Who Did Derive First the Division by Zero $1/0$ and the Division by Zero Calculus $\tan(\pi/2) = 0, \log 0 = 0$ as the Outputs of a Computer?

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Abstract: In this short paper, we will introduce an essence of the division by zero calculus and the situation from the viewpoint of computers that will contain a surprising news on the division by zero calculus.

Key Words: Division by zero, division by zero calculus, $0/0 = 1/0 = z/0 = \tan(\pi/2) = \log 0 = 0$, infinity, discontinuous, Isabelle/HOL.

2010 Mathematics Subject Classification: 00A05, 00A09

1 Introduction

In this short paper, we will introduce an essence of the division by zero calculus and the situation in connection with computers based on [50].

The Institute of Reproducing Kernels is dealing with the theory of division by zero calculus and declares that the division by zero was discovered as $0/0 = 1/0 = z/0 = 0$ **in a natural sense** on 2014.2.2. The result shows a new basic idea on the universe and space since Aristotele (BC384 - BC322) and Euclid (BC 3 Century -), and the division by zero is since Brahmagupta (598 - 668 ?).

For the details, see the references.

A simple and essential introduction of the division by zero is given by the **division by zero calculus**:

For any Laurent expansion around $z = a$,

$$f(z) = \sum_{n=-\infty}^{-1} C_n(z-a)^n + C_0 + \sum_{n=1}^{\infty} C_n(z-a)^n, \tag{1.1}$$

we define

$$f(a) = C_0, \tag{1.2}$$

as a value of the function f at the singular point $z = a$.

Here, we will note a naturality of the division by zero calculus.

Recall the Cauchy integral formula for analytic functions; for an analytic function $f(z)$ around $z = a$ and for a smooth simple Jordan closed curve γ enclosing one time the point a, we have

$$f(a) = \frac{1}{2\pi i}\int_\gamma \frac{f(z)}{z-a}dz.$$

Even when the function $f(z)$ has any singularity at the point a, we assume that this formula is valid as the division by zero calculus. We define the value $f(a)$ by the Cauchy integral. For this background idea, we can consider the value of a function with some mean value of the function.

For the importance of this definition, the division by zero calculus may be considered as a new axiom. This was discovered on May 8, 2014 from the value of the function $\exp(1/z)$ at the origin $z = 0$ ([2]).

In particular, for the function $W = f(z) = 1/z$, we have $f(0) = 0$. We will write this result as

$$\frac{1}{0} = 0,$$

from the form. Here, the definition of $\frac{1}{0}$ is given by this sense by means of the division by zero calculus. Of course, $\frac{1}{0}$ is not a usual fraction in the usual sense that $\frac{1}{0} = X$ if and only if $1 = 0 \times X$; this means a contradiction. See [21] for the details.

On February 16, 2019 H. Okumura introduced the surprising news in Research Gate:

José Manuel Rodríguez Caballero
Added an answer
In the proof assistant Isabelle/HOL we have $x/0 = 0$ for each number x. This is advantageous in order to simplify the proofs. You can download this proof assistant here: **https://isabelle.in.tum.de/**.

J.M.R. Caballero kindly showed surprisingly several examples by the system that

$$\tan\frac{\pi}{2} = 0,$$

$$\log 0 = 0,$$

$$\exp\frac{1}{x}(x = 0) = 1,$$

and others. Precisely:

Dear Saitoh,
In Isabelle/HOL, we can define and redefine every function in different ways. So, logarithm of zero depend upon our definition. The best definition is the one which simplify the proofs the most. According to the experts, z/0 = 0 is the best definition for division by zero.

$$\tan(\pi/2) = 0$$

$$\log 0 =$$

is undefined (but we can redefine it as 0)

$$e^0 = 1$$

(but we can redefine it as 0)

$$0^0 = 1$$

(but we can redefine it as 0).
In the attached file you will find some versions of logarithms and exponentials satisfying different properties. This file can be opened with the software Isabelle/HOL from this webpage: https://isabelle.in.tum.de/
Kind Regards,
José M.
(2017.2.17.11:09).

At 2019.3.4.18:04 for my short question, we received:

It is as it was programmed by the HOL team.

Jose M.

On Mar 4, 2019, Saburou Saitoh wrote:

Dear José M.

I have the short question.

For your outputs for the division by zero calculus, for the input, is it some direct or do you need some program???

With best regards, Sincerely yours,

Saburou Saitoh 2019.3.4.18:00

As we stated in [14], the important point in the division by zero problem is **on its definition (meaning of division.)**, because in the usual sense, we can not consider the division by zero.

L. C. Paulson stated that I would guess that Isabelle has used this **convention** $1/0 = 0$ since the 1980s and introduced his book [7] referred to this fact. However, in his group the importance of this fact seems to be entirely ignored at this moment as we see from the book.

The result $1/0 = 0$ has a long tradition of Isabelle, however, the result has not been accepted by the world.

Indeed, S. K. Sen and R. P. Agarwal [22] referred to the paper [2] in connection with division by zero, however, their understandings on the paper seem to be not suitable (not right) and their ideas on the division by zero seem to be traditional, indeed, they stated as a conclusion of the introduction of the book that:

"Thou shalt not divide by zero" remains valid eternally.

However, in [20] we stated simply based on the division by zero calculus that

We Can Divide the Numbers and Analytic Functions by Zero with a Natural Sense.

In these situations, the results of J.M.R. Caballero will be very interested. For some precise information, we would like to ask for the question that

Who did derive first the division by zero $1/0$ **and the division by zero calculus** $\tan(\pi/2) = 0, \log 0 = 0$ **as the outputs of a computer?**

If it is possible, we would like to know the related details.

Furthermore, we got the important informations from Caballero's e-mails to some person who was saying $0/0 = 1$ for many years:

2019.3.6.15:23: To accept that x/0 = 0 produces no contradiction, using the rules of Isabelle/HOL. You could download the software and try to prove that 1 = 0 (this will be impossible). http://isabelle.in.tum.de/

There are millions of dollars invested in Isabelle/HOL, where x/0 = 0 (this is not a joke): https://www.cl.cam.ac.uk/ lp15/Grants/Alexandria/

Prof. Saitoh derived that x/0 = 0 from pure geometric intuition and he was right.

2019.3.8.22:23: The deduction that z/0 = 0, for any z, is based in Saitoh's geometric intuition and it is currently applied in proof assistant technology, which are useful in industry and in the military.

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