Divergence-Free Vector Gauge Field Theory

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Abstract

We present results of applying our divergence-free effective action quantum field theory techniques, with powerful implementation of the principle of gauge covariance, to the theory of non-Abelian (Yang-Mills) gauge field theory. This describes the self interactions of a massless vector field. Results of two-loop computations are given demonstrating the simplicity and the viability of the underlying framework.

1 Introduction

The utility of the principle of momentum-space gauge and/or coordinate covariance, as implemented within the divergence-free effective action approach to quantum field theory[1],...,[9], finds supreme expression in non-Abelian gauge theories as well as in quantum gravity. In this letter, we demonstrate this fact with an application to a system of non-Abelian (massless) vector gauge field theory.

After presenting the Lagrangian and the associated Feynman rules, we give the results of applying our divergence-free methods to two loop computations. The underlying gauge algebra of the present example is simply SO_3, which is the same as SU_2. However, extensions of the results to an arbitrary simple Lie algebra should be straightforward.

We shall give results, suppressing much of the detailed derivations (given in comprehensive reports elsewhere[10]).

2 The Lagrangian of Non-Abelian Gauge Field Theory and Graphical Rules

The Lagrangian of non-Abelian gauge field theory is given by

$$\mathcal{L} = -\frac{1}{4} F_{\mu\nu}{}^{a} F_{\mu\nu}{}^{a} \tag{1}$$

where we have the curvature tensor $F_{\mu\nu}{}^{a}$ is given in terms of the vector potentials $A_{\mu}{}^{a}$,

$$F_{\mu\nu}{}^{a} = \partial_\mu A_\nu{}^{a} - \partial_\nu A_\mu{}^{a} + g\epsilon^{abc} A_\mu{}^{b} A_\nu{}^{c} \tag{2}$$

Here, g is a dimensionless coupling constant, ϵ^{abc} is the totally antisymmetric structure constant of SU_2.

Now according to the effective action scheme development, we make the shift ${A_\mu}^a \to {A_\mu}^a + {\mathcal{V}_\mu}^a$, where $\mathcal{V}$ corresponds to the virtual vector components. Accordingly, we have the corresponding shift in curvature components:

$$ {F_{\mu\nu}}^a \to {F_{\mu\nu}}^a + \nabla_\mu {\mathcal{V}_\nu}^a - \nabla_\nu {\mathcal{V}_\mu}^a + g\epsilon^{abc}{\mathcal{V}_\mu}^b{\mathcal{V}_\nu}^c \tag{3} $$

Notice that the covariant derivative of the virtual part is given by

$$ \nabla_\mu {\mathcal{V}_\nu}^a = \partial_\mu {\mathcal{V}_\nu}^a + g\epsilon^{abc}{A_\mu}^b{\mathcal{V}_\nu}^c \tag{4} $$

The shifted Lagrangian provides the needed gauge-covariant bilinears and couplings of the virtual vector components. The pertinent terms are:

$$ \left\{ \begin{array}{l} -\frac{1}{2}\epsilon^{abc}{F_{\mu\nu}}^a{\mathcal{V}_\mu}^b{\mathcal{V}_\nu}^c - \frac{1}{2}\nabla_\mu {\mathcal{V}_\nu}^a \left(\nabla_\mu {\mathcal{V}_\nu}^a - \nabla_\nu {\mathcal{V}_\mu}^a\right) \\ \\ -\epsilon^{abc}\nabla_\mu {\mathcal{V}_\nu}^a{\mathcal{V}_\mu}^b{\mathcal{V}_\nu}^c - \frac{1}{4}\left(\epsilon^{abc}{\mathcal{V}_\mu}^b{\mathcal{V}_\nu}^c\right)^2 \end{array} \right. \tag{5} $$

Manipulating the terms that are bilinear in the virtual part, we obtain

$$ \frac{1}{2}{\mathcal{V}_\mu}^a \left\{\nabla^2\delta^{ab}\eta_{\mu\nu} - 2g\epsilon^{abc}{F_{\mu\nu}}^c\right\}{\mathcal{V}_\nu}^b \tag{6} $$

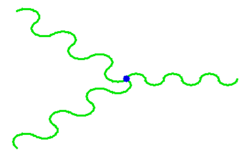
Notice that we have dropped terms that involve $\nabla_\mu {\mathcal{V}_\mu}^a$. The procedure preserves the effective gauge invariance.[1]

Hence, we derive the following basic graphical rules (in Minkowskian momentum space):

- For each internal line or propagator of the virtual vector (with momentum p) we must write
$$ \frac{1}{-p^2+m^2}\delta^{ab}\eta_{\mu\nu} $$
Notice that this is just the bare propagator. But how is gauge invariance guaranteed in the perturbative development via appropriate insertions, and related matters, is explained in detail, in earlier letters, and other comprehensive reports[1],...,[10]. This is depicted like a wavy line. Notice that we have included a mass-regulating parameter. This prescription is gauge invariant, as far as the effective field is concerned.

- For the bare trilinear vertex, we write
$$ ig\epsilon^{abc}\left\{\eta_{\mu\lambda}(p_\nu - r_\nu) + \eta_{\nu\lambda}(r_\mu - q_\mu) + \eta_{\mu\nu}(q_\lambda - p_\lambda)\right\} $$
Here, p, q, r are the momenta of the incoming vectors, μ, ν, λ the respective vector indices, and a, b, c the respective SU_2 indices, and we have the following depiction:

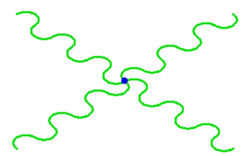
[1] ***Important Note***: There is a numerical error in Eqs. (7) and (8) of Ref. [4], namely the factor 3/2 associated with the curvature term in the bilinear. Whereas that error had no consequences in that article, it is important to understand its origin. In the operator notation used there, we have used the wrong expression $[\nabla_\mu, \nabla_\nu]\Phi = -iF_{\mu\nu}\Phi$, whereas the correct thing to use is $[\nabla_\mu, \nabla_\nu]\Phi = -i[F_{\mu\nu}, \Phi]$



- For the bare quartilinear vertex, we write

$$g^2 \left\{ \begin{array}{l} \eta_{\mu\nu}\eta_{\lambda\rho}\left(\delta^{ad}\delta^{bc} + \delta^{ac}\delta^{bd} - 2\delta^{ab}\delta^{cd}\right) \\ \\ \eta_{\mu\lambda}\eta_{\nu\rho}\left(\delta^{ad}\delta^{bc} - 2\delta^{ac}\delta^{bd} + \delta^{ab}\delta^{cd}\right) \\ \\ \eta_{\mu\rho}\eta_{\lambda\nu}\left(-2\delta^{ad}\delta^{bc} + \delta^{ac}\delta^{bd} + \delta^{ab}\delta^{cd}\right) \end{array} \right\}$$

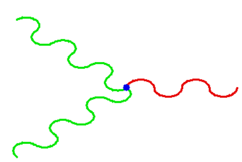
  Here, μ, ν, λ, ρ, are the respective vectorial indices, while a, b, c, d the respective SU_2 counterparts, and with the following depiction:



- Whereas the above couplings concern the virtual vector field only, we also have an external field-strength insertion (in terms of ${F_{\mu\nu}}^a$ rather than the potential ${A_\mu}^a$) on virtual propagators,

$$-2g\epsilon^{abc}{F_{\mu\nu}}^c$$

  corresponding to the following depiction

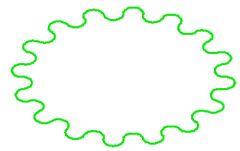


- We must associate a factor of i for each propagator, a factor of i for each vertex, and an overall factor of $-i$ for each graph.
- We must supply the appropriate combinatoric factors for each graph.
- Most importantly, we must supply the appropriate *regularizing parameters* and the corresponding *pole-removing operators*, together with the gamma functions factors, and Feynman parameter combinations, all according to our divergence-free methods.[1]

In the following section, we shall display associated graphics and computational results suppressing all details.

3 Vacuum Contributions

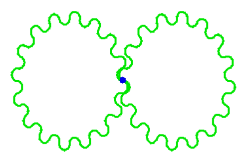
We first have the 1-loop vacuum contribution corresponding to the graph



This gives

$$\frac{3m^4}{32\pi^2}\left\{3 - 2\ln(m^2)\right\} \tag{7}$$

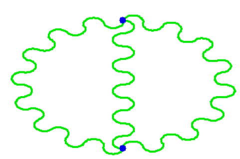
There are two, 2-loop vacuum contributions. The 1st corresponds to the graph



and gives

$$-\frac{9g^2m^4}{128\pi^4}\left\{-1 + \ln(m^2)\right\}^2 \tag{8}$$

The 2nd corresponds to the graph



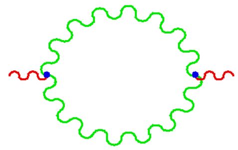
and gives (approx.)

$$\frac{g^2m^4}{1433600\pi^4}\left\{-33461 - 114900\ln(m^2) + 126000\ln^2(m^2)\right\} \tag{9}$$

The alert readers need not be told over and over again[1],...,[9] about the importance of using the above contributions in order to fix the vacuum and determine iteratively the value of $\ln(m^2) = 3/2 + O(g^2)$.

4 Bilinear Contributions

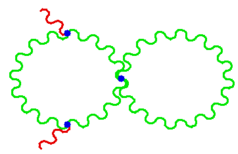
Here we present the 1-loop and 2-loop contributions to the effective bilinears. These are the coefficients of the Lagrangian term of the form $(F_{\mu\nu}{}^a)^2$. The 1-loop contribution corresponds to the graph



which gives

$$-\frac{g^2}{8\pi^2}\ln(m^2) \tag{10}$$

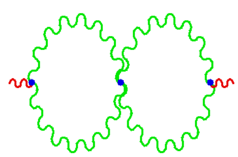
On the other hand, we have four, 2-loop contributions. The 1st corresponds to the graph



giving

$$-\frac{3g^4}{64\pi^4}\{-1+\ln(m^2)\} \tag{11}$$

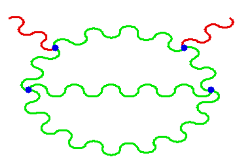
The 2nd corresponds to the graph



giving

$$-\frac{3g^4}{128\pi^4}\ln^2(m^2) \tag{12}$$

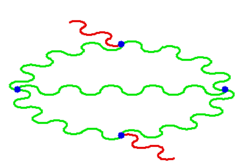
The 3rd corresponds to the graph



giving (approx.)

$$\frac{g^4}{860160\pi^4}\left\{-6689+11940\ln(m^2)\right\} \tag{13}$$

The 4th corresponds to the graph



giving (approx.)

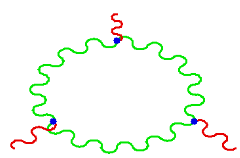
$$\frac{g^4}{286720\pi^4}\left\{-2825 + 7428\ln(m^2) + 3360\ln^2(m^2)\right\} \tag{14}$$

5 Trilinear Contributions

Here we present the 1-loop and 2-loop contributions to the effective trilinears. These are the coefficients of the Lagrangian term of the form

$$\epsilon^{abc} F_{\mu\nu}{}^{a} F_{\mu\lambda}{}^{b} F_{\nu\lambda}{}^{c} \tag{15}$$

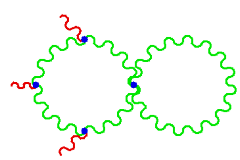
The 1-loop contribution corresponds to the graph



which gives

$$\frac{g^3}{24m^2\pi^2} \tag{16}$$

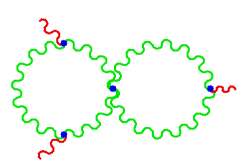
On the other hand, we have five, 2-loop contributions. The 1st corresponds to the graph



giving

$$-\frac{g^5}{64m^2\pi^4}\{-1 + \ln(m^2)\} \tag{17}$$

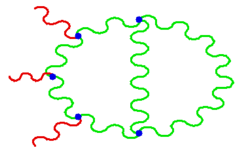
The 2nd corresponds to the graph



giving

$$\frac{3g^5}{128m^2\pi^4}\ln(m^2) \tag{18}$$

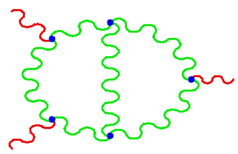
The 3rd corresponds to the graph



giving (approx.)

$$\frac{57g^5}{14336m^2\pi^2} \tag{19}$$

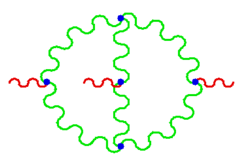
The 4th corresponds to the graph



giving (approx.)

$$-\frac{2179g^5}{215040m^2\pi^4} \tag{20}$$

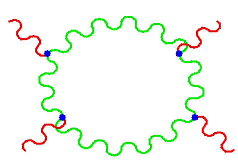
The 5th corresponds to the graph



This gives a vanishing result!

6 Quartilinear Contribution

Here we give the 1-loop contribution to the Lagrangian term that is quartilinear in the curvature tensor. This corresponds to the graph



This gives

$$\frac{g^4}{24m^4\pi^2}F_{\mu\nu}{}^{a}F_{\nu\lambda}{}^{a}F_{\lambda\rho}{}^{b}F_{\rho\mu}{}^{b} \tag{21}$$

7 Discussion

This letter demonstrates that the divergence-free effective action scheme for quantum field theory, combined with the principle of gauge-covariance, is a powerful and a very economical approach to the development of Feynman-like perturbative development. Whereas we have given results that pertain to non-Abelian vector gauge theory, similar results may be given to Einstein-like gravidynamic theories.

Readers who believe they have understood well our theoretical framework, and who think they had become proficient in applying the associated techniques, should be able to reproduce all the preceding results, as well as the results of all our preceding letters, without great toil. In any case, readers who are serious about their pursuit of divergence-free quantum field theory, should not be satisfied with the general perspective, but should get into the finer elements of the development. [10]

References

[1] N.S. Baaklini, "Effective Action Framework for Divergence-Free Quantum Field Theory", *N.S.B. Letters*, **NSBL-QF-010**; http://www.vixra.org/abs/1312.0056

[2] N.S. Baaklini, "The Divergence-Free Effective Action for a Scalar Field Theory", *N.S.B. Letters*, **NSBL-QF-014**, http://www.vixra.org/abs/1312.0065

[3] N.S. Baaklini, "The Divergence-Free Effective Action for Quantum Electrodynamics", *N.S.B. Letters*, **NSBL-QF-015**, http://www.vixra.org/abs/1401.0013

[4] N.S. Baaklini, "Framework for the Effective Action of Quantum Gauge and Gravitational Fields", *N.S.B. Letters*, **NSBL-QF-011**, http://www.vixra.org/abs/1401.0121

[5] N.S. Baaklini, "Graphs and Expressions for Higher-Loop Effective Quantum Action", *N.S.B. Computing*, **NSBC-QF-005**, http://www.vixra.org/abs/1402.0085

[6] N.S. Baaklini, "Divergence-Free Quantum Gravity in a Scalar Model", *N.S.B. Letters*, **NSBL-QF-041**, http://www.vixra.org/abs/1604.0115

[7] N.S. Baaklini, "Divergence Free Non-Linear Scalar Model", *N.S.B. Letters*, **NSBL-QF-043**, http://www.vixra.org/abs/1604.0156

[8] N.S. Baaklini, "Divergence-Free Scalar Electrodynamics", *N.S.B. Letters*, **NSBL-QF-045**, http://www.vixra.org/abs/1604.0341

[9] N.S. Baaklini, "Divergence-Free Quantum Electrodynamics", *N.S.B. Letters*, **NSBL-QF-047**,
http://www.vixra.org/abs/1605.0202

[10] *Divergence-Free Quantum Field Theory* by N.S. Baaklini (Complete Updated Development)

For Those Who Seek True Comprehension of Fundamental Theoretical Physics