Improving the Koide Formula

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Abstract. Koide formula has improved on the result 2/3 in the formula (14). The tau lepton mass is also calculated. This is third version of the article.

Introduction

The famous Koide formula [1] connects the masses of charged leptons with a simple equation:

$$\left(m_e + m_\mu + m_\tau\right) \Big/ \left(\sqrt{m_e} + \sqrt{m_\mu} + \sqrt{m_\tau}\right)^2 \approx 2/3 \quad \textbf{(1)}$$

We are using the following CODATA values [2]:
The inverse of the fine-structure constant ά=137.035 999 074 (44),
Mass ratio of protons and electrons μ= 1836.152 672 45 (75)
Proton mass m_p=1.672 621 777 (74) e-27 kg
Electron mass m_e=9.109 382 91 (40) e-31 kg
Muon mass m_μ=1.883 531 475(96) e-28 kg
Mass of tau particle m_τ= 3.167 47 (29) e-27 kg

We have:

$$\left(m_e + m_\mu + m_\tau\right) \Big/ \left(\sqrt{m_e} + \sqrt{m_\mu} + \sqrt{m_\tau}\right)^2 = 0.666658 \quad \textbf{(2)}$$

Analysis

Let’s start with the Boscovich force curve, figure 1, [5]:

Boscovich emphasized the importance of distances at which the curve crosses the abscissa: R, N, I and E represent the stable, but P, L and G are the unstable positions. It is a logical assumption that the relation is simpler between the particles of the same order, and even simpler between the neighboring points.

Suppose that all our leptons are in the stable positions, i.e. at point E on the curve (a). If that were indeed the case, then they would have no connection with their surroundings. Therefore, let’s assume that at least one member of the Koide formula is oscillating. Let's name the virtual particle a wave lepton. We can determine its mass, $\mathbf{m_w}$ by using the formula found in [3]:

$$\xi = 2\pi\alpha' * 2^{(2/3)*\left[(\mu/\alpha'+1)/(\mu/\alpha'+2)+1\right]} / \mu = 1.146691715 \quad \textbf{(3)}$$

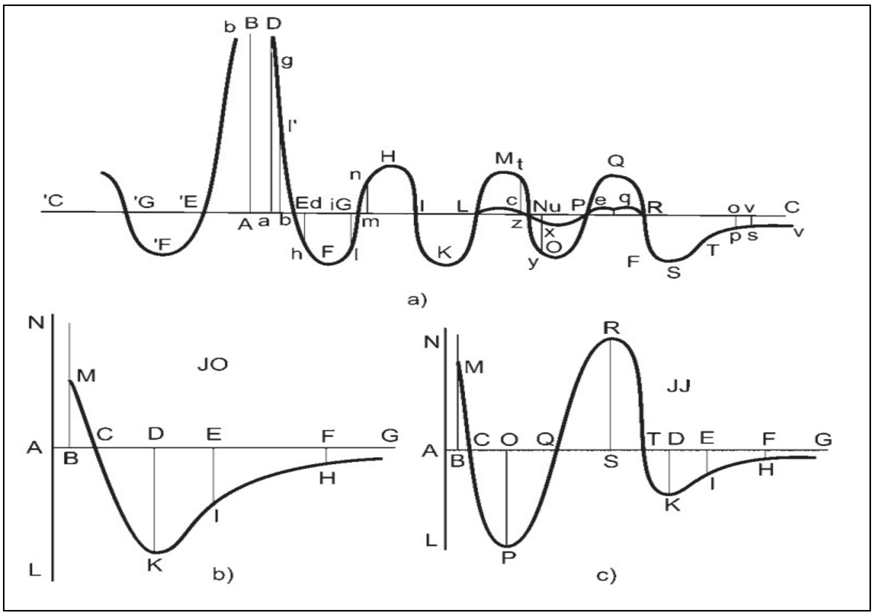

Figure 1 General (a) and particular (b, c) shapes of curves that present the attractive and repulsive forces (F) (bottom and upper ordinates, respectively) vs. distance (r) (abscissa) between the elementary points and particles of matter [6].

Then, wave lepton virtual mass is:

$$m_w = m_e / \xi = 7.944055747\text{E-}31\,\text{kg} \quad \textbf{(4)}$$

Now let's introduce the first correction to the Koide formula, assuming that in (1) instead of the electron mass in the root there should be the virtual lepton mass $\mathbf{m_w}$. Then we have:

$$\left(m_e + m_\mu + m_\tau\right) \Big/ \left(\sqrt{m_w} + \sqrt{m_\mu} + \sqrt{m_\tau}\right)^2 = 0.6678466747 \quad \textbf{(5)}$$

Here I owe a clarification for why I did not use the same value for muon and tau particle in the root. Because the influence of the electron is dominant, so the virtual lepton is surrounded by two remaining corpuscular leptons. The situation in which $\mathbf{m_w}$ would be assigned to muon, while the electron mass would be left in the root is by several orders of magnitude less frequent. The situation is even rarer for tau lepton. From the preceding discussion it is clear that even if we find a correction, in this way it would not be exact, since we neglected the correction for muon and tau. Since we have to start from somewhere, we will not worry about an error here, if there is one.

Correction

Let's assume here that the Koide formula is ideally applicable in the quasi-universe that has no substance, or which is perfectly empty. The universe is indeed almost empty, but of course if it were absolutely empty, it would not exist. Therefore, here we are introducing an assumption that the difference is the result of the existence of matter. Then it is reasonable to assume that the difference is **Δ=f (m_p)**, implying that Δ can most easily be expressed via the proton mass. It is also reasonable to assume that the correction can be expressed via the Planck mass, **Δ=f (m_{pl})**.

$$\Delta = x - 2/3 = 0.6678466747\ldots - 2/3 = 0.001180008\ldots \qquad (6)$$

Is the correction we have to determine for the Koide formula to be perfectly correct, implying the previous replacement of the electron mass in the root with the virtual mass of wave leptons? Therefore, let's define:

$$g = \log(M_u/m_{pl},2) = 202.3142277\ldots \qquad (7)$$

Where the mass of the universe and Planck mass are:

M_u=1.73944912E+53 kg , m_{pl} =2.1765099035E-08 [3]

Let's calculate:

$$y = 1/[2\pi*(2/3)*g] = 1/[2\pi*134.876151788675] = 1/847.4518552 = 0.001180008\ldots \qquad (8)$$

We can see that y=Δ. If we apply the replacements, from (5) and (8) we get:

$$x-\Delta = x-y = (m_e + m_\mu + m_\tau)/(\sqrt{m_w}+\sqrt{m_\mu}+\sqrt{m_\tau})^2 - y =$$
$$0.667846674704 - 0.001180008037 = 0.6666666666666\ldots \qquad (9)$$

Now let's define the mathematical constants:

$t=\log(2\pi,2)=2.6514961295\ldots$, **C**ycle, **$cy=e^{2\pi}=535.491655525\ldots$** and half cycle, **$z=cy/2$**

Now instead of (7), we can express g in the following way:

$$g = 3cy/8 + 3t/4 - (zp+1)/4 = 202.3142277 \qquad \textbf{(10)}$$

Where zp is the proton shift from **[3]**:

$$zp = \log(m_p/m_z,2) = 1.935060943 5 \qquad \textbf{(11)}$$

Or the proton shift from **[4, table 1]**:

$$zp = (\mu/\alpha'+1)/(\mu/\alpha'+2) + 1 = 1.935060943 5 \qquad \textbf{(12)}$$

Or:

$$zp = \frac{1}{1+\frac{1}{\mu/\alpha'+1}} + 1 \qquad \textbf{(12b)}$$

Then, if we put (10) into (8) we get:

$$y=1/[2\pi^*(2/3)^*g]=1/[2\pi^*(cy/4+t/2-zp/6)]=0.001180008\ldots \qquad (13)$$

Namely, from (5), (6), (8) and (12), the Koide formula with corrections is the following:

$$x=\left(m_e+m_\mu+m_\tau\right)/\left(\sqrt{m_w}+\sqrt{m_\mu}+\sqrt{m_\tau}\right)^2-1/\left[\pi*\left(e^{2\pi}/2+\log(2\pi,2)-zp/3\right)\right]=2/3 \quad \mathbf{(14)}$$

We have obtained the Koide formula with the corrections, which besides the masses of charged leptons, contains the mathematical constants, the fine structure constant and the proton. Therefore:

$$x=f(m_e,m_\mu,m_\tau,m_w,\mu,\acute{\alpha})=2/3$$

Hence, since $m_w=f(\mu,\acute{\alpha})$ and $\mu=m_p/m_e$ we can write:

$$x=f(m_e,m_\mu,m_\tau,m_p,\acute{\alpha})=2/3 \qquad (15)$$

Tau lepton mass

Formula (14) can be used for determining the tau lepton mass with the same accuracy as for the other two leptons. The result is: $m_p/m_\tau=0.52805989288087$. For this calculation a more precise values than in [2] were used:

(μ=1836.15267245005, $\acute{\alpha}$= 137.035999073361).

Table 1 presents the calculation of the tau lepton mass from the previously set values of physical constants, which are much more easily determined experimentally.

Table 1 Calculating the tau lepton mass (all masses in kg)

$\acute{\alpha}$=	137.035999073361
μ=	1836.15267245005
m_e=	9.1093829075E-31
m_μ=	1.8835314739E-28
$A=m_e+m_\mu$	1.8926408568E-28
$zp=(\mu/\acute{\alpha}+1)/(\mu/\acute{\alpha}+2)+1$	1.935060944
$\zeta=2\pi\acute{\alpha}*2^{2zp/3}/\mu$	1.146691714861000
$B=\sqrt{(m_e/\zeta)}+\sqrt{(m_\mu)}$	1.4615475444512E-14
$t=\log(2\pi,2)$	2.6514961295
$cy=\exp(2\pi)$	535.4916555248
$g=3cy/8+3t/4-zp/4=$	202.314227683011
$K=2/3+1/(4\pi g/3)=$	0.667846674703530
$x=(m_e+m_\mu+m_\tau)/(\sqrt{m_w}+\sqrt{m_\mu}+\sqrt{m_\tau})^2-1/[2\pi(cy/4+t/2-zp/6)]=$	0.66666666667
$m_\tau=((2KB/(1-K)+sq((-2KB/(1-K))^2-4*(A-KB^2)/(1-K)))/2)^2$	3.167484975E-27

Or if we simplify:

$$m_\tau = \left[\frac{KB + \sqrt{KB^2 + A(K-1)}}{1-K} \right]^2 = 3.16748497 \quad (28) \;\; \text{x } 10^{-27} \text{ kg} \qquad \textbf{(16)}$$

By analyzing the progress in the accuracy of determining the tau lepton mass throughout the CODATA history, while at the same time using the procedure shown in Table 1, the validity of formula **(16)** can be tested**.** If we write the Koide formula so that two square roots of lepton masses are constants and one the unknown, we get a quadratic equation. The root of the tau lepton mass ($\sqrt{m_\tau}$) is then a variable, therefore there are two solutions. The relation between these two values is interesting. Research and discussion of these relations would be useful and interesting, for all the three leptons.

Accuracy of the tau formula - Hugh Matlock's Analysis

Notice that CODATA did not publish a mass_tau until 1998. The value that formula (16) could predict in 1969 appears to be more precise than the current 2010 CODATA value for mass_tau.

Table 2 Comparison of the tau lepton mass determined by two methods

Year	CODATA tau *e-27 kg	Zivlak tau *e-27 kg	CODATA error	Zivlak error	C Error / Z Error
1969	0	3.167550(113)	1	1.27E-04	7857.805
1973	0	3.167542(33)	1	1.02E-04	9812.244
1986	0	3.1674871(33)	1	9.26E-05	10800.26
1998	3.16788(52)	3.16748435(47)	2.56E-04	9.17E-05	2.788674
2002	3.16777(52)	3.16748485(97)	2.56E-04	9.19E-05	2.78383
2006	3.16777(52)	3.16748468(32)	2.56E-04	9.17E-05	2.790086
2010	3.167470(290)	3.167484977(281)	1.83E-04	9.16E-05	1.998053

Conclusion

We started from the assumption that the Koide formula misses a part related to the proton mass, in order for the result to be an ideal 2/3. We have reached such a solution, provided that mathematical constants and the fine-structure constant are also included in the formula. Is the Koide formula with the correction developed in this way speculative?

- By defining the virtual mass of wave lepton we introduced corpuscular/wave nature of electron in the formula;
- Assumption that the material world expressed via protons should also influence the relation among three charged leptons is rational;
- Introduction of the Planck mass via (7) and (10) is also rational;
- The result confirms those assumptions;
- The obtained formulas (14) and (16) show that the Koide formula with corrections is in function of the all the charged particles, which is more rational than for the formula to be in the function of only three negatively charged particles;
- Determining the tau particle mass through the CODATA history (Table 2), also supports formula (14);

- Introducing the points from the Boscovich force curve is probably as successful as his prediction of atom orbitals [5].

The presented developed formulas and conclusions should be further explained by other rational approaches. A physical interpretation of the obtained results requires a new article with more concrete details. One such approach is by using the Boscovich force curve [5].

The advantages of using formula (16) are evident from the analysis in Table 2. The results obtained are much more accurate than the ones obtained by the CODATA methods.

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References:

1. http://en.wikipedia.org/wiki/Koide_formula
2. http://physics.nist.gov/cuu/Constants/
3. Branko Zivlak - Calculate Universe 1, viXra: 1303.0209
4. Branko Zivlak - Calculate Universe 2, viXra: 1304.0051
5. 13. Boscovich J. R.: (a) "Theoria philosophia naturalis redacta ad unicam legem virium in natura existentium", first (Wien, 1758) and second (Venetiis, 1763) edition in Latin language; (b) "A Theory of Natural Philosophy", in English, The M.I.T. Press, Massachusetts Institute of Technology, Cambridge, Massachusetts and London, England, first edition 1922, second edition 1966
6. Stoiljković D., "Ruđer Bošković - utemeljivač savremene nauke", Istraživačka stanica Petnica, 2010.