6 \quad 0.7 \quad 0.6 \quad 0.2 \quad 0.7].$$

As analysed from resultant R^t_5 the expert feels least preference is given for the food prepared in their presence and much preference is given for all the remaining constrains.

Now using

$$Q^t_{11} = [0.9 \quad 0.8 \quad 0.9 \quad 0.8 \quad 0.7 \quad 0.7 \quad 0.6]$$

in the equation $P_5 \circ Q^t_{11} = \text{MaxMin}(p_{ij}, q_{jk}) = R^t_{51}$ we get

$$R^t_{51} = [0.7 \quad 0.6 \quad 0.7 \quad 0.7 \quad 0.6 \quad 0.2 \quad 0.7].$$

As analysed from resultant R^t_{51} the expert feels least preference is given for the food prepared in their presence and much preference is given for all the remaining constrains.

Now using

$$Q^t_{12} = [0.4 \quad 0.2 \quad 0.3 \quad 0.4 \quad 0.5 \quad 0.4 \quad 0.3]$$

in the equation $P_5 \circ Q^t_{12} = \text{MaxMin}(p_{ij}, q_{jk}) = R^t_{52}$ we get

$$R^t_{52} = [0.5 \quad 0.4 \quad 0.5 \quad 0.4 \quad 0.3 \quad 0.2 \quad 0.4].$$

As analysed from resultant R^t_{52} the expert feels least preference is given for all the constrains.

The New Average Fuzzy Relation Equations(NAFREs) defined and developed in section 2 of the paper is constructed using five experts which gives the opinion of all the 5 experts feeling. As a law of large number the average taken for all the five experts give approximately a sensitive opinion.

$$(P_1 + P_2 + P_3 + P_4 + P_5)/5 =$$

$$P = \begin{array}{c} S_1 \\ S_2 \\ S_3 \\ S_4 \\ S_5 \\ S_6 \\ S_7 \end{array} \begin{bmatrix} 0.06 & 0.64 & 0.02 & 0.68 & 0.8 & 0.7 & 0.68 \\ 0.34 & 0.2 & 0.58 & 0.62 & 0.12 & 0.5 & 0.68 \\ 0.62 & 0.78 & 0.18 & 0.64 & 0.56 & 0.6 & 0.6 \\ 0.26 & 0.22 & 0.68 & 0.64 & 0.06 & 0.12 & 0.12 \\ 0.36 & 0.18 & 0.6 & 0.44 & 0.1 & 0.22 & 0.28 \\ 0.24 & 0.16 & 0.16 & 0.32 & 0.1 & 0.12 & 0.14 \\ 0.2 & 0.08 & 0.66 & 0.54 & 0.06 & 0.5 & 0.66 \end{bmatrix}.$$

Now using

$$Q^t_1 = [0.7 \ 0.5 \ 0.7 \ 0.4 \ 0.8 \ 0.6 \ 0.3]$$

in the equation $P \circ Q^t_1 = \text{MaxMin}(p_{ij}, q_{jk}) = R^t$ we get

$$R^t = [0.8 \quad 0.58 \quad 0.62 \quad 0.68 \quad 0.6 \quad 0.32 \quad 0.66]$$

As analysed from resultant the R^t, expert feels least preference for the food which is being prepared in their presence and much preference is given for all the remaining constrains.

Now using

$$Q_{11}^t = [0.9 \quad 0.8 \quad 0.9 \quad 0.8 \quad 0.7 \quad 0.7 \quad 0.6]$$

in the equation $P \circ Q_{11}^t = \text{MaxMin}(p_{ij}, q_{jk}) = R_a^t$ we get

$$R_a^t = [0.8 \quad 0.67 \quad 0.78 \quad 0.68 \quad 0.6 \quad 0.32 \quad 0.66].$$

As analysed from resultant R_a^t, expert feels least preference for the food which is being prepared in their presence and much preference for all the remaining constrains.

Now using

$$Q_{12}^t = [0.4 \quad 0.2 \quad 0.3 \quad 0.4 \quad 0.5 \quad 0.4 \quad 0.3]$$

in the equation $P \circ Q_{11}^t = \text{MaxMin}(p_{ij}, q_{jk}) = R_b^t$ we get

$$R_b^t = [0.5 \quad 0.4 \quad 0.5 \quad 0.4 \quad 0.4 \quad 0.32 \quad 0.4].$$

As analysed from resultant R_b^t expert feels least preferences is given for all the constrains.

Next we consider the opinion of 5 experts who wish to use the NREs to the same problem. Now we work with first expert.

Let N_1 be the membership matrix given by first expert

$$N_1 = \begin{array}{c} S_1 \\ S_2 \\ S_3 \\ S_4 \\ S_5 \\ S_6 \\ S_7 \end{array}\begin{bmatrix} 0.2I & 0.6 & 0.3I & 0.7 & 0.8 & 0.6 & 0.8 \\ 0.3 & 0.7I & 0.6 & 0.6 & 0.3I & 0.5 & 0.7 \\ 0.6 & 0.8 & 0.7I & 0.7 & 0.7 & 0.5 & 0.7 \\ 0.5I & 0.2 & 0.7 & 0.6 & 0.3 & 0.2 & 0.7 \\ 0.7 & 0.4I & 0.6 & 0.3 & 0.2I & 0.7 & 0.5 \\ 0.1 & 0.5 & 0.3 & 0.2 & 0.7 & 0.1 & 0.5I \\ 0.2 & 0 & 0.7 & 0.6 & 0 & 0.5 & 0.7 \end{bmatrix}$$

and the expert wishes to work with

$$Q_1^t = [0.7 \ 0.5 \ 0.7 \ 0.4 \ 0.8 \ 0.6 \ 0.3].$$

Now we find solution for the following neutrosophic equations. That is $N_1 \circ Q_1^t = \text{MaxMin}(n_{ij} q_{jk}) = B_1^t$; which gives $B_1{}^t = [0.8 \quad 0.6 \quad 0.7I \quad 0.7 \quad 0.7 \quad 0.7 \quad 0.7]$.

As analysed from resultant B_1^t expert feels that much preference is given to all the constrains and not able to express the constrain quantity of foodis about 70%.

Now using

$$Q_{11}^t = [0.9 \quad 0.8 \quad 0.9 \quad 0.8 \quad 0.7 \quad 0.7 \quad 0.6]$$

in the equation $N_1 \circ Q_{11}^t = \text{MaxMin}(n_{ij}, q_{jk}) = B_{11}^t$ we get

$$B_{11}^t = [0.7 \quad 0.7I \quad 0.8 \quad 0.7 \quad 0.7 \quad 0.7 \quad 0.7].$$

As analysed from resultant B_{11}^t expert feels that much preference is given to all the constrains and not able to express the constrain quality of food.

Now using

$$Q_{12}^t = [0.4 \quad 0.2 \quad 0.3 \quad 0.4 \quad 0.5 \quad 0.4 \quad 0.3]$$

in the equation $N_1 \circ Q_{12}^t = \text{MaxMin}(n_{ij}, q_{jk}) = B_{12}^t$ we get

$$B_{12}^t = [0.5 \quad 0.4 \quad 0.5 \quad 0.4 \quad 0.4 \quad 0.5 \quad 0.4].$$

As analysed from resultant B_{12}^t the expert feels least preference is given to all the constrains expect the food prepared in their presence.

Next we work with the second expert. Let N_2 be the membership matrix given by second expert which is as follows:

$$N2 = \begin{array}{c} S_1 \\ S_2 \\ S_3 \\ S_4 \\ S_5 \\ S_6 \\ S_7 \end{array}\begin{bmatrix} 0.2 & 0.7 & 0.3 & 0.7I & 0.8 & 0.7 & 0.7 \\ 0.6 & 0.2 & 0.7I & 0.6 & 0.2I & 0.3 & 0.5 \\ 0.7 & 0.5 & 0.4 & 0.6 & 0.2 & 0.6 & 0.3 \\ 0.4 & 0.2I & 0.6 & 0.6 & 0.1 & 0.2 & 0.1 \\ 0.4 & 0.1I & 0.5 & 0.4 & 0.1 & 0.3 & 0.2 \\ 0.1 & 0.2 & 0.3 & 0.4 & 0.1 & 0.2 & 0.4 \\ 0.3 & 0.1 & 0.6I & 0.3 & 0 & 0.4 & 0.8 \end{bmatrix}.$$

Now using

$$Q_1^t = [0.7 \ 0.5 \ 0.7 \ 0.4 \ 0.8 \ 0.6 \ 0.3]$$

in the equation $N_2 \circ Q_1^t = \text{MaxMin}(n_{ij}, q_{jk}) = B_2^t$ we get

$$B_2^t = [0.8 \quad 0.7I \quad 0.7 \quad 0.6 \quad 0.5 \quad 0.4 \quad 0.6I].$$

As analysed from resultant B_2^t the expert feels inability to express about the quality of food and less waiting time and much preference is given for all the remaining constrains.

Now using

$$Q_{11}^t = [0.9 \quad 0.8 \quad 0.9 \quad 0.8 \quad 0.7 \quad 0.7 \quad 0.6]$$

in the equation $N_2 \circ Q_{11}^t = \text{MaxMin}(p_{ij}, q_{jk}) = B_{21}^t$ we get

$$B_{21}^t = [0.7I \quad 0.7I \quad 0.7 \quad 0.6 \quad 0.5 \quad 0.4 \quad 0.6I].$$

As analysed from resultant B_{21}^t the expert feels inability to express about the reasonable cost, quality of food and less waiting time and least preference is given for better service and much preference for quantity of the food, better hygiene and for food prepared in their presence.

Now using

$$Q_{12}^t = [0.4 \quad 0.2 \quad 0.3 \quad 0.4 \quad 0.5 \quad 0.4 \quad 0.3]$$

in the equation $N_2 \circ Q_{22}^t = \text{MaxMin}(n_{ij}, q_{jk}) = B_{22}^t$ we get

$$B_{22}^t = [0.5 \quad 0.4 \quad 0.4 \quad 0.4 \quad 0.4 \quad 0.4 \quad 0.4].$$

As analysed from resultant B_{22}^t the expert feels much preference is given for food prepared in their presence and least preferences is given for all the remaining constrains.

Next we work with the third expert. Let N_3 be the membership matrix given by third expert which as follows:

$$N_3 = \begin{array}{c} S_1 \\ S_2 \\ S_3 \\ S_4 \\ S_5 \\ S_6 \\ S_7 \end{array}\begin{bmatrix} 0.1 & 0.6 & 0 & 0.4 & 0.7 & 0.8 & 0.9 \\ 0.3I & 0.1 & 0.4 & 0.6I & 0.1 & 0.5 & 0.7 \\ 0.5 & 0.9 & 0.2I & 0.5 & 0.6 & 0.5 & 0.6 \\ 0.3 & 0.4I & 0.6 & 0.4 & 0.2 & 0.1 & 0.2 \\ 0.5 & 0.5 & 0.6 & 0.5 & 0.1 & 0.2 & 0.5 \\ 0.3 & 0.2 & 0.2 & 0.3 & 0.2 & 0.1 & 0.3 \\ 0.1 & 0.5 & 0.2 & 0.4 & 0.1 & 0.4 & 0.5 \end{bmatrix}.$$

Now using

$$Q_1^t = [0.7\ \ 0.5\ \ 0.7\ \ 0.4\ \ 0.8\ \ 0.6\ \ 0.3]$$

in the equation $N_3 \circ Q_1^t = \text{MaxMin}(n_{ij}, q_{jk}) = B_3^t$ we get

$$B_3^t = [0.7\ \ 0.5\ \ 0.6\ \ 0.6\ \ 0.6\ \ 0.3\ \ 0.5].$$

As analysed from resultant B_3^t the expert feels least preference is given to the food prepared in their presence, less waiting time, and better service. Much preference is given for remaining constrains.

Now using

$$Q_{11}^t = [0.9\ \ 0.8\ \ 0.9\ \ 0.8\ \ 0.7\ \ 0.7\ \ 0.6]$$

in the equation $N_3 \circ Q_{11}^t = \text{MaxMin}(n_{ij}, q_{jk}) = B_{31}^t$ we get

$$B'_{31} = [0.7\ \ 0.6I\ \ 0.6\ \ 0.6\ \ 0.6\ \ 0.3\ \ 0.5].$$

As analysed from resultant B_{31}^t the expert feels least preference is given to the food prepared in their presence and for less waiting time and the expert is not able to express about the quality of food .

Now using

$$Q_{12}^t = [0.4\ \ 0.2\ \ 0.3\ \ 0.4\ \ 0.5\ \ 0.4\ \ 0.3]$$

in the equation $N_3 \circ Q_{12}^t = \text{MaxMin}(p_{ij}, q_{jk}) = B_{32}^t$ we get

$$B_{32}^t = [0.5\ \ 0.4\ \ 0.5\ \ 0.4\ \ 0.4\ \ 0.3\ \ 0.4].$$

As analysed from resultant B_{32}^t the expert feels least preference is given to all the constrains.

Next we work with fourth expert. Let N_4 be the membership matrix given by third expert which as follows:

$$N_4 = \begin{array}{c} S_1 \\ S_2 \\ S_3 \\ S_4 \\ S_5 \\ S_6 \\ S_7 \end{array} \begin{bmatrix} 0.1 & 0.5 & 0.2 & 0.7 & 0.6 & 0.9 & 0.5 \\ 0.6 & 0.8 & 0.8 & 0.9 & 0.2 & 0.6 & 0.5 \\ 0.4 & 0.2 & 0.5 & 0.4 & 0.7 & 0.6 & 0.3I \\ 0.5 & 0.4 & 0.2 & 0.5 & 0.2 & 0.2 & 0.3 \\ 0.5 & 0.4 & 0.5 & 0.8 & 0.6 & 0.4 & 0.5 \\ 0.4 & 0.2 & 0.6 & 0.5 & 0.2 & 0.1 & I \\ 0.2 & 0.3 & 0.6I & 0.4 & 0 & 0.2 & 0.2 \end{bmatrix}.$$

Now using

$$Q_1^t = [0.7\ \ 0.5\ \ 0.7\ \ 0.4\ \ 0.8\ \ 0.6\ \ 0.3]$$

in the equation $N_4 \circ Q_1^t = \text{MaxMin}(n_{ij}, q_{jk}) = B_4^t$ we get

$$B_4^t = [0.6\ \ 0.7\ \ 0.7\ \ 0.5\ \ 0.6\ \ 0.6\ \ 0.6I].$$

As analysed from the resultant B_4^t the expert is not able to express about the less waiting time and given much preference to all the constrains.

Now using

$$Q_{11}^t = [0.9\ \ 0.8\ \ 0.9\ \ 0.8\ \ 0.7\ \ 0.7\ \ 0.6]$$

in the equation $N_4 \circ Q_{11}^t = \text{MaxMin}(n_{ij}, q_{jk}) = B_{41}^t$ we get

$$B_{41}^t = [0.7\ \ 0.8\ \ 0.7\ \ 0.5\ \ 0.8\ \ 0.6\ \ 0.6I].$$

As analysed from the resultant B_{41}^t the expert is not able to express about the less waiting time and least preference is given to better hygiene and much preference is given to all the remaining constrains.

Now using

$$Q_{12}^t = [0.4\ \ 0.2\ \ 0.3\ \ 0.4\ \ 0.5\ \ 0.4\ \ 0.3]$$

in the equation $N_4 \circ Q_{12}^t = \text{MaxMin}(n_{ij}, q_{jk}) = B_{42}^t$ we get

$$B_{42}^t = [0.5\ \ 0.4\ \ 0.4\ \ 0.4\ \ 0.5\ \ 0.4\ \ 0.4].$$

As analysed from the resultant B_{42}^t the expert gives least preference to all the constrains expect about the food prepared in their presence.

Next we work with fourth expert. Let N_5 be the membership matrix given by third expert which as follows:

$$N_5 = \begin{array}{c} S_1 \\ S_2 \\ S_3 \\ S_4 \\ S_5 \\ S_6 \\ S_7 \end{array} \begin{bmatrix} 0.2 & 0.9 & 0.1 & 0.5 & 0.6 & 1 & 0.8 \\ 0 & 0.2 & 0.1I & 0.6 & 0.5 & 0 & 0.5 \\ 0.6 & 0.6 & 0.2 & 0.5 & 0.6 & 0.4 & 0.8 \\ 0.1 & 0.1 & 0.7 & 0.6 & 0.1 & 0.1 & 0.2 \\ 0.1 & 0 & 0.6 & 0.3 & 0 & 0.2 & 0.1 \\ 0.1 & 0.1 & 0.1 & 0.2 & 0.1 & 0.1 & 0.2 \\ 0.3 & 0.3 & 0.7 & 0.6 & 0.1 & 0.5 & 0.6 \end{bmatrix}.$$

Now using

$$Q_1^t = [0.7\ \ 0.5\ \ 0.7\ \ 0.4\ \ 0.8\ \ 0.6\ \ 0.3]$$

in the equation $N_5 \circ Q_1^t = \text{MaxMin}(n_{ij}, q_{jk}) = B_5^t$ we get

$$B_5^t = [0.6\ \ 0.5\ \ 0.6\ \ 0.7\ \ 0.6\ \ 0.2\ \ 0.7].$$

As analysed from the resultant B_5^t the expert feels least preference is given to the food prepared in their presence and quality of food.

Now using

$$Q_{11}^t = [0.9\ \ 0.8\ \ 0.9\ \ 0.8\ \ 0.7\ \ 0.7\ \ 0.6]$$

in the equation $N_5 \circ Q_{11}^t = \text{MaxMin}(n_{ij}, q_{jk}) = B_{51}^t$ we get

$$B_{51}^t = [0.8\ \ 0.6\ \ 0.8\ \ 0.7\ \ 0.6\ \ 0.2\ \ 0.7].$$

As analysed from the resultant B_{51}^t the expert feels least preference is given to the food prepared in their presence.

Now using

$$Q_{12}^t = [0.4\ \ 0.2\ \ 0.3\ \ 0.4\ \ 0.5\ \ 0.4\ \ 0.3]$$

in the equation $N_5 \circ Q_{12}^t = \text{MaxMin}(n_{ij}, q_{jk}) = B_{52}^t$ we get

$$B_{52}^t = [0.5\ \ 0.5\ \ 0.5\ \ 0.4\ \ 0.3\ \ 0.2\ \ 0.4].$$

As analysed from the resultant B_{52}^t the expert feels least preferences given to all the constrains.

The New Average Neutrosophic Relation Equation (NANRE) defined and developed in section 2 of the paper is constructed using five experts which gives the opinion of all the 5 experts feeling. As a law of large number the average taken for all the five experts gives the approximately a sensitive opinion. Using the special type of average mentioned in section two of this paper we find the average of $N_1, N_2, \ldots, N_5$ and denote it by N.

$$N=\begin{bmatrix} .15+.2I & 0.66 & .15+.3I & .68 & 0.8 & .7 & 0.68 \\ .38+.3I & .33+.7I & 0.6+.4I & .62 & 0.12 & .54 & 0.67 \\ 0.56 & 0.6 & .37+.45I & .64 & 0.62 & .6 & 0.6 \\ .33+.5I & .23+.3I & 0.56 & .54 & 0.18 & .16 & 0.3 \\ 0.44 & .3+.25I & 0.56 & .46 & .2+.2I & .36 & 0.36 \\ 0.2 & 0.24 & 0.3 & .32 & 0.26 & .12 & .3+.75I \\ 0.22 & 0.24 & .53+.6I & .46 & 0.04 & .4 & 0.56 \end{bmatrix}$$

Now using

$$Q_1^t = [0.7 \;\; 0.5 \;\; 0.7 \;\; 0.4 \;\; 0.8 \;\; 0.6 \;\; 0.3]$$

in the equation $N \circ Q_1^t = \text{MaxMin}(n_{ij}, q_{jk}) = B^t$ we get

$$B^t = [0.7 \quad 0.4I \quad 0.56 \quad 0.56 \quad 0.56 \quad 0.32 \quad 0.533].$$

As analysed from the resultant B^t the expert feels least preference is given to all the constrains except the reasonable cost and expert is not able to express about the quality of food.

Now using

$$Q_{11}^t = [0.9 \quad 0.8 \quad 0.9 \quad 0.8 \quad 0.7 \quad 0.7 \quad 0.6]$$

in the equation $N \circ Q_{11}^t = \text{MaxMin}(n_{ij}, q_{jk}) = B_a^t$ we get

$$B_a^t = [0.7 \quad 0.6I \quad 0.6 \quad 0.56 \quad 0.56 \quad 0.32 \quad 0.56].$$

As analysed from the resultant B_a^t the expert feels least preference is given to better hygiene, better service, food prepared in their presence and for less waiting time and expert is not able to express the quality of food.

Now using

$$Q_{12}^t = [0.4 \quad 0.2 \quad 0.3 \quad 0.4 \quad 0.5 \quad 0.4 \quad 0.3]$$

in the equation

$$N \circ Q_{12}^t = \text{MaxMin}(n_{ij}, q_{jk}) = B_b{}^t$$

$$B_b{}^t = [0.5 \quad 0.38 \quad 0.5 \quad 0.4 \quad 0.4 \quad 0.32 \quad 0.4].$$

As analysed from the resultant B_b^t the expert feels least preference to all the constrains.

6. Conclusions

Hence using the experts opinion for the Fuzzy Relational Equations. The performance that the most of the owners of the street shops choose this as their profession is more profit with reasonable investment and unemployment problem is solved since many owners are from 25 yrs to 35yrs. Many owners do this job as a part-time. Even retired person with less investment do this job and they have job satisfaction. These street shops even give jobs for more Handicapped persons. The following conclusion are not only derived from the five experts described here but all 32 tuck shops and their opinion are used and some of the owners whom we have interviewed are also ingrained in this analysis. Now we consolidate the opinion given by the five experts working with FRE model in the following tables and the eighth column of each of these tables gives the average of each of the R_i 's $1 \le i \le 5$ calculated for each of the five experts using the FRE model. We see readily from the table that the average R_{Ai} ($1 \le i \le 3$) so found and the resultant R_{Xi} ($1 \le i \le 3$) calculated using NAFREs given in column seven of all the tables do not differ. In fact the values are very close. So we are justified in the construction of this model as it can save both time and economy.

Q_1	R_1	R_2	R_3	R_4	R_5	R_{X1}	R_{A1}
0.7	0.8	0.7	0.8	0.8	0.7	0.76	0.8
0.5	0.6	0.5	0.5	0.6	0.5	0.54	0.58
0.7	0.7	0.7	0.7	0.7	0.6	0.68	0.62
0.4	0.7	0.4	0.7	0.6	0.7	0.62	0.68
0.5	0.6	0.8	0.6	0.6	0.6	0.64	0.6
0.6	0.2	0.6	0.5	0.4	0.2	0.38	0.32
0.3	0.7	0.3	0.6	0.7	0.7	0.6	0.66

Q_2	R_1	R_2	R_3	R_4	R_5	R_{X2}	R_{A2}
0.9	0.7	0.8	0.7	0.7	0.7	0.72	0.8
0.8	0.6	0.7	0.6	0.7	0.6	0.66	0.64
0.9	0.8	0.8	0.8	0.7	0.7	0.64	0.76
0.8	0.7	0.8	0.7	0.6	0.7	0.74	0.7
0.7	0.6	0.6	0.6	0.7	0.6	0.62	0.6
0.7	0.2	0.3	0.5	0.5	0.2	0.32	0.34
0.6	0.7	0.8	0.6	0.7	0.7	0.7	0.66

Q_3	R_1	R_2	R_3	R_4	R_5	R_{X3}	R_{A3}
0.4	0.5	0.5	0.5	0.5	0.5	0.44	0.5
0.2	0.4	0.4	0.4	0.4	0.4	0.32	0.4
0.3	0.5	0.4	0.5	0.5	0.5	0.42	0.46
0.4	0.4	0.4	0.4	0.4	0.4	0.42	0.4
0.5	0.3	0.4	0.4	0.4	0.3	0.3	0.36
0.4	0.2	0.3	0.4	0.4	0.2	0.34	0.3
0.3	0.4	0.4	0.4	0.4	0.4	0.38	0.4

Now we consolidate the opinion given by the five experts working with NRE model in the following tables and the eighth column of each of these tables gives the average of each of the B_i 's $1 \le i \le 5$ calculated for each of the five experts using the NRE model. We see readily from the table that the average taken so found and the resultant B_{Xi} ($1 \le i \le 3$) given in column seven of all the tables calculated using NANREs . From the column seven and eight of the table four, five and six we see when both indeterminacy and real values occur there is a deviation which is proper. For an expert who does not consider a relation values as an indeterminate may not agree upon the occurrence of indeterminate. Likewise the experts who feels a relational value to be an indeterminate cannot compromise with the occurrence of real numbers. Hence this justifies the deviation. So we are justified in the construction of this new model as it can save both time and economy.

Q_1	B_1	B_2	B_3	B_4	B_5	B_{X1}	B_{A1}
0.7	0.8	0.8	0.7	0.6	0.6	0.7	0.7
0.5	0.6	0.7I	0.5	0.7	0.5	0.4I	0.575+.7I
0.7	0.7I	0.7	0.6	0.7	0.6	0.56	0.65+.7I
0.4	0.7	0.6	0.6	0.5	0.7	0.56	0.62
0.5	0.7	0.5	0.6	0.6	0.6	0.56	0.6
0.6	0.7	0.4	0.3	0.6	0.2	0.32	0.44
0.3	0.7	0.6I	0.5	0.6I	0.7	0.533	0.633+.6I

Q_2	B_1	B_2	B_3	B_4	B_5	B_{X2}	B_{A2}
0.9	0.7	0.7I	0.7	0.7	0.8	0.72	0.725+.7I
0.8	0.7I	0.7I	0.6I	0.8	0.6	0.6I	0.7+.6I
0.9	0.8	0.7	0.6	0.7	0.8	0.6	0.72
0.8	0.7	0.6	0.6	0.5	0.7	0.56	0.62
0.7	0.7	0.5	0.6	0.8	0.6	0.56	0.64
0.7	0.7	0.4	0.3	0.6	0.2	0.32	0.44
0.6	0.7	0.6I	0.5	0.6I	0.7	0.56	0.633+.6I

Q_3	B_1	B_2	B_3	B_4	B_5	B_{X3}	B_{A3}
0.4	0.5	0.5	0.5	0.5	0.5	0.5	0.5
0.2	0.4	0.4	0.4	0.4	0.5	0.38	0.42
0.3	0.5	0.4	0.5	0.4	0.5	0.5	0.46
0.4	0.4	0.4	0.4	0.5	0.4	0.4	0.42
0.5	0.4	0.4	0.4	0.5	0.3	0.4	0.4
0.4	0.5	0.4	0.3	0.4	0.2	0.32	0.36
0.3	0.4	0.4	0.4	0.4	0.4	0.4	0.4

Hence we conclude both the new models serves not only the purpose of saving time and economy but also gives equal importance to each and every expert and avoids bias by choice which is vital.

References:

[1]. George J.Klir and Bo Yuan. Fuzzy Sets and Fuzzy logic Theory and Applications, Prentice-Hall of India, 2000.

[2]. Florentin Smarandache. A unifying Field in Logics: Neutrosophic Logic, Preface by charles Le, American Research Press, Rehoboth, 1999,2000.

[3]. Second edition of the proceedings of the First International Conference on Neutrosophy, Neutrosophic Logic, Neutrosophic Set.

[4]. Vasantha Kandasamy W.B. and Praseetha R. New Fuzzy Relation Equations to estimate the peak hours of the day for Transport systems, J.of Bhiar math.soc.20(2000), 1-14

[5]. Vasantha Kandasamy W.B, Neelakandan, N.R and Kannan SR. Replacement of Algebraic Linear Equations by FRE in chemical Engineering, In recent trends in mathematical sciences, Proc. Of Inst.conf.on recent advances in mathematical sciences held at IIT Kharagpur on Dec.20-22(2001) published by Narosa Publishing House(2001) 161-168.

[6]. Vasantha Kandasamy and Florentin Smarandache. Fuzzy Relation Equations and Neutrosophic Relation Equations,Hexis,2004.

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Abstract

This volume is a collection of ten papers, written by different authors and co-authors (listed in the order of the papers): F. Yuhua, A. A. Salama, F. Smarandache, S. A. Alblowi, M. Ali, M. Shabir, M. Naz, A. A. A. Agboola, S. A. Akinleye, M. Dhar, S. Broumi, P. Biswas, S. Pramanik, B. C. Giri, H. A. El-Ghareeb, A. M. Maine, V. Kandasamy, P. Sekar and J. Vidhyalakshmi.

In first paper, the author proposed Expanding Newton Mechanics with Neutrosophy and Quad-stage Method-New Newton Mechanics Taking Law of Conservation of Energy as Unique Source Law. The Characteristic Function of a Neutrosophic Set is proposed in the second paper. Neutrosophic Left Almost Semigroup is studied in third paper. In fourth paper Neutrosophic Hypercompositional Structures defined by Binary Relations are introduced. Similarly in fifth paper A Note on Square Neutrosophic Fuzzy Matrices are discussed. In paper six, A New Methodology for Neutrosophic Multi-Attribute Decision-Making with Unknown Weight Information is presented by the authors. Introduction to Develop Some Software Programs for dealing with Neutrosophic Sets are given in seventh paper. Paper eight is about to Soft Neutrosophic Ring and Soft Neutrosophic Field. In the next paper Rough Neutrosophic Sets are discussed. The authors introduced New type of Fuzzy Relational Equations and Neutrosophic Relational Equations-To Analyze Customer Preference to street shops in the last paper.

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