Wherefrom comes the missing baryon number in the Eightfoldway model?

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Abstract

An extremely puzzling problem of particle physics is, how come, no baryon number arises mathematically to describe the spin-1/2 octet baryons in the Eightfold way model. Recently the author has shown that all the canonical proposals to provide a baryon number to solve the above problem, are fundamentally wrong. So what is the resolution of this conundrum? Here we show that the topological Skyrme-Witten model which takes account of the Wess-Zumino anomaly comes to our rescue. In contrast to the two flavour model, the presence of this anomaly term for three flavours, shows that the quantal states are monopolar harmonics, which are not functions but sections of a fiber bundle. This generates a profoundly significant "right hypercharges", which lead to making the adjoint representation of SU(3) as being the ground state. This provides a topologically generated baryon number for the spin-1/2 baryons in the adjoint representation, to connect to the Eightfold way model baryon octet states. This solves the mystery of the missing baryon number in the Eightfold way model.

Keywords: Eightfold way model, missing baryon number, Skyrme-Witten model, Wess-Zumino anomaly, QCD, colour

The doublet $\binom{p}{n}$ corresponds to the fundamental representation of the isospin $SU(2)_I$ group. Hence, when the proliferation of hadrons in the late 1950's promted that the next higher group may be $SU(3)$, then it was natural to assume that it would manifest itself through it's fundamental representation of say $\begin{pmatrix} p \\ n \\ \Lambda \end{pmatrix}$. This was the Sakata model. This assumption was successfull in explaining the structure of the 0^- *and the* 1^+ mesons, but failed to explain the structure of the baryons. This led Ne'eman [1] and Gell-Mann [2] to suggest that when one went from SU(2) to the bigger SU(3) group, then the fundamental representaion of "three" looses its sacrosanct and basic character, and instead in SU(3), the "eight" of the adjoint representation becomes sacrosanct and basic. That was the reason for the highbrow word "Eightfold way" model [3], which was used by Gell-Mann to christen the eight spin 1/2 baryon multiplet and the meson octets. He was influenced by the "Eightfold way" ideology of Gautam Budha's philosophy for humans to lead a complete, wholesome and successful life (or attain "nirvana", so to say).

In the eightfold way model, the octet meson is given as [4],

$$P = \begin{pmatrix} \frac{\pi^0}{\sqrt{2}} + \frac{\eta^0}{\sqrt{6}} & \pi^+ & K^+ \\ \pi^- & -\frac{\pi^0}{\sqrt{2}} + \frac{\eta^0}{\sqrt{6}} & K^0 \\ K^- & \bar{K}^0 & -\frac{2}{\sqrt{6}}\eta^0 \end{pmatrix} \tag{1}$$

In terms of the cartesian coordinates $P_j, j = 1, ..8$ the above states can be written as

$$P = \begin{pmatrix} \frac{P_8}{\sqrt{6}} + \frac{P_3}{\sqrt{2}} & \frac{P_1 - iP_2}{\sqrt{2}} & \frac{P_4 - iP_5}{\sqrt{2}} \\ \frac{P_1 + iP_2}{\sqrt{2}} & \frac{P_8}{\sqrt{6}} - \frac{P_3}{\sqrt{2}} & \frac{P_6 - iP_7}{\sqrt{2}} \\ \frac{P_4 + iP_5}{\sqrt{2}} & \frac{P_6 + iP_7}{\sqrt{2}} & -\frac{2}{\sqrt{6}}P_8 \end{pmatrix} = \frac{1}{\sqrt{2}}\lambda_j P_j \tag{2}$$

In the same manner, the $1/2^+$ baryons octet can be described either by an 8-dimensional vector or by a traceless matrix $\frac{1}{\sqrt{2}}\lambda_j \mathcal{B}_j$. Here the components of $\mathcal{B}_j$ are related to the physical baryons N, Λ, Σ, Ξ in the same manner as for the above mesons as:

$$B = \begin{pmatrix} \frac{\Lambda^0}{\sqrt{6}} + \frac{\Sigma^0}{\sqrt{2}} & \Sigma^+ & p \\ \Sigma^- & \frac{\Lambda^0}{\sqrt{6}} - \frac{\Sigma^0}{\sqrt{2}} & n \\ \Xi^- & \Xi^0 & -\frac{2}{\sqrt{6}}\Lambda^0 \end{pmatrix} \tag{3}$$

Note that here there are no quark fields, which have been abandoned [4, p. 79] in favour of "physical" P_j and $\mathcal{B}_j$ fields reaspectively. Let us write the baryon octet as follows:

$$B = \begin{pmatrix} \frac{\Sigma^+ + \Sigma^-}{\sqrt{2}} \\ i\frac{\Sigma^+ - \Sigma^-}{\sqrt{2}} \\ \Sigma^0 \\ \frac{p - \Xi^-}{\sqrt{2}} \\ i\frac{p + \Xi^-}{\sqrt{2}} \\ \frac{n + \Xi^0}{\sqrt{2}} \\ i\frac{n - \Xi^0}{\sqrt{2}} \\ \Lambda \end{pmatrix} \tag{4}$$

Note that the group of the eightfold way is the factor group $\frac{SU(3)}{Z_3}$, where Z_3 is the centre of SU(3). The eightfold way adjoint representation is a subgroup of the orthogonal group SO(8) [5]. This is only true for the factor group and not for the universal covering group.

$$\frac{SU(3)}{Z_3} \subset SO(8) \tag{5}$$

Later Gell-Mann switched from the eight of the "Eightfold way model" as being a basic entity, to going back to the three being more basic in SU(3). He saw that by taking three quarks, as fictitious or mathematical entities, of spin half with fractional charges 2/3 and -1/3, he could provide an entity to act as a fundamental representation of SU(3). Thereby in the product of three such fundamental quarks, he could obtain octet baryons as a composite entity [6]. The additional benefit was that this quark model also gave a decuptet spin 3/2 baryons as well. Independently Zweig also came up with the same idea [7].

The point to note is that now the octet representation has a dual description. It is being described as a fundamental adjoint representation of the Eightfold way model and as a composite entity of three quarks in the SU(3)-flavour model. Both of them are valid descriptions of the octet baryons. The quark model describes other representations like spin 3/2 etc. as well. But the octet represenatation is unique in that it has a dual description - somewhat akin to the wave-particle duality in quantum mechanics.

Now in the quark based SU(3) model, the electric charge was defined in terms of the two diagonal generators as,

$$Q = T_3 + \frac{Y}{2} \tag{6}$$

where the hypercharge Y = B + S. Here the baryon number B = 1/3 arises from within the hypercharge itself. Thus the baryon number is an internally created quantum number in SU(3). So the baryon number of the octet is an interanally created quantum number in the SU(3) quark model.

But this is completely different from what occurs in the Eightfold way model. Therein, remember that for the octet baryon, the hypercharge and the charge are integral. And the most important fact is that in the Eightfold way model, the octet baryon has **no baryon number** arisisng in the group $\frac{SU(3)}{Z_3}$ [4]. Hence one tries to generate a baryon number in the product group $\frac{SU(3)}{Z_3} \otimes T$ where T is represented by integral numbers. Canonically this is then associated with the baryon number [5, p. 27]. of the octet baryon. Thus in this model the baryon number is created externally to the group SU(3) itself. But this is in conflict with the internal baryon number of the SU(3) quark model, as discussed above. Thus the external baryon for the Eightfold way model baryon is a non-starter. This point has been discussed in detail recently by the author [8].

So though the adjoint representaion of the Eiightfold way model and the octet represenattion of the SU(3) quark model provide dual description of the same super-multiplet, the Eightfold model is handicapped - having no baryon number to distingush it from the corresponding meson octet representation.

This lack of baryon number of the octet baryon in the Eightfold way model, is a serious shortcoming of this model. It demands rectification urgently [8]. In this paper we try to do so.

To do so, let us go to the Skyrme Lagrangian [9]

$$L_S = \frac{{f_\pi}^2}{4} Tr(L_\mu L^\mu) + \frac{1}{32e^2} Tr[L_\mu, L_\nu]^2 \tag{7}$$

where $L_\mu = U^\dagger \partial_\mu U$. The U field for the three flavour case is

$$U(x) = exp[\frac{i\lambda^a \phi^a(x)}{f_\pi}] \tag{8}$$

Here ϕ^a is the pseudoscalar octet of π, K and η mesons. In the full topological Skyrme-Witten model this is supplemented with a Wess-Zumino effective action gievn as,

$$\Gamma_{WZ} = \frac{-i}{240\pi^2} \int_\Sigma d^5x \epsilon^{\mu\nu\alpha\beta\gamma} Tr[L_\mu L_\nu L_\alpha L_\beta L_\gamma] \tag{9}$$

Thus with this anomaly term, the total action is.

$$S = \int d^4x \; L_S + N_c \; \Gamma_{WZ} \tag{10}$$

where N_c is the number of colours.

Remarkable consequences follow when the above action is quantized [10,11]. The presence of the above anomaly term brings about profound differences. In contrast to the two flavour case, for the three flavours, the quantal states are monopolar harmonics [12], which turn out not to be functions, but be sections of a fiber bundle. The quantization of the zero modes is obtained by inroducing time dependent spatial rotations and $SU(3)_F$ transformations:

$$U(x) \rightarrow U(t,x) = A(t) U(R(t)x) A^\dagger(t) \tag{11}$$

Let us recall how the quantum numbers of this Skyrme-Witten model is determined. Due to eqn. (11), it turns out that the manifold for the quantal solution A(t) is not SU(3) but is the coset space $\frac{SU(3)}{U(1)}$, which is a seven dimensional space. One finds that the left translations of this manifold gives SU(3)-flavour group. Also due to equivariance, the spin group SU(2) is provided by the right translations.

Most significantly, the anomaly implies that the right hypercharge is, $Y_R = N_c \frac{B}{3}$. If we take the fact that the nucleon has hypercharge 1, and baryon number 1, then $Y_R = 1$, and the number of colours is 3.

Most remarkably the ground state energy irreducible representation (while ignoring the others irreps as being much higher up in energy) of $SU(3)_F$ for baryon number 1, is an octet representation [210]. Its spin is 1/2, as the right translations on the irreducible representation [210] has to be $I_R = \frac{1}{2} = J$.

Thus lo and behold, the missing baryon number of the adjoint represenation of the Eightfold way model is right here. The Skyrme-Witten model is providing the above missing baryon number as a complementary property of the baryon - as being topological in nature. Thus the topological Skyrme-Witten model is essential to understand the structure of the baryon octet in the Eightfold way model in its compeleteness. Now it can stand parallel to the composite octet model of the $SU(3)_F$ quarks model, to produce a dual description of the same reality.

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