

Real Clifford Algebras and Our Universe: VOID to Cl(16) Lagrangian to Cl(1,25) AQFT

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Abstract

This paper is an outline of basic ideas of Cl(16) and Cl(1,25) Physics.
For details see my papers listed at
http://vixra.org/author/frank_dodd_tony_smith_jr
including <http://vixra.org/abs/1903.0485>
about the Nambu-Jona-Lasinio Higgs-Truth Quark System

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VOID to CI(16) - Octonionic Lagrangian

The **Real Clifford Algebra CI(16)** is the culmination of David Finkelstein's process of Iteration of Clifford Algebras that began when Our Universe emerged from an Empty Set Void in its Parent Universe by Quantum Fluctuation:

$$\text{CI(Void)} = \text{Clifford Algebra of Void} = 0$$

$$\text{CI}(0) = 2^0 \text{ dimensions} = 1$$

$$\text{CI}(1) = 2^1 = 2$$

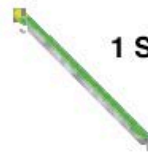
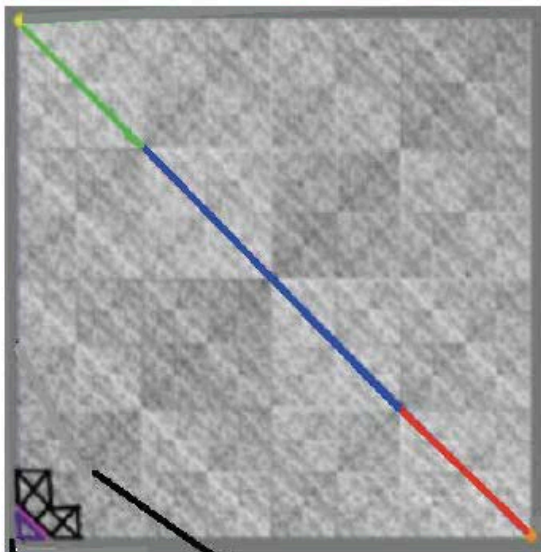
$$\text{CI}(2) = 2^2 = 4$$

$$\text{CI}(4) = 2^4 = 16$$

$$\text{CI}(16) = 2^{16} = 65,536 = 256 \times 256 \text{ Real Matrices } M(256, R):$$

$$\text{CI}(16) \text{ BiVectors} + \text{CI}(16) \text{ half-Spinors} = 248\text{-dim } E8$$

CI(16) 256x256 Matrix Representation



1 Scalar + 63 8-Vectors

128 half-Spinors

63 8-Vectors + 1 Pseudo-Scalar

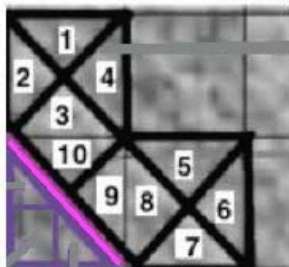


16 Vectors

120 BiVectors

560 TriVectors

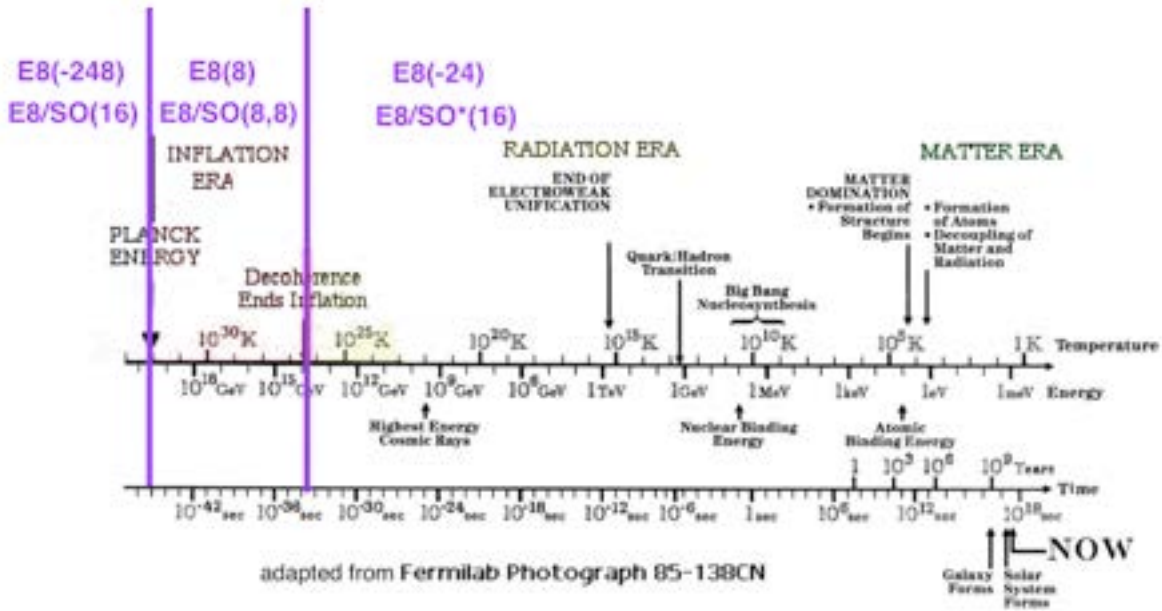
$$120 \text{ BiVectors} + 128 \text{ half-Spinors} = 248 \text{ } E8$$



$$65,536 - 128 - 120 - 16 - 560 = 64,712$$

Information Carriers

E8 has 3 Real Forms

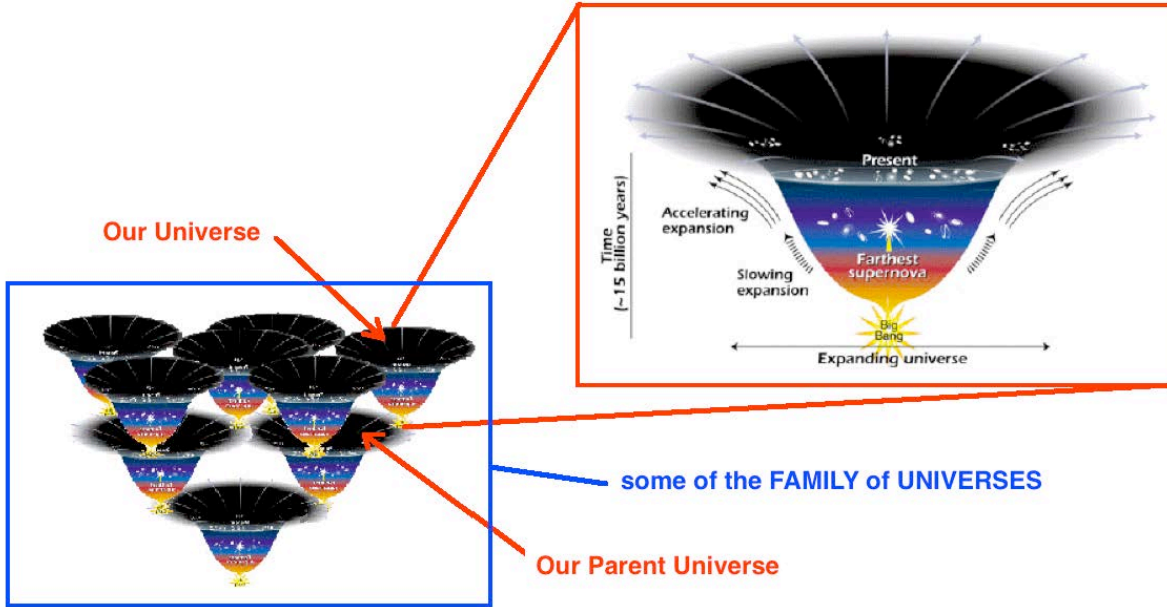


The compact form ... E8(-248) ... has symmetric space **E8/SO(16)**
for Clifford Iteration from VOID to CI(16)

The split form, EVIII (or E8(8)) ... has symmetric space **E8/SO(8,8)**
for Octonionic Inflation

EIX (or E8(-24)) ... has symmetric space **E8 / Sk(8,H) = E8 / SO*(16)**
for Quaternionic Expansion

First Stage of the Evolution of Our Universe - from VOID to Cl(16) -
 was when Our Universe emerged from a VOID in Our Parent Universe



in a **Finkelstein Clifford Iteration Big Bang leading to Cl(16)**



which contains **E8** as 120 BiVectors + 128 half-Spinors = 248-dim E8

E8 is a recipe for a Lagrangian of
8-dimensional Spacetime = Shilov Boundary of Cl(16) Vectors
 and
First-Generation Fermions = Cl(16) half-Spinors
 and
Gauge Bosons and Ghosts = 28+28 of 120 Cl(16) BiVectors

64 = 120-28-28 of 120 Cl(16) BiVectors = A7+R = Unimodular Gravity =
 = Central grade-0 Element of Maximal Contraction of E8

Maximal Contraction of E8 was shown by Rutwig Campoamor-Stursberg in Acta Physica Polonica B 41 (2010) 53-77 , “Contractions of Exceptional Lie Algebras and SemiDirect Products” , to be semi-direct product $H_{92} \times A_7 + R$ where H_{92} is $(8+28+56 +1+ 56+28+8)$ -dim Heisenberg Creation/Annihilation Algebra and E8 Max Contraction has graded structure $8 + 28 + 56 + 64 + 56 + 28 + 8$

grade -3 = Creation of 1 fermion (tree-level massless neutrino) with 8 SpaceTime Components for a total of 8 fermion component creators

grade -2 = Creation of $8+3+1 = 12$ Ghosts for Standard Model and 16 Conformal $U(2,2)$ Bosons for MacDowell-Mansouri Gravity

grade -1 = Creation of 7 massive Dirac fermion each with 8 SpaceTime Components for a total of 56 fermion component creators

grade 0 = $A_7 + R = 63 + 1 = 64$ -dim Creation/Annihilation of 8-dim SpaceTime of HyperVolume Elements

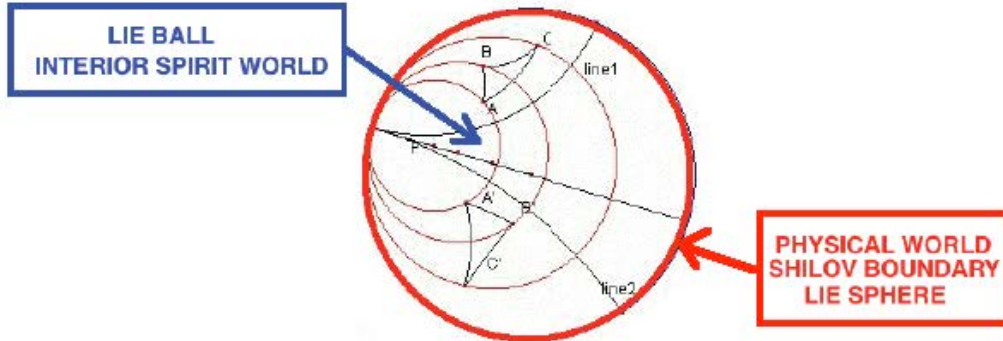
grade 1 = Annihilation of 7 massive Dirac fermions each with 8 SpaceTime Components for a total of 56 fermion component annihilators

grade 2 = Annihilation of $8+3+1 = 12$ Bosons for Standard Model and 16 Conformal $U(2,2)$ Ghosts for MacDowell-Mansouri Gravity

grade 3 = Annihilation of 1 fermion (tree-level massless neutrino) with 8 SpaceTime Components for a total of 8 fermion component annihilators

CI(16) Physical Interpretation - Vectors Spacetime as Shilov Boundary described by Hua

16 Vectors = 16-dim Lie Ball Spin(10) Spin(8) x U(1)
with Bounded Complex Domain of Hua Type IV(8)
whose Lie Sphere Shilov Boundary is 8-dim Spacetime
that is the Base Manifold for the CI(16) Lagrangian



The 8-dim Spacetime has a Kaluza-Klein $M4 \times CP2$ structure
with $CP2 = SU(3) / SU(2) \times U(1)$ Internal Symmetry Space
and $M4 =$ Minkowski Physical Spacetime that is seen by Gauge Groups as
 $S4$ by Gravity
 $CP2$ by Color Force
 $S2 \times S2$ by Weak Force
 $T4 = S1 \times S1 \times S1 \times S1$ by Electromagnetism

Loo-Keng Hua in his book Harmonic Analysis of Functions of Several Complex Variables in the Classical Domains describes the structure of Bounded Complex Domains and their Shilov Boundaries such as:

- Weak Force
- SU(2) for Spin(5) / $SU(2) \times U(1)$ - Type IV Domain with Shilov Boundary = $RP^1 \times S^2$
Color Force
- SU(3) for $SU(4) / SU(3) \times U(1)$ - Type I Domain B^6 (ball) with Shilov Boundary S^5
Gravity
- Spin(5) for Spin(7) / $Spin(5) \times U(1)$ - Type IV Domain with Shilov Boundary $RP^1 \times S^4$

Hua calculated their Poisson Kernels and Bergman Kernels that act as Green's Function Propagators for Schwinger Sources.

Note - Hua uses the term "Characteristic Manifold" for "Shilov Boundary".

Cl(16) Physical Interpretation - BiVectors + half-Spinors = E8

For Cl(8) Triality gives

8-dim Vectors $V = 8$ -dim +half-Spinors $S_+ = 8$ -dim -half-Spinors S_- which are also the off-diagonal terms of 26-dim traceless $J_3(O)$ that defines the structure of 26-dim Cl(1,25) Strings=World-Lines Theory of Cl(16) TriVectors = 10 copies of 56-dim Fr₃(O) Freudenthal Algebra each of which is a complexification of the 27-dim $J_3(O)$ Jordan Algebra.

For Cl(16) = tensor product Cl(8)xCl(8) the Relation is a bit more complex. Cl(16) contains E8 which is a recipe for a Lagrangian with base manifold 8-dimSpacetime = Shilov Boundary of Cl(16) Vectors and with these Lagrangian Density terms:

Gauge Bosons = 28 of 120 Cl(16) BiVectors

Ghosts = 28 of 120 Cl(16) BiVectors

First-Generation Fermion Particles = Cl(16) ++quarter-Spinors

First-Generation Fermion AntiParticles = Cl(16) --quarter-Spinors

The Cl(16) Relation is between

Components of 28 Gauge Bosons of the 120-dim BiVectors

$8 \times 28 = 224$ each with Lagrangian weight 1

for total of 224

and

Components of 28 Ghosts of the 120-dim BiVectors

$8 \times 28 = 224$ each with Lagrangian weight 1

for total of 224

and

Components of ++quarter-Spinor Fermion Particles

$8 \times 8 = 64$ each with Lagrangian weight $7/2$

for a total of $32 \times 7 = 224$

and

Components of --quarter-Spinor Fermion AntiParticles

$8 \times 8 = 64$ each with Lagrangian weight $7/2$

for a total of $32 \times 7 = 224$

**The Lagrangian weight equality between
Gauge Bosons plus Ghosts and Particles plus AntiParticles
gives UltraViolet Finiteness by a Subtle Version of SuperSymmetry**

Cl(16) Physical Interpretation - half-Spinors First-Generation Fermions

1 Scalar + 63 8-Vectors = 8 spacetime components of 8 fermion particles with the 1 Scalar being the time component of the neutrino.

alf-

1 Pseudo-Scalar + 63 8-Vectors = 8 spacetime components of 8 fermion anti-particles with the 1 Pseudo-Scalar being the time component of the anti-neutrino.

$$\begin{aligned} \mathbf{1\ Scalar + 63\ 8-Vectors + 63\ 8-Vectors + 1\ Pseudo-Scalar} &= \mathbf{128} \\ &= \mathbf{half-Spinors\ of\ Cl(16)} \\ \mathbf{128} &= \mathbf{E8 / D8\ Symmetric\ Space} \end{aligned}$$

E8 is made up of Bosonic D8 and half-Spinor 128

Lie Algebras are usually represented by Bosonic commutators

Bosonic D8 part of E8 is represented by commutators

and half-spinor 128 can be represented by commutators

but

half-Spinor 128 can also be represented by anti-commutators

and therefore **half-Spinor 128 can physically represent Fermions.**

This property of E8 is inherited from F4, for which this property was shown by Pierre Ramond in hep-th/0112261, because 52-dim F4 lives in Cl(8) as 8-dim Vector spacetime + 28-dim BiVector Gauge Bosons + 16-dim Spinors and 8-Periodicity of Real Clifford Algebras shows that the tensor product $Cl(8) \times Cl(8) = Cl(16)$ so the F4 parts of $Cl(8) \times Cl(8) = Cl(16)$ give E8 with the half-Spinor part of E8 inheriting from the Spinor parts of the F4s the property of being representable by Fermionic anti-commutators.

The **half-Spinor Fermions are physically not point particles**

because as soon as they appear in Spacetime

the Tachyons of World-Line = String Theory of Cl(16) Fr3(O) TriVectors

form a particle-antiparticle cloud around the original Valence Particle so

that **they become Schwinger Sources**

whose Green's Function Propagator is determined by Bergman Kernel of Hua Domains of Gauge Forces.

Hua's calculations applied to Schwinger Sources allowed Armand Wyler to calculate their properties with realistic results:

Calculation Results

Here is a summary of Cl(16) Physics model calculation results. Since ratios are calculated, values for one particle mass and one force strength are assumed. Quark masses are constituent masses. Most of the calculations are tree-level, so more detailed calculations might be even closer to observations.

Dark Energy : Dark Matter : Ordinary Matter = 0.75 : 0.21 : 0.04

Fermions as Schwinger Sources have geometry of Complex Bounded Domains with Kerr-Newman Black Hole structure size about 10^{-24} cm.

Particle/Force	Tree-Level	Higher-Order
e-neutrino	0	0 for nu_1
mu-neutrino	0	9×10^{-3} eV for nu_2
tau-neutrino	0	5.4×10^{-2} eV for nu_3
electron	0.5110 MeV	
down quark	312.8 MeV	charged pion = 139 MeV
up quark	312.8 MeV	proton = 938.25 MeV
		neutron - proton = 1.1 MeV
muon	104.8 MeV	106.2 MeV
strange quark	625 MeV	
charm quark	2090 MeV	
tauon	1.88 GeV	
beauty quark	5.63 GeV	
truth quark (low state)	130 GeV	(middle state) 174 GeV (high state) 220 GeV
W+	80.326 GeV	
W-	80.326 GeV	
W0	98.379 GeV	Z0 = 91.862 GeV
Mplanck	1.217×10^{19} GeV	
Higgs VEV (assumed)	252.5 GeV	
Higgs (low state)	125 GeV	(middle state) 195 GeV (high state) 250 GeV

Particle/Force	Tree-Level	Higher-Order
Gravity Gg (assumed) 1 (Gg)(Mproton ² / Mplanck ²)		5 x 10 ⁽⁻³⁹⁾
EM fine structure	1/137.03608	
Weak Gw Gw(Mproton ² / (Mw+ ² + Mw- ² + Mz0 ²))	0.2535	1.05 x 10 ⁽⁻⁵⁾
Color Force at 0.245 GeV	0.6286	0.106 at 91 GeV

Kobayashi-Maskawa parameters for W+ and W- processes are:

	d	s	b
u	0.975	0.222	0.00249 -0.00388i
c	-0.222 -0.000161i	0.974 -0.0000365i	0.0423
t	0.00698 -0.00378i	-0.0418 -0.00086i	0.999

The phase angle d13 is taken to be 1 radian.

Coleman-Mandula

Lagrangian has 8-dim Lorentz structure satisfying Coleman-Mandula because its Fermionic fundamental spinor representations are built with respect to spinor representations for 8-dim Spin(1,7) spacetime - see Steven Weinberg, "The Quantum Theory of Fields" Volume III

Chirality

Lagrangian is Chiral because

E8 contains Cl(16) half-spinors (64+64) for a Fermion Generation but does not contain Cl(16) Mirror Fermion AntiGeneration half-spinors.

Fermion +half-spinor Particles with high enough velocity are seen as left-handed.

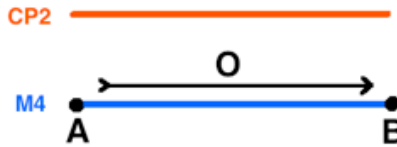
Fermion -half-spinor AntiParticles with high enough velocity are seen as right-handed.

3 Generations

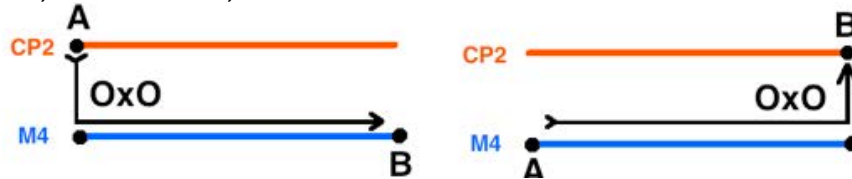
When 8-dim Octonionic Symmetry of Spacetime breaks down below Planck Energy to (4+4)-dim Quaternionic M4 x CP2 Kaluza-Klein with CP2 = SU(3) / SU(2) x U(1) the Higgs and Second and Third Generation Fermions emerge.

In Kaluza-Klein M4 x CP2 there are 3 possibilities for a Fermion represented by an Octonion O basis element to go from point A to point B:

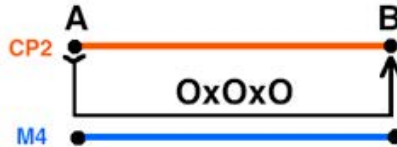
1 - A and B are both in M4: First Generation Fermion = Octonion O



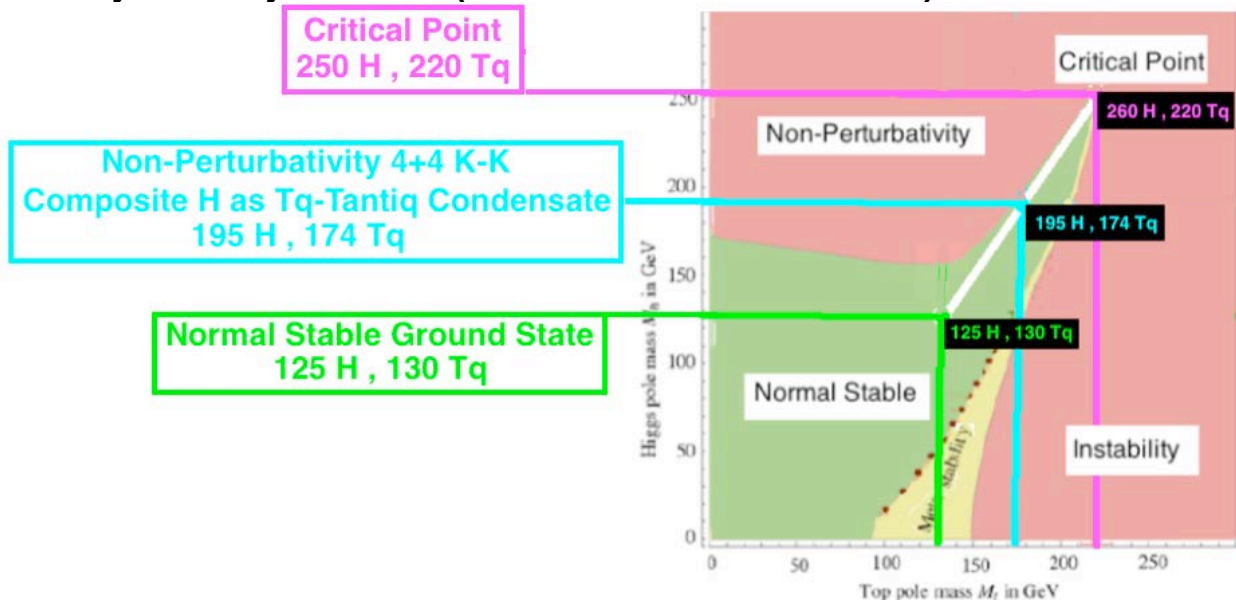
2 - Either A or B, but not both, is in CP2: Second Generation Fermion = OxO Pair



3 - Both A and B are in CP2: Third Generation Fermion = OxOxO Triple of Octonions



The Higgs and the Truth Quark form a Nambu-Jona-Lasinio System with 3 Mass States and the Higgs as Fermion Condensate dominated by the very massive (relative to other Fermions) Truth Quark



Spin-Statistics

Lagrangian obeys Spin-Statistics because the CP2 part of M4xCP2 Kaluza-Klein has index structure Euler number $2+1 = 3$ and Atiyah-Singer index $-1/8$ which is not the net number of generations because CP2 has no spin structure but you can use a generalized spin structure (Hawking and Pope (Phys. Lett. 73B (1978) 42-44)) to get (for integral m) the generalized CP2 index $n_R - n_L = (1/2) m (m+1)$

Prior to Dimensional Reduction from Octonionic Spacetime:

$$m = 1, n_R - n_L = (1/2) \times 1 \times 2 = 1 \text{ for 1 generation}$$

After Reduction to Quaternionic 4+4 Kaluza-Klein:

$$m = 2, n_R - n_L = (1/2) \times 2 \times 3 = 3 \text{ for 3 generations}$$

Hawking and Pope say: "Generalized Spin Structures in Quantum Gravity ...what happens in CP2 ... one could replace the electromagnetic field by a Yang-Mills field whose group G had a double covering $G\sim$. The fermion field would have to occur in representations which changed sign under the non-trivial element of the kernel of the projection ... $G\sim \rightarrow G$ while the bosons would have to occur in representations which did not change sign ...".

For E8 physics gauge bosons are in the $28+28=56$ -dim D4xD4 subalgebra. D4 = SO(8) is the Hawking-Pope G with double covering $G\sim = \text{Spin}(8)$. The 8 fermion particles / antiparticles are D4 half-spinors represented within E8 by anti-commutators and so do change sign while the 28 gauge bosons are D4 adjoint represented within E8 by commutators and so do not change sign.

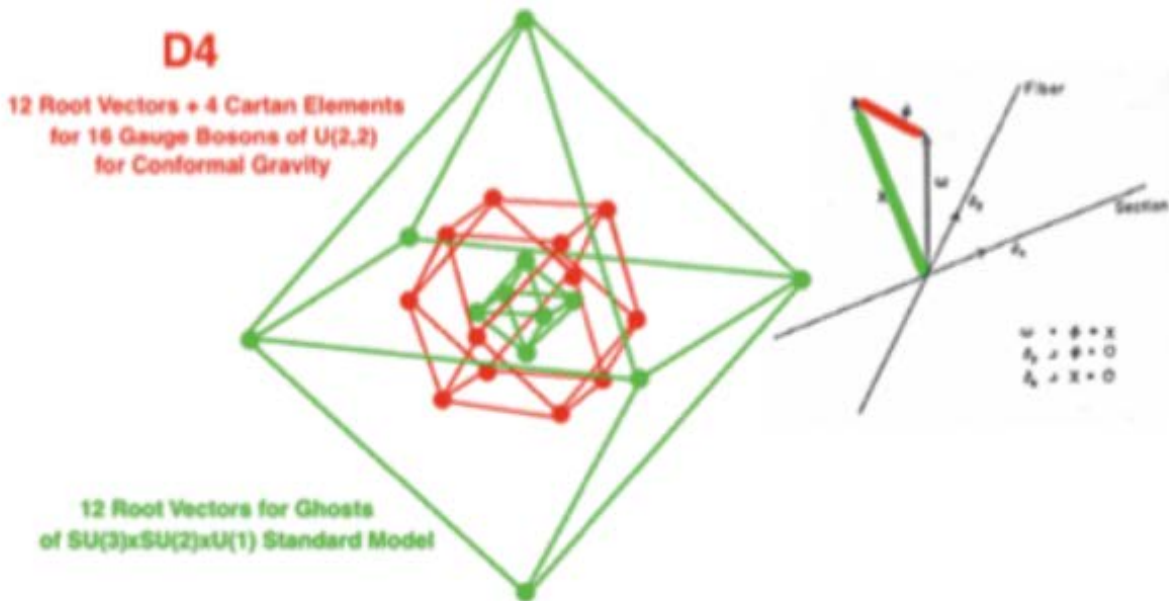
E8 Lagrangian inherits from F4 the property whereby its Spinor Part need not be written as Commutators but can also be written in terms of Fermionic AntiCommutators - see Pierre Ramond hep-th/0112261 - also, F4 lives in Cl(8) as Vectors + BiVectors + Spinors and by 8-Periodicity Cl(16) = tensor product Cl(8) x Cl(8) and E8 lives in Cl(16) as BiVectors + half-Spinors.

CI(16) Physical Interpretation - BiVectors Gauge Bosons and Ghosts and Unimodular Spacetime

120 BiVectors = D8 subalgebra of E8

D8 is made up of two copies of 28-dim D4 plus 64-dim A7+R

One D4 contains 16-dim U(2,2) of Conformal Gravity+Dark Energy
 28-16 = 12-dim = Ghosts of SU(3)xSU(2)xU(1) of the Standard Model



Jean Thierry-Mieg in J. Math. Phys. 21 (1980) 2834-2838 said:

“... The ghost and the gauge field:

The single lines represent a local coordinate system
of a principal fiber bundle of base space-time.

The double lines are 1 forms.

The connection of the principle bundle w is assumed to be vertical.
Its contravariant components ϕ and X are recognized, respectively,
as the Yang-Mills gauge field and the Faddeev-Popov ghost form ...”.

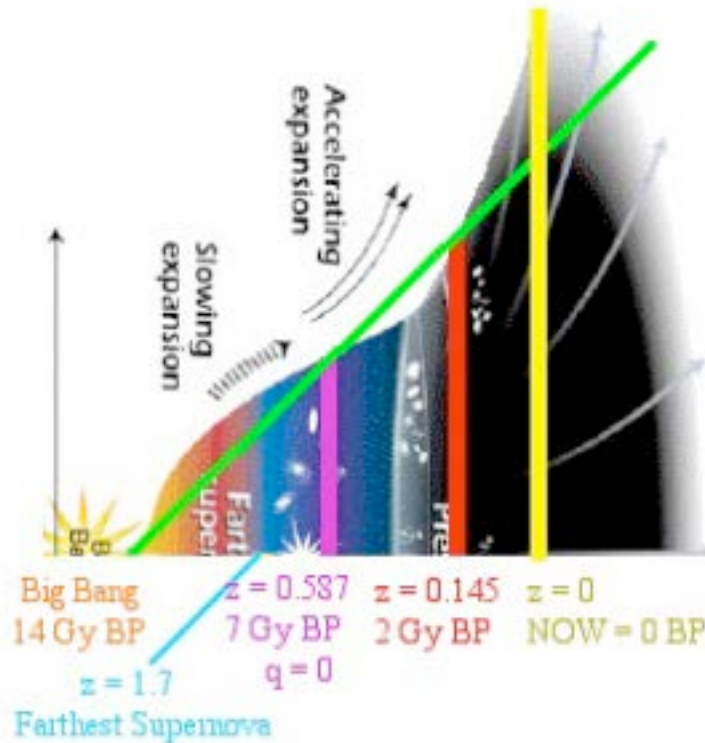
The ratio **Dark Energy : Dark Matter : Ordinary Matter** comes from the structure of the Conformal Group $SU(2,2) = Spin(2,4)$ whose 15 generators are:

- 10 = 6 Lorentz + 4 Special Conformal for Dark Energy
- 4 = Translations for Primordial Black Hole Dark Matter
- 1 = Dilation for Higgs Ordinary Matter

giving a tree-level ratio of

$$10 : 4 : 1 = 0.667 : 0.267 : 0.067$$

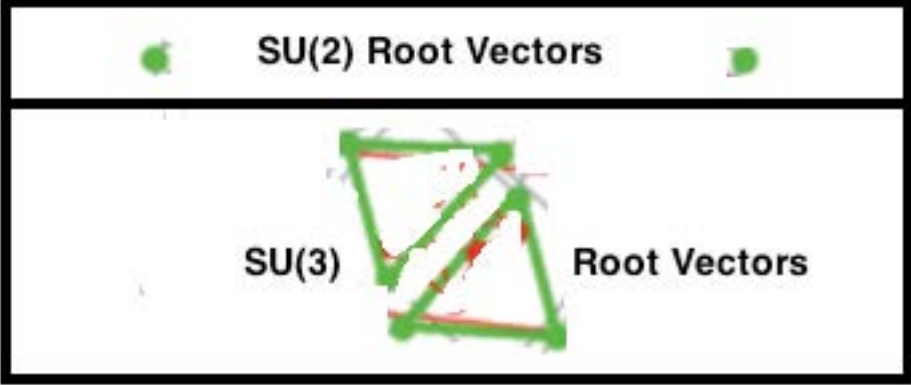
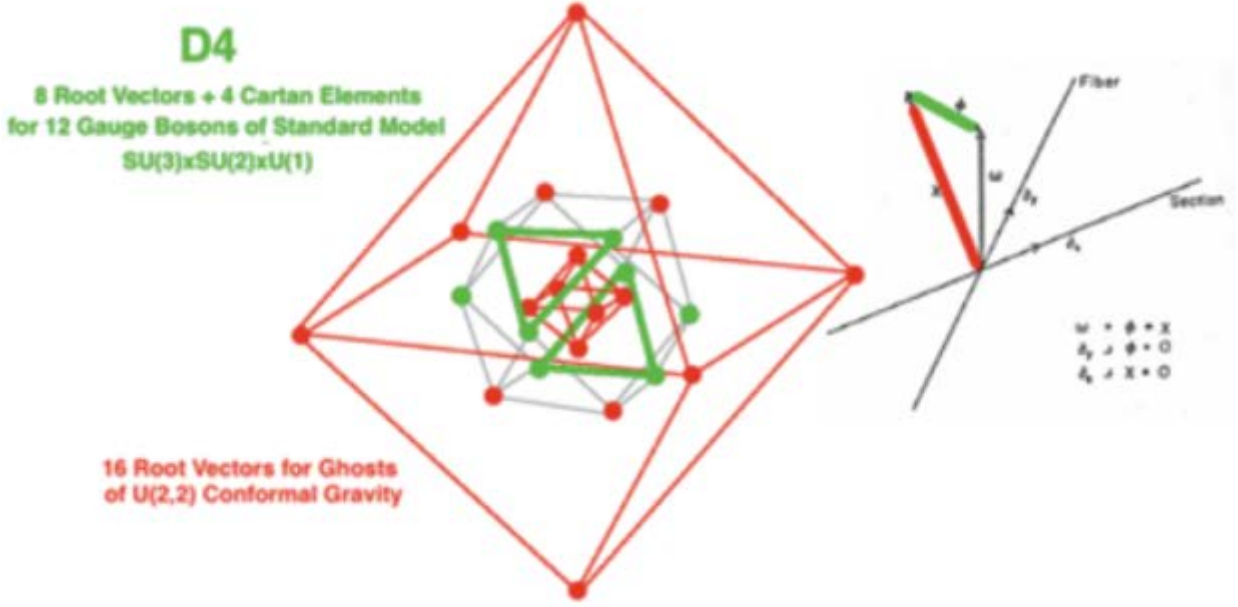
Taking Account of differences between Radiation and Matter Eras in the Evolution of Our Universe



gives

$$\begin{aligned} \text{Dark Energy : Dark Matter : Ordinary Matter} &= \\ &= 0.75 : 0.21 : 0.04 \end{aligned}$$

Other D4 contains SU(3) of Standard Model with SU(2)xU(1) from CP2
 $28 - 12(SU(3) \times SU(2) \times U(1)) = 16\text{-dim} = \text{Ghosts of Gravity} + \text{Dark Energy}$



A7+R = D8 / D4 x D4 Symmetric Space

Rutwig Campoamor-Stursberg in Acta Physica Polonica B 41 (2010) 53-77 ,
"Contractions of Exceptional Lie Algebras and SemiDirect Products" , showed that
the E8 Maximal Contraction of E8 = semi-direct product H92 x A7+R
where H92 is (8+28+56 +1+ 56+28+8)-dim Heisenberg Creation/Annihilation Algebra
and E8 Max Contraction has graded structure 8 + 28 + 56 + 64 + 56+ 28 + 8

grade -3 = Creation of 1 fermion (tree-level massless neutrino)
with 8 SpaceTime Components for a total of 8 fermion component creators

grade -2 = Creation of 8+3+1 = 12 Ghosts for Standard Model
and 16 Conformal U(2,2) Bosons for MacDowell-Mansouri Gravity

grade -1 = Creation of 7 massive Dirac fermion
each with 8 SpaceTime Components for a total of 56 fermion component creators

grade 0 = A7+R = 63 +1= 64-dim
Creation/Annihilation of 8-dim SpaceTime of HyperVolume Elements
A7 = Unimodular Relativity

Bradonjic and Stachel in arXiv 1110.2159 said: "... in ... Unimodular relativity ... the
metric tensor ... break[s up] ... into the conformal structure represented by a conformal
metric ... with $\det = -1$ and a ... volume element ... at each point of space-time ...
[that]... may be the remnant, in the ... continuum limit, of a more fundamental discrete
quantum structure of space-time itself ...".

Frampton, Ng, and Van Dam in J. Math. Phys. 33 (1992) 3881-3882 said:
"... Because of the existence of topologically nontrivial solutions, instantons, of the
classical field equations associated with quantum chromodynamics (QCD), the
quantized theory contains a dimensionless parameter ϑ ($0 < \vartheta < 2\pi$) ... Since ϑ
multiplies an expression odd in CP, QCD predicts violation of ... CP ... symmetry
unless the phase ϑ takes one of the special values ... $0 \pmod{\pi}$... this fine tuning is
the strong CP problem ... the quantum dynamics of ... unimodular gravity ... may lead to
the relaxation of ϑ to $\vartheta = 0 \pmod{\pi}$ without the need ... for ... the axion ...".

grade 1 = Annihilation of 7 massive Dirac fermions
each with 8 SpaceTime Components for a total of 56 fermion component annihilators

grade 2 = Annihilation of 8+3+1 = 12 Bosons for Standard Model
and 16 Conformal U(2,2) Ghosts for MacDowell-Mansouri Gravity

grade 3 = Annihilation of 1 fermion (tree-level massless neutrino)
with 8 SpaceTime Components for a total of 8 fermion component annihilators

**Conformal Unimodular Relativity is consistent
with Conformal Gravity+Dark Energy**

Cl(16) Physical Interpretation - TriVectors
World-Lines = Strings Bohm Quantum Potential
Schwinger Sources

560 TriVectors of Cl(16) represent 10 copies of 56-dim Fr3(O)
 56-dim Fr3(O) = Freudenthal Algebra of Zorn vector-matrices

$$\begin{array}{ccc}
 & & \begin{array}{ccc} d & S+ & V \end{array} \\
 & & \begin{array}{ccc} S+^* & e & S- \end{array} \\
 a & & \begin{array}{ccc} V^* & S-^* & f \end{array} \\
 \\
 \begin{array}{ccc} d' & S'+^* & V'^* \end{array} & & \\
 \begin{array}{ccc} S'+ & e' & S'-^* \end{array} & & b \\
 \begin{array}{ccc} V' & S'- & f' \end{array} & &
 \end{array}$$

where a, b, d, e, and f are real numbers; **S+**, **V**, **S-**, **S'+**, **V'**, and **S'-** are Octonions; and * denotes conjugation.

$$\begin{array}{ccc}
 d & S+ & V \\
 S+^* & e & S- \\
 V^* & S-^* & f
 \end{array}$$

is 27-dim J3(O) = 3x3 Hermitian Octonion Matrices whose traceless part is 26-dim J3(O) that describes

26D String Theory with Strings = World-Lines and

V = 8-dim Spacetime

the 8-real-dim space $RP^1 \times S^7$ that is the Shilov Boundary of the 16-real-dim $IV(8,2)$ Bounded Domain (tube type) of the BDI Symmetric Space $Spin(10) / Spin(8) \times U(1)$

S+ = 8 +half-Spinor Fermion Particles

the real part $RP^1 \times S^7$ of the Complex Shilov Boundary S of the 32-real-dim V non-tube type bounded Domain $(C \times O)P_2$ of the EIII Symmetric Space $E_6 / Spin(10) \times U(1)$.

S- = 8 - half-Spinor Fermion AntiParticles

$RP^1 \times S^7$ in the Complex part of the Shilov Boundary S of the 32-real-dim V non-tube type bounded Domain $(C \times O)P_2$ of the EIII Symmetric Space $E_6 / Spin(10) \times U(1)$

$Fr_3(O)$ has two copies of $J_3(O)$
and is a Complexification of $J_3(O)_o$ and of 26D String Theory
so

$Fr_3(O)$ is the structural basis for E8 World-Lines = Strings Theory

Strings = World-Lines of Particles interact by entire fine-grained histories.

Andrew Gray (quant-ph/9712037v2) said:

“... A new formulation of quantum mechanics ... assign[s] ... probabilities ... to entire fine-grained histories ... [It] is fully relativistic and applicable to multi-particle systems ... [and]... makes the same experimental predictions as quantum field theory ... consider space and time cut up into small volume elements ... and then take the limit as ... volume ... $\rightarrow 0$... get the final amplitude ... by considering all possible distributions at a time t earlier ... the interference factor ... is a measure of how much interference between the different possible histories that contain the distribution of interest there is at each time ... This result is the ... Feynman amplitude squared times the product of all the interference factors ...”.

Luis E. Ibanez and Angel M. Uranga in “String Theory and Particle Physics” said:

“... As a string evolves in time, it sweeps out a two-dimensional surface in spacetime, known as the worldsheet, which is the analog of the ... worldline of a point particle ... for the bosonic string theory ... the classical string action is the total area spanned by the worldsheet ... This is the ... Nambu– Goto action ...”.

The Gray Fine-Grained History Quantum Theory with World-Line = String is equivalent



to the Nambu-Goto action of 26D String Theory. Ibanez and Uranga also said:

“... The string groundstate corresponds to a 26d spacetime tachyonic scalar field $T(x)$. This tachyon ... is ... unstable ...

The massless two-index tensor splits into irreducible representations of $SO(24)$...

Its trace corresponds to a scalar field, the dilaton ϕ , whose vev fixes the string interaction coupling constant g_s ...

the antisymmetric part is the 26d 2-form field BMN ...

The symmetric traceless part is the 26d ... GMN ...”.

Closed string tachyons localized at orbifolds of fermions produce virtual clouds of particles / antiparticles that dress fermions as Schwinger Sources

The antisymmetric $SO(24)$ little group is related to the Monster automorphism group that is the symmetry of each cell of Planck-scale local lattice structure.

The symmetric traceless part is the Quantum Bohmion.

Cl(1,25) AQFT Quantum Consciousness

The 560 TriVectors of Cl(16) are 10 copies of $56 = \text{Fr}_3(\text{O})$

$\text{Fr}_3(\text{O})$ is Complexification of $\text{J}_3(\text{O})$

$\mathbf{V} = \text{Fr}_3(\text{O})$ Spacetime is a Superposition of 8 E8 8-dim Spacetime Lattices
(7 being Integral Domains)

Each Fermion Type propagates within its own E8 Lattice within the Superposition which accounts for 8 of the 10 copies of $\text{Fr}_3(\text{O})$

The other 2 copies of $\text{Fr}_3(\text{O})$ correspond to the 2 diagonal elements d and f which describe the 10-dim $\text{R}(1,9)$ space that is Conformal over 8-dim $\text{R}(0,8)$ space which has Clifford Algebra $\text{Cl}(0,8) = \text{Cl}(1,7)$ of $\text{RP}^1 \times \text{S}^7$

The two copies of $\text{J}_3(\text{O})$ within each $\text{Fr}_3(\text{O})$ correspond to Real and Imaginary 26D String Theories where

the Real Part represents the Shilov Boundary
of the Bulk Complex Bounded Domain.

The Clifford Algebra of 26D String Theory is $\text{Cl}(1,25)$

$\text{Cl}(1,25) = \text{M}(2, \text{Cl}(0,24)) = 2 \times 2$ matrices with entries in $\text{Cl}(0,24) =$
 $=$ Conformal Structure over $\text{Cl}(0,24)$

$\text{Cl}(0,24) = \text{Cl}(0,8) \times \text{Cl}(0,16)$

$\text{Cl}(0,16)$ has 16-dim Barnes-Wall Lattice Structure

$\text{Cl}(0,8)$ has 8-dim E8 Lattice Structure

so

$\text{Cl}(0,24)$ has 24-dim Leech Lattice Structure

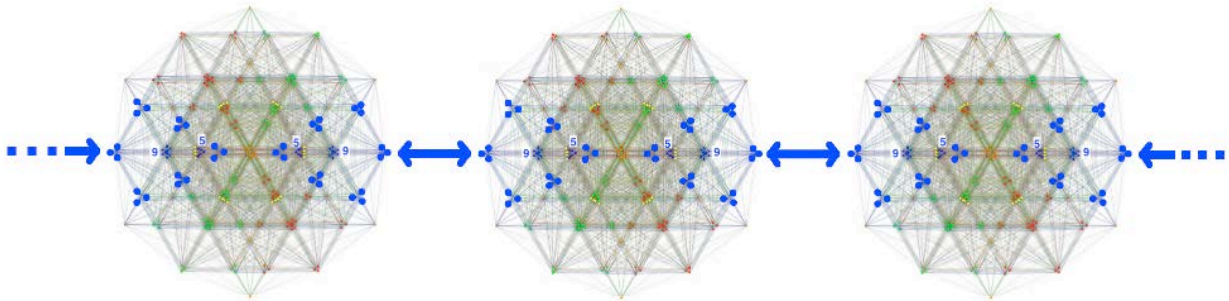
and

$\text{Cl}(1,25) = \text{M}(2, \text{Cl}(0,24))$ has Lorentz Leech Lattice Structure
with Monster Group Symmetry of each Lattice Cell

Since all matrix entries are $\text{Cl}(0,24) =$ tensor product of 3 copies of $\text{Cl}(0,8)$
8-Periodicity allows formation of the tensor products of copies of $\text{Cl}(1,25)$

One $Cl(1,25)$ containing one $Cl(0,16)$ containing one E_8 gives a Lagrangian description of one local spacetime neighborhood.

To get a realistic global spacetime structure, take the tensor product $Cl(1,25) \times \dots \times Cl(1,25)$ with all E_8 local 8-dim Octonionic spacetimes consistently aligned as described by [64-dim \$D_8 / D_4 \times D_4\$](#) (this visualization uses hexagonal 2D projection of the 240 E_8 root vectors)



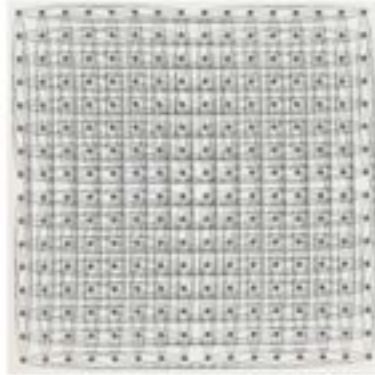
**Completion of the Union of all Tensor Products
of the form $Cl(1,25) \times \dots$ (N times tensor product) $\dots \times Cl(1,25)$
gives an Algebraic Quantum Field Theory (AQFT)**

For $N = 2^8 = 256$ the copies of $Cl(1,25)$ are on the 256 vertices of the 8-dim HyperCube

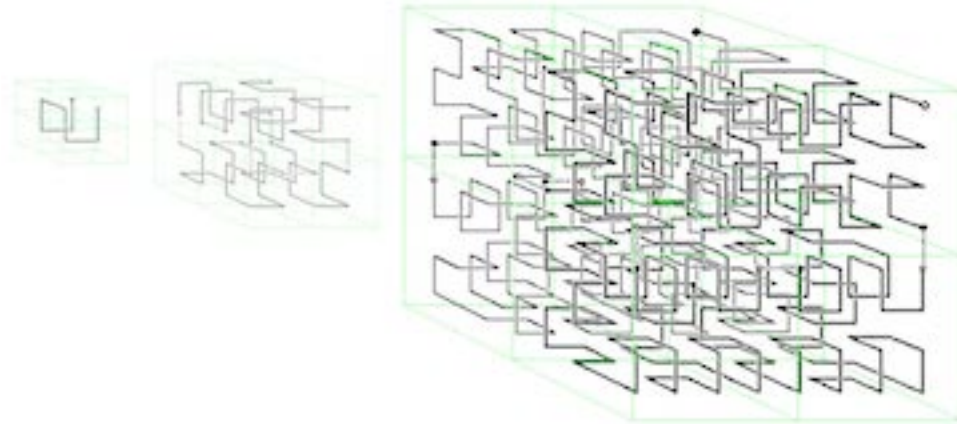
For $N = 2^{16} = 65,536 = 4^8$ the copies of $Cl(1,25)$ fill in the 8-dim HyperCube as described by William Gilbert's web page: "... The n-bit reflected binary [Gray](#) code will describe a path on the edges of an n-dimensional cube that can be used as the initial stage of a Hilbert curve that will fill an n-dimensional cube. ...".

The vertices of the Hilbert curve are at the centers of the 2^8 sub-8-HyperCubes whose edge lengths are $1/2$ of the edge lengths of the original 8-dim HyperCube

As N grows, the copies of $Cl(1,25)$ continue to fill the 8-dim HyperCube of E_8 SpaceTime using higher Hilbert curve stages from the 8-bit reflected binary Gray code subdividing the initial 8-dim HyperCube into more and more sub-HyperCubes.



If edges of sub-HyperCubes, equal to the distance between adjacent copies of $Cl(1,25)$, remain constantly at the Planck Length, then the full 8-dim HyperCube of our Universe expands as N grows to 2^{16} and beyond similarly to the way shown by this 3-HyperCube example for $N = 2^3, 4^3, 8^3$ from Wiliam Gilbert's web page:



The Union of all $Cl(1,25)$ tensor products is the Union of all subdivided 8-HyperCubes and their Completion is a huge superposition of 8-HyperCube Continuous Volumes which Completion belongs to the Third Grothendieck Universe and is an AQFT that is a Real Clifford Algebra generalization of the Complex Clifford Algebra Fock Space Hyperfinite II_1 von Neumann factor Algebra

**The Third Grothendieck Universe AQFT contains
 within each $CI(1,25)$
 a realistic $E8$ Lagrangian within $CI(16)$**

and

contains

**10 copies of 56-dim $Fr_3(O)$
 are 560 TriVectors of $CI(16)$
 to describe a World-Lines = Strings String Theory
 with Bohm Quantum Potential
 arising from Nambu-Goto Action**



Similarity of the Bohmion carrier of the Bohm Quantum Potential to the spin 2 Graviton accounts for the Bohmion's ability to support Penrose Consciousness with Superposition Separation Energy Difference $G m^2 / a$ where, for a Human Brain,

m = mass of electron and a = 1 nanometer in Tubulin Dimer

“... Bohm's Quantum Potential can be viewed as an internal energy of a quantum system ...”

according to Dennis, de Gosson, and Hiley (arXiv 1412.5133)
 and

Bohm Quantum Potential inherits Sarfatti Back-Reaction
 from its similarity to General Relativity

Peter R. Holland says in "The Quantum Theory of Motion": "... the total force ... from the quantum potential ... does not ... fall off with distance ... because ... the quantum potential ... depends on the form of ... [the quantum state]... rather than ... its ... magnitude ...".

Penrose-Hameroff-type Quantum Consciousness is due to Resonant Quantum Potential Connections among Quantum State Forms.

The Quantum State Form of a Conscious Brain is determined by the configuration of a subset of its 10^{18} to 10^{19} Tubulin Dimers described by a large Real Clifford Algebra.

Paola Zizzi in gr-qc/0007006 describes the Octonionic Inflation Era of Our Universe as a Quantum Consciousness Superposition of States ending with Self-Decoherence after 64 doublings of Octonionic Inflation, at which time Our Universe is "... a superposed state of quantum ... [qubits]. the self-reduction of the superposed quantum state is ... reached at the end of inflation ... [at]... the decoherence time ... [$T_{\text{decoh}} = 10^9 T_{\text{planck}} = 10^{(-34)}$ sec] ... and corresponds to a superposed state of ... [$10^{19} = 2^{64}$ qubits]. ...".

64 doublings to 2^{64} qubits corresponds to the Clifford algebra $Cl(64) = Cl(8 \times 8) = Cl(8) \times Cl(8) \times Cl(8) \times Cl(8) \times Cl(8) \times Cl(8) \times Cl(8) \times Cl(8)$. By the periodicity-8 theorem of Real Clifford algebras, $Cl(64)$ is the smallest Real Clifford algebra for which we can reflexively identify each component $Cl(8)$ with a basis vector in the $Cl(8)$ vector space. This reflexive identification causes our universe to decohere at $N = 2^{64} = 10^{19}$.

The End of Inflation time was at about $10^{(-34)}$ sec = $2^{64} T_{\text{planck}}$ and the size of our Universe was then about $10^{(-24)}$ cm which is about the size of a Schwinger Source Kerr-Newman Cloud.

What about information in the Microtubules of Human Consciousness ?

The information in one Microtubule is based on Cl(16)
which is contained in the Cl(1,25) of 26D String Theory E8 Physics

How does this give rise to Penrose-Hameroff Quantum Consciousness ?

Consider the Superposition of States State 0 and State 1 involving one
Tubulin Dimer with Conformation Electron mass m and State1 / State 0
position separation a .

The Superposition Separation Energy Difference is the internal energy

$$E_{\text{ssediff}} = G m^2 / a$$

that can be seen as the energy of 26D String Theory Bohmions
which physically represent the Bohm Quantum Potential internal energy.

For a given Tubulin Dimer $a = 1$ nanometer = 10^{-7} cm so that
 $T = h / E_{\text{electron}} = (\text{Compton} / \text{Schwarzschild}) (a / c) = 10^{26}$ sec =
 10^{19} years

Now consider the case of N Tubulin Dimers with total mass Nm in Coherent
Superposition connected by the Bohm Quantum Potential Force that does
not fall off with distance

$$E_{\text{ssediff}} = G (Nm)^2 / a$$

Jack Sarfatti defines coherence length L by $L^3 = N a^3$ so that
the Superposition Energy E_N of N superposed Conformation Electrons is

$$\begin{aligned} E_N &= G (Nm)^2 / L = G (Nm)^2 / (a N^{1/3}) = N^{5/3} Gm^2 / a = \\ &= N^{5/3} E_{\text{ssediff}} \end{aligned}$$

The decoherence time for the system of N Tubulin Electrons is

$$T_N = h / E_N = h / N^{5/3} E_{\text{ssediff}} = N^{-5/3} 10^{26} \text{ sec}$$

so we have the following rough approximate Decoherence Times T_N

Number of Involved
Tubulin Dimers

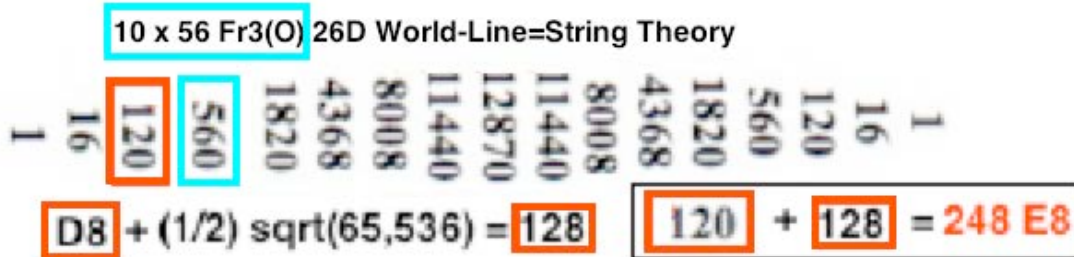
Time
 T_N

$10^{(11+9)} = 10^{20}$ $10^{(-33 + 26)} = 10^{(-7)}$ sec 10^{11} neurons x 10^9 TD / neuron
 10^{20} Tubulin Dimers in Human Brain

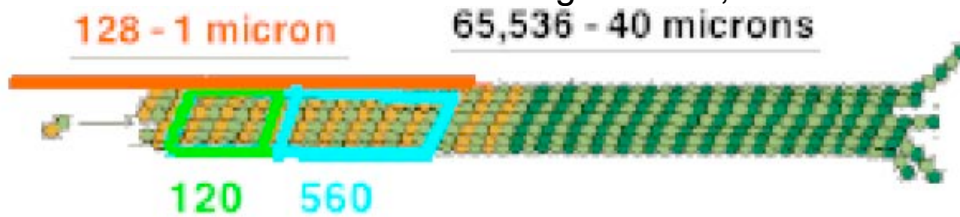
10^{16} $10^{(-27 + 26)} = 10^{(-1)}$ sec - 10 Hz
Human Alpha EEG is 8 to 13 Hz
Fundamental Schumann Resonance is 7.8 Hz

Time of Traverse by a String World-Line Quantum Bohmion of a Quantum
Consciousness Hamiltonian Circuit of 10^{16} TD separated from nearest neighbors
by 10 nm is $10^{16} \times 10 \text{ nm} / c = (10^{16} \times 10^{(-6)}) \text{ cm} / c = 10^{10} \text{ cm} / c = 0.3 \text{ sec}$

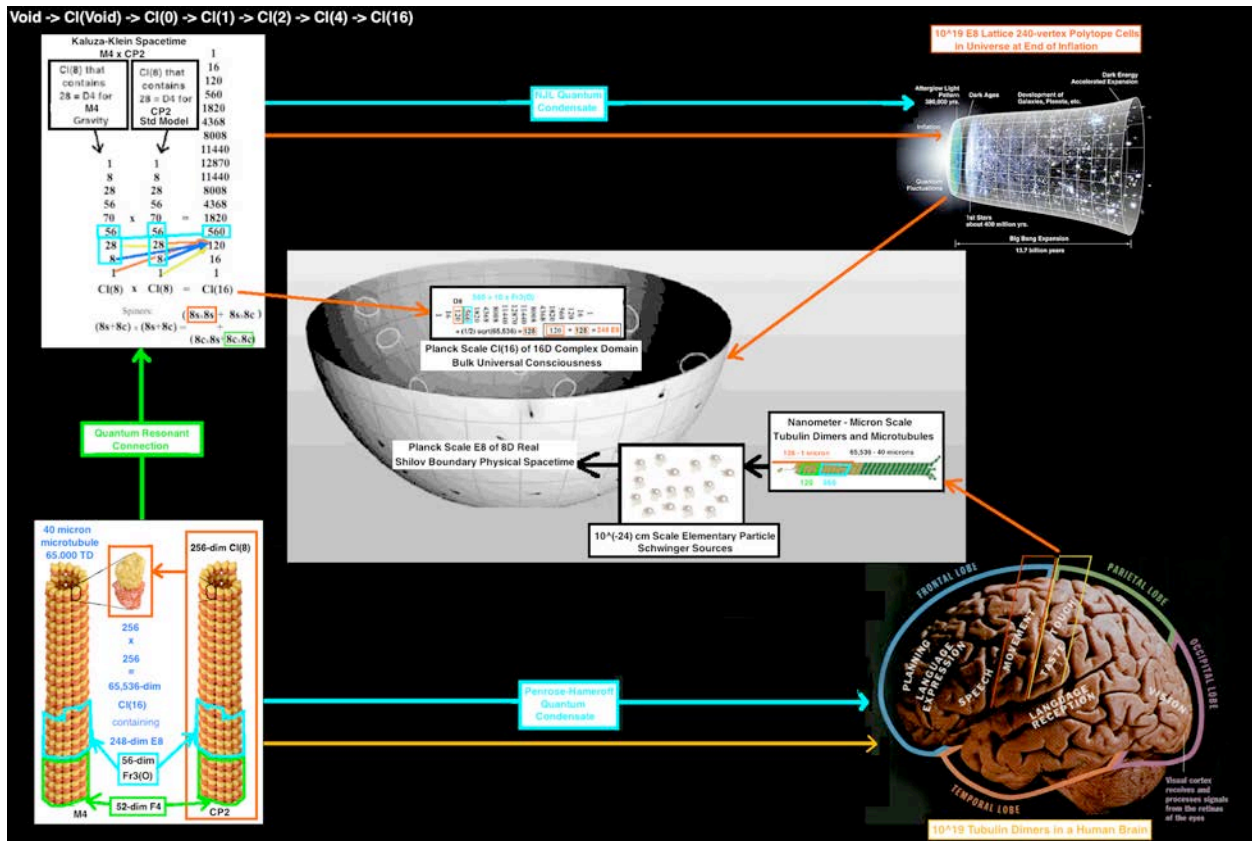
Cl(16) has 65,536 elements.



Human Brain Microtubules 40 microns long have 65,536 Tubulin Dimers



so can have Bohm Quantum Resonance with Cl(16) Spacetime cells



so that at any and all Times the State of Consciousness of a Human is in exact resonant correspondence with a subset of the cells of E8 Classical Lagrangian Spacetime Therefore E8 Classical Lagrangian Spacetime Nis effectively the Spirit World in which the Human States of Consciousness = Souls exist. After the death of the Human Physical Body the Spirit World interactions with its Soul are no longer constrained by Physical World interactions with its Body so that the Spirit World can harmonize the individual Soul with the collective Universal Soul. William Kingdon Clifford, who invented Real Clifford Algebras, called them “mind-stuff”, saying: “...

When matter takes the complex form of a living human brain, the corresponding mind-stuff takes the form of a human consciousness ...”.

Appendix: History of Real Clifford Algebras

Real Clifford Algebras have 8-Periodicity whereby

$$Cl(8N) = Cl(8) \times \dots (N \text{ times tensor product}) \dots Cl(8)$$

so that $Cl(8) = M(16, \mathbb{R}) = 16 \times 16$ Real Matrices is their basic building block.

Ancient (over 50,000 years ago) African IFA Divination has the structure of $Cl(8)$ (dimension $2^8 = 256 = 16 \times 16$)

National Geographic Genographic Y-DNA project showed that around 50,000 to 40,000 years ago humans began to leave Africa:

M96 went up the Nile River to Giza

Two Giza Pyramids and Sphinx represent $Cl(8) \times Cl(8)$ and $Cl(16)$

M89 went across the Red Sea to the Persian Gulf (then fertile land Eden)

Torah has $613 = 248 + 365$ commandments

248 E8 is in $Cl(16)$ - 365 center of 27×27 Magic Square of $Fr_3(O)$ in $C(16)$

M174 went around the Asian Coast to the Sunda Shelf (then fertile land) and on to Japan and Tibet

Angkor Temples Mt. Meru Binomial Triangle of Clifford Algebra Gradings

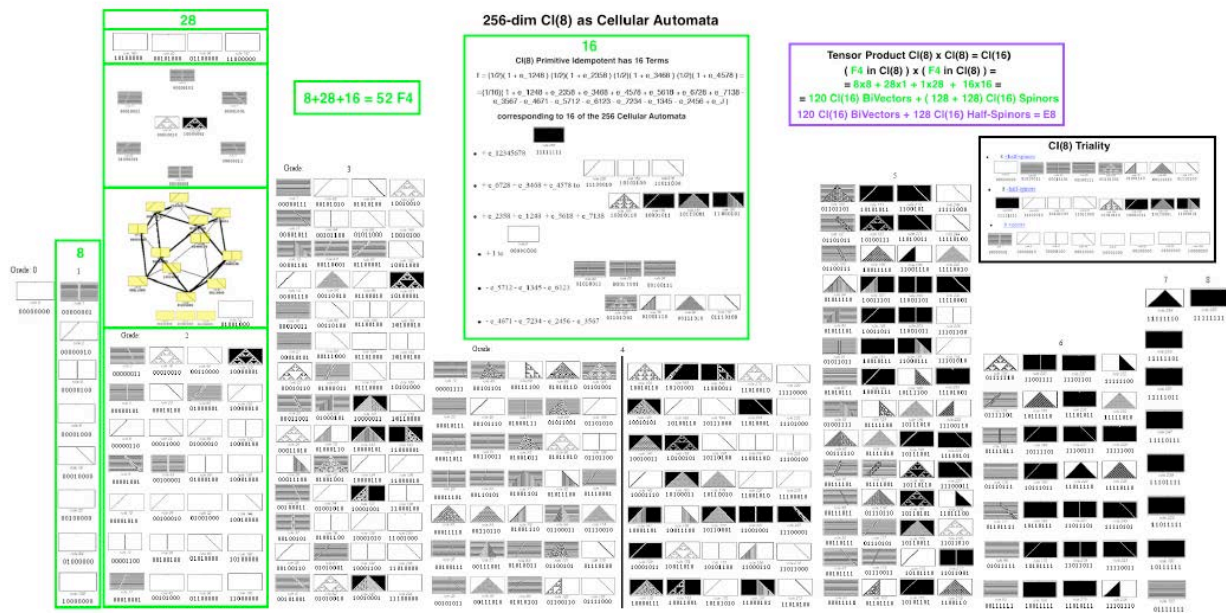
Rg Veda = $24 + 24 + 64 + 64 = 64 = 240$ Root Vectors of E8 in $Cl(16)$



Ron Eglash (in his book "African Fractals" (Rutgers 1999) and on his web site) says: "... a historical path for base-2 calculation ... begins with African divination, runs through the geomancy of European alchemists, and is finally transformed into binary calculation, where it is now applied in every digital circuit ...".

Raymond Aschheim (email May 2015) said, about Cellular Automata (CA):
 "... An elementary CA is defined by the next value (either 0 or 1) for a cell, depending on its ... value, and the ... value of it[s] left and of it[s] right neighbor cell (it is one dimensional, and involve only the first neighbors, and the cell itself) ... So the next value depends [on] 3 bits ... eight possible combination of three bits, and for each ... combination... the next value is either zero or one.
 So the[re] are 256 ... CAs ...".

The 256 Elementary Cellular Automata correspond to the 256-dim Cl(8) Clifford Algebra with graded structure 1 8 28 56 (35+35=70) 56 28 8 1



28

Grade: 0

8

1

Grade: 2

2

Grade: 3

3

$$8 + 28 + 16 = 52 \text{ F4}$$

Grade: 3

3

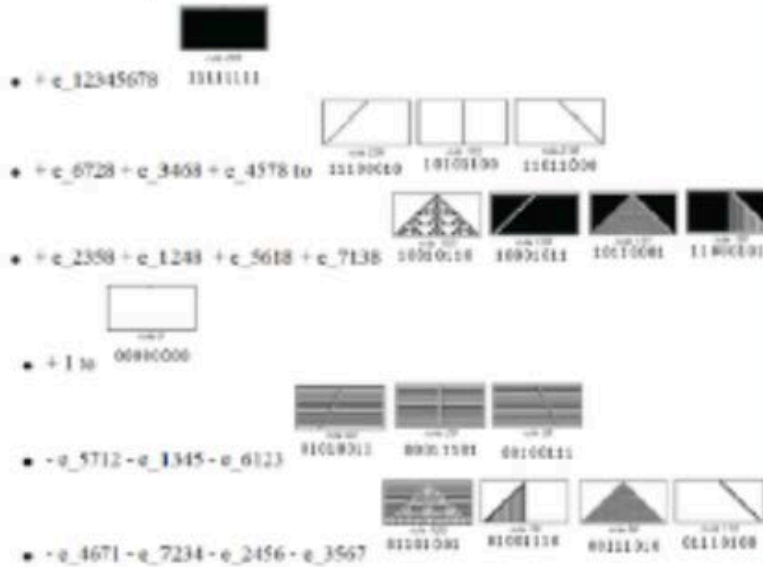
256-dim Cl(8) as Cellular Automata

16

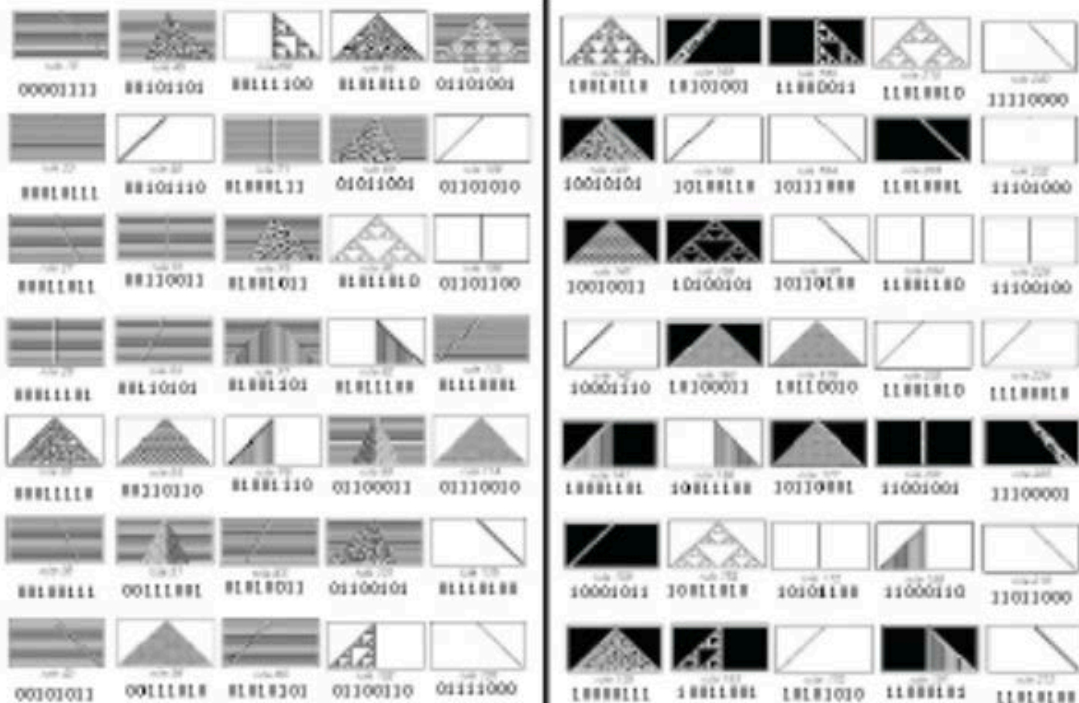
Cl(8) Primitive Idempotent has 16 Terms

$$f = (1/2)(1 + e_{1248}) (1/2)(1 + e_{2358}) (1/2)(1 + e_{3468}) (1/2)(1 + e_{4578}) = \\ = (1/16)(1 + e_{1248} + e_{2358} + e_{3468} + e_{4578} + e_{5618} + e_{6728} + e_{7138} - \\ - e_{3567} - e_{4671} - e_{5712} - e_{6123} - e_{7234} - e_{1345} - e_{2456} + e_{\dots})$$

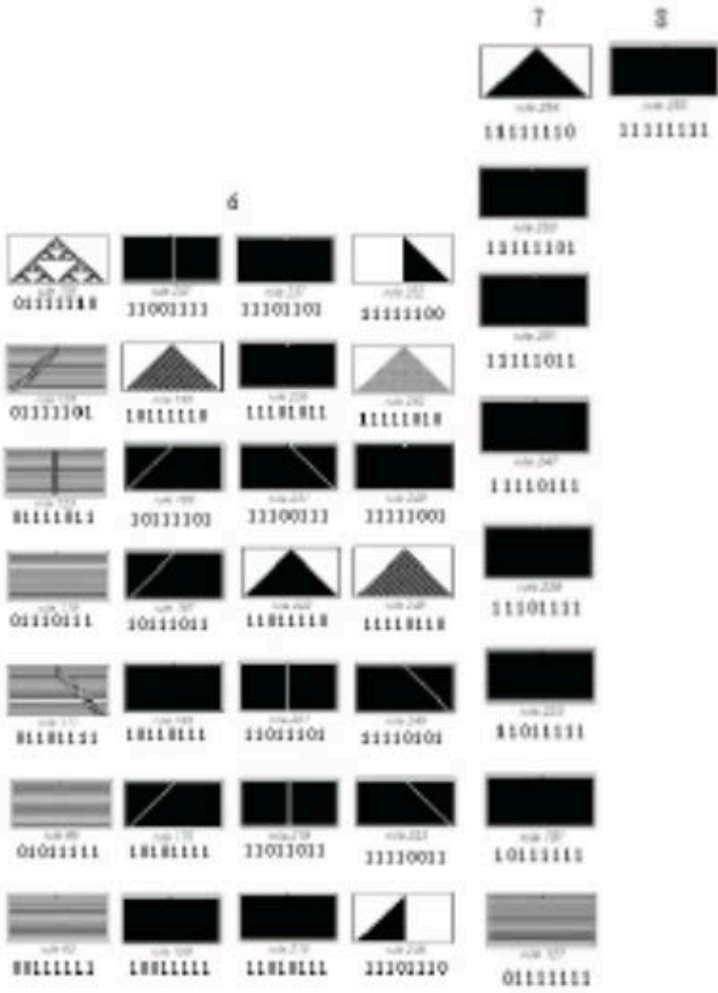
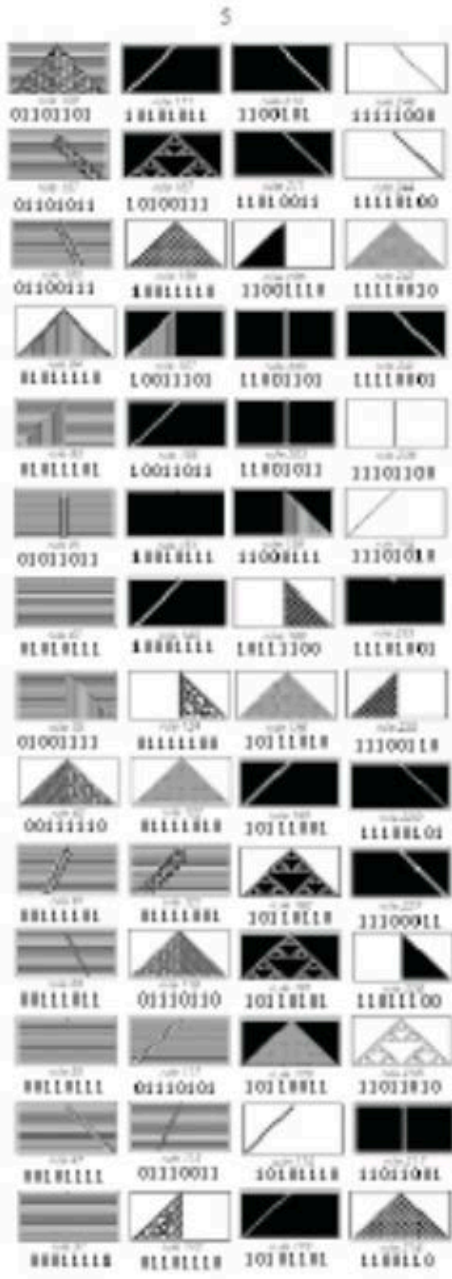
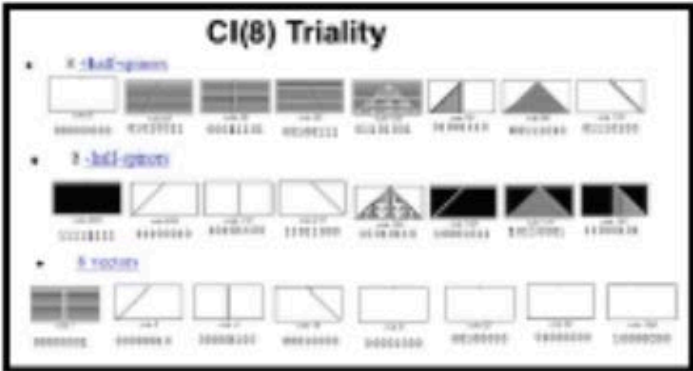
corresponding to 16 of the 256 Cellular Automata



Outside:



Tensor Product $Cl(8) \times Cl(8) = Cl(16)$
 $(F_4 \text{ in } Cl(8)) \times (F_4 \text{ in } Cl(8)) =$
 $= 8 \times 8 + 28 \times 1 + 1 \times 28 + 16 \times 16 =$
 $= 120 Cl(16) \text{ BiVectors} + (128 + 128) Cl(16) \text{ Spinors}$
 $120 Cl(16) \text{ BiVectors} + 128 Cl(16) \text{ Half-Spinors} = E_8$



CI(8) CA with U(2,2) D4 Gauge Bosons

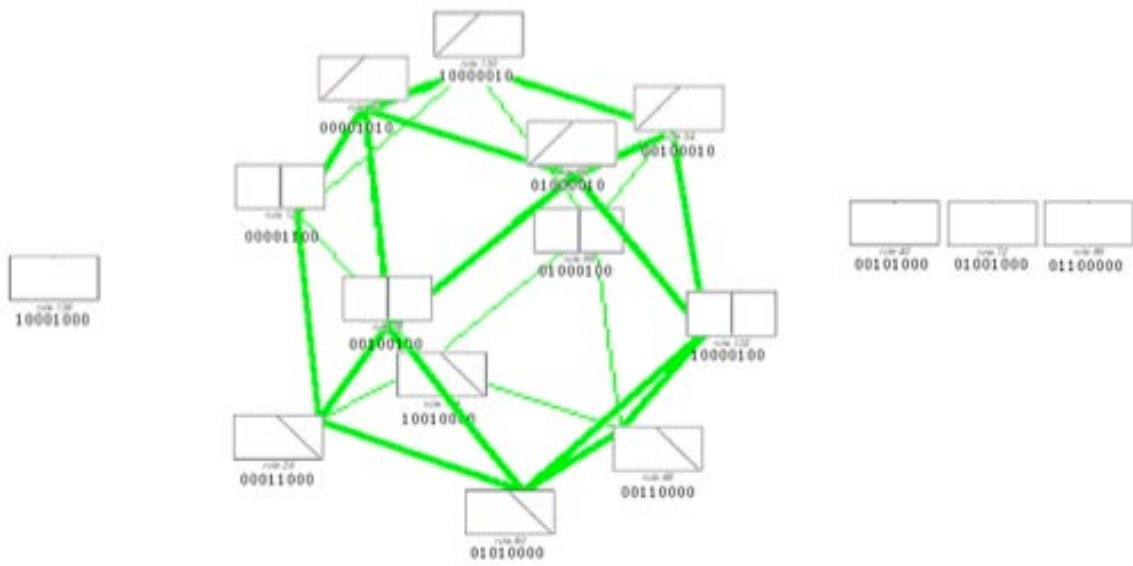
8
28 B-Vectors of Second D4
16 U(2,2) Gauge Bosons

8+28+16 = 62 F4

16
C80 Primitive Components Has 16 Terms
 $4 \times (1001 + \dots + 1001) + 4 \times (1010 + \dots + 1010) + 4 \times (1100 + \dots + 1100) + 4 \times (1110 + \dots + 1110)$
 corresponding to 16 of the 28 Cellular Automata

Tensor Product C(8) x C(8) = C(16)
 $(F_4 \text{ in } C(8)) \times (F_4 \text{ in } C(8)) = 8 \times 8 + 28 \times 1 + 14 \times 2 + 16 \times 10 = 120$
 $120 \text{ C}(16) \text{ B-Vectors} + (128 + 128) \text{ C}(16) \text{ Spinors} = 376$
 $120 \text{ C}(16) \text{ B-Vectors} + 128 \text{ C}(16) \text{ Half-Spinors} = 248$

C(8) Triviality



Great Giza Pyramid

1
8
28
56
70
56
28
8
1

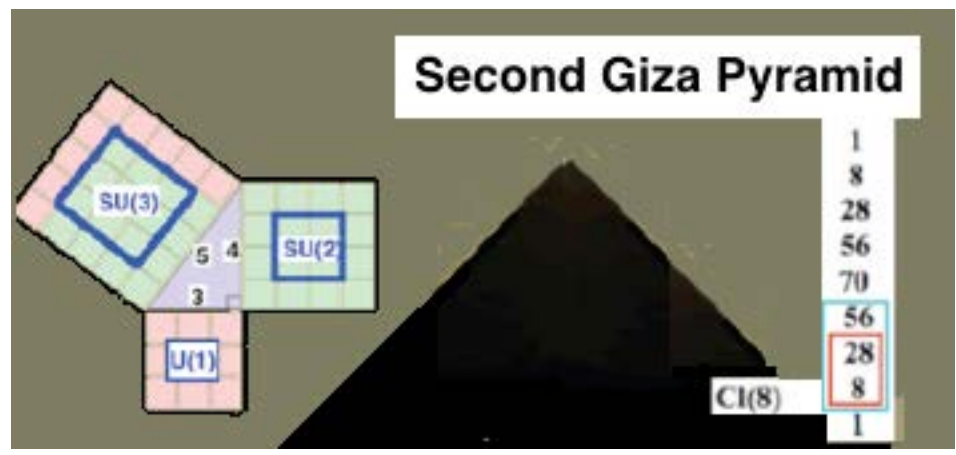
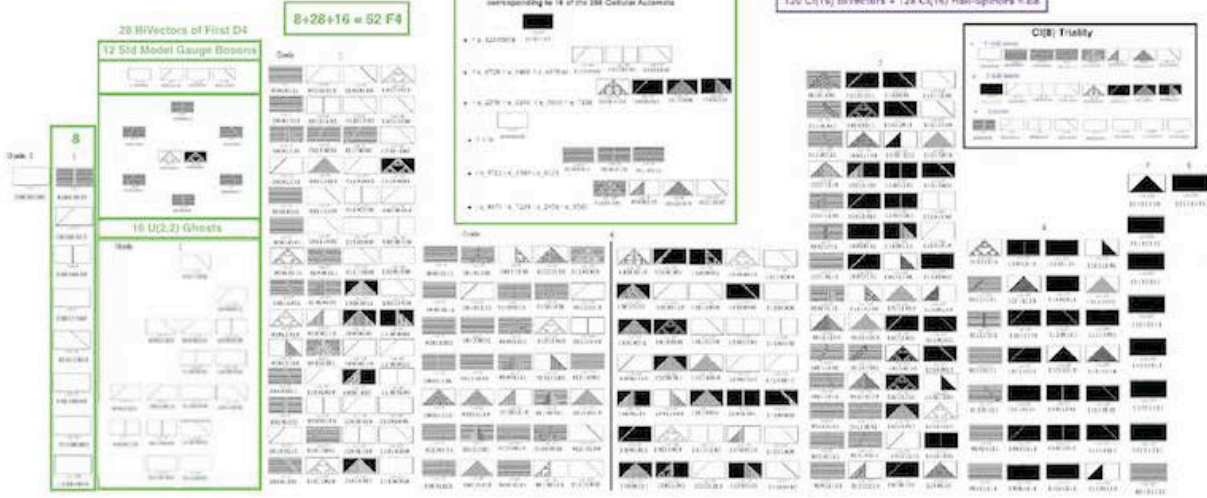
CI(8)

ϕ^2

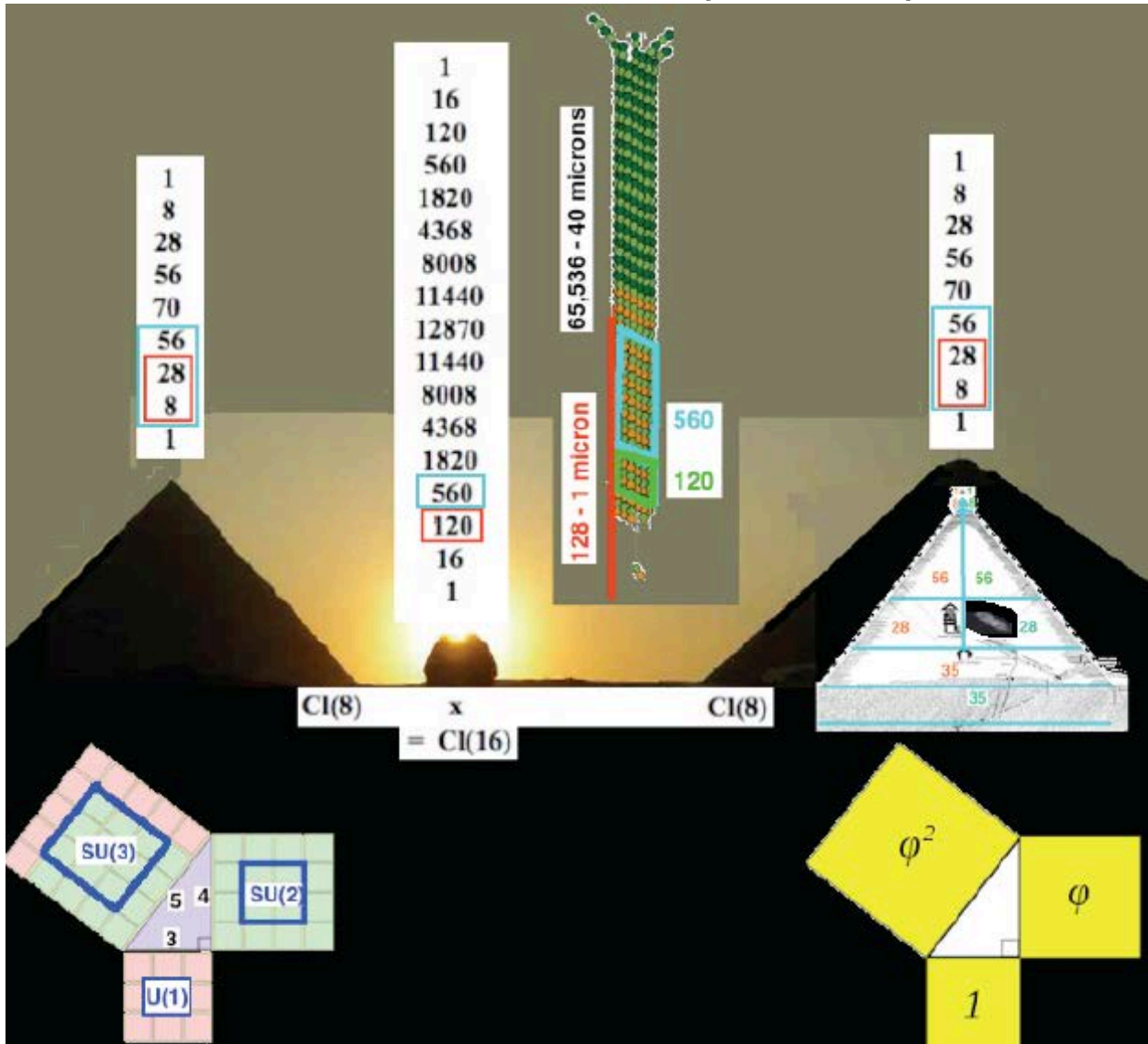
ϕ

1

CI(8) CA with Std Model D4 Gauge Bosons



Here are some details about the Giza Pyramids and Sphinx:



The **Great Pyramid** (right in image above) has

203 layers + 35 underground + 1+8+8+1 capstone = 256 = $Cl(8)$

It has a Grand Gallery for Physical World and a Grand Void for Spirit World

Its slope is Golden Ratio Triangle (H4 in E8 has Golden Ratio structure)

The Golden Ratio governs the Gravitational Spiral of Galaxies

The **Second Pyramid** (left in image above) has slope of 3-4-5 Triangle

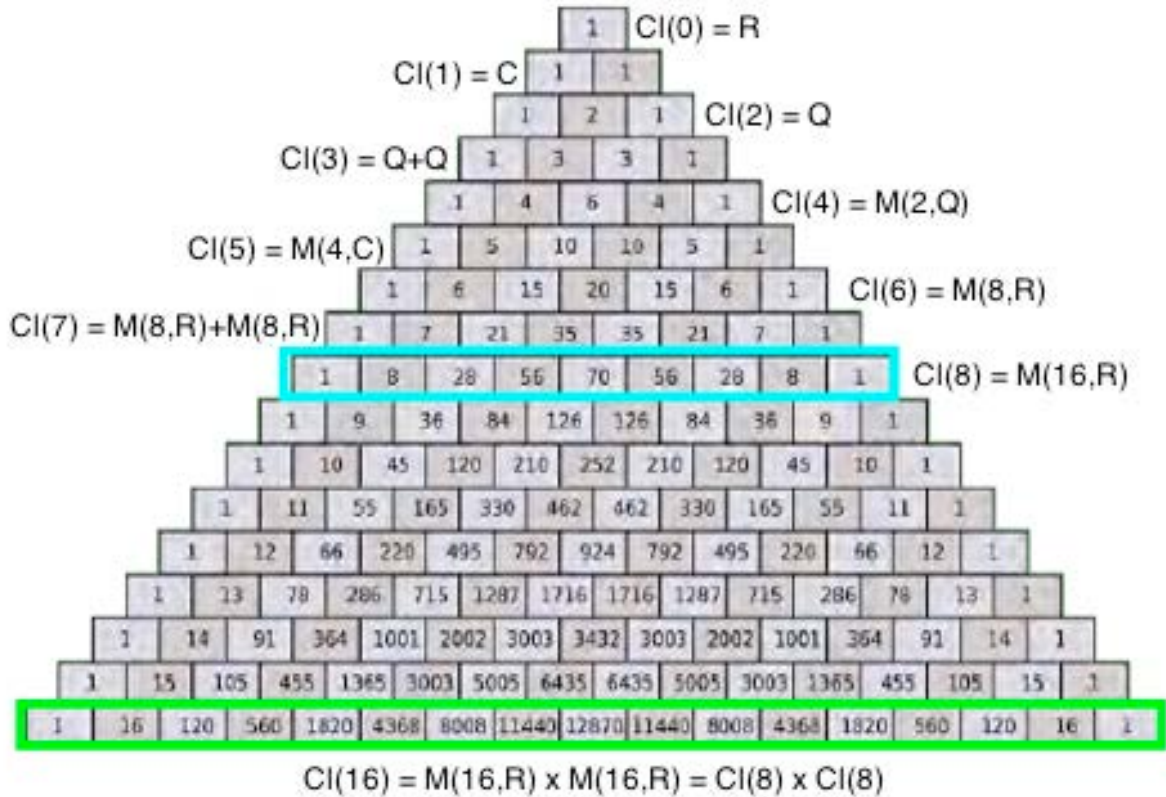
corresponding to U(1) - SU(2) - SU(3) Standard Model Gauge Groups

The **Sphinx** represents $Cl(16) = Cl(8) \times Cl(8) = \text{Great} \times \text{Second Pyramids}$

and shows correspondence between 65,536-dim $Cl(16)$ Physics cell and 65,536 Tubulins of maximal Microtubule Quantum Consciousness cell

Here are some details about Mt. Meru Binomial Graded Structure and the Rg Veda:

Angkor Temples contain representations of Mt. Meru which corresponds to the Binomial Triangle and the Grading Structure of Clifford Algebras



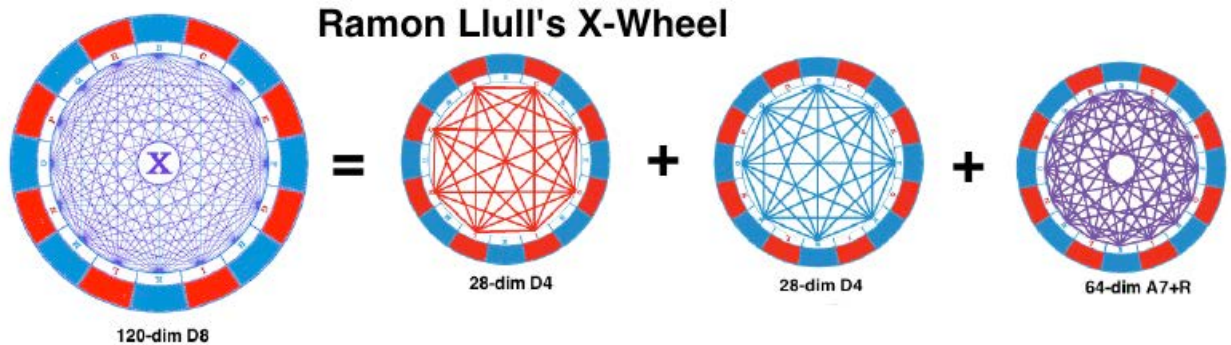
Rg Veda in Sanskrit is the oldest human document:

Akshar	Budhi	Manas	Akash	Vayu	Agni	Jal	Prithvi	Akshar	Budhi	Manas	Akash	Vayu	Agni	Jal	Prithvi	Akshar	Budhi	Manas	Akash	Vayu	Agni	Jal	Prithvi
अ	न	म	ल	पु	र	ह	त	य	उ	स्व	दु	व	म	त्वि	जम्	हो	ता	र	र	लु	धा	त	मम्
अ	नि	मी	ले	पु	रो	हि	तं	या	या	स्या	दे	वा	मि	त्वि	जम्	हो	ता	र	र	लु	धा	त	मम्
बु	भिः	पु	रै	भिः	भ	भि	भि	वी	ड	यो	नु	रं	दे	रु	व	स	दे	र	ए	ह	रं	व	रि
भ	भिः	पु	रै	भिः	भ	भि	भि	व्यो	व	पु	व	दि	वे	दि	वे	यु	श	रै	यो	र	व	त	मम्
भ	भिः	पु	रै	भिः	भ	भि	भि	द्वि	व	ने	प	दि	पु	र	दि	स	ह	दे	वे	पु	ग	रु	ति
भ	भिः	पु	रै	भिः	भ	भि	भि	सु	त्पश्	वि	व	श्रं	व	स्त	मः	दे	वो	दे	वे	वि	रा	गं	मम्
य	ड	ङ	दा	तु	षे	तु	ब	रु	तं	द	रं	क	रि	प्य	भि	व	वेत्	त्प	सु	त्प	मं	रि	रु
उ	पं	त्वा	दे	दि	वे	दि	वे	रो	भं	व	स्वर	दि	या	वु	सम्	न	मो	भ	रं	नु	ए	मं	रि
रा	रं	त्वा	म	ख	रा	र्यो	मो	पा	पु	त	खु	दी	दि	वि	प	व	धं	मा	तं	सु	वे	द	धं
स	रं	पु	ते	व	सु	न	वे	अ	मं	सु	षा	यु	नो	रं	व	ध	रं	सु	शा	नः	स्त	धं	धं

24 First Richa Syllables + 24 First Richa Gaps = D4sm + D4gde (purple)
 8x8 = 64 Last-8 Syllables of Last 8 lines = D8 / D4sm x D4gde (blue)
 8x8 = 64 (red) plus 8x8 = 64 (green) give 128 = E8 / D8 = Fermions
 Middle-8 Syllables of Last 8 lines plus First-8 Syllables of Last 8 Lines
 24+24+64+64+64 = 240 Root Vectors of E8 in Cl(16)

Here are some details about Ramon Llull's X-wheel and D8:

Around 1300 AD Ramon Llull constructed his X-wheel with 16 vertices (Vectors of $Cl(16)$ and $D8$) and 120 diameter lines (BiVectors of $C(16)$ and $D8$)



X-wheel D8 is made up of two copies of 28-dim D4 plus 64-dim A7+R

$A7+R = D8 / D4 \times D4$ Symmetric Space

One D4 contains $D3 = Spin(2,4)$ of Conformal Gravity+Dark Energy

Other D4 contains $SU(3)$ of Standard Model with $SU(2) \times U(1)$ from CP^2

Here are some details about Clifford's 1878 ideas bringing Real Clifford Algebras to the Europeans:

William Kingdon Clifford (1845 - 1879), according to Wikipedia - (1878, "On the Nature of Things-in-Themselves", Mind, Vol. 3, No. 9, pp. 57–67), said:

“... That element of which ... even the simplest feeling is a complex, I shall call **Mind-stuff**. A moving molecule of inorganic matter does not possess mind or consciousness ; but it possesses a small piece of mind-stuff. ... When molecules are ... combined together ... **the elements of mind-stuff** which go along with them ... combine ... to form the ... beginnings of Sentience. When the molecules are so combined as to form the brain and nervous system ... the corresponding elements of mind-stuff are so combined as to form some kind of consciousness ... changes in the complex which take place at the same time get so linked together that the repetition of one implies the repetition of the other. When matter takes the complex form of a living human brain,

the corresponding mind-stuff takes the form of a human consciousness ...”.

Here is a table of Real Clifford Algebras $Cl(p,q)$ as Matrices of Reals, Complex Numbers, or Quaternions for spaces with signature (p,q)

The $p=0$ line is $Cl(0,q) = Cl(q)$

Tensor Product $Cl(0,8) \times Cl(p,q) = M(8,16) \times Cl(p,q) = Cl(p,q+8)$

Real Clifford Algebras $Cl(p,q)$

8	$M_{16}(R)$	$M_{16}(C)$	$M_{16}(H)$	$M_{16}(H) \oplus M_{16}(H)$	$M_{16}(H) \oplus M_{16}(C)$	$M_{16}(H) \oplus M_{16}(R)$	$M_{16}(H) \oplus M_{16}(R) \oplus M_{16}(R)$	$M_{128}(H)$									
7	$M_8(C)$	$M_8(H)$	$M_8(H) \oplus M_8(H)$	$M_{16}(H)$	$M_{16}(C)$	$M_{16}(H) \oplus M_{16}(H)$	$M_{16}(H) \oplus M_{16}(R)$	$M_{128}(H)$	$M_{128}(C)$								
6	$M_4(H) \oplus M_4(H)$	$M_4(H) \oplus M_4(C)$	$M_4(H) \oplus M_4(H) \oplus M_4(H)$	$M_8(H)$	$M_8(C)$	$M_8(H) \oplus M_8(H)$	$M_8(H) \oplus M_8(R)$	$M_{64}(H)$	$M_{64}(C)$	$M_{64}(H)$							
5	$M_2(H) \oplus M_2(H)$	$M_4(H)$	$M_4(C)$	$M_4(H) \oplus M_4(H)$	$M_8(H)$	$M_8(C)$	$M_8(H) \oplus M_8(H)$	$M_{32}(H)$	$M_{32}(C)$	$M_{32}(H)$	$M_{32}(R)$						
4	$M_2(H)$	$M_4(C)$	$M_4(H)$	$M_4(H) \oplus M_4(H)$	$M_8(H)$	$M_8(C)$	$M_8(H) \oplus M_8(H)$	$M_{16}(H)$	$M_{16}(C)$	$M_{16}(H)$	$M_{16}(R)$	$M_{16}(R)$					
3	$M_2(C)$	$M_4(H)$	$M_4(C)$	$M_4(H) \oplus M_4(H)$	$M_8(H)$	$M_8(C)$	$M_8(H) \oplus M_8(H)$	$M_{8}(H) \oplus M_{8}(H)$	$M_{16}(H)$	$M_{16}(C)$	$M_{16}(H)$	$M_{16}(R)$	$M_{16}(R)$				
2	$M_2(R) \oplus M_2(R)$	$M_4(H)$	$M_4(C)$	$M_4(H) \oplus M_4(H)$	$M_8(H)$	$M_8(C)$	$M_8(H) \oplus M_8(H)$	$M_{16}(H)$	$M_{16}(C)$	$M_{16}(H)$	$M_{16}(R)$	$M_{16}(R)$	$M_{128}(H)$				
1	$R \oplus R$	$M_2(C)$	$M_2(H)$	$M_2(H) \oplus M_2(H)$	$M_4(H)$	$M_4(C)$	$M_4(H) \oplus M_4(H)$	$M_8(H)$	$M_8(C)$	$M_8(H)$	$M_8(R)$	$M_8(R)$	$M_{32}(H)$	$M_{32}(C)$			
0	R	C	H	$H \oplus H$	$M_2(C)$	$M_2(H)$	$M_2(H) \oplus M_2(H)$	$M_4(H)$	$M_4(C)$	$M_4(H)$	$M_4(R)$	$M_4(R)$	$M_{16}(H)$	$M_{16}(C)$	$M_{16}(R)$	$M_{16}(R)$	
	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16

Legend:
 H = Quaternion
 C = Complex
 R = Real