Part II - Gravity, Anomaly Cancellation, Anomaly Matching, and the Nucleus

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Abstract

Here we provide a consistent solution of the baryon asymmetry problem. The same model is also able to tell us as to what is the mathematical basis of the "equivalence principle" (i.e. the inertial mass being equal to the gravitational mass). We are also able to see as to wherefrom arises the semi-simple group structure of hadrons as $SU(2)_I \otimes U(1)_B$ (of the pre-eightfold-way-model period). Thus we are able to understand the origin of the Gell-Mann-Nishijima expression for the electric charges, $Q = I_3 + \frac{B}{2}$. This paper is in continuation of my recent paper, "Gravity, Anomaly Cancellation, Anomaly Matching, and the Nucleus" (syedafsarabbas.blogspot.in).

Keywords: Baryon asymmetry, equivalence principle, gravity, mixed gauge gravitational anomaly, Standard Model, anomaly cancellation,'t Hooft anomaly matching condition, nuclear isospin, baryon number

It is well known now that the electric charge, separately for each generation, is consistently quantized in the Standard Model (SM) [1,2]. Only within the periphery of just the anomaly cancellations (without spontaneous symmetry breaking) there arises the issue of the so called "bizarre solution, which is discarded for reasons given in ref. [3]. In a recent paper, "Gravity, Anomaly Cancellation, Anomaly Matching, and the Nucleus" (drafsarabbas.blogspot.in) [4], we utilized the 't Hooft anomaly matching condition to find a resolution of the above bizarreness problem. We found that the first generation of quarks and leptons in the SM, is special in the manner that due to the anomaly matching, it goes over to a unique single nucleon-lepton family. We discussed there [4], the issue of charge quantization in this unique single generation, arsising as a result of anomaly cancellations, plus spontaneous symmetry breaking (SSB), and also without SSB arising therein. Here we continue with these reasonings. First we find how the semi-simple group $SU(2)_I \otimes U(1)_B$ is realized. Also we see how the Gell-Mann-Nishijima expression of the electric charge $Q = I_3 + \frac{B}{2}$ becomes relevant. We also find, in this model, an amazing explanation of the baryon asymmetry conundrum. This model also explains the basis and the origin of the equivalence principle ($m_I = m_G$).

The particle physics Standard Model (SM) is based on the group structure $SU(3)_C \otimes SU(2)_L \otimes U(1)_Y$. This symmetry is spontaneously broken (SSB) to $SU(3)_C \otimes U(1)_{em}$ by an Englert-Brout-Higgs (EBH) field which is an $SU(2)_L$ group doublet [1,2],

$$\phi = \begin{pmatrix} \phi^+ \\ \phi^0 \end{pmatrix} \tag{1}$$

In the SM, the first generation of quarks and leptons is special (i.e. non-repetitive) in a manner that the same quaks form the $SU(2)_F$ doublet of this flavour group. And only this quark doublet is amenable to the 't Hooft anomaly matching condition formalism [5]. Chirality ensures that the fermions are massless. So composites of fundamental entities in the chiral limit may match each other through the 't Hooft anomaly matching condition [5]. We took the first generation as unique. It is unique as the coloured massless u-, d- quarks form an isospin doublet in the SM. Then the only colourless composites that we can create in the ground state are proton (uud - quarks) and neutron (udd - quarks). Now (p,n) do form a massless chiral isopin-doublet. Thus the 't Hooft matching condition is indeed satisfied. (Note the same argument does not go though for u, d and s, the three quark flavours).

Thus due to the above, the first generation of quark-lepton goes over to a unique single (non-repetitive) generation of massless chiral nucleon-lepton as follows [4],

$$N_L = \begin{pmatrix} p \\ n \end{pmatrix}_L, (1, 2, Y_N) \tag{2}$$

$$p_R, (1, 1, Y_p), \quad n_R, (1, 1, Y_n) \tag{3}$$

$$l_L = \begin{pmatrix} \nu_e \\ e \end{pmatrix}_L, (1, 2, Y_l) \quad (4)$$

$$e_R, (1, 1, Y_e) \quad (5)$$

Including the above Y_ϕ, there are six unknown hypercharges here,
Let us define the electric charge operator as,

$$Q = T_3 + b\,Y \quad (6)$$

To start with we have three massless generators W_1, W_2, W_3 of $SU(2)_L$ and X of $U(1)_Y$. SSB by EBH mechanism provides mass to the $W^\pm$ and Z^0 gauge particles while ensuring zero mass for photons [1,2]. Let $T_3 = -\frac{1}{2}$ of the EBH field develop a nonzero vacuum expectation value $< \phi >_0$. Thus,

$$Q < \phi >_0 = 0 \quad (7)$$

The unknown b is thereby fixed,

$$Q = T_3 + (\frac{1}{2Y_\phi})Y \quad (8)$$

In the SM we have to ensure that all the anomalies vanish. Thus we have three anomalies called A, B and C as below [1,2,4]

$$Anomaly\ A: \quad TrY[SU(N_C)]^2 = 0\ ;\ 2Y_N = Y_p + Y_n \quad (9)$$

$$Anomaly\ B: \quad TrY[SU(2)_L]^2 = 0\ ;\ giving\ \ Y_N = -Y_l \quad (10)$$

$$Anomaly\ C: \quad Tr[Y^3] = 0 \quad (11)$$

giving

$$2Y_N^3 - Y_p^3 - Y_n^3 + 2Y_l^3 - Y_e^3 = 0 \quad (12)$$

We still need to have more terms to determine all the hypercharges. The Yukawa mass terms in the SM is.

$$\mathcal{L} = -\phi^\dagger \bar{q}_L \bar{u}_R + \phi q_L \bar{d}_R + \phi e_L \bar{e}_R \quad (13)$$

These (fixing the nucleon mass to be about 940 MeV) bring in

$$Y_p = Y_N + Y_\phi, \quad Y_n = Y_N - Y_\phi, \quad Y_e = Y_e - Y_\phi \quad (14)$$

Thus we find

$$Y_l = -Y_\phi \quad (15)$$

Finally, we get quantized electric charges for this unique nucleon-lepton single generation as,

$$Q(p) = 1, \quad Q(n) = 0 \tag{16}$$

$$Q(\nu_e) = 0, \quad Q(e) = -1 \tag{17}$$

In the above we saw that the three anomalies A,B and C together with SSB through the EBH mechanism and generation of Yukawa masses for the matter particles, do give consistent and complete charge quantization for this unique single generation of nucleon-lepton. Most important to see that these nucleons are taken as fundamental particles and not composites of quarks - the 't Hooft anomaly matching had made these nucleons massless and point-like chiral fermions as fundamental particles belonging to a single generation.

In the SM $SU(3)_C \otimes SU(2)_L \otimes U(1)_Y \to SU(3)_C \otimes U(1)_{em}$. Hence as now $U(1)_{em}$ is functional, the L-handed $\binom{p}{n}$ - doublet, also becomes a $\binom{p}{n}$ - doublet, of the $SU(2)_F$, the flavour group. This is due to the fact that the colour group gives colour-singlet for both the L-handed and the non-chiral $\binom{p}{n}$ - doublet, (i.e. the isospin-group). Next due to the 't Hooft anomaly matching condition, the initial chiral (u d) as colour triplet representation of the $SU(3)_C$ group and also having baryon number $\frac{1}{3}$, goes over to the colour singlet represenation of $SU(3)_C$ and having baryon number "1". Clearly now as to the $SU(2)_I$ isospin group, the baryon number is coming from "outside" itself. Thus the 't Hooft anomaly matching condition induces a new group $U(1)_B$ which gives the baryon number to the nucleon. There exist anti-baryons too in this structure. Thus the group structure for the nucleon is $SU(2)_I \otimes U(1)_B$. Now this also gives the famous Gell-Mann-Nishijima formula for the charges as

$$Q = T_3 + \frac{B}{2} \tag{18}$$

This is actually the pre-eight-fold-way-model structure. Now in our model, we see where the baryon number comes from in the Gell-Mann-Nishijima formula.

Now this is clear that the above single generation with proton and neutron leads to a non-chiral basis and gets its 940 MeV mass through Yukawa coupling. Thus the nucleon as isospin doublet of fundamental particles - proton and neutron, provides a basis for Charge Independence (CI) in nuclear physics. One needs a Generalized Pauli Exclusion Principle (GPEP) which along with the use of the Bruckner-Hartree-Fock analysis brings about and justifies the Independent Particle Model (IPM). This of course is the most successful shell model of the nucleus.

This gives a rationale for the IPM success, and also an understanding of the structure of the nucleus in terms of nucleons treated as fundamental particles - like in the pre-meson and pre-quark days.

In the above we used only the three Anomalies A, B, and C. There is another anomaly arising from the use of gravity, called the mixed-gravitational-gauge-anomay. It is a triangular anomaly with a mixture of one chiral current vertex, with two enegy-momentum tensor (gravitational) vertices [5,6]. This is the way that gravity is brought into play in the long wave length limit [3]. Thus a necessary consequence that the SM couples to the gravity consistently is that the sum of $U(1)_Y$ hypercharges of the Weyl fermions, $TrY = 0$ must vanish. What is the role of this in our model structure? It is clear that the hyperchsarges obtained above automatically satisfy this additional constraint from gravity. This in itself had played no role in the model itself, but the hypercharges obtained are consistent with the mixed-gravitational-gauge-anomaly.

Note that the Yukawa masses generated in our model above, are actually the "inertial masses", call it m_I. Thus the additional consistency with respect to the mixed-gravitational-gauge-anomay just says that this mass is consistent with gravity. Thus it is consistent with the gravitational mass, call it m_G. Thus $m_I = m_G$. This is the celebrated "equivalence principle". This is a phenomenologically justifed principle. However theoretically, as Rindler stated, "the equality of inertial and active gravitational mass (...) remains as puzzling as ever" [9, p.22]. However, this model here, is able to provide a consistent solution and a proper mathematical basis behind the equivalence principle

Next we studied the charge quantization issue in the SM entirely within the anomalies framework. This we do with no SSB through an EBH kind of mechanism [4]. However, these three anomaly conditions A,B and C above are not constraining enough for this purpose [3] . The Amomalies A,B and C give constraints as given above in eqns. (9), (10) and (11). We call the additional fourth mixed-gravitational-gauge-anomay as Anomaly D which gives,

$$Anomaly\ D: \quad TrY = 0 \ ; \ 2\,Y_N - \ Y_p - \ Y_n + 2\,Y_l - Y_e = 0 \tag{19}$$

Putting together all the anomalies [4],

$$Y_p + Y_n + Y_e = 0 \tag{20}$$

Finally we get

$$Y_p = -Y_n \tag{21}$$

$$Y_N = Y_l = Y_e = 0 \tag{22}$$

To identify this solution, and to distinguish it from the others [3], we christen it as the "BIZ" solution [4].

Now the electric charge is defined as in eqn.(6). The parameter 'b' is now an undetermined quantity. However, in the above solution, this 'b' got fixed by the EBH field and SSB arising therein, as in eqn(8). This option is not present in the BIZ model. So using eqn.(6) the charges in the BIZ model are:

$$Q(\dot{p}_L) = T_z(\dot{p}_L) = \frac{1}{2}\ ; \quad Q(\dot{n}_L) = T_z(\dot{n}_L) = -\frac{1}{2} \tag{23}$$

$$Q(\dot{\nu}_L) = \frac{1}{2}\ ; \quad Q(\dot{e}_L) = -\frac{1}{2} \tag{24}$$

$$Q(\dot{p}_R) = bY_p \ ; \quad Q(\dot{n}_R) = -bY_p \tag{25}$$

$$Q(\dot{e}_R) = 0 \tag{26}$$

To distinguish these states from the ones given above in eqns. (16) and (17) we put a dot on the states here and call these "dot-nucleons" and "dot-leptons" [4],

The charges in eqn. (25), we divide off by Y_p,

$$Q'(\dot{p}_R) = b \ ; \quad Q'(\dot{n}_R) = -b \tag{27}$$

In matrix notation we get,

$$Q' \begin{pmatrix} \dot{p_R} \\ \dot{n_R} \end{pmatrix} = \begin{pmatrix} b & 0 \\ 0 & -b \end{pmatrix} \begin{pmatrix} \dot{p_R} \\ \dot{n_R} \end{pmatrix} = b \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} \begin{pmatrix} \dot{p_R} \\ \dot{n_R} \end{pmatrix} \tag{28}$$

Notice that "b" is actually quantized if treated as the diagonal generator of SU(2). Hence the R-handed dot-nucleons, the quantized R-handed charges are [4].

$$Q' \begin{pmatrix} \dot{p_R} \\ \dot{n_R} \end{pmatrix} = \begin{pmatrix} \frac{1}{2} & 0 \\ 0 & -\frac{1}{2} \end{pmatrix} \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} \begin{pmatrix} ,\dot{p_R} \\ \dot{n_R} \end{pmatrix} = \begin{pmatrix} \frac{1}{2} & 0 \\ 0 & \frac{1}{2} \end{pmatrix} \begin{pmatrix} \dot{p_R} \\ \dot{n_R} \end{pmatrix} \tag{29}$$

These are the R-handed charges.

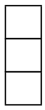
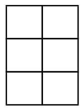
$$Q(\dot{p}_R) = \frac{1}{2} \ ; \quad Q(\dot{n}_R) = \frac{1}{2} \tag{30}$$

Thus the charges in the BIZ model are as in eqns. (23), (24), (30) ands (26). Now both the L-handed and the R-handed doublets exist. Note that for the first case the L-handed doublets are present but no R-handed doublets. However in the dot-nucleon case, both the L-handed and the R-handed doublets are present in equal measure.

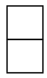
We'd like to point out that the Dirac equation separates out into the L- and R- chiral parts. But the negative energy sea associated with the L- and R- fermions are not separately defined in a gauge invariant way, for the L- and R- fermions [3 p. 423, 8]. But above we saw that the cancellation of the four anomalies ensures that this classical property of the Dirac equation is indeed recovered (see eqns. (23) and (30)) for the case of the dot-nucleons. This is indeed a consistency check on the dot-nucleons solution of the BIZ model.

Now in the first case, we saw that the nucleon-lepton single generation with the EBH field and the SSB, gave a consistent descrition of the IPM (shell-model) of the nucleus. Clearly, this new picture with no Yukawa term, should be simultaneously and dually, be valid for the description of the nucleus. In addition, it should also sit outside and be independent of the IPM model structure of the nucleus. In Ref.[4], we dsicussed how these two independent solutions gave dual and successfull description of the nucleus.

Note that unlike the SSB model above, in the BIZ model case, the group atructure before and after the application of the four anomalies, remains the same i.e. $SU(3)_C \otimes SU(2)_L \otimes U(1)_Y$. Thus the $\binom{p}{n}$ - doublet is colour singlet representation of the $SU(3)_C$ group. This is a baryon. What about antibaryon in this BIZ model? These are shown in the following Young Diagram (YD) (with baryon numbers indicated) [10]

colour singlet baryon $B = 1$  ; *colour singlet antibaryon* $B = -1$  (31)

Now remember that for a single quark and for a single antiquark the colours are represented as

colour 3 *quark* $B = \frac{1}{3}$  ; *colour* $\bar{3}$ *antiquark* $B = -\frac{1}{3}$  (32)

Note the same YD for anti-quark stands for two quarks too. However in that case these two quarks should be anti-symmetric to each other, Then we associate a baryon number $\frac{2}{3}$ with the same YD (eq. (32)),

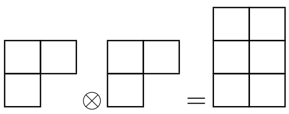
$$\phi^\alpha = \epsilon^{\alpha\beta\gamma}(\phi_\beta\phi_\gamma - \phi_\gamma\phi_\beta) \tag{33}$$

Which for the colour singlet baryon in $3 \otimes 3 \otimes 3 \to 3 \otimes \bar{3} \oplus .. = 1 \oplus 8 \oplus ..$ is a consequence of the above

$$\phi_\alpha\phi^\alpha = \epsilon^{\alpha\beta\gamma}\phi_\alpha(\phi_\beta\phi_\gamma - \phi_\gamma\phi_\beta) \tag{34}$$

Thus in eqn. (31) the colour singlet baryon corresponds to first YD as given. Then the B=-1 representation there should correspond to an antisymmetric state of two baryons with baryon number two which then would couple to B=1 state to give a baryon with baryon number 3.

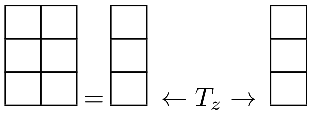
Note that

 $\oplus ...$ $(8 \otimes 8 = 1 \oplus ...)$ (35)

Clearly the B=-1 state in en. (31) corresponds to the singlet above in eqn. (35). This does not simply break up into an antisymmetric state of two colour singlet baryons. If this be so, then it shows that the 't Hooft anomaly matching as used above to go from quarks to nucleon, is untenable. But this should not be so. Thus for anomaly matching to hold, the second YD for B=-1 should break up into antisymmetric state of two colour singlet baryons.

Hence as this break can not occur in the colour space, thus another space should get induced to provide this antisymmetric two baryon structure. Here we argue that indeed the nuclear T_z space is that particular space. It does not exist in the case of isolated protons and neutrons, and comes into existence when a n-p pair forms a bound nucleus - which is the deutron.

Thus B=-1 state in eqn. (31) gets reduced to a B=2 state in the nuclesar T_z space. This happens as the YD provides this freedom along the horizontal direction of the YD. That is,



(36)

This breakup leads to an antisymmetric state in the T_Z space as,

$$\Psi = \frac{1}{\sqrt{2}}(\phi\,\chi - \chi\,\phi) \tag{37}$$

which is the deutron wave function. It is antisymmetric B=2 state of baryons. The point here is that such a state is uniquely arising in the BIZ model, and is outside the purview of the IPM model above. It is well known that in IPM, for two-nucleons, the isospin T=1 state is bound while the T=0 state is unbound. (However one invokes an additional spurious condition, to make T=0 state come down, to be able to explain the existence of the deutron in the IPM - but this is no-go for a single deutron nucleus).

Now as we saw in ref. [4], the charge of the nucleus is,

$$Q = [\frac{1}{2}(Z - N)]_L + [\frac{1}{2}(Z + N)]_R \tag{38}$$

where the third component of the nuclear sispin is given as,

$$T_z = [\frac{1}{2}(Z - N)]_L \tag{39}$$

Thus for N=Z light nuclei the nuclear structure is,

$$(N = Z)_A = (n - p) + (n - p) + (n - p) + (n - p) + ... \tag{40}$$

Here for every (n-p) pair the isospin is zero [4]. Thus for these nuclei the nuclear isospin T_z sinks to the lowest value - that is zero.

Thus as per the BIZ model, there are no antibaryons left in the universe. This is due to the fact that all of them have disappeared inside nuclei, to appear as a dibaryon (actually deutron) with an antisymmetric wave function, This is the solution of the baryon asymmetry problem as to why there are so few antibaryons in the universe. As most of the matter in the universe is there in the form of bound nuclei, this model is a resolution of the baryon asymmetry conundrum. However as single nucleons arising in the IPM model would allow antibaryons to exist. But these will be negligible in number as these would be only available in the vicinity of single nucleon gases in the Univesre.

The fact that the T_z state (in the form of deuteron) plays such a basic role in wiping out the antibaryons, should not be too surprising. Afterall it is the singlet state of the nuclear isospin - and whose origin still is the colour space which induces this T_z. We may say that it is another colour singlet state with a twist!

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