Revival of the Sakaton

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Abstract

Sakaton, $\mathcal{S} = \begin{pmatrix} p \\ n \\ \Lambda \end{pmatrix}$ with integral charges, 1,0,0, respectively, and treated as forming the fundamental representation of $SU(3)_F$ group, was successful in explaining the octet mesons but failed to describe the structure of baryons. This was replaced by fractionally charged quarks. $\mathcal{Q} = \begin{pmatrix} u \\ d \\ s \end{pmatrix}$, providing the fundamental representation of the $SU(3)_F$ group. This has been a thumping success. Thus a decent burial was given to the concept of the Sakaton. However, there is another model, the Topological Skyrme model, which has been providing a parallel and successful descrpition of the same hadrons. Nevertheless, sometimes this other model gives tantalizing hints of new structures in hadrons. In this paper we prove that this topological Skyrme model, leads to a clear revival of the above concept of Sakaton, as a real and a genuine physical entity. This provides a new perspective to the hypernuclei. 't Hooft anomaly matching gives an unambiguous support to this revival of the Sakaton.

Keywords: Quark model, Sakata model, Eightfold Way model, Sakaton, Topological Skyrme model, hypernuclei, electric charge, 't Hooft anomaly matching

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In the Fermi-Yang model [1] the fundamental representaion of $SU(2)_I$ isospin group with N = $\binom{p}{n}$, the newly discovered pions were treated as composites of the $N - \bar{N}$ pair. Sakata, [2] by extendng the group to $SU(2)_I \times U(1)_Y$, and including Λ as a representation of the U(1) group, assumed that hadrons could be treated to be composites of these. Thus it was natural to take

$$\mathcal{S} = \begin{pmatrix} p \\ n \\ \Lambda \end{pmatrix} \tag{1}$$

as the fundamenatl representation of a larger $SU(3)_F$ group [3]. It is called Sakaton in anology with Nucleon of the isospin group. Note that the charges in Sakaton are all integral: 1,0,0 respectively. The Sakata Model predicted the mesons correctly as composites: $3 \times \bar{3} = 1 + 8$. However it failed to describe the baryons as $3 \times 3 \times \bar{3} = 3 + 3 + 6 + 15$. In 1961, Gell-Mann and Ne'eman showed that spin-1/2 integral charge entities behaved as per the Eightfold Way model rather than like a Sakaton (see e.g. [4]). Later in 1963/64, Gell-Mann and Zweig realized that the fundamental representation of the $SU(3)_F$ group were quarks with fractional charges and thereby ruling out the Sakatons with integral charges. Subsequent successes of the quark model demolished the Sakaton concept completely.

The next major development in the theory of hadrons was the topological Skyrme model (see e.g. [4,5]). It is believed to give a parallel or a complementary description of hadrons. Though the phenomenological predictions of this model are not as accurate as those of the quark model, still this model tends to give tantalizing new perspective and to open up new avenues in the study of hadrons. In this paper we shall show as to how the Topological Skyrme model for three flavours, allows us to revive the Sakaton as a real and a relevant physical entity.

The Skyrme Lagrangian [4,5] is,

$$L_S = \frac{{f_\pi}^2}{4} Tr(L_\mu L^\mu) + \frac{1}{32e^2} Tr[L_\mu, L_\nu]^2 \tag{2}$$

where $L_\mu = U^\dagger \partial_\mu U$. Here $U(x)$ is an element of the group $SU(2)_F$,

$$U(x)^{SU(2)} = exp((i\tau^a \phi^a / f_\pi), \;\; (a = 1, 2, 3) \tag{3}$$

The solitonic structure present in the Lagrangian is obtained on making Skyrme ansatz as follows [4,5].

$$U_c(x)^{SU(2)} = exp((i/f_\pi \theta(r) \hat{r}^a \tau^a), \;\; (a = 1, 2, 3) \tag{4}$$

This $U_c(x)$ is called the Skyrmion. But on quantization, the two flavour model Skyrmion has a well known boson-fermion ambiguity [4-6]. This is rectified by going to three flavours. In that case we take,

$$U(x)^{SU(3)} = exp[\frac{i\lambda^a \phi^a(x)}{f_\pi}] \;\; (a = 1, 2..., 8) \tag{5}$$

with ϕ^a the pseudoscalar octet of π, K and η mesons. In the full topological Skyrme model this is supplemented with a Wess-Zumino (WZ) effective action [5-10]

$$\Gamma_{WZ} = \frac{-i}{240\pi^2} \int_\Sigma d^5x \epsilon^{\mu\nu\alpha\beta\gamma} Tr[L_\mu L_\nu L_\alpha L_\beta L_\gamma] \tag{6}$$

on surface Σ. Thus with this anomaly term, the effective action is.

$$S_{eff} = \int d^4x \; Tr \; [L_\mu L^\mu] + n \; \Gamma_{WZ} + quartic \; term \tag{7}$$

where the winding numbber n is an integer $n \in Z$, the homotopy group of mapping being $\Pi_5(SU(3)) = Z$.

Next we embed the SU(2) Skyrme ansatz into $U(x)^{SU(3)}$ as follows for the SU(3) Skyrmion [5-10],

$$U_c(x)^{SU(2)} \; \rightarrow \; U_c(x)^{SU(3)} = \begin{pmatrix} U_c(x)^{SU(2)} & \\ & 1 \end{pmatrix} \tag{8}$$

Let us insert the identity,

$$U(\vec{r},t)^{SU(3)} = A(t) U(\vec{r})_c^{SU(3)} A^{-1}(t) \qquad A \in SU(3)_F \tag{9}$$

where A is the collective coordinate. Note that $U(\vec{r},t)$ is invariant under,

$$A \rightarrow A e^{iY\alpha(t)} \tag{10}$$

where

$$Y = \frac{1}{3} \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & -2 \end{pmatrix} \tag{11}$$

With this the quantum degrees of freedom manifest themselves in the WZ term (eqn. (6)) as,

$$L_{WZ} = -\frac{1}{2} N_c B(U_c) tr(Y A^{-1} A) \tag{12}$$

where $B(U_c)$ is the baryon number (winding number) of the classical configuration U_c. The gauge invariance leads to changing L_{WZ} to

$$L_{WZ} \rightarrow L_{WZ} + \frac{1}{3} N_c B(U_c) \dot{\alpha} \tag{13}$$

In the quantized theory A and Y are operators and from Noether's theorem one obtains (with Ψ as allowed quantum state)

$$\hat{Y}\Psi = \frac{1}{3} N_c B \Psi \tag{14}$$

This gives the right-hypercharge,

$$Y_R = \frac{1}{3} N_c B \tag{15}$$

where the baryon number B is necssarily an integer and colour N_c is an integer too.

With B = 1 and $N_c = 3$ one gets $Y_R = 1$. This identifies the nucleon hypercharge with the body-fixed hypercharge Y_R. This shows that the SU(3) irreducible representation which is the lowest is the octet: [p,q] = [2,1]. Next one finds that the right translation on [2,1] with $Y_R = 1$, gives correct spin and isospin of the octet as $I_R = \frac{1}{2} = J$.

This is standard manner of determining the lowest dimensional irreducible representation to be [2,1] in SU(3) [5-10]. Note that this method determines the hypercharges, isospins and spins of the various members of the octet. However it has one shortcoming, and that is of not talking explicitly about the individual electric charges of the members of the octet. Note that we can say that the lowest member is the octet with confidence, only when we can also show that the charges are as per the Gell-MannNishijima expression (as in the Eightfold Way model [4]),

$$Q = I_3 + \frac{Y}{2} \tag{16}$$

This shortcoming was rectified by Balachandran et. al. [5]. Next we bring in the electric charge explicitly, in the three-flavour Skyrme model [5,11], and which we discuss in some detail below.

But before that, we study the structure of the electric charge in the $SU(2)_F$ model, where one defines the electric charge operator as,

$$Q = \begin{pmatrix} q_1 & 0 \\ 0 & q_2 \end{pmatrix} \tag{17}$$

Upon quantization, one obtains inconsistent electric charges of proton and neutron to be 1/2 and -1/2 respectively [5, p. 178]. This problem is rectified by including the Wess-Zumino anomaly term for the $SU(2)_F$ electric charges. As well known, even though the WZ anomaly term vanishes for two flavours, its contribution to the electric charge current does not. This extra contribution to electric charge was not present in the original Skyrme model. They find that the $SU(2)_F$ Skyrme model with the WZ anomaly term included, does indeed lead to properly quantized charge as [5, p. 209],

$$q_1 = \frac{2}{3} \quad , \quad q_2 = -\frac{1}{3} \tag{18}$$

Next, in going to $SU(3)_F$, let the field U in eqn. (5) be transformed by an electric charge operator Q as,

$$U(x) \rightarrow e^{i\Lambda Q} U(x) e^{-i\Lambda Q} \tag{19}$$

where all the charges are counted in units of the absolute value of the electronic charge.

Making $\Lambda = \Lambda(x)$ a local transformation the Noether current is [5,11]

$$J_\mu{}^{em}(x) = j_\mu{}^{em}(x) + j_\mu{}^{WZ}(x) \tag{20}$$

where the first one is the standard Skyrme term and the second is the Wess-Zumino term

$$j_\mu{}^{WZ}(x) = \frac{N_c}{48\pi^2}\epsilon_{\mu\nu\lambda\sigma} Tr L^\nu L^\lambda L^\sigma (Q + U^\dagger Q U) \tag{21}$$

In the standard way [5,11], we take the U(1) of electromagnetism as a subgroup of the three flavour SU(3). Its generators can be found by the canonical methods. As the charge operator can be simultaneously diagonalized along with the third component of isospin and hypercharge, we write it as

$$Q = \begin{pmatrix} q_1 & 0 & 0 \\ 0 & q_2 & 0 \\ 0 & 0 & q_3 \end{pmatrix} \tag{22}$$

The electric charge of pseudoscalar octet mesons demand,

$$q_1 - q_2 = 1, \quad q_2 = q_3 \tag{23}$$

Hence one obtains

$$Q = (q_2 + \frac{1}{3})\mathbf{1}_{3x3} + \frac{1}{2}\lambda_3 + \frac{1}{2\sqrt{3}}\lambda_8 \tag{24}$$

Now we use $U = A(t)U_c(\mathbf{x})A(t)^{-1}$ where A is the collective coordinate. We obtain the B=1 electric charge from the Skyrme term in terms of the left-handed generators L_α only as

$$Q^{em} = \frac{1}{2}(L_3 - (A^\dagger\lambda_3 A)_8 \frac{N_c B(U_c)}{\sqrt{3}}) + \frac{1}{2\sqrt{3}}(L_8 - (A^\dagger\lambda_8 A)_8 \frac{N_c B(U_c)}{\sqrt{3}}) \tag{25}$$

The Wess-Zumino term contributes

$$Q^{WZ} = N_c B(U_c)(q_2 + \frac{1}{3} + \frac{1}{2\sqrt{3}}(A^\dagger\lambda_3 A)_8 + \frac{1}{6}(A^\dagger\lambda_3 A)_8) \tag{26}$$

Hence the total electric charge is [5,11]

$$Q = \frac{1}{2}L_3 + \frac{1}{2\sqrt{3}}L_8 + (q_2 + \frac{1}{3})N_c B(U_c) \tag{27}$$

This equation was used by the author [11] to obtain the proper colour dependent electric charge of quarks [12], by ensuring that the proton charge was integral.

Here we study what was done by Balachandran et. al. [5, p. 210]. They put $q_2 = -\frac{1}{3}$ in eqn. (27) above. Thus the last term in eqn. (27) goes to zero. They then claimed that this gave them the expression of eqn. (16) which is the Gell-Mann Nishijima expression $Q = T_3 + \frac{Y}{2}$ of the electric charges of the octet baryons.

Thus as per this result, the shortcoming of the above analysis of the octet repesentation, was taken care of. As such this completes their proof that the lowest dimensional irreducible representation to be [2,1] in SU(3) [5-10].

However this result of Balachandran et. al. [5, p. 210] giving final touch to the earlier demonstartion [5-10] of (2,1) octet as being the lowest representation of the complete Skyrme model for $SU(3)_F$, is unfortunately wrong!

To understand this, let us point out a basic difference between the $SU(3)_F$ Sakata model and the quark model [4,13]. In the quark model, electric charge is defined as (note: in eqn. (16) $I_3 = F_3$ below)

$$Q = F_3 + \frac{1}{\sqrt{3}} F_8 = \begin{pmatrix} \frac{2}{3} & 0 & 0 \\ 0 & \frac{-1}{3} & 0 \\ 0 & 0 & \frac{-1}{3} \end{pmatrix} \tag{28}$$

While in the Sakata model it is,

$$Q = (F_3 + \frac{1}{\sqrt{3}} F_8) + \frac{B}{3} =$$

$$\begin{pmatrix} \frac{2}{3} & 0 & 0 \\ 0 & \frac{-1}{3} & 0 \\ 0 & 0 & \frac{-1}{3} \end{pmatrix} + \begin{pmatrix} \frac{1}{3} & 0 & 0 \\ 0 & \frac{1}{3} & 0 \\ 0 & 0 & \frac{1}{3} \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} \tag{29}$$

Where B is the baryon number. Note that in the Sakata model, the charges are "shifted" by $\frac{B}{3} = \frac{1}{3}$ with respect to the corresponding quark charges [4,13]. Thus a Sakaton has integral charges as above.

This shifting of charges is accounted for in a basic manner in the Gel'fand basis. The Gel'fand basis has an advantage over the Weyl basis, in that in general, while the Weyl basis are not orthonormal, the Gel'fand basis by construction (and hence by definition) are orthonormal [14].

The SU(3) irreducible representation given by [p,q] has a geometric Gel'fand pattern as [14],

$$\begin{pmatrix} p & & q & & 0 \\ & a & & b & \\ & & c & & \end{pmatrix} \tag{30}$$

where the integers a,b,c obey the between-ness rule: $a \geq c \geq b$. The isospin and hypercharge qunatum numbers are given as:

$$I = \frac{a-b}{2} \; ; \quad I_3 = c - \frac{a+b}{2} \; ; \quad Y = 2(\frac{a+b}{2} - \frac{p+q}{3}) \tag{31}$$

Now for example:

TRIPLET:

$$\begin{array}{rccc}
 & 100 & 100 & 100 \\
 & 10 & 10 & 00 \\
 & 1 & 0 & 0 \\
I: & 1/2 & 1/2 & 0 \\
I_3: & 1/2 & -1/2 & 0 \\
Y: & 1/3 & 1/3 & -2/3
\end{array} \tag{32}$$

OCTET:

$$\begin{array}{rcccccccc}
 & 210 & 210 & 210 & 210 & 210 & 210 & 210 & 210 \\
 & 21 & 21 & 20 & 20 & 20 & 10 & 10 & 11 \\
 & 2 & 1 & 2 & 1 & 0 & 1 & 0 & 1 \\
I: & 1/2 & 1/2 & 1 & 1 & 1 & 1/2 & 1/2 & 0 \\
I_3: & 1/2 & -1/2 & 1 & 0 & -1 & 1/2 & -1/2 & 0 \\
Y: & 1 & 1 & 0 & 0 & 0 & -1 & -1 & 0
\end{array} \tag{33}$$

The Gel'fand pattern can be expressed, first in terms of the I, I_3, Y quantum numbers, and subsequently in terms of the electric charge quantum number, which is given by the Gell-Mann Nishijima expression of eqn. (16) as,

$$\begin{aligned}
\begin{pmatrix} p & & q & & 0 \\ & a & & b & \\ & & c & & \end{pmatrix} &= \begin{pmatrix} p & & q & & 0 \\ & I+\frac{Y}{2}+\frac{p+q}{3} & & -I+\frac{Y}{2}+\frac{p+q}{3} & \\ & & I_3+\frac{Y}{2}+\frac{p+q}{3} & & \end{pmatrix} \\
&= \begin{pmatrix} p & & q & & 0 \\ & Q_{max}+\frac{p+q}{3} & & Q_{min}+\frac{p+q}{3} & \\ & & Q+\frac{p+q}{3} & & \end{pmatrix}
\end{aligned} \tag{34}$$

Thus the Gel'fand pattern directly involves the electric charges of a multiplet shifted by the value $\frac{p+q}{3}$. Interestingly this number $\frac{p+q}{3}$ is exactly of the form that the entries in the Gel'fand pattern are integers, like in the Sakaton charge eqn. (29).

So explicitly, the charge of the fundamental representation Sakaton [1,0], of the group $SU(3)_F$ is integral,

$$Q = I_3 + \frac{Y}{2} + \frac{p+q}{3} \tag{35}$$

As the octet and the decuptet representations have integral charges too, the Gel'fand pattern is able to accomodate both the integral charges in the Sakata model and the octet-decuptet, in a most remarkable manner as [14],

$$Q = I_3 + \frac{Y}{2} + \frac{1}{3}[(p+q)\ mod\ 3] \tag{36}$$

Most important to note that the last term is made up of product of two independent terms, viz $\frac{1}{3}$ and $(p+q)\ mod\ 3$. (Remember that these charges are for the group $SU(3)$ and not of the $\frac{SU(3)}{Z_3}$ sub-system). Note that the term

$[(p+q)\ mod\ 3]$ arises from the group $SU(3)$ itself, from within the canonical chain $SU(3) \supset SU(2) \times U(1) \supset SO(2)$ in the Gel'fand pattern.

Hence the first two terms would correspond to the Gell-Mann Nishijima charge for the octet [2,1] and the decuptet [3,0], only when in the last term, there exists dynamically a term $\frac{1}{3}$ outside the term, e.g. $(p+q)\ mod\ 3 = (2+1)\ mod\ 3 = 0$.

However this is in conflct with how the last term in eqn. (27), $(q_2 + \frac{1}{3})N_c B(U_c)$ was taken to zero by Balachandran et. al. [5, p. 210], on taking $q_2 = -\frac{1}{3}$. As we saw above, the charge $q_2 = -\frac{1}{3}$ is a pure $SU(2)_F$ result (eqn. (18)). Though it does sail through the quantization process in $SU(3)_F$, it is not a pure $SU(3)_F$ result per se. It is a kind of a memory of SU(2) sitting unchanged in SU(3). Hence their conclusion is erroneous. Thus the term left behind, $Q = T_3 + \frac{Y}{2}$, is not the octet Gell-Mann-Nishijima expression, but should be interpreted, say for neutron as,

$$Q(neutron) = -\frac{1}{2} + \frac{1}{2} = -\frac{1}{2} + (\frac{1}{6} + \frac{1}{3}) = (-\frac{1}{2} + \frac{1}{6}) + \frac{1}{3} \tag{37}$$

This is the charge of neutron in a Sakaton, as in eqn. (35). Thus the proton and neutron which arise here are part of a Sakaton and not that of the octet. Hence the proper interpretation of the $SU(3)_F$ Skyrmion is that it is not an octet, but a Sakaton. Therefore we are able to revive the Sakaton as a real physical entity. This arises on going beyond the quark model, and which demands a global topological interpretation as a Skyrmion. Hence a Sakaton should be taken as a basic physical entity; actually as much so as the quarks constituting a baryon..

How would this Sakaton help solve our current theoretical puzzles? Here we use it to solve a basic issue which arises as a result of a common physical application of the well known 't Hooft anomaly matching condition [15]

If a theory is weakly coupled when we are above a certain energy scale λ, 't Hooft showed [15] that then regardless of the strength of the interaction, anomaly must be present on both sides of λ. This allows one to identify the fermion sector of one's effective theory. So composites of fundamental entities in the chiral limit may match each other through the 't Hooft anomaly matching condition. This is possible if the sum of the anomaly coefficients A(r) for the composite fermions (below λ) is equal to that of fundamental fermions (above λ) [16]

$$\sum_r N_r A(r) = \sum_r n_r A(r) \tag{38}$$

(n_r are number of chiral fermions in representation r and N_r are number of massless composite fermions in representation r). Now (p,n) do form a massless chiral isospin-doublet. Thus the 't Hooft matching condition is indeed satisfied on going from $\binom{u}{d}$ to $\binom{p}{n}$. But conventionally this is rejected as [17, p. 325], "This could be merely a coincidence, though. Therefore let us not jump to conclusions." This is because the anomaly matching fails for the baryon octet $(p, n.\lambda, \Sigma^+, \Sigma^0, \Sigma^-, \Xi^0, \Xi^-)$.

However, for $\mathcal{Q} = \begin{pmatrix} u \\ d \\ s \end{pmatrix}$ to $\mathcal{S} = \begin{pmatrix} p \\ n \\ \Lambda \end{pmatrix}$ mapping works fine through the 't Hooft anomaly matching condition for the Sakatons. Hence our analysis in this paper, as to the Sakaton, sails through the scrutiny of the 't Hooft anomaly matching condition, and which therefore provides an unambiguous support to our conclusions.

Hence as one goes beyond the fractional charges of the three quarks in SU(3), then contrary to the Eightfold Way model asssertion [4,13], the integral charges of the octet is not the only option available, but on proper inclusion of the topological degrees of freedom, Sakaton turns out to be the correct path. Hence these Sakatons should manifest themsleves as real physical particles.

Physically as of now, one had assumed that hypernuclei reflect the presence of hyperons, arising in the spin 1/2 octet, in the nucleus. However, this picture is unable to explain as to why the hypernuclei observed experimentally upto now [18], are predominantly made up of Λ's only - fortyone have a single Λ present, three have two-Λ and only one has a Σ meson? Our model here shows that actually the hypernuclei are a manifestation of the presence of Sakatons in a nucleus. Hence it predicts that strangeness in nuclei should arise from the Sakatons. Thus the puzzling presence of only the Λ's in hypernuclei is actually a confirmation of our model.

Thus what we have shown is that Sakatons are physically as relevant as the quarks are in particle physics. But Sakatons are different from quarks in as much as they arise in the topological Skyrme model. Hence these are Skyrmions. Therefore the Skyrme model is not just one of the large number of phenomenological models of the hadrons, arising from some kind of an approximation of QCD. Our analysis here has shown that the Sakatons are as basic and as fundamental as the fractionally charged quarks are to QCD. Hence the prediction of a very heavy scalar meson which arises in the Topological Skyrme model [19], should be accorded a serious sonsideration as a genuine physical entity.

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