DS-VIKOR: A new methodology for supplier selection

Liguo Fei, Yong Deng and Yong Hu

Abstract—**How to select the optimal supplier is an open and important issue in supply chain management (SCM), which needs to solve the problem of assessment and sorting the potential suppliers, and can be considered as a multi-criteria decision-making (MCDM) problem. Experts' assessment play a very important role in the process of supplier selection, while the subjective judgment of human beings could introduce unpredictable uncertainty. However, existing methods seem powerless to represent and deal with this uncertainty effectively. Dempster-Shafer evidence theory (D-S theory) is widely used to uncertainty modeling, decision making and conflicts management due to its advantage to handle uncertain information. The VIKOR method has a great advantage to handle MCDM problems with non-commensurable and even conflicting criteria, and to obtain the compromised optimal solution. In this paper, a DS-VIKOR method is proposed for the supplier selection problem which expends the VIKOR method by D-S theory. In this method, the basic probability assignment (BPA) is used to denote the decision makers' assessment for suppliers, Deng entropy weight-based method is defined and applied to determine the weights of multi-criteria, and VIKOR method is used for getting the final ranking results. An illustrative example under real life is conducted to analyze and demonstrate the practicality and effectiveness of the proposed DS-VIKOR method.**

Index Terms—**Dempster-Shafer evidence theory, VIKOR, DS-VIKOR, Supply chain management, MCDM.**

I. Introduction

With the rapid progress of economic globalization and the swift development of information technology, supply chain management model has changed from the product-centric to customer demand as the center and become the major management model [1]. Supplier selection is a very important part of supply chain management (SCM) [2]. Therefore, in-depth study of the supplier selection problem is not only of large theoretical value, but also has a high practical significance. Supplier selection has become a very important research field and an increasing number of researchers have paid attention to this problem. How to select the appropriate suppliers according to the product price, delivery, product quality, technical ability and other criteria plays a crucial role for the survival and development of enterprises. And how to construct the evaluation criteria and integrate the evaluation results of decision experts are two important issues in the process of supplier selection.

With regard to construct evaluation criteria, there are two aspects need to be concerned about: criteria setting and weights obtaining [3], [4].

To set criteria, many researchers give the reference [5], [6], [7] from engineering application [8], [9], [10], environmental protection [11], [12] and sustainable development [13], [14], [15]. On the other hand, in order to evaluate the alternatives comprehensively, many set criteria need to be considered, while the importance of each criterion is not the same, so each criterion should be given a special weight. But how to finish this job? Most of methods let decision experts to determine the weights, but this may lead to the inaccurate results because of too many subjective factors. To avoid such errors, a new Deng entropy weight-based method is proposed in this paper that can calculate the weight of each criterion more objective from the viewpoint of mathematics.

Another important part for selecting the optimal supplier is to determine the aggregation algorithm for integrating decision experts' assessment. More and more methods have been developed by researchers in the related fields. Analytic hierarchy process (AHP) [16], [17], [18] is a main part, and the AHP models are constructed for market and garment evaluation in [19]. Fuzzy set theory is also important and used in [20], [21], [22]. The fuzzy extended AHP method is introduced for supplier selection in washing machine company [23] and word sense disambiguation [24]. In addition, Vector estimation [25], [26], Dempster-Shafer evidence theory [14], [27], [28], [29] and uncertainty theory [30], [31], [32] are widely considered to solve this problems. Hybrid MCDM method [33], [34] is also effective for selecting green supplier and many other methods [35], [36], [37], [38] are also presented. Fuzzy multiple criteria decision-making techniques and applications are reviewed in [39]. With respect to this problem, the fuzzy extended AHP method [40] is proposed, but there still exist some shortcomings about the inconsistency of the comparison matrix [41]. The fuzzy preference relation has been studied widely [42], but it still seems powerless for incomplete information even though it can express the preference relation using a membership function under fuzzy environment.

The general procedure of supplier selection is to establish a panel according to the specific problem at first.

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Then they will construct the evaluation criteria against actual conditions and select the potential suppliers that satisfy general conditions. Next, the decision experts score all the alternatives based on their performance under different criteria. After this step, the appropriate aggregation algorithm is needed to integrate the experts' assessment. Finally, using the selected sort algorithm to obtain the final ranking results. Experts' assessment are critical in the process of supplier selection. Because these evaluations are from human subjective judgment, so it is inevitable that there will appear some uncertainty, for example, vagueness, ambiguity and incompleteness. So how to represent the uncertain information should become the focus of attention [43], [44], [45] in supplier selection.

In order to effectively deal with uncertain information in the supplier selection problem, Dempster-Shafer evidence theory (D-S theory) is applied in this paper due to its great advantage in handling uncertain information. D-S theory is proposed by Dempster and developed later by Shafer [46], [47]. This theory extends the elementary event space in probability theory to its power set named as frame of discernment and constructs the basic probability assignment(BPA) on it. D-S theory is the generalization of probability theory with the purpose of handling uncertainty and is widely used to uncertainty modeling [48], [49], [50], decision making [51], [17], [38], [52], [53], information fusion [54], [55] and uncertain information processing [56], [57], [58]. In this paper, decision experts' assessment will be represented by BPA, and a new fusion method is proposed which extends the traditional Dempster' combination rule based on Deng entropy [59] as an aggregation algorithm in supplier selection problem. In addition, Deng entropy weight-based method is defined for making up for the shortcomings of the Entropy-weight method, which has greater capacity to deal with uncertain information. It is also a new train of thought to determine the weights of the criteria.

Recently, the advantages of VIKOR method [60], [61] are more and more obvious in deal with the problems about multi-criteria optimization in complex systems. The VIKOR method can obtain the compromise feasible solution which has the nearest distance with the ideal solution. The premise is that criteria give way to each other. In the field of supplier selection, this method has been applied more and more. In [62], the VIKOR method is combined by DEMATEL-based ANP to explore smart phone improvements, which belongs to MCDM problem. A Multi-criteria Assessment Model of Technologies is proposed based on VIKOR method in [63]. A group multi-criteria supplier selection method using an extended VIKOR method with interval 2-tuple linguistic information is introduced in [26]. And many others method applied VIKOR are developed in [64], [65], [66]. In this paper, the VIKOR method is extended by D-S theory [46], [47] that has more prominent performance in dealing with uncertain problems.

The rest of this paper is organised as follows. In Section II, the basic concepts of D-S theory and VIKOR method are introduced briefly. In Section III, the proposed DS-VIKOR method is introduced, including a new combination rule for BPA, the Deng entropy weight-based method and the detailed steps for DS-VIKOR method. An illustrative example under real life is conducted in Section IV to illustrate the effectiveness of the proposed DS-VIKOR mthod. Section V concludes this paper.

II. Preliminaries

A. Dempster-Shafer evidence theory

Definition II.1. *Dempster-Shafer evidence theory [67], [68] is also called D-S theory, which supposes the definition of a set of elementary hypotheses called the frame of discernment, defined as:*

$$\theta = \{H_1, H_2, ..., H_N\} \tag{1}$$

That is, θ is a set of mutually exclusive and collectively exhaustive events. Let us denote 2^θ the power set of θ.

Definition II.2. *When the frame of discernment is determined, a mass function m is defined as follows.*

$$m : 2^\theta \to [0, 1] \tag{2}$$

which satisfies the following conditions:

$$m(\phi) = 0 \tag{3}$$

$$\sum_{A \in 2^\theta} m(A) = 1 \tag{4}$$

In D-S theory, a mass function is also called a basic probability assignment (BPA).

Definition II.3. *The discounting operation is used when an evidence provides a BPA, but the evidence is believed by probability α. In this circumstances, The BPA m^α is redefined based on the probability of reliability α as follows*

$$m^\alpha(A) = \alpha \times m(A), \quad A \subset \theta \tag{5}$$

$$m^\alpha(\theta) = (1 - \alpha) + \alpha \times m(\theta) \tag{6}$$

where A *is the focal element, and* m *is the mass function.*

Definition II.4. *Suppose m_1 and m_2 are two mass functions. The Dempster's rule of combination denoted by $m = m_1 \oplus m_2$ is defined as follows:*

$$m(A) = \frac{\sum_{B \bigcap C = A} m_1(B) m_2(C)}{1 - K} \tag{7}$$

with

$$K = \sum_{B \bigcap C = \phi} m_1(B) m_2(C) \tag{8}$$

Note that the Dempster's rule of combination is only applicable to such two BPAs which satisfy the condition $K < 1$.

B. Deng entropy

Deng entropy [59] is presented to measure the uncertainty degree of BPA as a generalized Shannon entropy in D-S theory.

Definition II.5. *The Deng entropy can be described as follows*

$$E = -\sum_i m(F_i)\log\frac{m(F_i)}{2^{|F_i|}-1} \tag{9}$$

where F_i is a proposition in mass function m, and $|F_i|$ is the cardinality of F_i.

It is the generalization of Shannon entropy. Specially, the Deng entropy can definitely degenerate to the Shannon entropy if the belief is only assigned to single elements. The process is shown as follows

$$E_d = -\sum_i m(\theta_i)\log\frac{m(\theta_i)}{2^{|\theta_i|}-1} = -\sum_i m(\theta_i)\log m(\theta_i) \tag{10}$$

C. The maximum Deng entropy

In the previous section, the definition of Deng entropy [59] has been introduced. However, what conditions should be satisfied to get the maximum value of Deng entropy? This problem has been resolved in [69] and details are as follows:

Definition II.6. *The Deng entropy will get the maximum value when satisfies the following requirements.*

The maximum Deng entropy:

$$E_{max} = -\sum_i m(F_i)\log\frac{m(F_i)}{2^{|F_i|}-1} \tag{11}$$

if and only if $m(F_i) = \frac{2^{F_i}-1}{\sum_i 2^{F_i}-1}$.

D. The pignistic probability function $BetP_m$

Definition II.7. *Let m be a BPA on Θ. Its associated pignistic probability function $BetP_m$ [70] is defined as follows:*

$$BetP_m(w) = \sum_{A\subseteq\Theta, w\in A}\frac{1}{|A|}\frac{m(A)}{1-m(\emptyset)}, \quad m(\emptyset)\neq 1, \tag{12}$$

where $|A|$ is the cardinality of subset A and w is the subset proposition in A. The main aim of $BetP_m$ is to translate a BPA into probability in order to make a decision.

E. VIKOR method

Vlsekriterijumska Optimizacija I Kompromisno Resenje (i.e. VIKOR) method was developed by Opricovic in 1998 for multi-criteria optimization of complex systems [60], [61]. This is a sort of compromise sorting method, which compromises ranking of a finite decision scheme by maximizing group utility and minimizing individual regret. This method is an powerful tool for multi-criteria decision making, and it can solve the following problems effectively: 1) Decision makers can not or do not determine how to express their preferences accurately 2) There are conflicts and incommensurability between evaluation criteria (different measure units) 3) Decision makers that deal with the conflict can accept the compromise solution.

The key idea of the VIKOR method is to determine positive ideal solution (PIS) and negative ideal solution (NIS) firstly. Then selecting the optimal solution according to the closeness degree between the evaluation value of each solution and PIS under the conditions that acceptable advantages and decision process stability. The solution obtained by the VIKOR method is usually a compromise solution, which is the feasible solution of the most close to the optimal solution in all solutions. The VIKOR algorithm obtains the compromise solution which can be accepted by decision makers by maximizing the group benefit and minimizing the individual losses. The VIKOR and TOPSIS both are the compromise methods which are most close to the ideal solution. But the VIKOR algorithm does not need to consider the problem that the closest solution should be the closest to the ideal solution and the most distant from the negative ideal point. It can sort the solutions directly, is an excellent multi-attribute decision making method. The optimal solution obtained by VIKOR is closer to the ideal scheme, but the TOPSIS method is not [71].

For the synthesis method, VIKOR uses an aggregate function developed by $L_p - metric$ [72], [73]. The $L_{p,j}$ measure the distance between the best ideal solution and the alternative A_j that is proposed by Duckstein and Opricovic in 1980. The value obtained from jth alternative under ith criteria is denoted by f_{ij}.

The VIKOR method is derived from the following form of $L_p - metric$:

$$L_{p,j} = \{\sum_{i=1}^{n}[w_i(f_i^* - f_{ij})/(f_i^* - f_i^-)]^p\}^{1/p} \tag{13}$$

where $1 \leq p \leq \propto, j = 1, 2, ...J$.

F^c is the compromise solution which is the "closest" feasible solution to the ideal solution F^*. It established on the premise that mutual concessions, which is shown in Fig. 1 with $\Delta f_1 = f_1^* - f_1^c$ and $\Delta f_2 = f_2^* - f_2^c$.

III. THE PROPOSED DS-VIKOR METHOD FOR SUPPLIER SELECTION

In this section, the detailed steps of the proposed DS-VIKOR method will be introduced. At first, two new method based on Deng entropy [59] in D-S theory [46], [47] is presented, they are the new combination rule for BPA and Deng entropy weight-based method, respectively.

A. The Deng entropy-based combination rule for BPA

In this part, a new combination method is proposed for BPA in D-S theory based on Deng entropy [59] introduced in section II-B.

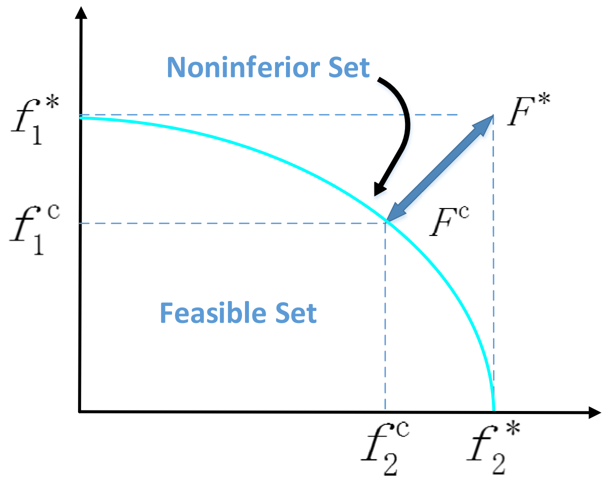

Fig. 1. Ideal and compromise solutions

Definition III.1. *As mentioned in Definition II.4, the Dempster's combination rule can be used to fuse two different evidences. However, the evidence has a certain extent uncertainty by itself resulting in a decline in the degree of trust. Therefore, it is necessary to quantify somehow the quality of information and to consider the uncertainty degree when combining evidences. Obviously, the greater the uncertainty degree, the lower the accuracy of evidence and the larger confusion to us to combine them. Based on these findings we define weights of evidences according to Deng entropy [59] and the maximum Deng entropy [69] as follows*

$$w_i(BPA) = 1 - \frac{E(BPA_i)}{E_{max}(BPA_i)} \tag{14}$$

where E is the Deng entropy expression of the BPA, i.e. The weights are different from one evidence to another depending on how much belief degree the BPA has provided from each evidence.

Definition III.2. *Based on the Definition II.3 and its corresponding weight $w_i(BPA)$, we can obtain the discounted mass function before combining them, expressed as follows:*

$$\begin{aligned} m_i^w(A) &= w_i(BPA) \times m_i^w(A) \\ m_i^w(\theta) &= (1 - w_i(BPA)) + w_i(BPA) \times m_i(\theta) \end{aligned} \tag{15}$$

where θ is the universal set of mass function.

Definition III.3. *As of now the weighted BPAs have been determined for individual evidence. Next, we devote to combine all the BPAs originating, based on the combining rule of D-S theory [46], [47], to determine an overall mass function for making the final fusion result. The final mass function can be calculated for the expression as follows*

$$m_i(BPA) = \oplus_{i=1}^{R}(m_i^w(BPA)) \tag{16}$$

where $\otimes$ is a combination operator.

B. The proposed Deng entropy weight-based methodology

The classical entropy weight method is applied to multi-criteria decision-making problems based on Shannon entropy, and the elements for decision matrix represented by scores or assessments from experts. In this part, the Deng entropy weight-based methodology is proposed to calculate the weights based on BPAs in D-S theory [46], [47].

Definition III.4. *For determining weights, unlike the entropy-weight method, which considers the values of all the alternatives under a criterion as probability distribution, the values are supposed to be a BPA in D-S theory according to Deng entropy weight-based methodology, and all the alternatives construct the frame of discernment as $\Theta = \{A_1, A_2, ..., A_m\}$. The decision matrix in Deng entropy weight-based methodology can be defined as follows where each BPA can be considered as the contribution degree of all the alternatives for jth criterion.*

Definition III.5. *According to the mentioned above, the amount of contribution degree of all the alternatives for criterion C_j can be defined as BPA_j*

$$\begin{aligned} BPA_j = \{m_j(\{A_1\}), m_j(\{A_2\}), ..., \\ m_j(\{A_m\}), m_j(\{A_1, A_2\})..., m_j(\{A_1, A_2, ..., A_m\})\} \end{aligned} \tag{18}$$

next, the Deng entropy is also applied to calculate the uncertainty of jth criterion (Eq. (9)) as follows

$$E_j = -\sum_i m_j(F_i) \log \frac{m_j(F_i)}{2^{|F_i|} - 1} \tag{19}$$

The obtained Deng entropy E_j need to be normalized that it can satisfy $0 \le E_j \le 1$.

Definition III.6. *Let D_j represent the consistency of each alternative for jth criterion.*

$$D_j = 1 - E_j \tag{20}$$

Definition III.7. *Based on the analysis above, the weight of each criterion W_j can be described as*

$$W_j = \frac{D_j}{\sum_{j=1}^{n} D_j} \tag{21}$$

note that the weight of jth criterion is 0 if $D_j = 0$.

After obtained the weights, normalized operation is need to determine final weights. The maximal improvement of the Deng entropy weight-based method is reflected in that it can represent uncertainties more effective because the decision matrix is constructed by BPAs, which is not restricted to the subjective assessments of experts. The relation and comparison have been given in Fig. 2.

$$\begin{array}{c} \\ A_1 \\ A_2 \\ \vdots \\ A_m \\ A_1, A_2 \\ \vdots \\ A_1, A_2, ..., A_m \end{array} \begin{array}{c} \begin{array}{cccc} C_1 & C_2 & \cdots & C_n \end{array} \\ \left[\begin{array}{cccc} m_1(\{A_1\}) & m_2(\{A_1\}) & \cdots & m_n(\{A_1\}) \\ m_1(\{A_2\}) & m_2(\{A_2\}) & \cdots & m_n(\{A_2\}) \\ \vdots & \vdots & \ddots & \vdots \\ m_1(\{A_m\}) & m_2(\{A_m\}) & \cdots & m_n(\{A_m\}) \\ m_1(\{A_1, A_2\}) & m_2(\{A_1, A_2\}) & \cdots & m_n(\{A_1, A_2\}) \\ \vdots & \vdots & \ddots & \vdots \\ m_1(\{A_1, A_2, ..., A_m\}) & m_2(\{A_1, A_2, ..., A_m\}) & \cdots & m_n(\{A_1, A_2, ..., A_m\}) \end{array} \right] \end{array} \quad (17)$$

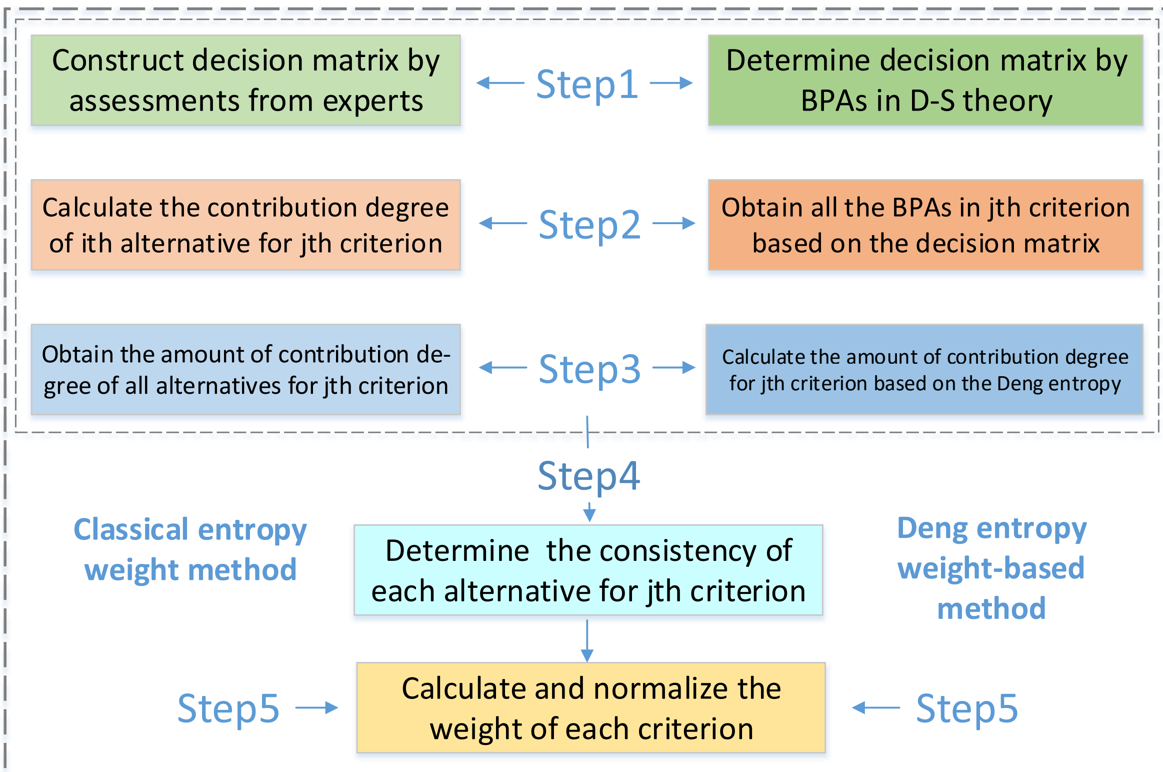

Fig. 2. The comparison between classical entropy weight method and the Deng entropy weight-based method

C. The proposed DS-VIKOR method for supplier selection

In this section, the traditional VIKOR method [60], [61] is extended based on D-S theory [46], [47] which can deal with uncertain information. How to select the best supplier in the supply chain system can be considered as the multi-criteria decision making problem, and it can be denoted by the sets as follows:

(I). The set that includes m alternatives: $A = \{A_1, A_2, ..., A_m\}$;

(II). The set that includes n possible criteria: $C = \{C_1, C_2, ..., C_n\}$;

(III). The set that includes k decision experts: $E = \{E_1, E_2, ..., C_k\}$;

(IV). The set of evaluation values X: $X = \{x_{ij}, i = 1, 2, ..., m; j = 1, 2, ..., n\}$, which the rating for ith alternative under jth criterion.

The main steps of the proposed DS-VIKOR method for supplier selection has shown in Fig. 3, and the details are as follows:

Step1: Determine the Linguistic terms.

In the process of the supplier selection, the decision experts will score all the alternatives under different criteria based on their performances. Therefore, the evaluation level and corresponding values need to be defined. Table I gives the linguistic terms, abbreviation, linguistic judgment and their corresponding values.

Step2: Set the Scale of the confidence level.

In the process of evaluation, experts score each alternative based on their experience and subjective judgment. This is likely to lead to the emergence of ambiguity and uncertainty. In this paper, a more flexibility method for the experts' judgement of uncertainty is presented. A scale of [0,1] is used to denote the confidence of experts' assessment. The numerical scale for representing experts' confidence levels is shown in Table II. The number 1 denotes complete confidence in the judgement while 0 represents no confidence with his/her judgement.

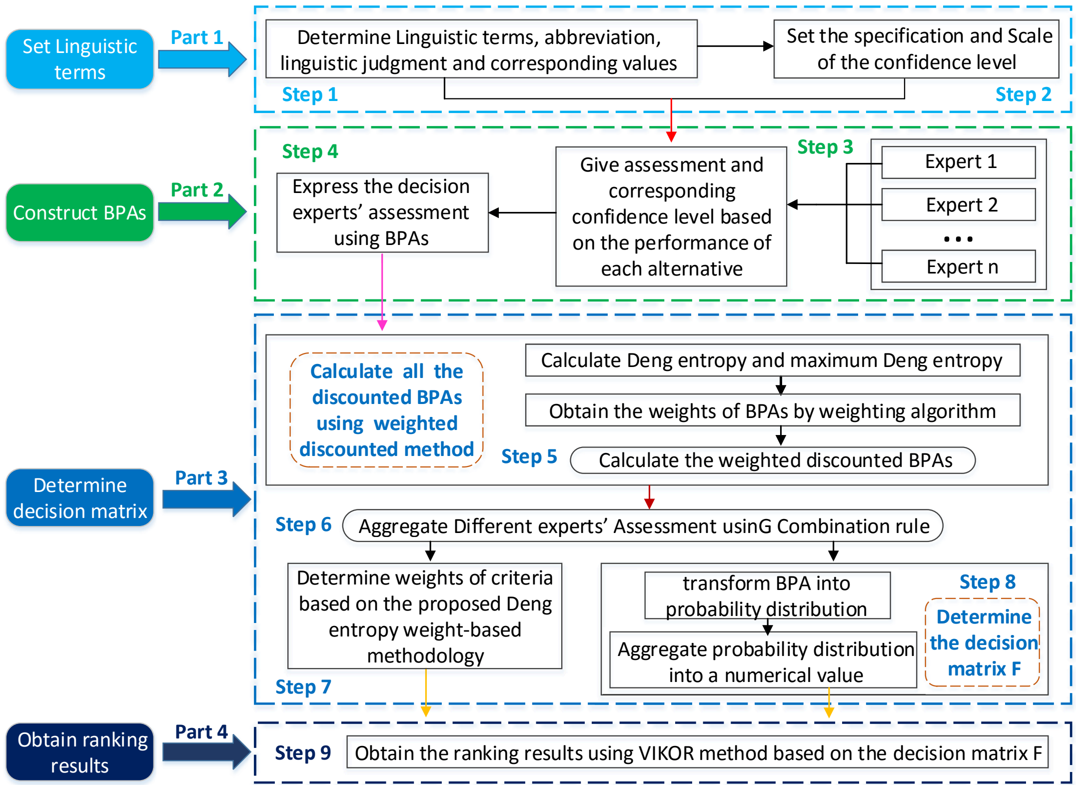

Fig. 3. The comparison between classical entropy weight method and the Deng entropy weight-based method

TABLE I
LINGUISTIC TERMS AND CORRESPONDING VALUE

Importance	Abbreviation	Linguistic Judgment	Corresponding Values
Very Low	VL	Almost no recognition to the performance	1
Low	L	Low evaluation to the performance	2
Medium Low	ML	A low and middle level of performance	3
Medium	M	The level of the performance is medium	4
Medium High	MH	A middle and high level of performance	5
High	H	High evaluation to the performance	6
Very High	VH	Almost fully recognized this performance	7

TABLE II
SCALE OF THE CONFIDENCE LEVEL

Specification of the confidence level	Scale
Fully convinced	1.0
Almost convinced	0.8
Properly convinced	0.6
Some convinced	0.4
Almost not convinced	0.2
Completely not convinced	0.0
Intermediate values between two adjacent levels	0.9, 0.7, 0.5, 0.3, 0.1

Step3: The decision experts give assessment and corresponding confidence level based on the performance of each alternative under different criteria. The evaluation results will be denoted by the importance defined in Table I and the confidence level will be expressed by the scale determined in Table II.

Step4: Express the decision experts' assessment using BPAs.

There are 7 elements in the importance defined in Table I, and these elements can be considered as the frame of discernment in D-S theory [46], [47], the same way, the evaluation importance from experts can be seen as focal element and the confidence level is the corresponding belief. In addition, the remaining confidence equals to the belief which is assigned to the universal set. Let x_{ij} represent the evaluation result from experts for *ith* alternative under *jth* criterion. Suppose the evaluation important in x_{ij} are A, B,$\cdots$, and the corresponding confidence level are a, b,$\cdots$. So the mass function of x_{ij} can be denoted as:

$$\begin{aligned} m(A) &= a \\ m(B) &= b \\ &\vdots \\ m(\Phi) &= 1 - a - b - \cdots \end{aligned}$$

Step5: Calculate the discounted BPAs using the proposed weighted discounting method.

In the last step, experts' assessment have been expressed using BPAs. In order to reduce the uncertainty of BPAs, in this step, the proposed weighted discounting method is used. At first, the Deng entropy [59] and maximum Deng entropy [69] need to be calculated based on Eq. (9) and (11). Then, the weights of BPAs can be obtained by Eq. (14). Finally, Eq .(15) is used to calculate the weighted discounting BPAs.

Step6: Aggregate different experts' assessment using combination rule.

According to experience, there are more than one expert in the panel, and these experts will score each alternative under different criteria. Therefore, with regard to the same alternative under the same criterion, the assessment need to be aggregated for the further decision-making, and the combination rule is used to aggregate different experts' assessment introduced in Eq. (16).

Step7: Determine weights of criteria based on the proposed Deng entropy weight-based methodology.

As mentioned above, the importance of each criterion is different. In this paper, a new method to determine weights is proposed, called Deng entropy weight-based methodology. At first, combining BPAs under the same criterion by the Deng entropy-based combination rule in section III-A. The obtained combined result of each criterion is a BPA, which equals to the probability distribution in traditional entropy weight method, but the advantages lie in the ability to handle uncertain information. Then, the weights of criteria can be calculation by the new method defined in section III-B.

Step8: Determine the decision matrix $F = (f_{ij})_{m\times n}$.

In the step 6, the aggregated BPAs for each alternative under each criterion have been obtained. For determining the decision matrix, the pignistic probability function $BetP$ is used to transform BPA into probability distribution by Eq. (12). Next, we define a aggregate function to integrate each probability distribution into a numerical value.

Definition III.8. *Suppose the importance in Linguistic terms as $I_1, I_2, ..., I_n$ and the corresponding values as $W = (w_1, w_2, ..., w_n)^T$. And the probability distribution is $P = (p_1, p_2, ..., p_n)$.*

$$F(I_1, ..., I_n) = PW = p_1w_1 + p_2w_2 + ... + p_nw_n \tag{22}$$

where P is the probability distribution from BetP, and corresponds to the importance $I_1, I_2, ..., I_n$.

Finally, the decision matrix $F = (f_{ij})_{m\times n}$ will be calculated by Eq. (22).

Step9: Obtain the ranking results using VIKOR method based on the decision matrix $F = (f_{ij})_{m\times n}$.

In the Step 8, the decision matrix has been obtained, then the VIKOR method will be used to sore all the alternatives. At first, determine the best f_i^* and worst f_i^- values in all the criteria.

$$\begin{aligned} &f_i^* = max_j(f_{ij}), f_i^- = min_j(f_{ij}) \quad && i\text{th function denotes a benefit} \\ &f_i^* = min_j(f_{ij}), f_i^- = max_j(f_{ij}) \quad && i\text{th function denotes a cost} \end{aligned} \tag{23}$$

Then, calculate the values S_j and R_j as follows:

$$S_j = \sum_{i=1}^{n} w_i \frac{(f_i^* - f_{ij})}{(f_i^* - f_i^-)} \tag{24}$$

$$R_j = max[w_i \frac{(f_i^* - f_{ij})}{(f_i^* - f_i^-)}] \tag{25}$$

where w_i represents the weight of ith criterion.

Next, compute Q_j values according to S_j and R_j as follows:

$$Q_j = v\frac{(S_j - S^*)}{(S^- - S*)} + (1 - v)\frac{(R_j - R^*)}{(R^- - R*)} \tag{26}$$

where $S^* = min_jS_j$, $S^- = max_jS_j$,$R^* = min_jR_j$ and $R^- = max_jR_j$. v is the weight of the strategy of the majority of criteria, and $1 - v$ is the weight of the individual regret. In this paper, the value of v is set to 0.5.

Then, rank the alternatives, sorting by the values of S, R and Q with decreasing order. The compromise solution is determined to be $A^{(1)}$, which obtains the minimum value by Q when the following two conditions are satisfied:

Condition 1. $Q(A^{(2)}) - Q(A^{(1)}) \geq DQ$, where $A^{(2)}$ is the alternative with second position in the ranking list by Q, and $DQ = 1/(n-1)$.

Condition 2. If alternative $A^{(1)}$ is also the best one ranked by S and R, then it is the most stable optimal selection in the process of decision-making.

If one of the conditions is not satisfied, then the compromise solutions will be divided into two cases:

- Alternatives $A^{(1)}$ and $A^{(2)}$ can be considered as the compromise solution if only Condition 2 is not satisfied, or
- The compromise solution is alternatives $A^{(1)}$, $A^{(2)}$,...,$A^{(N)}$ if Condition 1 is not satisfied, and $A^{(N)}$ is determined by the relation $Q(A^{(N)}) - Q(A^{(1)}) < DQ$ for maximum N.

IV. Illustrative example

In this section, an illustrative case will be conducted for illustrating the effectiveness of the proposed DS-VIKOR method, which draws on the application of the real life in [74]. A manufacturing enterprise wants to select more competitive suppliers in the market as part of its supply chain. So how to select the best suppliers from a number of alternatives with multi-criteria is where the critical problem lies. Applying the proposed algorithm in the last section, this problem can be solved scientific and effective. The detailed problem will be introduced in Section IV-A including the decision experts, evaluation criteria and other relevant knowledge. The Section IV-B shows the specific solutions using the proposed DS-VIKOR method step by step.

A. Problem description

The enterprise arranges three decision experts (k=3) to participate in the process of supplier selection. In order to select the most suitable supplier, the enterprise defines 5 criteria from different considerations (n=5). The details are as follows:

- C_1: Product quality
- C_2: Difficulty to establish cooperation
- C_3: Service performance
- C_4: Risk factor
- C_5: Price/Cost

After careful screening, 4 candidates (m=4) were selected from a large number of suppliers for further in-depth research.

B. The solution

The proposed DS-VIKOR method is used to solve the above problem for supplier selection, and the detailed steps are shown as follows:

Step1: After decision experts discussion, the Linguistic terms defined in Table I will be used to evaluate the performance of alternatives.

Step2: The scale of the confidence level determined in Table II will be applied to represent the confidence of experts' judgements.

Step3: According to the above definitions, the importance and corresponding confidence level will be determined by decision experts based on the performance of each alternative under different criteria. The evaluation results are shown in Table III.

Step4: In the last step, we have obtained the importance and confidence of each alternative, so we denote the evaluation results by basic probability assignment (BPA), and the results are shown in Table IV.

Step5: Calculate the Deng entropy of each BPA which represents the evaluation results of experts based on Eq. (9). And the results are shown in Table V, which are key for the next step to calculate the weights. According to the actual situation of this problem, there are 7 level of the important, that is, 7 focal elements are considered in the BPA. Then the maximum Deng entropy [69] can be calculated based on Eq. (11), and details are as follows:

Based on the above description and definition II.6 [69], the distribution of propositions in BPA should be satisfy:

$$
\begin{aligned}
&(\{VL\}, 1/2059), (\{L\}, 1/2059), \cdots, (\{VH\}, 1/2059)\\
&(\{VL, L\}, 3/2059), (\{VL, ML\}, 3/2059), \cdots, (\{H, VH\}, 3/2059)\\
&(\{VL, L, ML\}, 7/2059), (\{VL, L, M\}, 7/2059), \cdots, (\{MH, H, VH\}, 7/2059)\\
&(\{VL, L, ML, M\}, 15/2059), \cdots, (\{M, MH, H, VH\}, 15/2059)\\
&(\{VL, L, ML, M, MH\}, 31/2059), \cdots, (\{ML, M, MH, H, VH\}, 31/2059)\\
&(\{VL, L, ML, M, MH, H\}, 63/2059), \cdots, (\{L, ML, M, MH, H, VH\}, 63/2059)\\
&(\{VL, L, ML, M, MH, H, VH\}, 127/2059)
\end{aligned}
$$

Calculate the Deng entropy of this BPA based on Eq. (9), we obtain the maximum Deng entropy is 11.0077. Then, the weight of each BPA can be calculated based on Eq. (14) and the discounted BPAs can be obtained by Eq. (15). And the results are shown in Table V.

Step6: Because there are a number of experts to evaluate the same alternative under the same criterion, next, we will combine the multi-evaluations based on Eq. (16) to obtain a combined BPA to make better decisions for the next step. The results are shown as Table VI.

Step7: Determine weights of the 5 criteria based on the proposed Deng entropy weight-based methodology.

In the last step, we have obtained the discounted BPAs of all the alternatives, now the combination algorithm in Definition III.3 will be used again to combine different BPAs of alternatives under the same criterion. And the results are shown as Table VII. Then, the Deng entropy, divergences and weights can be calculated by Eq. (20) and (21), and the results are shown as Table VII.

Step8: Obtain the decision matrix F.

For the BPAs obtained in Table VI, we will convert them into probability distributions using pignistic probability function based on Eq. (12). Next, the probability distributions will be aggregated to get the elements of the decision matrix based on Eq. (22), and the results are shown as in Table VIII. And the decision matrix F is shown in Table IX.

Step9: Use the VIKOR method to make the final decision.

At first, determine the best values f_i^* and the worst values f_i^- of each criterion based on Eq. (23). And the results are shown in Table X. Secondly, calculate the

TABLE III
EVALUATION RESULTS OF ALTERNATIVES FROM DECISION EXPERTS

Experts	Criteria	Alternatives	Importance	Confidence
E1	C11	A1	{MH, H}	0.2
		A2	{MH}	0.3
		A3	{H, VH}	0.5
		A4	{H, VH}	1
	C12	A1	{M}	0.4
		A2	{MH}	0.3
		A3	{MH, H}	0.3
		A4	{H, VH}	0.2
	C13	A1	{MH}	0.5
		A2	{H}:{VH}=1:3	0.4
		A3	{M}	0.3
		A4	{M, H}	1
	C14	A1	{H, VH}	1
		A2	{H}	0.2
		A3	{MH}:{H, VH}=3:2	0.5
		A4	{MH}	0.2
	C15	A1	{VH}	0.1
		A2	{MH, H}	0.2
		A3	{VH}	0.5
		A4	{H}:{VH}=4:1	0.5
E2	C21	A1	{H, VH}	0.5
		A2	{MH, H}	0.3
		A3	{H}	0.2
		A4	{H, VH}	1
	C22	A1	{MH}:{H}=4:1	0.5
		A2	{H}	0.2
		A3	{H, VH}	0.2
		A4	{HM}	0.3
	C23	A1	{H, VH}	0.2
		A2	{MH}	0.3
		A3	{M}:{MH}=4:1	0.5
		A4	{M, H}	1
	C24	A1	{H, VH}	0.2
		A2	{MH}	0.2
		A3	{MH, H}	0.3
		A4	{H}	0.3
	C25	A1	{H, VH}	0.2
		A2	{M, MH}	1
		A3	{H}:{VH}=1:4	0.5
		A4	{H}	0.3
E3	C31	A1	{MH, H}	0.3
		A2	{MH}:{VH}=3:1	0.4
		A3	{H, VH}	0.2
		A4	{H, VH}	1
	C32	A1	{VH}	0.3
		A2	{H, VH}	0.3
		A3	{MH, H}:{VH}=4:1	0.5
		A4	{MH}	0.2
	C33	A1	{H, VH}	0.3
		A2	{H}:{VH}=1:4	0.5
		A3	{M, MH}	0.3
		A4	{H}	1
	C34	A1	{M, MH}	0.2
		A2	{M, H}	0.3
		A3	{MH, H}	0.3
		A4	{MH}	0.2
	C35	A1	{M, MH}	1
		A2	{H}	0.3
		A3	{MH, H}	0.2
		A4	{MH}	0.3

TABLE IV
CONSTRUCT BPA BASED ON EXPERTS' EVALUATIONS FOR CRITERIA

Experts	**Criteria**	**Alternatives**	**Basic Probability Assignment (BPA)**
E1	C11	A1	m({MH, H})=0.2, m()=0.6
		A2	m({MH})=0.3, m()=0.7
		A3	m({H, VH})=0.5,m()=0.5
		A4	m({H, VH})=1
	C12	A1	m({M})=0.4, m()=0.6
		A2	m({MH})=0.3, m()=0.7
		A3	m({MH, H})=0.7, m()=0.7
		A4	m({H, VH})=0.2, m()=0.8
	C13	A1	m({MH})=0.5, m()=0.5
		A2	m({H})=0.1, m({VH})=0.3, m()=0.6
		A3	m({M})=0.3, m()=0.7
		A4	m({M, H})=1
	C14	A1	m({H, VH})=1
		A2	m({H})=0.2, m()=0.8
		A3	m({MH})=0.3, m({H, VH})=0.2, m()=0.5
		A4	m({MH})=0.2, m()=0.8
	C15	A1	m({VH})=0.1, m()=0.9
		A2	m({MH, H})=0.2, m()=0.8
		A3	m({VH})=0.5, m()=0.5
		A4	m({H})=0.4, m({VH})=0.1, m()=0.5
E2	C21	A1	m({H, VH})=0.5, m()=0.5
		A2	m({MH, H})=0.3, m()=0.7
		A3	m({H})=0.2, m()=0.8
		A4	m({H, VH})=1
	C22	A1	m({MH})=0.4, m({H})=0.1, m()=0.5
		A2	m({H})=0.2, m()=0.8
		A3	m({H, VH})=0.2, m()=0.8
		A4	m({HM})=0.3, m()=0.7
	C23	A1	m({H, VH})=0.2, m()=0.8
		A2	m({MH})=0.3, m()=0.7
		A3	m({M})=0.4, m({MH})=0.1, m()=0.5
		A4	m({M, H})=1
	C24	A1	m({H, VH})=0.2, m()=0.8
		A2	m({MH})=0.2, m()=0.8
		A3	m({MH, H})=0.3, m()=0.7
		A4	m({H})=0.3, m()=0.7
	C25	A1	m({H, VH})=0.2, m()=0.8
		A2	m({M, MH})=1
		A3	m({H})=0.1, m({VH})=0.4, m()=0.5
		A4	m({H})=0.3, m()=0.7
E3	C31	A1	m({MH, H})=0.3, m()=0.7
		A2	m({MH})=0.3, m({VH})=0.1, m()=0.7
		A3	m({H, VH})=0.2, m()=0.8
		A4	m({H, VH})=1
	C32	A1	m({VH})=0.3, m()=0.7
		A2	m({H, VH})=0.3, m()=0.7
		A3	m({MH, H})=0.4, m({VH})=0.1, m()=0.5
		A4	m({MH})=0.2, m()=0.8
	C33	A1	m({H, VH})=0.3, m()=0.7
		A2	m({H})=0.1, m({VH})=0.4, m()=0.5
		A3	m({M, MH})=0.3, m()=0.7
		A4	m({H})=1
	C34	A1	m({M, MH})=0.2, m()=0.8
		A2	m({M, H})=0.3, m()=0.7
		A3	m({MH, H})=0.3, m()=0.7
		A4	m({MH})=0.2, m()=0.8
	C35	A1	m({M, MH})=1
		A2	m({H})=0.3, m()=0.7
		A3	m({MH, H})=0.2, m()=0.8
		A4	M({MH})=0.3, m()=0.7

TABLE V
THE VALUES OF DENG ENTROPY, WEIGHTS AND DISCOUNTED BPA

Experts	Criteria	Alternatives	Deng entropy	Weights	Discounted BPA
E1	C11	A1	6.6299	0.3977	m({MH, H})=0.0795, m()=0.9205
		A2	5.7734	0.4755	m({MH})=0.1427, m()=0.8573
		A3	5.2868	0.5197	m({H, VH})=0.2599,m()=0.7401
		A4	0.0000	0.8560	m({H, VH})=0.8560, m()=0.1440
	C12	A1	5.1642	0.5309	m({M})=0.2123, m()=0.7877
		A2	5.7734	0.4755	m({MH})=0.1427, m()=0.8573
		A3	6.2489	0.4323	m({MH, H})=0.1297, m()=0.8703
		A4	6.6299	0.3977	m({H, VH})=0.0795, m()=0.9205
	C13	A1	4.4943	0.5917	m({MH})=0.2959, m()=0.7041
		A2	5.4887	0.5014	m({H})=0.0501, m({VH})=0.0150, m()=0.9349
		A3	5.7734	0.4755	m({M})=0.1427, m()=0.8573
		A4	0.0000	0.8560	m({M, H})=8560, m()=0.1440
	C14	A1	1.5850	0.8560	m({H, VH})=0.8560, m()=0.1440
		A2	6.3129	0.4265	m({H})=0.0853, m()=0.9147
		A3	5.2968	0.5188	m({MH})=0.1566, m({H, VH})=0.1038, m()=0.7406
		A4	6.3129	0.4265	m({MH})=0.0853, m()=0.9147
	C15	A1	6.7588	0.3860	m({VH})=0.0386, m()=0.9614
		A2	6.6299	0.3977	m({MH, H})=0.0795, m()=0.9205
		A3	4.4943	0.5917	m({VH})=0.2959, m()=0.7041
		A4	4.8553	0.5589	m({H})=0.2236, m({VH})=0.0559, m()=0.7205
E2	C21	A1	5.2868	0.5197	m({H, VH})=0.2599, m()=0.7401
		A2	6.2489	0.4323	m({MH, H})=0.1297, m()=0.8703
		A3	6.3129	0.4265	m({H})=0.0853, m()=0.9147
		A4	0.0000	0.8560	m({H, VH})=8560, m()=0.1440
	C22	A1	4.8553	0.5589	m({MH})=0.2236, m({H})=0.0559, m()=0.7205
		A2	6.3129	0.4265	m({H})=0.0853, m()=0.9147
		A3	6.6299	0.3977	m({H, VH})=0.0795, m()=0.9205
		A4	5.7734	0.4755	m({HM})=0.1427, m()=0.8573
	C23	A1	6.6299	0.3977	m({H, VH})=0.0795, m()=0.9205
		A2	5.7734	0.4755	m({MH})=0.1427, m()=0.8573
		A3	4.8553	0.5589	m({M})=0.2236, m({MH})=0.0559, m()=0.7205
		A4	0.0000	0.8560	m({M, H})=0.8560, m()=0.1440
	C24	A1	6.6299	0.3977	m({H, VH})=0.0795, m()=0.9205
		A2	6.3129	0.4265	m({MH})=0.0853, m()=0.9147
		A3	6.2489	0.4323	m({MH, H})=0.1297, m()=0.8703
		A4	5.7734	0.4755	m({H})=0.1427, m()=0.8573
	C25	A1	6.6299	0.3977	m({H, VH})=0.0795, m()=0.9205
		A2	1.5850	0.8560	m({M, MH})=0.8560, m()=0.1440
		A3	4.8553	0.5589	m({H})=0.0559, m({VH})=0.2236, m()=0.7205
		A4	5.7734	0.4755	m({H})=0.1427, m()=0.8573
E3	C31	A1	6.2489	0.4323	m({MH, H})=0.1297, m()=0.8703
		A2	5.4887	0.5014	m({MH})=0.1504, m({VH})=0.0150, m()=0.8346
		A3	5.4168	0.5079	m({H, VH})=0.1016, m()=0.8984
		A4	0.0000	0.8560	m({H, VH})=0.8560,m()=0.1440
	C32	A1	5.7734	0.4755	m({VH})=0.1427, m()=0.8573
		A2	6.2489	0.4323	m({H, VH})=0.1297, m()=0.8703
		A3	5.4893	0.5013	m({MH, H})=0.2005, m({VH})=0.0201, m()=0.7794
		A4	6.3129	0.4265	m({MH})=0.0853, m()=0.9147
	C33	A1	6.2489	0.4323	m({H, VH})=0.1297, m()=0.8703
		A2	4.8553	0.5589	m({H})=0.0559, m({VH})=0.2236, m()=0.7205
		A3	6.2489	0.4323	m({M, MH})=0.1297, m()=0.8703
		A4	0.0000	0.8560	m({H})=0.8560, m()=0.1440
	C34	A1	6.6299	0.3977	m({M, MH})=0.0795, m()=0.9205
		A2	6.2489	0.4323	m({M, H})=0.1297, m()=0.8703
		A3	6.2489	0.4323	m({MH, H})=0.1297, m()=0.8703
		A4	6.3129	0.4265	m({MH})=0.0853, m()=0.9147
	C35	A1	1.5850	0.8560	m({M, MH})=8560, m()=0.1440
		A2	5.7734	0.4755	m({H})=0.1427, m()=0.8573
		A3	6.6299	0.3977	m({MH, H})=0.0795, m()=0.9205
		A4	5.7734	0.4755	M({MH})=0.1427, m()=0.8573

TABLE VI
OBTAIN THE COMBINED RESULTS OF MULTI-EXPERTS' EVALUATION FOR THE SAME ALTERNATIVE UNDER THE SAME CRITERION

Criteria	Alternatives	Combined BPA
C1	A1	m({H})=0.0517, m({MH, H})=0.1472, m({H, VH})=0.2082, m()=0.5929
	A2	m({MH})=0.2705, m({MH, H})=0.0932, m({VH})=0.0112, m()=0.6251
	A3	m({H})=0.0853, m({H,VH})=0.3065, m()=0.6082
	A4	m({H, VH})=0.8560, m()=0.1440
C2	A1	m({M})=0.1478, m({MH})=0.1702, m({H})=0.0425, m({VH})=0.0913, m()=0.5483
	A2	m({MH})=0.1170, m({H})=0.0753, m({H,VH})=0.1048, m()=0.7029
	A3	m({H})=0.0240, m({VH})=0.0175, m({MH,H})=0.2783, m({H,VH})=0.0541, m()=0.6260
	A4	m({H,VH})=0.0634, m({MH})=0.2021, m()=0.7344
C3	A1	m({MH})=0.2519, m({H,VH})=0.1488, m()=0.5993
	A2	m({H})=0.0829, m({VH})=0.2029, m({MH})=0.1019, m()=0.6123
	A3	m({MH})=0.1639, m({M})=0.1980, m({M, MH})=0.0828, m()=0.5553
	A4	m({M, H})=8560, m()=0.1440
C4	A1	m({H, VH})=0.8576, m({M, MH})=0.0113, m()=0.1311
	A2	m({H})=0.0794, m({MH})=0.0691, m({M, H})=0.1104, m()=0.7411
	A3	m({MH})=0.1556, m({H})=0.0252, m({H,VH})=0.0786, m({MH,H})=0.1797, m()=0.5609
	A4	m({MH})=0.1434, m({H})=0.1222, m()=0.7344
C5	A1	m({VH})=0.0062, m({H,VH})=0.0122, m({M, MH})=0.8403, m()=0.1414
	A2	m({MH})=0.0665, m({H})=0.0234, m({MH,H})=0.0112, m({M,MH})=0.7695, m()=0.1294
	A3	m({VH})=0.4238, m({H})=0.0415, m({MH, H})=0.0425, m()=0.4922
	A4	m({H})=0.2981, m({VH})=0.0438, m({MH})=0.0939, m()=0.5642

TABLE VII
THE COMBINED BPAS AND WEIGHTS FOR EACH CRITERION

Criteria	Combined BPA	Deng entropy	Divergence	Weights
C1	m({H})=0.3308, m({MH})=0.0236, m({MH, H})=0.0190, m({VH})=0.0110, m({H,VH})=0.5720, m()=0.0436	2.7352	0.8282	0.2070
C2	m({M})=0.0639, m({MH})=0.3253, m({H})=0.1202, m({VH})=0.0632, m({H,VH})=0.0845, m({MH, H})=0.1055, m()=0.2372	4.4950	0.7176	0.1794
C3	m({MH})=0.0828, m({H})=0.1765, m({VH})=0.0276, m({H,VH})=0.0165, m({M})=0.2240, m({M,MH})=0.0099, m({M,H})=0.3960, m()=0.0666	3.4543	0.7830	0.1958
C4	m({MH})=0.0489, m({M})=0.0008, m({M,MH})=0.0049, m({H})=0.4300, m({H,VH})=0.4319, m({M,H})=0.0085, m({MH,H})=0.0182, m()=0.0568	2.8354	0.8219	0.2055
C5	m({VH})=0.0171, m({H})=0.0174, m({H,VH})=0.0013, m({MH})=0.2613, m({M,MH})=0.1861, m({MH,H})=0.0026, m()=0.0145	2.3993	0.8493	0.2123

TABLE VIII
THE BETP AND AGGREGATE VALUES FOR EACH ALTERNATIVE UNDER DIFFERENT CRITERIA

Criteria	Alternatives	BetP	Aggregate values
C1	A1	BetP(VL)=0.0847,BetP(L)=0.0847, BetP(ML)=0.0847, BetP(M)=0.0847, BetP(MH)=0.1583, BetP(H)=0.3141, BetP(VH)=0.1888	4.8447
	A2	BetP(VL)=0.0893,BetP(L)=0.0893, BetP(ML)=0.0893, BetP(M)=0.0893, BetP(MH)=0.4064, BetP(H)=0.1359, BetP(VH)=0.1005	4.4439
	A3	BetP(VL)=0.3254,BetP(L)=0.0869, BetP(ML)=0.0869, BetP(M)=0.0869, BetP(MH)=0.0869, BetP(H)=0.3254, BetP(VH)=0.2401	4.9368
	A4	BetP(VL)=0.0206,BetP(L)=0.0206, BetP(ML)=0.0206, BetP(M)=0.0206, BetP(MH)=0.0206, BetP(H)=0.4486, BetP(VH)=0.4486	6.1400
C2	A1	BetP(VL)=0.0783,BetP(L)=0.0783, BetP(ML)=0.0783, BetP(M)=0.2261, BetP(MH)=0.2485, BetP(H)=0.1208, BetP(VH)=0.1696	4.5295
	A2	BetP(VL)=0.1004,BetP(L)=0.1004, BetP(ML)=0.1004, BetP(M)=0.1004, BetP(MH)=0.2174, BetP(H)=0.2281, BetP(VH)=0.1528	4.5296
	A3	BetP(VL)=0.0894,BetP(L)=0.0894, BetP(ML)=0.0894, BetP(M)=0.0894, BetP(MH)=0.2286, BetP(H)=0.2796, BetP(VH)=0.1340	4.6528
	A4	BetP(VL)=0.1049,BetP(L)=0.1049, BetP(ML)=0.1049, BetP(M)=0.1049, BetP(MH)=0.3070, BetP(H)=0.1366, BetP(VH)=0.1366	4.3602
C3	A1	BetP(VL)=0.0856,BetP(L)=0.0856, BetP(ML)=0.0856, BetP(M)=0.0856, BetP(MH)=0.3375, BetP(H)=0.1600, BetP(VH)=0.1600	4.6239
	A2	BetP(VL)=0.0875,BetP(L)=0.0875, BetP(ML)=0.0875, BetP(M)=0.0875, BetP(MH)=0.1894, BetP(H)=0.1704, BetP(VH)=0.2904	4.8764
	A3	BetP(VL)=0.0793,BetP(L)=0.0793, BetP(ML)=0.0793, BetP(M)=0.3187, BetP(MH)=0.2846, BetP(H)=0.0793, BetP(VH)=0.0793	4.2053
	A4	BetP(VL)=0.0206,BetP(L)=0.0206, BetP(ML)=0.0206, BetP(M)=0.4486, BetP(MH)=0.0206, BetP(H)=0.4486, BetP(VH)=0.0206	4.8560
C4	A1	BetP(VL)=0.0187,BetP(L)=0.0187, BetP(ML)=0.0187, BetP(M)=0.0244, BetP(MH)=0.0244, BetP(H)=0.4475, BetP(VH)=0.4475	6.1497
	A2	BetP(VL)=0.1059,BetP(L)=0.1059, BetP(ML)=0.1059, BetP(M)=0.1611, BetP(MH)=0.1750, BetP(H)=0.2405, BetP(VH)=0.1059	4.3383
	A3	BetP(VL)=0.0801,BetP(L)=0.0801, BetP(ML)=0.0801, BetP(M)=0.0801, BetP(MH)=0.3256, BetP(H)=0.2345, BetP(VH)=0.1194	4.6721
	A4	BetP(VL)=0.1049,BetP(L)=0.1049, BetP(ML)=0.1049, BetP(M)=0.1049, BetP(MH)=0.2483, BetP(H)=0.2271, BetP(VH)=0.1049	4.3878
C5	A1	BetP(VL)=0.0202,BetP(L)=0.0202, BetP(ML)=0.0202, BetP(M)=0.4404, BetP(MH)=0.4404, BetP(H)=0.0263, BetP(VH)=0.0325	4.4697
	A2	BetP(VL)=0.0185,BetP(L)=0.0185, BetP(ML)=0.0185, BetP(M)=0.4032, BetP(MH)=0.4753, BetP(H)=0.0475, BetP(VH)=0.0185	4.5149
	A3	BetP(VL)=0.0703,BetP(L)=0.0703, BetP(ML)=0.0703, BetP(M)=0.0703, BetP(MH)=0.0916, BetP(H)=0.1331, BetP(VH)=0.4941	5.4182
	A4	BetP(VL)=0.0806,BetP(L)=0.0806, BetP(ML)=0.0806, BetP(M)=0.0806, BetP(MH)=0.1745, BetP(H)=0.3787, BetP(VH)=0.1244	4.8215

TABLE IX
DECISION MATRIX F

	A1	A2	A3	A4
C1	4.8447	4.4439	4.9368	6.1400
C2	4.5295	4.5296	4.6528	4.3602
C3	4.6239	4.8764	4.2053	4.8560
C4	6.1497	4.3383	4.6721	4.3878
C5	4.4697	4.5149	5.4182	4.8215

values of S_j, R_j and Q_j based on Eq. (24), (25) and (26), and the results are shown in Table XI. Then, sort the alternatives based on the values of S_j, R_j and Q_j in descending order, and the results are shown in Table XII. According to the ranking results shown in Table XII, the ranking of suppliers is $A_3 \succ A_4 \succ A_2 \succ A_1$ by Q. And A_3 is still the best selection based on S and R, so it can satisfy Condition 2. However, $Q(A_4) - Q(A_3) = 0.1956 < 1/3$, so it cannot satisfy Condition 1. Let $Q(A^{(N)}) - Q(A_3) = 0.1956 < 1/3$, we have $N = 4$, so the compromise solution is A_3, A_4.

TABLE X
BEST VALUES f^* AND THE WORST VALUES f^- OF EACH CRITERION

	C1	C2	C3	C4	C5
f^*	6.1400	4.6528	4.8764	6.1497	5.4182
f^-	4.4439	4.3602	4.2053	4.3383	4.4697

TABLE XI
THE VALUES OF S, R AND Q FOR EACH SUPPLIER

	A1	A2	A3	A4
S_j	0.5197	0.6902	0.5103	0.5188
R_j	0.2123	0.2070	0.1958	0.1999
Q_j	0.5260	0.8394	0.0000	0.1474

TABLE XII
THE RANKING OF SUPPLIERS IN ASCENDING ORDER BY S, R AND Q

The ranking index	The ascending ranks
S	$A_3 \succ A_4 \succ A_1 \succ A_2$
R	$A_3 \succ A_4 \succ A_2 \succ A_1$
Q	$A_3 \succ A_4 \succ A_1 \succ A_2$

A lot of work has been tried by scholars to deal with supplier selection problems under uncertain environment. In the following, we will compare our approach with some other existing methods from the aspects of the application environment, type of criteria weights, type of problem, main idea, etc., separately. As Table XIII shows, the following conclusions can be easily obtained:

(i) The proposed approach can be applied synchronously in the DS evidence theory and Linguistic variables environment, while other methods than [14] can not work, for example, method [30] can only be used with Fuzzy numbers and method [26] can only work on Interval 2-tuple linguistic information.

(ii) With regard to the expression and determination of criterion weights, except for the approach we proposed, the weights of other methods all come from the decision maker, which undoubtedly increases the subjectivity of decision making. Our approach determines the weights according to the decision information, doing so can effectively weaken the subjective factors of human decision making and make the result of decision-making more accurately.

(iii) The proposed approach and method [26] can effectively deal with supplier selection problems with VIKOR method, which can not be done by method [14], [30] and [23]. From the view of main idea and contribution, our work is more prominent than other papers, mainly in the following aspects: our paper extended the VIKOR method with D-S evidence theory, and the decision information can be represented by BPAs under uncertain environment. In addition, a new combination rule for BPAs is defined based on Deng entropy, which also is used to determine criterion weights.

V. CONCLUSION

In this paper, a new VIKOR method called DS-VIKOR is proposed to handle the supplier selection problem using D-S theory to extend the traditional VIKOR method. In our proposed method, the assessment from experts are represented by BPAs in D-S theory that can handle uncertain information effectively. The Deng entropy-based combination rule for BPA is proposed in this paper to eliminate uncertainty from human being's subjective judgment and aggregate the assessment from different experts. The Deng entropy weight-based methodology is also proposed to determine weights of criteria, which extends the classical entropy weight method. What's more, with the advantage that providing a maximum group utility of the majority and a minimum of the individual regret of the opponent, VIKOR method can give the compromise solution which can be accepted by decision makers in MCDM problem. The combination between D-S theory and VIKOR method ont only can deal with uncertain information effectively, but also can obtain the compromise solution reasonably in supplier selection problem. An illustrative example of supplier selection under real life is conducted to demonstrate the effectiveness of the proposed DS-VIKOR method. In the future research, the theoretical framework of the DS-VIKOR method needs to be increasingly perfected. And the proposed method should be applied to more applications to further verity its feasibility.

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TABLE XIII
COMPARISON ANALYSIS BETWEEN THE PROPOSED APPROACH AND THE EXISTING METHODS

The method	Environment	Type of criteria weights	Type of problem	Main idea
Method [26]	Interval 2-tuple linguistic information	Crisp numbers	Supplier selection	VIKOR & interval 2-tuple linguistic information
Method [14]	Fuzzy numbers & BPA	Crisp numbers	Supplier selection	TOPSIS method
Method [30]	Fuzzy numbers	Crisp numbers	Supplier selection	Fuzzy AHP
Method [23]	Fuzzy numbers	Grey linguistic variables	Supplier selection	Combination of grey system theory & uncertainty theory
Our method	BPA & Linguistic variables	BPA	Supplier selection	DS-VIKOR

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