Charge Quantization as a Quantum Theory Law in sub-Planckian Spacetime

Joseph F. Messina*†

Abstract

It is shown that the quantization of electric charge can be explained, in a fundamentally consistent manner, as a manifestation of the quantization of the *intrinsic* vibrational energy of the *fabric* of spacetime by a *non-Planckian* "action" in *sub-Planckian* spacetime. It is found that this conceptualization of the elementary charge provides a natural explanation of some of the more vexing questions that have plagued quantum electrodynamics since its inception. A possible experiment is suggested that might test for the presence of such a non-Planckian "action" in gravitational radiation.

Keywords: Electric Charge; Quantization; Fabric of Spacetime; non-Planckian "Action" Constant; sub-Planckian Spacetime

PACS (2010): 41.20.Cv; 12.90.+b; 03.65.Ca; 03.65.Ta; 04.80.Nm

1 Introduction

Electric charge has been described as that certain *je ne sais quoi* that makes a particle attractive to the opposite kind of particle [1]. Richard Feynman used to refer to it as that experimentally determined number (1/137.036) that all good theoretical physicists put up on their wall and worry about [2]. While it is true that the conservation of charge is understood, its physical nature and dimensionality are not.

*APS Division of Gravitational Physics, P.O.Box 130520, The Woodlands, TX 77393, USA
†jfmessina77@yahoo.com

The object of this paper is to show that a satisfactory understanding of the quantization of electric charge may be possible based on the fundamental assumption that electric charges are a manifestation of the *intrinsic* vibrational energy of the *fabric* of spacetime, whose quantization is governed by a *non-Planckian* "action" in *sub-Planckian* spacetime.

The Planck length 1.616×10^{-33} cm has been the subject of much speculation since it was introduced into physics by Max Planck over a century ago. It has long been assumed that this is the shortest possible distance at which quantum mechanical processes have any meaning, which is clearly unsettling since it implies that quantum theory breaks down at sub-Planckian distances. It should be emphasized, however, that this assumption is so far only an extrapolated hypothesis unsupported by experimental evidence. In face, as we shall see below, it is possible, utilizing dimensional analysis, to formulate a *viable* system of absolute units, more *diminutive* than Planck's, whose *size* suggests that they may be the key to some essential structure.

2 Charge Quantization in sub-Planckian Spacetime

In order to give physical meaning to this hypothesis, it is necessary to formulate a system of absolute units based on the elementary charge (e), the gravitational constant (G), and the velocity of light (c), in the form:

$$M_0 = \left(\frac{e^2}{G}\right)^{1/2} = 1.859 \times 10^{-6} \text{ g} \tag{1}$$

$$L_0 = \left(\frac{e^2 G}{c^4}\right)^{1/2} = 1.380 \times 10^{-34} \text{ cm} \tag{2}$$

$$T_0 = \left(\frac{e^2 G}{c^6}\right)^{1/2} = 4.605 \times 10^{-45} \text{ sec} \tag{3}$$

which, for convenience, are expressed in CGS units. It will be seen at once that the magnitude of the absolute unit of length, L_0, deriving from Eq. (2), is an order of magnitude *smaller* than the Planck length. Hence, it is at a scale *below* the Planck length that we might expect to find a connection between the *elementary* charge e and the *intrinsic* vibrational energy of the *fabric* of spacetime.

It is a remarkable fact that the properties of the elementary processes imposed by this more diminutive system of absolute units makes the formulation of a quantum theoretic conceptualization of the elementary charge appear obligatory. Its consistency is most eloquently communicated by its mathematical simplicity as evidenced by Eq. (1), which we shall take as our starting point by expressing the gravitational mass equivalent, M_0, of the *electrostatic* potential energy, in terms of Einstein's mass-energy relation

$$E_0 = M_0 c^2. \tag{4}$$

Let us now assume that the *fabric* of the spacetime vibrates with an *intrinsic* vibrational energy M_0c^2, and frequency $1/T_0$, which is the reciprocal of the period, T_0, deriving from Eq. (3). It will then be seen that these two quantities are linked by a *non-Planckian* constant that has the *same* dimensions as Planck's quantum of "action" $\hbar$. Its magnitude is given by

$$\frac{M_0 c^2}{T_0^{-1}} = 7.695 \times 10^{-30} \text{ erg·sec} \tag{5}$$

which leads to the relation

$$M_0 c^2 = j\nu_0 \tag{6}$$

where for simplicity of presentation we have denoted the "action" constant M_0c^2/T_0^{-1} by the symbol j, and put the reciprocal of the period T_0 equal to the frequency ν_0. The corresponding wavelength, denoted by λ_0, can then be expressed in terms of the momentum M_0c,

$$\begin{aligned} \lambda_0 &= \frac{j}{M_0 c} \\ &= 1.380 \times 10^{-34} \text{ cm} \end{aligned} \tag{7}$$

which is consistent with the absolute unit of length of Eq. (2). The energy *per cycle* can then be expressed in the form

$$\begin{aligned} E_{pc} &= (j\nu_0)\,\lambda_0 \\ &= 2.306 \times 10^{-19} \text{ erg·cm} \end{aligned} \tag{8}$$

which yields

$$\begin{aligned} e &= \sqrt{(j\nu_0)\,\lambda_0} \\ &= 4.802 \times 10^{-10} \text{ esu} \end{aligned} \tag{9}$$

in quantitative agreement with the experimental value from which it draws its justification. We have thus achieved an easily interpreted expression for the quantization of electric charge as a *quantum theory law*.

3 Discussion

This conceptualization of the elementary charge as a quantized vibration of the fabric of spacetime is of the utmost significance as evidenced by the fact that in addition to affirming the applicability of quantum theory down to a distance of 10^{-34} cm, it has revealed some of the oddities indigenous to the fabric of spacetime *below* the Planck cutoff (10^{-33} cm). Most notably, the revelation that at a length scale of 10^{-34} cm the quantum of "action" (denoted by j) is *smaller* than Planck's reduced "action" constant, $\hbar$, by a factor of 0.00729; recognizable as the dimensionless coupling constant of quantum electrodynamics, the so-called fine-structure constant α, which is quantitatively expressed as $e^2/\hbar c = 1/137.036$. This insight is particularly satisfying since it provides us with a theoretical understanding of the numerical *value* and *origin* of this dimensionless number. Indeed, it strongly supports this hypothesis.

Most importantly, as evidenced by the results presented above, the disruptive effects that were expected to occur below the Planck length were found to be baseless. Quite the opposite, it turned out to be an entry way to an unexplored level of reality whose governing laws are dictated by a more *diminutive* "action" constant; an intrinsic property that is fundamentally reassuring since, in addition to underscoring the *dynamic* role of the fabric of spacetime, it relegates to the quantum the *primary* role of describing the fundamental constituents of the physical world.

4 Appendix — Possible Experiment

To determine unequivocally if electric charges are a vibratory property of the fabric of spacetime, similar to the vibrations we refer to as gravitational waves, one must be able to determine experimentally if this newly derived *non-Planckian* quantum of "action," in *sub-Planckian* spacetime, is an intrinsic property of the fabric of spacetime.

A relatively straightforward experiment is suggested by the above considerations. To be more specific, if, as we have seen, at a distance of 10^{-34} cm the quantum of "action" is *smaller* than Planck's reduced constant, $\hbar$, by a factor of 0.00729, then it should be possible to differentiate between these two elementary "actions" from the displacement *amplitude* produced by a gravitational wave when it interacts with a resonant-mass. For a given bandwidth the best solution for measuring such an effect is a spherically shaped detector. In addition to maximizing gravitational wave absorption, spherical detectors are omnidirectional, which means that they have the same sensitivity in any direction of observation. As a result, only a single detector is needed to determine the direction and polarizations of the incoming wave. One of the two more innovative of these detectors is the Schenberg resonant-mass telescope in Brazil [3], which is designed to sense multipole modes of vibration. When fully operational it will provide information regarding a wave's amplitude, polarization, and direction of source. The detector program, which we shall presently exploit, uses an 1150 Kg spherical resonant-mass made of CuAl (6%) alloy, and has a *resonance* frequency ν of 3200 Hz. We may then express the energy, E, for a single quantum of excitation, as a function of Planck's constant, $\hbar$, in the form

$$E = \hbar\omega \tag{1}$$

where $\hbar = 1.055 \times 10^{-34} J \cdot s$ and ω is the *angular* frequency ($2\pi\nu$). We can then profit from the fact that the *vibrational* energy induced in the spherical mass by a gravitational wave can be converted into a value for the *actual* displacement of the sphere by making use of the relation between amplitude x, energy E, and the total mass M for a harmonic oscillator, in the familiar form

$$E = 1/2 M\omega^2 x^2. \tag{2}$$

It is then possible, using Eq. (2), to calculate the displacement caused by a single quantum of excitation by putting energy$=\hbar\omega$, and substituting the designated values,

$$\begin{aligned} x &= \left(\frac{2\hbar}{M\omega}\right)^{1/2} \\ &\simeq 3.02 \times 10^{-21} m \end{aligned} \tag{3}$$

which corresponds to the detector's *quantum limit* (expressed in meters for purely practical reasons). A comparison with the *derived* non-Planckian "action" constant, j, is then possible by putting energy$= j\omega$, and, once again, substituting the designated values,

$$\begin{aligned} x &= \left(\frac{2j}{M\omega}\right)^{1/2} \\ &\simeq 2.58 \times 10^{-22} m \end{aligned} \tag{4}$$

where $j = 7.695 \times 10^{-37} J \cdot s$. It will thus be seen that if Eq. (4) corresponds to reality the resulting displacement will be *smaller* than the detector's quantum limit by a factor of 0.0854, which is simply the square root of the ratio of these two elementary "action" constants, $j/\hbar$.

A similar spherical detector known as MiniGRAIL is presently under development in the Netherlands [4]. It consists of a 1400 Kg spherical test mass, made of CuAl (6%) alloy, which has a resonance frequency ν of 3000 Hz. In order to facilitate a comparison of these two elementary "actions" in the experimental arena we shall utilize the same procedure as before by substituting the designated value for the spherical test mass and the angular frequency ω in Eqs. (3) and (4), respectively,

$$\begin{aligned} x &= \left(\frac{2\hbar}{M\omega}\right)^{1/2} \\ &\simeq 2.82 \times 10^{-21} m \end{aligned} \tag{5}$$

and

$$\begin{aligned} x &= \left(\frac{2j}{M\omega}\right)^{1/2} \\ &\simeq 2.41 \times 10^{-22} m \end{aligned} \tag{6}$$

which is consistent with the Schenberg results.

To determine unequivocally which of these two "action" constants corresponds to reality will require ultra-high sensitive measurements that extend beyond the detector's quantum limit. Fortunately, such measurements are now possible utilizing quantum *squeezing* technology. It is anticipated that these spherical detectors will be highly sensitive in the 2–4kHz range, suitable for detecting gravitational waves from neutron star instabilities and small black hole mergers.

References

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