Self-Consistent Generation of Quantum Fermions in Theories of Gravity

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Abstract

I search for concepts that would allow self-consistent generation of dressed fermions in theories of gravitation. Self-consistency means here having the Compton wave lengths of the same order of magnitude for all particles and the four interactions. To build the quarks and leptons of the standard model preons of spin 1/2 and charge 1/3 or 0 have been introduced by the author. Classification of preons, quarks and leptons is provided by the two lowest representations of the quantum group SLq(2). Three extensions of general relativity are considered for self-consistency: (a) propagating and (b) non-propagating torsion theories in Einstein-Cartan spacetime and (c) a Kerr-Newman metric based theory in general relativity (GR). For self-consistency, the case (a) is not excluded, (b) is possible and (c) has been shown to provide it, reinforcing the preon model, too. Therefore I propose that semiclassical GR with its quantum extension (c) and the preon model will be considered a basis for unification of physics. The possibility remains that there are 'true' quantum gravitational phenomena at or near the Planck scale.

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Contents

1 Introduction

The purpose of this note is to to study possible connections between three subjects: (i) a preon model, (ii) the standard model (SM) of particles, and (iii) gravity. More exactly, I attempt to provide group theoretic structure for basic matter particles and to embed the preon model [1] into a theory of gravity. Embedding means here to determine whether pointlike preons would obtain in a gravity theory dressed structure similar to the SM particles. By similar I mean Compton wave lengths being roughly of the same order of magnitude.

Merging gravity with quantum theory has been a major problem in theoretical physics for more than half a century. Several attempts have been made using various formalisms - but without hoped for success. Therefore a simple phenomenological model building may be worth trying for the moment.

In [1] the quarks and leptons were formed of three fermionic preons. Secondly, the preon concept was introduced as an object which would provide a connecting link between the SM fermions and micro black holes (BH).[1] The preon model was further developed in [2, 3].

Key issues in general relativity (GR) which I wish to address here are the singularity at the center of black holes and the quantization. Black holes at

[1]The main stream modern theories consider the AdS/CFT duality as a holographic connection between gravity and quantum theory.

accelerator length scales, and smaller, are an open, admittedly unsafe territory for model building.

BHs can be described by three quantum numbers: mass, spin and charge. Therefore a conservative choice is that preons take the very same quantities. I assume for preons charge values $\pm 1/3$ and zero, to make available the known quark charges. Preons have a light mass and spin $1/2$. The preon states are doublets $\Psi = \begin{pmatrix} \psi^{+1/3} \\ \psi^{0} \end{pmatrix}$, both components $\psi(x)$ being four component spinors. Preons have only electromagnetic and gravitational interactions. In order to find a way to introduce gravity in the preon model, three candidate theories of gravity are considered: the extended Einstein-Cartan (EC) gravity with torsion, including both the non-propagating and propagating torsion. And thirdly, a Kerr-Newman (KN) metric based construction in Einstein-Hilbert (EH) gravity. Our model with the KN completion suggests unification of particle theory with four interactions in terms of matter fields (preons) rather than gauge interactions, as traditionally.

The present preon model [1, 2, 3] is described in section 2. It suggests coherently the gauge group structures $SU(2)$ and $SU(3)$ for the weak and strong interactions, respectively. Group theoretic global structure for preons, quarks and leptons has been given by Finkelstein [4, 5] using the quantum group $SLq(2)$. This is reviewed in section 3. For gravity two extensions of general relativity with torsion are discussed in section 4. We will see that towards Planck scale and high matter density torsional effects become important both in the non-propagating theory of Poplawski [6] and in the propagating theory of Fabbri [7] (see also [8]). In section 5 a model by Burinskii [9], based on the Kerr-Newman over-spinning metric, is reviewed. Desired, interesting results are obtained. Finally, conclusions are given in section 6. Briefly, preons are connected to SM by $SLq(2)$ and self-consistently to gravity by the Kerr-Newman metric based bag construction.

2 Preon Model

Starting from the known quark and lepton charges it is natural to try the following charge quantization $\{0,\, 1/3,\, 2/3,\, 1\}$. Fermionic permutation antisymmetry for same charge preons must be included. These arguments lead to four bound states of three light preons which form the first generation quarks and leptons [1, 2]

$$\begin{aligned} u_k &= \epsilon_{ijk} m_i^+ m_j^+ m^0 \\ \bar{d}_k &= \epsilon_{ijk} m^+ m_i^0 m_j^0 \\ e &= \epsilon_{ijk} m_i^- m_j^- m_k^- \\ \bar{\nu} &= \epsilon_{ijk} \bar{m}_i^0 \bar{m}_j^0 \bar{m}_k^0 \end{aligned} \tag{2.1}$$

A useful feature in (2.1) with two same charge preons is that the construction provides a three-valued index for quark $SU(3)$ color, as it was originally discovered [10], the corresponding gauge bosons being in the adjoint representation. The weak $SU(2)$ left handed doublets can be read from the first two and last two lines in (2.1). The standard model (SM) gauge structure $SU(N), N = 2, 3$ is emergent in this sense from the present preon model. In the same way quark-lepton transitions between lines 1↔3 and 2↔4 in (2.1) are possible.

The preon and SM fermion global group structure is better illuminated using the representations of the $SLq(2)$ group in the next section 3.

The above gauge picture is supposed to hold in the present scheme up to the energy of about 10^{16} GeV. The electroweak interaction has the spontaneously broken symmetry phase below an energy of the order of 100 GeV and symmetric phase above it. The electromagnetic and weak forces take separate ways at higher energies (100 GeV$\ll E \ll 10^{16}$ GeV), the latter restores its symmetry but melts away due to ionization of quarks and leptons into preons. The electromagnetic interaction, in turn, stays strong towards Planck scale, $M_{\rm Pl} \sim 1.22 \times 10^{19}$ GeV. Likewise, the quark color and leptoquark interactions suffer the same destiny as the weak force. One is left with the electromagnetic and gravitational forces only near Planck scale.

The proton, neutron, electron and ν can be constructed of 12 preons and 12 anti-preons. The construction (2.1) is matter-antimatter symmetric on preon level, which is desirable for early universe. The model makes it possible to create from vacuum a universe with only matter: combine e.g. six m^+, six m^0 and their antiparticles to make the basic β-decay particles. Corresponding antiparticles may occur equally well, but the matter dominance case seems to have been made. Neutral dark matter is formed of preon-antipreon pairs more likely than ordinary matter when the temperature of the universe is lowered to a proper free mean path value between preon collisions.

The baryon number (B) is not conserved in this model: a proton may decay at Planck scale temperature by a preon rearrangement process into a positron and a pion. This is expected to be independent of the details of the preon interaction. Baryon number minus lepton number (B-L) is conserved.

I suppose the preon-preon interaction is attractive, non-confining and strong enough to keep together the charged preons but weak enough to liberate the preons at high temperature. A candidate for this interaction is the axial-vector field mediated force discussed in section 4.3. On the other hand, it has been suggested [4] that preons may not appear as free particles, i.e. have any independent degree of freedom, but are concentrations of energy-momentum at the crossings of a flux tube.

One may now consider the case that, as far as there is an ultimate unified field theory within the standard model, it is a preon theory with gravitational and electromagnetic interactions only.

3 Knot Theory: Preons, Quarks and Leptons

Early work on knots in physics goes back in time to 19th and 18th century [11, 12]. More recently Finkelstein has proposed a model based on the quantum group $SLq(2)$ [4, 5]. The idea is that Lie groups can be considered as degenerate forms of quantum groups [13]. Therefore it is of interest to study a physical theory by replacing its Lie group by the corresponding quantum group. Finkelstein has introduced the global group $SLq(2)$ as an extension to the SM gauge group obtaining the group structure SU(3) × SU(2) × U(1) × SLq(2).

Knots are objects in three dimensional space. Their projections onto two dimensional plane are considered here. Oriented knots can be characterized by three numbers as follows. Where two dimensional curves cross there is an overline and an underline at each point, the vertex. A vertex has a crossing sign +1 or -1 depending on whether the overline direction is carried into the underline direction by a counterclockwise or clockwise rotation, respectively. The sum of all crossing signs is the writhe w which is a topological invariant. The number of rotations of the tangent of the curve in going once around the knot is a second topological invariant and it is called the rotation r. An oriented knot can be labeled by the number of crossings N, the writhe w and rotation r. The writhe and rotation are integers of opposite parity.

One can transform to quantum coordinates (j, m, m'). These indices label the irreducible representations of $D^j_{mm'}$ of the symmetry algebra of the knot, $SLq(2)$, by defining

$$j = N/2, \quad m = w/2, \quad m' = (r + o)/2 \tag{3.1}$$

This linear transformations makes half-integer representations possible. The knot constraints require w and r to be of opposite parity, therefore o is an odd integer.

The standard model field operators $\psi(x)$ are complemented in his model by knot factors D as follows [5]

$$\psi(x) \to \hat{\psi}(x) D^j_{mm'} \tag{3.2}$$

where $D^j_{mm'}$ is a 2j+1 dimensional representation of the $SLq(2)$ algebra ($\hat{\psi}(x)$ also has the (j,m, m') indices, see [5]).

Any knot (N, w, r) may be labeled by $D^{N/2}_{w/2,(r+o)/2}(a, b, c, d)$. Therefore, to the (N, w, r) knot the following expression of the algebra is associated

$$D^j_{mm'}(a.b, c, d) = \sum_{\substack{\delta(n_a+n_b,n_+)\\ \delta(n_c+n_d,n_-)}} A^j_{mm'}(q, n_a, n_c)\delta(n_a + n_b, n'_+) a^{n_a} b^{n_b} c^{n_c} d^{n_d} \tag{3.3}$$

where (j, m, m') is given by (3.1), $n_\pm = j \pm m$, $n'_\pm = j \pm m'$ and $A^j_{mm'}(q, n_a, n_c)$ is given by

$$A^j_{mm'}(q, n_a, n_c) = \left[\frac{\langle n'_+\rangle_1 \langle n'_-\rangle_1}{\langle n_+\rangle_1 \langle n_-\rangle_1}\right]^{1/2} \frac{\langle n_+\rangle_1!}{\langle n_a\rangle_1! \langle n_b\rangle_1!} \frac{\langle n_-\rangle_1!}{\langle n_c\rangle_1! \langle n_d\rangle_1!} \tag{3.4}$$

where $n_+ = n_a + n_b$, $n_- = n_c + n_d$, $\langle n\rangle_q = \frac{q^{n-1}}{q-1}$ and $\langle\rangle_1 = \langle\rangle_{q_1}$.

One assigns physical meaning to the $D^j_{mm'}$ in (3.3) by interpreting the a, b, c, and d as creation operators for spin 1/2 preons. These are the four elements of the fundamental $j = 1/2$ representation $D^{1/2}_{mm'}$ as indicated in table 1.

m	m'	preon
1/2	1/2	a
1/2	-1/2	b
-1/2	1/2	c
-1/2	-1/2	d

Table 1.

For notational clarity, I use in the tables 1. and 2. the preon names of [4]. The preon dictionary from the notation of [1] is the following:

$$\begin{aligned} m^+ &\mapsto a, \quad m^0 \mapsto c \\ m^- &\mapsto d, \quad \bar{m}^0 \mapsto b \end{aligned} \tag{3.5}$$

The standard model particles are the following $D^{3/2}_{mm'}$ representations

m	m'	particle	preons
3/2	3/2	electron	aaa
3/2	3/2	neutrino	ccc
3/2	-1/2	d-quark	abb
-3/2	-1/2	u-quark	cdd

Table 2.

The preon, quark and lepton knot structures are presented graphically in figure 1.

All details of the $SLq(2)$ extended standard model are discussed in [4], including the gauge and Higgs bosons and a candidate for dark matter. I do not, however, see much advantage for introducing composite gauge bosons in the model (gauge invariance is a local property). Introduction of color is done slightly differently in [5]. In the early universe developments there is similarity between the knot and the present preon model. Therefore the model of [1] and the knot algebra of [4] are equivalent in the fermion sector.

Preon binding into bound states is not completely clear. The trefoil field structure may be regarded as a trefoil flux tube carrying energy, momentum and charge so that all three are concentrated at the three crossings. Then one can regard these three concentrations at the three crossings as actually defining the three preons, without postulating their existence with independent degrees of freedom. In the next section 4 I describe the possibility that the preon interactions are gravitationally caused by torsion.

The most elementary configuration of type considered above is a simple loop having $j = 0$. Some pairs of these loops with opposite rotation may be brought

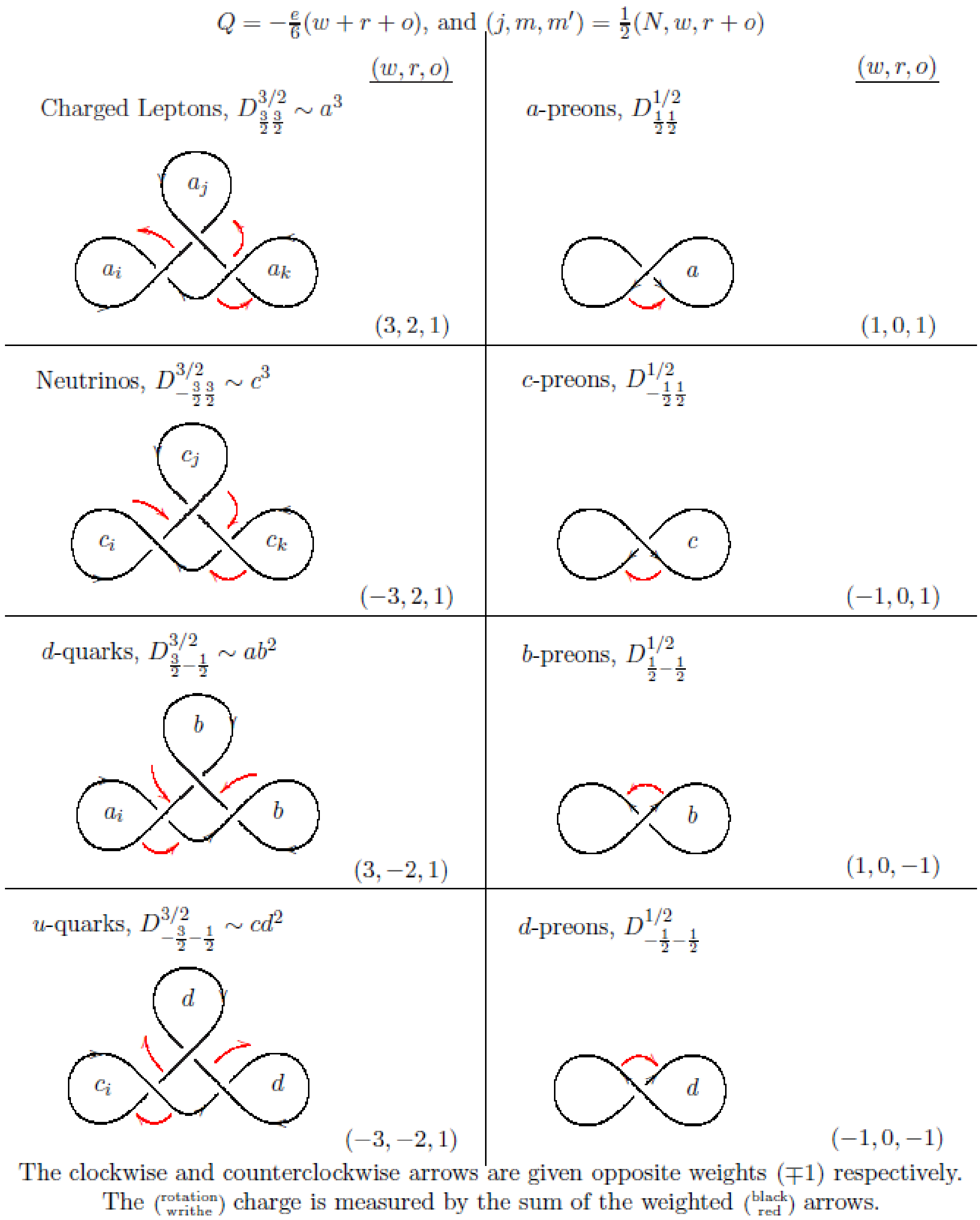

Figure 1: Preonic structure of elementary fermions (taken from [4]. Preprint of an article submitted for consideration in Int.J.Mod.Phys.A. © 2017 http://www.worldscientific.com)

together, e.g. in the early universe, by gravitational attraction making two opposing $j = 1/2$ twisted loops as indicated in figure 2.

In summary, knots having odd number of crossings are fermions and knots with even number of crossings are correspondingly bosons. Instead of considering spin 1 bound states of six preons I assume that the SM gauge bosons are genuine point like gauge fields. The leptons and quarks are simple quantum knots, the quantum trefoils, with three crossings and $j = 3/2$. At each crossing

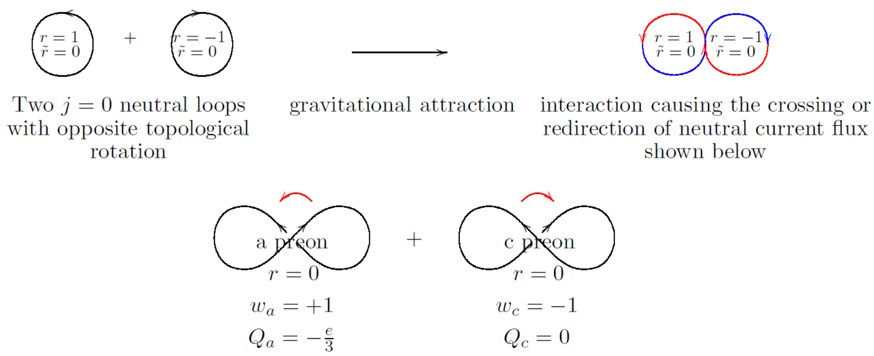

Figure 2: Creation of preons as twisted loops (Origin of picture as in figure 1).

there is a preon. The preons are twisted loops with one crossing and $j = 1/2$. The $j = 0$ states are simple neutral loops with zero crossings, called yons in [4].
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4 Einstein-Cartan Gravity with Torsion

4.1 Preliminaries

The basic quantity of general relativity (GR) is the metric tensor field $g_{\mu\nu}(x)$ the variations of which are caused by the matter fields. Covariant derivatives are defined with respect to the Christoffel connection, which is the unique metric-compatible and no-torsion connection. A simple modification to GR is to consider the connection a fundamental variable rather than an expression of some function of the metric. There are several forms of these type theories. An early modified GR is the Palatini formulation (for a review, see [14]) of Einstein-Hilbert gravity: the action is the same but the connection is varied independently of the metric. The equations of motion include the usual expression plus extra terms depending algebraically on the matter fields. The new terms contributing to the connection include a new tensor, called the torsion tensor. The Palatini formulation gives rise to non-propagating torsion.

From gauge theory point of view, the Einstein-Hilbert theory of gravity provides rotational curvature (cf. rolling a piece of dough) to spacetime in terms of the metric tensor. This is the prevalent dogma in gravity. It is not, however, the most general case of gauge symmetry available. The EH theory can be generalized by including in the action terms of torsion, which leads to translational curvature (cf. turning a screw) in spacetime. This way the full symmetry of the ten parameter Poincaré group can be taken into account.

From a different point of view, curvature arises in the form of metric from energy density and torsion in the form of a connection from spin density. Torsion is therefore defined on microscopic scales only. Torsion requires extension of the Riemann geometry to Riemann-Cartan (RC) geometry [15]. RC gravity, or Einstein-Cartan-Kibble–Sciama (ECKS) [16, 17, 18] gravity can be reduced to general relativity gravity plus torsional contributions.

Poplawski has considered the case of non-propagating torsion in ECKS thery [6]. Free fermions in the ECKS theory extend in two spatial dimensions at least on the scale of their Cartan radii $r_C \sim \sqrt[3]{G_N \hbar^2/mc^4}$. The Cartan density for an electron, $\rho_C \sim m_e/r_C^3 \sim 10^{49} g/cm^3$, approximates the order of the maximum density of matter composed of standard model particles [6]. This gives an idea of how high spinorial matter densities may be required for torsional effects to occur.

Fabbri [7] has developed a theory with propagating torsion and spinor matter fields, which yields a massive axial-vector coupled to spinors. His goal is to explain most of the open problems in the standard model of particles (and cosmology) as well as to analyze the nature of spinor fields. Here I consider the axial-vector coupling of [7] as a possible preon binding interaction.

The field equations of the EH theory of gravity are

$$R^{\mu\nu} - \frac{1}{2} g^{\mu\nu} R - g^{\mu\nu}\Lambda = \frac{1}{2} k T^{\mu\nu} \tag{4.1}$$

where $R^{\mu\nu}$ is the Ricci tensor, R its trace, $T^{\mu\nu}$ is the matter energy density and $k = 8\pi G$. They will now be extended to include torsion.

Connections are used to define covariant derivatives. A suitable connection must be constructed. In general form, a covariant derivative of a vector is defined by

$$D_\alpha V^\mu = \partial_\alpha V^\mu + V^\rho \Gamma^\mu_{\rho\alpha} \tag{4.2}$$

The connection $\Gamma^\mu_{\rho\alpha}$ has three indices: μ and ρ shuffle, or transform, the components of the vector V^ρ and α indicates the coordinate in the partial derivative.

Metric and connection should be unrelated. This is implemented by demanding that the covariant derivative of the metric vanishes. In this case the connection is metric-compatible. Metric-compatible connections can be divided into antisymmetric part, given by the torsion tensor, and symmetric part which includes a combination of torsion tensors plus a symmetric, metric dependent connection.

In a general Riemannian spacetime $\mathbf{R}$, at each point p with coordinates x^μ, there is a Minkowski tangent space $\mathbf{M} = T_p\mathbf{R}$, the fiber, on which the local gauge transformation of the $T_{x^\mu}\mathbf{R}$ coordinates x^a takes place

$$x'^a = x^a + \epsilon^a(x^\mu) \tag{4.3}$$

where ϵ^a are the transformation parameters, μ is a spacetime index and a a fiber frame index.

The dynamics of the theory is based on vierbeins (tetrads) $e^a{}_\mu$, not on the metric tensor $g_{\mu\nu}$. The Cartan, or affine, connection has a primary role and it is

$$\Gamma_{\mu\lambda\nu} = e^a{}_\mu \partial_\lambda e_{a\nu} \tag{4.4}$$

The tensor associated with this connection is torsion tensor

$$T^{\mu}_{\ \lambda\nu} = e_a^{\ \mu}(\partial_\lambda e^a_{\ \nu} - \partial_\nu e^a_{\ \lambda}) \tag{4.5}$$

The connection

$$\Lambda^{\rho}_{\alpha\beta} = \tfrac{1}{2} g^{\rho\mu} \left(\partial_\beta g_{\alpha\mu} + \partial_\alpha g_{\mu\beta} - \partial_\mu g_{\alpha\beta}\right) \tag{4.6}$$

is symmetric and written entirely in terms of the partial derivatives of the metric tensor, and it is called metric connection, or Christollel symbol, while the torsion tensor with all lower indices is taken to be completely antisymmetric.

Unfortunate for the development of gravitation theory, spin was not discovered in the laboratory before 1916. Spinors were introduced in mathematics by Cartan in the 1920's and spinor wave equation was found by Dirac in 1928.

4.2 Non-propagating Torsion

The dynamical variables describing the spacetime of ECKS theory are the tetrad e^i_a and the spin connection [6]

$$\omega^a_{bk} = e^a_j(\partial_k e^j_b + \Gamma^j_{ik} e^i_b) \tag{4.7}$$

where Γ^j_{ik} is the affine connection. The torsion tensor is $T^i_{jk} = \Gamma^i_{[j,k]}$, where [] denotes antisymmetrization. The spin connection ω^a_{bk} is a generalization of Γ^i_{jk}. The tetrad e^a_j connects the spacetime coordinates i, j, ... to the local Lorentz coordinates a, b, ... as follows $V^a = V^i e^a_i$.

The dynamical energy-momentum density is obtained from the Lagrangian density of matter $\mathscr{L}_m$ (without fermions for the present) with respect to tetrad

$$\Sigma_{ab}^{\ \ i} = 2\frac{\delta \mathscr{L}_m}{\delta \omega^{ab}_{\ \ i}} \tag{4.8}$$

Conservation of spin density follows from the Lorentz invariance of $\mathscr{L}_m$

$$\partial_k \Sigma^{ijk} - \Gamma^i_{lk}\Sigma^{jlk} + \Gamma^j_{lk} - 2\Theta^{[ij]} = 0 \tag{4.9}$$

The ECKS Lagrangian density is

$$\mathscr{L} = \mathscr{L}_m - \frac{c^4}{2k}\mathbf{e}R \tag{4.10}$$

where $\mathbf{e} = det^a_i$, $R = R^b_{\ j} e^j_b$ is the Ricci scalar. The energy-momentum density is determined locally through the Einstein equation

$$\mathbf{e}\big(R^a_{\ i} - \frac{1}{2}Re^a_i\big) = \frac{k}{c^4}\Theta_i^{\ a} \tag{4.11}$$

which follows from variation of $\mathscr{L}$ with respect to the tetrad. The torsion of spacetime is locally related to the spin density (4.8) by the Cartan equation

$$\mathbf{e}\big(S^i_{\ \ ab} - S_a E^i_b + S_b e^i_a\big) = -\frac{k}{c^4}\Sigma_{ab}^{\ \ i} \tag{4.12}$$

where $S_i = S^k{}_{ik}$ is the torsion vector. This follows from the stationarity of the action under variation of the spin connection. Combining (4.11) and (4.12) gives

$$G_{ik} = \frac{k}{c^4} T_{ik} + U_{ik} \tag{4.13}$$

where $G_{ik} = R_{ik} - \frac{1}{2} R g_{ik}$ is the Einstein tensor and $T_{ik} = (2/\mathbf{e}) \delta \mathscr{L}_m / \delta g^{ik}$ is the metric energy-momentum tensor. The tensor U_{ik} is

$$\begin{aligned} U_{ik} = -\big(S^l_{ij} + 2S_{\{ij\}}{}^l\big)\big(S^j_{kl} + 2S_{\{kl\}}{}^j\big) + 4S_i S_k + \\ \frac{1}{2} g_{ik}\big(S^{mjl} + 2S^{\{jl\}m}\big)\big(S_{ljm} + 2S_{\{jm\}l}\big) - 2g_{ik} S^j S_j \end{aligned} \tag{4.14}$$

where $\{\}$ means symmetrization. U_{ik} is quadratic in $\Sigma_{ij}{}^k$. The Lagrangian density (4.10) choice for a torsional theory. When torsion vanishes (4.13) reduces to the usual Einstein equation.

The Cartan equation (4.12) is linear algebraic equation. Therefore torsion is simply proportional to spin density and it vanishes in vacuum, outside material bodies. In other words, torsion is non-propagating.

Now introduce fermions obeying the Dirac equation $i\gamma_i(\hbar\partial_i\psi + (iq/c)A_i\psi) - mc\psi = 0$, where A_i is the electromagnetic potential and the γ_i are the 4×4 Dirac matrices. The Dirac Lagrangian is

$$\begin{aligned} \mathscr{L}_m = \mathscr{L}_\psi = \frac{1}{2} i\hbar c \mathbf{e}\big(\bar{\psi}\gamma^i\partial_i\psi - \partial_i\bar{\psi}\gamma^i\psi\big) - \\ \frac{1}{2}\bar{\psi}\big(\gamma^i\Gamma_i + \Gamma_i\gamma^i\big)\psi - q\mathbf{e}\bar{\psi}\gamma^i\psi A_i - mc^2\mathbf{e}\bar{\psi}\psi \end{aligned} \tag{4.15}$$

where the spinor connection is $\Gamma_i = -(1/4)\omega_{abi}\gamma^a\gamma^b$, which is called the Fock-Ivanenko expression. The spin density from (4.15) is the totally antisymmetric form

$$\Sigma^{ijk} = \frac{1}{2} i\hbar c \mathbf{e}\bar{\psi}\gamma^{[i}\gamma^j\gamma^{k]}\psi \tag{4.16}$$

The definition (4.8) indicates that only the totally antisymmetric part of the torsion tensor couples to Dirac fields. The spin density (4.16) does not include m and q, it does not depend on weak and strong interactions of fermions either. Therefore our assumption that the weak and strong interactions fade away at high enough energy like, 10^{16} GeV, is not that restrictive. As to the present scheme, results based on (4.16) apply to both charged and neutral preons. Substituting (4.16) into (4.15) yields a four fermion axial self-interaction term, called the Heisenberg-Ivanenko term, in the Lagrangian density

$$\mathscr{L}_S = \frac{3}{2}\pi G\mathbf{e}(\hbar c)^2\big(\bar{\psi}\gamma^i\gamma^5\psi\big)\big(\bar{\psi}\gamma_i\gamma^5\psi\big) \tag{4.17}$$

Now in [6] the author assumes for tests three types of solutions (i) a point particle, (ii) field of string form, and (iii) toroid with certain inner and outer radii. Conservation of spin density excludes solution types (i) and (ii). The type

(iii) solution can be assessed as follows. One has to choose between EH and ECKS theories. In ECKS a Dirac field cannot form a singular Kerr-Newman ring because the ring would have to have spatial extension along the r- and z-coordinates. In ECKS the extension is expected to be of the order of the Cartan radius. The value of the this size is obtained from the condition when the two terms on the right hand side of (4.13) are of the same order of magnitude. In other words, the size is determined by the condition when the repulsive four-fermion term equals the attractive gravitational mass term. The value of Cartan radius is $r_C \sim \sqrt[3]{G_N \hbar^2/mc^4}$. Poplawski concludes (i) that torsion may modify Burinskii's model [9] replacing the Dirac–Kerr-Newman ring singularity with a non-singular toroid with the inner radius of the order of Cartan radius, for electrons $r_C \sim 10^{-25}$ cm, and outer radius of the order of Compton wave length $\lambda = \hbar/mc \sim 2.4 \times 10^{-10}$ cm, which is valid also for neutral fermions, (ii) the full Einstein-Maxwell-Yang-Mills-Dirac-Heisenberg-Ivanenko field equations would have to be solved. But (ii) is just what one wants to avoid in the present preon model. In section 5 we take a close look at the Burinskii model singularity.

4.3 Propagating Torsion

In this subsection the propagating torsion in ECKS spacetime iis discussed. The system of field equations can be derived by the variational method from a dynamical action, whose Lagrangian function is [7]

$$\begin{aligned}\mathscr{L} = &-\tfrac{1}{4}(\partial B)^2 + \tfrac{1}{2}M^2 B^2 - \tfrac{1}{k}R - \tfrac{2}{k}\Lambda - \tfrac{1}{4}F^2 + \\ &+ i\overline{\psi}\boldsymbol{\gamma}^\mu \boldsymbol{\nabla}_\mu \psi - g_B \overline{\psi}\boldsymbol{\gamma}^\mu \boldsymbol{\pi}\psi B_\mu - m\overline{\psi}\psi\end{aligned} \tag{4.18}$$

where F is the electromagnetic tensor

$$F_{\alpha\beta} = \partial_\alpha A_\beta - \partial_\beta A_\alpha \tag{4.19}$$

Torsion has the important property that it can be separated from gauge and metric factors. Let us start from the metric connection

$$\Lambda^\rho_{\alpha\beta} = \tfrac{1}{2} g^{\rho\mu} \left(\partial_\beta g_{\alpha\mu} + \partial_\alpha g_{\mu\beta} - \partial_\mu g_{\alpha\beta}\right) \tag{4.20}$$

The torsion tensor is completely antisymmetric only if some restrictions are imposed, called the metric-hypercompatibility conditions [19]. Then it can be written in the form

$$Q_{\alpha\sigma\nu} = \tfrac{1}{6} B^\mu \varepsilon_{\mu\alpha\sigma\nu} \tag{4.21}$$

where B^μ is torsion pseudo-vector, obtained from the torsion tensor after a Hodge dual. With the metric connection and the torsion pseudo-vector the most general connection can be written as a sum of $\Lambda^\rho_{\alpha\beta}$ and $Q_{\alpha\sigma\nu}$ as follows

$$\Gamma^\rho_{\alpha\beta} = \tfrac{1}{2} g^{\rho\mu} \left[(\partial_\beta g_{\alpha\mu} + \partial_\alpha g_{\mu\beta} - \partial_\mu g_{\alpha\beta}) + \tfrac{1}{6} B^\nu \varepsilon_{\nu\mu\alpha\beta}\right] \tag{4.22}$$

Functions $\Omega^{a}_{b\mu}$ that transform under a general coordinate transformation like a lower Greek index vector and under a Lorentz transformation as

$$\Omega'^{a'}_{b'\nu} = \Lambda^{a'}_{a}\left[\Omega^{a}_{b\nu} - (\Lambda^{-1})^{a}_{k}(\partial_{\nu}\Lambda)^{k}_{b}\right](\Lambda^{-1})^{b}_{b'} \tag{4.23}$$

are called a spin connection. The torsion in coordinate formalism is defined as follows

$$Q^{a}{}_{\mu\nu} = -(\partial_{\mu}e^{a}_{\nu} - \partial_{\nu}e^{a}_{\mu} + e^{b}_{\nu}\Omega^{a}_{b\mu} - e^{b}_{\mu}\Omega^{a}_{b\nu}) \tag{4.24}$$

and the spin connection is given by

$$\Omega^{a}_{b\mu} = e^{\nu}_{b}e^{a}_{\rho}\left(\Gamma^{\rho}_{\nu\mu} - e^{\rho}_{k}\partial_{\mu}e^{k}_{\nu}\right) \tag{4.25}$$

which is antisymmetric in the two Lorentz indices after both of them are brought in the same upper or lower position. The most general spinorial connection is

$$\boldsymbol{\Omega}_{\mu} = \tfrac{1}{2}\Omega_{ab\mu}\boldsymbol{\sigma}^{ab} + iqA_{\mu}\mathbb{I} \tag{4.26}$$

where A_{μ} is the gauge potential. The spinorial curvature is using the spinorial connection

$$\boldsymbol{F}_{\alpha\beta} = \partial_{\alpha}\boldsymbol{\Omega}_{\beta} - \partial_{\beta}\boldsymbol{\Omega}_{\alpha} + [\boldsymbol{\Omega}_{\alpha}, \boldsymbol{\Omega}_{\beta}] \tag{4.27}$$

Let us define the decomposition of the spinor field in its left and right parts

$$\boldsymbol{\pi}_{L}\psi = \psi_{L} \qquad \overline{\psi}\boldsymbol{\pi}_{R} = \overline{\psi}_{L} \tag{4.28}$$

$$\boldsymbol{\pi}_{R}\psi = \psi_{R} \qquad \overline{\psi}\boldsymbol{\pi}_{L} = \overline{\psi}_{R} \tag{4.29}$$

so that

$$\overline{\psi}_{L} + \overline{\psi}_{R} = \overline{\psi} \qquad \psi_{L} + \psi_{R} = \psi \tag{4.30}$$

Now one has 16 linearly-independent bi-linear spinorial quantities

$$2\overline{\psi}\boldsymbol{\sigma}^{ab}\boldsymbol{\pi}\psi = \Sigma^{ab} \tag{4.31}$$

$$2i\overline{\psi}\boldsymbol{\sigma}^{ab}\psi = S^{ab} \tag{4.32}$$

$$\overline{\psi}\boldsymbol{\gamma}^{a}\boldsymbol{\pi}\psi = V^{a} \tag{4.33}$$

$$\overline{\psi}\boldsymbol{\gamma}^{a}\psi = U^{a} \tag{4.34}$$

$$i\overline{\psi}\boldsymbol{\pi}\psi = \Theta \tag{4.35}$$

$$\overline{\psi}\psi = \Phi \tag{4.36}$$

To have the most general connection decomposed into the simplest symmetric connection plus torsion terms we substitute (4.22) in (4.25) and this in (4.26). The field equations reduce to the following

$$\nabla_{\rho}(\partial B)^{\rho\mu} + M^{2}B^{\mu} = g_{B}\overline{\psi}\boldsymbol{\gamma}^{\mu}\boldsymbol{\pi}\psi \tag{4.37}$$

for torsion axial-vector and

$$
\begin{aligned}
R^{\rho\sigma}-\tfrac{1}{2}Rg^{\rho\sigma}-\Lambda g^{\rho\sigma}= \\
=\tfrac{k}{2}[\tfrac{1}{4}F^2g^{\rho\sigma}-F^{\rho\alpha}F^{\sigma}{}_{\alpha}+ \\
+\tfrac{1}{4}(\partial B)^2g^{\rho\sigma}-(\partial B)^{\sigma\alpha}(\partial B)^{\rho}{}_{\alpha}+ \\
+M^2(B^{\rho}B^{\sigma}-\tfrac{1}{2}B^2g^{\rho\sigma})+ \\
+\tfrac{i}{4}(\overline{\psi}\boldsymbol{\gamma}^{\rho}\boldsymbol{\nabla}^{\sigma}\psi-\boldsymbol{\nabla}^{\sigma}\overline{\psi}\boldsymbol{\gamma}^{\rho}\psi+\overline{\psi}\boldsymbol{\gamma}^{\sigma}\boldsymbol{\nabla}^{\rho}\psi-\boldsymbol{\nabla}^{\rho}\overline{\psi}\boldsymbol{\gamma}^{\sigma}\psi)- \\
-\tfrac{1}{2}g_B(B^{\sigma}\overline{\psi}\boldsymbol{\gamma}^{\rho}\boldsymbol{\pi}\psi+B^{\rho}\overline{\psi}\boldsymbol{\gamma}^{\sigma}\boldsymbol{\pi}\psi)]
\end{aligned}
\tag{4.38, 4.39}
$$

for the torsion-spin and curvature-energy coupling, and

$$\nabla_{\sigma}F^{\sigma\mu}=q\overline{\psi}\boldsymbol{\gamma}^{\mu}\psi \tag{4.40}$$

for the gauge-current coupling; and finally

$$i\boldsymbol{\gamma}^{\mu}\boldsymbol{\nabla}_{\mu}\psi-g_BB_{\sigma}\boldsymbol{\gamma}^{\sigma}\boldsymbol{\pi}\psi-m\psi=0 \tag{4.41}$$

for the spinor field equations.

From (4.37) one sees that torsion behaves like a massive axial-vector field satisfying Proca field equations. It is noted that torsion does not couple to gauge fields. Torsion and gravitation seem to have the same coupling constant. However, in [7] it is shown that using the Einstein-Kibble-Sciama field equations these two independent fields with independent sources can have independent coupling constants.

The preon-preon interaction is attractive and of short range due to the mass of the axial-vector field. The interaction includes two free parameters, the coupling constant g_B and the mass M of the axial-vector. Therefore, bound states of preons may be formed by the axial-vector interaction.

The preons interact by coupling to an axial-vector boson B arising in Einstein-Kibble-Sciama theory of gravity. The preon-preon interaction is attractive [7] providing the binding for three preon states. The mass of the axial-vector boson is estimated to be of the order of the grand unified theory (GUT) scale 10^{16} GeV (see below in this section). This makes the torsion interaction range very short. At all scales the B couples to preons relatively strongly but to the standard model particles always weakly. The role of curvature, or gravitons, is beyond the scope of this study.

The field equation for torsion axial-vector is (4.37), from subsection 4.3

$$\nabla_{\rho}(\partial B)^{\rho\mu}+M^2B^{\mu}=g_B\overline{\psi}\boldsymbol{\gamma}^{\mu}\boldsymbol{\pi}\psi \tag{4.42}$$

where M is the axial-vector mass, g_B the preon–axial-vector coupling and ψ the preon wave function. The coupling g_B must be larger than the electromagnetic coupling α to keep the charged preons bound. In EKS gravity, g_B is independent of the gravitational coupling [7]. The key point of this note is that (4.42) depends only on the axial-vector B and preon field ψ, not on gauge and metric factors.

Couplings in GUT theory are of the order 0.02 at the GUT scale. With a Yukawa potential in the Schrödinger equation $V(r) = -V_0\ exp(-ar)/r$ [20], or in our notation $-g_B exp(r/M)/r$ with the physicality condition $n+l+1 \leq \sqrt{g_B m M}$, one may estimate that large M correlates with small preon mass $m \ll m_{proton}$. These matters deserve naturally quantitative attention.

5 General Relativity and the Kerr-Newman Metric

5.1 Kerr-Newman Metric

In this subsection we consider stationary rotating black holes which have axial symmetry aound the axis of rotation. The solution of Einstein equation for rotating black holes was discovered by Kerr as late as 1963 [21]. The Kerr metric is

$$
\begin{aligned}
ds^2 = -\left(1 - \frac{2GMr}{\rho^2}\right)dt^2 - \frac{2GMar\ sin^2\theta}{\rho^2}(dtd\phi + d\phi dt) \\
+\frac{\rho^2}{\Delta}dr^2 + \rho^2 d\theta^2 + \frac{sin^2\theta}{\rho^2}\left[(r^2+a^2)^2 - a^2\Delta sin^2\theta\right]d\phi^2
\end{aligned}
\tag{5.1}
$$

where

$$\Delta(r) = r^2 - 2GMr + a^2 \tag{5.2}$$

and

$$\rho^2(r,\theta) = r^2 + a^2 cos^2\theta \tag{5.3}$$

The quantities mass M and angular momentum J determine the possible solutions. The angular momentum per unit mass $a = J/M$ is also important for elementary particles. When $a = 0$ the metric goes to the Schwarzschild metric. In (5.1) there are two Killing vectors $K = \partial_t$ and $R = \partial_\phi$ because the corresponding metric coefficients are independent of t and ϕ.

Electric and magnetis charges Q and P can be included by replacing $2GMr$ with $2GMr - G(Q^2 + P^2)$. The resulting metric is called the Kerr-Newman (KN) metric [22].

The coordinates (t, r, θ, ϕ) are called the Boyer-Lindquist coordinates. In the limit a fixed and $M \to 0$ we find flat spacetime but not in ordinary polar coordinates but with line element

$$ds^2 = -dt^2 + \frac{r^2 + a^2cos^2\theta}{r^2+a^2}dr^2 + (r^2 + a^2cos^2\theta)^2 d\theta^2 + (r^2+a^2)sin^2\theta d\phi^2 \tag{5.4}$$

The coordinates have been chosen so that the event horizons occur for fixed values of r for which $g^{rr} = \Delta/\rho^2 = 0$, or

$$\Delta(r) = r^2 - 2GMr + a^2 = 0 \tag{5.5}$$

There are three cases: (i) $GM > a$, $GM = a$ and $GM < a$. Usually the case $GM > a$ is of interest, (ii) is unstable and (iii) has a (ring) singularity, which we will meet in subsection 5.2. In case (i) there are two solutions for $\Delta = 0$

$$r_{\pm} = GM \pm \sqrt{G^2M^2 - a^2} \tag{5.6}$$

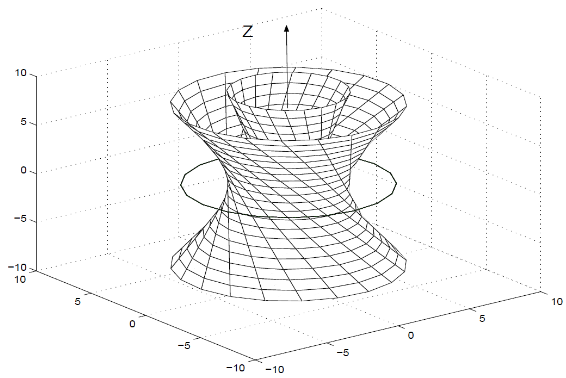

Figure 3: The Kerr singular ring and the Kerr congruence of twistors (PNC). Singular ring is a branch line of space, and PNC propagates from the "negative" sheet of the Kerr space to the "positive" one, covering the space-time twice (taken from [25]).

5.2 The Dirac–Kerr-Newman Fermion

The Kerr-Newman solution has been used as a model for the electron after the discovery [23] that it has the gyromagnetic ratio $g = 2$. This leads to the question are the Dirac equation and the KN solution somehow connected? In this subsection we review a model which connects the Dirac equation and the spinor (twistor) structure of the KN solution [24]. We consider the Burinskii model based on the assumption that the Dirac equation and the KN solution are complementary to each other. The Dirac spinors fit together with the spinor structure of the KN spinning particles. Tho role of the Dirac equation is to play as an order parameter and to control the system. The combined Dirac–Kerr-Newman system is indistinguishable from the behavior of the Dirac electron.

The angular momentum $J = \hbar/2$ of an electron is so big, compared to its mass, that the black hole horizons disappear. This can be called over-rotating Kerr geometry. The source of the KN spinning particle is a naked singularity ring. The ring represents a string which is able to have excitations generating the spin and mass of the extended object. The ring is a focal line of the principal null congruence which is a bundle of light-like rays, or twistors. The form of the metric, the Kerr-Schild form of KN metric, is determined by a null vector field $k^\mu(\mathrm{x})$ which is tangent to the vortex of light-like rays which are twistors, see figure 3.

$$g^{\mu\nu} = \eta^{\mu\nu} + 2Hk^\mu k^\nu \tag{5.7}$$

where $\eta^{\mu\nu}$ is the Minkowski metric, the vector potential for the charged KN solution is

$$A_\mu = \mathcal{A}(x) k_\mu \tag{5.8}$$

and the function H is

$$H = \frac{mr - e^2/2}{r^2 + a^2 cos^2\theta} \tag{5.9}$$

where r, θ are ellipsoidal coordinates

$$x + iy = (r + ia)e^{i\phi} sin\theta, \quad z = r cos\theta, \quad t = \rho - r \tag{5.10}$$

The vector field k^μ is tangent to the principal null congruence (PNC) which is geodesic and shear-free (GSF). The PNC is obtained from a complex function $Y(x)$

$$k = du + \bar{Y} d\zeta + Y d\bar{\zeta} - Y\bar{Y} dv \tag{5.11}$$

where in the null Cartesian coordinates

$$\begin{aligned} \sqrt{2}\zeta = & \quad x + iy, \quad \sqrt{2}\bar{\zeta} = x - iy \\ \sqrt{2}u = & \quad z + t, \quad \sqrt{2}v = z - t \end{aligned} \tag{5.12}$$

The twisting Kerr congruence is determined by the Kerr theorem. It represents a technical instrument which allows one to obtain the KN solution (and its generalizations). The general geodesic and shear-free congruence on Minkowski spacetime M^4 is generated by the algebraic equation

$$F(Y, \lambda_1, \lambda_2) = 0 \tag{5.13}$$

where $F(Y, \lambda_1, \lambda_2)$ is a holomorphic function of the projective twistor coordinates

$$Y, \quad \lambda_1 = \zeta - Yv, \quad \lambda_2 = u + Y\bar{\zeta} \tag{5.14}$$

The solution of (5.13) is a function $Y(x)$ which allows to obtain PNC using (5.11). Function F is called the generating function of the Kerr theorem.

The complexification $(x, y, z) \to (x, y, z + ia)$ to the source of the Coulomb potential at origin

$$\Phi(x, y, z) = \mathcal{R}e \frac{q}{\tilde{r}} \tag{5.15}$$

where $\tilde{r} = \sqrt{x^2 + y^2 + (z - ia)^2}$ is complex. On the real slice (x, y, z) this solution gains a singular ring for $\tilde{r} = 0$. The radius of the ring is a and it is located in the plane $z = 0$. The solution can be presented in the oblate spheroidal coordinate system (r, θ) where $\tilde{r} = r + ia\ cos\theta$. The space is seen to have twofold structure with the ring-like singularity as the branch line. For each real point $(t, x, y, z) \subset \mathbf{M^4}$ there are two points, one lying on the positive sheet with $r > 0$ and the other on the negative sheet with $r < 0$.

The potential (5.15) corresponds exactly to the electromagnetic field of the KN solution. The complex shift $\vec{a} = (a_x, a_y, a_z)$ corresponds to the angular momentum of the KN solution.

5.3 The Dirac Equation in the Weyl Basis

The Dirac equation in the Weyl basis reads

$$\left(\gamma^\mu \hat{\Pi}_\mu + m\right)\Psi = 0 \tag{5.16}$$

where $\Psi = \begin{pmatrix} \phi_\alpha \\ \chi^{\dot{\alpha}} \end{pmatrix}$, and $\hat{\Pi}_\mu = -i\partial_\mu - ea_\mu$. It splits into

$$\begin{aligned} \sigma^\mu_{\alpha\dot{\alpha}}\left(i\partial_\mu + eA_\mu\right)\chi^{\dot{\alpha}} &= m\phi_\alpha \\ \bar{\sigma}^{\mu\dot{\alpha}\alpha}\left(i\partial_\mu + eA_\mu\right)\phi_\alpha &= m\chi^{\dot{\alpha}} \end{aligned} \tag{5.17}$$

The Dirac current is

$$J_\mu = e\left(\bar{\Psi}\Psi\right) = e(\bar{\chi}\sigma_\mu\chi + \bar{\phi}\bar{\sigma}_\mu\phi) \tag{5.18}$$

where $\bar{\Psi} = (\chi^+, \phi^+)$ is a sum two light-like components of opposite chirality

$$\begin{aligned} J^\mu_L &= e\bar{\chi}\sigma^\mu\chi \\ J^\mu_R &= e\bar{\phi}\bar{\sigma}_\mu\phi \end{aligned} \tag{5.19}$$

The products of the null vectors $k^\mu_L = \bar{\chi}\sigma\mu\chi$ and $k^\mu_R = \bar{\phi}\bar{\sigma}^\mu\phi$ is

$$\begin{aligned} k^\mu_L k_{R\mu} = & \quad (\bar{\chi}\sigma^\mu\chi)(\bar{\phi}\bar{\sigma}^\mu\phi) \\ = & \; -2(\phi\bar{\chi})(\chi\bar{\phi} = 2(\bar{\chi}\phi)(\bar{\chi}\phi)^+ \end{aligned} \tag{5.20}$$

In addition, two more Dirac spinor vector combinations are available, $m^\mu = \phi\sigma^\mu\chi$ and $\bar{m}^\mu = (\phi\sigma^\mu\chi)^+ = \bar{\chi}\sigma^\mu\bar{\phi}$. Their scalar product is

$$m^\mu\bar{m}_\mu = 2(\bar{\chi}\phi)(\bar{\chi}\phi)^+ \tag{5.21}$$

All other products between k_L, k_R, m and $\bar{m}$ are null.

The four normalized null vectors

$$n^a = \frac{1}{\sqrt{2(\bar{\chi}\phi)(\bar{\chi}\phi)^+}}, \quad a = 1, 2, 3, 4 \tag{5.22}$$

provide a field of the quasi orthogonal null tetrad determined by the solution $\Psi(x)$ of the Dirac equation.

The Dirac equations for a plane wave $\Psi = \begin{pmatrix} \phi_\alpha \\ \chi^{\dot{\alpha}} \end{pmatrix} = \begin{pmatrix} \breve{\phi}_\alpha \\ \breve{\chi}^{\dot{\alpha}} \end{pmatrix}$ is of the form

$$-\Pi_\mu\sigma^\mu_{\alpha\dot{\alpha}}\chi^{\dot{\alpha}} = \; m\phi_\alpha \tag{5.23}$$

$$-\Pi_\mu\bar{\sigma}^{\mu\dot{\alpha}\alpha}\phi_\alpha = \; m\chi^{\dot{\alpha}} \tag{5.24}$$

where $\Pi_\mu = p_\mu - eA_\mu$.

The complex vectors m and $\bar{m}$ are modulated by the phase factor $exp(2ip_\mu x^\mu)$ from Dirac spinors. They also carry oscillations and de Broglie periodicity of a moving particle. For real null vectors k_L and k_R the phase factor cancels.

The vector Π_μ is spanned by the real vectors k_L and k_R

$$\Pi^\mu = -\frac{m}{2\bar{\phi}\chi}\left(k_L^m + k_R^m\right) \tag{5.25}$$

In the rest frame

$$k_L^0 = k_R^0, \quad k_L^i = -k_R^i \tag{5.26}$$

The spatial components of Π_i vanish.

The polarization vector of the electron is

$$S^\mu = i\bar{\Psi}\gamma^\mu\gamma^5\Psi = k_L^\mu - k_R^\mu \tag{5.27}$$

In the rest frame $S = (0, S^i)$.

It has been shown that the complex KN geometry is related to two null vectors k_L and k_R. They determine the momentum, angular momentum and spin of the KN particle.

5.4 Dirac Equation and the Kerr-Newman Twistorial Structure

The Kerr-Schild ansatz for the metric (5.1) determined by the null vector field $l_\mu(x)$. The vector field is tangent to the Kerr PNC and is determined by the complex function $Y(x)$

$$k_\mu dx^\mu = 1/P(du + \bar{Y}d\zeta + Y d\bar{\zeta} - Y\bar{Y}dv) \tag{5.28}$$

where $P(Y)$ is a normalization factor and $(u, v, \zeta, \bar{\zeta})$ are the null Cartesian coordinates (5.12).

The Kerr theorem says that all the geodesic and shear-free congruences are determined by the function $Y(x)$ which is a solution of $F = 0$ where F is holomorphic function of the projective twistor coordinates (5.14).

A twistor is a pair $Z^\alpha = \{\psi_\alpha, \mu^{\dot{\alpha}}\}$ where $\mu^{\dot{\alpha}} = x^\mu\bar{\sigma}_\mu\psi_\alpha$. The projective twistor is $Z^\alpha/\psi_1 = \{1, Y, \lambda_1, \lambda_2\}$. Therefore the function $Y(x)$ is a projective spinor coordinate $Y = \psi_2/\psi_1$. F may be chosen as homogenous function of Z^α. The following form can be derived for F

$$F(\psi, x^\mu) = x^\mu\left[(\phi\psi)(\bar{\phi}\psi) + (\bar{\chi}\psi)(\chi\bar{\sigma}_\mu\psi)\right]/(\bar{\phi}\chi) - 2ia(\phi\psi)(\bar{\chi}\psi) \tag{5.29}$$

The function (5.29) posesses all the properties of a Kerr generating function.

The Dirac wave function $\Psi = (\phi, \chi)$ has the role of an order parameter which controls the dynamics of the Dirac–Kerr-Newman particle, momentum, spin and deformation of the Kerr congruence caused by the external electromagnetis field A_μ.

Conclusions of the present section so far include (i) the electron (or a preon) has an extended space-time structure in accordance with QED, (ii) the Kerr-Newman metric twistorial structure is controlled by the Dirac equation, not conflicting QED, (iii) the KN model has a geometric structure indicating close relationship to quantum theory and (iv) gravity renormalizes and regularizes the Dirac particle [24].

5.5 The Gravitating Bag Model

The Dirac equation inside the KN soliton source has been analyzed [26] with the results that the KN solution shares many features with the hadronic bag models developed at MIT and SLAC. The gravitating bag has to preserve the external KN field. The bag models are based on semiclassical theory including elements of quantum theory based on Minkowski spacetime without gravity. To resolve the conflict between gravity and quantum theory the following solution is proposed [27, 28]: inside the bag there is flat spacetime and outside the bag there is exact KN model solution.

The Kerr-Schild form of metric is (5.7) and (5.9). The variables r and θ are ellipsoidal coordinates and the null vector field $k_\mu(x)$ forms a vortex polarization of Kerr spacetime. Between the negative sheet $r < 0$ and the positive sheet $r > 0$ there is the surface $r = 0$, a bridge connecting the two sheets. The disk $r = 0$ is spanned by the Kerr singular ring $r = 0$ and $cos\theta = 0$, see figure 3. The null vector fields are different on these sheets and are thus denoted as $k^{\mu\pm}(x)$ making two different congruences $\mathcal{K}^\pm$ with metrics $g^{pm}_{\mu\nu} = \eta_{\mu\nu} + 2Hk^\pm_\mu k^\pm_\nu$.

A regularization of the two-sheeted Kerr geometry was suggested by Lopez [29]. The singular region and the negative sheet were excised and replaced by a regular core having flat metric $\eta_{\mu\nu}$. This core forms a vacuum bubble. It must match the external KN solution at the boundary $r = R$ as follows

$$H_{r=R}(r) = 0 \tag{5.30}$$

which gives

$$R = r_e = \frac{e^2}{2m} \tag{5.31}$$

The bubble covers the Kerr singular ring and forms a thin rotating disk of radius

$$r_C \sim a = \frac{\hbar}{mc} \tag{5.32}$$

The oblateness of the disk $r_e/r_C \sim e^2$ is the fine structure constant $\alpha \sim 1/137$. The bubble model is a soliton structure of a vacuum bubble with domain wall boundary, see figure 4. Classical gravity controls the external spacetime and quantum theory makes a supersymmetric pseudo-vacuum state inside the soliton with the Higgs mechanism breaking the symmetry.

The discrepancy between gravity and quantum theory is avoided by three principles: (i) spacetime is flat inside the core, (ii) outside the core there is the Kerr-Newman spacetime and (iii) the boundary between inside and outside of the core is determined by the Lopez condition (5.30).

The effectiveness of these principles (i)-(iii) define uniquely the form of the soliton and the following properties [27, 28, 30, 31, 32]: (a) The Higgs field is an oscillon, oscillating with frequency $\omega = 2m$ and (b) angular momentum is quantized as $J = \frac{n}{2}, n = 1, 2, 3, ...$

In fact, the KN bubble forms a Bogomolnyi-Prasad-Somerfield (BPS) saturated soliton [33, 34] and both properties (a) and (b) are uniquely determined

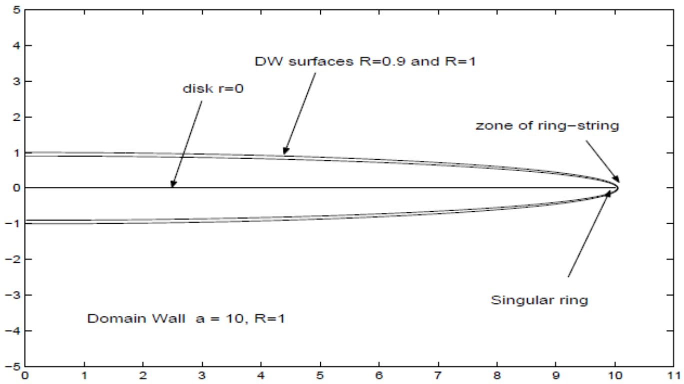

Figure 4: Axial section of the spheroidal domain wall phase transition (taken from [25]).

by the Bogomolnyi equations. These equations give also the shape of the soliton and its dynamics and stability.

In [31] it is shown that a quartic potential usually used for the Higgs field

$$V(|\Phi|) = g(\bar{\sigma}\sigma - \eta^2)^2, \quad \sigma = \langle|\Phi|\rangle \tag{5.33}$$

is not a good source for the Kerr-Newman solution. The external Higgs field contradicts the electromagnetic field's long range. Instead the Higgs must be enclosed inside the bag. Therefore a more complex scheme must be introduced which includes three chiral fields $\Phi^{(i)}, i = 1, 2, 3$. This is a supersymmetric generalization of the Landau-Ginzburg model [35]. Let the Higgs field be $\Phi^{(1)}$ and define the following notation

$$\big(\Phi, Z, \Sigma\big) = \big(\Phi^1, \Phi^2, \Phi^3\big) \tag{5.34}$$

The bag must be placed in the flat geometry region and the domain wall phase transition can be considered with this flat background metric. Therefore the wall boundary and the bag are not dragged by rotation, and the chiral part of the Hamiltonian takes the simple form

$$H^{(ch)} = T_0^{0(ch)} = \frac{1}{2}\sum_{i=1}^{3}\big[\sum_{\mu=0}^{3}|\mathcal{D}_{mu}^{(i)}\Phi^i|^2 + |\partial_i W|^2\big] \tag{5.35}$$

where the derivative is $\mathcal{D}_\mu^{(i)} = \partial_\mu - ieA_\mu^i$. The potential V is

$$V(r) = \sum_i |\partial_i W|^2 \tag{5.36}$$

and the superpotential is

$$W(\Phi^i, \bar{\Phi}^i) = Z(\Sigma\bar{\Sigma} - \eta^2) + (Z + \mu)\Phi\bar{\Phi} \tag{5.37}$$

where μ and η are real constants. (5.37) gives the necessary concentration of the Higgs field inside the bag. Using the condition $\partial_i W = 0$ one gets two vacuum states

internal vacuum: $r < R - \delta$

$$V(r) \neq 0,\ |\Phi| = \eta = constant,\ Z = -\mu,\ \Sigma = 0,\ W_{in} = \mu\eta^2, \tag{5.38}$$

external vacuum: $r > R + \delta$

$$V(r) = 0,\ \Phi = 0,\ Z = 0,\ \Sigma = \eta,\ W_{ext} = 0 \tag{5.39}$$

and transition area $R - \delta < r < R + \delta$ where internal and external vacua are separated by a spike of potential $V > 0$.

It has been shown that the requirements (i)-(iii) above concerning the structure of the vacua establish the stability of the bag. A supersymmetric ad BPS-saturated source of the Kerr-Newman solution has been obtained. This is gives substantial support to the present preon scheme.

6 Conclusions

The spin 1/2 and charge {0, ±1/3} preon model discussed above has a sound group theoretical basis. It is hoped that the present preon scheme has provided a way towards better understanding of the roles of all interactions, including gravity. Gravity and electromagnetism are the 'original' interactions of early cosmology in this scenario. The weak and strong interactions are 'emergent' from the basic fermion structure of the model (2.1).

The SLq(2) group provides a solid basis for the fermion sector of the present preon model. In [5] it was shown that the SLq(2) preon model agrees with the Harari-Shupe (H-S) rishon model [36, 37].[2] Unlike the said preon/rishon authors, I do not think that (i) SM gauge bosons should be treated as bound states of several preons, or (ii) hypercolor is realistic for preon interactions. In any case, a mechanism for preon binding into quarks and leptons is yet to be developed.

Torsion in ECKS theory gives interesting results for the non-propagating case. The fermions are suggested to have geometric structure of finite size, down to a scale of roughly $r_C \sim 10^{-25}$ cm. Propagating torsion may bring attractive interaction for preon bound state formation but this could not be adequately shown. Details of the torsional theories have to be studied further [7, 38] e.g. for consequences of zitterbewegung.

The case of the Kerr-Newman metric based bag model of the electron has the desired properties for the present preon model. The central singularity is removed by construction and gravitational and electromagnetic interactions

[2] The basic idea of the present model was originally conceived during the week of the ψ discovery in November 1974 at SLAC. I proposed that the c-quark is a gravitationally excited u-quark, both consisting of three spin 1/2 and charge {0, 1/3} heavy constituents. This idea met resistance. Therefore the model was not developed further until years later.

have long range. The size of the semiclassical KN object is self-consistently of a typical quantum value like Compton wave length, or α times it, far above the Cartan radius r_C or Planck scale. The Dirac equation is integrated in the core of the model. This substantiates the matter field unification scheme proposed above meaning that the preon matter field could be considered as a basis for unification of (i) quarks and leptons and (ii) these particles with gravitational physics of section 5. Any 'true' quantum gravitational phenomena at or near the Planck scale remain an open question.

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