0-Branes of SU(2) Lattice Gauge Theory: First Numerical Results

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Abstract

The site reduction of SU(2) lattice gauge theory is employed to model the magnetic monopoles of SU(2) gauge theory. The site reduced theory is a matrix model on discrete world-line for the angle-valued coordinates of 0-branes. The Monte Carlo numerical analysis introduces the critical temperature $T_c \simeq 0.25\ a^{-1}$ and the critical coupling $g_c \simeq 1.56$, above which the free energy does not exhibit a minimum leading to a phase transition.

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According to string theory, the dynamics of Dp-branes as p-dimensional solitonic solutions of the effective field theory is captured by the dimensional reduction of gauge theory from the target space down to the world-volume of Dp-branes [1,2]. In the process of dimensional reduction the gauge fields in transverse directions appear as coordinates of the Dp-branes, leading to the correspondence:

$$A_i \longleftrightarrow X_i/l_s^2 \tag{1}$$

in which l_s is the string theory length. Dp-branes as charged solitonic objects possess mass proportional to the inverse string coupling λ_s, similar to gauge theory magnetic monopoles with $m \propto g_{YM}^{-2} = \lambda_s^{-1}$. In the case of N Dp-branes, the transverse coordinates would appear as N dimensional hermitian matrices [3]. In the case of D0-branes all spatial components of the gauge field would appear as the time dependent space coordinates of D0-branes [1]

In a recent work the above correspondence is generalized to the pure U(1) lattice gauge theory [4] to model the dynamics of magnetic monopoles [5]. The action of site reduced theory in the imaginary time formalism comes to the form

$$S_{\mathrm{U}(1)} = \frac{1}{g^2} \sum_{n,i} \left(\cos \frac{x_{n+1}^i - x_n^i}{R} - 1 \right) \tag{2}$$

which is the sum of three copies of the Hamiltonian of the 1D plane-rotator model of magnetic systems. The close relation between lattice gauge theories and the Villain approximation [6] of plane-rotator model of spin systems is discussed in [7–10]. In the continuum limit $S_{\mathrm{U}(1)} \simeq -m_0 \dot{x}^2/2$ with mass $m_0 \propto g^{-2}$ just as expected for the mass of monopoles. The transfer matrix between time steps is used to read the exact energy levels as [5]

$$E_s^{\mathrm{U}(1)}(\kappa) = -\frac{1}{a} \ln \left[(2\pi\kappa)^{1/2} e^{-\kappa} I_s(\kappa) \right] \tag{3}$$

with $\kappa = g^{-2}$ and $I_s(\kappa)$ as the modified Bessel function of the first kind. Interestingly the lowest energy E_0 exhibits a minimum at $\kappa_{c\mathrm{U}(1)} = 0.790$, leading to a phase transition at low temperatures [5]. As the consequence, at $T \to 0$ and below the critical coupling $g_{c\mathrm{U}(1)} = \kappa_{c\mathrm{U}(1)}^{-0.5} = 1.125$ the kinetic term is negative, and the 0-branes are not detectable, like the situation for low energy particles inside a potential

barrier. For $g > g_{c\mathrm{U}(1)}$ the 0-branes are available to be detected. In the gauge theory side the phase with detectable 0-branes corresponds

$$\left\langle \vec{B}^2 \right\rangle > \left\langle \vec{E}^2 \right\rangle \qquad \text{for} \qquad g > g_c \tag{4}$$

which is interpreted as the dominance of the density of magnetic monopoles in the defining vacuum of the theory. This in fact supports the initial guess that the reduced model might represent the dynamics of the magnetic monopoles of the theory. The above results are in agreement with the dual Meissner effect scenario for the confined phase of gauge theories [11–13]. According to the scenario, whenever the monopoles are available they prevent the wide spreading of the electric fluxes originated from electric charges, leading to the confinement of these charges. For higher temperatures there is a critical temperature $T_{c\mathrm{U}(1)} = 0.247\ a^{-1}$ above which for any coupling constant the monopoles are available to be detected, and the theory is in confined phase [5].

The aim at present work is to generalize the above to 0-branes of SU(2) lattice gauge theory. Setting $\kappa = g^{-2}$ the site reduced model for the group SU(2) ($\mathrm{Tr}\,\mathbb{1} = 2$) takes the form [5]:

$$S_{\mathrm{SU}(2)} = \kappa \sum_n \left(-12 + \sum_{i=1}^{3} \mathrm{Re}\Big[\mathrm{Tr}\, e^{f_{n,0i}}\Big] + \sum_{i,j=1}^{3} \mathrm{Re}\Big[\mathrm{Tr}\, e^{\mathrm{i} f_{n,ij}}\Big] \right) \tag{5}$$

in which "Re" is for the real part and

$$e^{\mathrm{i} f_{n,0i}} = e^{\mathrm{i}\, a A_n^0}\, e^{\mathrm{i}\, X_{n+1}^i/R}\, e^{-\mathrm{i}\, a A_n^0}\, e^{-\mathrm{i}\, X_n^i/R}, \tag{6}$$

$$e^{\mathrm{i} f_{n,ij}} = e^{\mathrm{i}\, X_n^i/R}\, e^{\mathrm{i}\, X_n^j/R}\, e^{-\mathrm{i}\, X_n^i/R}\, e^{-\mathrm{i}\, X_n^j/R}, \tag{7}$$

The model enjoys the gauge symmetry

$$X_n^i \to U_n\, X_n^i\, U_n^\dagger, \qquad e^{\mathrm{i}\, a A_n^0} \to U_n\, e^{\mathrm{i}\, a A_n^0}\, U_{n+1}^\dagger \tag{8}$$

in which U_n's are unitary matrices depending on discrete time. As expected, it can be shown that at continuum limit the action (5) reduces to the matrix model for 0-branes in imaginary time formalism [5]

$$S_{\mathrm{SU}(2)} \simeq \frac{-a}{g^2 R^2} \int dt\ \mathrm{Tr} \left(\frac{1}{2} \dot{X}_i^2 + \frac{1}{4a^2 R^2} [X^i, X^j]^2 \right). \tag{9}$$

For the SU(2) gauge theory one can use expansions based on Pauli matrices

$$\exp\left(\frac{\mathrm{i}}{2}\,\sigma\cdot Y^i_n\right)=\cos\frac{|Y^i_n|}{2}\,\mathbb{1}+\mathrm{i}\,\sin\frac{|Y^i_n|}{2}\sum_{a=1}^{3}\sigma_{(a)}\widehat{Y}^i_{n(a)} \tag{10}$$

in which $Y^i_n:=X^i_n/R$, and $|Y^i_n|=\sqrt{Y^{i\,2}_{n(1)}+Y^{i\,2}_{n(2)}+Y^{i\,2}_{n(3)}}$, and $\widehat{Y}^i_{n(a)}=Y^i_{n(a)}/|Y^i_n|$; to avoid any confusion all group indices are coming as "(a)" with $a=1,2,3$. The similar expressions can be used for $B^0_n:=aA^0_n$. We have for all group components $-\pi\le B^0_{n(a)}\,\&\,Y^i_{n(a)}\le\pi$. All the trace parts in action (5) can be expressed as

$$\begin{aligned}\mathrm{Tr}\Big[e^{\frac{\mathrm{i}}{2}\sigma\cdot U}e^{\frac{\mathrm{i}}{2}\sigma\cdot V}e^{\frac{-\mathrm{i}}{2}\sigma\cdot U}e^{\frac{-\mathrm{i}}{2}\sigma\cdot W}\Big]=2\Bigg(&\cos\frac{V}{2}\cos\frac{W}{2}\\&+\Big[2(\widehat{U}\cdot\widehat{W})(\widehat{U}\cdot\widehat{V})\sin^2\frac{U}{2}+(\widehat{V}\cdot\widehat{W})\cos U\\&+\widehat{U}\cdot(\widehat{W}\times\widehat{V})\sin U\Big]\sin\frac{V}{2}\sin\frac{W}{2}\Bigg)\end{aligned} \tag{11}$$

At temperature $T=\beta^{-1}$ one has for the free energy $\mathcal{A}(\kappa,T)$

$$e^{-\beta\mathcal{A}(\kappa,T)}=\int_{-\pi}^{\pi}\prod_{m=0}^{\beta-1}\prod_{a=1}^{3}\left[\frac{dB^0_{m(a)}}{2\pi}\prod_{i=1}^{3}\sqrt{2\pi\kappa}\,\frac{dY^i_{m(a)}}{2\pi}\right]e^{S_{\mathrm{SU}(2)}} \tag{12}$$

accompanied by the periodic boundary condition $Y^i_{0(a)}=Y^i_{\beta(a)}$ and $B^0_{0(a)}=B^0_{\beta(a)}$. The normalization factor $\sqrt{2\pi\kappa}$ is inserted to match the contribution from kinetic term with the continuum limit for ordinary particles [5] (like the factor $\sqrt{m_0}$ in path-integral). So there is a factor of $(2\pi\kappa)^{9/2}$ for each $m=0,\cdots,\beta-1$ step of integration.

In contrast to U(1) case in [5] the analytic expressions is not available for the SU(2) case. However one can use the numerical methods to study the thermodynamics by (12). The results presented below are based on the Monte Carlo method for evaluation of (12).

The plots of free energy $\mathcal{A}(\kappa,T)$ versus κ at high temperatures (low $\beta=2,3,4,5\;a$) are presented in Fig. 1. The plots suggest that at high temperatures down to $T_c\simeq 0.25\;a^{-1}$ the free energy does not exhibit a minimum, leading to a phase transition. Comparing to U(1) case ($T_{c\mathrm{U}(1)}=0.247\;a^{-1}$ [5]) proposes the least relevance of the potential term among the matrix coordinates at high temperatures.

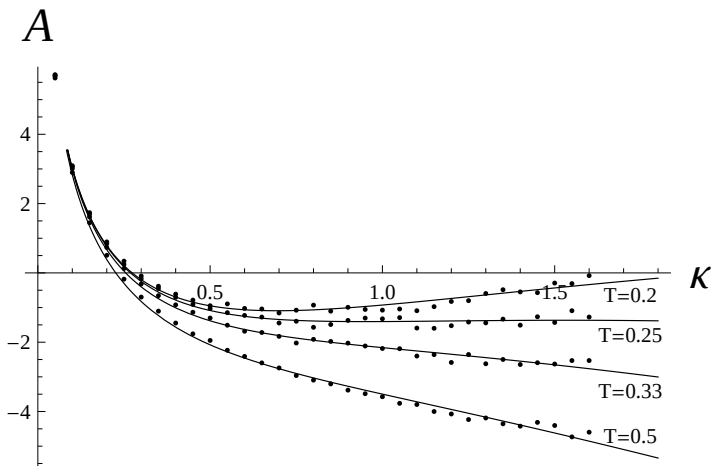

Figure 1: The free energy $\mathcal{A}$ versus κ for different high temperatures (unit: a^{-1}), suggesting that the appearance of minimum occurs around $T \simeq 0.25$.

The results at low temperatures (high $\beta = 10, 20, 40, 100\ a$) are presented in Fig. 2. As for the low temperatures the leading contribution to the partition function comes from the ground state, we can have an estimation for the behavior of E_0:

$$\lim_{T\to 0} \mathcal{A}(\kappa, T) \to -T\ \ln e^{-E_0(\kappa)/T} = E_0(\kappa) \tag{13}$$

The results for temperature $T = 0.01\ a^{-1}$ is best fitted by a slight modification of the expression (3) for the U(1) theory as

$$E_0^{\mathrm{SU(2)}}(\kappa) = -\frac{1}{a} \ln \left[(2\pi\kappa)^{9/2} e^{-12\, b\, \kappa} (I_0(\kappa))^c \right] \tag{14}$$

with the below for values of b and c

$$b = 0.928[\pm 0.005], \qquad c = 0.219[\pm 0.048]. \tag{15}$$

We mention that the extra powers "9/2" and "12" in (14) are originated from expressions (12) and (5). The minimum of (14) is at $\kappa_c = 0.41$ corresponding to the critical coupling $g_c = \kappa_c^{-0.5} = 1.56$, above which no phase transition occurs. Comparing with the U(1) case for which we have an asymptotic form as [5]

$$\lim_{\kappa\to\infty} E_0^{\mathrm{U(1)}} \to 0^- \tag{16}$$

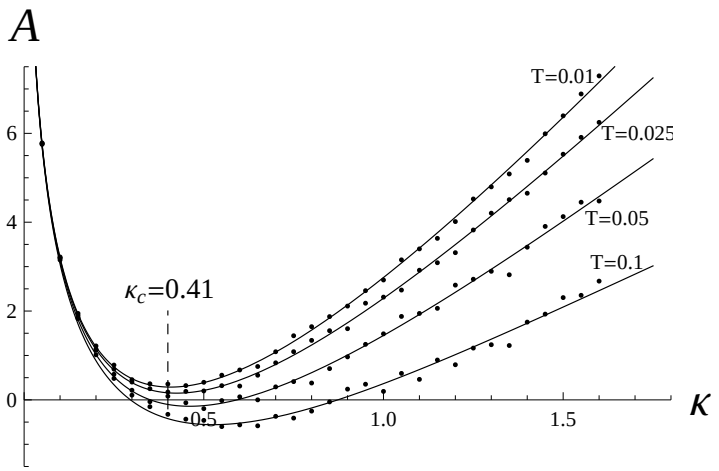

Figure 2: The free energy $\mathcal{A}$ versus κ for different low temperatures (unit: a^{-1}).

here E_0 has a much deeper minimum. The different behaviors are due to the definite contribution of the potential term of matrix coordinates in SU(2) theory at low temperatures.

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