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A possibility of CPT violation in the Standard Model

S.A. Larin

Institute for Nuclear Research of the Russian Academy of Sciences,
60th October Anniversary Prospect 7a, Moscow 117312, Russia

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Abstract

It is shown that there is a possibility of violation of CPT symmetry in the Standard Model which does not contradict to the famous CPT theorem. To check this possibility experimentally it is necessary to increase the precision of measurements of the proton and antiproton mass difference by an order of magnitude.

1 Introduction

Invariance under the combined transformation CPT is considered as one of the most fundamental symmetries of local quantum field theory. Here C is the operator of charge conjugation, P and T are operators of space and time reflections. The famous CPT-theorem was proved in the papers [1, 2]. The content of the CPT-theorem is approximately as follows: a Lagrangian of any local Lorentz invariant quantum field theory with usual connection between spin and statistics is invariant with respect to an antiunitary operator Θ which coincides with the CPT-operator up to a phase (the complete formulation of the CPT-theorem will be given below).

In the present paper we demonstrate that there is a possibility to violate CPT symmetry of the Lagrangian of the Standard Model which does not contradict to the CPT-theorem. To check this possibility experimentally it is necessary to increase the current precision of measurements of mass difference of a proton and an antiproton $|m_p - m_{\overline{p}}|/m_p$ by an order of magnitude.

2 Short review of CPT invariance

Let us remind the definition of the operator CPT which we denote for shortness as Θ :

$$\Theta \equiv CPT. \tag{1}$$

For simplicity we will consider only fields with spins $0, 1/2$ and 1. We will use (until the opposite case is underlined) the interaction representation for fields which in particular allows to define the operators C, P and T not depending on time even if the corresponding symmetries are violated.

Transformations for scalar, vector and spinor fields are

$$\begin{aligned} \Theta\phi(x)\Theta^{-1} &= \eta_\Theta(\phi)\phi^+(-x), \\ \Theta V_\mu(x)\Theta^{-1} &= \eta_\Theta(V)V_\mu^+(-x), \\ \Theta\psi(x)\Theta^{-1} &= \eta_\Theta(\psi)\gamma_5\psi^+(-x), \end{aligned} \tag{2}$$

where η_Θ are some phases and γ_5 is the Dirac γ-matrix.

It is well known that the operator Θ can be defined only as an antiunitary operator because the time reversal operator T is antiunitary. An untiunitary operator has the specific property

$$\Theta\lambda\Theta^{-1} = \lambda^*, \tag{3}$$

where λ is an arbitrary c-number.

The CPT-theorem in the Lagrangian formalism is as follows. If quantum field theory satisfies the following six postulates:

1) field equations are local;

2) the Lagrangian is invariant with respect to the proper Lorentz group;

3) one has usual connection between spin and statistics;

4) boson fields commute with all other fields, kinematically independent fermion fields anticommute;

5) any product of field operators is symmetrized in cases of boson fields and antysimmetrized in cases of fermion fields (normal ordering of operators posesses this property);

6) the Lagrangian is Hermitian;

then the Lagrangian of any such theory of interacting fields with spins 0, 1/2 and 1 is invariant with respect to the following antiunitary operator Θ in the Hilbert space:

$$\begin{aligned} \Theta\phi(x)\Theta^{-1} &= \phi^{+}(-x), \\ \Theta V_{\mu}(x)\Theta^{-1} &= -V_{\mu}^{+}(-x), \\ \Theta\psi(x)\Theta^{-1} &= -i\gamma_5\psi^{+}(-x). \end{aligned} \tag{4}$$

One can see that this operator Θ up to phase factors coinsides with the product of three operators C, P and T defined in eq.(2).

Let us now consider the Jost CPT-theorem [2] in terms of Wightman functions (in terms of vacuum expectations of fields) [3].

The postulates of this formalism are:
a) invariance of the theory with respect to the proper Lorentz group;
b) positivity of energy, the existing of vacuum;
c) weak causality:

$$< 0|\Phi_1(x_1)\Phi_2(x_2)...\Phi_n(x_n)|0 >= (-1)^{\sigma} < 0|\Phi_n(x_n)...\Phi_2(x_2)\Phi_1(x_1)|0 > \tag{5}$$

for all $(x_1, x_2, ...x_n)$ for which $\sum_i \lambda_i(x_i - x_{i+1})$ is always a spacelike vector if $\lambda_i \geq 0$ and $\sum_i \lambda_i = 1$, where σ is the number of permutations of fermionic fields. Weak causality is valid if usual causality conditions of postulates 3) and 4) of the CPT-theorem in the Lagrangian formalism are valid, but it is essentially weaker of these postulates.

The Jost theorem is: for any quantum field theory satisfying the postulates a)-c), vacuum expextations are invariant with respect to the operator Θ defined in (4), that is for any set $(x_1, x_2, ...x_n)$ one has

$$< 0|\Phi_1(x_1)\Phi_2(x_2)...\Phi_n(x_n)|0 >= \tag{6}$$
$$< 0|\Theta^{-1}\Theta\Phi_1(x_1)\Theta^{-1}\Theta\Phi_2(x_2)\Theta^{-1}\Theta...\Theta^{-1}\Theta\Phi_n(x_n)\Theta^{-1}\Theta|0 >=$$
$$< 0|\Theta\Phi_1(x_1)\Theta^{-1}\Theta\Phi_2(x_2)\Theta^{-1}\Theta...\Theta^{-1}\Theta\Phi_n(x_n)\Theta^{-1}|0 >^* .$$

For example, for scalar fields it means

$$< 0|\phi_1(x_1)\phi_2(x_2)...\phi_n(x_n)|0 >=< 0|\phi_1^+(-x_1)\phi_2^+(-x_2)...\phi_n^+(-x_n)|0 >^* . \tag{7}$$

Let us stress that in general theory of interacting fields formulated in terms of Wightman functions, one takes full field operators in the Heisenberg representation.

The check of the equality of masses of particles and antiparticles is one of fundamental tests of CPT invariance. Let us consider the case of a proton which as a stable particle is most appropriate for precise direct mass measurements. The results of experiments are [4]

$$m_p = 938.272081 \pm 0.000006 \;\; MeV,$$
$$|m_p - m_{\overline{p}}|/m_p < 7 \times 10^{-10} \;\; at \;\; CL = 90\%. \tag{8}$$

The most impressive test of the CPT-symmetry comes [4] from the limit on the mass difference between neutral kaons K^0 and $\overline{K}^0$:

$$|m_{\overline{K}^0} - m_{K^0}|/m_{K^0} \leq 0.8 \times 10^{-18} \;\; at \;\; CL = 90\%. \tag{9}$$

But our further considerations will not concern this special case of neutral kaon system (and will not contradict to it).

3 Main part

In spite of the theoretical perfectness of the CPT-theorem one can still assume that the operator Θ defined by this theorem in eq.(4) is unphysical, i.e. it does not transform physical states into physical ones.

One can assume that the physical CPT operator Θ_{ph} differs from the theoretical operator Θ by another choice of the CPT phases η_Θ in eq.(4). And it turns out that it is possible to violate CPT invariance of the Lagrangian of the Standard model with a non-standard choice of phases η_Θ for quark fields.

The Standard Model Lagrangian dencity is

$$L_{SM}(x) = \frac{g}{2\sqrt{2}} W_\mu^+(x)\overline{u}\gamma_\mu(1-\gamma_5)\left(d(x)\ cos\theta_c + s(x)\ sin\theta_c\right) + h.c. + ..., \quad (10)$$

where we have written explicitly only interesting for us terms of interactions of light u, d and s quarks with the W bosons.

Here the weak coupling constant g is connected with the Fermi constant G_F in the usual way

$$\frac{g^2}{8M_W^2} = \frac{G_F}{\sqrt{2}}, \qquad G_F \approx \frac{10^{-5}}{m_p^2}. \quad (11)$$

The main point is that one can assume that e.g. d and s quarks have opposite η_Θ phases with respect to the physical operator $CPT_{physical} \equiv \Theta_{ph}$:

$$\Theta_{ph} d(x) \Theta_{ph}^{-1} = \eta_\Theta(d)\gamma_5 d^+(-x), \qquad \Theta_{ph} s(x) \Theta_{ph}^{-1} = \eta_\Theta(s)\gamma_5 s^+(-x),$$
$$\eta_\Theta(s) = -\eta_\Theta(d), \quad (12)$$

where $\eta_\Theta(d) = -i$, as in eq.(4). It can be arranged e.g. in models of composite quarks.

In this case the Standard Model Lagrangian will not be any more invariant under the physical CPT operator Θ_{ph} but will consist of the CPT even and CPT odd parts.

At first glance this extra minus in eq.(12) contradicts to the powerful Jost theorem which does not allow extra minuses in eqs.(6),(4). But one should remember that the fields there are the full operators in the Heisenberg representation. The operator Θ_{ph} does not commute with the Hamiltonian, hence it depends on time and is not restricted by the Jost theorem.

Let us consider the influence of this CPT violation on the difference of proton and antiproton masses. For this purpose we will use the simplified quantum mechanical picture of a proton as a superposition

$$|p> = |uud> + \xi|uus>, \quad (13)$$

where $|uud>$ and $|uus>$ are the eigenvectors of the Hamiltonian of strong interactions consisting of the corresponding light quarks u, d and s.

The amplitude ξ appears after the mixing of these states due to the Hamiltonian of weak interactions H_w:

$$\xi \approx \frac{< uud|H_w|uus >}{m_\Sigma - m_p} \approx \frac{G_F sin\theta_c}{m_\Sigma - m_p} \approx 0.8 \times 10^{-5}, \quad (14)$$

here $m_\Sigma \approx 1189\ MeV$, the Σ^+-hyperon mass.

The full Hamiltonian is the sum of the CPT even and CPT odd parts:

$$H = H_+ + H_-. \tag{15}$$

The masses of a proton and an antiproton are

$$m_p =< p|H_+|p> + < p|H_-|p>, \tag{16}$$

$$m_{\overline{p}} =< \overline{p}|H_+|\overline{p}> + < \overline{p}|H_-|\overline{p}> .$$

Applying the CPT- operator Θ_{ph} we get

$$m_{\overline{p}} =< \overline{p}|\Theta_{ph}^{-1}\Theta_{ph}H\Theta_{ph}^{-1}\Theta_{ph}|\overline{p}>=< p|H_+|p> - < p|H_-|p> . \tag{17}$$

Subtracting (16) and (17) one has

$$m_p - m_{\overline{p}} = 2 < p|H_-|p>=$$

$$2 < uud + \xi\ uus|H_-|uud + \xi\ uus> \approx 4\xi < uud|H_-|uus>,$$

$$|m_p - m_{\overline{p}}|/m_p \approx 4\xi sin\theta_c G_F \approx 6 \times 10^{-11}, \tag{18}$$

which should be compared with present experiments, see eq.(8).

Thus to check the possibility of CPT violation it is necessary to improve the current experimental precision for $|m_p - m_{\overline{p}}|/m_p$ approximately by an order of magnitude.

4 Conclusions

We have shown that there is still the possibility of violation of CPT symmetry in the Standard Model. It can be achived by the proper choice of the CPT-phases for quarks. To check this possibility experimentally it is necessary to increase the accuracy of measurements of the proton and antiproton mass difference by an order of magnitude.

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