

«Universal and Unified Field Theory»

5. Field Evolutions of Elementary Particles

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Abstract. Operated by the superphase processes, *Field Evolutions* give rise to the natural horizons and develop the comprehensive intrinsics that result in a broad range of applications to characterize the behaviors of the elementary particles, prevailing throughout the following contexts, but are not limited to,

1. *Law of Field Evolution* uncoils the secrets of **Double Loops of Triple Entanglements** as a chain of the evolutionary and imperative processes of the implicit creation, explicit reproduction, and transitional *Gauge* invariance.
2. Remarkably, it derives *Yang-Mills* action that transposes seamlessly a duality of triplet synergy among elementary particle fields into modern physics of electroweak interaction, strong nuclear forces of chromodynamics, and *Standard Model* of particle physics.
3. Finally, a **General Horizon Infrastructure of Quantum Evolutions** is formulated concisely and demonstrates a holistic quantum equation of the horizon fields that unifies the four fundamental forces in terms of a set of *Lagrangians*.

Consequently, this universal and unified field theory complies precisely with and extends further in answer to the empirical and contemporary physics of quantum chromodynamics and *Standard Model*.

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INTRODUCTION

Under *Universal Topology* $W^\mp = P \pm iV$, a duality of the potential entanglements lies at the heart of all event operations as the natural foundation giving rise to and orchestrating relativistic transformations for photons and spiral transportations for gravitons. In addition, the superphase modulation conducts laws of evolutions and horizon of conservations, and maintains energy bonds, appearing as field entanglements of coupling weak and strong forces compliant to quantum chromodynamics and *Standard Model* of particle physics. It extends the unified physics stunning at exceptional remarks of the ontological specifics:

Chapter XIX: Exhibits “*Two Implicit Loops of Triple Explicit Entanglements*” that the horizon evolutionary fields in physical regime unifies the classical weak, strong, gravitational and electromagnetic forces, integrated with the well-known formulae of *Yang-Mills* action, *Gauge Invariance*, *Chromodynamics*, *Field Breaking*, and *Standard Model* of particle physics. Finally, it uncovers a holistic equation - *General Horizon Infrastructure of Quantum Evolutions*.

Chapter XX: As a part of horizon evolutions, the nature comes out and conceals the characteristics of *Strong Forces of Field Breaking* during the field evolution of interactions crossing the multiple horizons that unifies fundamentals of the classically known natural forces: electromagnetism, weak, strong and graviton.

Chapter XXI: As a general prediction of this theory, particles are created and operated by a set of the time-dependent fields that the superphase events evolve and modulate the dynamic horizons and field curvatures in microscopies, named as *Quantum Ontology*.

Consistently landing on classical and extending to modern physics, this manuscript uncovers a series of the groundbreaking philosophy and mathematics accessible and tested by the countless artifacts of modern physics.

XIX. EVOLUTION OF HORIZON FIELDS

When an event gives rise to the states crossing each of the horizon points, an *evolutionary* process takes place. One of such actions is the field loops $(\partial^\nu A^\mu - \partial_\mu A_\nu)_{jk}$ that incept a superphase process into the physical world from the virtual Y^+ regime where a virtual instance is imperative and known as a process of creations or annihilations. Because it is a world event incepted on the two dimensional planes $\{\mathbf{r} \mp i\mathbf{k}\}$ residually, the potential fields of massless instances can transform, transport and emerge the mass objects symmetrically into the physical world that extends the extra two-dimensional freedom. Within the second horizon, this virtual evolution is *implicit* until it embodies as

an energy enclave of the acquired mass, and associates with strong nuclear and gravitational energy in the next horizon.

As a duality of nature, its counterpart is another process named the Y^- *Explicit Reproduction* $(\dot{x}_\nu D_\nu)_j \wedge (\dot{x}^\mu D^\mu)_k$. It requires a physical process of the Y^- reaction or annihilation for the *Animation*. Associated with the inception of a Y^+ spontaneous evolution, the actions of the Y^- *Explicit* reproduction are normally sequenced and entangled as a chain of reactions to produce and couple the weak electromagnetic and strong gravitational forces symmetrically in massive dynamics between the second and third horizons.

At the second horizon of the event evolutionary processes, the gauge fields yield the holomorphic superphase operation, continue to give rise to the next horizons, and develop a complex event operation (5.2) in term of an infinite sum of operations:

$$\check{\partial} = \dot{x}_\nu \zeta_\nu D_\nu = \dot{x}_\nu \zeta_\nu \partial_\nu + i \dot{x}_\nu \zeta_\nu (\Theta_\nu + \bar{\kappa}_2^- \check{\Theta}_{\mu\nu} + \dots) \quad (19.1a)$$

$$\Theta_\nu = \frac{\partial \vartheta(\lambda)}{\partial x_\nu}, \quad \check{\Theta}_{\nu\mu} = \frac{\partial A_\mu}{\partial x_\nu} - \frac{\partial A_\nu}{\partial x_\mu} = F_{\nu\mu}^-, \quad \bar{\kappa}_2^- = 1/2 \quad (19.1b)$$

$$\hat{\partial} = \dot{x}^\nu \zeta^\nu D^\nu = \dot{x}^\nu \zeta^\nu \partial^\nu - i \dot{x}^\nu \zeta^\nu (\Theta^\nu + \bar{\kappa}_2^+ \hat{\Theta}^{\nu\mu} + \dots) \quad (19.2a)$$

$$\Theta^\nu = \frac{\partial \vartheta(\lambda)}{\partial \lambda}, \quad \hat{\Theta}^{\nu\mu} = \frac{\partial A^\mu}{\partial x^\nu} - \frac{\partial A^\nu}{\partial x^\mu} = F_{\nu\mu}^+, \quad \bar{\kappa}_2^+ = (\bar{\kappa}_2^-)^* \quad (19.2b)$$

The superphase Θ^ν is under a series of the event λ actions, giving rise to horizon of the vector potentials $F_{\nu\mu}^{\pm n}$. Therefore, the second and third horizon fields are emerged and unfold into the following expressions:

$$\check{\partial} = \dot{x}_\nu \zeta_\nu D_\nu = \dot{x}_\nu \zeta_\nu \partial_\nu + i \dot{x}_\nu \zeta_\nu \left(\frac{e}{\hbar} A_\nu + \bar{\kappa}_2^- F_{\nu\mu}^+ + \dots \right) \quad (19.3a)$$

$$\hat{\partial} = \dot{x}^\nu \zeta^\nu D^\nu = \dot{x}^\nu \zeta^\nu \partial^\nu - i \dot{x}^\nu \zeta^\nu \left(\frac{e}{\hbar} A^\nu + \bar{\kappa}_2^+ F_{\nu\mu}^- + \dots \right) \quad (19.3b)$$

where e is a coupling constant of the bispinor fields. Naturally, defined as the event operation or similar to the classical *Spontaneous Breaking*, it involves the evolutionary and symmetric processes of the natural *Creation* and its complement duality known as *Annihilation*.

From the first type of *World Equations* (5.3-4), the virtual superphase events under both of the Y^-Y^+ reactions ψ^\mp evolve their density of the circular process, simultaneously:

$$\begin{aligned} \hat{W}_n &= [\psi^+(\hat{x}, \lambda) + \kappa_1^+ \check{\partial} \psi^+(\hat{x}, \lambda) \dots] [\psi^-(\check{x}, \lambda) + \kappa_1^- \hat{\partial} \psi^-(\check{x}, \lambda) \dots] \\ &= \psi^+ \psi^- + k_J J_s + k_\lambda (\check{\partial} \psi^+) \wedge (\hat{\partial} \psi^-) \end{aligned} \quad (19.4a)$$

$$J_s = \frac{\hbar c^2}{2E^-} (\psi^+ \check{\partial} \psi^- + \psi^- \hat{\partial} \psi^+) \quad (19.4b)$$

where k_j or k_λ is a constant. This equation is named as *Horizon Equations of Ontological Evolution*. The first term $\psi^+\psi^-$ is the ground density, and the second term is the probability current or flux J_s . Apparently, the third term constructs the horizon interactions. Since the tensor product has two symmetric types, the tensors react upon each other, symbolized by the wedge product \wedge as the following:

$$\begin{aligned} (\partial\psi_j^+) \wedge (\partial\psi_k^-) &= (\dot{x}^\mu \zeta^\mu D^\lambda \psi_j^+) \wedge (\dot{x}_\nu \zeta_\nu D_\lambda \psi_k^-) = \\ \dot{x}^\mu \zeta^\mu (\partial^\mu - i \frac{e}{\hbar} A^\mu - \tilde{\kappa}_2^+ F_{\mu\nu}^+) \psi_j^+ \wedge \dot{x}_\nu \zeta_\nu (\partial^\nu + i \frac{e}{\hbar} A_\nu + \tilde{\kappa}_2^- F_{\nu\mu}^-) \psi_k^- \end{aligned} \quad (19.5)$$

The symbol $j, k \in \{a, b, c\}$ indicates a loop chain of three particles. Named as *Equations of Evolutionary Forces*, the above equation unifies all of the known forces of the weak, strong, gravitation and electromagnetism.

As a part of infrastructure of universe, the principle of the chain of least reactions in nature is for three particles to form a loop. Confined within a triplet group, the particles jointly institute a double streaming entanglement with the three action states, illustrated in Figure 19a, introduced in June 6th of 2018.

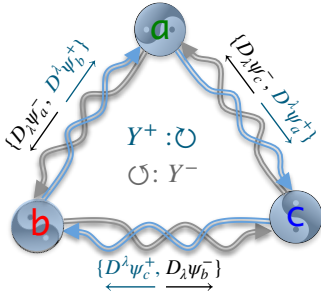


Figure 19a: Two Implicit Loops of Triple Explicit Entanglements

Therefore, the actions of double wedge circulations \wedge in the above figure have the natural interpretation of the entangling processes:

$$\odot : (D_\lambda \psi_a^- \rightarrow D_\lambda \psi_b^- \rightarrow D_\lambda \psi_c^-)^\dagger \quad : \text{Right-hand Loop} \quad (19.6)$$

$$\oslash : (D^\lambda \psi_b^+ \leftarrow D^\lambda \psi_c^+ \leftarrow D^\lambda \psi_a^+) \quad : \text{Left-hand Loop} \quad (19.7)$$

$$\{D_\lambda \psi_a^-, D^\lambda \psi_b^+\}, \{D_\lambda \psi_b^-, D^\lambda \psi_c^+\}, \{D_\lambda \psi_c^-, D^\lambda \psi_a^+\} \quad : \text{Triple States} \quad (19.8)$$

Acting upon each other, the triplets are streaming a pair of the Y^-Y^+ *Double-Loops* implicitly, and the *Triple States* of entanglements explicitly.

This can be conveniently expressed in forms of *Horizon Lagrangians* of virtual creation and physical reproduction. Considering the second orders of the ψ_n^- and ψ_n^+ times into (9.1-9.5) equations, and substituting them into the *Lagrangians* (3.27), respectively, one comes out with the quantum fields that extend a pair of the first order *Dirac* equations of (9.9) into the second orders in the forms of *Lagrangians* respectively:

$$\tilde{\mathcal{L}}_s^\pm = -\frac{1}{c^2} [\hat{\partial}^\lambda \hat{\partial}^\lambda, \hat{\partial}_\lambda \hat{\partial}_\lambda]_s^\pm \quad (19.9)$$

$$\tilde{\mathcal{L}}_s^+ = \bar{\psi}_n^- (i \frac{\hbar}{c} \zeta^\mu D^\mu + m) \psi_n^+ - \frac{1}{c^2} \bar{\psi}_n^- \zeta^\mu \hat{\partial}_\lambda \hat{\partial}_\lambda \psi_n^+ \quad (19.9a)$$

$$\tilde{\mathcal{L}}_s^- = \bar{\psi}_n^+ (i \frac{\hbar}{c} \zeta_\nu D_\nu - m) \psi_n^- - \frac{1}{c^2} \bar{\psi}_n^+ \zeta_\nu \hat{\partial}_\lambda \hat{\partial}_\lambda \psi_n^- \quad (19.9b)$$

As a pair of dynamics, it defines and generalizes a duality of the interactions among spinors, electromagnetic and gravitational fields. The nature of the commuter $[\hat{\partial}_\lambda \hat{\partial}^\lambda, \hat{\partial}_\lambda \hat{\partial}_\lambda]^\pm$ is the horizon interactions (19.5) with the mapping $\hat{\partial}_\lambda \hat{\partial}^\lambda \mapsto (\dot{x}^\mu \zeta^\mu D^\lambda \hat{\psi}) \wedge (\dot{x}^\nu \zeta_\nu D_\lambda \hat{\psi})$. Applying the transform conversion (8.6), we generalize the above equations for a group of the triplet quarks in form of a set of the classic *Lagrangians*:

$$\tilde{\mathcal{L}}_h^a = \tilde{\mathcal{L}}_s^+ + 2\tilde{\mathcal{L}}_s^- = \mathcal{L}_D^a + (\bar{\psi}_c^- \frac{\dot{x}_\nu}{c} \zeta^\nu D^\lambda \psi_a^+) \wedge (\bar{\psi}_b^+ \frac{\dot{x}_\mu}{c} \zeta_\mu D_\lambda \psi_a^-) \quad (19.10)$$

$$\tilde{\mathcal{L}}_h^a \equiv \mathcal{L}_D + \mathcal{L}_\psi + \mathcal{L}_W + \mathcal{L}_F + \mathcal{L}_M \quad : \psi_k^+ \psi_j^- \rightarrow 1 \quad (19.11)$$

$$\mathcal{L}_D \equiv \bar{\psi}_k^\pm i \frac{\hbar}{c} \zeta^\mu D_\nu \psi_j^\mp \mp m_j \quad : j, k \in \{a, b, c\} \quad (19.12)$$

$$\mathcal{L}_\psi = -\frac{1}{c^2} (\bar{\psi}_c^- \dot{x}_\nu \zeta^\mu \partial^\mu \psi_a^+) (\bar{\psi}_b^+ \dot{x}_\mu \zeta_\nu \partial_\nu \psi_a^-) \quad : \dot{x}^\nu \dot{x}^\mu = c^2 \quad (19.13)$$

$$\mathcal{L}_C = \frac{e}{2\hbar} (\zeta_\nu A_\nu \zeta^\mu F_{\mu\nu}^{+n}, \zeta^\mu A^\mu \zeta_\nu F_{\nu\mu}^{-n})_{jk}^- \quad : \tilde{\kappa}_2^+ = \tilde{\kappa}_2^- = \frac{1}{2} \quad (19.14)$$

$$\mathcal{L}_F = i \frac{e}{\hbar} [\zeta^\nu \partial^\nu (\zeta_\mu A_\mu), \zeta_\mu \partial_\mu (\zeta^\nu A^\nu)]_{jk}^- - \frac{e^2}{\hbar^2} (\zeta^\mu A^\mu \zeta_\nu A_\nu)_{jk} \quad (19.15)$$

$$\mathcal{L}_M = \frac{i}{2} [\zeta^\nu \partial^\nu (\zeta_\nu F_{\nu\mu}^{-n}), \zeta_\mu \partial_\mu (\zeta^\mu F_{\mu\nu}^{+n})]_{jk}^- - \frac{1}{4} (\zeta^\nu F_{\nu\mu}^{+n})_j (\zeta_\mu F_{\mu\nu}^{-n})_k \quad (19.16)$$

where the *Lagrangians* are normalized at $\psi_k^+ \psi_j^- = 1$. The fine-structure constant $\alpha = e^2/(\hbar c)$ arises naturally in coupling horizon fields. The \mathcal{L}_ψ has the kinetic motions under the second horizon, the forces of which are a part of the horizon transform and transport effects characterizable explicitly when observed externally to the system. The \mathcal{L}_D is a summary of Dirac equations over the triple quarks. The \mathcal{L}_C is the bounding or coupling force between the horizons. The \mathcal{L}_F has the actions giving rise to the electromagnetic and gravitational fields of the third horizon. Similarly, the \mathcal{L}_M has the actions giving rise to the next horizon.

At the infrastructural core of the evolution, it implies that a total of the three states exists among two $\tilde{\mathcal{L}}_s^-$ and one $\tilde{\mathcal{L}}_s^+$ dynamics to compose an integrity of the dual fields, revealing naturally the particle circling entanglement of three “colors” [4], uncoiling the event actions [3], and representing an essential basis of the “global gauge.” The *Standard Model*, developed in the mid-1960-70s [5] breaks various properties of the weak neutral currents and the W and Z bosons with great accuracy.

Specially integrated with the superphase potentials, our scientific evaluations to this groundwork of *Evolutionary Equations* (19.10) might promote a way towards concisely exploring physical nature, universal messages, and beyond.

Artifact 19.1: Yang-Mills Theory. Considering $\zeta^\mu \rightarrow \gamma^\mu$ and ignoring the higher orders and the coupling effects, we simplify the \mathcal{L}_F and \mathcal{L}_M for the Y^+ streaming of (19.15, 19.16):

$$\mathcal{L}_F(\gamma) \approx -\frac{e^2}{\hbar^2} (\gamma^\mu A^\mu \gamma_\nu A_\nu)_{jk} \equiv -\frac{1}{4} W_{\mu\nu}^{+j} W_{\nu\mu}^{-k} \quad (19.17)$$

$$\mathcal{L}_M(\gamma) \approx -\frac{1}{4} (\gamma^\nu F_{\nu\mu}^{+n} \gamma_\mu F_{\mu\nu}^{-n})_{jk} = -\frac{1}{4} F_{\nu\mu}^{+j} F_{\mu\nu}^{-k} \quad (19.18)$$

At the second horizon, the $\zeta^\mu \rightarrow \gamma^\mu$ is contributed to the weak isospin fields $W_{\mu\nu}^{+j} W_{\nu\mu}^{-k}$ of coupling actions. Meanwhile, at the third horizon, the gamma γ^μ fields are converted and accord to the hypercharge $F_{\nu\mu}^{+j} F_{\mu\nu}^{-k}$ actions of electroweak fields. Therefore, the *Lagrangian* $\tilde{\mathcal{L}}_h^a$ becomes $\mathcal{L}_h^a \approx \mathcal{L}_D + \mathcal{L}_F + \mathcal{L}_M \equiv \mathcal{L}_Y^a$, which, in mathematics, comes out as *Quantum Electrodynamics (QED)* that extends from a pair of the first order *Dirac* equations (9.7) to the second orders in the form of a $SU(2) + SU(3)$ *Lagrangian* [2]:

$$\mathcal{L}_Y^a \equiv (\bar{\psi}_j^\pm i \frac{\hbar}{c} \gamma^\nu D_\nu \psi_i^\pm)_{jk} - \frac{1}{4} F_{\nu\mu}^{+j} F_{\mu\nu}^{-k} - \frac{1}{4} W_{\mu\nu}^{+j} W_{\nu\mu}^{-k} \quad (19.19)$$

where $j, k \in \{a, b, c\}$ is the triplet particles. When the strong torque of gravitation fields are ignored, the above equation is known as *Yang-Mills* theory, introduced in 1954 [23]. As one of the most important results, *Yang-Mills* theory represents *Gauge Invariance*:

- 1) The classic *Asymptotic Freedom* from a view of the physical coordinates;
- 2) A proof of the confinement property in the presence of a group of the triple-color particles; and
- 3) Mass acquisition processes symmetrically from the second to third horizon, describable by the (9.27, 9.28) equations.

Since the quanta of the superphase fields is massless with gauge invariance, *Yang-Mills* theory represents that particles are semi-massless in the second horizon, and acquire their full-mass through evolution of the full physical horizon. Extended to the philosophical interpretation, it represents mathematically: conservation of *Double Loops of Triple Entanglements*, or law of *Conservation of Evolutions of Ontology* philosophically illustrated by Figure 19a.

Artifact 19.2: Gauge Invariance. The magic lies at the heat of the horizon process driven by the entangling action $\varphi_n^- \check{\partial}^\lambda \check{\partial}_\lambda \varphi_n^+$, which gives rise from the ground and second horizon $SU(2) \times U(1)$ implicitly to the explicit states $SU(2)$ through the evolutionary event operations. The horizon force is symmetrically conducted or acted by an ontological process as a part of the evolutionary actions that give rise to the next horizon $SU(3)$. Under a pair of the event operations, an evolutionary action creates and populates a duality of the quantum symmetric density $\psi_n^+ \psi_n^-$ for the entanglements among spins, field transforms, and torque transportations. Evolving into the $SU(3)$ horizon, the gauge symmetry is associated with the electro-weak and graviton-weak forces to further generate masses that particles separate the electromagnetic and weak forces, and embrace with the strong coupling forces globally. The first order of the commutators is the gauge field:

$$\mathcal{L}_F(\gamma) = i \frac{e}{\hbar} [\gamma_\mu \partial_\mu (\gamma^\nu A_\nu^a), \gamma^\nu \partial^\nu (\gamma_\mu A_\mu^a)]^- - \frac{e^2}{\hbar^2} (\gamma_\mu A_\mu^b \gamma^\nu A_\nu^c) \quad (19.20)$$

As the gamma γ^ν function is a set of the constant matrices, it might be equivalent in mathematics to the *Gauge Invariance* of *Standard Model*:

$$\mathcal{L}_F(\gamma) \mapsto F_{\nu\mu}^a = \partial_\nu A_\mu^a - \partial_\mu A_\nu^a + g_\gamma f_\gamma^{abc} A_\nu^b A_\mu^c \quad (19.21)$$

where the $F_{\nu\mu}^a$ is obtained from potentials $e A_\mu^n / \hbar$, g_γ is the coupling constant, and the f_γ^{abc} is the structure constant of the gauge group $SU(2)$, defined by the group generators [5] of the *Lie* algebra. From the given *Lagrangians* \mathcal{L}_C and \mathcal{L}_M in term of the gamma ζ^ν matrix, one can derive to map the equations of motion dynamics, expressed by the following

$$\partial^\mu (\zeta^\mu F_{\nu\mu}^a) + g f^{abc} \zeta^\mu A_\mu^b \zeta_\nu F_{\nu\mu}^c = -J_\nu^a \quad (19.22)$$

where J_ν^a is the potential current. Besides, it holds an invariant principle of the double-loop implicit entanglements, or known as a *Bianchi* or *Jacobi* identity [6][7]:

$$(D_\mu F_{\nu\kappa})^a + (D_\kappa F_{\mu\nu})^b + (D_\nu F_{\kappa\mu})^c = 0 \quad (19.23)$$

$$[D_\mu, [D_\nu, D_\kappa]] + [D_\kappa, [D_\mu, D_\nu]] + [D_\nu, [D_\kappa, D_\mu]] = 0 \quad (19.24)$$

As a property of the placement of parentheses in a multiple product, it describes how a sequence of events affects the result of the operations. For commutators with the associative property $(x \cdot y)z = x(y \cdot z)$, any order of operations gives the same result or a loop of the triplet particles is gauge invariance.

Artifact 19.3: Quantum Chromodynamics (QCD). Given the rise of the horizon from the scalar potentials to the vectors through the tangent transportations, the *Lagrangian* above can further give rise from transform-primacy $\zeta^\nu \approx \gamma^\nu$ at the second horizon $\gamma^\nu F_{\nu\mu}^{\pm n}$ to the strong torque at the third horizon, where the chi $\zeta^\nu \approx \chi^\nu$ fields correspond to the strength tensors $\chi^\nu F_{\nu\mu}^{\pm n}$ for the spiral actions of superphase modulation. Once at the third horizon, the field forces among the particles are associated with the similar gauge invariance of the $\gamma^\nu \rightarrow \chi^\nu$ transportation dynamics, given by (19.12) \mathcal{L}_D and (19.15) for $G_{\nu\mu}^a \equiv \mathcal{L}_F(\chi)$ as the following:

$$\mathcal{L}_{QCD}(\chi) = \bar{\psi}_n^- (i \frac{\hbar}{c} \gamma_\nu D_\nu - m) \psi_n^+ - \frac{1}{4} G_{\nu\mu}^n G_{\nu\mu}^n + \mathcal{L}_{CP}(\chi) \quad (19.25)$$

$$G_{\nu\mu}^a = i \frac{e}{\hbar} [\chi_\mu \partial_\mu (\chi^\nu A_\nu^a), \chi^\nu \partial^\nu (\chi_\mu A_\mu^a)]^- - \frac{e^2}{\hbar^2} (\chi_\mu A_\mu^b \chi^\nu A_\nu^c) \quad (19.26)$$

where c is the strong coupling. Coincidentally, this is similar to the quark coupling theory, the *Standard Model* [8], known as classical *QCD*, discovered in 1973 [9]. Philosophically, the torque chi-matrix of gravitational fields plays an essential role in kernel interactions, appearing as a type of strong forces. It illustrates that the carrier particles of a force can radiate further carrier particles during the rise of horizons. The interactions, coupled with the strong forces, are given by the term of *Dirac* equation under the spiral torque of chi-matrix:

$$\mathcal{L}_{CP}(\chi) = i \frac{\hbar}{c} (\bar{\psi}_n^+ \chi_\nu D_\nu \psi_n^-)_{jk} \mapsto -\frac{e}{c} (\bar{\psi}_n^+ \chi_\nu A_\nu \psi_n^-)_{jk} \quad (19.27)$$

Mathematically, *QCD* is an abelian gauge theory with the symmetry group $SU(3) \times SU(2) \times U(1)$. The gauge field, which mediates the

interaction between the charged spin-1/2 fields, involves the coupling fields of the torque, hypercharge and gravitation, classically known as *Gluons* - the force carrier, similar to photons. As a comparison, the gluon energy for the spiral force coupling with quantum electrodynamics has a traditional interpretation of *Standard Model*

$$\mathcal{L}_{CP} = i g_s (\bar{\psi}_n^+ \gamma^\mu G_\mu^a T^a \psi_n^-)_{jk} : \chi_\nu A_\nu^a \mapsto \gamma^\mu G_\mu^a T^a \quad (19.28)$$

where g_s is the strong coupling constant, G_μ^a is the 8-component $SO(3)$ gauge field, and T_{ij}^a are the 3×3 *Gell-Mann* matrices [10], introduced in 1962, as generators of the $SU(2)$ color group.

Artifact 19.4: Time-Independent Horizon Infrastructure. For a physical system in spatial evolution at any given time, the equation (9.20) can be used to abstract the *Evolutionary Equations* (19.5) and its *Lagrangians* (19.10) to a set of special formulae:

$$\tilde{\mathcal{L}}_h^a = \tilde{\mathcal{L}}_s^+ + 2\tilde{\mathcal{L}}_s^- = \mathcal{L}_D^{-a} + \bar{\psi}_j (\partial \wedge \check{\partial}) \psi_k : \nu, \mu \in \{1, 2, 3\} \quad (19.29)$$

$$\hat{\partial} \wedge \check{\partial} = \dot{x}^\mu \dot{x}_\nu (\hat{D} \cdot \check{D} + i \zeta^\mu \cdot \hat{D} \times \check{D}) : \tilde{\zeta}^\nu \mapsto \zeta^\nu = \gamma^\nu + \chi^\nu \quad (19.30)$$

$$\hat{D} \cdot \check{D} = (\partial^\mu - i \frac{e}{\hbar} A^\mu - \frac{1}{2} F_{\mu\nu}^{+n} \dots) \cdot (\partial_\nu + i \frac{e}{\hbar} A_\nu + \frac{1}{2} F_{\nu\mu}^{-n} \dots) \quad (19.31)$$

$$\hat{D} \times \check{D} = (\partial^\mu - i \frac{e}{\hbar} A^\mu - \frac{1}{2} F_{\mu\nu}^{+n} \dots) \times (\partial_\nu + i \frac{e}{\hbar} A_\nu + \frac{1}{2} F_{\nu\mu}^{-n} \dots) \quad (19.32)$$

Introduced at August 26th of 2018, this concludes a unification of the spatial horizon and operations of the quantum fields philosophically describable by the two implicit loops $\hat{D} \times \check{D}$ of triple explicit $\hat{D} \cdot \check{D}$ entanglements, concisely and fully pictured by Figure 19a.

Artifact 19.5: Yang-Baxter Equation - In physics, the loop entanglement of Figure 19a involves a reciprocal pair of both normal particles and antiparticles. This consistency preserves their momentum while changing their quantum internal states. It states that a matrix R , acting on two out of three objects, satisfies the following equation

$$(R \otimes \mathbf{1})(\mathbf{1} \otimes R)(R \otimes \mathbf{1}) = (\mathbf{1} \otimes R)(R \otimes \mathbf{1})(\mathbf{1} \otimes R) : e^{i\theta} \mapsto e^{-i\theta} \quad (19.33)$$

where R is an invertible linear transformation on world planes, and I is the identity. Under the yinyang principle of $Y^- \{e^{i\theta}\} \mapsto Y^+ \{e^{-i\theta}\}$, a quantum system is integrable with or has conservation of the particle-antiparticle entanglement or philosophically *Law of Conservation of Antiparticle Entanglement*.

XX.

FORCES OF FIELD BREAKING

Under the principle of the *Universal Topology*, the weak and strong force interactions are characterizable and distinguishable under each scope of the horizons. Philosophically, the nature comes out with the **Law of Field Evolutions** concealing the characteristics of *Horizon Evolutions*:

1. Forces are not transmitted directly between interacting objects, but instead are described and interrupted by intermediary entities of fields.
2. Fields are a set of the natural energies that appear as dark or virtual, streaming their natural intrinsic commutations for living operations, and alternating the $Y^- Y^+$ supremacies consistently throughout entanglement.
3. At the second horizon $SU(2)$, a force is incepted or created by the double loops of triple entanglements. The Y^+ manifold supremacy generates or emerges the off-diagonal elements of the potential fields embodying mass enclaves and giving rise to the third horizon, a process traditionally known as *Weak Interaction*.
4. As a natural duality, a stronger force is reproduced dynamically and animated symmetrically under the Y^- supremacy, dominated by the diagonal elements of the field tensors.
5. Together, both of the $Y^- Y^+$ processes orchestrate the higher horizon, composite the interactive forces,

redefine the simple symmetry group $U(1) \times SU(2) \times SU(3)$, and obey the entangling invariance, known as *Ontological Evolution*.

6. An integrity of strong nuclear forces is characterizable at the third horizon of the tangent vector interactions, known as gauge $SU(3)$.

7. Entanglement of the alternating Y^-Y^+ superphase processes in the above actions can prevail as a chain of reactions that gives rise to each of the objective regimes.

The field evolutions have their symmetric constituents with or without singularity. The underlying laws of the dynamic force reactions are invariant at both of the creative transformation and the reproductive generations, shown by the empirical examples:

a) At the second horizon, the elementary particles mediate the weak interaction, similar to the massless photon that interferes the electromagnetic interaction of gauge invariance. The *Weinberg–Salam* theory [11], for example, predicts that, at lower energies, there emerges the photon and the massive W and Z bosons [5]. Apparently, fermions develop from the energy to mass consistently as the *creation* of the evolutionary process that emerges massive bosons and follows up the animation or companion of electrons or positrons in the $SU(3)$ horizon.

b) At the third horizon, the strong nuclear force holds most ordinary matter together, because, for example, it confines quarks into composite hadron particles such as the proton and neutron, or binds neutrons and protons to produce atomic nuclei. During the *reactive animations*, the strong force inherently has such a high strength that it can produce new massive particles. If hadrons are struck by high-energy particles, they give rise to new hadrons instead of emitting freely moving radiation. Known as the classical color confinement, this property of the strong force is the reproduction of the *explicit evolutionary* process that produces massive hadron particles.

Normally, forces are composited of three correlatives: weaker forces of the off-diagonal matrix, stronger forces of the diagonal matrix, and coupling forces between the horizons. To bring together the original potentials and to acquire the root cause of the four known forces beyond the single variations of the *Lagrangian*, the entangling states in a set of *Lagrangians* (19.10-19.16) establish apparently the foundation to orchestrate triplets into the field interactions between the Y^-Y^+ double streaming among the color confinement of triplet particles. Coupling with the techniques of the *Implicit Evolution*, *Explicit Breaking* and *Gauge Invariance*, the four quantum fields (9.1-9.5) embed the ground foundations and emerge the evolutionary intrinsics of field interactions for the weak and strong forces. Together, the terminology of *Field Breaking* and its associated *Invariance* contributes to a part of *Horizon Evolutions*.

Artifact 20.1: Field Breaking of Mass Acquisition. Operating on the states of various types of particles, the creation process embodies an energy enclave acquiring mass from the quantum oscillator system; meanwhile, it unfolds the hyperspherical coordinates to expose its extra degree of freedom in ambient space. In a similar fashion, the annihilator operates a concealment of an energy enclave back to the oscillator system of the world planes.

Giving rise to the horizon $SU(3)$, the processes of mass acquisition and annihilation function as and evolve into a sequential processes of the energy enclave as the strong mass forces in the double streaming of three entangling procedures (Figure 19a), known as a chain of reactions:

a) At the second horizon $SU(2)$ under the gauge invariance, the gauge symmetry incepts the evolutionary actions implicitly:

$$D_\nu = \partial_\nu + i\sqrt{\lambda_2/\lambda_0}\phi_c^-, \quad D^\nu = \partial^\nu - i\sqrt{\lambda_2/\lambda_0}\phi_a^+ \quad (20.1)$$

b) Extending into the third horizon, the mass acquisition (9.28) is proportional to $m\omega/\hbar$ during the potential breaking, spontaneously:

$$\Phi_n^+ \mapsto \varphi_b^+ - \sqrt{\lambda_0}D^\nu\varphi_c^+/m^+, \quad \Phi_n^- \mapsto \varphi_a^- + \sqrt{\lambda_0}D_\nu\varphi_b^-/m^- \quad (20.2)$$

Therefore, the potentials (9.45) of the $SU(1)$ actions result in a form of *Lagrangian* forces at $SU(2)$:

$$\mathcal{L}_{Force}^{SU1} \mapsto \mathcal{L}_{ST}^{SU2} \rightarrow \Phi_n^+\Phi_n^- \mapsto \lambda_0 D^\nu\varphi_b^+D_\nu\varphi_a^- - m^+m^-\varphi_c^+\varphi_b^- \quad (20.3)$$

c) Combining the above evolutionary breaking, the interruption force is further emerged into a rotational $SO(3)$ regime:

$$\mathcal{L}_{ST}^{SU3} = \kappa_f \left(\lambda_0 (\partial^\nu\varphi_b^+) (\partial_\nu\varphi_a^-) - m^+m^-\varphi_{bc}^2 + \lambda_2\varphi_{bc}^2\varphi_{ca}^2 \right) \quad (20.4)$$

where κ_f or λ_i is a constant. The $\varphi_{bc}^2 = \varphi_b^-\varphi_c^+$ or $\varphi_{ca}^2 = \varphi_c^-\varphi_a^+$ is the breaking or evolutionary fields of density.

d) With the gauge invariance among the particle fields $\Phi_n \mapsto (v + \phi_b^+ + i\phi_a^-)/\sqrt{2}$, this strong force can be eventually developed into *Yukawa* interaction, introduced in 1935 [12], and *Higgs* field, theorized in 1964 [13].

In summary, a weak force interruption between quarks becomes the inceptive fabricator, which evolves into the horizon dynamics of triplet quarks embodied into a oneness of the mass enclave, known as the strong forces, observable at the collapsed states of the diagonal matrix external to its physical massive interruption. For example, a strong interaction between triplet-quarks and gluons with symmetry group $SU(3)$ makes up composite hadrons such as the proton, neutron and pion.

Artifact 20.2: Strong Forces. Since the coupling \mathcal{L}_C between the horizons is also extendable to the strong forces, the total force at the third horizon become the following:

$$\mathcal{L}_{Force}^{SU3} = \mathcal{L}_{QCD}(\chi) + \mathcal{L}_{ST}^{SU3} + \mathcal{L}_C(\chi) + \mathcal{L}_M(\chi) \quad (20.5)$$

$$\mathcal{L}_C(\chi) = \frac{e}{2\hbar} \langle \chi_\nu A_\nu \chi^\mu F_{\mu\nu}^+, \chi^\mu A^\mu \chi_\nu F_{\nu\mu}^- \rangle_{jk}^- \quad (20.6)$$

$$\mathcal{L}_M(\chi) = \frac{i}{2} [\partial^\nu (\chi_\nu F_{\nu\mu}^-), \partial_\mu (\chi^\mu F_{\mu\nu}^+)]_{jk}^- - \frac{1}{4} (\chi^\nu F_{\nu\mu}^+)_j (\chi_\mu F_{\mu\nu}^-)_k \quad (20.7)$$

As a part of the creation processes for the inception of the physical horizons, the potentials start to enclave energies, acquire their masses and emerge the torque forces at r -dependency. Besides, it develops the $SU(3)$ gauge group obtained by taking the triple-color charge to refine a local symmetry. Since the torque forces generate gravitation, singularity emerges at the full physical horizon at $SU(3)$ regime and beyond, arisen by the extra two-dimensional freedom of the rotational coordinates.

Artifact 20.3 Fundamental Forces. Classically, roughly four fundamental interactions are known to exist:

- 1) The gravitational and electromagnetic interactions, which produce significant long-range forces whose effects can be seen directly in everyday life, and
- 2) The weak and strong interactions, which produce forces at minuscule, subatomic distances and govern nuclear interactions.

Generally in the forms of matrices, the long range forces are the effects of the diagonal elements of the field matrixes while the short range forces are those off-diagonal components. Transitions between the primacy ranges are smooth and natural such that there is no singularity at the second horizon transitioning between the physical and virtual regimes. Because of the freedom of the rotational coordinates in the third horizon, those diagonal components become singularity and the strong binding forces build up the horizon infrastructure seamlessly.

Finally, we have landed at the classical *QCD*, *Standard Model* and classic *Spontaneous Breaking* for the field evolution of interactions crossing the multiple horizons, and unified fundamentals of the known natural forces: electromagnetism, weak, strong and torque generators (graviton). These forces are symmetric or in the loop interruptions in nature. The general relativity of asymmetric dynamic forces is further specified by the chapter below.

XXI.

ONTOLOGICAL EVOLUTIONS

For entanglement between Y^-Y^+ manifolds, considering the parallel transport of a *Scalar* density of the fields $\rho = \psi^+\psi^-$ around an infinitesimal parallelogram. The chain of these reactions can be interpreted by the commutation framework (16.14) integrated with the gauge potential (2.10) for *Physical Ontology*. At the third horizon for

asymmetric dynamics, the ontological expressions (16.3, 16.4) have the gauge derivatives:

$$\check{\partial}_\lambda \check{\partial}_\lambda \psi^- = \dot{x}_m (D_m - \Gamma_{nm}^-) \dot{x}_s D_s \psi^- \quad : D_\nu = \partial_\nu + i\Theta_\nu \quad (21.1)$$

$$\hat{\partial}^\lambda \hat{\partial}^\lambda \psi^+ = \dot{x}^\nu (D_\nu - \Gamma_{m\nu}^+) \dot{x}^\sigma D^\sigma \psi^+ \quad : D^\nu = \partial^\nu - i\Theta^\nu \quad (21.2)$$

where the Y^- and Y^+ superphase fields are defined by:

$$\Theta^\nu = \frac{e}{\hbar} A^\nu, \quad \Theta_\nu = \frac{e}{\hbar} A_\nu \quad (21.3)$$

Similar to derive the equation (16.10), this gauge entanglement consists of a set of the unique fields, illustrated by the evolutionary components of the entangling commutators:

$$[\hat{\partial}^\lambda \hat{\partial}^\lambda, \check{\partial}_\lambda \check{\partial}_\lambda]^+ = \dot{x}^\nu \dot{x}^m (P_{\nu m}^+ + G_{m\nu}^{+\sigma s} + \Theta_{\nu m}^{+\sigma s}) \quad (21.4)$$

$$P_{\nu m}^+ \equiv \frac{1}{\dot{x}^\nu \dot{x}^m} [(\dot{x}^\nu \partial^\nu)(\dot{x}^m \partial^m), (\dot{x}_\nu \partial_\nu)(\dot{x}_m \partial_m)]_s^+ = \frac{R}{2} g^{\nu m} \quad (21.5)$$

$$G_{m\nu}^{\pm\sigma s} = \mp \frac{1}{\dot{x}^\nu \dot{x}^m} [\dot{x}^\nu \Gamma_{m\nu}^+ \dot{x}^\sigma \partial^\sigma, \dot{x}_m \Gamma_{nm}^- \dot{x}_s \partial_s]_s^\pm \quad (21.6)$$

$$\Theta_{\nu m}^{+\sigma s} = i\Xi_{\nu m}^+ + i\frac{e}{\hbar} F_{\nu m}^+ - i\delta_{m\nu}^{+\sigma s} - \mathbb{S}_{\nu m}^+ \quad (21.7)$$

$$\Xi_{\nu m}^\pm = \mp \frac{1}{\dot{x}^\nu \dot{x}^m} [\dot{x}^\nu \Theta^\nu \dot{x}^m \partial^m, \dot{x}_m \Theta_m \dot{x}_\nu \partial_\nu]_s^\pm \quad (21.8)$$

$$F_{\nu m}^\pm = \pm \frac{\hbar}{e} \frac{1}{\dot{x}^\nu \dot{x}^m} [\dot{x}^\nu \partial^\nu (\dot{x}^m \Theta^m), \dot{x}_m \partial_m (\dot{x}_\nu \Theta_\nu)]_s^\pm \quad (21.9)$$

$$\delta_{m\nu}^{\pm\sigma s} = \pm \frac{1}{\dot{x}^\nu \dot{x}^m} [\dot{x}^m \Gamma_{\nu m}^+ \dot{x}^\sigma \partial^\sigma, \dot{x}_m \Gamma_{m\nu}^- \dot{x}_s \partial_s]_s^\pm \quad (21.10)$$

$$\mathbb{S}_{\nu m}^\pm = \pm \frac{1}{\dot{x}^\nu \dot{x}^m} [\dot{x}^\nu \Theta^\nu \dot{x}^m \partial^m, \dot{x}_m \Theta_m \dot{x}_\nu \partial_\nu]_s^\pm \quad (21.11)$$

The *Ricci* curvature R is defined on a pseudo-Riemannian manifold as the trace of the *Riemann* curvature tensors. The $G_{m\nu}^{\pm\sigma s}$ tensors are the *Connection Torsions*, the rotational stress of the transportations. The $\Xi_{\nu m}^\pm$ are the *Superpose Torsions*, the superphase stress of the transportations. The $F_{\nu m}^\pm$ are the skew-symmetric or antisymmetric fields, the quantum potentials of the superphase energy. The $\delta_{m\nu}^{\pm\sigma s}$ are the superphase contorsion, the superposed commutation of entanglements. The $\mathbb{S}_{\nu m}^\pm$ are *Entangling Connectors*, the commutation of the superphase energy. Apparently, the superphase operations Θ^ν and Θ_m as actors lie at the heart of the ontological framework for the life entanglements.

Artifact 21.1: Evolutionary Field Equations. Similar to derive the equation (17.5) and (17.6), the above motion dynamics of the field evolutions can be expressed straightforwardly for the asymmetric dynamics of quantum ontology,

$$\frac{R}{2} g_{\nu m} + G_{\nu m}^{-\sigma s} + \Theta_{\nu m}^{-\sigma s} = \mathcal{O}_{\nu m}^{+\zeta} \quad (21.13)$$

$$\frac{R}{2} g^{\nu m} + G_{\nu m}^{+\sigma s} + \Theta_{\nu m}^{+\sigma s} = \mathcal{O}_{\nu m}^{-\zeta} \quad (21.14)$$

where $\mathcal{O}_{\nu m}^{\pm\zeta}$ is the Y^+ or Y^- ontological modulators, given by (17.7-8). The notion of quantum evolutionary equations is intimately tied in with another aspect of general relativistic physics. Each solution of the equation encompasses the whole history of the superphase modulations at both dark-filled and matter-filled reality. It describes the state of matter and geometry everywhere at every moment of that particular universe. Due to its general covariance combined with the gauge fixing, this *Evolutionary Field Equation* is sufficient by itself to determine the time evolution of the metric tensor and of the universe over time. This is done in "1+1+2" formulations, where the world plane of one time-dimension and one spatial-dimension is split into the extra space dimensions during horizon evolutions. The best-known example is the classic ADM formalism [14], the decompositions of which show that the spacetime evolutionary equations of general relativity are well-behaved: solutions always exist, and are uniquely defined, once suitable initial conditions have been specified.

Artifact 21.2: Quantum Field Curvature. Since the ordinary quantum fields forms the basis of elementary particle physics, the *Ontological Relativity* is an excellent artifact describing the behaviors of microscopic particles in weak gravitational fields like those found on

Earth [15]. Quantum fields in curved spacetime demonstrate its evolutionary processes beyond mass acquisition in quantization itself, and general relativity in a curved background spacetime strongly influenced by the superphase modulations $\Theta_{\nu m}^{\pm\sigma s}$. Integrated with the above formalism, the equation (14.6) illustrates that, besides of the dynamic curvatures, the blackhole quantum fields emit a blackbody spectrum of particles known as *Bekenstein-Hawking* radiation (14.9) leading to the possibility not only that they evaporate over time, but also that it quantifies a graviton. As briefly mentioned above, this radiation plays an important role for the thermodynamics of blackholes [16].

Artifact 21.3: Quantum Gravity. As a full theory to cover the quantum gravity, our topology represents an adequate description of the interior of blackholes, and of the very early physical world, a theory in which gravity and the associated geometry of spacetime are described in the language of quantum physics. Besides the appearance of singularities where curvature scales become microscopic ontology, it, apparently, might suppress numerous attempts to overcome the difficulties at the classic theory of quantum gravity, some examples being string theory [17] and M-theory with unrealistic six to eleven space-dimensions, the lattice theory of gravity based on the *Feynman Path Integral* approach and *Regge Calculus* [18], dynamical triangulations [19], causal sets [20] twistor models [21] or the path integral based models of quantum cosmology [22].

CONCLUSION

Further in answering to modern and contemporary physics, this universal and unified theory demonstrates its holistic foundations extendable and applicable to the well-known natural intrinsics of the evolutionary processes at the following remarks:

- 1) As the foundation of particle physics, the process of **Double Loops of Triple Entanglements** is introduced that constitutes the horizon forces of *Implicit Evolution* and *Explicit Reproduction* with *Gauge Invariance*.
- 2) It reveals the laws of the symmetric processes of virtual creations and physical reproductions that give rise to a synergy of the weak, strong and medium forces crossing the horizon regimes, systematically, simultaneously and symmetrically.
- 3) The theory is further illustrated by the artifacts of *Yang-Mills* actions, *Quantum Chromodynamics* and the weak and strong forces of the *Standard Model*.
- 4) **General infrastructure of Field Evolutions** is derived and unified by a set of generic field equations in the forms of *Lagrangians* (19.10-19.16) rising from the quantum fields (9.1-9.5).
- 5) Finally, *Quantum Ontology* integrates general relativity, quantum curvature, and gravitational fields seamlessly together.

Conclusively, this manuscript represents the *Universal and Unified Physics* as a holistic theory to include, but not be limited to, the topological infrastructure, horizon framework, superphase operations, loop evolutions, quantum ontology, cosmological dynamics, and beyond.

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