Methods for Derivation of Generalized Equations in the $(S,0)\oplus(0,S)$ Representations of the Lorentz Group[1]

Valeriy V. Dvoeglazov

UAF, Universidad Autónoma de Zacatecas
Apartado Postal 636, Suc. 3, Zacatecas 98061 Zac., México
E-mail: valeri@fisica.uaz.edu.mx

Abstract. We continue the discussion of several explicit examples of generalizations in relativistic quantum mechanics. We discussed the generalized spin-1/2 equations for neutrinos and the spin-1 equations for photon. The equations obtained by means of the Gersten-Sakurai method and those of Weinberg for spin-1 particles have been mentioned. Thus, we generalized the Maxwell and Weyl equations. Particularly, we found connections of the well-known solutions and the dark 4-spinors in the Ahluwalia-Grumiller elko model. They are also not the eigenstates of the chirality and helicity. The equations may lead to the dynamics which are different from those accepted at the present time. For instance, the photon may have non-transverse components and the neutrino may be *not* in the energy states and in the chirality states. The second-order equations have been considered too. They have been obtained by the Ryder method.

1. Introduction. The Gersten Method and its Relation to Relativistic Quantum Equations.

The content of this talk is the following. We use the van der Waerden-Sakurai procedure for derivation of the Dirac equation and its generalizations. As a consequence, for example, the scalar fields appear in the Maxwell-like equations. Next, we have massless particles ($p^2=0$) with massive parameters, which may be considered as the measures of chirality. We also discuss the massive/massless 2nd order equations.

The first part is based on recent papers [1, 2, 3]. Gersten [1a] writes: "We have shown how all Maxwell equations can be derived simultaneously from first principles, similar to those which have been used to derive the Dirac relativistic electron equation" and concludes: "…Maxwell equations should be used as a guideline for proper interpretation of quantum theories".

In fact, he used a method presented by van der Waerden and Sakurai [4]. Let us begin with the Klein-Gordon equation:

$$(E^2-c^2\mathbf{p}^2-m^2c^4)\Psi_{(2)}=0\,. \tag{1}$$

[1] Some parts of this paper have been presented at the XI Escuela de DGFM SMF, Dec. 5-9, 2016, Playa del Carmen, QRoo, México, the IARD2018, June 4-7, 2018, Mérida, Yuc., México and the MG15 Meeting, July 1-7, Rome, Italy. To be published in the Proceedings of the 11th Vigier Symposium, Aug. 6-9, 2018. Liege, Belgium.

Hence, for Ψ with *two* components and $c = \hbar = 1$ we have

$$(EI^{(2)} - \boldsymbol{\sigma}\cdot\mathbf{p})(EI^{(2)} + \boldsymbol{\sigma}\cdot\mathbf{p})\Psi_{(2)} = m^2\Psi_{(2)}\,. \tag{2}$$

Let us denote $\Psi_{(2)} = \eta$, then we have

$$\begin{aligned} \frac{(EI^{(2)} + \boldsymbol{\sigma}\cdot\mathbf{p})}{m}\eta &= \chi\,, & (3)\\ (EI^{(2)} - \boldsymbol{\sigma}\cdot\mathbf{p})\chi &= m\eta\,. & (4)\end{aligned}$$

In the 4-component form one has

$$\begin{pmatrix} -mI^{(2)} & EI^{(2)} + \boldsymbol{\sigma}\cdot\mathbf{p} \\ EI^{(2)} - \boldsymbol{\sigma}\cdot\mathbf{p} & -mI^{(2)} \end{pmatrix}\begin{pmatrix}\chi\\ \eta\end{pmatrix} = 0 \tag{5}$$

With the quantum-operator substitutions $E \to i\hbar\frac{\partial}{\partial t}$ and $\mathbf{p} \to -i\hbar\boldsymbol{\nabla}$ we recover the Dirac equation![2]

In the $S = 1$ and $m = 0$ case one can proceed in a similar way:

$$\begin{aligned}\left(\frac{E^2}{c^2} - \mathbf{p}^2\right)\boldsymbol{\Psi}_{(3)} &= \left(\frac{E}{c}I^{(3)} - \mathbf{S}\cdot\mathbf{p}\right)\left(\frac{E}{c}I^{(3)} + \mathbf{S}\cdot\mathbf{p}\right)\boldsymbol{\Psi}_{(3)} - & (6)\\ &- \begin{pmatrix}p_x\\ p_y\\ p_z\end{pmatrix}\left(\mathbf{p}\cdot\boldsymbol{\Psi}_{(3)}\right) = 0\,, \quad \text{Eq. (9) of [1a]}\,,\end{aligned}$$

where $(\mathbf{S}^i)^{jk} = -i\epsilon^{ijk}$.[3] Gersten found that solutions are defined by

$$\begin{aligned}&\left(\frac{E}{c}I^{(3)} + \mathbf{S}\cdot\mathbf{p}\right)\boldsymbol{\Psi}_{(3)} = 0 \qquad \text{Eq. (10) of [1a]}\\ &\left(\mathbf{p}\cdot\boldsymbol{\Psi}_{(3)}\right) = 0 \qquad \text{Eq. (11) of [1a]}\end{aligned}$$

and their complex conjugates. The latter may be interpreted as the solutions of opposite helicity. If one assumes $\boldsymbol{\Psi}_{(3)} = \mathbf{E} - i\mathbf{B}$ then after quantum operator substitutions:

$$\begin{aligned}&\frac{i\hbar}{c}\frac{\partial\boldsymbol{\Psi}_{(3)}}{\partial t} = -\hbar\boldsymbol{\nabla}\times\boldsymbol{\Psi}_{(3)}\,, & (7)\\ &-i\hbar\boldsymbol{\nabla}\cdot\boldsymbol{\Psi}_{(3)} = 0\,, & (8)\end{aligned}$$

or

$$\begin{aligned}&\boldsymbol{\nabla}\times(\mathbf{E} - i\mathbf{B}) = -\frac{i}{c}\frac{\partial(\mathbf{E} - i\mathbf{B})}{\partial t}\,, & (9)\\ &\boldsymbol{\nabla}\cdot(\mathbf{E} - i\mathbf{B}) = 0\,. & (10)\end{aligned}$$

[2] One can also decompose (1) into the 4-component form from the beginning

$$(EI^{(4)} + \boldsymbol{\alpha}\cdot\mathbf{p} + m\beta)(EI^{(4)} - \boldsymbol{\alpha}\cdot\mathbf{p} - m\beta)\Psi_{(4)} = 0$$

and then look for $\boldsymbol{\alpha}$ and β.

[3] Please note the difference $(\boldsymbol{\sigma}\cdot\mathbf{p})^2 = \mathbf{p}^2$, but $(\mathbf{S}\cdot\mathbf{p})^3 = (\mathbf{S}\cdot\mathbf{p})$; the spin-1 matrices are *singular*.

Please note that the Planck constant was cancelled out! The procedure of separation into real and imaginary parts leads to

$$\boldsymbol{\nabla}\times\vec{\mathbf{E}} = -\frac{1}{c}\frac{\partial\vec{\mathbf{B}}}{\partial t}\,, \tag{11}$$

$$\boldsymbol{\nabla}\times\vec{\mathbf{B}} = \frac{1}{c}\frac{\partial\vec{\mathbf{E}}}{\partial t}\,, \tag{12}$$

$$\boldsymbol{\nabla}\cdot\vec{\mathbf{E}} = 0\,, \tag{13}$$

$$\boldsymbol{\nabla}\cdot\vec{\mathbf{B}} = 0\,. \tag{14}$$

The situation is the same if one starts with the complex conjugate function $\boldsymbol{\Psi}^*_{(3)} = \mathbf{E} + i\mathbf{B}$.

The Lagrangian for this theory has recently been presented in [1, 5]:

$$\mathcal{L} = -c\boldsymbol{\Psi}^\dagger_{(3)}(\frac{E}{c}I^{(3)} + \mathbf{S}\cdot\mathbf{p})\ \boldsymbol{\Psi}_{(3)}\,. \tag{15}$$

However, it was shown in [6] that such a form of Lagrangians is *not* a scalar (it is a zero component of a 4-vector; see also [7]). Next, we have

$$W_\mu W^\mu = -s(s+1)p_\mu p^\mu I^{(s)} \tag{16}$$

and

$$W^\mu p_\mu = 0 \tag{17}$$

by definition. Hence,

$$(W_\mu - sp_\mu)(W^\mu + sp^\mu) = -s(2s+1)p_\mu p^\mu I^{(s)}\,, \tag{18}$$

which is similar to Eq. (9) of Ref. [1a] if one imposes $p_\mu p^\mu = m^2 = 0$.

Therefore, on using the definition of the Pauli-Lubanski operator we have:

$$(\mathbf{S}\cdot\mathbf{p} - sp_0)\boldsymbol{\Psi}_{(s)} = 0\,, \tag{19}$$

$$(\mathbf{S}p_0 + i\mathbf{S}\times\mathbf{p} - s\mathbf{p})\boldsymbol{\Psi}_{(s)} = 0\,. \tag{20}$$

It is easy to see that in the $S = 1$ case one can recover the previous consideration [1b]. The second equation can be considered as a subsidiary condition

$$\begin{pmatrix} 0 & 0 & 0 \\ p_x & p_y & p_z \\ 0 & 0 & 0 \end{pmatrix}\boldsymbol{\Psi}_{(3)} = 0\,. \tag{21}$$

In [2] we corrected the Gersten's claim. The equation (9) of Ref. [1a] (the equation (6) above) is satisfied also under the choice

$$\left(\frac{E}{c}I^{(3)} + \mathbf{S}\cdot\mathbf{p}\right)\boldsymbol{\Psi}_{(3)} = \mathbf{p}\chi\,, \tag{22}$$

$$\left(\mathbf{p}\cdot\boldsymbol{\Psi}_{(3)}\right) = \frac{E}{c}\chi\,, \tag{23}$$

due to $(\mathbf{S}\cdot\mathbf{p})\mathbf{p}=0$. The above set leads to

$$\boldsymbol{\nabla}\times\mathbf{E}=-\frac{1}{c}\frac{\partial\mathbf{B}}{\partial t}+\boldsymbol{\nabla}Im\chi\,, \tag{24}$$

$$\boldsymbol{\nabla}\times\mathbf{B}=\frac{1}{c}\frac{\partial\mathbf{E}}{\partial t}+\boldsymbol{\nabla}Re\chi\,, \tag{25}$$

$$\boldsymbol{\nabla}\cdot\mathbf{E}=-\frac{1}{c}\frac{\partial}{\partial t}Re\chi\,, \tag{26}$$

$$\boldsymbol{\nabla}\cdot\mathbf{B}=\frac{1}{c}\frac{\partial}{\partial t}Im\chi\,, \tag{27}$$

with an additional scalar field χ. It is also possible that

$$\left(\frac{E}{c}I^{(3)}-\mathbf{S}\cdot\mathbf{p}\right)\boldsymbol{\Psi}'_{(3)}=\mathbf{p}\chi' \tag{28}$$

$$\left(\mathbf{p}\cdot\boldsymbol{\Psi}'_{(3)}\right)=\frac{E}{c}\chi'\,. \tag{29}$$

If $\chi'=\chi^*$ then $\boldsymbol{\Psi}'_{(3)}=\mathbf{E}+i\mathbf{B}$ and we recover the generalized Maxwell equations (24-27). Kruglov found relations of $\chi-$ functions with QED and the Riemann tensor [8].

Thus,

- We obtain $\mathbf{p}\cdot\boldsymbol{\Psi}_{(3)}\neq 0$, therefore the free photon may have a non-transverse component [9];
- The χ-fields may be function(al) of higher-rank tensor fields, thus leading to equations which are non-linear in $\mathbf{E}$ and $\mathbf{B}$ (cf. Ref. [10]);
- One can find possible relations to the Ogievetskiĭ-Polubarinov-Kalb-Ramond field [11, 12, 13]. After performing the Bargmann-Wigner procedure for the spin-1 field we obtain

$$\partial_\alpha F^{\alpha\mu}+\frac{m}{2}A^\mu=0\,, \tag{30}$$

$$2mF^{\mu\nu}=\partial^\mu A^\nu-\partial^\nu A^\mu\,, \tag{31}$$

instead of the well-known Proca set:

$$\partial_\alpha F^{\alpha\mu}+m^2A^\mu=0\,, \tag{32}$$

$$F^{\mu\nu}=\partial^\mu A^\nu-\partial^\nu A^\mu\,, \tag{33}$$

In fact, these sets are related one to another by the re-normalization transformation: $A^\mu\rightarrow 2mA^\mu$ or $F^{\mu\nu}\rightarrow\frac{1}{2m}F^{\mu\nu}$. Ogievetskiĭ and Polubarinov [11] wrote: "In the massless limit the system of $2s+1$ states is no longer irreducible; it decomposes and describes a set of different particles with zero mass and helicities $\pm s,\pm(s-1),\pm 1,0$ (for integer spin and if parity is conserved); the situation is analogous for half-integer spins." One can also see this after performing the Lorentz transformation in the $(1/2,1/2)$ representation, $A^\mu=\Lambda^\mu\;{}_\nu A^\nu_{basis}$, and looking for massless limit. Thus, we find

$$u^\mu(\mathbf{p},+1)=-\frac{N}{\sqrt{2}m}\begin{pmatrix}-p_r\\ m+\frac{p_xp_r}{E_p+m}\\ im+\frac{p_yp_r}{E_p+m}\\ \frac{p_zp_r}{E_p+m}\end{pmatrix}\,,\quad u^\mu(\mathbf{p},-1)=\frac{N}{\sqrt{2}m}\begin{pmatrix}-p_l\\ m+\frac{p_xp_l}{E_p+m}\\ -im+\frac{p_yp_l}{E_p+m}\\ \frac{p_zp_l}{E_p+m}\end{pmatrix}\,, \tag{34}$$

and

$$u^\mu(\mathbf{p},0)=\frac{N}{m}\begin{pmatrix}-p_z\\ \frac{p_xp_z}{E_p+m}\\ \frac{p_yp_z}{E_p+m}\\ m+\frac{p_z^2}{E_p+m}\end{pmatrix}\,,\quad u^\mu(\mathbf{p},0_t)=\frac{N}{m}\begin{pmatrix}E_p\\ -p_x\\ -p_y\\ -p_z\end{pmatrix}\,. \tag{35}$$

Please note that for helicities $\sigma = \pm 1, 0$ one has $p_\mu u^\mu(\mathbf{p}, \sigma) = 0$ (an analogue of the Lorentz condition [13]). This is not the case for the "time-like" photons. In view of the fact that in the case $N = 1$ we have divergent behaviour of certain parts of the 4-vector momentum-space functions in $m \to 0$, the first degree of m in the equations (30,31) can cancel this divergent term in the denominators. The massless limits of the Proca-like equations are actually

$$\partial_\alpha F^{\alpha\mu} = -\frac{m}{2} A^\mu \Longrightarrow \partial_\alpha F^{\alpha\mu} = \partial^\mu \chi \,. \tag{36}$$

I want to present some comments:

- Of course, when we pass over to the second quantization, the commutation relations for $F^{\mu\nu}$ and A^μ may be changed in order to keep the correct dimensions of the fields and in order the action to be *dimensionless*.
- Ogievetskiĭ and Polubarinov, Kalb and Ramond [11, 12] analized the scalar Lagrangian of the antisymmetric tensor field [6, 13, 14] and "gauge out" the *transverse* components by means of

$$F_{\mu\nu} \to F_{\mu\nu} + \partial_\nu \Lambda_\mu - \partial_\mu \Lambda_\nu \,, \tag{37}$$

  the new "gauge" transformation. Therefore, they obtained a *pure* longitudinal field, the *notoph* (or, the *Kalb-Ramond field*, as it is frequently called in the US literature).
- In [13, 14] we found a map between the Ogievetskiĭ-Polubarinov formulation and the Weinberg $2(2s+1)$ theory [15]. In the latter case the Lagrangian is given by

$$\mathcal{L} = \partial_\mu \overline{\Psi}_{(6)} \gamma^{\mu\nu} \partial_\nu \Psi_{(6)} \pm m^2 \overline{\Psi}_{(6)} \Psi_{(6)} \,, \tag{38}$$

  (or its analogues for fields of different dimensions). The $\gamma^{\mu\nu}$ is a set of covariant matrices of the $(1,0) \oplus (0,1)$ representation; $\Psi_{(6)}$ and $\overline{\Psi}_{(6)}$ are bivectors. In general, various Ψ can be used, which differ each other by discrete symmetry transformations. The map exists between the equations obtained from (38)

$$[\gamma^{\mu\nu} \partial_\mu \partial_\nu \mp m^2] \Psi_{(6)} = 0 \tag{39}$$

  and the equation

$$\partial_\mu \partial^\alpha F_\alpha^{\ \ \nu} - \partial^\nu \partial^\alpha F_{\alpha\mu} - \frac{1}{2}(m^2 + \partial_\lambda \partial^\lambda) F_{\mu\nu} = -m^2 F_\mu^{\ \ \nu} \tag{40}$$

  and its dual. See also [16].

Next, we return to the van der Waerden-Sakurai derivation of the Dirac equation:

$$(E^2 - c^2 \vec{\mathbf{p}}^{\,2}) I^{(2)} \Psi_{(2)} = \left[E I^{(2)} - c\boldsymbol{\sigma} \cdot \mathbf{p}\right] \left[E I^{(2)} + c\boldsymbol{\sigma} \cdot \mathbf{p}\right] \Psi_{(2)} = m_2^2 c^4 \Psi_{(2)} \,. \tag{41}$$

If one denotes $\Psi_{(2)} = \eta$ one can define $\chi = \frac{1}{m_1 c}(i\hbar \frac{\partial}{\partial x_0} - i\hbar \boldsymbol{\sigma} \cdot \boldsymbol{\nabla})\eta$. Please note that we introduced the second mass parameter m_1. The corresponding set of 2-component equations is

$$(i\hbar \frac{\partial}{\partial x_0} - i\hbar \boldsymbol{\sigma} \cdot \boldsymbol{\nabla})\eta = m_1 c \chi \,, \tag{42}$$

$$(i\hbar \frac{\partial}{\partial x_0} + i\hbar \boldsymbol{\sigma} \cdot \boldsymbol{\nabla})\chi = \frac{m_2^2 c}{m_1} \eta \,. \tag{43}$$

In the 4-component form we have

$$\begin{pmatrix} i\hbar(\partial/\partial x_0) & i\hbar\boldsymbol{\sigma}\cdot\boldsymbol{\nabla} \\ -i\hbar\boldsymbol{\sigma}\cdot\boldsymbol{\nabla} & -i\hbar(\partial/\partial x_0)\end{pmatrix}\begin{pmatrix}\chi+\eta\\ \chi-\eta\end{pmatrix} = \tag{44}$$

$$= \frac{c}{2}\begin{pmatrix} (m_2^2/m_1+m_1) & (-m_2^2/m_1+m_1) \\ (-m_2^2/m_1+m_1) & (m_2^2/m_1+m_1)\end{pmatrix}\begin{pmatrix}\chi+\eta\\ \chi-\eta\end{pmatrix}, \tag{45}$$

which results in

$$\left[i\hbar\gamma^\mu\partial_\mu - \frac{m_2^2 c}{m_1}\frac{1-\gamma_5}{2} - m_1 c\frac{1+\gamma_5}{2}\right]\Psi_{(4)} = 0\,. \tag{46}$$

The "new" massless equation is ($m_2\rightarrow 0$, $p^2=0$, $m_1\neq 0$)

$$\left[i\gamma^\mu\partial_\mu - \frac{m_1 c}{\hbar}\frac{1+\gamma_5}{2}\right]\Psi_{(4)} = 0\,. \tag{47}$$

It is easy to check that dispersion relations are $E=\pm|\mathbf{p}|$, that give us rights to call it *massless*, even though there is a "mass" parameter in (47).

In the 2-component formalism we do not know the parity properties of the 2-spinors $\Psi_{(2)}$. It is possible to obtain yet another equation differing from (47) by the sign at the γ_5 term. The equation is

$$[i\gamma^\mu\partial_\mu - \frac{m_3 c}{\hbar}\frac{1-\gamma_5}{2}]\Psi^\prime_{(4)} = 0\,. \tag{48}$$

Moreover, instead of $(1\pm\gamma^5)/2$ one can use any singular 4×4 matrix of appropriate physical dimension and still have massless particles. The relevant equations can be found in the old [17] and new literature [18, 19] literature.

Is the physical content of the generalized $S=1/2$ *massless* equations the same as that of the Weyl equation? Our answer is 'No'. The excellent discussion can be found in [17a,b]. First of all, the theory does *not* have chiral invariance. Those authors call the additional parameters as measures of the degree of chirality. Apart of this, Tokuoka introduced the concept of the gauge transformations (not to confuse with phase transformations) for the 4-spinor fields. He also found some strange properties of the anti-commutation relations (see §3 in [17a]). And finally, the equation describes *four* states, two of which answer for the positive energy $E=|\mathbf{p}|$, and two others answer for the negative energy $E=-|\mathbf{p}|$. I just want to add the following to the discussion. The operator of the *chiral-helicity* $\hat\eta = (\boldsymbol{\alpha}\cdot\hat{\mathbf{p}})$ does *not* commute, *e.g.*, with the Hamiltonian of the equation (47):[4]

$$[\mathcal{H}, \boldsymbol{\alpha}\cdot\hat{\mathbf{p}}]_- = 2\frac{m_1 c}{\hbar}\frac{1-\gamma^5}{2}(\boldsymbol{\gamma}\cdot\hat{\mathbf{p}})\,. \tag{49}$$

For the eigenstates of the *chiral-helicity* the system of corresponding equations can be read ($\eta=\uparrow,\downarrow$)

$$i\gamma^\mu\partial_\mu\Psi_\eta - \frac{m_1 c}{\hbar}\frac{1+\gamma^5}{2}\Psi_{-\eta} = 0\,. \tag{50}$$

The conjugated eigenstates of the Hamiltonian $|\Psi_\uparrow+\Psi_\downarrow>$ and $|\Psi_\uparrow-\Psi_\downarrow>$ are connected, in fact, by γ^5 transformation $\Psi\rightarrow\gamma^5\Psi\sim(\boldsymbol{\alpha}\cdot\hat{\mathbf{p}})\Psi$ (or $m_1\rightarrow -m_1$). However, the γ^5 transformation is related to the PT ($t\rightarrow -t$ only) transformation [17b], which, in its turn, can be interpreted as the change of the energy sign $p_0\rightarrow -p_0$, if one accepts the Stueckelberg idea about antiparticles. For example, we may associate $|\Psi_\uparrow+\Psi_\downarrow>$ with the positive-energy eigenvalue of the Hamiltonian and $|\Psi_\uparrow-\Psi_\downarrow>$, with the negative-energy eigenvalue of the Hamiltonian. Thus, the free chiral-helicity

[4] Do not confuse with the Dirac Hamiltonian.

massless eigenstates may oscillate one to another with the frequency $\omega = E/\hbar$ (as the massive chiral-helicity eigenstates, see [20] for details). Moreover, a special kind of interaction which is not symmetric with respect to the chiral-helicity states (for instance, if the left chiral-helicity eigenstates interact with the matter only) may induce changes in the oscillation frequency, like in the Wolfenstein (MSW) formalism.

2. The Second-Order Equations.

A correct equation for an adequate description of neutrinos was sought for a long time [17, 21, 22]. This problem is, in general, connected with the problem of taking the massless limit of relativistic equations. For instance, it has been known for a long time that *"one cannot simply set the mass equal to zero in a manifestly covariant massive-particle equation, in order to obtain the corresponding massless case"*, e. g., Ref. [17d].

Secondly, in the seventies the second-order equation in the 4-dimensional representation of the $O(4,2)$ group was proposed by Barut *et al.* in order to solve the problem of the mass splitting of leptons [23, 24, 25] and by Fushchich *et al.*, for describing various spin states in this representation [26, 27]. The equations (they proposed) may depend on two parameters. Recently we derived the Barut-Wilson equation on the basis of the first principles [28]. Briefly, the scheme for derivation of the equation

$$[i\gamma^\mu \partial_\mu + \alpha_2 \partial^\mu \partial_\mu - \kappa]\, \phi(x) = 0 \tag{51}$$

is the following. First, apply the generalized Ryder relation [29] (see also below, Eq. (61)) and the standard scheme for the derivation of relativistic wave equations [30, footnote # 1], [16]. Then form the Dirac 4-spinors; the left- and right parts of them are connected as follows:

$$\phi_L^\uparrow(p^\mu) = -\Theta_{[1/2]}[\phi_R^\downarrow(p^\mu)]^* \quad , \quad \phi_L^\downarrow(p^\mu) = +\Theta_{[1/2]}[\phi_R^\uparrow(p^\mu)]^* \; , \tag{52}$$

$$\phi_R^\uparrow(p^\mu) = -\Theta_{[1/2]}[\phi_L^\downarrow(p^\mu)]^* \quad , \quad \phi_R^\downarrow(p^\mu) = +\Theta_{[1/2]}[\phi_L^\uparrow(p^\mu)]^* \; , \tag{53}$$

in order to obtain

$$\left[a\,\frac{i\gamma^\mu\partial_\mu}{m} + b\,\mathcal{C}\mathcal{K} - 1\right]\Psi(x^\mu) = 0 \tag{54}$$

in the coordinate space. Transfer to the Majorana representation with the unitary matrix

$$U = \frac{1}{2}\begin{pmatrix} 1 - i\Theta_{[1/2]} & 1 + i\Theta_{[1/2]} \\ -1 - i\Theta_{[1/2]} & 1 - i\Theta_{[1/2]} \end{pmatrix} \quad , \quad U^\dagger = \frac{1}{2}\begin{pmatrix} 1 - i\Theta_{[1/2]} & -1 - i\Theta_{[1/2]} \\ 1 + i\Theta_{[1/2]} & 1 - i\Theta_{[1/2]} \end{pmatrix} \; . \tag{55}$$

Finally, one obtains the set

$$\left[a\frac{i\gamma^\mu\partial_\mu}{m} - 1\right]\phi - b\,\chi = 0 \; , \tag{56}$$

$$\left[a\frac{i\gamma^\mu\partial_\mu}{m} - 1\right]\chi - b\,\phi = 0 \tag{57}$$

for $\phi(x) = \Psi_1 + \Psi_2$ or $\chi(x) = \Psi_1 - \Psi_2$ (where $\Psi^{^{MR}}(x) = \Psi_1 + i\Psi_2$). With the identification $a/2m \to \alpha_2$ and $m(1-b^2)/2a \to \kappa$ the above set leads to the second-order equation of the Barut type.

Thirdly, we found the possibility of generalizations of the $(1,0)\oplus(0,1)$ equations (namely, the Maxwell's equations and the Weinberg-Tucker-Hammer equations[5]) also on the basis of including two independent constants [14].

[5] In general, the latter does *not* completely reduce to the former after taking the massless limit in the accustomed way.

The following definitions and postulates are used in this Section: The operators of the discrete symmetries are defined as follows: a) the space inversion operator:

$$P_{[1/2]} = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \qquad (58)$$

is the 4×4 anti-diagonal matrix; b) the charge conjugation operator:

$$C_{[1/2]} = \begin{pmatrix} 0 & i\Theta_{[1/2]} \\ -i\Theta_{[1/2]} & 0 \end{pmatrix} \mathcal{K}, \quad \Theta_{[1/2]} = \begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix}, \qquad (59)$$

with $\mathcal{K}$ being the operation of complex conjugation.

The left- and the right- spinors transform to the frame with the momentum p^μ (from the zero-momentum frame) according to the Wigner rules, with $\Lambda_{R,L} = \exp(\pm \mathbf{S} \cdot \boldsymbol{\varphi})$ being the matrices of the Lorentz boosts. $\mathbf{S}$ are the spin matrices for spin s, e. g., Ref.[31]; $\boldsymbol{\varphi}$ are parameters of the given boost. They are defined, *e. g.*, Refs. [29, 16], by means of

$$\cosh(\varphi) = \gamma = \frac{1}{\sqrt{1-v^2}} = \frac{E}{m}, \quad \sinh(\varphi) = v\gamma = \frac{|\mathbf{p}|}{m}, \quad \hat{\boldsymbol{\varphi}} = \mathbf{n} = \frac{\mathbf{p}}{|\mathbf{p}|} . \qquad (60)$$

The Ryder relation between spinors in the zero-momentum frame [29] is established[6]

$$\phi_L^h(\mathring{p}^\mu) = a(-1)^{\frac{1}{2}-h} e^{i(\vartheta_1+\vartheta_2)} \Theta_{[1/2]} [\phi_L^{-h}(\mathring{p}^\mu)]^* + b e^{2i\vartheta_h} \Xi^{-1}_{[1/2]} [\phi_L^h(\mathring{p}^\mu)]^* , \qquad (61)$$

$\mathring{p}^\mu = (m, \mathbf{0})$ with the *real* constant a and b being arbitrary at this stage; h is the polarization index. Next,

$$\Xi_{[1/2]} = \begin{pmatrix} e^{i\phi} & 0 \\ 0 & e^{-i\phi} \end{pmatrix} , \qquad (62)$$

ϕ is here the azimuthal angle related to $\mathbf{p} \to \mathbf{0}$.[7]

One can form either Dirac 4-spinors:

$$u_h(p^\mu) = \begin{pmatrix} \phi_R(p^\mu) \\ \phi_L(p^\mu) \end{pmatrix} \quad , \quad v_h(p^\mu) = \gamma^5 u_h(p^\mu) , \qquad (64)$$

or the second-type spinors [30, 33], see also [34, 35, 36]:

$$\lambda(p^\mu) = \begin{pmatrix} (\zeta_\lambda \Theta_{[j]}) \phi_L^*(p^\mu) \\ \phi_L(p^\mu) \end{pmatrix}, \quad \rho(p^\mu) = \begin{pmatrix} \phi_R(p^\mu) \\ (\zeta_\rho \Theta_{[j]})^* \phi_R^*(p^\mu) \end{pmatrix} , \qquad (65)$$

or even more general forms of 4-spinors depending on the phase factors between their left- and right- parts and helicity sub-spaces that they belong to. For the second-type spinors several forms of the field operators have been proposed. For example,

$$\begin{aligned} \nu^{DL}(x^\mu) &= \sum_\eta \int \frac{d^3\mathbf{p}}{(2\pi)^3} \frac{1}{2E_p} \Big[\lambda^S_\eta(p^\mu) c_\eta(p^\mu) \exp(-ip\cdot x) + \\ &+ \lambda^A_\eta(p^\mu) d^\dagger_\eta(p^\mu) \exp(+ip\cdot x) \Big] . \end{aligned} \qquad (66)$$

[6] It can be derived from the Faustov work [32] too.

[7] In general, one can connect also $\phi_L^\uparrow$ and $\phi_L^\downarrow$. with using the Ω matrix (see formulas (22a,b) in Ref. [30]):

$$\phi_L^\uparrow(0^\mu) = \Omega \phi_L^\downarrow(0^\mu) , \quad \Omega = \begin{pmatrix} cotan(\theta/2) & 0 \\ 0 & -tan(\theta/2) \end{pmatrix} = \frac{|\mathbf{p}|}{\sqrt{\mathbf{p}^2 - p_3^2}} (\sigma_3 + \frac{p_3}{|\mathbf{p}|}) . \qquad (63)$$

We did not yet find the explicitly covariant form of the resulting equation.

The Dirac equation has been derived by this method (the relation between 2-spinors at rest. $\phi_R(\mathbf{0}) = \pm\phi_L(\mathbf{0})$, and boosts). Next, the coupled Dirac equations for $\lambda-$ and $\rho-$ spinors have also been presented [20, 35]. The corresponding Lagrangian, projection operators, and the Feynman-Dyson-Stueckelberg propagator have been found later. However, we have surprisingly:

$$d_\kappa^\dagger(p) = -\frac{ip_y}{p}\sigma^y_{\kappa\tau}c_\tau(-p)\,, \tag{67}$$

$$c_\kappa(-p) = -\frac{ip_y}{p}\sigma^y_{\kappa\tau}d^\dagger_\tau(p)\,. \tag{68}$$

In the Majorana-like case ($c_\eta(p) = e^{-i\varphi}d_\eta(p)$) we have difficulties in the construction of field operators.

On the basis of these definitions on using the standard rules [30, footnote # 1] one can derive: In the case $\vartheta_1 = 0$, $\vartheta_2 = \pi$ the following equations are obtained for $\phi_{_L}(p^\mu)$ and $\chi_{_R} = \zeta_\lambda\Theta_{[1/2]}\phi^*_{_L}(p^\mu)$:[8]

$$\begin{aligned}&\phi^h_{_L}(p^\mu) = \Lambda_{_L}(p^\mu \leftarrow \mathring{p}^\mu)\phi^h_{_L}(0^\mu) = \frac{a}{\zeta_\lambda}(-1)^{\frac12+h}\Lambda_{_L}(p^\mu \leftarrow \mathring{p}^\mu)\Lambda^{-1}_{_R}(p^\mu \leftarrow \mathring{p}^\mu)\chi^h_{_R}(p^\mu) + \\ &+\frac{b}{\zeta_\lambda}\Lambda_{_L}(p^\mu \leftarrow 0^\mu)\Xi^{-1}_{[1/2]}\Theta^{-1}_{[1/2]}\Lambda^{-1}_{_R}(p^\mu \leftarrow 0^\mu)\chi^{-h}_{_R}(p^\mu)\;,\end{aligned} \tag{69}$$

$$\begin{aligned}&\chi^{-h}_{_R}(p^\mu) = \Lambda_{_R}(p^\mu \leftarrow 0^\mu)\chi^{-h}_{_R}(0^\mu) = a\zeta_\lambda(-1)^{\frac12-h}\Lambda_{_R}(p^\mu \leftarrow 0^\mu)\Lambda^{-1}_{_L}(p^\mu \leftarrow 0^\mu)\phi^{-h}_{_L}(p^\mu) + \\ &+b\zeta_\lambda\Lambda_{_R}(p^\mu \leftarrow 0^\mu)\Theta_{[1/2]}\Xi_{[1/2]}\Lambda^{-1}_{_L}(p^\mu \leftarrow 0^\mu)\phi^h_{_L}(p^\mu)\;.\end{aligned} \tag{70}$$

Hence, the equations for the 4-spinors $\lambda^{S,A}_\eta(p^\mu)$ take the forms:

$$ia\frac{\widehat{p}}{m}\lambda^S_\uparrow(p^\mu) - (b\mathcal{C}\mathcal{K} - 1)\lambda^S_\downarrow(p^\mu) = 0\;, \tag{71}$$

$$ia\frac{\widehat{p}}{m}\lambda^S_\downarrow(p^\mu) + (b\mathcal{C}\mathcal{K} - 1)\lambda^S_\uparrow(p^\mu) = 0\;, \tag{72}$$

$$ia\frac{\widehat{p}}{m}\lambda^A_\uparrow(p^\mu) - (b\mathcal{C}\mathcal{K} + 1)\lambda^A_\downarrow(p^\mu) = 0\;, \tag{73}$$

$$ia\frac{\widehat{p}}{m}\lambda^A_\downarrow(p^\mu) + (b\mathcal{C}\mathcal{K} + 1)\lambda^A_\uparrow(p^\mu) = 0\;, \tag{74}$$

$a = \pm(b-1)$ if we want to have $E_p^2 - \mathbf{p}^2 = m^2$ for massive particles. We can write several forms of equations in the coordinate representation depending on the relations between creation/annihilation operators. For example, provided that we imply $d_\uparrow(p^\mu) = +ic_\downarrow(p^\mu)$ and $d_\downarrow(p^\mu) = -ic_\uparrow(p^\mu)$; the $\mathcal{K}$ operator acts on $q-$ numbers as hermitian conjugation, then the first generalized equation in the coordinate space reads

$$\left[ia\frac{\gamma^\mu\partial_\mu}{m} - (b-1)\gamma^5\mathcal{C}\mathcal{K}\right]\Psi(x^\mu) = 0\;. \tag{75}$$

Transferring into the Majorana representation one obtains two real equations:[9]

$$ia\frac{\gamma^\mu\partial_\mu}{m}\Psi_1(x^\mu) - i(b-1)\gamma^5\Psi_2(x^\mu) = 0\;, \tag{76}$$

$$ia\frac{\gamma^\mu\partial_\mu}{m}\Psi_2(x^\mu) - i(b-1)\gamma^5\Psi_1(x^\mu) = 0\;. \tag{77}$$

[8] The phase factors ζ are defined by various constraints imposed on the 4-spinors, e. g., the condition of the self/anti-self charge conjugacy gives $\zeta^{S,A}_\lambda = \pm i$. But, one should still note that phase factors also depend on the phase factor in the definition of the charge conjugation operator (59). The “mass term” of resulting dynamical equations may also be different.

[9] This procedure can be carried out for any spin, cf. [37].

for real and imaginary parts of the field function $\Psi^{MR}(x^\mu) = \Psi_1(x^\mu) + i\Psi_2(x^\mu)$. In the case of $a = 1 - b$ and considering the field function $\phi = \Psi_1 + \Psi_2$ we come to the equation for the spinors of the *second kind* [34, Eq.(8)] and Ref. [36]. Next, we come to the second-order equation in the coordinate representation for massive particles

$$\left[a^2 \frac{\partial_\mu \partial^\mu}{m^2} + (b-1)^2\right] \begin{cases} \Psi_1(x^\mu) \\ \Psi_2(x^\mu) \end{cases} = 0 \ . \tag{78}$$

Of course, it may be reduced to the Klein-Gordon equation. In general, there may exist mass splitting between various $CP-$ conjugate states.

One can find the relation between creation/annihilation operators for another equation ($\beta_1,\, \beta_2 \in \Re e$)

$$\left[ia \frac{\gamma^\mu \partial_\mu}{m} - e^{i\alpha_1} \beta_1 \gamma^5 \mathcal{C}\mathcal{K} + e^{i\alpha_2}\beta_2\right] \Psi(x^\mu) = 0 \ , \tag{79}$$

which would be consistent with the equations (71-74).[10] In the Majorana representation the resulting set of the real equations are

$$\left[ia \frac{\gamma^\mu \partial_\mu}{m} + i\beta_1 \sin\alpha_1 \gamma^5 + \beta_2\right] \Psi_1 - i\beta_1 \cos\alpha_1 \gamma^5 \Psi_2 = 0 \ , \tag{80}$$

$$\left[ia \frac{\gamma^\mu \partial_\mu}{m} - i\beta_1 \sin\alpha_1 \gamma^5 + \beta_2\right] \Psi_2 - i\beta_1 \cos\alpha_1 \gamma^5 \Psi_1 = 0 \ . \tag{81}$$

For instance in the $\alpha_1 = \frac{\pi}{2}$ we obtain

$$\left[ia \frac{\gamma^\mu \partial_\mu}{m} + i\beta_1 \gamma^5 + \beta_2\right] \Psi_1 = 0 \ , \tag{82}$$

$$\left[ia \frac{\gamma^\mu \partial_\mu}{m} - i\beta_1 \gamma^5 + \beta_2\right] \Psi_2 = 0 \ . \tag{83}$$

But, in any case one can recover the Klein-Gordon equation for both real and imaginary parts of the field function, Eq. (78).

We are able to consider other constraints on the creation/annihilation operators, introduce various types of fields operators (as in [14]) and/or generalize the Ryder relation even more. In the general case, we suggest to start from

$$(a \frac{\widehat{p}}{m} - 1) u_h(p^\mu) + ib(-1)^{\frac{1}{2}-h} \gamma^5 \mathcal{C} u^*_{-h}(p^\mu) = 0 \ ; \tag{84}$$

i. e., the equation (11) of [28]. But, as opposed to the cited paper, we write the coordinate-space equation in the form:

$$\left[a\,\frac{i\gamma^\mu \partial_\mu}{m} + b_1\,\mathcal{C}\mathcal{K} - 1\right] \Psi(x^\mu) + b_2 \gamma^5 \mathcal{C}\mathcal{K} \widetilde{\Psi}(x^\mu) = 0 \ , \tag{85}$$

thus introducing the third parameter. Then we can perform the same procedure as in Ref. [28]. Implying $\Psi^{MR} = \Psi_1 + i\Psi_2$ and $\widetilde{\Psi}^{MR} = \Psi_3 + i\Psi_4$, one obtains real equations in the Majorana representation:

$$(a \frac{i\gamma^\mu \partial_\mu}{m} - 1)\phi - b_1 \chi + ib_2 \gamma^5 \widetilde{\phi} = 0\,, \tag{86}$$

$$(a \frac{i\gamma^\mu \partial_\mu}{m} - 1)\chi - b_1 \phi - ib_2 \gamma^5 \widetilde{\chi} = 0\,, \tag{87}$$

[10] As one can expect from this consideration the equation (79) may be reminiscent of the old works, Refs. [17, 18].

for $\phi = \Psi_1 + \Psi_2$, $\chi = \Psi_1 - \Psi_2$ and $\widetilde{\phi} = \Psi_3 + \Psi_4$, $\widetilde{\chi} = \Psi_3 - \Psi_4$. After algebraic transformations we have:

$$\begin{aligned}
&(a\frac{i\gamma^\mu \partial_\mu}{m} - b_1 - 1)\left[2ia\frac{\gamma^\nu \partial_\nu}{m} + a^2\frac{\partial^\nu \partial_\nu}{m^2} + b_1^2 - 1\right]\Psi_1 - \\
&\qquad -ib_2\gamma^5\left[2ia\frac{\gamma^\mu \partial_\nu}{m} - a^2\frac{\partial^\mu \partial_\mu}{m^2} - b_1^2 + 1\right]\Psi_4 = 0\,; \qquad (88)\\
&(a\frac{i\gamma^\mu \partial_\mu}{m} + b_1 - 1)\left[2ia\frac{\gamma^\nu \partial_\nu}{m} + a^2\frac{\partial^\nu \partial_\nu}{m^2} + b_1^2 - 1\right]\Psi_2 - \\
&\qquad -ib_2\gamma^5\left[2ia\frac{\gamma^\mu \partial_\mu}{m} - a^2\frac{\partial^\mu \partial_\mu}{m^2} - b_1^2 + 1\right]\Psi_3 = 0\,, \qquad (89)
\end{aligned}$$

the third-order equations. However, the field operator $\widetilde{\Psi}$ may be linear dependent on the states included in the Ψ. So, relations may exist between $\Psi_{3,4}$ and $\Psi_{1,2}$. If we apply the simplest constraints $\Psi_1 = -i\gamma^5\Psi_4$ and $\Psi_2 = i\gamma^5\Psi_3$ one should recover the Dirac-Barut-like equation with *three* mass eigenvalues:

$$\left[i\gamma^\mu\partial_\mu - m\frac{1 \pm b_1 \pm b_2}{a}\right] \times \left[i\gamma^\nu\partial_\nu + \frac{a}{2m}\partial^\nu\partial_\nu + m\frac{b_1^2 - 1}{2a}\right]\Psi_{1,2} = 0\,. \qquad (90)$$

Furthermore, we apparently note that the similar results can be obtained by consecutive applications of the generalized Ryder relations. As indicated by Barut himself, several ways for introdcution of interaction with 4-vector potential exist in second-order equations. Only considering the correct one (and, probably, taking into account γ^5 axial currents), we shall be able to answer the question of why the α_2 parameter of the Barut works is fixed by means of the use of the *classical* value of the anomalous magentic moment; and on what physical basis we have to fix other parameters we introduced above.

Concerning the massless limit please note that the mass appears in the denominator within this method. So, we multiply by it in order to obtain the Dirac-like equations that is not an obvious procedure. Next, we have the d'Alembert operator $\partial_\mu\partial^\mu$, whose eigenvalues may be different from m, and even be zeros. Anyway, a) in every equations we should calculate the characteristic determinant to find out the dispersion relations; b) we should not forget about the possibility of divergent (in $m \to 0$) terms in the solutions of the corresponding equations, and treat them properly.

3. Conclusions.

In conclusion, we presented two very natural ways of deriving the massive/massless equations in the $(S,0) \oplus (0,S)$ representation space, which lead to the equations given by other researchers in the past. It is known that present-day neutrino physics has come across serious difficulties. Experiments and observations are not in agreement with theoretical predictions of the standard model. That was a motivation for the present work.

The Barut's way of solving the hierarchy problem, which was almost forgotten, has been analized here from different viewpoint.

The terms $m(1 \pm \gamma_5)/2$ in the massless equation violate chiral invariance of the theory (due to $[\hat{p}, \gamma_5]_- \neq 0$). The solutions are *not* eigenstates of γ_5 operator, but in the case $m_{1,3} \to 0$ the chiral invariance is restored. So, they have been called *measures of chirality*.

In fact, the solutions of the Dirac massless/massive equation represent a mixture of various polarization states. This fact may be related to recent research of the Majorana-like constructs ($m \neq 0$) when we also mixed solutions of the Dirac equation in order to obtain self/anti-self charge conjugate states [38, 36].

Dynamics of massless particles and neutrino may differ from those derived from the well-known Weyl and Maxwell equations.

Acknowledgements. I greatly appreciate discussions with Profs. A. Gersten, A. Raspini and A. F. Pashkov. This work has been partly supported by the Mexican Sistema Nacional de Investigadores and the ESDEPED. Next, as a reader probably know that I have 86 papers in arxiv.org (in the primary submission categories hep-th, hep-ph, quant-ph, math-ph) and several more with the physics primary category; 52 articles in the mainstream journals with impact index, more articles in proceedings and local journals; 29 plenary invited talks at the international conferences (and many contributed talks); 22 editorial books and special issues (and several more invited chapters). However, the moderators of arxiv.org began to supress and misclassify even my submissions. Obviously, this moderator activity is the scientific crime and racism (and many other well-known researchers complainted about it to me). This system in the US science is even worse than the censorship system in the ex-Soviet Union. However, it is well known in the history. And it may be useful for my future aims. So, I want to acknowledge the administrators and moderators of arxiv.org for their mentality and capacity, for their Sisyphus ethernal work, while, of course, disagree with them.

[1] Gersten A 1999 *Found. Phys. Lett.* **12** 291; 2000 ibid. **13** 185.
[2] Dvoeglazov V V 2000 *J. Phys A: Math. Gen.* **33** 5011.
[3] Dvoeglazov V V 2002 *Spacetime and Substance*, **3** 28 (*Preprint* quant-ph/0012107v1); 2016 *Adv. Appl. Clifford Algebras.* **26** 1173.
[4] Sakurai J J 1967 *Advanced Quantum Mechanics* (Reading:Addison-Wesley), §3.2 (see also footnote on p. 91 and Dvoeglazov V V 2000 *Investigación Científica* **2** 5).
[5] Abreu E M C and Hott M 2000 *Phys. Rev. D* **62** 027702.
[6] Dvoeglazov V V 1993 *Hadronic J.* **16** 459.
[7] Sudbery A. 1986 *J. Phys. A: Math. Gen.* **19** L33.
[8] Kruglov S. I. 2011 *EJTP-Zacatecas Proc. II*, 107 (*Preprint* ArXiV: 1204.6709).
[9] Imaeda K 1950 *Prog. Theor. Phys.* **5** 133; Ohmura T (Kikuta) 1956 *Prog. Theor. Phys.* **16** 684; *ibid* 685.
[10] Donev S and Tashkova M 1998 *Ann. Fond. L de Broglie* **23** 89; *ibid* 103; *ibid* 109; Donev S 2000 Extended Electrodynamics: Basic Equations and Photon-like (3+1)-Soliton Solutions in Vacuum. In *Photon: Old Problems in Light of New Ideas* ed V V Dvoeglazov, pp. 32-56. Series *Contemporary Fundamental Physics* (Huntington: Nova Science Publishers).
[11] Ogievetskiĭ V I and Polubarinov I V 1966 *Yadern. Fiz.* **4** 216 [English translation: 1967 *Sov. J. Nucl. Phys.* **4** 156].
[12] Hayashi K 1973 *Phys. Lett.* **B44** 497; Kalb M and Ramond P. 1974 *Phys. Rev.* **D9** 2273.
[13] Dvoeglazov V V 2000 *Czech. J. Phys.* **50** 225; 2000 Longitudinal Nature of Antisymmetric Tensor Field After Quantization and Importance of the Normalization. In *Photon: Old Problems in Light of New Ideas* ed V V Dvoeglazov, pp. 319-337. (Huntington: Nova Science Publishers); 2015 *Int. J. Theor. Phys.* **54** 161.
[14] Dvoeglazov V V 1997 *Helv. Phys. Acta* **70** 677; *ibid* 686; *ibid* 697; 1998 *Ann. Fond. L. de Broglie* **23** 116; 1998 *Int. J. Theor. Phys.* **37** 1915.
[15] Weinberg S 1964 *Phys. Rev.* **B133** 1318; *ibid* **B134** 882; *ibid* **181** 1893.
[16] Ahluwalia D V *et al.* 1993 *Phys. Lett.* B**316** 102.
[17] Tokuoka Z 1967 *Prog. Theor. Phys.* **37** 603; Gupta N D S 1967 *Nucl. Phys.* **B4** 147; Santhanam T S and Chandrasekaran P S 1969 *Prog. Theor. Phys.* **41** 264; Simon M T 1971 *Lett. Nuovo Cim.* **2** 616; Santhanam T S and Tekumalla A R 1972 *Lett. Nuovo Cim.* **3** 190; Seetharaman M, Simon M T and Mathews P M 1972 Nuovo Cimento **12**A 788.
[18] Raspini A 1994 *Int. J. Theor. Phys.* **33** 1503; 1996 *Fizika B* **5** 159; 1997 *Fizika B* **6** 123; 1998 *Fizika B* **7** 83.
[19] Raspini A 1999 A Review of Some Alternative Descriptions of Neutrino. In *Photon and Poincaré Group* ed V V Dvoeglazov. (Commack: Nova Science Pubs.), pp. 181-188.
[20] Dvoeglazov V V 1995 *Int. J. Theor. Phys.* **34** 2467.
[21] Landau L 1957 *Sov. Phys. JETP* **5** 337; 1957 *Nucl. Phys.* **3** 127; Lee T D and Yang C N 1957 *Phys. Rev.* **105** 1671; Salam A 1957 *Nuovo Cimento* **5** 299; Sokolov A A 1958 *Sov. Phys. JETP* **6** 611.
[22] Fushchich V I and Grishchenko A L 1970 *Lett. Nuovo Cim.* **4** 927; Fushchich V I 1970 *Nucl. Phys.* B**21** 321; 1972 *Lett. Nuovo Cim.* **4** 344.
[23] Barut A O, Cordero P and Ghirardi G C 1969 *Phys. Rev.* **182** 1844; 1970 *Nuovo Cim.* **66**A 36.
[24] Wilson R 1974 Nucl. Phys. B**68** 157.

[25] Barut A O 1978 *Phys. Lett.* **73**B 310; 1979 *Phys. Rev. Lett.* **42** 1251.

[26] Fushchich V I and Nikitin A 1978 Teor. Mat. Fiz. **34** 319; 1978 *Fiz. Elem. Chast. At. Yadra* **9** 501.

[27] Fushchich V I 1975 *Lett. Nuovo Cim.* **14** 435.

[28] Dvoeglazov V V 1998 *Int. J. Theor. Phys.* **37** 1909.

[29] Ryder L H 1985 *Quantum Field Theory.* (Cambridge: University Press).

[30] Ahluwalia D V 1996 *Int. J. Mod. Phys.* A**11** 1855.

[31] Varshalovich D A, Moskalev A N and Khersonskiĭ V K 1988 *Quantum Theory of Angular Momentum.* (Singapore: World Scientific).

[32] Faustov R N 1971 *Relyativistskie preobrazovaniya odnochastichnykh volnovykh funktsiĭ.* (*Preprint* ITF-71-117P, Kiev). The equation (22a) of this paper reads $Bu_\lambda(0) = u_\lambda(0)$, where $u_\lambda(\vec{p})$ is a $2(2s+1)-$ spinor and $B^2 = 1$ is an arbitrary $2(2s+1)\times 2(2s+1)$ matrix with the above-mentioned property.

[33] Wigner E P 1965 *Group theoretical concepts and methods in elementary particle physics – Lectures of the Istanbul Summer School of Theoretical Physics, 1962* ed F Gürsey, (Philadelphia:Gordon & Breach), p. 37.

[34] Sokolik G A 1958 *Sov. Phys. JETP* **6** 1170.

[35] Dvoeglazov V V 1995 *Nuovo Cimento* **108**A 1467.

[36] Dvoeglazov V V 1996 *Nuovo Cim.* B**111** 483; 1997 *Fizika* **B6** 111; 1997 *Hadronic J.* **20** 435.

[37] Dvoeglazov V V 1997 *Int. J. Theor. Phys.* **36** 635.

[38] Ziino G 1989 *Ann. Fond. L. de Broglie* **14** 427; 1991 ibid. **16** 343; Barut A O and G Ziino 1993 *Mod. Phys. Lett.* A**8** 1011; Ziino G 1996 *Int. J. Mod. Phys.* A**11** 2081.