Up and Down-Quark Masses

P. R. Silva – Retired associate professor – Departamento de Física – ICEx – Universidade Federal de Minas Gerais (UFMG) – email: prsilvafis@gmail.com

Abstract- The Gell-Mann, Oakes and Renner and the Goldberger-Treiman relations jointly with a bold hypothesis about virtual thermal equilibrium, leads to the determination of the up and down-quarks current masses. The obtained results show god agreement with the best ones of the literature.

1 – Introduction

Besides the constituent mass, acquired due to the confinement explained by the Quantum Chromodynamics [1], down-up quarks current-mass difference seems to be related to the neutron-proton mass difference [2]. In this work we intend to estimate the up and down-quarks current (bare) masses. First we will use in section 2, the Gell-Mann, Oakes and Renner [3] and the Goldberger and Treiman [4] relations to determine the average of the up and down quarks bare masses. In section 3, we make a bold hypothesis which links the down-up quarks bare masses difference to that of the neutron-proton. In section 4 we combine the results of sections 2 and 3, as a means to determine the up and down-quarks current masses. Section 5 gives some concluding remarks.

2 – The averaged up-down quarks masses

The Gell-Mann, Oakes, Renner relation [3] can be written as

$$m_\pi^2\, f_\pi^2 = -\tfrac{1}{2}\,(m_u + m_d) < \underline{\Psi}\Psi >. \qquad (1)$$

In (1) we have: the pion, the up and down-quarks masses and the quark condensate $< \underline{\Psi}\Psi >$. We will take the quark condensate as given by (being m the constituent-mass of the quarks)

$$< \underline{\Psi}\Psi > = - m^3. \qquad (2)$$

For compactness, we also define

$$½ (m_u + m_d) = m_{ud}. \qquad (3)$$

Inserting (2) and (3) into (1), we get

$$m_\pi^2 f_\pi^2 = m_{ud} m^3. \qquad (4)$$

Now, we will make use of the Goldberger-Treiman relation

$$g_{\pi qq} f_\pi = m. \qquad (5)$$

In (5), $g_{\pi qq}$ is the pion-quarks coupling [5], and f_π is the pion decay constant [6].

Inserting (5) into (4) and solving for m_{ud}, we find

$$m_{ud} = m_\pi^2 / (f_\pi g_{\pi qq}^3) = (3m_\pi) / (2g_\pi^3). \qquad (6)$$

In the last equality of (6), we took into account that $f_\pi = \frac{2}{3} m_\pi$ [7], and used a simplified notation for the g-coupling, making the identification

$$g_\pi \equiv g_{\pi qq}. \qquad \text{(I)}$$

3 – Virtual Thermal Equilibrium

In this section we aim to compare the up-down quarks mass difference with that of the neutron-proton. As a means to do this, let us write in a symbolic way and by considering only quarks and hadrons, in the "reactions" occurring in the neutron decay. First we look inside the nucleons and write

$$d + d + u \rightarrow u + u + d \qquad (7)$$

In (7) we describe a "transmutation" of a down-quark to an up-quark, at the neutron decay process. Meanwhile, in a "global" looking at the hadrons in this reaction, we can write

$$n \rightarrow p \qquad (8)$$

Now we imagine that just at the intermediate step of the process, we can define a "virtual temperature (T_v)", and we think in terms of a virtual thermal equilibrium where an energy equal-partition principle is at work.

With respect to the quarks, their confinement suggests to consider the whole system as a three dimensional harmonic oscillator, and we have

$$\Delta m = m_d - m_u = 3k_B\, T_v. \qquad (9)$$

The nucleon (proton or neutron) behave as a single particle, and we consider in this case the equal-partition applied to an ideal gas, leading to

$$\Delta M = M_n - M_p = (3/2)\, k_B\, T_v. \qquad (10)$$

Combining (9) and (10) leads to

$$\Delta m = 2\, \Delta M. \qquad (11)$$

4 – Determining the Quarks Masses

From the neutron-proton mass difference [8], and by using (11), we can write

$$\Delta m = m_d - m_u = 2.58 \text{ MeV}. \qquad (12)$$

Putting in numbers in relation (6), by taking: g_π =3.752 [5] and m_π = 139.57 MeV [8], we get

$$m_{ud} = ½\, (m_u + m_d) = 3.96 \text{ MeV}. \qquad (13)$$

Finally from (12) and (13) we obtain

$$m_d = 5.25 \text{ MeV}, \qquad \text{and} \qquad m_u = 2.67 \text{ meV}. \qquad (14)$$

Manohar and Sachrajda in reference [9], exhibit some data displaying the results of estimates of the up and down quarks masses. There, they show a diagram with a window for the allowed values of these masses, and the results of (14) are found to be inside this window.

5 – Concluding remarks

A look at the literature [1, 6,10], shows that previous estimates of up and down-quarks masses present values which differs significantly from the updated ones, as those exhibited in reference [9]. It seems that the fact of the present calculations to match the accepted update values of quark masses could be considered as a nice achievement of this work.

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