Topological Skyrme Model with Wess-Zumino Anomaly term and their Representations

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Abstract

Our model here, the Skyrme-Wess-Zumino model, is Skyrme lagrangian supplemented with the Wess-Zumino anomaly term. It is commonly believed that spin-half octet and spin three-half decuptet are the lowest dimensional representations that the three-flavour Skyrmions would correspond to. We study the effect of including the electric charges consistently in these analysis. We show that indeed this leads to significant improvement in our understanding of proper reprentations of two-flavour and three-flavour Skyrmionic representations.

Keywords: Topological Skyrme model, Wess-Zumino anomaly, Chiral symmetry, electric charge, Quark model, Sakaton, hypernuclei

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The topological Skyrme model of the 1960's [1] has been focus of much activity in recent years [2-12. The original Skyrme lagrangian needs to be supplemented with a Wess-Zumino anomaly term to ensure proper quantzation [2-6,10,11]. Our model here for three flavours shall be the original Skyrme lagrangian plus the Wess-Zumino anomaly term and which we call the Skyrme-Wess-Zumino model. As well known, Wess-Zumino anomaly term is non-vanishing for three flavours and vanishes for two flavours. In this paper we discuss the basic significance of this fact as to the octet and the decuptet representations obtained in the three flavour Skyrme-Wess-Zumino model. We shall study the electric charge for two-flavours and three-flavours and extract some interesting and useful new information about their role in identifying the proper representations in our Skyrme-Wess-Zumino model.

Given an element U of SU(2),

$$L_\mu = U^\dagger \partial_\mu U \tag{1}$$

the Skyrme Lagrangian is given as [2-6],

$$L_S = \frac{{f_\pi}^2}{4} Tr(L_\mu L^\mu) + \frac{1}{32e^2} Tr[L_\mu, L_\nu]^2 \tag{2}$$

Here the Skyrme topological current is,

$$W_\mu = \frac{1}{24\pi^2} \epsilon_{\mu\nu\alpha\beta} Tr[L_\nu L_\alpha L_\beta] \tag{3}$$

On most general grounds this topological current is conserved, i.e. $\partial^\mu W_\mu = 0$ and giving a conserved topological charge $q = \int W_0 d^3x$. This current is independent of any WZ term that shall added below.

Here $U(x)$ is an element of the group $SU(2)_F$,

$$U(x)^{SU(2)} = exp((i\tau^a \phi^a / f_\pi), \ \ (a = 1, 2, 3) \tag{4}$$

The solitonic structure present in the Lagrangian is obtained on making Skyrme ansatz as follows [2-6].

$$U_c(x)^{SU(2)} = exp((i/f_\pi \theta(r) \hat{r}^a \tau^a), \ \ (a = 1, 2, 3) \tag{5}$$

This $U_c(x)$ is called the Skyrmion. But on quantization, the two flavour model Skyrmion has a well known boson-fermion ambiguity [2-6]. This is rectified by going to three flavours. In that case we take,

$$U(x)^{SU(3)} = exp[\frac{i\lambda^a \phi^a(x)}{f_\pi}] \ \ (a = 1, 2..., 8) \tag{6}$$

with ϕ^a the pseudoscalar octet of π, K and η mesons. In the full topological Skyrme model this is supplemented with a Wess-Zumino (WZ) effective action [2-6]

$$\Gamma_{WZ} = \frac{-i}{240\pi^2} \int_\Sigma d^5x \epsilon^{\mu\nu\alpha\beta\gamma} Tr[L_\mu L_\nu L_\alpha L_\beta L_\gamma] \tag{7}$$

on surface Σ. Thus with this anomaly term, the effective action is.

$$S_{eff} = \frac{f_\pi^2}{4} \int d^4x \, Tr \, [L_\mu L^\mu] + n \, \Gamma_{WZ} \tag{8}$$

where the winding number n is an integer $n \in Z$, the homotopy group of mapping being $\Pi_5(SU(3)) = Z$.

Write effective action as,

$$S_{eff} = \frac{f_\pi^2}{4} \int d^4x \, Tr \, [\partial_\mu U \partial^\mu U^\dagger] + n \, \Gamma_{WZ} \tag{9}$$

Taking Q as charge operator, under a local electro-magnetic gauge transformation $h(x) = \exp(i\theta(x)Q)$ with small θ, one finds

$$\Gamma_{WZ} \rightarrow \Gamma_{WZ} - \int d^4x \partial_\mu x J^\mu(x) \tag{10}$$

whee J^μ is the Noether current arising from the WZ term. This coupling to the photon field is like,

$$J_\mu = \frac{1}{48\pi^2} \epsilon^{\mu\nu\alpha\beta} Tr[Q(L_\nu L_\alpha L_\beta - R_\nu R_\alpha R_\beta)] \tag{11}$$

where $L_\mu = U^\dagger \partial_\mu U$, $R_\mu = U \partial_\mu U^\dagger$. With the electromagnetic field A_μ present, the gauge invariant form of eqn. (8) is,

$$\hat{S_{eff}} = \frac{f_\pi^2}{4} \int d^4x \, Tr \, [L_\mu L^\mu] + n \, \hat{\Gamma_{WZ}} \tag{12}$$

This means that when replacing the LHS in eqn. (10) by $\hat{\Gamma_{WZ}}$, then the RHS has two new terms involving $F_{\mu\nu}F^{\mu\nu}$. This allows us to interpret J_μ with the current carried by quarks [2-6]. With the charge operator Q, J_μ is found to be isoscalar. To obtain the baryon current from eqn. (11), one replaces Q by $\frac{1}{N_c}$ (where N_c is the number of colours in $SU(N_c)$ - QCD for arbitrary number of colours), which is the baryon charge carried by each quark making up the baryon. For total antisymmetry, N_c number of quarks are needed to make up a baryon. Then $nJ_\mu \rightarrow J_\mu^B$ gives,

$$\begin{aligned} nJ_\mu^B(x) &= \frac{1}{48\pi^2} \left(\frac{n}{N_c}\right) \epsilon^{\mu\nu\alpha\beta} Tr[(L_\nu L_\alpha L_\beta - R_\nu R_\alpha R_\beta)] \\ &= \frac{1}{24\pi^2} \epsilon^{\mu\nu\alpha\beta} Tr[L_\nu L_\alpha L_\beta] \end{aligned} \tag{13}$$

This is the same as the topological current of Skyrme as given by eqn. (3). Thus the gauged WZ term gives rise to $J_\mu(x)$ which in turn gives the baryon charge. Thus though the WZ term Γ_{WZ} is zero for two-flavour case, but $J_\mu(x)$ still contibutes to the two-flavour case.

Next we embed the SU(2) Skyrme ansatz into $U(x)^{SU(3)}$ as follows for the SU(3) Skyrmion [10],

$$U_c(x)^{SU(2)} \; \rightarrow \; U_c(x)^{SU(3)} = \begin{pmatrix} U_c(x)^{SU(2)} & \\ & 1 \end{pmatrix} \tag{14}$$

Next we insert the identity,

$$U(\vec{r},t)^{SU(3)} = A(t) U(\vec{r})_c^{SU(3)} A^{-1}(t) \qquad A \in SU(3)_F \tag{15}$$

where A is the collective coordinate. Note that $U(\vec{r},t)$ is invariant under,

$$A \rightarrow A e^{iY\alpha(t)} \tag{16}$$

where

$$Y = \frac{1}{3} \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & -2 \end{pmatrix} \tag{17}$$

With this the quantum degrees of freedom manifest themselves in the WZ term (eqn. (7)) as,

$$L_{WZ} = -\frac{1}{2} N_c B(U_c) tr(Y A^{-1} A) \tag{18}$$

where $B(U_c)$ is the baryon number (winding number) of the classical configuration U_c. The gauge invariance leads to changing L_{WZ} to

$$L_{WZ} \rightarrow L_{WZ} + \frac{1}{3} N_c B(U_c) \dot{\alpha} \tag{19}$$

In the quantized theory A and Y are operators and from Noether's theorem one obtains (with Ψ as allowed quantum state)

$$\hat{Y}\Psi = \frac{1}{3} N_c B \Psi \tag{20}$$

This gives the right-hypercharge,

$$Y_R = \frac{1}{3} N_c B \tag{21}$$

where the baryon number B and colour N_c are integers [2-6,10].

It is important to remember that this right-hypercharge eqn (21) was dictated by having defined SU(2) embedding in SU(3) as given in eqn. (14) [2,3]. With B = 1 and $N_c = 3$ one gets $Y_R = 1$. This identifies the nucleon hypercharge with the body-fixed hypercharge Y_R. Ultimately one obtains a tower of irreducible represenatations: (8,1/2), (10,3/2), 10,1/2), 27,3/2), ... of which the lowest octet and decuptet are identified with the observed low energy baryons [2-6,10,11]. Hence we get all the low dimensional fermions as in the quark model.

We now study the significance of the above fact, that the Wess-Zumino term provides only isoscalar electric charge. Hence let us next look at the structure of the electric charge in the $SU(2)_F$ SWZ model. It has been pointed out by

Balachandran et. al. [10, p. 176] that this has not been paid the attention it deserves. This because as we show below, it presents a serious challenge to the Skyrme lagrangian for two flavours. Following Balachandran et. al. [10], we define the electric charge operator in SU(2) as,

$$Q = \begin{pmatrix} q_1 & 0 \\ 0 & q_2 \end{pmatrix} \tag{22}$$

It induces the following transformation,

$$U(x) \to e^{i\epsilon_0 \Lambda Q} U(x) e^{-i\epsilon_0 \Lambda Q} = e^{\frac{i\epsilon_0 \Lambda \tau_3 (q_1 - q_2)}{2}} U(x) e^{\frac{-i\epsilon_0 \Lambda \tau_3 (q_1 - q_2)}{2}} \tag{23}$$

where ϵ_0 is the electromagnetic coupling constant. The Noether current associated with the above symmetry is,

$$\frac{J_\mu^{em}}{\epsilon_0} = \frac{iF_\pi^2}{8} Tr\ L_\mu (Q - U^\dagger Q U) - \frac{i}{8\epsilon_0^2} Tr\ [L_\nu, Q - U^\dagger Q U][L_\mu, L_\nu] \tag{24}$$

We obtain the gauge theory by replacing

$$\partial_\mu U \to D_\mu U = \partial_\mu U - i\epsilon_0 \Lambda_\mu [Q, U] \tag{25}$$

To obtain constraints on charges in eqn. (22), first expand on pion fields as,

$$J_\mu^{em} = -i\epsilon_0 (q_1 - q_2)(\pi_- \partial_\mu \pi_+ - \pi_+ \partial_\mu \pi_-) + ... \tag{26}$$

From pion charges one gets

$$(q_1 - q_2) = 1 \tag{27}$$

Next the charges of baryons $N\ and\ \Delta$ with B=1 charge on using eqn. (15),

$$Q = \int d^4x\ J_0^{em}(\vec{x}, t) = \epsilon_0 L_\alpha\ Tr \tau_\alpha Q \tag{28}$$

From eqn. (22) we get,

$$Q = \epsilon_0 (q_1 - q_2) L_3 \tag{29}$$

On using eqn. (27),

$$Q = \epsilon_0 L_3 \tag{30}$$

L_3 is the third component of the isospiun operator, we get (in units of ϵ_0),

$$Q(proton) = +\frac{1}{2}\ \ and\ \ Q(neutron) = -\frac{1}{2} \tag{31}$$

This is in disagreement with experiment. Thus the Skyrme Lagrangian eqn. (2) fails to provide correct electric charges to proton and neutron. As such this should be construed to mean that just the Skyrme lagrangian in itself, is not enough to give consistent description of the B=1 nucleon. However note that the Skyrme lagrangian provides pure isovector charges of proton and neutron.

Thus as electric charge of proton and neutron are more than what is provided above, it needs another term to pull it out of this conundrum. And indeed we have the additional WZ term to do the job. Again let the field U be transformed by an electric charge operator Q as, $U(x) \rightarrow e^{i\Lambda\epsilon_0 Q} U(x) e^{-i\Lambda\epsilon_0 Q}$,

Making $\Lambda = \Lambda(x)$ a local transformation the Noether current is [10]

$$J_\mu{}^{em}(x) = j_\mu{}^{em}(x) + j_\mu{}^{WZ}(x) \tag{32}$$

where the first one is the standard Skyrme term and the second is the Wess-Zumino term

$$j_\mu{}^{WZ}(x) = \frac{\epsilon_0 N_c}{48\pi^2} \epsilon_{\mu\nu\lambda\sigma} Tr V^\nu V^\lambda V^\sigma (Q + U^\dagger Q U) \tag{33}$$

Remember that even though the WZ term vanishes for two flavours, its resulting contribution to electric chaege does not. This term was of course missing in the original version of the Skyrme Lagrangian (eqn. (2)).

One finally obtains [10, p. 208],

$$j_\mu{}^{WZ}(x) = \frac{\epsilon_0}{2}(q_1 + q_2) N_c J_\mu(x) \tag{34}$$

The WZ term correction to the electric charge is therefore,

$$\frac{\epsilon_0}{2}(q_1 + q_2) N_c \int J_0(x) d^3x \tag{35}$$

Using eqn. (15) above,

$$\frac{\epsilon_0}{2}(q_1 + q_2) N_c B(U_c) \tag{36}$$

Remember the right hypercharge $Y_R = 1$ in eqn. (21) and subsequently B=1 for $N_c = 3$. Note also the baryon in the Skyrme model with B=1 now as per eqn. (13) has three quarks. We thus obtain the charges of $N \ and \ \Delta$ if we put,

$$q_1 + q_2 = \frac{1}{3} \tag{37}$$

Along with eqn. (27), we obtain the charges as,

$$q_1 = \frac{2}{3}\ , \quad q_2 = -\frac{1}{3} \tag{38}$$

It is amazing that we are getting the fractional quark charges in the $SU(2)_F$ group itself. These fractional charges are those of u- and d- quarks which form fundamenatal representation of the group SU(2). This is opposite to what happens in the $SU(3)_F$ quark model. In the quark model, in the smaller SU(2)-isospin group, one gets no fractional charges, and one has integral charges for nucleon N = $\binom{p}{n}$. One has to go to higher group $SU(3)_F$, to be able to get fractional charges for the quarks in the quark model. This is a major difference between the quark model and the topological Skyrme-Wess-Zumino model here.

However most important to note that for the SU(2) case, the Skyrme Laganguian (without the Wess-Zumino term) gave us pure isovector charges for proton and neutron (as clear from eqns. (27) and (31)). And next ,the Wess-Zumino term, as clear from eqn. (37), gave us pure isoscalar charge. Then the correct quark charges (as in eqn. (38) are obtained only after including both the original Skyrme lagrangian plus the Wess-Zumino anomaly term.

Rememeber above in eqn. (11) and (13) the baryon number B was related to charge Q because as stated there, the electric charge in the Wess-Zumino term was pure isoscalar. This is what we have found for two flavours as above.

Next as $Q_p = q_1 + q_1 + q_2$; $Q_n = q_1 + q_2 + q_2$, hence necessarily due to eqns. (27) and (37) one has the main result for nucleon charge in Skyrme-Wess-Zumino model:

$$Q_p - Q_n = 1 \;\; (isovector); \quad Q_p + Q_n = 1 \;\; (isoscalar) \tag{39}$$

Again note that here no Gell-Mann-Nishijima expression (of quark model) for electric charge of proton and neutron, but quantized isovector and isoscalar charges of the nucleon. From these skyrmions, using Z=1 for proton and N=1 or neutron the charges are

$$\begin{aligned} Q(p) &= \left(\frac{Z=1}{2}\right)_{isovector} + \left(\frac{Z=1}{2}\right)_{isoscalar} \\ Q(n) &= -\left(\frac{N=1}{2}\right)_{isovector} + \left(\frac{N=1}{2}\right)_{isoscalar} \end{aligned} \tag{40}$$

Hence as per these skyrmions, this model gives right away the charge of a nucleus for arbitrary number of Z protons and N neutrons as,

$$Q = \frac{Z-N}{2} + \frac{Z+N}{2} \quad = \quad T_3 + \frac{A}{2} \tag{41}$$

This well known charge of the nucleus is obtained here, as nucleus is treated as made up of Z-protonic skyrmions and N-neutronic skyrmions. Note that we have obtained the fundamental nuclear charge equation (eqn. (41)), directly in terms of the atomic mass number A [12], as a direct and basic result of the Skyrmion in the Skyrme-Wess-Zumino model. To belabour the point, this cannot be done for pure Skyrme model without the addition of the Wess-Zumino anomaly term. This is thus the proper representation of the Nucleon in the nucleus as per the SWZ model.

Next, in going to $SU(3)_F$, let the field U in eqn. (6) be transformed by an electric charge operator Q as,

$$U(x) \rightarrow e^{i\Lambda Q} U(x) e^{-i\Lambda Q} \tag{42}$$

where the charges are counted in units of the absolute value of the electronic charge.

Making $\Lambda = \Lambda(x)$ a local transformation, the Noether current is [10]

$$J_\mu{}^{em}(x) = j_\mu{}^{em}(x) + j_\mu{}^{WZ}(x) \tag{43}$$

where the first one is the standard Skyrme term and the second is the Wess-Zumino term

$$j_\mu{}^{WZ}(x) = \frac{N_c}{48\pi^2}\epsilon_{\mu\nu\lambda\sigma} Tr L^\nu L^\lambda L^\sigma (Q + U^\dagger Q U) \tag{44}$$

In the standard way [10], we take the U(1) of electromagnetism as a subgroup of the three flavour SU(3). Its generators can be found by the canonical methods. As the charge operator can be simultaneously diagonalized along with the third component of isospin and hypercharge, we write it as

$$Q = \begin{pmatrix} q_1 & 0 & 0 \\ 0 & q_2 & 0 \\ 0 & 0 & q_3 \end{pmatrix} \tag{45}$$

The electric charge of pseudoscalar octet mesons demand,

$$q_1 - q_2 = 1, \quad q_2 = q_3 \tag{46}$$

Hence one obtains

$$Q = (q_2 + \frac{1}{3})\mathbf{1}_{3x3} + \frac{1}{2}\lambda_3 + \frac{1}{2\sqrt{3}}\lambda_8 \tag{47}$$

Now we use $U = A(t)U_c(\mathbf{x})A(t)^{-1}$ where A is the collective coordinate. We obtain the B=1 electric charge from the Skyrme term in terms of the left-handed generators L_α only as

$$Q^{em} = \frac{1}{2}(L_3 - (A^\dagger\lambda_3 A)_8 \frac{N_c B(U_c)}{\sqrt{3}}) + \frac{1}{2\sqrt{3}}(L_8 - (A^\dagger\lambda_8 A)_8 \frac{N_c B(U_c)}{\sqrt{3}}) \tag{48}$$

The Wess-Zumino term contributes

$$Q^{WZ} = N_c B(U_c)(q_2 + \frac{1}{3} + \frac{1}{2\sqrt{3}}(A^\dagger\lambda_3 A)_8 + \frac{1}{6}(A^\dagger\lambda_3 A)_8) \tag{49}$$

Hence the total electric charge is [10]

$$Q = \frac{1}{2}L_3 + \frac{1}{2\sqrt{3}}L_8 + (q_2 + \frac{1}{3})N_c B(U_c) \tag{50}$$

The last term vanishes once we take the down quark charge $q_2 = -\frac{1}{3}$ [10, p. 210] and one is left with the Gell-Mann-Nishijima expression of charge as

$$Q = t_3 + \frac{Y}{2} \tag{51}$$

This gives the electric charges of all the members of the baryon octet.

But this is precisely what we do not want! As we saw above, the SU(2) nucleon charges are given in the Skyrme-Wess-Zumino model as isoscalar and isovector charges. Those are the properly quantized charges. The SU(3) representation which should carry along these nucleon charges - demands non-compliance with Gell-Mann-Nishijima electric charges and should be consistent with separately quantized isovector and isoscalar charges. Out of all the members of the octet representation, the only one which has this property is Λ. Thus the SU(3) skyrmion is not octet or decuptet, or any member of the infinite ladder, but the spin half fermion $\mathcal{S} = \begin{pmatrix} p \\ n \\ \Lambda \end{pmatrix}$ This is the long-ago discarded Sakaton of SU(3) [13,14].

Note that Sakata [13] had extended the group $SU(2)_I$ to $SU(2)_I \times U(1)_Y$, and had taken Λ as a representation of the U(1) group. Thus it was natural to take $\mathcal{S} = \begin{pmatrix} p \\ n \\ \Lambda \end{pmatrix}$ as the fundamental representation of a larger $SU(3)_F$ group [14]. It is called Sakaton in anology with Nucleon of the isospin group. Note that the charges in Sakaton are all integral: 1,0,0 respectively. The Sakata Model predicted the mesons correctly as composites: $3 \times \bar{3} = 1 + 8$. However it failed to describe the baryons as $3 \times 3 \times \bar{3} = 3 + 3 + 6 + 15$. Also as both $p \ and \ \Lambda$ are neutral members of the fundamental triplet in Sakata model, they should have the same magnetic moment, $\mu_\Lambda = \mu_n$. This fails to match the exapriment where, $\mu_\Lambda = -0.613 \ and \ \mu_n = -1.913$ in units of $\frac{e\hbar}{2m_p c}$, where mass is that of proton. Thus the fundamental triplet Sakaton was rejected.

However in our Skyrme-Wess-Zumino model with minimal symmetry breaking [2,5], the masses are: $m_s = m_0$, and $(m_u = m_d) = m_o + aY$. With 'a' as negative in magnitute, $m_s > (m_u = m_d)$. Hence magnetic moments of our skyrmionic Sakaton, $\mathcal{S} = \begin{pmatrix} p \\ n \\ \Lambda \end{pmatrix}$ are succssfully obtained as [7],

$$\begin{array}{ccc} Baryons & SWZ\ model & experiment \\ p & \frac{(4\mu_u - \mu_d)}{3} & 2.793 \\ n & \frac{(4\mu_d - \mu_u)}{3} & -1.913 \\ \Lambda & \mu_s & -0.614 \end{array} \tag{52}$$

Physically as of now, one had assumed that hypernuclei reflect the presence of hyperons, arising in the spin 1/2 octet, in the nucleus. However, this picture is unable to explain as to why the hypernuclei observed experimentally upto now [15], are predominantly made up of Λ's only - fortyone have a single Λ present, three have two-Λ and only one has a Σ meson? Our model here shows that actually the hypernuclei are a manifestation of the presence of Sakatons in a nucleus. Hence it predicts that strangeness in nuclei should arise from the Sakatons. Thus the puzzling presence of only the Λ's in hypernuclei is actually a confirmation of our model.

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