IEEE Access
Multidisciplinary | Rapid Review | Open Access Journal

Date of publication xxxx 00, 0000, date of current version xxxx 00, 0000.
Digital Object Identifier 10.1109/ACCESS.2017.DOI

Divergence measure of belief function

YUTONG SONG[1], YONG DENG[2].
[1]Glasgow College, University of Electronic Science and Technology of China, Chengdu, 610054, China.
[2]Institute of Fundamental and Frontier Science, University of Electronic Science and Technology of China, Chengdu, 610054, China
Corresponding author: Yong Deng (e-mail:dengentropy@uestc.edu.cn; prof.deng@hotmail.com).

The work is partially supported by National Natural Science Foundation of China (Grant Nos. 61573290, 61503237).

ABSTRACT Dempster Shafer evidence theory (D-S theory) is widely applied to information fusion for its efficiency in dealing with uncertain information. Furthermore, it is important to measure the divergent or conflicting degree among pieces of information for information preprocessing in case for the unreliable results which come from the combination of conflicting bodies of evidence using Dempsterąŕs combination rules. However, how to measure the divergence of different evidence is still an open issue. In this paper, a new divergence measure of belief function based on Deng entropy is proposed in order to measure the divergence of different belief function. The divergence measure is the generalization of Kullback-Leibler divergence for probability since when the basic probability assignment (BPA) is degenerated as probability, divergence measure is equal to Kullback-Leibler divergence. Numerical examples are used to illustrate the effectiveness of the proposed divergence measure.

INDEX TERMS Likelihood function, Soft likelihood function, OWA operator, POWA operator, Geometric mean

I. INTRODUCTION

Dempster-Shafer evidence theory [1], [2] is widely used in real applications like information fusion [3], decision making [4], target recognition [5] and so on for its efficiency in handling the combining uncertain information. However, it counter-intuitive results [6], [7] may come from the combination of conflicting different evidence.

To address this problem, one widely accepted solution is to revise the Dempster's combination rules and thus re-allocate the conflict. For example, Deng [8] proposed to averaged weight different evidences based on Jousselme distance [9] to measure the divergent degree among the evidences.In this paper, a new divergence measure called Song-Deng divergence(D-S divergence)is proposed based on Deng entropy [10] in this paper. The divergence measure is the generalization of Kullback-Leibler divergence [11], [12] for probability since when the basic probability assignment (BPA) is degenerated as probability, divergence measure is equal to Kullback-Leibler divergence. The new divergence measure could allocate masses not only to the propositions consisting of single objects, but also to the unions of such objects so it has a better performance in modelling both of the uncertainty and impresion.

The remainder of this paper is constituted as follows. Section 2 introduces some necessary preliminaries of D-S evidence theory, and Deng entropy. In Section 3, a new divergence measure for belief function is proposed. There is an example for the new method and some properties of it are given in Section 4. Finally, the conclusion is given in Section 5.

II. PRELIMINARIES

In this section, some basic preliminaries on Dempster-Shafer evidence theory [1], [2], belief function and Deng entropy are introduced.

A. DEMPSTER-SHAFER EVIDENCE THEORY

Dempster-Shafer evidence theory(D-S theory) [1], [2] is firstly proposed by Dempster and then developed by Shafer. Dempster-Shafer evidence theory extends the elementary event space in probability theory to its power set and the basic probability assignment(BPA) is constructed in the frame of discernment. The basic definitions about D-S theory is shown as follows:

1) Frame of discernment

Definition II.1. *D-S theory supposes the definition of a set of elementary hypotheses called the frame of discernment.*

$$\theta = \{H_1, H_2, ..., H_N\} \tag{1}$$

θ is a set of mutually exclusively exhaustive events, and 2^θ is the power set of θ.

2) Mass function

In the frame of discernment, the mass function m is defined as follows.

Definition II.2. *D-S theory supposes the definition of a set of elementary hypotheses called the frame of discernment.*

$$m = 2^{\theta} \rightarrow [0, 1] \tag{2}$$

And there are some conditions for mass function to satisfy.

$$m(\phi) = 0 \tag{3}$$

$$\sum_{A \in 2^{\theta}} m(A) = 1 \tag{4}$$

In D-S theory, a mass function is also called a basic probability assignment (BPA).

B. DENG ENTROPY

Deng entropy [10] is a uncertainty measure proposed by Deng to measure the uncertainty of a BPA. Deng entropy is the generalization of Shannon entropy and the value of its is equal to Shannon entropy [13] when BPAs degenerate to probabilities. Deng entropy is defined as follows. Deng entropy is defined as follows.

Definition II.3. *For m is a mass function defined on the frame of discernment X.*

$$E_d(m) = -\sum_{A \in X} m(A) \frac{m(A)}{2^{|A|} - 1} \tag{5}$$

III. A NEW DIVERGENCE MEASURE OF BELIEF FUNCTION

A new divergence measure of belief function is proposed in this section. Considering the relationship between BPA and belief function, in this section, the main gaol is to measure the divergence based on BPAs. The new divergence measure is defined as follows.

Definition III.1. *D-S theory supposes the definition of a set of elementary hypotheses called the frame of discernment.*

$$Div(m_1, m_2) = \frac{D_d(m_1 \parallel m_2) + D_d(m_2 \parallel m_1)}{2} \tag{6}$$

In which the fuction $D_d(m_1 || m_2)$ is defined as follows.

Definition III.2. *D-S theory supposes the definition of a set of elementary hypotheses called the frame of discernment.*

$$D_d(m_1 \parallel m_2) = \sum_i \frac{1}{2^{|F_i|} - 1} m_1(F_i) \log(\frac{m_1(F_i)}{m_2(F_i)}) \tag{7}$$

The divergence measure is the generalization of Kullback-Leibler divergence for probability since when the basic probability assignment (BPA) is degenerated as probability, divergence measure is equal to Kullback-Leibler divergence. The new divergence measure could allocate masses not only to the propositions consisting of single objects, but also to the unions of such objects so it has a better performance in modelling both of the uncertainty and impresion.

A. AN EXAMPLE OF THE NEW DIVERGENCE MEASURE

There is an example to show the properties of S-D divergence.

1) The application statement

Assume there are several sources(N) of evidence from sensors. The BPAs given by the sensors are shown in Table.1.i and j are the numbers under the amount of all sensors and i is smaller than j .

2) The properties of new divergence measure

As the the amount of masses in the last BPA increases, the value of S-D divergence varies as the following figure shows.It shows that value of D-S divergence decreases as the number of focal elements increases. It shows that value of D-S divergence decreases as the number of focal elements increases. When the number of focal elements increases, it means that the sensor's reliability for any target decreases while the uncertainty of the all target increases. Moreover, the value of D-S divergence is positive for different sizes of masses.

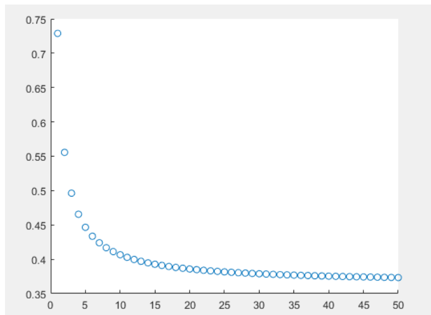

FIGURE 1: The value of S-D divergence for different sizes of masses

IV. CONCLUSION

In order to address the problem of measuring the divergent degree among different evidence, a new divergence measure of BPA is proposed in the this paper. The divergence measure is the generalization of Kullback-Leibler divergence for probability since when the basic probability assignment (BPA) is degenerated as probability, S-D divergence is equal to Kullback-Leibler divergence when all the belief are assigned to single elements. Making use of Deng entropy, the new divergence measure can greatly measure the divergent degree among evidences.

CONFLICT OF INTERESTS

The authors declare that there is no conflict of interests regarding the publication of this paper.

BPA	X_1	X_2	X_3	X_4	X_5	X_6	$X_i, ..., X_j$
0.3	0.3	0.01	0.1	0.1	0.05	0.04	0.4
0.3	0.4	0.2	0.15	0.04	0.06	0.1	0.05

TABLE 1: The BPAs for N sensors

REFERENCES

[1] A. P. Dempster, Upper and lower probabilities induced by a multivalued mapping, in: Classic Works of the Dempster-Shafer Theory of Belief Functions, Springer, 2008, pp. 57–72.

[2] G. Shafer, A mathematical theory of evidence, Vol. 42, Princeton university press, 1976.

[3] J. Schubert, Conflict management in dempster–shafer theory using the degree of falsity, International Journal of Approximate Reasoning 52 (3) (2011) 449–460.

[4] R. R. Yager, N. Alajlan, Decision making with ordinal payoffs under dempster–shafer type uncertainty, International Journal of Intelligent Systems 28 (11) (2013) 1039–1053.

[5] Y. Deng, A threat assessment model under uncertain environment, Mathematical Problems in Engineering 2015.

[6] E. Lefevre, O. Colot, P. Vannoorenberghe, Belief function combination and conflict management, Information fusion 3 (2) (2002) 149–162.

[7] W. Liu, Analyzing the degree of conflict among belief functions, Artificial Intelligence 170 (11) (2006) 909–924.

[8] D. Yong, S. WenKang, Z. ZhenFu, L. Qi, Combining belief functions based on distance of evidence, Decision support systems 38 (3) (2004) 489–493.

[9] A.-L. Jousselme, D. Grenier, É. Bossé, A new distance between two bodies of evidence, Information fusion 2 (2) (2001) 91–101.

[10] D. Yong, Deng entropy, Chaos Solitons & Fractals the Interdisciplinary Journal of Nonlinear Science & Nonequilibrium & Complex Phenomena 91 (2016) 549–553.

[11] S. Kullback, R. A. Leibler, On information and sufficiency, The annals of mathematical statistics 22 (1) (1951) 79–86.

[12] S. Kullback, Information theory and statistics, Courier Corporation, 1997.

[13] C. E. Shannon, A mathematical theory of communication, Bell system technical journal 27 (3) (1948) 379–423.

YONG DENG received the Ph.D. degree in Precise Instrumentation from Shanghai Jiao Tong University, Shanghai, China, in 2003. From 2005 to 2011, he was an Associate Professor in the Department of Instrument Science and Technology, Shanghai Jiao Tong University. From 2010, he was a Professor in the School of Computer and Information Science, Southwest University, Chongqing, China. From 2012, he was an Visiting Professor in Vanderbilt University, Nashville, TN, USA. From 2016, he was also a Professor in School of Electronic and Information Engineering, Xi'an Jiaotong University, Xiąŕ an, China. From 2017, he is the full professor of Institute of Fundamental and Frontier Science, University of Electronic Science and Technology of China, Chengdu, China. Professor Deng has published more than 200 papers in referred journals such as IEEE Trans. On Fuzzy Systems, IEEE Trans. On Cybernetics, Decision Support Systems, European Journal of Operational Research and Scientific Reports. His research interests include evidence theory, decision making, information fusion and complex system modelling. He severed as the program member of many conferences such as International Conference on Belief Functions. He severed as many editorial board members such as Academic Editor of the PLOS ONE and Applied Intelligence. He severed as the reviewer for more than 30 journals such as IEEE Transaction on Fuzzy Systems. Professor Deng has received numerous honors and awards, including the Elsevier Highly Cited Scientist in China in 2014 to 2018.

YUTONG SONG is a junior student at the Glasgow College of the University of Electronic Science and Technology of China, Chengdu, China. Yutong was appointed as research assistant in Institute of Fundamental and Frontier Science at the University of Electronic Science and Technology of China, Chengdu, China. Yutong does scientific research with Prof. Deng on information fusion.