Formula for the mass spectrum of charged fermions and bosons

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We present the formula for the mass spectrum of the charged composite particles (CP). This formula includes the renormalized fine-structure constant α =1/128.330593928, the rest mass of a new electrically charged particle m = 156.3699214 eV/c^2 and two quantum numbers of n and k. The half–integer and integer quantum number n is the projection of an orbital angular momentum electrically charged particle on the symmetry axis of the CP, and the integer k defines the magnetic charges of two Dirac magnetic monopoles, which have opposite signs of magnetic charges and masses. The presented model predicts the values of spins, masses, charge orbit radii and magnetic moments for an infinite number of charged fermions and bosons in the infinite range of mass.

1. Introduction

120 years ago J. J. Thomson discovered the first subatomic particle – the electron [1]. Now several hundred subatomic particles are experimentally known, however, the question about the nature of the mass and the mass spectrum of subatomic particles is one of the most fundamental problems in physics, which unfortunately is not solved yet.

By the early 20th century J. J. Thomson, A. M. Abraham, H. A. Lorentz and H. H. Poincaré had put forward a very promising idea concerning the electromagnetic nature of the electron mass in the framework of classical electrodynamics [1–4]. Lorentz has applied this idea for the extended electron model and has estimated quantity named the classical electron radius r_e equating electrostatic energy of the charge e to the electron rest energy: $r_e = e^2/m_ec^2 = 2.817{*}10^{-12}$ cm, where e and m_e are the electron charge and mass and c is the speed of light. However, the problem of electron mass has not found any consistent solution neither in classical electrodynamics nor later in quantum electrodynamics, mainly due to the divergence of electron self-energy in a point particle models and the problems with electron stability in extended particle models [5, 6].

The presently successful standard model (SM) of modern particle physics considers tree known families of the charged leptons (e, μ, τ) and the quarks (u, d, s, c, b, t) as the fundamental (elementary) point-like particles [7]. However, in this approach the SM cannot explain nor predict the possible numbers of the families of the fundamental particle, its masses and spins and it takes all of them outside the theory as experimentally known parameters. Thus, not all of us understand yet the reason of the muon existence and the muon-electron universality. Another theoretical problem is the parameters proliferation. The SM depends on more than twenty parameters, which are a priori arbitrary. The masses of leptons and quarks in the structure of this theory, as is assumed, are proportional to the coupling constants of the charged fermions with the scalar Higgs field [8]. As these coupling constants cannot be consistently defined in the Higgs theory, this mechanism actually does not provide any deeper understanding of the nature and the hierarchy of the mass of fundamental particles.

It is reasonable that the solutions of the problem of mass can lie in the developing of a new fundamental theory on the basis of a new fundamental structural level which allows to explain naturally the known experimental results and in addition to predict new phenomena. As it is known from the history of physics, a similar problem was successfully solved at the beginning of the last century thanks to the discovery of the structure of atom and the development of the quantum theory.

Various phenomenological models of composite subatomic particles have been proposed with the aim to reproduce quantum numbers of leptons and quarks [9–12]. An excellent analysis of this field has been presented by Lyons [13]. These models, however, in spite of some success in the classification of possible quantum states of composite particles (Quantum Numerology), have not solved the mass spectrum problem, because they have not contained any convincing dynamics at the fundamental level.

To find the way to the future fundamental theory of subatomic particles it would be useful to find for particles at first an analogous to the Balmer-like formula or the Bohr theory for hydrogen atom. In this line Barut [14] has proposed the following formula for the muon mass calculation: $m_\mu = m_e(1+(3/2)\alpha^{-1})$, where input parameters are the electron mass m_e and the fine–structure constant α. This formula has been extended further by him, predicting in addition the masses of two heavy leptons [15]. Other empirical mass formulas for charged leptons and quarks have also been proposed [16, 17].

In this work we present the formula for the mass spectrum of charged composite particles, which we have derived by using the Bohr–Sommerfeld quantization rule for an electrically charged particle moving on the circular orbit at relativistic velocity in the magnetic field of two Dirac magnetic monopoles. The details of our model and derivation of the mass formula will be presented in a future publication. This formula includes the renormalized fine-structure constant $\alpha = 1/128.330593928$, the rest mass $m = 156.3699214$ eV/c^2 of a new electrically charged particle and two quantum numbers n and k. The half-integer and integer quantum number n is the projection of the orbital angular momentum of an electrically charged particle on the symmetry axis of the CP and the integer number k defines the magnetic charges of two Dirac's magnetic monopoles (MM), which have opposite signs of magnetic charges and masses.

2. Mass spectrum formula of charged fermions and bosons

Our model of the charged composite particles includes two massive spineless Dirac's magnetic monopoles (which having opposite signs of magnetic charges and masses) and a light spineless electrically charged particle moving on the circular orbit at relativistic velocity in magnetic field of two Dirac magnetic monopoles on the symmetry plain perpendicular to the magnetic monopoles axis.

The following form represents the formula for a mass spectrum of the charged CP, received by us from this structure of particles:

$$\frac{E(n,k)}{mc^2} = \frac{n^2 - 2\alpha k^2}{2^8\alpha^4\left[\left|\frac{k}{4n}\right|^{2/3} - 1\right]^{3/2}} + \left\{1 + \frac{n^2}{2^6\alpha^4\left[\left|\frac{k}{4n}\right|^{2/3} - 1\right]^2}\right\}^{1/2} . \tag{1}$$

Here $E(n, k)$ is the rest energy of the charged CP, m is the rest mass of the new spineless electrically charged particle, c is the velocity of light, $\alpha = e^2/\hbar c$ is the fine-structure constant ($\hbar$ is Plank's constant), $n = 0, \pm 1/2, \pm 1, \pm 3/2\ldots$ is the projection of the orbital angular momentum of the electrically charged particle on the symmetry axes of the CP and $k = 0, \pm 1, \pm 2, \pm 3\ldots$ is the quantum number, which defines the magnetic monopole charge of g through the Dirac relation modified by us $eg/\hbar c = k/4$ [16].

The first term in equation (1) is the sum of two terms of the contribution of energy: the repulsion energy $E_r \propto n^2/\alpha^4$ between the moving charge and two magnetic monopoles and the Coulomb attraction energy $E_c \propto -k^2/\alpha^3$ between two magnetic monopoles. The second term in equation (1) $E_k \propto |n|/\alpha^2$ is the total relativistic energy of the moving charged particle with the rest mass m. There is an obvious inequality E_r, $|E_c| >> E_k$, because $\alpha \sim 10^{-2}$ and thus the sum of repulsion and attraction energy terms gives the main contribution to the rest energy $E(n, k)$ of the charged CP. In our model we have neglected the kinetic energy of two magnetic monopoles because of a large value of its masses $|M| \approx 10^8$ GeV/ c^2 [19]. Due to the opposite signs of the mass of two Dirac magnetic monopoles both masses cancel each other and therefore do not give any contribution to the rest energy of the charged CP.

According to equation (1) the rest energy $E(n, k)$ of the charged CP is independent of the sign of the electric charge of the moving particle (because $\alpha \propto e^2$) and of the signs of the quantum numbers n and k. Therefore, we have a twofold degeneracy connected with the opposite signs of $\pm n$, corresponding to the clockwise and counterclockwise rotations of the electrically charged particle. This behaviour of the charged CP is similar to a two-dimensional quantum-rigid rotator, because both systems for each state with $|n| > 0$ have twofold degeneracy connected with the opposite signs of the projections of the orbital angular momentum $\pm n$. Taking into account two possible signs of a moving particle mass $\pm m$ and a projection of the orbital angular momentum $\pm n$, our model predicts two degenerate states $\pm n$ for the particle with positive rest energy $E(n, k) > 0$ for $m > 0$ and two degenerate states $\pm n$ for the antiparticle with negative rest energy $E(n, k) < 0$ for $m < 0$, which is consistent with the Dirac theory for the electron [20].

Further, accepting (in agreement with the experiments) that the elementary particles in the quantum theory can be considered as scattering (metastable) states with positive rest energy and the antiparticles – as scattering states with negative rest energy (see also [21]), we receive the following condition from the numerator of the first term in equation (1)

$$n^2 - 2\alpha k^2 \geq 0. \tag{2}$$

Substituting in the first approximation value $\alpha = 1/137$ in inequality (2) and having cut off the factor $(1/(2\alpha))^{1/2}$ up to the closest integer 8, we find $|k| \leq 8|n|$. From the denominator of the first term in equation (1) we find the second condition of $|k/(4n)| > 1$. Summing up these two inequalities, we obtain the following constraints on the values k for the given value n:

$$4|n| < |k| \leq 8|n|. \tag{3}$$

For the minimum possible value of the quantum number $n = 0$ inequalities (3) have no any possible values for the quantum number k and, thus, in our model there is no charged CP with zero spin. The first family of the charged CP begins for $n = 1/2$ with 2 possible values for $k = 3, 4$ providing 2 fermions. The second family of the charged CP corresponds to $n = 1$ with 4 possible bosons states for $k = 5, 6, 7, 8$. For the next value of $n = 3/2$ there are 6 possible values for $k = 7, 8, 9, 10, 11, 12$ for 6 fermions. In a general case for the given value of the quantum number n there are $\mathrm{N} = 4n$ states of the charged CP. Therefore, within the framework of our model, there are infinite numbers of charged composite particles – fermions and bosons, connected with half-integer and integer values of n, respectively.

As follows from equation (1), the rest energy $E(n, k)$ of the charged CP quickly enhanced with the increasing of the projection of the orbital angular momentum of the mowing charged particle n, mainly due to the increasing of the repulsion energy $E_r \propto n^2/\alpha^4$. For the given n the rest energy $E(n, k)$ of the charged CP decreased with the increasing of k due to the increasing of the attraction energy $E_c \propto -k^2/\alpha^3$ between two magnetic monopoles. Thus, on the basis on equation (1) we expect a periodic change of the rest energy $E(n, k)$ of the charged CP as a function of the magnetic monopole charge k for different values of an orbital angular momentum n of the moving charged particle.

The global minimum of $E(n, k)$ clearly belongs to the charged CP with the minimal possible value of $n = 1/2$ together with the maximal possible value of $k = 4$, which is to the $E(1/2, 4)$ state. This state can be naturally interpreted as the lightest spin 1/2 charged fermion – the electron. The second and the last possible state in this family is the $E(1/2, 3)$ state, which can be connected to one more massive spin 1/2 charged lepton - the muon. On the basis of this interpretation we can have the value of the fine-structure constant determined as compared to the theoretical and experimental values of muon and electron mass

$$\frac{E(1/2,3)}{E(1/2,4)} = \frac{m_\mu}{m_e} = 206.7682826, \tag{4}$$

where m_e = 0.5109989461(31) MeV/c^2 and m_μ = 105.6583745(24) MeV/c^2 are the electron and muon masses received from CODATA [22]. Having solved numerically equation (4), we have found the value of the renormalized fine-structure constant α = 1/128.330593928 = 0.00779237412836. This value of α is about 6.8% larger than the standard value of $\alpha = e^2/\hbar c$ = 1/137.035999139(31), obtained in the low energy limit [22]. The obtained value of α is in agreement with the value α = 1/128.5(25) extracted from the analysis of the electron-positron scattering at the momentum transfer $Q^2 = (57.66\ \mathrm{GeV}/c)^2$ [23] and also with the value α = 1/128.7862(87), obtained from the analysis of the empirical formulas for charged leptons [18].

Using equation (1), the obtained value of the renormalized fine-structure constant α and the experimentally known electron mass, we have determined the rest mass of a new electrically charged particle m = 156.3699214 eV/c^2. Thus, this new particle is very light even in comparison with the electron mass ($m/m_e \approx 0.000306$).

After estimating the values of these two parameters in equation (1) and assuming in the first approximation their constant values for all allowed values of quantum numbers *n* and *k*, we can calculate the rest masses $E(\mathrm{n, k})/c^2$ of the charged fermions and bosons for any allowed pair *n* and *k*.

The results of calculation for n = 1/2, 1, 3/2, 2, 5/2, 3 and the corresponding values of *k* are listed in table 1 and also shown in Figure 1. As we can see from Figure 1, the rest energy $E(n, k)$ of the charged CP has a characteristic periodic dependence as a function of the magnetic charge of magnetic monopoles. The rest energy of the charged CP $E(n, k)$ achieves the maximum value at the beginning of every period when *k* has the minimum possible value for the fixed value of *n*. The increasing of *k* along the period the rest energy $E(n, k)$ of the charged CP quickly decreases due to the negative contribution of the attraction energy $E_c \propto -k^2/\alpha^3$ between two magnetic monopoles. The transition to the next period through the increasing of *n* repeats the general behaviour of the rest energy $E(n,k)$ and also includes the increasing of the maximal and minimal values of $E(n, k)$ due to the bigger contribution of the repulsion energy $E_r \propto n^2/\alpha^4$ between the moving of electrically charged particle and two magnetic monopoles. Note also that a similar periodicity exists also for the orbit radii $R(n, k)$ of the mowing charge.

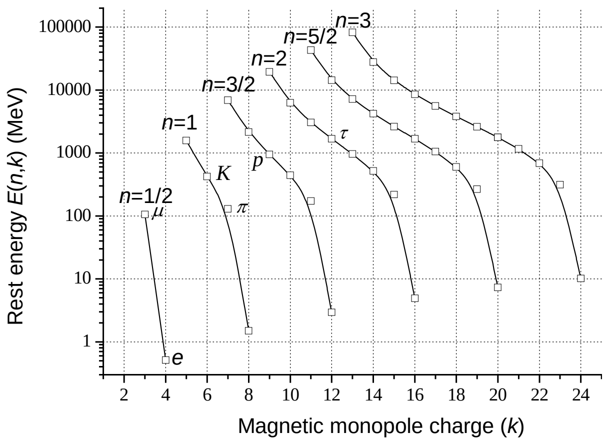

Fig. 1: The rest energy $E(n, k)$ of the charged CP as a function of the quantum number *k* (which defines the magnetic charge of magnetic monopoles) for six values of the orbital angular momentum n = 1/2, 1, 3/2, 2, 5/2, 3 electrically charged particle. Symbols e, μ, π, K, p, τ represent the position of the corresponding charged elementary particles on the graph.

The orbit radius of the mowing charge $R(n, k)$ is defined by the form:

$$R(n,k) = 2^3 \alpha^2 \frac{\hbar}{|m|c}\left[\left|\frac{k}{4n}\right|^{2/3} - 1\right]. \tag{5}$$

Using equation (5) and the known values of parameters for α and m, we have calculated the charge orbit radii $R(n, k)$ of a moving electrically charged particle for 6 periods of the charged CP for n = 1/2, 1, 3/2, 2, 5/2, 3. The results of calculations are listed in table 1 and shown in Figure 2. Figure 2 illustrates the characteristic periodicity of the charge orbit radii $R(n, k)$ as a function of magnetic monopoles charge *k* for different values of *n*. The charge orbit radius $R(n, k)$ has the minimum value at the beginning of every period when k has the minimum possible value for the fixed value of n. With the increasing of k along the period the charge orbit radius $R(n, k)$ as a function of k is increased and reaches the same maximum value $R_{max} = 0.360078795\times10^{-10}$ cm in every period for the same ratio value $k/n = 8$, when the rest energy $E(n, k)$ of the charge CP reaches the minimum value, i. e. at the end of the period (see also Figures. 1, 2 and table 1).

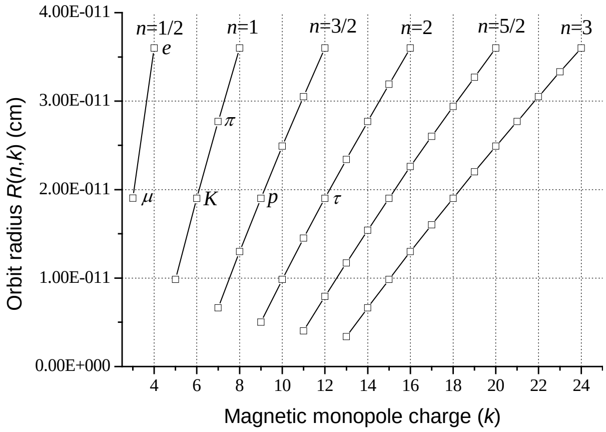

Fig. 2: The charge orbit radius $R(n, k)$ of the charged CP as function of the quantum number k (which defines the magnetic charge of magnetic monopoles) for six values of the orbital angular momentum n = 1/2, 1, 3/2, 2, 5/2, 3 electrically charged particle. Symbols e, μ, π, K, p, τ represent the position of the corresponding charged elementary particles on the graph.

Table 1: The values of the rest energy $E(n, k)$ and the charge orbit radius $R(n, k)$ for the charged CP calculated according to equation (1) and equation (5), where n is the orbital angular momentum electrically charged particle and quantum number k defines the magnetic charge of magnetic monopoles.

n	k	$E(n,k)$ (MeV)	$R(n,k)$ (10^{-10} cm)
1/2	3	105.6583743	0.190258268
1/2	4	0.510998946	0.360078795
1	5	1576.135023	0.098324021
1	6	421.5963438	0.190258268
1	7	129.4757510	0.277197915
1	8	1.495985078	0.360078795
3/2	7	6919.793736	0.066347368
3/2	8	2136.991621	0.129597318
3/2	9	947.8139087	0.190258268
3/2	10	444.4930987	0.248708387
3/2	11	172.7064655	0.305238602
3/2	12	2.954958477	0.360078795
2	9	19433.67801	0.050074511
2	10	6300.526284	0.098324021
2	11	3047.688652	0.144988686
2	12	1684.311069	0.190258268
2	13	959.575117	0.234286009

2	14	516.479278	0.277197915
2	15	218.890831	0.319099184
2	16	4.887919133	0.360078795
5/2	11	43041.66493	0.040214389
5/2	12	14288.95571	0.079226532
5/2	13	7173.263627	0.117168489
5/2	14	4201.192819	0.154148849
5/2	15	2631.087824	0.190258268
5/2	16	1679.690103	0.225573426
5/2	17	1048.094258	0.260159915
5/2	18	600.2540707	0.294074394
5/2	19	266.3548445	0.327366236
5/2	20	7.294867044	0.360078795
3	13	82205.67430	0.033599455
3	14	27670.25249	0.066347368
3	15	14173.17378	0.098324021
3	16	8543.398636	0.129597318
3	17	5577.586735	0.160225341
3	18	3788.144175	0.190258268
3	19	2607.036691	0.219739819
3	20	1775.592174	0.248708387
3	21	1161.010580	0.277197915
3	22	688.8864579	0.305238602
3	23	314.6542934	0.332857465
3	24	10.17580221	0.360078795

Note also that the distance between the two magnetic monopoles of the charge CP is given by $d(n, k) = 2*R(n, k)*(|k/(4n)|^{2/3}-1)^{1/2}$ and it shows the periodicity similar to a charge orbit radius $R(n, k)$.

The periodicity of the rest energy $E(n, k)$, the charge orbit radii $R(n, k)$ and the distance between the two magnetic monopoles $d(n, k)$ as the functions of the magnetic charge of magnetic monopoles for the charged CP are analogous to the periodicity of the first ionisation energy and atomic radii on the nucleus electric charge in the Mendeleev periodic table of chemical elements [24].

On the basis of analogy with the Mendeleev periodic table of chemical elements it is useful to introduce the definition of the groups in addition to the periods of the charged CP, which are characterized by the same values of charge orbit radii and the distance between magnetic monopoles, caused by the identical value of ratio k/n. In this connection we assume that the charged CP of the same group can have good wave functions overlapping and thus would have a large resonance cross section for various reactions of elementary particles belonging to the same group. The first group – the group of the ground states of the charged CP, has the maximum value $k/n = 8$ and begins in the first period including the electron and another bosons and fermions from other periods. The second group for the value $k/n = 6$ begins also in the first period and it includes the muon and other fermions and bosons from other periods. The other two new groups of the charged CP with the values $k/n = 5$ and 7 begin in the second period (see table 1 and Figure 2).

As we already mentioned, the first period ($n = 1/2$) of the charged CP has only 2 possible fermions states, $k = 3$, 4 belonging to the muon and to the electron, respectively. Our model represents correctly the values of its spins and masses. The distinction between the values of the magnetic charges of the magnetic monopoles for the electron and muon causes a large difference between their rest masses due to the effect of cancellation the repulsive and attractive energy terms in equation (1). Thus, for the electron the sum of repulsion and attraction energies is $E_r + E_c = 91.997 - 91.760 = 0.237$ MeV, which is comparable with the total relativistic energy 0.274 MeV of a moving electrically charged particle. For the muon the sum of repulsion and attraction energies is $E_r + E_c = 239.526 - 134.386 = 105.14$ MeV, giving the main contribution to the muon rest energy and being much larger than the total relativistic energy $E_k = 0.519$ MeV of a moving electrically charged particle. This cancellation effect is also present for the ground states of other periods of the charged CP.

The electron charge orbit radius in our model is $0.360078795\times10^{-10}$ cm and it is only 7% less than the electron Compton wavelength $\lambda_c = \hbar/m_e c = 0.3861592674\times10^{-10}$ cm. For comparison, the radius of the muon charge is equal to $0.190258268\times10^{-10}$ cm and it is approximately a hundred times bigger than the muon Compton wavelength $0.1857594307\times10^{-12}$ cm [22].

Insofar there is no place for the $\tau^{\pm}$ lepton in the first period of the charged CP, but we have found the place for it in the middle part of the fourth period (see below).

The second period of the charged CP for $n = 1$ includes 4 bosons states in the energy interval 1.5 – 1576 MeV for $k = 5, 6, 7, 8$. This interval contains masses of $\pi^{\pm}$ and $K^{\pm}$ mesons at 139.57018(35) and 493.677(13) MeV/c^2 [25], respectively. We accept for assignment $\pi^{\pm}$ and $K^{\pm}$ mesons to the charge CP states $E(1, 7) = 129.5$ MeV and $E(1, 6) = 421.6$ MeV, respectively. The distinction between the experimental and the predicted mass values for $\pi^{\pm}$ and $K^{\pm}$ mesons can be reduced through the parameters renormalization in equation 1, i.e. by decreasing the fine-structure constant α at a level of 1.06% and 3.06% or by increasing the rest mass of a moving electrically charged particle at a level of 7.80% and 17.1%, respectively.

As is known, the SM considers $\pi^{\pm}$ and $K^{\pm}$-mesons as zero spin bosons. However, our model does not include the charged CP with zero spin because of the absence of a repulsive force between the moving electrically charged particle and two magnetic monopoles, which makes them unstable. The charged CP with the non-zero orbital angular momentum n of an electrically charged particle has the magnetic moment distinct from zero (see below), which in principle can be measured for $\pi^{\pm}$ and $K^{\pm}$ mesons in a strong magnetic field.

The third period of the charged CP ($n = 3/2$) contains 6 fermions in the energy interval 3-6920 MeV for $k = 7, 8, 9, 10, 11, 12$. The rest energy of the state $E(3/2, 9) = 947.81$ MeV matches with the proton rest energy 938.2720813(58) MeV with the accuracy of 1% [25], and we naturally connect this state with the proton. The small difference between the theoretical and experimental proton mass values can be reduced by means of tiny increasing in fine-structure constant α at a level of 0.192% or a small decreasing of the rest mass m of the electrically charged particle at a level of 1.02%.

The ratio of the masses of proton and electron is an important dimensionless physical constant with the experimental value $m_p/m_e = 1836.15267389(17)$ [22]. According to equation (1), this ratio in our model is defined by the value of the renormalized fine-structure constant α, the values of quantum numbers n and k for proton and electron and it provides the theoretical value $m_p/m_e = 1854.8$ in good agreement with the experimental data.

As is known, the SM considers the proton as a strongly bounded state of three quarks (uud) and the electron as the structureless point-like particle and in this approach it cannot give the theoretical prediction of m_p/m_e ratio. On the contrary, our model, being in line with [26], defines this ratio through the dimensionless constant of the electromagnetic interaction α, which in our model is a universal interaction constant for all charged CP.

The other two problems are connected with the proton's spin and magnetic moment. The SM describes the proton as a spin 1/2 particle with two possible spin projections $s_z = \pm 1/2$. In our model the proton also has only two projections of the orbital angular momentum (spin) $n = \pm 3/2$. As is known, a point-like particle with electric charge e, rest mass m, and spin 1/2 in the Dirac theory has the intrinsic magnetic moment $\mu_D = e\hbar/(2mc)$. CODATA gives for the proton experimental value of the magnetic moment $\mu_p = 2.7938473508(85)\mu_D \approx 3\mu_D$ [22]. Our model predicts for the charged CP with an orbital angular momentum of the moving electrically charged particle n the value of the magnetic moment $\mu = 2n\mu_D$ and, thus, for the proton ($n = 3/2$) the magnetic moment is given by $\mu_p = 3\mu_D$ in agreement with experimental data.

The proton charge orbit radius predicted in our model is $R(3/2, 9) = 1.90258268\times10^{-11}$ cm, but the proton charge radius extracted from the analyses of the electron-proton elastic scattering is only $R_p = 0.8759(77)\times10^{-13}$ cm [22, 27]. However, we assume that this value of the proton charge radius is necessary for increasing by the dimensionless factor $1/\alpha \sim 10^2$. This factor can naturally occur through the change of the proton electromagnetic form factors due to the possible influence of the large magnetic charge of magnetic monopoles $g \sim e/\alpha$ existing inside of every charged CP. Thus, after a correction by the factor $1/\alpha$ the proton charge radius extracted from the electron-proton scattering data would reach the value $R_p \approx 1\times10^{-11}$ cm in agreement with the result of our model.

Experimentally are known only two practically stable charged fermions – the electron and the proton, which have the mean lifetimes $\tau \geq 6.6\times10^{28}$ years [28] and $\tau \geq 6.6\times10^{33}$ years [29], respectively. However, their positions in table 1 of the charged CP are different. The electron is located in the ground state of the first period of the charged CP and it has the absolute minimum of rest energy. But the proton is located in the middle part of the third period in the muon group (see table 1 and Figure 1). What is the physical reason of proton stability among six other states of this period and what dynamical mechanism prevents proton's decay to the lower energy states of the charged CP, which have a common structure. This important problem deserves a further theoretical investigation, including our model as well.

The fourth period ($n = 2$) includes 8 bosons in the rest energy interval 4.89 – 19434 MeV for k = 9, 10, 11, 12, 13, 14, 15, 16. We assume the state $E(2, 12) = 1684.3$ MeV in the middle part of this period to be assigned on the $\tau^\pm$ particle with the experimental value of rest mass 1776.82(16) MeV/c^2 [30]. The distinction (5.49%) between the theoretical and the experimental mass values of the $\tau^\pm$ particle can be reduced by decreasing the fine-structure constant α at a level of 1.02% or by increasing the rest mass m of the electrically charged particle at the level of 5.49%.

The fifth period of the charged CP ($n = 5/2$) has 10 fermions in the rest energy interval 7.3 – 43042 MeV for k = 11, 12, 13, 14, 15, 16, 17, 18, 19, 20.

The sixth period ($n = 3$) of the charged CP includes 12 bosons states in the rest energy interval 10.2–82206 MeV for k = 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24.

Three bosons states in the muon group (k/n = 6) at E(3, 18) = 3.79, E(4, 24) = 6.73 and E(6, 36) = 15.1 GeV are close to the rest mass of 3 new particles at 3.6, 7.3 and 15 GeV/c^2 obtained by the CDF collaboration in the analysis of the multi-muon events [31]. These multi-muon events were produced in proton-antiproton interactions and discovered at the Fermilab Tevatron collider by the CDF collaboration [32]. It was supposed that the heavier new particles cascade-decay into lighter ones, whereas the lightest state decays into $\tau^{\pm}$ pair with a lifetime of the order of 20 ps. In the framework of our model these processes are quite reasonable because all the interacting particles belong to some muon group and thus have the resonance condition for the processes of cascade-decay, because all particles have the same charge orbit radii of the moving charge and the distance between magnetic monopoles. Thus, the mass predictions from our model are in agreement with the results of the phenomenological analysis of multi-muon events by the CDF collaboration.

In addition to bosons, the muon group contains also fermions (see table 1 and Figure 2). We have already connected the second fermion's state of this group E(3/2, 9) = 0.9478 GeV to the proton. Another 4 fermions states in the muon group in the rest energy interval 2.5–13 GeV are: E(5/2, 15) = 2.63 , E(7/2, 21) = 5.16, E(9/2, 27) = 8.59 and E(11/2, 33) = 12.7 GeV, respectively.

The standard model of cosmology indicates that 3 stable particles - protons, neutrons and electrons, form the ordinary (baryonic) matter, which constitutes only ~5% of the Universe mass. The remaining non-baryonic part of the Universe (with the unknown nature) belongs to dark matter ~27% and dark energy ~68% [33].

During the search of dark matter (DM) particles the CoGeNT and DAMA collaboration [34] has found an excess of low energy events and the possibility that these events originate from the elastic scattering of a light m_{DM} ~5–10 GeV/c^2 weakly interacting with the massive particle has been discussed. This mass interval is in agreement with the asymmetric models for dark matter generation, which predict the values of the DM mass at m_{DM} ~5–15 GeV/c^2 [35].

Thus, if 4 charged fermions from the muon group in the rest energy interval 2.5–13 GeV are stable particles (similar to the proton) and can create the stable neutral partners (similar to the neutron), then these new neutral particles can be the possible candidates for dark matter particles, predicted by our model.

Let us go back to the group of ground states (k/n = 8) of the charged CP, i. e. the electron group. As we already mentioned, all these fermions and bosons of this group have the same charge orbit radii of the electrically charged particle and the same distance between magnetic monopoles. The rest energy for 13 members of this group is located in the range of 0.511 – 43.6 MeV. Below we consider the results of various experiments, which can be considered as the evidence of the emergence of particles of this group.

The so-called heavy electrons with the rest mass $m \approx 3m_e \approx 1.5$ MeV/c^2 and $m \approx 6m_e \approx 3$ MeV/c^2 were found in air showers as particles that were more penetrating than electrons [36]. These particles in our model can be connected to the first boson state of the electron group with the rest energy E(1, 8) = 1.50 MeV and to the second fermion state of this group E(3/2, 12) = 2.95 MeV, respectively.

A new particle with the mass $m \approx 11.4m_e \approx 5.8$ MeV/c^2 was observed together with electrons in the cloud chamber [37]. We hope to connect this particle to the second boson state in the electron group E(2, 16) = 4.89 MeV.

Further, when studying the optical characteristics of the pulsed laboratory synchrotron LIS-2 (the orbit radius 6 cm) [38], we noticed that a considerable part of the accelerated electrons (up to 100%) can drop out from synchronous mode acceleration.

This effect was observed as sharp intensity reduction of synchrotron radiation when electrons reached the region of the maximal energy in the range $E_{max} \approx$ 37-40 MeV [39]. Now we are trying to explain this effect in the framework of our model as the revealing of the resonant transition of relativistic electrons, receiving the total energy equal to the rest energy of the charge CP, which belongs to the boson state of the electron group E(6, 48) = 37.4 MeV. The momentum excess of the accelerated electrons in this resonant transition is transferred to the magnetic field of the synchrotron practically without any energy transmission. This resonant transition is similar to the Mössbauer effect when in resonance recoil-free absorption of gamma photons by atomic nucleus the photon momentum is transferred to the whole crystal.

3. Conclusions

In this work we have presented the formula for the mass spectrum of the composite charged fermions and bosons.

This formula includes only two parameters – the mass of the new electrically charged particle m = 156.3699214 eV/c^2 and the renormalized fine-structure constant α = 1/128.330593928, and also two quantum numbers n and k. The half-integer and integer quantum number n is the projection of the orbital angular momentum of electrically charged particle on the symmetry axis of the CP, and the integer quantum number k defines the magnetic charges of two Dirac magnetic monopoles.

Taking into account two possible signs of the mass $\pm m$ of the moving particle and two projections of the orbital angular momentum (spin) $\pm n$, our model predicts two degenerate states $\pm n$ for the particles with positive rest energy $E(n, k) > 0$ for $m > 0$ and two degenerate states $\pm n$ for the antiparticles with negative rest energy $E(n, k) < 0$ for $m < 0$ in agreement with the Dirac theory for the electron.

In addition, our model shows the characteristic periodic dependence of the predicted masses and the charge orbit radius as the function of the magnetic charges of magnetic monopoles. Both periodic dependences are analogous to the periodicity of the first ionization energy and the atomic radius on the electric charge of the nucleus in the Mendeleev periodic table of chemical elements.

Thus, the presented model predicts the values of spins, masses, charge orbit radii and magnetic moments for the infinite numbers of the charged fermions and bosons in the infinite range of mass.

4. References

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