Bekenstein-Hawking Entropy from Holography

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Bekenstein-Hawking entropy takes discrete values proportional to 2 to the power n in a holographic model. Horizon numbers n are calculated for specific black holes and for subatomic particles, which have been shown to be the analogues of black holes.

The Bekenstein-Hawking entropy S_{BH} of a black hole of horizon area A is given, in natural units[1], by

$$S_{BH} = \frac{A}{4} \tag{1}$$

Since $A = 4\pi r_S{}^2$, where r_S is the Schwarzschild radius, and $r_S = 2m_{BH}$, where m_{BH} is the mass of the black hole, it follows that $S_{BH} = 4\pi m_{BH}{}^2$. We can then write

$$S_{BH} = 8\pi.\frac{m_{BH}{}^2}{2} \tag{2}$$

Building on an analysis of vacuum energy [1, 2] and the Big Bang timeline of events [3], we have formulated a 10D/4D correspondence, which for black holes relates the length scale $l_{BH} < l_{Planck}$ on a boundary in AdS_5 spacetime and the mass $m_{BH} > m_{Planck}$ of a black hole [4]:

$$l_{BH}{}^{-5} = \frac{m_{BH}{}^2}{2} \tag{3}$$

By regarding $l_{BH}{}^{-5}$ as the volume of a 5-sphere and conjecturing that

$$\frac{m_{BH}{}^2}{2} = 2^n \tag{4}$$

where 2^n is the number of states of n quantum bits, it follows from (2) that

$$S_{BH} = 8\pi.2^n \tag{5}$$

To be consistent with Bekenstein-Hawking entropy, the *horizon number* n must take both integer and fractional values. Such a scenario is consistent with the Planck Model [5], in which mass and length scales are related to Planck scale through multiplication by a common ratio raised to both integer and fractional (half-integer, quarter-integer etc) powers; integer powers are favoured, though, and are found for the quarks as doublets, the electron and the proton.

The horizon numbers of two black holes will differ by 1 if they are related in mass by a factor $2^{½}$, which severely limits the number of such bodies suitable for analysis because of uncertainty in the mass measurement. However, the limitation is moderated for some black holes, since, in line with the

[1] c = G = ħ = 1 throughout

Planck Model, horizon numbers that are multiples of 5, and particularly 25, are favoured. For subatomic particles, which have been shown to be the analogues of black holes [6] and for which horizon numbers will also be calculated, the limitation does not apply in most cases as particle masses are known with sufficient precision.

First, horizon numbers n have been calculated for black holes and for the Hubble sphere (the photon horizon). The value of Planck Mass used is $1.220910(20)\times10^{19}$ GeV [7].

The Tolman-Oppenheimer-Volkoff (TOV) limit, as originally calculated for non-interacting neutrons [8], set an upper bound of ~0.7 M_{Θ} on the mass of a neutron star that is able, through neutron degeneracy pressure, to resist collapse to a black hole. The horizon number n of a black hole of mass 0.7±0.05 M is equal to 250.18±0.20. A black hole of the fundamental mass scale 0.7 M_{Θ} is of horizon number 250.

The Milky Way (MW) and M31 galaxies, the two largest galaxies in the Local Group, form a 'galactic doublet' in the Planck Model [6]. The value of n calculated for the supermassive black hole (SMBH), Sgr A*, of the Milky Way, of mass 4.1×10^{6} M_{Θ} [9], is 295.1. The value of n calculated for the SMBH of M31, of mass 1.4×10^{8} M_{Θ} [10], is 305.3. The MW-M31 SMBH doublet is centred on horizon number 300.

To calculate the horizon number n of the Hubble sphere (HS), we rewrite (4) as

$$\frac{\Omega_m {m_{HS}}^2}{2} = 2^n \tag{6}$$

or

$$\frac{\Omega_m {r_{HS}}^2}{8} = 2^n \tag{7}$$

where Ω_m is the matter density parameter of the universe. We can then write

$$\frac{\Omega_m}{8{H_0}^2} = 2^n \tag{8}$$

where H_0 is the Hubble constant. With H_0 = 67.8±0.9 km/s/Mpc and Ω_m = 0.308±0.012 [11], the horizon number of the Hubble sphere calculated from (10) is 400.09±0.10. The Hubble sphere is of horizon number 400.

The horizon numbers calculated above are shown in Figure 1.

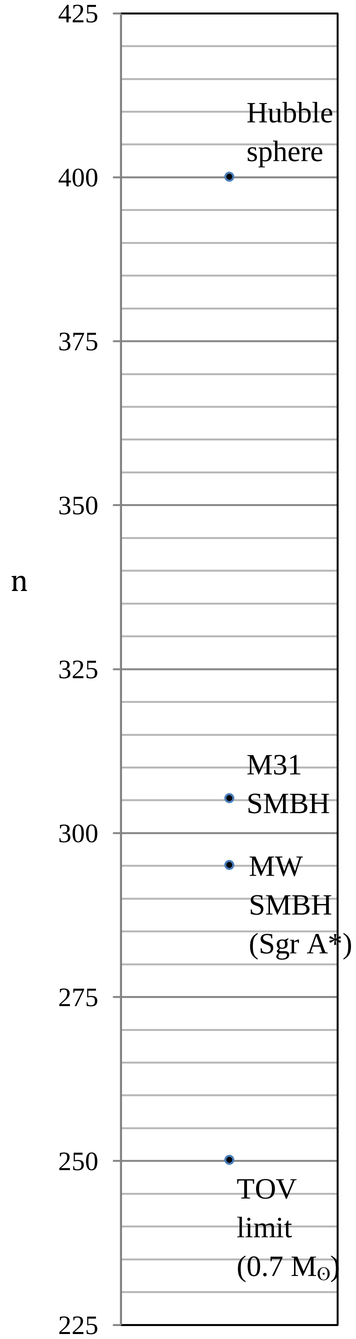

Figure 1: The occupation of *horizon levels* ($m > m_P$)

For subatomic particles (sp), the analogues of black holes [6], we write

$$\left(\frac{{m_{sp}}^2}{2}\right)^{-5/2} = 2^{n'} \tag{9}$$

where n′ is the particle horizon number. The horizon numbers n and n′, of a black hole and its particle analogue, will be equal.

Atomic nuclei have been shown to be the analogues of supermassive black holes [6]. Atomic nuclei with mass number A = 1, 2, 4, 8 etc are shown on horizon levels in Figure 2. All but one of the lighter nuclei shown (up to ^{64}Ni) are stable; there is no stable nucleus with A=8. Three of the nuclei shown are of double magic number.

Nuclear masses, from which the horizon numbers have been calculated, have been computed from tables of nuclide mass [12].

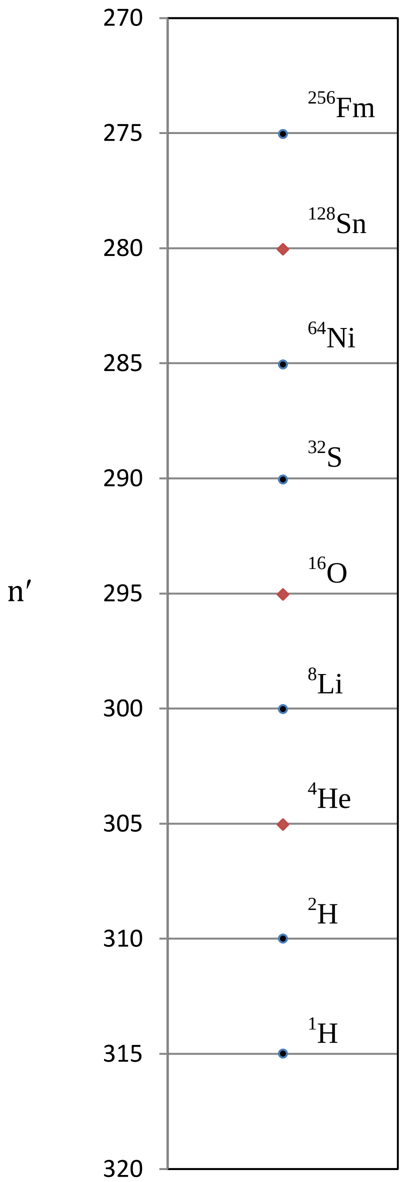

Figure 2: Atomic nuclei in geometric sequence of mass number on horizon levels ($m < m_P$). Double magic number nuclei are marked by red diamonds.

The lightest ground state baryons, N (uud, udd), Λ (uds) and Ξ (uss, dss) are all associated with horizon levels and low order[2] sublevels, as shown in Figure 3. The uds baryon Σ^0 also occupies a sublevel and forms a symmetric pairing with Λ about horizon level 313.5. The two uds baryons are also arranged symmetrically about level 97 of Sequence 2 in the Planck Model [5]. Sequence 2 descends geometrically from the Planck Mass with common ratio 2/π.

The particle masses used to calculate horizon numbers are the evaluations of the Particle Data Group [13].

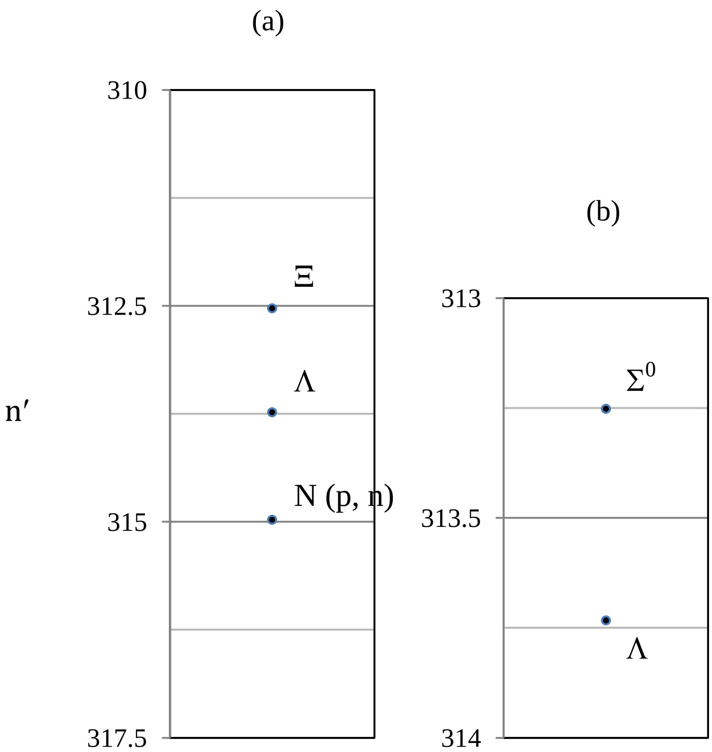

Figure 3: (a) Ground state N, Λ and Ξ baryons on horizon levels. Isospin doublets are represented by the geometric mean of the two masses. (b) The Λ and Σ^0 uds baryons on horizon levels.

[2] Half-levels and quarter-levels

The pseudoscalar mesons $\pi^{\pm}$, $D^{\pm}$, D^{0}, $B^{\pm}$, B^{0} and B_c^{+} are all associated with horizon levels and low order sublevels, as shown in Figure 4. The K-mesons lie close to horizon level 320 but precisely on level 39 of Sequence 1 (of common ratio $1/\pi$) in the Planck Model, as shown in Figure 5.

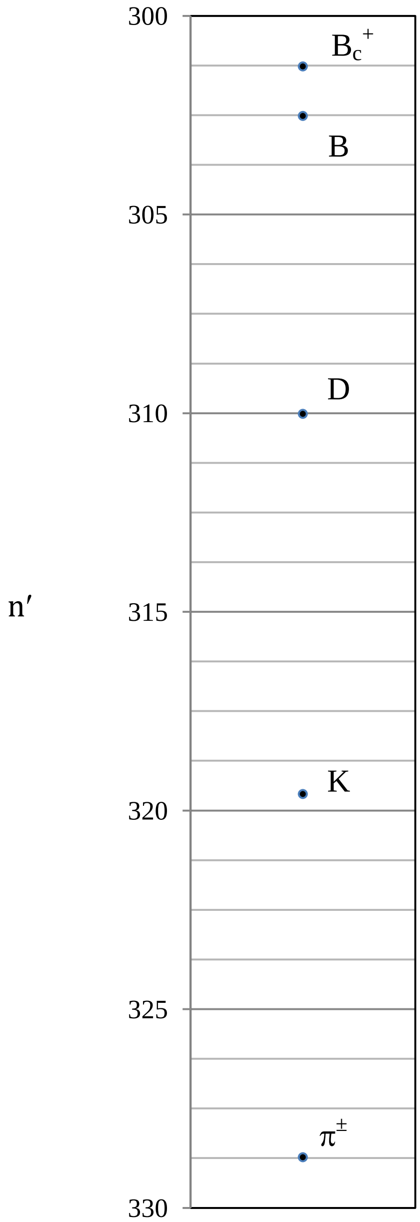

Figure 4: Pseudoscalar mesons on horizon levels. Isospin doublets are represented by the geometric mean of the two masses.

The occupation of mass levels by ground state uds, udc and udb baryons is also shown in Figure 5. We will return to these particles later.

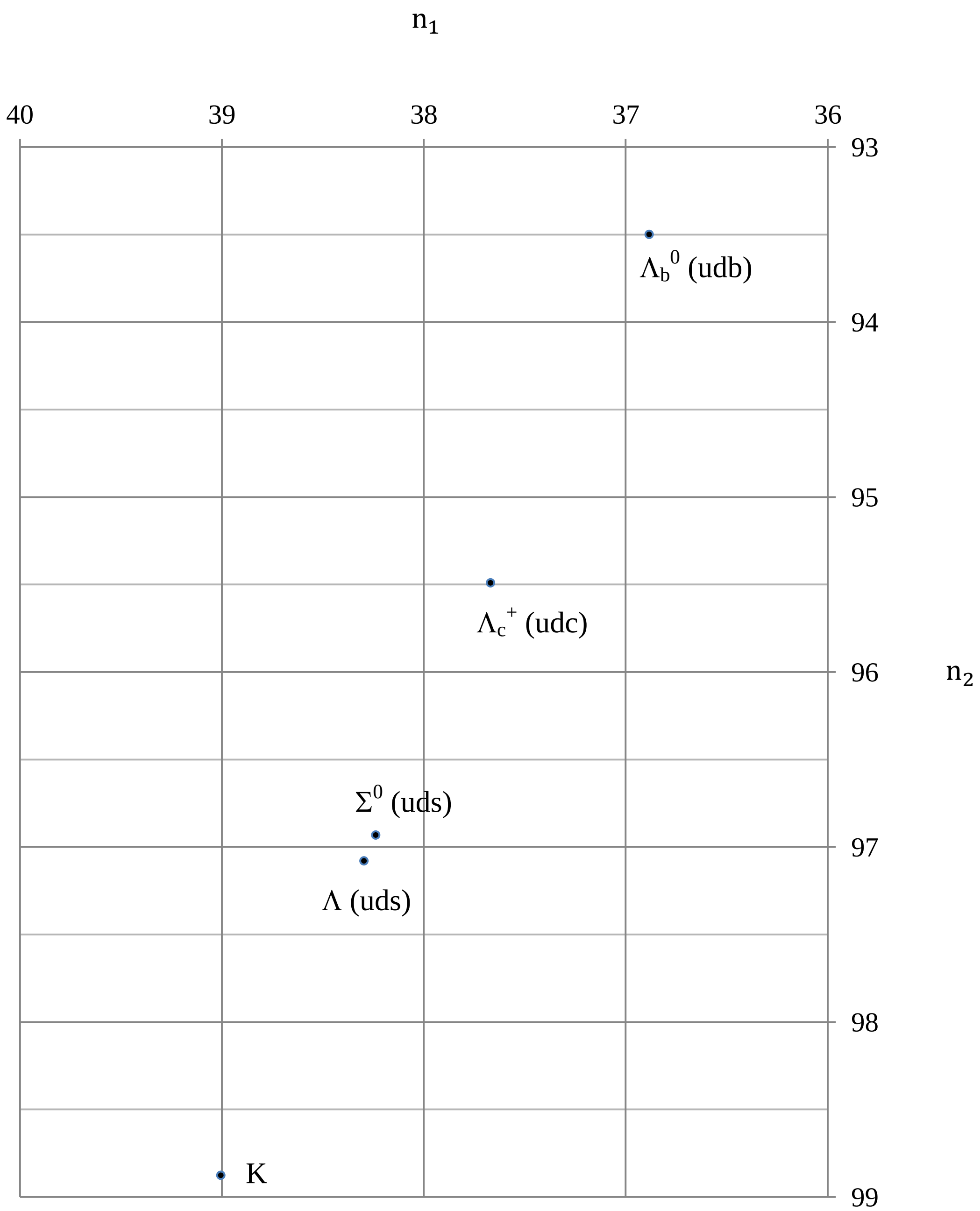

Figure 5: The occupation of mass levels in Sequences 1 and 2 by selected hadrons. The K-meson isospin doublets are represented by the geometric mean of the two masses. Sequences 1 and 2 descend from Planck scale with common ratio $1/\pi$ and $2/\pi$, respectively.

Three pivotal scales of an extended Standard Model correspond to prominent horizon numbers. Those scales are the Higgs field vacuum expectation value (VEV) υ, of 246 GeV, the conjectured right-handed VEV υ_R (7.7 TeV) of a left-right supersymmetric model [14, 15], and the GUT scale of 2×10^{16} GeV. To these scales is added the mass scale, 7.1 keV, of the widely-conjectured dark matter particle that could explain an unidentified emission line at 3.5-3.6 keV in inner galactic X-ray spectra [16, 17]. Horizon numbers for these four scales are shown in Figure 6.

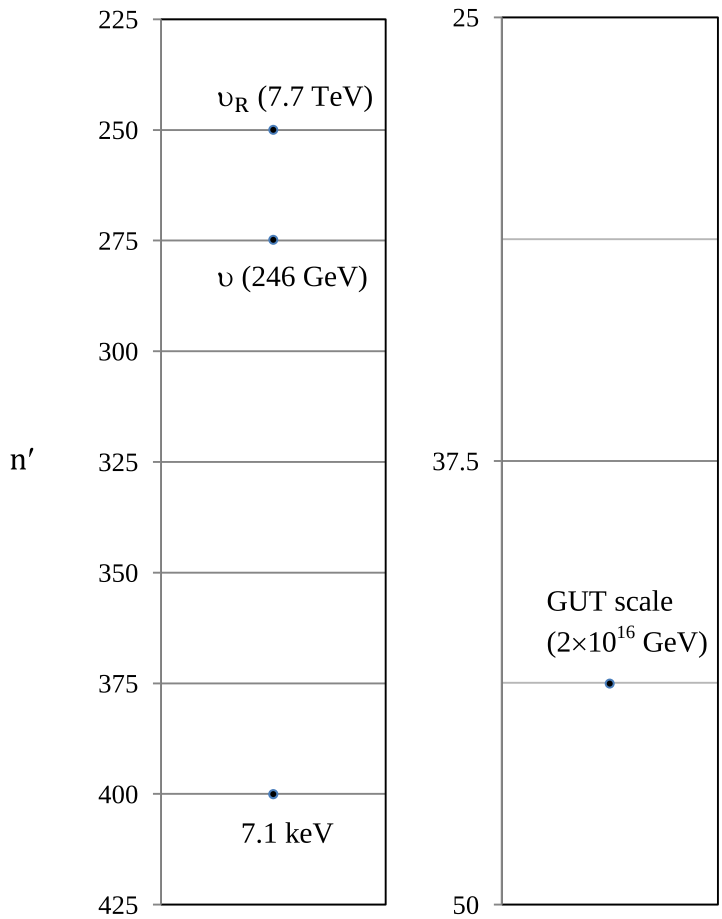

Figure 6: Scales of an extended Standard Model on horizon levels

Both VEVs coincide with horizon levels whose numbers n′ are multiples of 25, which is not surprising in the Planck Model. The right-handed VEV is the analogue of the TOV limit of 0.7 M_{Θ} (see Figure 1). The horizon number n′ of the 7.1 keV dark matter candidate is 400, i.e. the conjectured particle is the analogue of the Hubble sphere. We have previously suggested that the accelerating expansion of space is linked to this particle [18]. Clearly, though, there is a connection between the dark matter candidate and the Hubble constant. The precise value of horizon level 400 is 7.14 keV. The GUT scale coincides with a horizon sublevel.

As the Higgs VEV (246 GeV) lies on a horizon level we might wish to relate particle horizon numbers n′ to that (275) of the VEV. But since two particles with a horizon number difference of 5 are related in mass by a factor 2 we will relate particle masses to the VEV by factors 2^{-N}. Values of N for selected particles are shown in Figures 7 and 8. Note that the particles are related in mass to the right-handed VEV by factors $2^{-(N+5)}$.

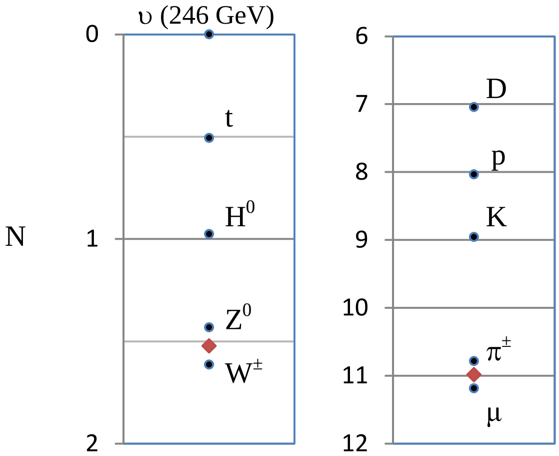

Figure 7: Values of N for selected particles. The gauge bosons $W^{\pm}$ and Z^0, and also the muon and the charged pions lie in doublet configuration (symmetrically about a mass level) in the Planck Model [5]. Each 'doublet' is represented in mass by the geometric mean of the two masses, which is marked by a red diamond.

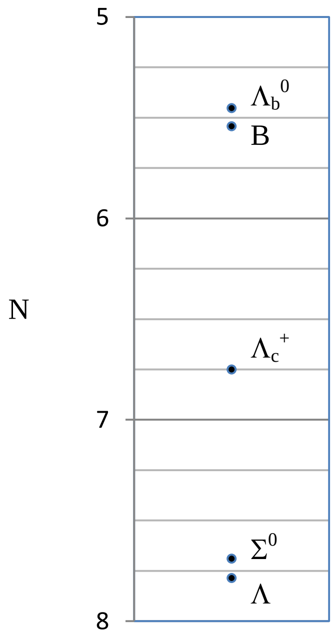

Figure 8: Values of N for Λ, Λ_c^+, Λ_b^0 and associated particles

It is clear from Figures 7 and 8, which include the most massive known particles[3] ($W^{\pm}$, Z^0, H^0 and the top quark, t) and the lightest unflavoured, strange, charmed and bottom baryons and mesons, that the horizon level structure forms a framework for the Standard Model and, from Figure 6, its extension.

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[3] Other than some atomic nuclei