<u>Multifractal Geometry and Stochastic Quantization: A Brief Comparison</u>

Ervin Goldfain

Advanced Technology and Sensor Group, Welch Allyn Inc., Skaneateles Falls, NY 13153

Abstract

We suggest that the multifractal geometry of the Standard Model (SM) near the electroweak scale may be placed on equal footing with the *stochastic quantization* method. This analogy gives support to earlier attempts by Beck to derive the SM parameters using the dynamics of *coupled map lattices*.

The object of this short note is to highlight the connection between multifractal geometry of the SM, introduced in [1-3], and *stochastic quantization*, the latter being built on the analogy between Euclidean QFT and equilibrium Statistical Mechanics [4-6].

Stochastic quantization identifies the Euclidean path integral measure of Quantum Field Theory, $\exp\{-S_E\}/\mathrm{D}\varphi\exp\{-S_E\}$, (with $\hbar=1$) with the stationary distribution of a stochastic process. This interpretation implies that Euclidean Green functions of Quantum Field Theory are indistinguishable from to the correlation functions of equilibrium statistical mechanics (see (3) and (4) below).

The key premise of stochastic quantization is that the fields present in the theory ($\varphi(x)$) are supplemented with an additional coordinate called "*fictitious*" time τ [4-6]. Fields become coupled to a fictitious heat reservoir which relaxes to thermal equilibrium as $\tau\to\infty$. In this picture, the fictitious time evolution of $\varphi(x)$ in four-dimensional

Euclidean space resembles a *continuous random walk*. It is described by a stochastic differential equation of the Langevin or Fokker-Planck type. For example,

$$\frac{\partial\varphi(x,\tau)}{\partial\tau} = -\frac{\partial S_E}{\partial\varphi(x,\tau)} + \eta(x,\tau) \tag{1}$$

Here, S_E is the Euclidean action of the system, obtained by integration over τ

$$S_E = \int d\tau\, d^4x\, L(\varphi(x,\tau), \frac{\partial\varphi(x,\tau)}{\partial x}) \tag{2}$$

and $\eta(x,\tau)$ stands for delta-correlated Gaussian noise. In the equilibrium limit $\tau\to\infty$, equal time correlation functions of the fields are shown to be identical to the corresponding quantum Green functions, i.e.

$$\lim_{\tau\to\infty}\langle\varphi(x_1,\tau)...\varphi(x_k,\tau)\rangle = \langle\varphi(x_1)...\varphi(x_k)\rangle \tag{3}$$

where

$$\langle\varphi(x_1)...\varphi(x_k)\rangle = \frac{\int \mathrm{D}\varphi \exp(-S_E)\varphi(x_1)...\varphi(x_k)}{\int \mathrm{D}\varphi \exp(-S_E)} \tag{4}$$

It was shown in [1, 2] that, near the electroweak scale M_{EW}, the spectrum of particle masses m_i entering the SM satisfies the closure constraint

$$\sum_{i=1}^{16} r_i^2 = \sum_{i=1}^{16}(\frac{m_i}{M_{EW}})^2 = 1 \tag{5}$$

Since (5) reflects a typical relationship in the theory of multifractal sets, it allows for a direct connection between multifractal geometry of the SM and stochastic quantization. To show this, let us start from

$$1-\varepsilon_i = 1-(\frac{m_i}{M_{EW}})^2(\frac{M_{EW}}{\Lambda_{UV}})^2 = 1-r_i^2\varepsilon_0 = O(1)\,, \quad \varepsilon_i = r_i^2\varepsilon_0 = O(4-D) << 1 \tag{6}$$

in which $\varepsilon_i, \varepsilon_0$ represent arbitrarily small deviations from the four-dimensionality of classical spacetime [1, 2]. The fictitious time of stochastic quantization can be then naturally interpreted as $\tau = \langle \tau_i \rangle$, with $\tau_i = O(\varepsilon_i^{-1})$. Conventional formulation of Quantum Field Theory is recovered in the deep infrared limit $\tau = \langle \tau_i \rangle \to \infty$, $\varepsilon_0, \varepsilon_i \to 0$, where spacetime dimensionality settles at $D = 4$.

The analogy sketched here supports the connection between stochastic quantization via the dynamics of *coupled map lattices* and the structure of SM parameters [7-9].

References

[1] This reference can be located at:

http://www.aracneeditrice.it/aracneweb/index.php/pubblicazione.html?item=9788854889972

A copy of the book is available at:

https://www.researchgate.net/publication/278849474_Introduction_to_Fractional_Field_Theory_consolidated_version

[2] http://www.prespacetime.com/index.php/pst/article/viewFile/638/636

[3] This reference can be located at:

http://www.ingentaconnect.com/content/asp/qm/2014/00000003/00000003/art00012?crawler=true

[4] https://homepage.univie.ac.at/helmuth.hueffel/PhysRep.pdf

[5] http://tkynt2.phys.s.u-tokyo.ac.jp/~hirano/files/lecturenote09_nakazato.pdf

[6] https://arxiv.org/pdf/0903.0732.pdf

[7] http://www.worldscientific.com/worldscibooks/10.1142/4853

[8] https://arxiv.org/abs/hep-th/0207081

[9] This reference can be located at:

http://citeseerx.ist.psu.edu/viewdoc/download?doi=10.1.1.139.8301&rep=rep1&type=pdf