From Maxwell's Equations to the String Theory and Particle Physics: New mathematical connections with some sectors of Number Theory

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Abstract

In this research thesis, we have described some new mathematical connections between Maxwell's Equations, some sectors of the String Theory and Particle Physics, and some sectors of Number Theory, precisely various Ramanujan's expressions and equations.

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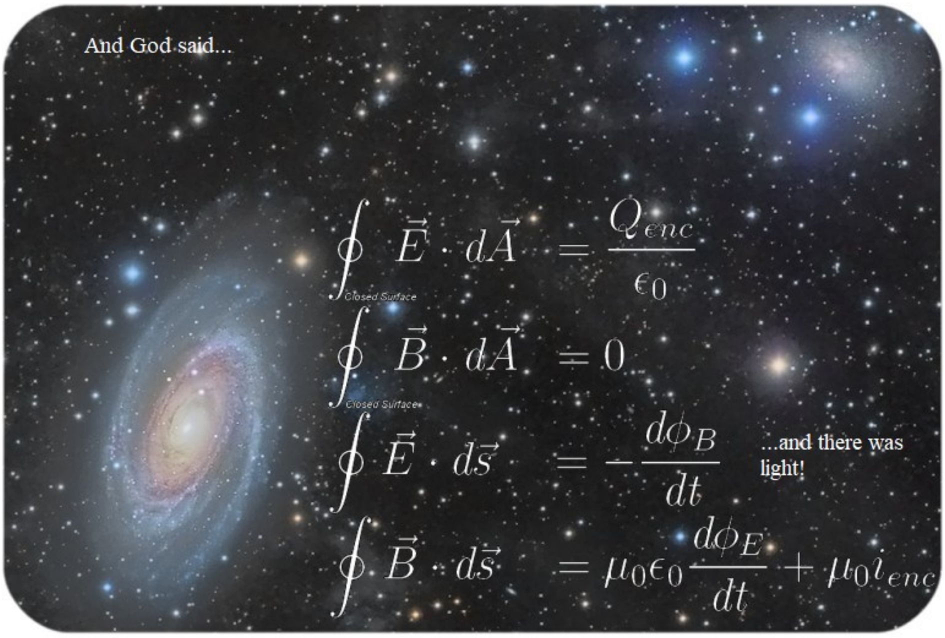
This paper is dedicated to the memory of the great genius J.C. Maxwell, a man of science, a man of God!



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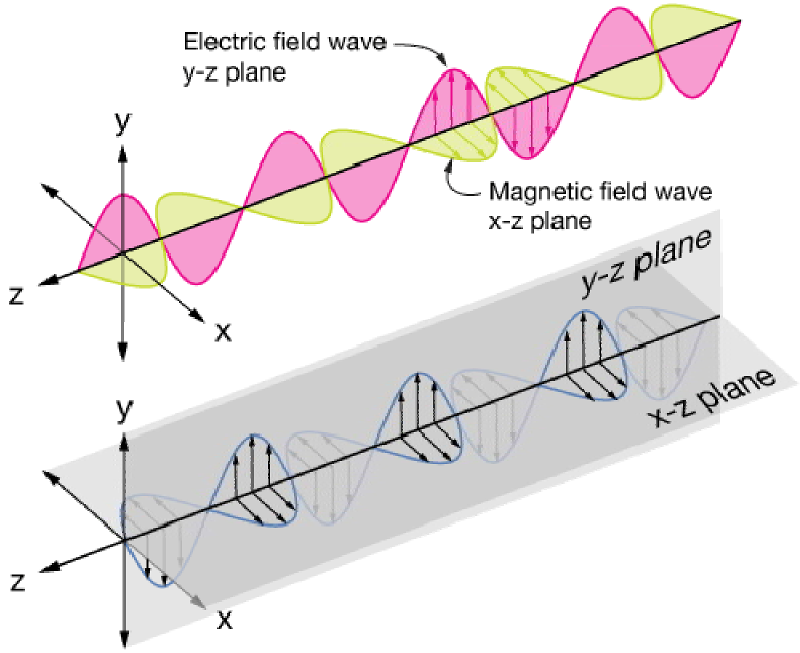

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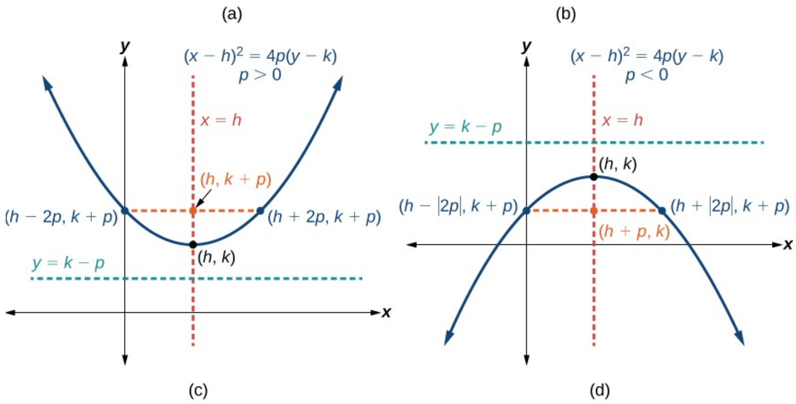


From:
http://web.mit.edu/8.02t/www/mitxmaterials/Presentations/Presentation_W13D2.pdf

Maxwell's Equations

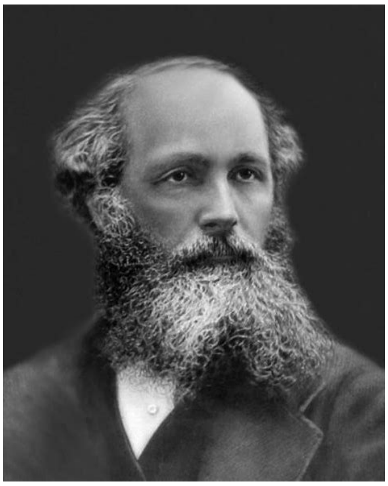
$$\oint\!\!\oint_S \vec{\mathbf{E}}\cdot\hat{\mathbf{n}}\,dA = \frac{1}{\varepsilon_0}\iiint_V \rho\,dV \qquad \text{(Gauss's Law)}$$

$$\oint\!\!\oint_S \vec{\mathbf{B}}\cdot\hat{\mathbf{n}}\,dA = 0 \qquad \text{(Magnetic Gauss's Law)}$$

$$\oint_C \vec{\mathbf{E}}\cdot d\vec{\mathbf{s}} = -\frac{d}{dt}\iint_S \vec{\mathbf{B}}\cdot\hat{\mathbf{n}}\,dA \qquad \text{(Faraday's Law)}$$

$$\oint_C \vec{\mathbf{B}}\cdot d\vec{\mathbf{s}} = \mu_0\iint_S \vec{\mathbf{J}}\cdot\hat{\mathbf{n}}\,dA + \mu_0\varepsilon_0\frac{d}{dt}\iint_S \vec{\mathbf{E}}\cdot\hat{\mathbf{n}}\,dA \qquad \text{(Maxwell - Ampere's Law)}$$

James Clerk Maxwell



Man of science, man of God.

Maxwell's faith also manifested itself in his approach to scientific activity. He declared himself a "reader of the book of nature". According to Maxwell, this book appears to the scientist as orderly and harmonious, revealing the infinite power and wisdom of God in his unattainable and eternal truth. Maxwell justified the knowable of nature and the success of science, that is man's ability to develop a science that knew how to preach some truths about nature, through an act of faith. In fact he claimed that God had created human mind and nature in correspondence.

Maxwell's Equations

Law	Equation	Physical Interpretation
Gauss's law for $\vec{\mathbf{E}}$	$\oint\!\!\oint_S \vec{\mathbf{E}} \cdot d\vec{\mathbf{A}} = \frac{Q}{\varepsilon_0}$	Electric flux through a closed surface is proportional to the charged enclosed
Faraday's law	$\oint \vec{\mathbf{E}} \cdot d\,\vec{\mathbf{s}} = -\frac{d\Phi_B}{dt}$	Changing magnetic flux produces an electric field
Gauss's law for $\vec{\mathbf{B}}$	$\oint\!\!\oint_S \vec{\mathbf{B}} \cdot d\vec{\mathbf{A}} = 0$	The total magnetic flux through a closed surface is zero
Ampere – Maxwell law	$\oint \vec{\mathbf{B}} \cdot d\,\vec{\mathbf{s}} = \mu_0 I + \mu_0 \varepsilon_0 \frac{d\Phi_E}{dt}$	Electric current and changing electric flux produces a magnetic field

Collectively they are known as Maxwell's equations. The above equations may also be written in differential forms as

$$\begin{aligned} \nabla \cdot \vec{\mathbf{E}} &= \frac{\rho}{\varepsilon_0} \\ \nabla \times \vec{\mathbf{E}} &= -\frac{\partial \vec{\mathbf{B}}}{\partial t} \\ \nabla \cdot \vec{\mathbf{B}} &= 0 \\ \nabla \times \vec{\mathbf{B}} &= \mu_0 \vec{\mathbf{J}} + \mu_0 \varepsilon_0 \frac{\partial \vec{\mathbf{E}}}{\partial t} \end{aligned} \tag{13.3.1}$$

where ρ and $\vec{\mathbf{J}}$ are the free charge and the conduction current densities, respectively. In the absence of sources where $Q = 0$, $I = 0$, the above equations become

$$\begin{aligned} \oint\!\!\oint_S \vec{\mathbf{E}} \cdot d\vec{\mathbf{A}} &= 0 \\ \oint \vec{\mathbf{E}} \cdot d\,\vec{\mathbf{s}} &= -\frac{d\Phi_B}{dt} \\ \oint\!\!\oint_S \vec{\mathbf{B}} \cdot d\vec{\mathbf{A}} &= 0 \\ \oint \vec{\mathbf{B}} \cdot d\,\vec{\mathbf{s}} &= \mu_0 \varepsilon_0 \frac{d\Phi_E}{dt} \end{aligned} \tag{13.3.2}$$

An important consequence of Maxwell's equations, as we shall see below, is the prediction of the existence of electromagnetic waves that travel with speed of light $c = 1/\sqrt{\mu_0 \varepsilon_0}$. The reason is due to the fact that a changing electric field produces a magnetic field and vice versa, and the coupling between the two fields leads to the generation of electromagnetic waves. The prediction was confirmed by H. Hertz in 1887.

From:

A Dynamical Theory of the Electromagnetic Field
J. Clerk Maxwell - *Phil. Trans. R. Soc. Lond.* 1865 **155**, 459-512, published 1 January 1865

Absolute Values of the Electromotive and Magnetic Forces called into play in the Propagation of Light.

(108) If the equation of propagation of light is

$$F = A \cos \frac{2\pi}{\lambda}(z - Vt),$$

the electromotive force will be

$$P = -A \frac{2\pi}{\lambda} V \sin \frac{2\pi}{\lambda}(z - Vt);$$

and the energy per unit of volume will be

$$\frac{P^2}{8\pi\mu V^2},$$

where P represents the greatest value of the electromotive force. Half of this consists of magnetic and half of electric energy.

The energy passing through a unit of area is

$$W = \frac{P^2}{8\pi\mu V};$$

so that

$$P = \sqrt{8\pi\mu VW},$$

where V is the velocity of light, and W is the energy communicated to unit of area by the light in a second.

According to POUILLET's data, as calculated by Professor W. THOMSON*, the mechanical value of direct sunlight at the Earth is

83·4 foot-pounds per second per square foot.

This gives the maximum value of P in direct sunlight at the Earth's distance from the Sun,

$$P = 60,000,000,$$

or about 600 DANIELL's cells per metre.

Now, we calculate the integral of half value of P, that represent the greatest value of the electromotive force. Indeed, half of this consists of magnetic and half of electric energy:

1.08643 * integrate [3*10^7] x, [0, Pi/10^4]

$$1.08643 \int_0^{\frac{\pi}{10^4}} 3 \times 10^7 \, x \, dx$$

Result:

1.6084

We calculate now the double integral of the value of P:

1.08643 * integrate integrate [6*10^7] [Pi/73.7*10^4]

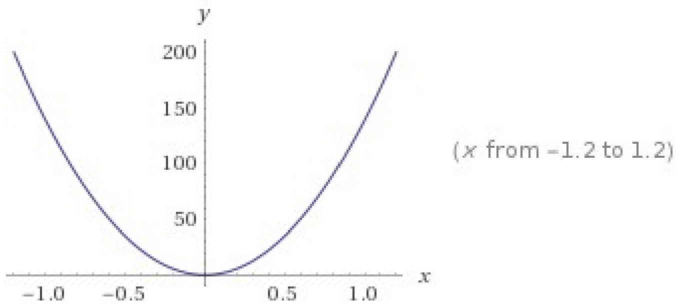
where Pi/73.7 = 0,0426267659 = 0,6740782248[8]

$$1.08643 \int\left(\int 6\times 10^{7}\times \frac{\pi}{73.7\times 10^{4}}\,dx\right)dx$$

Result:

$138.933\,x^2$

Plot:



Now we calculate the integral of the value of W = 477464.8292 that is the energy passing through the unit of area:

1.08643 * 1/10^25 integrate [477464.8292] [0, Pi]

$$1.08643\times \frac{1}{10^{25}}\int_0^{\pi} 477464.8292\,dx$$

Result:

1.62964×10^{-19}

Result that is a good approximation of the electric charge of positron.

Mechanical Force on an Electrified Body.

(79) If there is no motion or change of strength of currents or magnets in the field, the electromotive force is entirely due to variation of electric potential, and we shall have (§ 65)

$$P=-\frac{d\Psi}{dx}, \quad Q=-\frac{d\Psi}{dy}, \quad R=-\frac{d\Psi}{dz}.$$

Integrating by parts the expression (I) for the energy due to electric displacement, and remembering that P, Q, R vanish at an infinite distance, it becomes

$$\tfrac{1}{2}\Sigma\left\{\Psi\left(\frac{df}{dx}+\frac{dg}{dy}+\frac{dh}{dz}\right)\right\}dV,$$

or by the equation of Free Electricity (G), p. 485,

$$-\tfrac{1}{2}\Sigma(\Psi e)dV.$$

By the same demonstration as was used in the case of the mechanical action on a magnet, it may be shown that the mechanical force on a small body containing a quantity e_2 of free electricity placed in a field whose potential arising from other electrified bodies is Ψ_1, has for components

$$\left.\begin{aligned} X&=e_2\frac{d\Psi_1}{dx}=-P_1e_2,\\ Y&=e_2\frac{d\Psi_1}{dy}=-Q_1e_2,\\ Z&=e_3\frac{d\Psi_1}{dz}=-R_1e_2.\end{aligned}\right\} \quad \text{(D)}$$

So that an electrified body is urged in the direction of the electromotive force with a force equal to the product of the quantity of free electricity and the electromotive force.

If the electrification of the field arises from the presence of a small electrified body containing e_1 of free electrity, the only solution of Ψ_1 is

$$\Psi_1=\frac{k}{4\pi}\frac{e_1}{r}, \quad (43)$$

where r is the distance from the electrified body.

The repulsion between two electrified bodies e_1, e_2 is therefore

$$e_2\frac{d\Psi_1}{dr}=\frac{k}{4\pi}\frac{e_1e_2}{r^2}. \quad (44)$$

Measurement of Electrical Phenomena by Electrostatic Effects.

(80) The quantities with which we have had to do have been hitherto expressed in terms of the Electromagnetic System of measurement, which is founded on the mechanical action between currents. The electrostatic system of measurement is founded on the mechanical action between electrified bodies, and is independent of, and incompatible with, the electromagnetic system; so that the units of the different kinds of quantity have different values according to the system we adopt, and to pass from the one system to the other, a reduction of all the quantities is required.

According to the electrostatic system, the repulsion between two small bodies charged with quantities η_1, η_2 of electricity is

$$\frac{\eta_1\eta_2}{r^2},$$

where r is the distance between them.

Let the relation of the two systems be such that one electromagnetic unit of electricity contains v electrostatic units; then $\eta_1=ve_1$ and $\eta_2=ve_2$, and this repulsion becomes

$$v^2\frac{e_1e_2}{r^2}=\frac{k}{4\pi}\frac{e_1e_2}{r^2} \text{ by equation (44)}, \qquad (45)$$

whence k, the coefficient of "electric elasticity" in the medium in which the experiments are made, *i. e.* common air, is related to v, the number of electrostatic units in one electromagnetic unit, by the equation

$$k=4\pi v^2. \qquad (46)$$

The quantity v may be determined by experiment in several ways. According to the experiments of MM. Weber and Kohlrausch,

$$v=310{,}740{,}000 \text{ metres per second.}$$

PART VI.—ELECTROMAGNETIC THEORY OF LIGHT.

(91) At the commencement of this paper we made use of the optical hypothesis of an elastic medium through which the vibrations of light are propagated, in order to show that we have warrantable grounds for seeking, in the same medium, the cause of other phenomena as well as those of light. We then examined electromagnetic phenomena, seeking for their explanation in the properties of the field which surrounds the electrified or magnetic bodies. In this way we arrived at certain equations expressing certain properties of the electromagnetic field. We now proceed to investigate whether these properties of that which constitutes the electromagnetic field, deduced from electromagnetic phenomena alone, are sufficient to explain the propagation of light through the same substance.

(92) Let us suppose that a plane wave whose direction cosines are l, m, n is propagated through the field with a velocity V. Then all the electromagnetic functions will be functions of

$$w=lx+my+nz-\mathrm{V}t.$$

The equations of Magnetic Force (B), p. 482, will become

$$\mu\alpha=m\frac{d\mathrm{H}}{dw}-n\frac{d\mathrm{G}}{dw},$$
$$\mu\beta=n\frac{d\mathrm{F}}{dw}-l\frac{d\mathrm{H}}{dw},$$
$$\mu\gamma=l\frac{d\mathrm{G}}{dw}-m\frac{d\mathrm{F}}{dw}.$$

If we multiply these equations respectively by l, m, n, and add, we find

$$l\mu\alpha+m\mu\beta+n\mu\gamma=0, \quad \ldots \ldots \quad (62)$$

which shows that the direction of the magnetization must be in the plane of the wave.

(93) If we combine the equations of Magnetic Force (B) with those of Electric Currents (C), and put for brevity

$$\frac{d\mathrm{F}}{dx}+\frac{d\mathrm{G}}{dy}+\frac{d\mathrm{H}}{dz}=\mathrm{J}, \text{ and } \frac{d^2}{dx^2}+\frac{d^2}{dy^2}+\frac{d^2}{dz^2}=\nabla^2, \quad \ldots \ldots \quad (63)$$

$$\left.\begin{aligned} 4\pi\mu p' &= \frac{d\mathrm{J}}{dx}-\nabla^2\mathrm{F}, \\ 4\pi\mu q' &= \frac{d\mathrm{J}}{dy}-\nabla^2\mathrm{G}, \\ 4\pi\mu r' &= \frac{d\mathrm{J}}{dz}-\nabla^2\mathrm{H}. \end{aligned}\right\} \quad \ldots \ldots \quad (64)$$

If the medium in the field is a perfect dielectric there is no true conduction, and the currents p', q', r' are only variations in the electric displacement, or, by the equations of Total Currents (A),

$$p'=\frac{df}{dt}, \qquad q'=\frac{dg}{dt}, \qquad r'=\frac{dh}{dt}. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \qquad (65)$$

But these electric displacements are caused by electromotive forces, and by the equations of Electric Elasticity (E),

$$P=kf, \qquad Q=kg, \qquad R=kh. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \qquad (66)$$

These electromotive forces are due to the variations either of the electromagnetic or the electrostatic functions, as there is no motion of conductors in the field; so that the equations of electromotive force (D) are

$$\left.\begin{aligned} P&=-\frac{dF}{dt}-\frac{d\Psi}{dx}, \\ Q&=-\frac{dG}{dt}-\frac{d\Psi}{dy}, \\ R&=-\frac{dH}{dt}-\frac{d\Psi}{dz}. \end{aligned}\right\} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \qquad (67)$$

(94) Combining these equations, we obtain the following:—

$$\left.\begin{aligned} k\left(\frac{dJ}{dx}-\nabla^2F\right)+4\pi\mu\left(\frac{d^2F}{dt^2}+\frac{d^2\Psi}{dxdt}\right)&=0, \\ k\left(\frac{dJ}{dy}-\nabla^2G\right)+4\pi\mu\left(\frac{d^2G}{dt^2}+\frac{d^2\Psi}{dydt}\right)&=0, \\ k\left(\frac{dJ}{dz}-\nabla^2H\right)+4\pi\mu\left(\frac{d^2H}{dt^2}+\frac{d^2\Psi}{dzdt}\right)&=0. \end{aligned}\right\} \quad . \quad . \quad . \quad . \quad . \quad . \qquad (68)$$

If we differentiate the third of these equations with respect to y, and the second with respect to z, and subtract, J and Ψ disappear, and by remembering the equations (B) of magnetic force, the results may be written

$$\left.\begin{aligned} k\nabla^2\mu\alpha&=4\pi\mu\frac{d^2}{dt^2}\mu\alpha, \\ k\nabla^2\mu\beta&=4\pi\mu\frac{d^2}{dt^2}\mu\beta, \\ k\nabla^2\mu\gamma&=4\pi\mu\frac{d^2}{d^2t}\mu\gamma. \end{aligned}\right\} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \qquad (69)$$

(95) If we assume that α, β, γ are functions of $lx+my+nz-Vt=w$, the first equation becomes

$$k\mu\frac{d^2\alpha}{dw^2}=4\pi\mu^2V^2\frac{d^2\alpha}{dw^2}, \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \qquad (70)$$

or

$$V=\pm\sqrt{\frac{k}{4\pi\mu}}. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \qquad (71)$$

The other equations give the same value for V, so that the wave is propagated in either direction with a velocity V.

This wave consists entirely of magnetic disturbances, the direction of magnetization being in the plane of the wave. No magnetic disturbance whose direction of magnetization is not in the plane of the wave can be propagated as a plane wave at all.

Hence magnetic disturbances propagated through the electromagnetic field agree with light in this, that the disturbance at any point is transverse to the direction of propagation, and such waves may have all the properties of polarized light.

(96) The only medium in which experiments have been made to determine the value of k is air, in which $\mu=1$, and therefore, by equation (46),

$$V=v. \qquad \qquad (72)$$

By the electromagnetic experiments of MM. WEBER and KOHLRAUSCH *,

$$v=310,740,000 \text{ metres per second}$$

The value of k, that is the "coefficient of electric elasticity, of (46) is equal to 1213400548222332932.49

Now we calculate the following integral:

1.08643^2 * 0.61803398^3 * (1/(10^37)) integrate [1213400548222332932.49] x, [0, Pi/10^4]

$$1.08643^2 \times 0.61803398^3 \times \frac{1}{10^{37}} \int_0^{\frac{\pi}{10^4}} 1.21340054822233293249 \times 10^{18}\, x\, dx$$

Result:

1.66845×10^{-27}

value very near to the mass of the proton.

Now we calculate the following double integral:

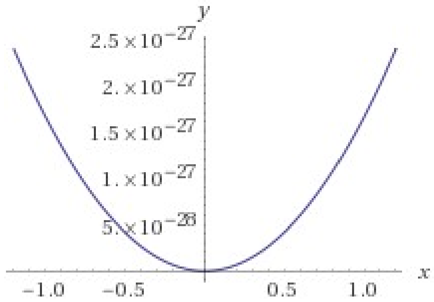
1.08643^2 * 0.61803398^e *e * (1/10^41) integrate integrate [1213400548222332932.49] [Pi/10^4]

$$1.08643^2 \times 0.61803398^e\, e \times \frac{1}{10^{41}} \int \left(\int 1.21340054822233293249 \times 10^{18} \times \frac{\pi}{10^4}\, dx \right) dx$$

Result:

$1.65324 \times 10^{-27}\, x^2$

Plot:



(x from -1.2 to 1.2)

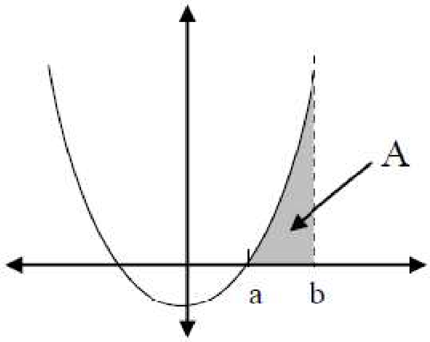
Results that, practically, are excellent approximations of the proton mass. Recall that when a proton collides with its antiparticle (and in general when any baryon collides with an antibarion), the reaction is not as simple as electron-positron annihilation. Unlike the electron, the proton is not an elementary particle. In fact it is a particle composed of three valence quarks and an indeterminate number of sea quarks, linked by gluons. So when a proton collides with an antiproton, one of the valence quarks that constitute it can annihilate itself with an antiquark, while the remaining quarks will rearrange into mesons (mainly pions and kaons) that will move away from the point where the annihilation occurred. The mesons created are unstable particles and will decay. In particle physics, mesons are a group of subatomic particles composed of a quark and an antiquark bound by a strong force. They are unstable particles and typically decay into photons or leptons.

From:

https://www.rit.edu/studentaffairs/asc/sites/rit.edu.studentaffairs.asc/files/docs/services/resources/handouts/C7_AreasbyIntegration_BP_9_22_14.pdf

Ex. 2. Find the area bounded by the following curves: $y = x^2 - 4,\ y = 0,\ x = 4,$

Graph:



Finding the boundaries:

$$y = x^2 - 4, \text{ and } y = 0 \text{ implies } x^2 - 4 = 0 \text{ so } (x-2)(x+2) = 0$$
$$x = -2 \text{ or } x = 2$$

From the graph we see that $x = 2$ is our boundary at a. The value $x = -2$ is a solution to the equation above but it is not bounding the area. (Here's why the graph is an important tool to help us determine correct results. *Don't skip this step*!)

The other boundary value is given by the equation of the vertical line $x = 4,$

Boundaries are: $a = 2$, and $b = 4,$

Set up the integral:

$$A = \int_a^b f(x)dx = \int_2^4 (x^2 - 4)dx$$

Solve:

$$\int_2^4 (x^2 - 4)dx = \left(\frac{1}{3}x^3 - 4x\right)\Bigg|_2^4 = \left(\frac{1}{3}\cdot(4)^3 - 4\cdot 4\right) - \left(\frac{1}{3}\cdot(2)^3 - 4\cdot 2\right)$$
$$= \left(\frac{64}{3} - 16\right) - \left(\frac{8}{3} - 8\right) = \frac{64}{3} - 16 - \frac{8}{3} + 8 = \frac{56}{3} - 8 = \frac{32}{3}$$

The area bounded by the curves $y = x^2 - 4,\ y = 0,\ x = 4,$ is equal to $\frac{32}{3}$ square units.

Now:

integrate [32/3] x, [0, e^2/14] * 1.08643

$$\left(\int_0^{\frac{e^2}{14}} \frac{32}{3}\, x\, dx\right) \times 1.08643$$

Result:

1.61407

the result is very near to the electric charge of the positron

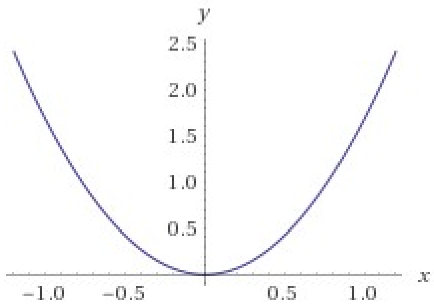
Pi/(4e) * integrate integrate [32/3] * 1.08643

$$\frac{\pi}{4e} \int \left(\int \frac{32}{3} \times 1.08643 \, dx \right) dx$$

Result:

$1.67416 x^2$

Plot:

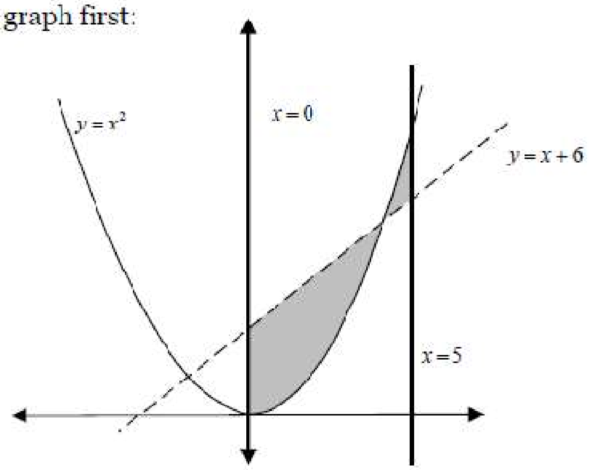


(x from −1.2 to 1.2)

and this is very near to the value of the mass of proton.

Ex.7. Find the area of the region enclosed by the following curves: $y = x^2$, $y = x+6$, $x = 0$ and $x = 5$

As usual – sketch a rough graph first:



In this case it is very important to draw the graph, since the functions intersect between the boundaries. This means that we will have to actually calculate two separate integrals and then add the results. Otherwise we would end up subtracting the two pieces from each other.

First we need the "middle" intersection point so we will solve the equation: $x^2 = x + 5$

$$x^2 - x - 5 = 0$$
$$(x-3)(x+2) = 0$$
$$x = 3 \text{ or } x = -2$$

The intersection point at $x = -2$ is outside our area. We are interested in $x = 3$, this is our "middle" boundary value.

In this case the integral set-up will look as follows:

$$A = \int_0^3 \left(x + 6 - x^2\right)dx + \int_3^5 \left(x^2 - (x+6)\right)dx = \left(\frac{1}{2}x^2 + 6x - \frac{1}{3}x^3\Big|_0^3\right) + \left(\frac{1}{3}x^3 - \frac{1}{2}x^2 - 6x\Big|_3^5\right) =$$

$$= \left(\frac{1}{2}\cdot 3^2 + 6\cdot 3 - \frac{1}{3}\cdot 3^3\right) + \left(\frac{1}{3}\cdot 5^3 - \frac{1}{2}\cdot 5^2 - 6\cdot 5\right) - \left(\frac{1}{3}\cdot 3^3 - \frac{1}{2}\cdot 3^2 - 6\cdot 3\right) =$$

$$\frac{9}{2} + 18 - 9 + \frac{125}{3} - \frac{25}{2} - 30 - 9 + \frac{9}{2} + 18 = \frac{157}{6} \text{ square units.}$$

So the area of the region enclosed by the curves: : $y = x^2$, $y = x + 6$, $x = 0$ and $x = 5$ is equal to $\frac{157}{6}$ square units.

Now:

integrate [157/6] x, [0, e^2/22] * 1.08643 $0.66940042^{2.71828} = 0.3358657\ldots = e^2/22$

$$\left(\int_0^{\frac{e^2}{22}} \frac{157}{6}\, x\, dx\right) \times 1.08643$$

Result:

1.60344

integrate [157/6] x, [0, e^2/21.54] * 1.08643 where 0,343039 one obtain from $0.66453^{2.61803398}$ =

$$1.08643 \int_0^{0.343039} \frac{157\,x}{6}\,dx = 1.67266$$

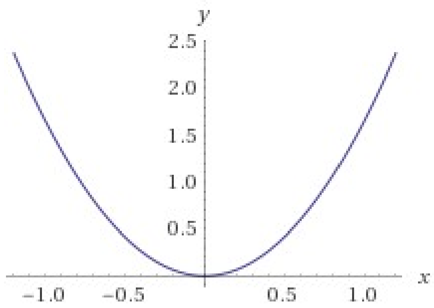
Pi/(10e) * integrate integrate [157/6] * 1.08643

$$\frac{\pi}{10\,e} \int\left(\int \frac{157}{6} \times 1.08643\,dx\right)dx$$

Result:

$1.64277\,x^2$

Plot:

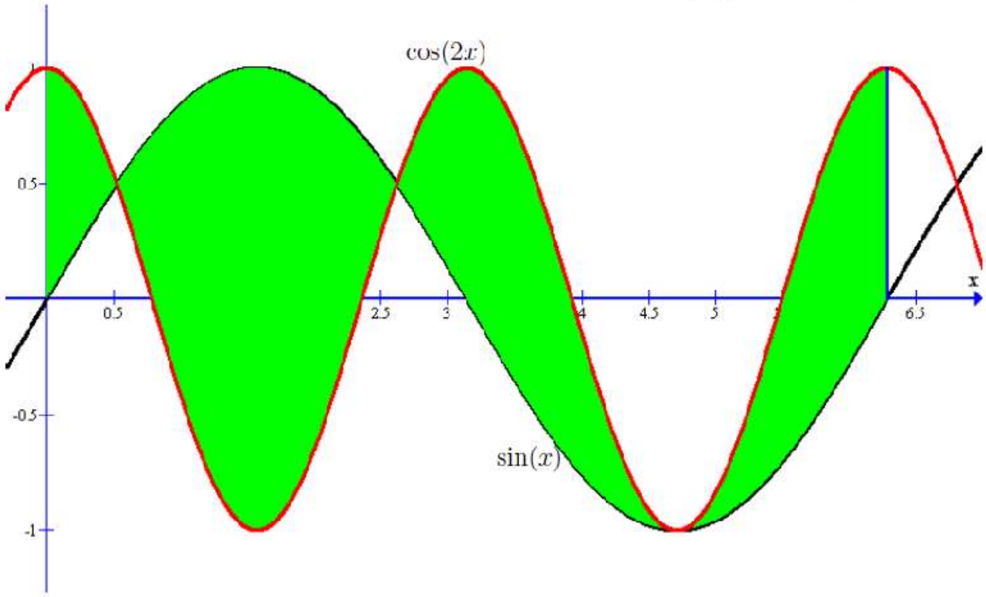


(x from −1.2 to 1.2)

value very near to the mass of the proton

From: https://www.berkeleycitycollege.edu/wp/wjeh/files/2012/08/calculus_note_area_bt_curve.pdf

Example: Find the area bounded between $x = 0$, $x = 2\pi$, $y = \cos(2x)$, $y = \sin(x)$



Ans: We are interested in $\displaystyle\int_0^{2\pi} |\cos(2x) - \sin(x)| \; dx$

Inside the interval $[0, 2\pi]$, for some values of x, $\cos(2x) > \sin(x)$, and for some other values of x, $\sin(x) > \cos(2x)$. We set the two functions equal to each other to find the points of intersection:

$$\begin{gathered}
\cos(2x) = \sin(x) \\
1 - 2\sin^2(x) = \sin(x) \\
2\sin^2 x + \sin(x) - 1 = 0 \\
(2\sin(x) - 1)(\sin(x) + 1) = 0 \\
\sin(x) = \frac{1}{2} \;\text{ or }\; \sin(x) = -1 \\
x = \frac{\pi}{6} \;\text{ or }\; x = \frac{5\pi}{6} \;\text{ or }\; x = \frac{3\pi}{2}
\end{gathered}$$

For x inside the intervals $\left(0, \frac{\pi}{6}\right) \cup \left(\frac{5\pi}{6}, 2\pi\right)$, $\cos(2x) > \sin(x)$ (at $x = \frac{3\pi}{2}$, the two functions intersect, but the relationship between which is greater is not changed at that point.)

For x inside the interval $\left(\frac{\pi}{6}, \frac{5\pi}{6}\right)$, $\sin(x) > \cos(2x)$

The definite integral can be evaluated as:

$$A = \int_0^{2\pi} |\cos(2x) - \sin(x)| \; dx =$$

$$\int_0^{\pi/6} \cos(2x) - \sin(x) \; dx + \int_{\pi/6}^{5\pi/6} \sin(x) - \cos(2x) \; dx + \int_{5\pi/6}^{2\pi} \cos(2x) - \sin(x) \; dx$$

$$= \left(\frac{\sin(2x)}{2} + \cos(x)\right)\Big|_0^{\pi/6} + \left(-\cos(x) - \frac{\sin(2x)}{2}\right)\Big|_{\pi/6}^{5\pi/6} + \left(\frac{\sin(2x)}{2} + \cos(x)\right)\Big|_{5\pi/6}^{2\pi}$$

$$= \left(\frac{\sin(2(\pi/6))}{2} + \cos(\pi/6)\right) - \left(\frac{\sin(2(0))}{2} + \cos(0)\right)$$

$$+ \left(-\cos(5\pi/6) - \frac{\sin(2(5\pi/6))}{2}\right) - \left(-\cos(\pi/6) - \frac{\sin(2(\pi/6))}{2}\right)$$

$$+ \left(\frac{\sin(2(2\pi))}{2} + \cos(2\pi)\right) - \left(\frac{\sin(2(5\pi/6))}{2} + \cos(5\pi/6)\right)$$

$$= \left(\frac{\sqrt{3}}{4} + \frac{\sqrt{3}}{2}\right) - (0+1) + \left(\frac{\sqrt{3}}{2} + \frac{\sqrt{3}}{4}\right) - \left(-\frac{\sqrt{3}}{2} - \frac{\sqrt{3}}{4}\right) + (0+1) - \left(-\frac{\sqrt{3}}{4} - \frac{\sqrt{3}}{2}\right)$$

$$= \left(\frac{3\sqrt{3}}{4} - 1\right) + \left(\frac{3\sqrt{3}}{2}\right) + \left(1 + \frac{3\sqrt{3}}{4}\right)$$

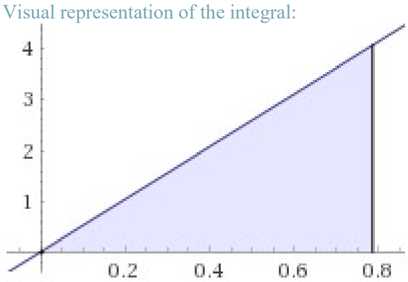
$$= 3\sqrt{3}$$

We calculate the following integral:

integrate [3*sqrt(3)] x, [0, Pi/4]

$$\int_0^{\frac{\pi}{4}} \left(3\sqrt{3}\right) x \, dx = \frac{3\sqrt{3}\,\pi^2}{32} \approx 1.6026$$

Visual representation of the integral:



Pi/(2e) * integrate integrate [3*sqrt(3)] * 1.08643   or:

Pi/(2e) * integrate integrate [(((3*sqrt(3)/4))-1))+(((3*sqrt(3)/2)))+((1+3*sqrt(3)/4))] * 1.08643

$$\frac{\pi}{2e} \int \left(\int \left(\left(3 \times \frac{\sqrt{3}}{4} - 1 \right) + 3 \times \frac{\sqrt{3}}{2} + \left(1 + 3 \times \frac{\sqrt{3}}{4} \right) \right) \times 1.08643 \, dx \right) dx$$

Result:

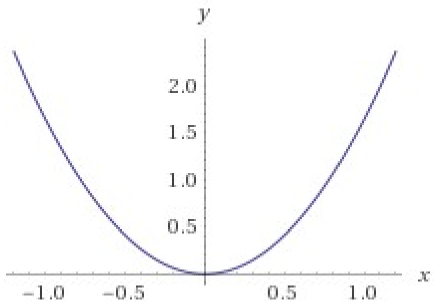
$1.63109\,x^2$

$$\frac{\pi}{2e}\int\left(\int\left(3\sqrt{3}\right)\times 1.08643\,dx\right)dx$$

Result:

$1.63109\,x^2$

Plot:



(x from −1.2 to 1.2)

All the values are very near to the electric charge of the positron

Wave function of the Universe concerning the Hartle-Hawking no-boundary proposal

Now we calculate the following integral regarding the wave function of the Universe of the Hartle-Hawking no-boundary proposal**,** that is:

$$\psi_0(a_0)=2\cos\left[\frac{(H^2a_0{}^2-1)^{3/2}}{3H^2}-\frac{\pi}{4}\right]$$

$\psi_0 a_0 = -3{,}357714479.$

1.08643 * integrate [-3.357714479] x,[0, -Pi^2/10.53] where 0,937284 is

$0{,}6354749^{1/7}$

$$1.08643\int_0^{-0.937284}-3.35771\,x\,dx=-1.60235$$

Now we calculate the following double integral:

1.08643 * integrate integrate [-3.357714479] [0, -Pi^2/11]

where – Pi^2/11 = – 0.89723676 that is

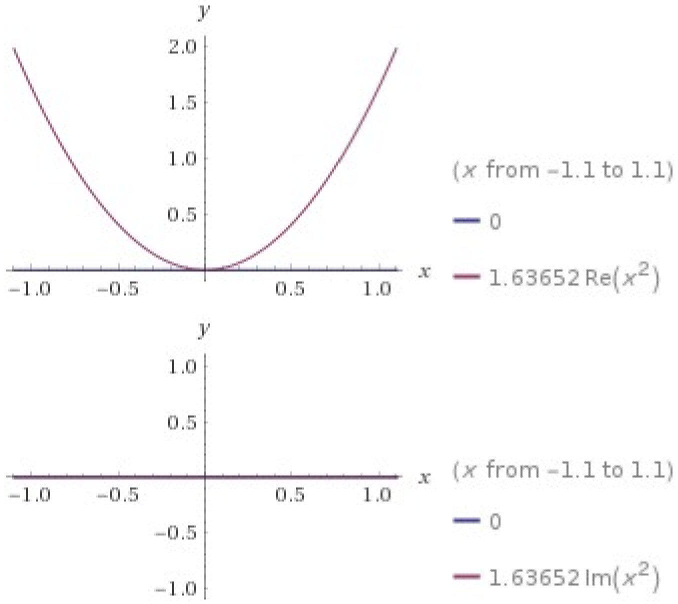
$0{,}6480794355^{1/4}$ (i^2) = 0.89723676 * – 1 = – 0.89723676

$$1.08643 \int \left(\int -3.357714479 \left\{0, -\frac{\pi^2}{11}\right\} dx \right) dx$$

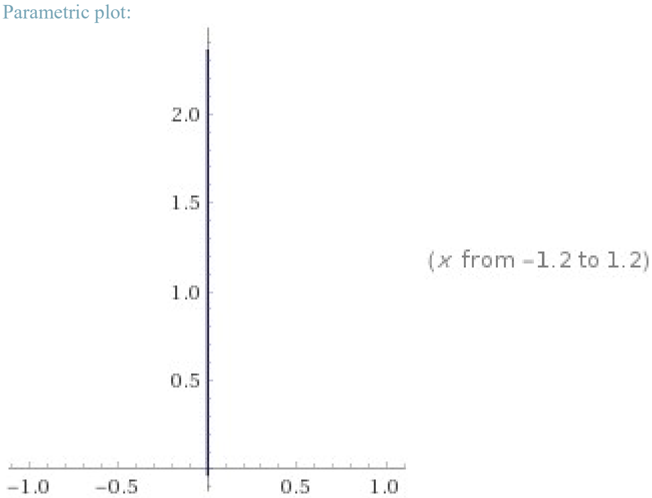
Result:

$\{0, 1.63652 x^2\}$

Plot:



Parametric plot:



Values very near to the electric charge of the electron and to the mass of the proton.

From these examples it can be seen that also from an integration of an integration on a parabola, with the appropriate definition range, we obtain solutions correspondent to the charges of the positron and mass of the proton. D-branes are usually classified by their size, which is indicated by a number written after D: a D0-brane represents a point, a D1-brane (also called D-string) a line, a D2 brane a plane , a D25-brane represents a possible space predicted by string theory.

From:

https://www.secret-bases.co.uk/wiki/History_of_Maxwell%27s_equations

Relationships among electricity, magnetism, and the speed of light

The relationships among electricity, magnetism, and the speed of light can be summarized by the modern equation:

$$c = \frac{1}{\sqrt{\mu_0 \varepsilon_0}}\ .$$

The left-hand side is the speed of light and the right-hand side is a quantity related to the constants that appear in the equations governing electricity and magnetism. Although the right-hand side has units of velocity, it can be inferred from measurements of electric and magnetic forces, which involve no physical velocities. Therefore, establishing this relationship provided convincing evidence that light is an electromagnetic phenomenon.

The discovery of this relationship started in 1855, when Wilhelm Eduard Weber and Rudolf Kohlrausch determined that there was a quantity related to electricity and magnetism, "the ratio of the absolute electromagnetic unit of charge to the absolute electrostatic unit of charge" (in modern language, the value $1/\sqrt{\mu_0 \varepsilon_0}$), and determined that it should have units of velocity. They then measured this ratio by an experiment which involved charging and discharging a Leyden jar and **measuring the magnetic force from the discharge current, and found a value** 3.107×10^8 m/s, **remarkably close to the speed of light, which had recently been measured at** 3.14×10^8 m/s by Hippolyte Fizeau in 1848 and at 2.98×10^8 m/s by Léon Foucault in 1850. However, Weber and Kohlrausch did not make the connection to the speed of light. ***Towards the end of 1861 while working on part III of his paper On Physical Lines of Force, Maxwell travelled from Scotland to London and looked up Weber and Kohlrausch's results. He converted them into a format which was compatible with his own writings, and in doing so he established the connection to the speed of light and concluded that light is a form of electromagnetic radiation***

The four equations we use today appeared separately in Maxwell's 1861 paper, *On Physical Lines of Force*:

1. Equation (56) in Maxwell's 1861 paper is $\nabla \cdot \mathbf{B} = 0$.
2. Equation (112) is Ampère's circuital law, with Maxwell's addition of displacement current. This may be the most remarkable contribution of Maxwell's work, enabling him to derive the electromagnetic wave equation in his 1865 paper A Dynamical Theory of the Electromagnetic Field, showing that light is an electromagnetic wave. This lent the equations their full significance with respect to understanding the nature of the phenomena he elucidated. (Kirchhoff derived the telegrapher's equations in 1857 without using displacement current, but he did use Poisson's equation and the equation of continuity, which are the mathematical ingredients of the displacement current. Nevertheless, believing his equations to be applicable only inside an electric wire, he cannot be credited with the discovery that light is an electromagnetic wave).

3. Equation (115) is Gauss's law.
4. Equation (54) expresses what Oliver Heaviside referred to as 'Faraday's law', which addresses the time-variant aspect of electromagnetic induction, but not the one induced by motion; Faraday's original flux law accounted for both.[10][11] Maxwell deals with the motion-related aspect of electromagnetic induction, **v** × **B**, in equation (77), which is the same as equation (D) in Maxwell's original equations as listed below. It is expressed today as the force law equation, **F** = *q*(**E** + **v** × **B**), which sits adjacent to Maxwell's equations and bears the name Lorentz force, even though Maxwell derived it when Lorentz was still a young boy.

The difference between the **B** and the **H** vectors can be traced back to Maxwell's 1855 paper entitled *On Faraday's Lines of Force* which was read to the Cambridge Philosophical Society. The paper presented a simplified model of Faraday's work, and how the two phenomena were related. He reduced all of the current knowledge into a linked set of differential equations.

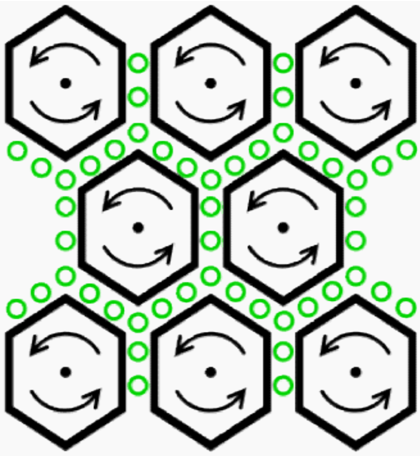


Figure of Maxwell's molecular vortex model. For a uniform magnetic field, the field lines point outward from the display screen, as can be observed from the black dots in the middle of the hexagons. The vortex of each hexagonal molecule rotates counter-clockwise. The small green circles are clockwise rotating particles sandwiching between the molecular vortices.

It is later clarified in his concept of a sea of molecular vortices that appears in his 1861 paper *On Physical Lines of Force*. Within that context, **H** represented pure vorticity (spin), whereas **B** was a weighted vorticity that was weighted for the density of the vortex sea. Maxwell considered magnetic permeability μ to be a measure of the density of the vortex sea. Hence the relationship, (a) **Magnetic induction current** causes a magnetic current density **B** = μ **H** was essentially a rotational analogy to the linear electric current relationship, (b) **Electric convection current J** = ρ **v** where ρ is electric charge density. **B** was seen as a kind of magnetic current of vortices aligned in their axial planes, with **H** being the circumferential velocity of the vortices. With μ representing vortex density, it follows that the product of μ with vorticity **H** leads to the magnetic field denoted as **B**.

The electric current equation can be viewed as a convective current of electric charge that involves linear motion. By analogy, the magnetic equation is an inductive current involving spin. There is no linear motion in the inductive current along the direction of the **B** vector. The magnetic inductive current represents lines of force. In particular, it represents lines of inverse-square law force.

The extension of the above considerations confirms that where **B** is to **H**, and where **J** is to ρ, then it necessarily follows from Gauss's law and from the equation of continuity of charge that **E** is to **D**. i.e. **B** parallels with **E**, whereas **H** parallels with **D**.

$$\left.\begin{aligned}\frac{dQ}{dz}-\frac{dR}{dy}&=\mu\frac{d\alpha}{dt}\\ \frac{dR}{dx}-\frac{dP}{dz}&=\mu\frac{d\beta}{dt}\\ \frac{dP}{dy}-\frac{dQ}{dx}&=\mu\frac{d\gamma}{dt}\end{aligned}\right\}\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots(54).$$

$$\frac{1}{4\pi}\left(\frac{d}{dx}\mu\alpha+\frac{d}{dy}\mu\beta+\frac{d}{dz}\mu\gamma\right)=m=0\ldots\ldots\ldots\ldots\ldots(56),$$

$$\left.\begin{aligned}P&=\mu\gamma\frac{dy}{dt}-\mu\beta\frac{dz}{dt}+\frac{dF}{dt}-\frac{d\Psi}{dx}\\ Q&=\mu\alpha\frac{dz}{dt}-\mu\gamma\frac{dx}{dt}+\frac{dG}{dt}-\frac{d\Psi}{dy}\\ R&=\mu\beta\frac{dx}{dt}-\mu\alpha\frac{dy}{dt}+\frac{dH}{dt}-\frac{d\Psi}{dz}\end{aligned}\right\}\ldots\ldots\ldots\ldots\ldots\ldots\ldots(77).$$

$$\left.\begin{aligned}p&=\frac{1}{4\pi}\left(\frac{d\gamma}{dy}-\frac{d\beta}{dz}-\frac{1}{E^2}\frac{dP}{dt}\right)\\ q&=\frac{1}{4\pi}\left(\frac{d\alpha}{dy}-\frac{d\gamma}{dx}-\frac{1}{E^2}\frac{dQ}{dt}\right)\\ r&=\frac{1}{4\pi}\left(\frac{d\beta}{dx}-\frac{d\alpha}{dy}-\frac{1}{E^2}\frac{dR}{dt}\right)\end{aligned}\right\}\ldots\ldots\ldots\ldots\ldots\ldots\ldots(112),$$

$$e=\frac{1}{4\pi E^2}\left(\frac{dP}{dx}+\frac{dQ}{dy}+\frac{dR}{dz}\right)\ldots\ldots\ldots\ldots\ldots\ldots\ldots(115),$$

From Wikipedia:

Superfluidity is the characteristic property of a fluid with zero viscosity which therefore flows without loss of kinetic energy. **When stirred, a superfluid forms cellular vortices that continue to rotate indefinitely.**
It is also a property of various other exotic states of matter theorized to exist in astrophysics, high-energy physics, and theories of quantum gravity.[1] The phenomenon is related to Bose–Einstein condensation, but neither is a specific type of the other: not all Bose-Einstein condensates can be regarded as superfluids, and not all superfluids are Bose–Einstein condensates.

Superfluid vacuum theory (SVT) is an approach in theoretical physics and quantum mechanics where the physical vacuum is viewed as superfluid. The ultimate goal of the approach is to develop scientific models that unify quantum mechanics (describing three of the four known fundamental interactions) with gravity. This makes SVT a candidate for the theory of quantum gravity and an extension of the Standard Model. It is hoped that development of such theory would unify into a single consistent model of all fundamental interactions, and to describe all known interactions and elementary particles as different manifestations of the same entity, superfluid vacuum.

Vortex-quantisation in a superfluid

A superfluid has the special property of having phase, given by the wavefunction, and the velocity of the superfluid is proportional to the gradient of the phase (in the parabolic mass approximation). The circulation around any closed loop in the superfluid is zero if the region enclosed is simply connected. The superfluid is deemed irrotational; however, **if the enclosed region actually contains a smaller region with an absence of superfluid, for example a rod through the superfluid or a vortex, then the circulation is:**

$$\oint_C \mathbf{v} \cdot d\mathbf{l} = \frac{\hbar}{m} \oint_C \nabla\phi_v \cdot d\mathbf{l} = \frac{\hbar}{m} \Delta^{tot} \phi_v,$$

where $\hbar$ is Planck's constant divided by 2π, m is the mass of the superfluid particle, and $\Delta^{tot} \phi_V$ is the total phase difference around the vortex. **Because the wave-function must return to its same value after an integer number of turns around the vortex** (similar to what is described in the Bohr model), **then** $\Delta^{tot} \phi_V = 2\pi n$**, where** n **is an integer. Thus, the circulation is quantized**:

$$\oint_C \mathbf{v} \cdot d\mathbf{l} \equiv \frac{2\pi\hbar}{m} n$$

See paper: "Bose-Einstein condensation of photons in an optical microcavity" - https://arxiv.org/ftp/arxiv/papers/1007/1007.4088.pdf),

For m = 6.7 * 10^{-36} kg e

$\hbar = 1{,}054\,571\,726(47) \times 10^{-34}\ \mathrm{J \cdot s} = 6{,}582\,119\,28(15) \times 10^{-16}\ \mathrm{eV \cdot s}$ from the $\frac{2\pi\hbar}{m} n$

We have: for n = 1 0,9889656 * 10^2 , for n = 2 1.9779312 * 10^2 , thence the two values 98,89656 and 197,79312

We carry out the following double integrals:

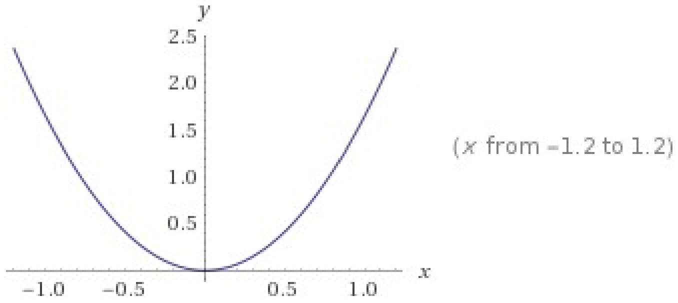
(1/(12e)) * 1.08643 * integrate integrate [98.89656]

$$\frac{1}{12\,e} \times 1.08643 \int \left(\int 98.89656\, dx \right) dx$$

Result:

$1.64694\, x^2$

Plot:



a result very near to the value of the mass of the proton

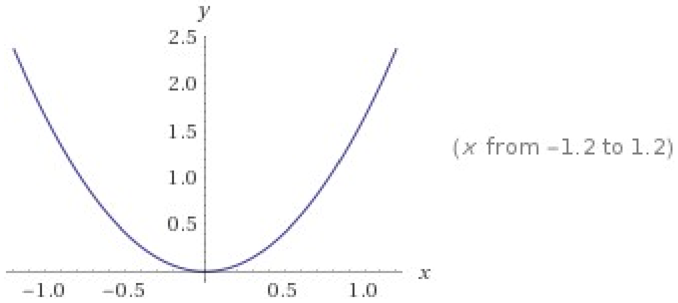
(1/(24e)) * 1.08643 * integrate integrate [197.79312]

$$\frac{1}{24\,e} \times 1.08643 \int \left(\int 197.79312\, dx \right) dx$$

Result:

$1.64694\, x^2$

Plot:



also this result is very near to the value of the mass of the proton

From:

VOLUME 44, NUMBER 8

Strings, superfluid vortices, and relativity

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(Received 29 December 1989; revised manuscript received 27 February 1991)

In the standard global string model [5,7], its dynamics are determined by an action of the form

$$S=-\int[\phi^{*}{}_{,\mu}\phi^{,\mu}+\tfrac{1}{2}\lambda(|\phi|^{2}-\phi_{0}{}^{2})^{2}]d^{4}x\ , \tag{2}$$

and the quantum-phenomenological description of superfluid vortices [10] is nothing but a nonrelativistic version of it [9]. Since most of the results of the present work (except for numerical values of some coefficients) are model independent, I shall use instead the general action [14]

$$S=-\int[\phi^{*}{}_{,\mu}\phi^{,\mu}+U(|\phi|^{2})]d^{4}x\ , \tag{3}$$

with the corresponding field equation $\phi^{,\mu}{}_{,\mu}=U'(|\phi|^{2})\phi$, whose real and imaginary parts are

$$|\phi|^{,\mu}{}_{,\mu}-(\varphi^{,\mu}+V^{\mu})(\varphi_{,\mu}+V_{\mu})|\phi|=U'(|\phi|^{2})|\phi|\ , \tag{4a}$$

$$[2|\phi|^{2}(\varphi^{,\mu}+V^{\mu})]_{,\mu}=0\ . \tag{4b}$$

It is assumed here that $U(|\phi|^{2})$ is a potential that allows vortex solutions under some conditions, such as a "Mexican hat," a Ginzburg-Landau potential, or a sine-Gordon potential. Since asymptotically $\varphi_{,\mu}\rightarrow 0$, we get by substitution of this limit in (4a) that there is a one-to-one correspondence between the chemical potential of the medium and the vacuum or ground-state expectation value of the field,

$$\omega^{2}=-V^{\mu}V_{\mu}=U'(\phi_{0}{}^{2})\ . \tag{5}$$

Using this combined model for superfluid vortices and global strings, we can now test the validity of the HIA for vortex arrays. Consider stationary arrays of straight and

$$\oint_c d\varphi = \oint_c v_\mu dx^\mu = 2\pi \sum_i \Gamma_i \ , \tag{6}$$

where the sum extends over all the vortices that are included in C. In order to bring Eq. (6) to a differential form, we assume that the vortices can be represented by two-dimensional world sheets; $x^\mu = \xi_i^\mu(\zeta^a)$, $a = 0, 1$. Then Eq. (6) leads to

$$v_{\nu,\mu} - v_{\mu,\nu} = \pi \sum_i \Gamma_i \cdot \epsilon_{\mu\nu\lambda\rho} \int\int \delta^4(x - \xi_i) d\Sigma_i^{\lambda\rho} \ , \tag{7}$$

where $\epsilon_{\mu\nu\lambda\rho}$ is the fully antisymmetric unit pseudotensor ($\epsilon_{0123} = 1$) and $d\Sigma_i^{\mu\nu} = d\xi_i^\mu \wedge d\xi_i^\nu$ is a surface element on the world sheet. The relativistic condition for incompressibility is that the sound velocity be equal to the velocity of light, leading to the equation [19]

$$v^\mu{}_{,\mu} = 0 \ . \tag{8}$$

Together Eqs. (7) and (8) are the fundamental equations for relativistic incompressible flow with isolated vortices. They were already obtained by Lund and Regge [2], who have shown their equivalence to the Kalb-Ramond field equations [1], and we see that they are naturally obtained from relativistic hydrodynamics. The general solution for v_μ is given by

$$\begin{aligned} v_\mu &= V_\mu + \sum_i v_{i\mu} \\ &= V_\mu + \tfrac{1}{4} \sum_i \Gamma_i \epsilon_{\mu\nu\lambda\rho} \int\int \partial^\nu G(x - \xi_i) d\Sigma_i^{\lambda\rho} \ , \end{aligned} \tag{9}$$

where $G(x - \xi_i)$ is a Green's function satisfying

$$\Box G(x - \xi_i) = -4\pi \delta^4(x - \xi_i) \ ,$$

and V_μ is an arbitrary constant vector, to be determined by boundary conditions. In the original Lund and Regge

$$J^\mu = \rho v^\mu = \frac{\rho_0}{4}(\epsilon^{\mu\nu\lambda\rho} A_{\lambda\rho})_{,\nu} = \frac{\rho_0}{12} F_{\nu\lambda\rho}\epsilon^{\mu\nu\lambda\rho} ,$$

$$F_{\mu\nu\lambda} = A_{\mu\nu,\lambda} + A_{\nu\lambda,\mu} + A_{\lambda\mu,\nu} . \quad (11)$$

Inverting Eq. (11), we obtain the relation

$$\frac{\rho_0}{\rho} F^{\mu\nu\lambda} = 2\epsilon^{\mu\nu\lambda\rho} v_\rho ,$$

which, upon taking its divergence and using (7), yields the field equation for $F^{\mu\nu\lambda}$:

$$\left[\frac{\rho_0}{\rho} F^{\mu\nu\lambda}\right]_{,\lambda} = 4\pi j^{\mu\nu}$$

$$\equiv 4\pi \sum_i \Gamma_i \int\int \delta^4(x-\xi_i) d\Sigma_i^{\mu\nu} . \quad (12)$$

An interesting interpretation of the vortices' field equation (12) is suggested by introducing the "medium vorticity" $j_m^{\mu\nu}$ defined by

$$j_m^{\mu\nu} \equiv \frac{-1}{4\pi}\left[\frac{\rho_0 - \rho}{\rho} F^{\mu\nu\lambda}\right]_{,\lambda} , \quad (28)$$

so that the field equation becomes

$$(F^{\mu\nu\lambda})_{,\lambda} = \Box A^{\mu\nu} = 4\pi(j^{\mu\nu} + j_m^{\mu\nu}) . \quad (29)$$

Ignoring for a moment the dependence of $j_m^{\mu\nu}$ on $F_{\mu\nu\lambda}$, Eq. (29) is in the form of a Kalb-Ramond field equation, with the right-hand side of it standing for the source term. In analogy with the electromagnetic theory, the Kalb-Ramond equations correspond to a free-space interaction [1]. Equation (29) is then formally analogous to electromagnetic theory in a medium [23]. It suggests, then, that the existence of the singular vortices induces some vorticity in the medium in which the vortices appear, which is significant only when the distances between the vortices or their radii of curvature are of the order of the core radius. The medium does not need to be material, since these results apply to global strings as well as to vortices in a fluid.

We carry out the following double integral on (29). We have:

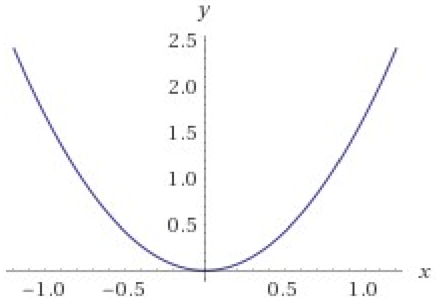
(2/(3e)) * 1.08643 * integrate integrate [4Pi]

$$\frac{2}{3e} \times 1.08643 \int\left(\int 4\pi \, dx\right) dx$$

Result:

$1.67416\,x^2$

Plot:



(x from −1.2 to 1.2)

Open code

value very near to the value of the mass of the neutron

From:

Vortices on world sheets of strings and superstrings

A. A. Abrikosov, Jr. and Ya. I. Kogan

Institute of Theoretical and ExperimentalPhysics

(Submitted ***15*** January 1989)

Zh. Eksp. Teor. Fiz. 96,418-436 (August 1989)

The defining role in the vortex system of the XY model is played by vortices with the smallest charge $Q=\pm 1$. Their contribution is described by the partition function of the two-dimensional Coulomb gas:

$$Z=\sum_{n_+=n_-=0}^{\infty}\int \frac{d^2\xi_1/a^2\dots d^2\xi_{n_+}/a^2}{n_+!}\,\frac{d^2z_1/a^2\dots d^2z_{n_-}/a^2}{n_-!}$$

$$\times\exp\Big[-2n\beta\mu+\pi\beta\sum_{i,j=1}^{n_+=n_-}\Big(\ln\frac{|\xi_i-\xi_j|^2+a^2}{a^2}+\ln\frac{|z_i-z_j|^2+a^2}{a^2}-\ln\frac{|\xi_i-z_j|^2+a^2}{a^2}\Big)\Big]. \qquad (3.1)$$

Expression (3.1) has a field representation. It can be shown that Eq. (3.1) is equivalent to the partition function for the sine-Gordon model[10]:

$$Z=N_0^{-1}\int D\varphi\,\exp\Big[-\int d^2\xi\Big(\frac{1}{2}(\partial_a\varphi)^2+\lambda\cos 2\pi(2\beta)^{1/2}\varphi\Big)\Big],$$

$$N_0=\int D\varphi\,\exp\Big[-\frac{1}{2}\int d^2\xi(\partial_a\varphi)^2\Big],\qquad \lambda=2e^{-\beta\mu}/a^2. \qquad (3.2)$$

Expansion of Eq. (3.2) in a power series in λ coincides with Eq. (3.1). Upon making in Eq. (3.2) the change of variable $y = 2\pi(2\beta)^{1/2}\varphi$ we obtain the Lagrangian

$$\mathcal{L} = (\partial_a y)^2/16\pi^2\beta + \lambda \cos y. \tag{3.3}$$

Let us consider the mean square value of the dipole moment of the vortex-antivortex pair, where the divergence corresponds to pair dissociation and BKT-transition:

$$\langle \mathbf{p}^2 \rangle = \int d^2\xi\, d^2\eta\, |\xi-\eta|^2 \exp\left(-2\pi\beta \ln \frac{|\xi-\eta|^2+a^2}{a^2}\right)\Omega_2(\xi,\eta)$$
$$\times\left[\int d^2\xi\, d^2\eta \exp\left(-2\pi\beta \ln \frac{|\xi-\eta|^2+a^2}{a^2}\right)\Omega_2(\xi,\eta)\right]^{-1}$$
$$\sim a^2 \frac{\pi\beta}{2\pi\beta-1}, \quad \beta_c{}' = \frac{1}{2}\pi^{-1} = \frac{1}{2}\beta_c, \tag{5.8}$$

i.e., the critical value for $\beta_c{}^f$, and therefore for $R_c{}^2$, is two times smaller than in the bosonic case.

There exists a field description of supervortices, analogous to Eq. (3.2), with the Lagrangian in this case being

$$\mathcal{L} = \frac{1}{2}(\partial_a\Phi)^2 + \frac{1}{2}\bar{\psi} i\gamma_a\partial_a\psi - \lambda\bar{\psi}\psi\cos[2\pi(2\beta)^{1/2}\Phi]. \tag{5.9}$$

The equivalence between the partition functions corresponding to Eqs. (5.9) and (5.6) is proved by term-by-term comparison of the series in powers of λ, just as in the bosonic case. The factor $\Omega_{2p}(\xi_i, z_j)$ [see Eq. (5.7)] arises from evaluation of thee average of the product $\bar{\Psi}\Psi$.

The Lagrangian, Eq. (5.9), differs from the supersymmetric Lagrangian in the sine-Gordon model by the absence of the term $(\lambda^2/4\pi^2\beta)\sin^2[2\pi(2\beta)\frac{1}{2}\Phi]$, which is of higher order in λ and immaterial in the limit of small λ.

The critical value $\beta_c{}^f = \frac{1}{2}\pi^{-1}$ can be obtained from Eq. (5.9) by, for example, bosonization $\bar{\Psi}\Psi \to \cos[2\pi(2\beta)\frac{1}{2}\Phi], i\bar{\Psi}\delta_a\partial_a\Psi \to (\partial_a\Phi)^2$. Comparing the Lagrangian

$$\mathcal{L} = (\partial_a\Phi)^2 + \lambda' \cos[4\pi(2\beta)^{1/2}\Phi]$$

with Eq. (3.3) we arrive at the value $\beta_c{}^f = \frac{1}{2}\pi^{-1}$. As in the bosonic case there is the duality (3.4), and the action (5.9) describes the superstring in the tachyonic field condensate.

We take the value of $\frac{1}{2}\pi^{-1} = 0.63661977 ...$ and carry out the following double integral:

(Pi^2*sqrt(2))/30 *(1/10^18)) * 1.08643 * integrate integrate [(Pi/2)^-1)]

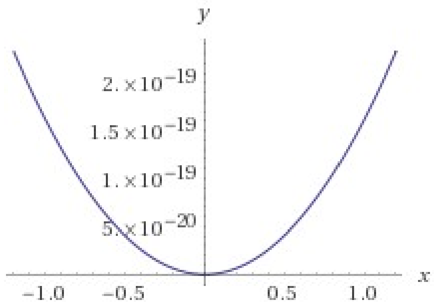
where $(\pi^2\sqrt{2})/30$, Korkin-Zolotarev constant

$$\left(\frac{1}{30}\left(\pi^2\sqrt{2}\right)\right)\times\frac{1}{10^{18}}\times 1.08643\int\left(\int\frac{1}{\frac{\pi}{2}}\,dx\right)dx$$

Result:

$1.60896\times 10^{-19}\,x^2$

Plot:



(x from −1.2 to 1.2)

value very near to the electric charge of the positron

From:

Rotating black strings in f(R)-Maxwell theory
A. Sheykhi, S. Salarpour and Y. Bahrampour

II. FIELD EQUATIONS AND SOLUTIONS

We start from the four-dimensional $R + f(R)$ theory coupled to the Maxwell field

$$I_G = -\frac{1}{16\pi}\int_{\mathcal{M}} d^4x\sqrt{-g}\left(R + f(R) - F_{\mu\nu}F^{\mu\nu}\right) - \frac{1}{8\pi}\int_{\partial\mathcal{M}} d^3x\sqrt{-h}\Theta(h), \tag{1}$$

where R is the Ricci scalar curvature, $F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$ is the electromagnetic field tensor, and A_μ is the electromagnetic potential. The last term in Eq. (1) is the Gibbons-Hawking boundary term. It is required for the variational principle to be well-defined. The factor Θ represents the trace of the extrinsic curvature for the boundary $\partial\mathcal{M}$ and h is the induced metric on the boundary. The equations of motion can be obtained by varying the action (1) with respect to the gravitational field $g_{\mu\nu}$ and the gauge field A_μ which yields the following field equations

$$R_{\mu\nu}\left(1 + f'(R)\right) - \frac{1}{2}g_{\mu\nu}\left(R + f(R)\right) + \left(g_{\mu\nu}\nabla^2 - \nabla_\mu\nabla_\nu\right)f'(R) = 8\pi T_{\mu\nu}, \tag{2}$$

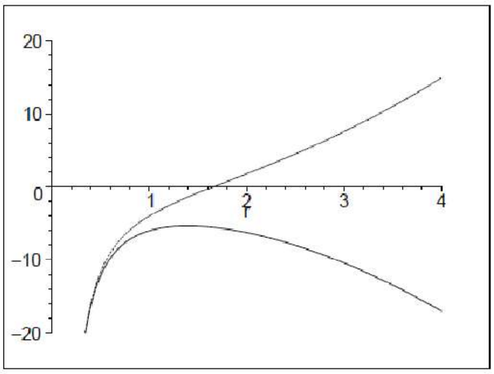


FIG. 3: The function $N(r)$ versus r for $m = 2$, $f'(R_0) = -2$ and $q = 1$. $R_0 = 12$ (bold line) and $R_0 = -12$ (continuous line).

$$\nabla_\mu F^{\mu\nu} = 0, \tag{3}$$

with the energy-momentum tensor

$$T_{\mu\nu} = \frac{1}{4\pi}\left(F_{\mu\eta}F_{\nu}^{\ \eta} - \frac{1}{4}g_{\mu\nu}F_{\lambda\eta}F^{\lambda\eta}\right). \tag{4}$$

The above energy-momentum tensor is traceless in four dimension, i. e., $T^{\mu}_{\ \mu} = 0$. As we mentioned already this property plays an important role in our derivation. In Eq. (2) the "prime" denotes differentiation with respect to curvature scalar R. Assuming the constant curvature scalar $R = R_0$, the trace of Eq. (2) yields

$$R_0\left(1 + f'(R_0)\right) - 2\left(R_0 + f(R_0)\right) = 0, \tag{5}$$

Solving the above equation for negative R_0, gives

$$R_0 = \frac{2f(R_0)}{f'(R_0) - 1} \equiv 4\Lambda_{\mathrm{f}} < 0. \tag{6}$$

Substituting the above relation into Eq. (2), we obtain the following equation for Ricci tensor

$$R_{\mu\nu} = \frac{1}{2}g_{\mu\nu}\left(\frac{f(R_0)}{f'(R_0) - 1}\right) + \frac{2}{1 + f'(R_0)}T_{\mu\nu}. \tag{7}$$

Now, we want to construct charged rotating black string solutions of the field equations (2) and (3) and investigate their properties. We are looking for the four-dimensional rotating solution with cylindrical or toroidal horizons. The metric which describes such a spacetime can be written in the following form [17, 18]

$$\begin{aligned} ds^2 &= -N(r)\left(\Xi dt - a d\phi\right)^2 + r^2\left(\frac{a}{l^2}dt - \Xi d\phi\right)^2 + \frac{dr^2}{N(r)} + \frac{r^2}{l^2}dz^2, \\ \Xi^2 &= 1 + \frac{a^2}{l^2}, \end{aligned} \tag{8}$$

where a is the rotation parameter. The function $N(r)$ should be determined and l has the dimension of length which is related to the constant Λ_{f} by the relation $l^2 = -3/\Lambda_{\mathrm{f}}$. The two dimensional space, t=constant and r =constant, can be (i) the flat torus model T^2 with topology $S^1 \times S^1$, and $0 \leq \phi < 2\pi$, $0 \leq z < 2\pi l$, (ii) the standard cylindrical model with topology $R \times S^1$, and $0 \leq \phi < 2\pi$, $-\infty < z < \infty$, and (iii) the infinite plane R^2 with $-\infty < \phi < \infty$ and $-\infty < z < \infty$. We will focus upon (i) and (ii). The Maxwell equation (3) can be integrated immediately to give

$$\begin{aligned} F_{tr} &= \frac{q\Xi}{r^2}, \\ F_{\phi r} &= -\frac{a}{\Xi}F_{tr}, \end{aligned} \tag{9}$$

where q is the charge parameter of the black string. Substituting the Maxwell fields (9) as well as the metric (8) in the field equation (2) with constant curvature, the non-vanishing independent components of the field equations for $a = 0$ reduce to

$$\left(1 + f'(R_o)\right)\left(2r^4\frac{d^2N(r)}{dr^2} + 4r^3\frac{dN(r)}{dr} + R_o r^4\right) - 4q^2 = 0, \tag{10}$$

$$\left(1 + f'(R_o)\right)\left(4r^3\frac{dN(r)}{dr} + 4r^2N(r) + R_0 r^4\right) + 4q^2 = 0. \tag{11}$$

One can easily show that the above equations have the following solution

$$N(r) = -\frac{2m}{r} + \frac{q^2}{(1 + f'(R_0))r^2} - \frac{R_0}{12}r^2, \tag{12}$$

where m is an integration constant which is related to the mass of the string. One can also check that these solutions satisfy equations (2)-(3) in the rotating case where $a \neq 0$. It is apparent that this spacetime is similar with asymptotically AdS black string. Indeed, with the following replacement

$$\frac{q^2}{(1 + f'(R_0))} \to Q^2 \tag{13}$$

$$\frac{R_0}{4} \to \Lambda \tag{14}$$

where N and V^i are the lapse function and shift vector. Then the electric field is $E^\mu = g^{\mu\rho}F_{\rho\nu}u^\nu$, and the electric charge per unit length of the string can be found by calculating the flux of the electric field at infinity,

$$Q = \frac{\Xi q}{4\pi l\sqrt{1+f'(R_0)}}. \tag{28}$$

The electric potential U, measured at infinity with respect to the horizon, is defined by [22]

$$U = A_\mu \chi^\mu \left|_{r\to\infty} - A_\mu \chi^\mu\right|_{r=r_+}, \tag{29}$$

where χ is the null generator of the event horizon given in Eq. (23). One can easily obtain the electric potential as

$$U = \frac{q}{\Xi r_+}\sqrt{1+f'(R_0)}. \tag{30}$$

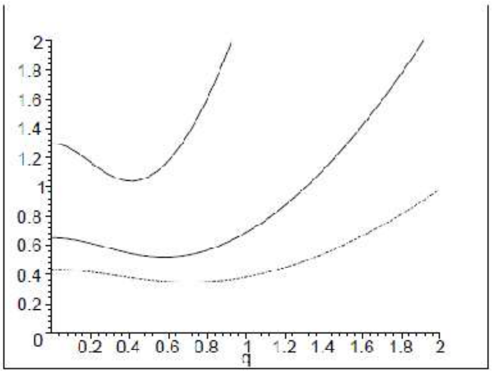


FIG. 4: The function $(\partial^2 M/\partial S^2)_{J,Q}$ versus q for $l = 1$, $\Xi = 1.25$, $r_+ = 0.7$ and $R_0 = -12$. $f'(R_0) = 0$ (bold line), $f'(R_0) = 1$ (continuous line) and $f'(R_0) = 2$ (dashed line).

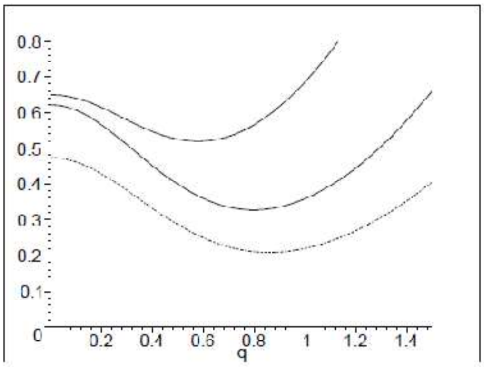


FIG. 6: The function $(\partial^2 M/\partial S^2)_{J,Q}$ versus q for $l = 1$, $f'(R_0) = 1$ and $R_0 = -12$. $\Xi = 1.25$, (bold line), $\Xi = 1.75$, (continuous line) and $\Xi = 2.25$, (dashed line).

From the eq. (28), we obtain:

$$Q = \frac{\Xi q}{4\pi l\sqrt{1+f'(R_0)}}.$$

Q = (2.25 * 1) / 4π*1*√(1+1) = 0,12660698195959304103119988623532;

We calculate the following double integral for Q:

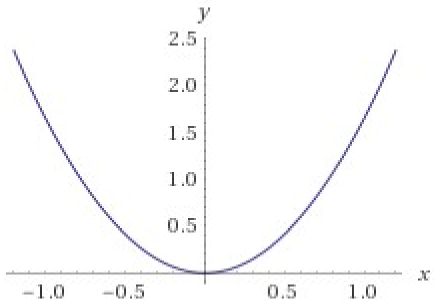
7Pi * 1.08643^2 * integrate integrate [0.12660698195959304103119988623532]

$$7\pi \times 1.08643^2 \int\left(\int 0.1266069819595930410311998862353 2\, dx\right) dx$$

Result:

$1.64316\, x^2$

Plot:



(x from −1.2 to 1.2)

Or:

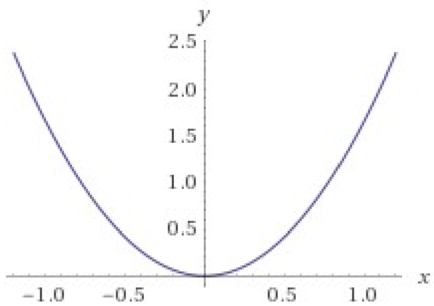
(24) * 1.08643 * integrate integrate [0.12660698195959304103119988623532]

$$24 \times 1.08643 \int\left(\int 0.12660698195959304103119988623532\, dx\right) dx$$

Result:

$1.6506\, x^2$

Plot:



(x from −1.2 to 1.2)

values very near to the mass of the proton.

And:

(53Pi*11) * 1.08643 * integrate integrate [0.12660698195959304103119988623532]

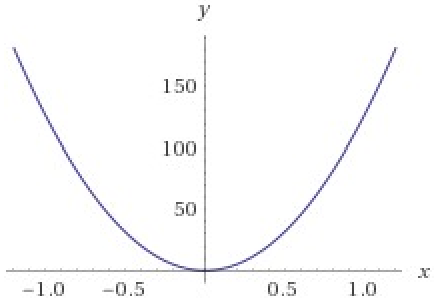
$$(53\pi \times 11) \times 1.08643 \int\left(\int 0.12660698195959304103119988623532\, dx\right) dx$$

Result:

$125.964\, x^2$

value practically equal to the value of the mass of the Higgs boson

Plot:



(x from -1.2 to 1.2)

Where 53 is a prime number and is the sum of five prime numbers

53 = 5 + 7 + 11 + 13 + 17.

Then:

$(Q)^{1/5}$ = 0,66144163923697434151492565583228

$(Q)^{1/5}$ * $(10)^{2/5}$ = 0,66144163923697434151492565583228 * 2,5118864315095801110850320677993 = 1,6614662788348105459451448294251

Now, we calculate the following double integrals of this result:

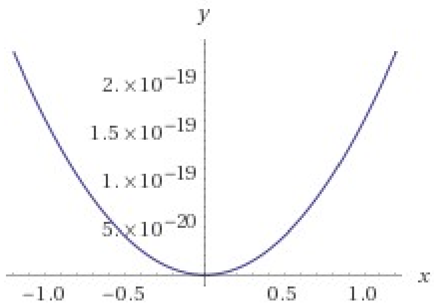
1/(10^19) e*(0.6530256) * 1.08643 * integrate integrate [1.6614662788348105459451448294251]

$$\frac{1}{10^{19}}\, e \times 0.6530256 \times 1.08643 \int\left(\int 1.6614662788348105459451448294251\, dx\right) dx$$

Result:

$1.60209 \times 10^{-19}\, x^2$

Plot:



(x from -1.2 to 1.2)

and:

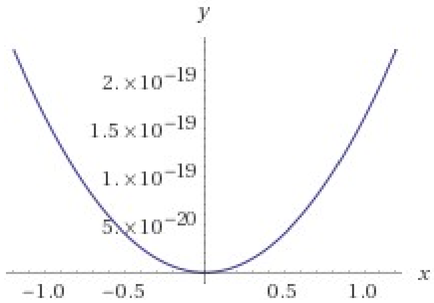
1/(10^-33) * 1/(10^52) * e*(0.64180256) * 1.1056 * integrate integrate [1.6614662788348105459451448294251]

$$\frac{1}{\frac{1}{10^{33}}} \times \frac{1}{10^{52}} e \times 0.64180256 \times 1.1056$$
$$\int\left(\int 1.6614662788348105459451448294251 \, dx\right) dx$$

Result:

$1.60234 \times 10^{-19} x^2$

Plot:



(x from −1.2 to 1.2)

values practically equals to the electric charges of the positron

Note that we have calculated the first double integral with the "Ramanujan new constant", while the second double integral with the value of Cosmological Constant given the Planck (2018), where Λ has the value of

$$\Lambda = 1.1056 \times 10^{-52} \, \mathrm{m}^{-2},$$

or: 4.33×10^{-66} eV^2 in natural units. Thus, look evident that the Ramanujan new constant is a good approximation of the Cosmological constant, perhaps also more precise than the given value from Planck satellite, being the result of precise mathematical calculations of a inspired genius.

We note that the eq. (3), i.e.

$$\nabla_\mu F^{\mu\nu} = 0,$$

is the fundamental following Maxwell's equation, i.e. the Gauss' law for magnetism:

$$\nabla \cdot \mathbf{B} = 0$$

Or in integral form

$$\int_S \mathbf{B} \cdot d\mathbf{a} = 0$$

Thence, evident mathematical connections between this Maxwell's equation and the charge of black strings (charged rotating black holes).

Now from the eq. (30) concerning the electric potential:

$$U = \frac{q}{\Xi r_+}\sqrt{1 + f'(R_0)}.$$

U = 1 / (1.25 * 0.7) √3 =

1,1428571428571428571428571428571 * 1,7320508075688772935274463415059 =

= 1,979486637221574049745652961721

Now we calculate the following integral:

(Pi*(ln1.606695)) * 1.08643 * integrate integrate [1.979486637221574049745652961721]

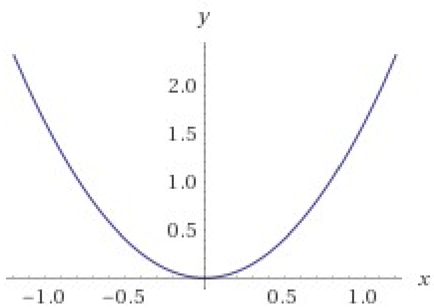
where 1.606 695 is the "Erdös - Borwein constant"

$$(\pi \log(1.606695)) \times 1.08643 \int\left(\int 1.979486637221574049745652961721\, dx\right) dx$$

Result:

$1.60183\, x^2$

Plot:



(x from −1.2 to 1.2)

Now, we calculate the following double integral:

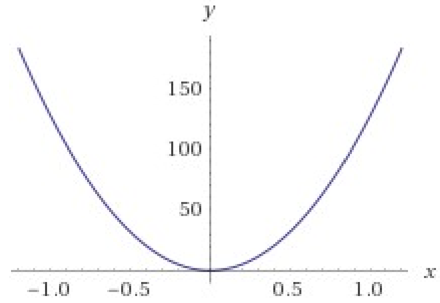
(11Pi^2) * 1.08643^2 * integrate integrate [1.979486637221574049745652961721]

$$(11\,\pi^2) \times 1.08643^2 \int\left(\int 1.979486637221574049745652961721\, dx\right) dx$$

Result:

$126.829\, x^2$

Plot:



(x from -1.2 to 1.2)

results very neat to the electric charge of positron and to the mass of the Higgs boson.

Maxwell's equation concerning the Gauss' law in differential and integral form:

$$\nabla \cdot \mathbf{E} = \frac{\rho}{\epsilon_0}$$

$$\int_S \mathbf{E} \cdot d\mathbf{a} = \frac{1}{\epsilon_0} \int \rho \, dV$$

From:

ELEVEN-DIMENSIONAL SUPERGRAVITY ON A MANIFOLD WITH BOUNDARY
Petr Horava - Joseph Henry Laboratories, Princeton University - Jadwin Hall, Princeton, NJ 08544, USA **and Edward Witten**⋆ School of Natural Sciences, Institute for Advanced Study * Olden Lane, Princeton, NJ 08540, USA

The supergravity multiplet consists of the metric g, the gravitino $\psi_{I\alpha}$, and a three-form C (with field strength G, normalized as in a previous footnote). The supergravity Lagrangian, up to terms quartic in the gravitino (which we will not need), is [8]

$$L_S = \frac{1}{\kappa^2} \int_{M^{11}} d^{11}x \sqrt{g} \Big(\; \frac{1}{2} R \quad \frac{1}{2} \overline{\psi}_I \Gamma^{IJK} D_J \psi_K \quad \frac{1}{48} G_{IJKL} G^{IJKL}$$
$$- \frac{\sqrt{2}}{192} \left(\overline{\psi}_I \Gamma^{IJKLMN} \psi_N + 12 \overline{\psi}^J \Gamma^{KL} \psi^M \right) G_{JKLM} \qquad (2.1)$$
$$- \frac{\sqrt{2}}{3456} \epsilon^{I_1 I_2 \dots I_{11}} C_{I_1 I_2 I_3} G_{I_4 \dots I_7} G_{I_8 \dots I_{11}} \Big) .$$

Note that: 3456 = 1728 * 2 and that $\frac{\sqrt{2}}{3456} = \frac{1}{1728\sqrt{2}}$

From Polchinski book "String Theory vol. I", we have that:

$$S = \frac{1}{2\kappa^2}\int d^DX\,(-\tilde{G})^{1/2}\Big[-\frac{2(D-26)}{3\alpha'}e^{4\tilde{\Phi}/(D-2)} + \tilde{R} - \frac{1}{12}e^{-8\tilde{\Phi}/(D-2)}H_{\mu\nu\lambda}\tilde{H}^{\mu\nu\lambda} - \frac{4}{D-2}\partial_\mu\tilde{\Phi}\tilde{\partial}^\mu\tilde{\Phi} + O(\alpha')\Big], \qquad (3.7.25)$$

where tildes have been inserted as a reminder that indices here are raised with $\tilde{G}^{\mu\nu}$. In terms of $\tilde{G}_{\mu\nu}$, the gravitational Lagrangian density takes the standard Hilbert form $(-\tilde{G})^{1/2}\tilde{R}/2\kappa^2$. The constant $\kappa = \kappa_0 e^{\Phi_0}$ is the gravitational coupling, which in four-dimensional gravity has the value

$$\kappa = (8\pi G_N)^{1/2} = \frac{(8\pi)^{1/2}}{M_P} = (2.43 \times 10^{18}\ \text{GeV})^{-1}. \qquad (3.7.26)$$

Thence: $(8\pi G_N)^{1/2} = \kappa = (2.43 * 10^{18}\ \text{GeV})^{-1} = 4{,}115226337448 * 10^{-19}$;

and $\kappa^2 = 1{,}693508780843 * 10^{-37}$.

We have that:

$$\Big(-\frac{1}{2}R - \frac{1}{2}\overline{\psi}_I\Gamma^{IJK}D_J\psi_K - \frac{1}{48}G_{IJKL}G^{IJKL} - \frac{\sqrt{2}}{192}\left(\overline{\psi}_I\Gamma^{IJKLMN}\psi_N + 12\overline{\psi}^J\Gamma^{KL}\psi^M\right)G_{JKLM} - \frac{\sqrt{2}}{3456}\epsilon^{I_1I_2\ldots I_{11}}C_{I_1I_2I_3}G_{I_4\ldots I_7}G_{I_8\ldots I_{11}}\Big).$$

$$-\frac{1}{2}-\frac{1}{2}-\frac{1}{48}-\frac{\sqrt{2}}{192}-\frac{12\sqrt{2}}{192}-\frac{\sqrt{2}}{3456}=$$

$$=\frac{-1728-1728-72-18\sqrt{2}-216\sqrt{2}-\sqrt{2}}{3456}=$$

$$=\frac{1728}{3456}-\frac{1728}{3456}-\frac{72}{3456}-\frac{18\sqrt{2}}{3456}-\frac{216\sqrt{2}}{3456}-\frac{\sqrt{2}}{3456}=$$

-3860,34018656 / 3456 = – 1,11699658

$1 / \kappa^2 = 5{,}9049 * 10^{36}$ that multiplied to – 1,11699658 = $-6{,}59575311 * 10^{36}$

Now the gravitational coupling is:

$$\alpha_G = \frac{Gm_e^2}{\hbar c} = \left(\frac{m_e}{m_P}\right)^2 \approx 1.751751596 \times 10^{-45}$$

(1) In weakly coupled heterotic string theory, the gauge and gravitational couplings unify at tree level to form one dimensionless string coupling constant g_{string} [10]

$$k_Y g_Y^2 = k_2 g_2^2 = k_3 g_3^2 = 8\pi \frac{G_N}{\alpha'} = g_{\text{string}}^2 \,, \qquad (1)$$

where g_Y, g_2, and g_3 are the gauge couplings for the $U(1)_Y$, $SU(2)_L$, and $SU(3)_C$, respectively, G_N is the gravitational coupling and α' is the string tension. Here, k_Y, k_2 and k_3 are the levels of the corresponding Kac-Moody algebras; k_2 and k_3 are positive integers while k_Y is a rational number in general [10].

In the paper "INTRODUCTION TO STRING THEORY∗ version 14-05-04

of Gerard 't Hooft" α' appeared to be universal, approximately 1 GeV^{-2} . Thence:

$g^2 = 8\pi\ (1{,}751751596 * 10^{-45}) = 4{,}40263196 * 10^{-44}$; $g = 2{,}09824497 * 10^{-22}$;

$\sqrt{g} = 1{,}44853201 * 10^{-11}$.

Now, we calculate the following integral:

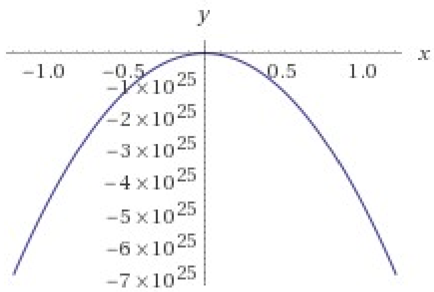
5.9049 * (10^36) integrate [(1.44853201 * 10^-11)*(-1.11699658)] x

$$5.9049 \times 10^{36} \int \left(\frac{1.44853201}{10^{11}} \times (-1.11699658)\right) x\, dx$$

Result:

$-4.77708 \times 10^{25}\, x^2$

Plot:



(x from -1.2 to 1.2)

Indefinite integral assuming all variables are real:

$-1.59236 \times 10^{25}\, x^3$ + constant

Now:

(1/10^54) * 1.08643^2 * 5.9049 * (10^36) integrate [(1.44853201 * 10^-11)*(-1.11699658)] x, [0, 34/(2Pi)]

$$\frac{1}{10^{54}} \times 1.08643^2 \times 5.9049 \times 10^{36} \int_0^{\frac{34}{2\pi}} \left(\frac{1.44853201}{10^{11}} \times (-1.11699658) \right) x \, dx$$

Result:

-1.65106×10^{-27}

This results is a good approximation to the value of the mass of the anti-proton.

We note that:

$$\sqrt{g}\left(-\frac{1}{2}R - \frac{1}{2}\overline{\psi}_I \Gamma^{IJK} D_J \psi_K - \frac{1}{48} G_{IJKL} G^{IJKL} - \frac{\sqrt{2}}{192}\left(\overline{\psi}_I \Gamma^{IJKLMN} \psi_N + 12 \overline{\psi}^J \Gamma^{KL} \psi^M\right) G_{JKLM} - \frac{\sqrt{2}}{3456} \epsilon^{I_1 I_2 \ldots I_{11}} C_{I_1 I_2 I_3} G_{I_4 \ldots I_7} G_{I_8 \ldots I_{11}}\right).$$

is equal to

$$\frac{1.44853201}{10^{11}} \times (-1.11699658)$$

Result:

$-1.6180053011905258 \times 10^{-11}$

Now, we calculate the following double integral:

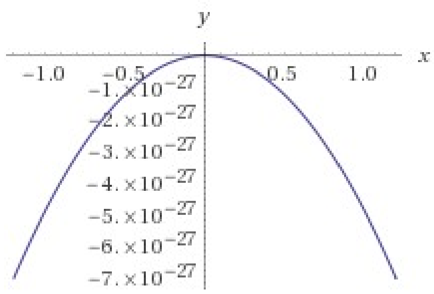
(1*10^-52) * (2*0.618)^3 * 1.08643 integrate integrate [-4.77708*10^25]

$$1 \times 10^{-52} (2 \times 0.618)^3 \times 1.08643 \int \left(\int -4.77708 \times 10^{25} \, dx \right) dx$$

Result:

$-4.89993 \times 10^{-27} x^2$

Plot:



(x from -1.2 to 1.2)

Indefinite integral assuming all variables are real:

$-1.63331\times10^{-27}\,x^3$ + constant

Also this result is a good approximation of the value of the mass of the anti-proton.

Now:

"**Ramanujan – Twelve lectures on subjects suggested by his life and work**" – by G. H. Hardy – Cambridge at the University Press - 1940

10.5. The congruences of § 10.4 are satisfied by all n of certain arithmetical progressions. There are also congruences satisfied by "almost all" n. For example

(10.5.1) $$\tau(n)\equiv 0 \pmod 5$$

for almost all n (in the sense of § 3.4).

We begin by proving that

(10.5.2) $$\tau(n)\equiv n\sigma(n) \pmod 5,$$

where $\sigma(n)$ is the sum of the divisors of n, for all n. This depends on two identities in the theory of the modular functions, viz.

(10.5.3) $$Q^3-R^2 = 1728g(x),$$

and

(10.5.4) $$Q-P^2 = 288\,\Sigma\frac{n^2x^n}{(1-x^n)^2},$$

where

(10.5.5) $$P = 1-24\left(\frac{x}{1-x}+\frac{2x^2}{1-x^2}+\frac{3x^3}{1-x^3}+\dots\right),$$

(10.5.6) $$Q = 1+240\left(\frac{x}{1-x}+\frac{2^3x^2}{1-x^2}+\frac{3^3x^3}{1-x^3}+\dots\right)$$

(10.5.7) $$R = 1-504\left(\frac{x}{1-x}+\frac{2^5x^2}{1-x^2}+\frac{3^5x^3}{1-x^3}+\dots\right).$$

The identity (10.5.3) is familiar, but I have not seen (10.5.4) anywhere except in Ramanujan's work.

We have that Q = 241, P = -23 $Q – P^2$ = 241 – 529 = – 288

$Q^3 – R^2 = (1 + 240)^3 – (1 – 504)^2$ = 13997521 – 253009 = 13744512;

Where 13744512 = 1728 * 7954; 1728 * 7954 = 13744512;

1728 = 13744512 / 7954

We calculate the following integral:

integrate [13744512] x, [0, 1/(1.644934^13*Pi)]

$$\int_0^{\frac{1}{1.644934^{13}\pi}} 13\,744\,512\,x\,dx = 1.67086$$

where 1,644934 = ζ(2) = $\pi^2/6$

And the following double integral:

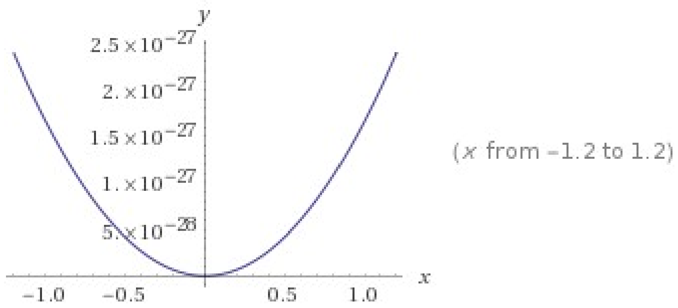
1/(10^34) * 1.08643 * (Pi/sqrt(2)) integrate integrate [13744512]

$$\frac{1}{10^{34}} \times 1.08643 \times \frac{\pi}{\sqrt{2}} \int\left(\int 13\,744\,512\,dx\right)dx$$

Result:

$1.65858 \times 10^{-27}\,x^2$

Plot:



Results that are very near to the value of the mass of the proton.

From:

INTRODUCTION TO STRING THEORY
version 14-05-04 - Gerard 't Hooft

12.2. Computing the spectrum of states.

The general method to compute the number of states consists of calculating, for the entire Hilbert space,

$$G(q) = \sum_{n=0}^{\infty} W_n q^n = \mathrm{Tr}\, q^N \ , \tag{12.12}$$

where q is a complex number corresponding to $1/z = e^{-i\tau}$, as in Eq. (10.3), W_n is the degree of degeneracy of the $n^{\rm th}$ level, and N is the number operator,

$$N = \sum_{\mu=1}^{D-2}\Big(\sum_{n=1}^{\infty}\alpha_{-n}\alpha_n + \sum_{r>0} r d_{-r}d_r\Big) = \sum_{\mu=1}^{D-2}\Big(\sum_{n=1}^{\infty} n\, N^{\rm Bos}_{\mu,n} + \sum_{r>0} r\, N^{\rm Ferm}_{\mu,r}\Big) \ , \tag{12.13}$$

where the sum over the fermionic operators is either over integers (Ramond) or integers $+\frac{1}{2}$ (Neveu-Schwarz). Since N receives its contributions independently from each mode, we can write $G(q)$ as a product:

$$G(q) = \prod_{\mu=1}^{D-2}\prod_{n=1}^{\infty}\prod_{r>0} f_n(q) g_r(q) \ , \tag{12.14}$$

with

$$f_n(q) = \sum_{m=0}^{\infty} q^{nm} = \frac{1}{1-q^n} \ , \tag{12.15}$$

while

$$g_r(q) = \sum_{m=0}^{1} q^{rm} = 1 + q^r \ . \tag{12.16}$$

We find that, for the purely bosonic string in 24 transverse dimensions:

$$G(q) = \prod_{n=1}^{\infty}(1-q^n)^{-24} \ . \tag{12.17}$$

The Taylor expansion of this function gives us the level density functions W_n. There are also many mathematical theorems concerning functions of this sort.

For the superstring in 8 transverse dimensions, we have

$$\begin{aligned} G(q) &= \prod_{n=1}^{\infty}\left(\frac{1+q^{n-\frac{1}{2}}}{1-q^n}\right)^8 \quad ({\rm NS}) \ ; \\ G(q) &= 16\prod_{n=1}^{\infty}\left(\frac{1+q^n}{1-q^n}\right)^8 \quad ({\rm Ramond}) \ , \end{aligned} \tag{12.18}$$

where, in the Ramond case, the overall factor 16 comes from the 16 spinor elements of the ground state.

Now let us impose the GSO projection. In the Ramond case, it simply divides the result by 2, since we start with an 8 component spinor in the ground state. In the NS case, we have to remove the states with even fermion number. This amounts to

$$G(q) = \tfrac{1}{2}{\rm Tr}\left(q^N - (-1)^F q^N\right) \ , \tag{12.19}$$

where F is the fermion number. Multiplying with $(-1)^F$ implies that we replace $g(r)$ in Eq. (12.16) by

$$\tilde{g}(r) = \sum_{m=0}^{1} (-q)^{rm} = 1 - q^r \ . \tag{12.20}$$

This way, Eq. (12.18) turns into

$$\begin{aligned} G_{\rm NS}(q) &= \frac{1}{2\sqrt{q}} \left[\prod_{n=1}^{\infty} \left(\frac{1+q^{n-\frac{1}{2}}}{1-q^n} \right)^8 - \prod_{n=1}^{\infty} \left(\frac{1-q^{n-\frac{1}{2}}}{1-q^n} \right)^8 \right] \quad \text{(NS)} \ ; \\ G_{\rm R}(q) &= 8 \prod_{n=1}^{\infty} \left(\frac{1+q^n}{1-q^n} \right)^8 \qquad \text{(Ramond)} \ . \end{aligned} \tag{12.21}$$

Here, in the NS case, we divided by $\sqrt{q}$ because the ground state can now be situated at $N = -\frac{1}{2}$, and it cancels out.

The mathematical theorem alluded to in the previous subsection says that, in Eq. (12.21), $G_{\rm NS}(q)$ and $G_{\rm R}(q)$ are equal. Mathematica gives for both:

$$\begin{aligned} G(q) &= 8 + 128\,q + 1152\,q^2 + 7680\,q^3 + 42112\,q^4 + 200448\,q^5 \\ &\quad + 855552\,q^6 + 3345408\,q^7 + 12166272\,q^8 + \cdots \ . \end{aligned} \tag{12.22}$$

We note that:

1152 / 288 = 4 12166272 / 288 = 42244 200448 / 1728 = 116

3345408 / 1728 = 1936; where 288 * 6 = 1728

We calculate the following double integral:

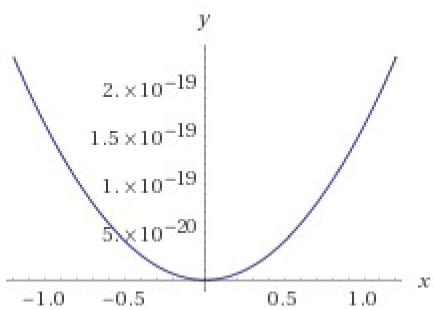
1/(10^22) * 1.08643^2 * (Pi/2) integrate integrate [1728]

$$\frac{1}{10^{22}} \times 1.08643^2 \times \frac{\pi}{2} \int \left(\int 1728\, dx \right) dx$$

Result:

$1.60191 \times 10^{-19}\, x^2$

Plot:



(x from -1.2 to 1.2)

And:

1.08643 integrate [1728] x, [0, Pi/((1.618)^9)]

Definite integral:

$$1.08643 \int_0^{0.0413374} 1728\, x\, dx = 1.60399$$

results very near to the values of the electric charges of the positron.

Now we take some parts of the following very interesting paper: "RAMANUJAN'S UNPUBLISHED MANUSCRIPT ON THE PARTITION AND TAU FUNCTIONS WITH PROOFS AND COMMENTARY - Bruce C. Berndt and Ken Ono"

PROPERTIES OF $p(n)$ AND $\tau(n)$ DEFINED BY THE FUNCTIONS $\sum_{n=0}^{\infty} p(n)q^n = (q;q)_\infty^{-1}$, $\sum_{n=1}^{\infty} \tau(n)q^n = q(q;q)_\infty^{24}$

S. RAMANUJAN

We take:

$$\sum_{n=1}^{\infty} \tau(n)q^n = q(q;q)_\infty^{24}$$

Modulus 5

1. Let

$$P := 1 - 24\sum_{n=1}^{\infty}\frac{nq^n}{1-q^n},$$
$$Q := 1 + 240\sum_{n=1}^{\infty}\frac{n^3q^n}{1-q^n}$$

and

$$R := 1 - 504\sum_{n=1}^{\infty}\frac{n^5q^n}{1-q^n},$$

so that[2]

$$Q^3 - R^2 = 1728q(q;q)_\infty^{24}. \tag{1.1}$$

Let $\sigma_s(n)$ denote the [sum of the] s^{th} powers of the divisors of n. Then it is easy to see that

$$Q = 1 + 5J; \qquad R = P + 5J. \tag{1.2}$$

Hence,

$$Q^3 - R^2 = Q - P^2 + 5J. \tag{1.3}$$

But[3]

$$Q - P^2 = 288\sum_{n=1}^{\infty} n\sigma_1(n)q^n; \tag{1.4}$$

and it is obvious that

$$(q;q)_\infty^{24} = \frac{(q^{25};q^{25})_\infty}{(q;q)_\infty} + 5J. \tag{1.5}$$

Now:

$$Q^3 - R^2 = 1728q(q;q)_\infty^{24}.$$

where

$$\sum_{n=1}^{\infty}\tau(n)q^n = q(q;q)_\infty^{24}$$

We note that: Q = 1 + 240 = 241; R = 1 – 504 = – 503; thence

$Q^3 - R^2 = 241^3 - (-503^2) = 13997521 - 253009 = 13744512$;

We have that: 13744512 / 1728 = 7954; thence $q(q;q)_\infty^{24} = 7954$. Indeed:

13744512 = 1728 * 7954.

Now, we calculate the following double integral:

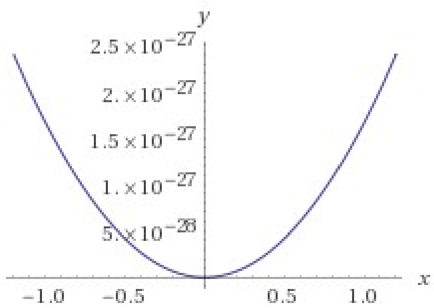
1/(10^33) * 1/((sqrt(e))^3) * 1.08643 * integrate integrate [13744512]

$$\frac{1}{10^{33}} \times \frac{1}{\sqrt{e}^3} \times 1.08643 \int \left(\int 13744512 \, dx \right) dx$$

Result:

$1.66594 \times 10^{-27} x^2$

Plot:



(x from -1.2 to 1.2)

results that is a good approximation to the mass of the proton.

$$(1.4) \qquad Q - P^2 = 288 \sum_{n=1}^{\infty} n\sigma_1(n) q^n;$$

We note that 288 / 144 = 2 and that 288 * 6 = 1728 and 1728 / 144 = 12

$$(3.2) \qquad Q^2 - PR = 1008 \sum_{n=1}^{\infty} n\sigma_5(n) q^n;$$

We note that 1008 / 144 = 7

We have:

(5.2) $$Q^2 = P + 7J; \qquad R = 1 + 7J;$$

and so

(5.3) $$(Q^3 - R^2)^2 = P^3 - 2PQ + R + 7J.$$

But[8]

(5.4) $$\begin{cases} PQ - R = 720 \sum_{n=1}^{\infty} n\sigma_3(n)q^n, \\ P^3 - 3PQ + 2R = -1728 \sum_{n=1}^{\infty} n^2\sigma_1(n)q^n; \end{cases}$$

We note that 720 / 144 = 5; 1728 / 288 = 6

Then: R = 1 + 7J; for R = - 503; 1 + 7J = - 503; 7J = - 504; J = - 504 / 7 = - 72.

Q^2 = P – 504; P – 504 = Q^2; P = Q^2 + 504 = 241^2 + 504 = 58585;

PQ – R = 58585 * 241 - (- 503) = 14118985 + 503 = 14119488;

Indeed: 720 * 19610,4 = 141194888;

P^3 – 3PQ + 2R = 201075567351625 – 3(14118985) – 1006 = 201075524993664;

Indeed: -116363151038 * -1728 = 201075524993664

Now, we calculate the following double integral:

1.1056^2 * 1/(10^40) * 1/(e)^2 integrate integrate [201075524993664]

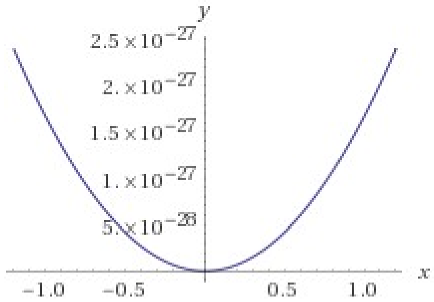
where 1.1056 is the value of the cosmological constant (Planck 2018)

$$1.1056^2 \times \frac{1}{10^{40}} \times \frac{1}{e^2} \int \left(\int 201075524993664 \, dx \right) dx$$

Result:

$1.66317 \times 10^{-27} x^2$

Plot:



(x from −1.2 to 1.2)

value that is a excellent approximation to the value of the mass of the proton

Now:

$$(7.1) \qquad \begin{cases} Q^3 \quad R^2 = 1728 \sum_{n-1}^{\infty} \tau(n) q^n, \\ 3Q^3 + 2R^2 - 5PQR - 1584 \sum_{n=1}^{\infty} n\sigma_9(n) q^n, \\ 5Q^3 + 4R^2 - 18PQR + 9P^2Q^2 = 8640 \sum_{n=1}^{\infty} n^2 \sigma_7(n) q^n; \end{cases}$$

We have that:

$5 * 241^3 + 4 * (-503)^2 – 18(58585*241* -503) + 9(58585^2 * 241^2) =$

= 69987605 + 1012036 + 127833290190 + 1794111636872025 =

= 1794239541161856. We have that

1038333067802 * (8640/5) = 1794239541161856 and 1728 * 5 = 8640

Now, we calculate the following double integral, where -0.165421 is ζ'(-1):

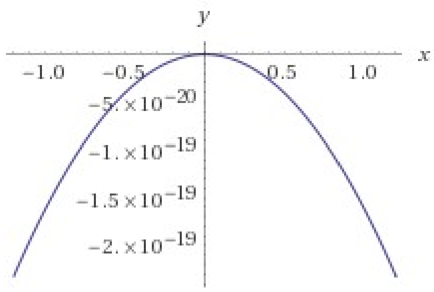
1/(10^33) * (-0.165421) * 1.08643 * integrate integrate [1794239541161856]

$$\frac{1}{10^{33}} \times (-0.165421) \times 1.08643 \int \left(\int 1794239541161856 \, dx \right) dx$$

Result:

$-1.61229 \times 10^{-19} x^2$

Plot:



(x from -1.2 to 1.2)

results that is a good approximation to the value of the electric charge of the electron.

Now:

$$(9.3)\quad \begin{cases} P^5 - 10P^3Q + 20P^2R - 15PQ^2 + 4QR = -20736\sum_{n=1}^{\infty} n^4\sigma_1(n)q^n, \\ P^3Q - 3P^2R + 3PQ^2 - QR = 3456\sum_{n=1}^{\infty} n^3\sigma_3(n)q^n, \\ P^2R - 2PQ^2 + QR = -1728\sum_{n=1}^{\infty} n^2\sigma_5(n)q^n, \\ PQ^2 - QR = 720\sum_{n=1}^{\infty} n\sigma_7(n)q^n; \end{cases}$$

$58585^2 * (-503) – 2(58585)(241)^2 + 241* (-503) =$

= 1726397719175 – 6805350770 – 121223 = 1719592247182:

We have that - 995134402,304398148 * - 1728 = 1719592247182

Now, we calculate the following double integral:

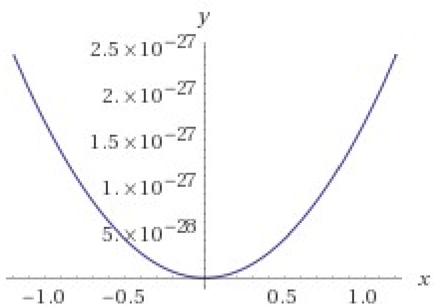
1.08643^2 * 1/(10^37) * 1/(4e^e) integrate integrate [1719592247182]

$$1.08643^2 \times \frac{1}{10^{37}} \times \frac{1}{4\,e^e} \int\left(\int 1\,719\,592\,247\,182\,dx\right)dx$$

Result:

$$1.67419\times10^{-27}\,x^2$$

Plot:



(x from -1.2 to 1.2)

results that is practically equal to the value of the mass of neutron.

And:

(13.3)

$$\begin{cases} 5(P^6 - 15P^4Q + 40P^3R - 45P^2Q^2 + 24PQR) \\ -(9Q^3 + 16R^2) = -248832 \sum_{n=1}^{\infty} n^5 \sigma_1(n) q^n, \\ 7(P^4Q - 4P^3R + 6P^2Q^2 - 4PQR) + (3Q^3 + 4R^2) = 41472 \sum_{n=1}^{\infty} n^4 \sigma_3(n) q^n, \\ 2(P^3R - 3P^2Q^2 + 3PQR) - (Q^3 + R^2) = -5184 \sum_{n=1}^{\infty} n^3 \sigma_5(n) q^n, \\ 9(PQ - R)^2 + 5(Q^3 - R^2) = 8640 \sum_{n=1}^{\infty} n^3 \sigma_7(n) q^n, \\ 5PQR - (3Q^3 + 2R^2) = -1584 \sum_{n=1}^{\infty} n \sigma_9(n) q^n, \\ Q^3 - R^2 = 1728 \sum_{n=1}^{\infty} \tau(n) q^n; \end{cases}$$

Where – 248832 / 1728 = – 144

Thence:

$-(9*241^3 + 16*(-503)^2) = -(125977689 + 4048144) = - 130025833$

-248832 * 522,54466065457818930041152263374 = - 130025833

(1728 * (-144)) * 522,54466065457818930041152263374 = - 130025833.

Now, we calculate the following double integral:

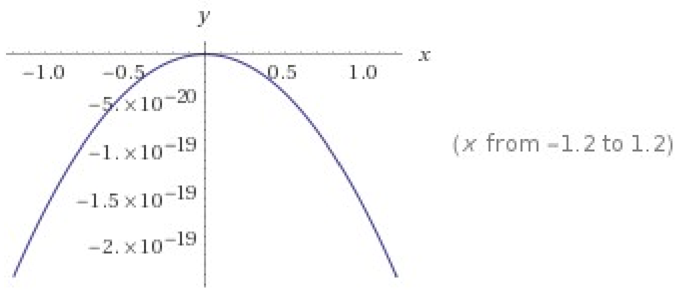
1.08643^2 * 1/(10^25) * 1/(Pi*e^e) integrate integrate [-130025833]

$$1.08643^2 \times \frac{1}{10^{25}} \times \frac{1}{\pi\, e^e} \int\left(\int -130\,025\,833\, dx\right) dx$$

Result:

$-1.61183 \times 10^{-19}\, x^2$

Plot:



value that is an excellent approximation to the electric charge of the electron.

Now:

$$6912\sum_{n=1}^{\infty} n^3\sigma_1(n)q^n = 6P^2Q - 8PR + 3Q^2 - P^4.$$

$6*58585^2 * 241 – 8*58585*(-503) + 3*241^2 – 58585^4 =$

= 4962964417350 + 235746040 + 174243 – 11780012113294950625 =

= -11780007150094769992

(4*1728) * -1704283441853988,7141203703703704 = -11780007150094769992

Now, we calculate the following double integral:

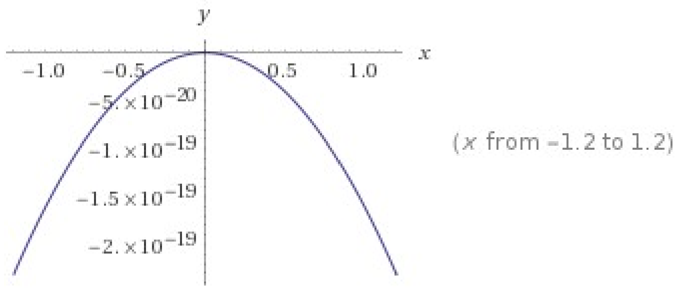
1.08643^2 * 1/(10^36) * 1/(Pi^2*1.61803398*e) integrate integrate [-11780007150094769992]

$$1.08643^2 \times \frac{1}{10^{36}} \times \frac{1}{\pi^2 \times 1.61803398\, e} \int\left(\int -11\,780\,007\,150\,094\,769\,992\, dx\right) dx$$

Result:

$-1.60154 \times 10^{-19}\, x^2$

Plot:



value that is an excellent approximation to the electric charge of the electron.

From: Canad. Math. Bull. Vol. **42** (4), 1999 pp. 427–440
"Ramanujan and the Modular *j*-Invariant" - Bruce C. Berndt and Heng Huat Chan

Now, we have the following Ramanujan function:

Except for four entries, the last two pages in Ramanujan's third notebook, pages 392 and 393 in the pagination of [21, vol. 2], are devoted to values of the modular j-invariant. Recall [14, p. 81], [15, p. 224] that the invariants $J(\tau)$ and $j(\tau)$, for $\tau \in \mathbb{H} := \{\tau : \operatorname{Im}\tau > 0\}$, are defined by

$$J(\tau) = \frac{g_2^3(\tau)}{\Delta(\tau)} \quad \text{and} \quad j(\tau) = 1728\,J(\tau), \tag{1.1}$$

where

$$\Delta(\tau) = g_2^3(\tau) - 27g_3^2(\tau), \tag{1.2}$$

$$g_2(\tau) = 60 \sum_{\substack{m,n=-\infty \\ (m,n)\neq(0,0)}}^{\infty} (m\tau + n)^{-4},$$

and

$$g_3(\tau) = 140 \sum_{\substack{m,n=-\infty \\ (m,n)\neq(0,0)}}^{\infty} (m\tau + n)^{-6}.$$

Furthermore, the function $\gamma_2(\tau)$ is defined by [15, p. 249]

$$\gamma_2(\tau) = \sqrt[3]{j(\tau)} \tag{1.3}$$

Theorem 1.1 *For $q = \exp(-\pi\sqrt{n})$, define*

$$t := t_n := \sqrt{3}q^{1/18}\frac{f(q^{1/3})f(q^3)}{f^2(q)}. \tag{1.13}$$

Then

$$t_n = \left(2\sqrt{64J_n^2 - 24J_n + 9} - (16J_n - 3)\right)^{1/6}. \tag{1.14}$$

Ramanujan then gives a table of polynomials satisfied by t_n, for five values of n.

Theorem 1.2 *For the values of n given below, we have the following table of polynomials $p_n(t)$ satisfied by t_n.*

n	$p_n(t)$
11	$t - 1$
35	$t^2 + t - 1$
59	$t^3 + 2t - 1$
83	$t^3 + 2t^2 + 2t - 1$
107	$t^3 - 2t^2 + 4t - 1$

Proof of Theorem 1.2 It is well known that $J_{11} = \mathbf{1}$ [15, p. 261]. Thus, we find that

$$t_{11} = (2 \cdot 7 - 13)^{1/6} = 1,$$

as desired.

Secondly, from a paper of W. E. Berwick [6],

$$J_{35} = \sqrt{5}\left(\frac{\sqrt{5}+1}{2}\right)^4.$$

Hence,

$$t_{35} = \left(2\sqrt{64 \cdot 5\left(\frac{\sqrt{5}+1}{2}\right)^8 - 24\sqrt{5}\left(\frac{\sqrt{5}+1}{2}\right)^4 + 9} - \left(16\sqrt{5}\left(\frac{\sqrt{5}+1}{2}\right)^4 - 3\right)\right)^{1/6}$$

$$= \left(2\sqrt{7349 + 3276\sqrt{5}} - 117 - 56\sqrt{5}\right)^{1/6}.$$

We have:

J_{35} = 15,3262379; and

t_{35} = (2*121,1377674149 – 117 – 125,219806739)$^{1/6}$ = (0,0557280908)$^{1/6}$ =

= 0,618033990227 = (√5 – 1) / 2

For

$$j(\tau) = 1728\,J(\tau),$$

we have: 1728 * 15,3262379 = 26483,7390912

We calculate the following double integral:

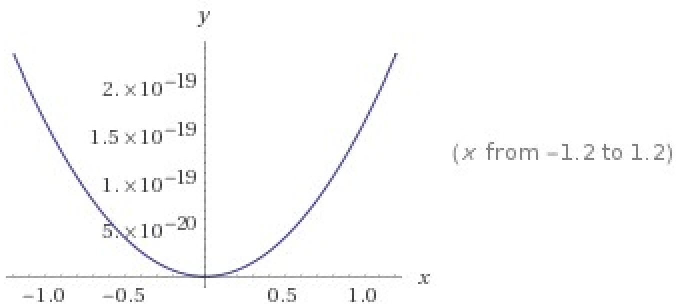
1.08643^2 * 1/(10^20) * 1/(Pi^6) integrate integrate [26483.7390912]

$$1.08643^2 \times \frac{1}{10^{20}} \times \frac{1}{\pi^6} \int\left(\int 26483.7390912\, dx\right) dx$$

Result:

$$1.62575 \times 10^{-19} x^2$$

Plot:



result that is a good approximation to the value of the electric charge of the positron.

It is easy to verify that if $a^2 - db^2 = c^2$, then

$$\sqrt{a \pm b\sqrt{d}} = \sqrt{\frac{a+c}{2}} \pm \sqrt{\frac{a-c}{2}}. \tag{4.1}$$

Now, since

$$7349^2 - 5 \cdot 3276^2 = 589^2,$$

we find that

$$\sqrt{7349 + 3276\sqrt{5}} = \sqrt{\frac{7349+589}{2}} + \sqrt{\frac{7349-589}{2}} = \sqrt{3969} + \sqrt{3380} = 63 + 26\sqrt{5},$$

by (4.1). Hence,

$$t_{35} = \left(2\left(63 + 26\sqrt{5}\right) - 117 - 56\sqrt{5}\right)^{1/6} = (9 - 4\sqrt{5})^{1/6} = \frac{\sqrt{5}-1}{2}.$$

Hence, t_{35} is a root of $t^2 + t - 1$, and the second result is established.

For $n = 59$, Greenhill [18] showed that u_{59}, defined by (1.4), is a root of the equation

$$u - 392 \cdot 2^{1/3} u^{2/3} + 1072 \cdot 4^{1/3} u^{1/3} - 2816 = 0.$$

We have that: $63 + 26\sqrt{5} = 121{,}13776741499453210663851538701$;

Note that $(121{,}1377674149)^{1/10} = 1{,}61557809657...$

We calculate the following double integral:

1.08643^2 * 1/(10^20) * 0.226 integrate integrate [121.13776741499453210663851538701]

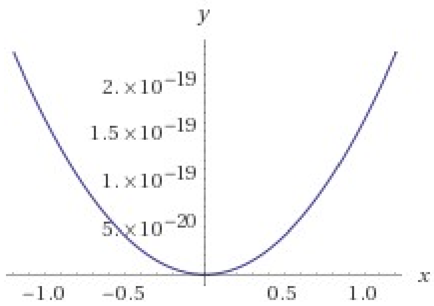
where 0.225791 = log(sqrt(π/2) = 0.226

$$1.08643^2 \times \frac{1}{10^{20}} \times 0.226 \int\left(\int 121.13776741499453210663851538701 \, dx\right) dx$$

Result:

$1.6157 \times 10^{-19} x^2$

Plot:



(x from −1.2 to 1.2)

result that is a good approximation to the electric charge of the positron.

From: "**On Faraday's Lines of Force**" – J.C.Maxwell (From the Transactions of the Cambridge Philosophical Society, VoL x. Part I) - [Read Dec. 10, 1855, and Feb. 11, 1856.]

THEOREM III.

Let U and V be two functions of x, y, z, then

$$\iiint U\left(\frac{d^2V}{dx^2}+\frac{d^2V}{dy^2}+\frac{d^2V}{dz^2}\right)dxdydz = -\iiint\left(\frac{dU}{dx}\frac{dV}{dx}+\frac{dU}{dy}\frac{dV}{dy}+\frac{dU}{dz}\frac{dV}{dz}\right)dxdydz$$

$$=\iiint\left(\frac{d^2U}{dx^2}+\frac{d^2U}{dy^2}+\frac{d^2U}{dz^2}\right)V dx\,dy\,dz\,;$$

where the integrations are supposed to extend over all the space in which U and V have values differing from 0.—(Green, p. 10.)

This theorem shews that if there be two attracting systems the actions between them are equal and opposite. And by making $U=V$ we find that the potential of a system on itself is proportional to the integral of the square of the resultant attraction through all space; a result deducible from Art. (30), since the volume of each cell is inversely as the square of the velocity (Arts. 12, 13), and therefore the number of cells in a given space is directly as the square of the velocity.

Since the mathematical laws of magnetism are identical with those of electricity, as far as we now consider them, we may regard α, β, γ as magnetizing forces, p as *magnetic tension*, and ρ as *real magnetic density*, k being the coefficient of resistance to magnetic induction.

The proof of this theorem rests on the determination of the minimum value of

$$Q=\iiint\left\{\frac{1}{k}\left(\alpha-\frac{dp}{dx}-k\frac{dV}{dx}\right)^2+\frac{1}{k}\left(\beta-\frac{dp}{dy}-k\frac{dV}{dy}\right)^2+\frac{1}{k}\left(\gamma-\frac{dp}{dz}-k\frac{dV}{dz}\right)^2\right\}dx\,dy\,dz;$$

where V is got from the equation

$$\frac{d^2V}{dx^2}+\frac{d^2V}{dy^2}+\frac{d^2V}{dz^2}+4\pi\rho=0,$$

and p has to be determined.

The meaning of this integral in electrical language may be thus brought out. If the presence of the media in which k has various values did not affect the distribution of forces, then the "quantity" resolved in x would be simply $\frac{dV}{dx}$ and the intensity $k\frac{dV}{dx}$. But the actual quantity and intensity are $\frac{1}{k}\left(\alpha-\frac{dp}{dx}\right)$ and $\alpha-\frac{dp}{dx}$, and the parts due to the distribution of media alone are therefore

$$\frac{1}{k}\left(\alpha-\frac{dp}{dx}\right)-\frac{dV}{dx} \text{ and } \alpha-\frac{dp}{dx}-k\frac{dV}{dx}.$$

Now the product of these represents the work done on account of this distribution of media, the distribution of sources being determined, and taking in the terms in y and z we get the expression Q for the total work done by that part of the whole effect at any point which is due to the distribution of conducting media, and not directly to the presence of the sources.

This quantity Q is rendered a minimum by one and only one value of p, namely, that which satisfies the original equation.

The integral throughout infinity

$$Q = \iiint (a_1\alpha_2 + b_1\beta_1 + c_1\gamma_1)\, dx\,dy\,dz,$$

where $a_1b_1c_1$, $\alpha_1\beta_1\gamma_1$ are any functions whatsoever, is capable of transformation into

$$Q = +\iiint\{4\pi p\rho_1 - (\alpha_0 a_2 + \beta_0 b_2 + \gamma_0 c_2)\}\, dx\,dy\,dz,$$

in which the quantities are found from the equations

$$\frac{da_1}{dx} + \frac{db_1}{dy} + \frac{dc_1}{dz} + 4\pi\rho_1 = 0,$$

$$\frac{d\alpha_1}{dx} + \frac{d\beta_1}{dy} + \frac{d\gamma_1}{dz} + 4\pi\rho_1' = 0;$$

$\alpha_0\beta_0\gamma_0 V$ are determined from $a_1b_1c_1$ by the last theorem, so that

$$a_1 = \frac{d\beta_0}{dz} - \frac{d\gamma_0}{dy} + \frac{dV}{dx};$$

$a_2b_2c_2$ are found from $\alpha_1\beta_1\gamma_1$ by the equations

$$a_2 = \frac{d\beta_1}{dz} - \frac{d\gamma_1}{dy} \text{ \&c.},$$

and p is found from the equation

$$\frac{d^2p}{dx^2} + \frac{d^2p}{dy^2} + \frac{d^2p}{dz^2} + 4\pi\rho_1' = 0.$$

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For, if we put a_1 in the form

$$\frac{d\beta_0}{dz} - \frac{d\gamma_0}{dy} + \frac{dV}{dx},$$

and treat b_1 and c_1 similarly, then we have by integration by parts through infinity, remembering that all the functions vanish at the limits,

$$Q = -\iiint\left\{V\left(\frac{d\alpha_1}{dx} + \frac{d\beta_1}{dy} + \frac{d\gamma_1}{dz}\right) + \alpha_0\left(\frac{d\beta_1}{dz} - \frac{d\gamma_1}{dy}\right) + \beta_0\left(\frac{d\gamma_1}{dx} - \frac{d\alpha_1}{dz}\right) + \gamma_0\left(\frac{d\alpha_1}{dy} - \frac{d\beta_1}{dx}\right)\right\} dx\,dy\,dz,$$

$$\text{or } Q = +\iiint\{(4\pi V\rho') - (\alpha_0 a_2 + \beta_0 b_2 + \gamma_0 c_2)\}\, dx\,dy\,dz,$$

and by Theorem III.

$$\iiint V\rho'\, dx\,dy\,dz = \iiint p\rho\, dx\,dy\,dz,$$

so that finally

$$Q = \iiint\{4\pi p\rho - (\alpha_0 a_2 + \beta_0 b_2 + \gamma_0 c_2)\}\, dx\,dy\,dz.$$

If $a_1b_1c_1$ represent the components of magnetic quantity, and $\alpha_1\beta_1\gamma_1$ those of magnetic intensity, then ρ will represent the *real magnetic density*, and p the magnetic potential or tension. $a_2b_2c_2$ will be the components of quantity of electric currents, and $\alpha_0\beta_0\gamma_0$ will be three functions deduced from $a_1b_1c_1$, which will be found to be the mathematical expression for Faraday's Electro-tonic state.

We calculate the following triple integrals before of 4π and after of 4πpρ for the values of 0.618 and 3 for p and ρ. We obtain:

(1/10^12)* 1.08643 * integrate 12.56637(x,y,z) dx dy dz, x=0..Pi/253.8 y=0..Pi/253.8 z=0..Pi/253.8

Input interpretation:

$$\frac{1}{10^{12}} \times 1.08643 \times \int_0^{\frac{\pi}{253.8}} \int_0^{\frac{\pi}{253.8}} \int_0^{\frac{\pi}{253.8}} 12.56637 \{x, y, z\} \, dx \, dy \, dz$$

Result:

$$\{1.60256 \times 10^{-19}, 1.60256 \times 10^{-19}, 1.60256 \times 10^{-19}\}$$

Or:

3/(2e) * 1/(10^20)*1.08643 * integrate (12.56637*0.618*3)(x,y,z) dx dy dz, x=0..Pi/253.8 y=0..Pi/253.8 z=0..Pi/253.8

$$\frac{3}{2e} \times \frac{1}{10^{20}} \times 1.08643 \int_0^{\frac{\pi}{253.8}} \int_0^{\frac{\pi}{253.8}} \int_0^{\frac{\pi}{253.8}} (12.56637 \times 0.618 \times 3) \{x, y, z\} \, dx \, dy \, dz$$

Result:

$$\{1.63954 \times 10^{-27}, 1.63954 \times 10^{-27}, 1.63954 \times 10^{-27}\}$$

Results that are very near to the values of the electric charge of the positron and of the mass of the proton.

a perfect differential of a function of x, y, z. On the principle of analogy we may call p_1 the magnetic tension.

The forces which act on a mass m of south magnetism at any point are

$$-m\frac{dp_1}{dx}, \quad -m\frac{dp_1}{dy}, \text{ and } -m\frac{dp_1}{dz},$$

in the direction of the axes, and therefore the whole work done during any displacement of a magnetic system is equal to the decrement of the integral

$$Q = \iiint \rho_1 p_1 dx dy dz$$

throughout the system.

Let us now call Q the *total potential of the system on itself.* The increase or decrease of Q will measure the work lost or gained by any displacement of any part of the system, and will therefore enable us to determine the forces acting on that part of the system.

By Theorem III. Q may be put under the form

$$Q = +\frac{1}{4\pi}\iiint (a_1\alpha_1 + b_1\beta_1 + c_1\gamma_1)\, dx dy dz,$$

in which $\alpha_1\beta_1\gamma_1$ are the differential coefficients of p_1 with respect to x, y, z respectively.

If we now assume that this expression for Q is true whatever be the values of α_1, β_1, γ_1, we pass from the consideration of the magnetism of permanent magnets to that of the magnetic effects of electric currents, and we have then by Theorem VII.

$$Q = \iiint \left\{ p_1\rho_1 - \frac{1}{4\pi}(\alpha_0 a_2 + \beta_0 b_2 + \gamma_0 c_2) \right\} dx dy dz.$$

So that in the case of electric currents, the components of the currents have to be multiplied by the functions α_0, β_0, γ_0 respectively, and the summations of all such products throughout the system gives us the part of Q due to those currents.

We have now obtained in the functions α_0, β_0, γ_0 the means of avoiding the consideration of the quantity of magnetic induction which *passes through* the circuit. Instead of this artificial method we have the natural one of considering the current with reference to quantities existing in the same space with the current itself. To these I give the name of *Electro-tonic functions*, or *components of the Electro-tonic intensity.*

The conductor is long l = 1.5m the magnetic field is equal to B = 0,5T, the speed v = 4m/s. Find the potential difference at the ends of the conductor

We use the formula

$\vec{E} = \vec{v} \times \vec{B} \longrightarrow E = vB\sin 90° = vB$ we know that the potential is

$V = \int \vec{E} \cdot d\vec{l} = El \longrightarrow V = vlB = 4 \cdot 1{,}5 \cdot 0{,}5 = 3\ V$

Formula for magnetic sphere flow density

Formula for the field B on the symmetry axis of a magnetic sphere axially magnetized:

$$B = B_r \frac{2}{3} \frac{R^3}{(z+R)^3}$$

$\boldsymbol{B_r}$: remanence field, independent of the magnet geometry

$\boldsymbol{z}$: distance on the axis of symmetry from the edge of the sphere

$\boldsymbol{R}$: half the diameter (radius) of the sphere

The unit of length measurement can be chosen at will, as long as it is the same for all lengths. For R = 26, Grade = N42, z = 3 we have a flow density of 0.618 T

From the equation:

$$Q=\iiint \rho_1 p_1 \, dx\,dy\,dz$$

For a magnetic density of 0,618 T and a magnetic potential of 3V, we calculate the following triple integral:

1.08643 * integrate (0.618*3)(x,y,z) dx dy dz, x=0..1.129 y=0..1.129 z=0..1.129 where 1.129 = $1.63047^{1/4}$

Input interpretation:

$$1.08643 \int_0^{1.129}\int_0^{1.129}\int_0^{1.129} (0.618\times 3)\{x, y, z\}\, dx\, dy\, dz$$

Result:

$\{1.63628, 1.63628, 1.63628\}$

result that is a good approximation to the value of the mass of the proton.

From the equation of the total potential of the system on itself:

$$Q=\iiint \left\{ p_1\rho_1 - \frac{1}{4\pi}(a_0 a_1 + \beta_0 b_1 + \gamma_0 c_1) \right\} dx\,dy\,dz.$$

for a magnetic density of 0.031416 T and a magnetic potential of 1.571V, we have the following triple integral:

1.08643 * integrate ((0.06*1.5)-(0.07957))(x,y,z) dx dy dz, x=0..4.1 y=0..4.1 z=0..4.1 where 4.1 = 1.60052063^3

Input interpretation:

$$1.08643 \int_0^{4.1}\int_0^{4.1}\int_0^{4.1} (0.06\times 1.5 - 0.07957)\{x, y, z\}\, dx\, dy\, dz$$

Result:

$\{1.601, 1.601, 1.601\}$

or:

1.08643 * integrate ((0.06*1.5)-(((0.07957)*αa+βb+γc)))(x,y,z) dx dy dz, x=0..4.1 y=0..4.1 z=0..4.1

Input interpretation:

$$1.08643 \int_0^{4.1}\int_0^{4.1}\int_0^{4.1} (0.06\times 1.5 - (0.07957\,\alpha\, a + \beta\, b + \gamma\, c))\{x, y, z\}\,dx\,dy\,dz$$

$$\{153.5\,(-0.07957\,a\,\alpha - b\,\beta - c\,\gamma + 0.09),\ 153.5\,(-0.07957\,a\,\alpha - b\,\beta - c\,\gamma + 0.09),\ 153.5\,(-0.07957\,a\,\alpha - b\,\beta - c\,\gamma + 0.09)\}$$

where 153.5(– 0.07957aα) = – 12.213995aα and 153.5(0.09) = 13.815   for which :

– 12.213995aα – 153.5bβ – 153.5cγ + 13.815 = 1.601005

(putting aα = 1, bβ = 1 e cγ = – 1)

The result is very near to the value of the electric charge of the positron.

From:

[From the Philosophical Magazine, Vol. xxi.] - XXIII. J.C.Maxwell
“On Physical Lines of Force”.

from $+x$ to $+y$ parallel to the plane of xy. Now if an electric current whose strength is r is traversing the axis of z, which, we may suppose, points vertically upwards, then, if the axis of x is east and that of y north, a unit north pole will be urged round the axis of z in the direction from x to y, so that in one revolution the work done will be $=4\pi r$. Hence $\frac{1}{4\pi}\left(\frac{d\beta}{dx}-\frac{d\alpha}{dy}\right)$ represents the *strength of an electric current parallel to z* through unit of area; and if we write

$$\frac{1}{4\pi}\left(\frac{d\gamma}{dy}-\frac{d\beta}{dz}\right)=p,\quad \frac{1}{4\pi}\left(\frac{d\alpha}{dz}-\frac{d\gamma}{dx}\right)=q,\quad \frac{1}{4\pi}\left(\frac{d\beta}{dx}-\frac{d\alpha}{dy}\right)=r \quad\text{..........(9)},$$

then p, q, r will be the quantity of electric current per unit of area perpendicular to the axes of x, y, and z respectively.

$$\overline{V}p=\tfrac{1}{2}\rho\left(\frac{d\gamma}{dy}-\frac{d\beta}{dz}\right)(V_1+V_2+\&c.)\quad\text{................(32)}$$

or dividing by $\bar{V} = V_1 + V_2 + \&c.,$

$$p = \tfrac{1}{2}\rho\left(\frac{d\gamma}{dy} - \frac{d\beta}{dz}\right) \qquad (33).$$

If we make

$$\rho = \frac{1}{2\pi} \qquad (34),$$

then equation (33) will be identical with the first of equations (9), which give the relation between the quantity of an electric current and the intensity of the lines of force surrounding it.

thence:

$$p = \frac{1}{4\pi}\left(\frac{d\gamma}{dy} - \frac{d\beta}{dz}\right)$$

Calculate the following double integral:

13e * 1.08643^2 * integrate integrate [1/(4Pi)]

[d(γ/y – β/z)] 13e * 1.08643^2 * integrate integrate [1/(4Pi)]

$$13\,e \times 1.08643^2 \int\left(\int \frac{1}{4\pi}\,dx\right)dx$$

$$\left(d\left(\frac{\gamma}{y} - \frac{\beta}{z}\right)\right) \times 13\,e \times 1.08643^2 \int\left(\int \frac{1}{4\pi}\,dx\right)dx$$

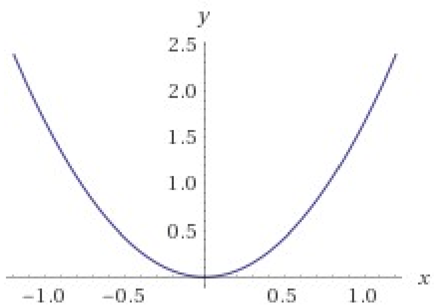
Result:

$1.65959\,x^2$

Result:

$1.65959\,d\,x^2\left(\frac{\gamma}{y} - \frac{\beta}{z}\right)$

Plot:



(x from −1.2 to 1.2)

result that is very near to the value of the mass of the proton.

Let α, β, γ be the components of the circumferential velocity, as in Prop. II., then the actual energy of the vortices in unit of volume will be proportional to the density and to the square of the velocity. As we do not know the distribution of density and velocity in each vortex, we cannot determine the numerical value of the energy directly; but since μ also bears a constant though unknown ratio to the mean density, let us assume that the energy in unit of volume is

$$E = C\mu\,(\alpha^2+\beta^2+\gamma^2),$$

where C is a constant to be determined.

Let us take the case in which

$$\alpha=\frac{d\phi}{dx},\quad \beta=\frac{d\phi}{dy},\quad \gamma=\frac{d\phi}{dz} \quad\text{.........................} (35).$$

Let

$$\phi=\phi_1+\phi_2 \quad\text{......................................} (36),$$

and let

$$\frac{\mu}{4\pi}\left(\frac{d^2\phi_1}{dx^2}+\frac{d^2\phi_1}{dy^2}+\frac{d^2\phi_1}{dz^2}\right)=m_1, \text{ and } \frac{\mu}{4\pi}\left(\frac{d^2\phi_2}{dx^2}+\frac{d^2\phi_2}{dy^2}+\frac{d^2\phi_2}{dz^2}\right)=m_2 \text{.....} (37);$$

then ϕ_1 is the potential at any point due to the magnetic system m_1, and ϕ_2 that due to the distribution of magnetism represented by m_2. The actual energy of all the vortices is

$$E=\Sigma C\mu\,(\alpha^2+\beta^2+\gamma^2)\,dV \quad\text{..............................} (38),$$

the integration being performed over all space.

This may be shewn by integration by parts (see Green's 'Essay on Electricity,' p. 10) to be equal to

$$E=-4\pi C\Sigma\,(\phi_1 m_1+\phi_2 m_2+\phi_1 m_2+\phi_2 m_1)\,dV \quad\text{.................} (39).$$

Or since it has been proved (Green's 'Essay,' p. 10) that

$$\Sigma\phi_1 m_2 dV=\Sigma\phi_2 m_1 dV,$$

$$E=-4\pi C\,(\phi_1 m_1+\phi_2 m_2+2\phi_1 m_2)\,dV \quad\text{..................} (40).$$

Now let the magnetic system m_1 remain at rest, and let m_2 be moved parallel to itself in the direction of x through a space δx; then, since ϕ_1 depends on m_1 only, it will remain as before, so that $\phi_1 m_1$ will be constant; and since ϕ_2 depends on m_2 only, the distribution of ϕ_2 about m_2 will remain the same, so that $\phi_2 m_2$ will be the same as before the change. The only part of E that will be altered is that depending on $2\phi_1 m_2$, because ϕ_1 becomes $\phi_1 + \frac{d\phi_1}{dx}\delta x$ on account of the displacement. The variation of actual energy due to the displacement is therefore

$$\delta E = -4\pi C \Sigma \left(2\frac{d\phi_1}{dx} m_2\right) dV\delta x \quad \text{(41)}.$$

But by equation (12) the work done by the mechanical forces on m_2 during the motion is

$$\delta W = \Sigma \left(\frac{d\phi_1}{dx} m_2 dV\right) \delta x \quad \text{(42)};$$

and since our hypothesis is a purely mechanical one, we must have by the conservation of force,

$$\delta E + \delta W = 0 \quad \text{(43)};$$

that is, the loss of energy of the vortices must be made up by work done in moving magnets, so that

$$-4\pi C \Sigma \left(2\frac{d\phi_1}{dx} m_2 dV\right)\delta x + \Sigma \left(\frac{d\phi_1}{dx} m_2 dV\right)\delta x = 0,$$

or

$$C = \frac{1}{8\pi} \quad \text{(44)};$$

so that the energy of the vortices in unit of volume is

$$\frac{1}{8\pi}\mu(\alpha^2 + \beta^2 + \gamma^2) \quad \text{(45)};$$

and that of a vortex whose volume is V is

$$\frac{1}{8\pi}\mu(\alpha^2 + \beta^2 + \gamma^2) V \quad \text{(46)}.$$

In order to produce or destroy this energy, work must be expended on, or received from, the vortex, either by the tangential action of the layer of particles in contact with it, or by change of form in the vortex. We shall first investigate the tangential action between the vortices and the layer of particles in contact with them.

$$E = \frac{1}{8\pi}\mu(\alpha^2 + \beta^2 + \gamma^2)$$

Now calculate the following double integral:

26e * 1.08643^2 * μ(α^2+β^2+γ^2) integrate integrate [1/(8Pi)]

$$26\, e \times 1.08643^2\, \mu(\alpha^2+\beta^2+\gamma^2) \int\left(\int \frac{1}{8\pi}\, dx\right) dx$$

Result:

$$1.65959\, x^2\, \mu(\alpha^2+\beta^2+\gamma^2)$$

result that is very near to the value of the mass of the proton.

Now:

Electromotive force acting on a dielectric produces a state of polarization of its parts similar in distribution to the polarity of the particles of iron under the influence of a magnet*, and, like the magnetic polarization, capable of being described as a state in which every particle has its poles in opposite conditions.

In a dielectric under induction, we may conceive that the electricity in each molecule is so displaced that one side is rendered positively, and the other negatively electrical, but that the electricity remains entirely connected with the molecule, and does not pass from one molecule to another.

The effect of this action on the whole dielectric mass is to produce a general displacement of the electricity in a certain direction. This displacement does not amount to a current, because when it has attained a certain value it remains constant, but it is the commencement of a current, and its variations constitute currents in the positive or negative direction, according as the displacement is increasing or diminishing. The amount of the displacement depends on the nature of the body, and on the electromotive force; so that if h is the displacement, R the electromotive force, and E a coefficient depending on the nature of the dielectric,

$$R = -4\pi E^2 h;$$

and if r is the value of the electric current due to displacement,

$$r = \frac{dh}{dt}.$$

PROP. XII. To find the conditions of equilibrium of an elastic sphere whose surface is exposed to normal and tangential forces, the tangential forces being proportional to the sine of the distance from a given point on the sphere.

Let the axis of z be the axis of spherical co-ordinates.

Let ξ, η, ζ be the displacements of any particle of the sphere in the directions of x, y, and z.

Let p_{xx}, p_{yy}, p_{zz} be the stresses normal to planes perpendicular to the three axes, and let p_{yz}, p_{zx}, p_{xy} be the stresses of distortion in the planes yz, zx, and xy.

Let μ be the coefficient of cubic elasticity, so that if

$$p_{xx} = p_{yy} = p_{zz} = p,$$

$$p = \mu\left(\frac{d\xi}{dx} + \frac{d\eta}{dy} + \frac{d\zeta}{dz}\right) \qquad (80).$$

Let m be the coefficient of rigidity, so that

$$p_{xx} - p_{yy} = m\left(\frac{d\xi}{dx} - \frac{d\eta}{dy}\right), \text{ \&c.} \qquad (81).$$

Then we have the following equations of elasticity in an isotropic medium,

$$p_{xx} = (\mu - \tfrac{1}{3}m)\left(\frac{d\xi}{dx} + \frac{d\eta}{dy} + \frac{d\zeta}{dz}\right) + m\frac{d\xi}{dx} \qquad (82);$$

with similar equations in y and z, and also

$$p_{yz} = \frac{m}{2}\left(\frac{d\eta}{dz} + \frac{d\zeta}{dy}\right), \text{ \&c.} \qquad (83).$$

In the case of the sphere, let us assume the radius $= a$, and

$$\xi = exz, \quad \eta = ezy, \quad \zeta = f(x^2 + y^2) + gz^2 + d \qquad (84).$$

PROP. XIII.—To find the relation between electromotive force and electric displacement when a uniform electromotive force R acts parallel to the axis of z.

Take any element δS of the surface, covered with a stratum whose density is ρ, and having its normal inclined θ to the axis of z; then the tangential force upon it will be

$$\rho R \delta S \sin\theta = 2\, T\delta S \qquad (99),$$

T being, as before, the tangential force on each side of the surface. Putting $\rho = \frac{1}{2\pi}$ as in equation (34)*, we find

$$R = -2\pi m a\,(e + 2f) \qquad (100).$$

The displacement of electricity due to the distortion of the sphere is

$$\Sigma \delta S \tfrac{1}{2} \rho t \sin\theta \text{ taken over the whole surface} \qquad (101);$$

and if h is the electric displacement per unit of volume, we shall have

$$\tfrac{4}{3}\pi a^3 h = \tfrac{2}{3} a^2 e \qquad (102),$$

or

$$h = \frac{1}{2\pi} a e \qquad (103);$$

so that

$$R = 4\pi^2 m \frac{e+2f}{e} h \qquad (104),$$

or we may write

$$R = -4\pi E^2 h \qquad (105),$$

provided we assume

$$E^2 = -\pi m \frac{e+2f}{e} \qquad (106).$$

Finding e and f from (87) and (90), we get

$$E^2 = \pi m \frac{3}{1 + \frac{5}{3}\frac{m}{\mu}} \qquad (107).$$

The ratio of m to μ varies in different substances; but in a medium whose elasticity depends entirely upon forces acting between pairs of particles, this ratio is that of 6 to 5, and in this case

$$E^2 = \pi m \qquad (108).$$

* *Phil. Mag.* April, 1861 [p. 471 of this vol.].

So E^2 is a coefficient dependent on the nature of the dielectric, m = 6 the coefficient of rigidity, and μ = 5 the cubic elasticity coefficient. By performing the following double integral on E, we obtain:

(e/4) *1.08643 integrate integrate [sqrt(6*Pi)]

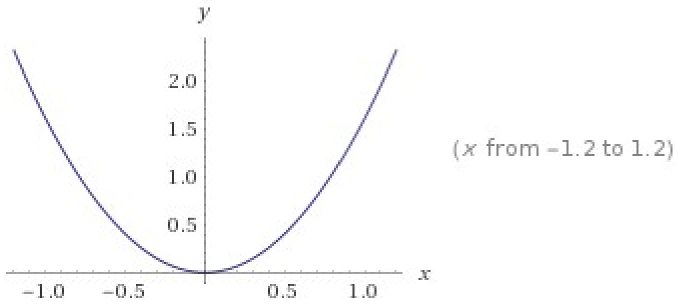
where e/4 = 0.67957 but we can also utilize the “Body-centered cubic” that is equal to 0.680174 = (π√3)/8

$$\frac{e}{4} \times 1.08643 \int \left(\int \sqrt{6\pi}\, dx \right) dx$$

Result:

$1.60272\, x^2$

Plot:



or:

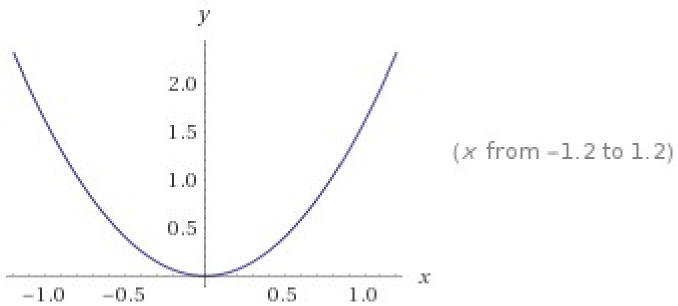
((π√3)/8) *1.08643 integrate integrate [sqrt(6*Pi)]

$$\left(\frac{1}{8}\left(\pi \sqrt{3}\right)\right) \times 1.08643 \int \left(\int \sqrt{6\pi}\, dx \right) dx$$

Result:

$1.60414\, x^2$

Plot:



results that are very near to the values of the electric charge of the positron.

We have seen that electromotive force and electric displacement are connected by equation (105). Differentiating this equation with respect to t, we find

$$\frac{dR}{dt} = -4\pi E^2 \frac{dh}{dt} \qquad (111),$$

shewing that when the electromotive force varies, the electric displacement also varies. But a variation of displacement is equivalent to a current, and this current must be taken into account in equations (9) and added to r. The three equations then become

$$\left.\begin{aligned} p &= \frac{1}{4\pi}\left(\frac{d\gamma}{dy} - \frac{d\beta}{dz} - \frac{1}{E^2}\frac{dP}{dt}\right) \\ q &= \frac{1}{4\pi}\left(\frac{d\alpha}{dy} - \frac{d\gamma}{dx} - \frac{1}{E^2}\frac{dQ}{dt}\right) \\ r &= \frac{1}{4\pi}\left(\frac{d\beta}{dx} - \frac{d\alpha}{dy} - \frac{1}{E^2}\frac{dR}{dt}\right) \end{aligned}\right\} \qquad (112),$$

where p, q, r are the electric currents in the directions of x, y, and z; α, β, γ are the components of magnetic intensity; and P, Q, R are the electromotive forces. Now if e be the quantity of free electricity in unit of volume, then the equation of continuity will be

$$\frac{dp}{dx} + \frac{dq}{dy} + \frac{dr}{dz} + \frac{de}{dt} = 0 \qquad (113).$$

* See Rankine "On Elasticity," *Camb. and Dub. Math. Journ.* 1851.

† *Phil. Mag.* March, 1861 [p. 462 of this vol.].

Differentiating (112) with respect to x, y, and z respectively, and substituting, we find

$$\frac{de}{dt} = \frac{1}{4\pi E^2}\frac{d}{dt}\left(\frac{dP}{dx} + \frac{dQ}{dy} + \frac{dR}{dz}\right) \qquad (114);$$

whence

$$e = \frac{1}{4\pi E^2}\left(\frac{dP}{dx} + \frac{dQ}{dy} + \frac{dR}{dz}\right) \qquad (115),$$

the constant being omitted, because $e = 0$ when there are no electromotive forces.

Then:

$-4\pi E^2 = -236,8705$ taken from (111) which shows that when the electromotive force changes, the electric displacement also varies.

$e = 1 / 4\pi E^2 = 1 / 236.8705 = 0.0042217$ taken from (115) which shows the equations of electric currents due to the effect due to the elasticity of the medium.

We calculate the following double integral for: $-4\pi * \pi m = -236,8705$

(e/216) *1.08643 integrate integrate [-(4*Pi)*6Pi]

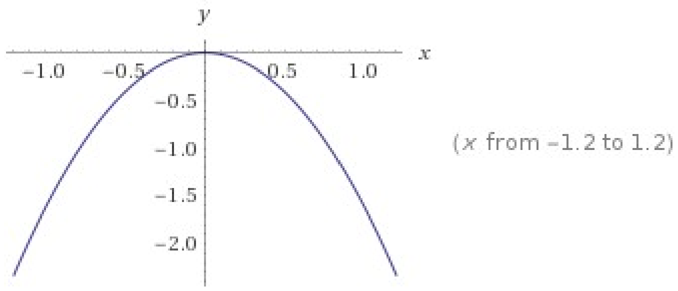
$$\frac{e}{216} \times 1.08643 \int \left(\int -(4\pi) \times 6\pi \, dx \right) dx$$

$$\frac{e}{216} \times 1.08643 \int \left(\int -236.8705 \, dx \right) dx$$

Result:

$-1.61929 x^2$

Plot:



Now for $1/4\pi E^2 = 1/236,8705 = 0,0042217$

(260e) *1.08643 integrate integrate [1/(4*Pi*6Pi)]

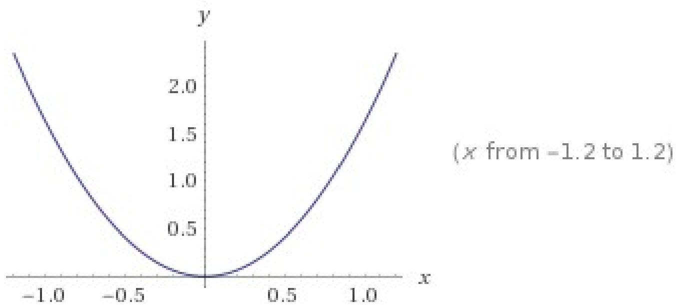
$$(260\, e) \times 1.08643 \int \left(\int \frac{1}{4\pi \times 6\pi} \, dx \right) dx$$

$$(260\, e) \times 1.08643 \int \left(\int 0.0042217 \, dx \right) dx$$

Result:

$1.6208 x^2$

Plot:



results that are very near to the values of the charges of the electron and positron.

Or also:

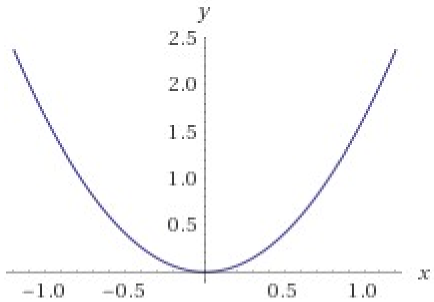
(264e) *1.08643 integrate integrate [1/(4*Pi*6Pi)]

$$(264\,e)\times 1.08643 \int\left(\int \frac{1}{4\pi\times 6\pi}\,dx\right)dx$$

Result:

$1.64573\,x^2$

Plot:



(x from −1.2 to 1.2)

result that is very near to the value of the mass of the proton.

Now:

PART III.

THE THEORY OF MOLECULAR VORTICES APPLIED TO STATICAL ELECTRICITY.

In the first part of this paper* I have shewn how the forces acting between magnets, electric currents, and matter capable of magnetic induction may be accounted for on the hypothesis of the magnetic field being occupied with innumerable vortices of revolving matter, their axes coinciding with the direction of the magnetic force at every point of the field.

The centrifugal force of these vortices produces pressures distributed in such a way that the final effect is a force identical in direction and magnitude with that which we observe.

In the second part† I described the mechanism by which these rotations may be made to coexist, and to be distributed according to the known laws of magnetic lines of force.

I conceived the rotating matter to be the substance of certain cells, divided from each other by cell-walls composed of particles which are very small compared with the cells, and that it is by the motions of these particles, and their tangential action on the substance in the cells, that the rotation is communicated from one cell to another.

I have not attempted to explain this tangential action, but it is necessary to suppose, in order to account for the transmission of rotation from the exterior to the interior parts of each cell, that the substance in the cells possesses elasticity of figure, similar in kind, though different in degree, to that observed in solid bodies. The undulatory theory of light requires us to admit this kind of elasticity in the luminiferous medium, in order to account for transverse vibrations. We need not then be surprised if the magneto-electric medium possesses the same property.

That electric current which, circulating round a ring whose area is unity, produces the same effect on a distant magnet as a magnet would produce whose strength is unity and length unity placed perpendicularly to the plane of the ring, is a unit current; and E units of electricity, measured statically, traverse the section of this current in one second,—these units being such that any two of them, placed at unit of distance, repel each other with unit of force.

We may suppose either that E units of positive electricity move in the positive direction through the wire, or that E units of negative electricity move in the negative direction, or, thirdly, that $\frac{1}{2}E$ units of positive electricity move in the positive direction, while $\frac{1}{2}E$ units of negative electricity move in the negative direction at the same time.

The last is the supposition on which MM. Weber and Kohlrausch* proceed, who have found

$$\tfrac{1}{2}E = 155{,}370{,}000{,}000 \quad\ldots\ldots\ldots\ldots\ldots\ldots\ldots (130),$$

the unit of length being the millimetre, and that of time being one second, whence

$$E = 310{,}740{,}000{,}000 \quad\ldots\ldots\ldots\ldots\ldots\ldots\ldots (131).$$

PROP. XVI.—To find the rate of propagation of transverse vibrations through the elastic medium of which the cells are composed, on the supposition that its elasticity is due entirely to forces acting between pairs of particles.

By the ordinary method of investigation we know that

$$V = \sqrt{\frac{m}{\rho}} \quad\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (132),$$

where m is the coefficient of transverse elasticity, and ρ is the density. By referring to the equations of Part I., it will be seen that if ρ is the density of the matter of the vortices, and μ is the "coefficient of magnetic induction,"

$$\mu = \pi\rho \quad\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (133);$$

whence

$$\pi m = V^2\mu \quad\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (134);$$

and by (108),

$$E = V\sqrt{\mu} \quad\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (135).$$

In air or vacuum $\mu = 1$, and therefore

$$\left.\begin{aligned} V &= E \\ &= 310{,}740{,}000{,}000 \text{ millimetres per second} \\ &= 193{,}088 \text{ miles per second} \end{aligned}\right\} \ldots\ldots\ldots\ldots (136).$$

We note that $\mu = 1$, $m = (310740000000\text{^}2)/Pi = 3{,}07357949E22$

for $\mu = 5$, $m = ((310740000000\text{^}2)*5))/Pi = 1{,}53678975*10^{23}$

$\rho = 1.591549430918954$

$V = \sqrt{9{,}65593478E22} = 310.740.000.000 = 310.740.000.280{,}3659973$

Calculate the following double integrals for $1{,}53678975*10^{23}$, for 1.591549430918954 , for 155370000000, and for 310740000000

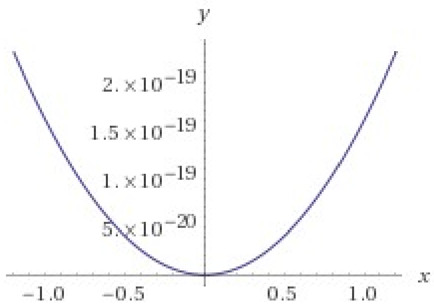
1/(2*10^40) * 1/(26) * 1.08643 * integrate integrate [1.53678975*10^23]

$$\frac{1}{2 \times 10^{40}} \times \frac{1}{26} \times 1.08643 \int \left(\int 1.53678975 \times 10^{23} \, dx \right) dx$$

Result:

$1.6054 \times 10^{-19} x^2$

Plot:



(x from −1.2 to 1.2)

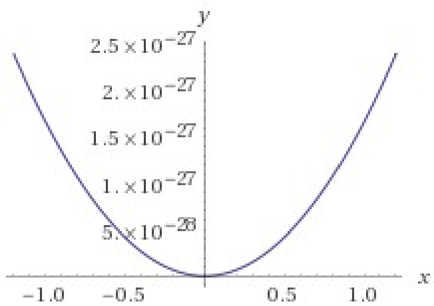
1/(2*10^25) * 1/(26) * 1.08643 * integrate integrate [1.591549430918954]

$$\frac{1}{2 \times 10^{25}} \times \frac{1}{26} \times 1.08643 \int \left(\int 1.591549430918954 \, dx \right) dx$$

Result:

$1.6626 \times 10^{-27} x^2$

Plot:



(x from −1.2 to 1.2)

1/(10^36) * (sqrt(6)/4)^8 * 1.08643 * integrate integrate [155370000140.18299865]

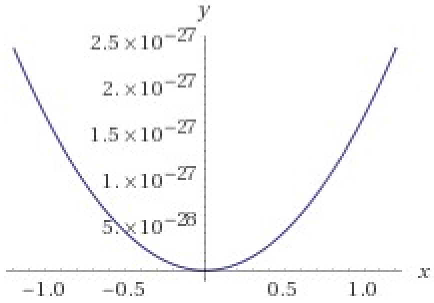
dove “(√6)/4 = Circumradius sphere, congruent with vertices (Tetrahedron)”

$$\frac{1}{10^{36}} \left(\frac{\sqrt{6}}{4} \right)^8 \times 1.08643 \int \left(\int 1.5537000014018299865 \times 10^{11} \, dx \right) dx$$

Result:

$1.66903 \times 10^{-27} x^2$

Plot:



(x from -1.2 to 1.2)

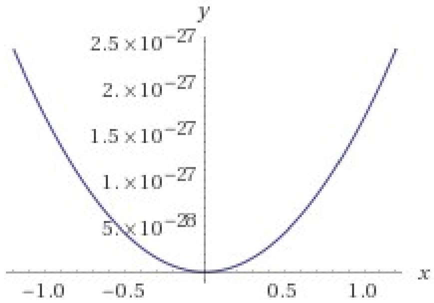
1/2 * 1/(10^36) * (sqrt(6)/4)^8 * 1.08643 * integrate integrate [310740000280.3659973]

$$\frac{1}{2} \times \frac{1}{10^{36}} \left(\frac{\sqrt{6}}{4}\right)^8 \times 1.08643 \int \left(\int 3.107400002803659973 \times 10^{11} \, dx \right) dx$$

Result:

$1.66903 \times 10^{-27} x^2$

Plot:



(x from -1.2 to 1.2)

Also these results are excellent approximations of the proton mass. Recall that when a proton collides with an antiproton, one of the valence quarks that constitute it can annihilate itself with an antiquark, while the remaining quarks will rearrange into mesons. The mesons created are a group of subatomic particles composed of a quark and an antiquark bound by the strong force. They are unstable particles and typically decay into photons or leptons. This means that the light, hence the electromagnetic waves concerning the Maxwell's equations, seems to have a fermionic origin (electron-positron / proton-antiproton annihilation), also if it consists of photons (bosons). In fact this is only the logical consequence of the formalization of the Einstein’s equation $E = mc^2$ in which the energy is closely connected with the mass, and which is closely related also to the Maxwell’s equations. In the our proposal, it is the mass that becomes energy, which in turn, following at the various breaks of symmetry, returns mass, in an cycle eternal in the time and infinite in the space. This is a further try to support of the theory of a cyclical oscillating universe (or eventually also a multiverse). But the most extraordinary thing is the possibility that the universe itself, which has neither beginning nor end, was born as a wave-particle of infinite density and energy (wave function of universe-model "no-boundary proposal" of S.W. Hawking).

The lines of gravitating force near two dense bodies are exactly of the same form as the lines of magnetic force near two poles of the same name; but whereas the poles are repelled, the bodies are attracted. Let E be the intrinsic energy of the field surrounding two gravitating bodies M_1, M_2, and let E' be the intrinsic energy of the field surrounding two magnetic poles, m_1, m_2 equal in numerical value to M_1, M_2, and let X be the gravitating force acting during the displacement δx, and X' the magnetic force,

$$X\delta x = \delta E, \qquad X'\delta x = \delta E';$$

now X and X' are equal in numerical value, but of opposite signs; so that

$$\delta E = -\delta E',$$

or

$$E = C - E'$$

$$= C - \Sigma \frac{1}{8\pi} (\alpha^2 + \beta^2 + \gamma^2)\, dV$$

where α, β, γ are the components of magnetic intensity. If R be the resultant gravitating force, and R' the resultant magnetic force at a corresponding part of the field,

$$R = -R', \text{ and } \alpha^2 + \beta^2 + \gamma^2 = R^2 = R'^2.$$

Hence

$$E = C - \Sigma \frac{1}{8\pi} R^2 dV \quad \text{.................................. (47).}$$

The intrinsic energy of the field of gravitation must therefore be less wherever there is a resultant gravitating force.

As energy is essentially positive, it is impossible for any part of space to have negative intrinsic energy. Hence those parts of space in which there is no resultant force, such as the points of equilibrium in the space between the different bodies of a system, and within the substance of each body, must have an intrinsic energy per unit of volume greater than

$$\frac{1}{8\pi} R^2,$$

where R is the greatest possible value of the intensity of gravitating force in any part of the universe.

The assumption, therefore, that gravitation arises from the action of the surrounding medium in the way pointed out, leads to the conclusion that every part of this medium possesses, when undisturbed, an enormous intrinsic energy, and that the presence of dense bodies influences the medium so as to diminish this energy wherever there is a resultant attraction.

Putting that R is equal to the Gravitational universal constant, that is

$G = 6,67 \times 10^{-11}$ N m² / kg² . We obtain:

$$E = \frac{1}{8\pi} \cdot (6,674 \times 10^{-11})^2 = 1,77228085 \times 10^{-22}$$

Now calculate the following double integral:

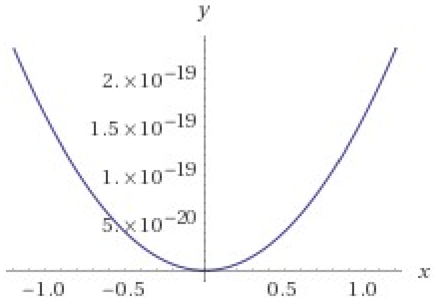
(0.63894^4) * 10^4 * 1.08643 * integrate integrate [(1.77228085) *(10^-22)]

$$0.63894^4 \times 10^4 \times 1.08643 \int \left(\int \frac{1.77228085}{10^{22}} \, dx \right) dx$$

Result:

$1.60452 \times 10^{-19} x^2$

Plot:



(x from -1.2 to 1.2)

result that is very near to the value of the electric charge of the positron.

From:

A TREATISE ON ELECTRICITY AND MAGNETISM
BY JAMES CLERK MAXWELL,
M.A - LLD. EDIN., P.E.SS. LONDON AND EDINBURGH HONORARY FELLOW OF TRINITY COLLEGE, AND PROFESSOR OF EXPERIMENTAL PHYSICS IN THE UNIVERSITY OF CAMBRIDGE ***VOL. I*** *AT THE CLARENDON PRESS 1873*

Finally, let $8\pi Q$ represent the triple integral

$$8\pi Q = \iiint \frac{1}{K}(a^2+b^2+c^2)\,dx\,dy\,dz, \tag{9}$$

extended over a space bounded by surfaces, for each of which either

$$V = \text{constant},$$

or
$$la+mb+nc = Kl\frac{dV}{dx} + Km\frac{dV}{dy} + Kn\frac{dV}{dz} = q, \tag{10}$$

where the value of q is given at every point of the surface; then, if a, b, c be supposed to vary in any manner, subject to the above conditions, the value of Q will be a *unique minimum*, when

$$a = K\frac{dV}{dx}, \quad b = K\frac{dV}{dy}, \quad c = K\frac{dV}{dz}. \tag{11}$$

Proof.

If we put for the general values of a, b, c,

$$a = K\frac{dV}{dx}+u, \quad b = K\frac{dV}{dy}+v, \quad c = K\frac{dV}{dz}+w; \tag{12}$$

then, by substituting these values in equations (5) and (7), we find that u, v, w satisfy the general solenoidal condition

$$(1) \quad \frac{du}{dx}+\frac{dv}{dy}+\frac{dw}{dz} = 0.$$

We also find, by equations (6) and (8), that at the surfaces of discontinuity the values of u_1, v_1, w_1 and u_2, v_2, w_2 satisfy the superficial solenoidal condition

$$(2) \quad l(u_1-u_2)+m(v_1-v_2)+n(w_1-w_2) = 0.$$

The quantities u, v, w, therefore, satisfy at every point the solenoidal conditions as stated in the preceding lemma.

We may now express Q in terms of u, v, w and V,

$$\pi Q = \iiint K \left(\overline{\frac{dV}{dx}}\Big|^2 + \overline{\frac{dV}{dy}}\Big|^2 + \overline{\frac{dV}{dz}}\Big|^2 \right) dx\,dy\,dz + \iiint \frac{1}{K} (u^2+v^2+w^2)\,dx\,dy\,dz$$

$$+ 2 \iiint \left(u\frac{dV}{dx} + v\frac{dV}{dy} + w\frac{dV}{dz} \right) dx\,dy\,dz. \quad (13)$$

The last term of Q may be written $2M$, where M is the quantity considered in the lemma, and which we proved to be zero when the space is bounded by surfaces, each of which is either equipotential or satisfies the condition of equation (10), which may be written

$$(4) \quad lu + mv + nw = 0.$$

Q is therefore reduced to the sum of the first and second terms.

In each of these terms the quantity under the sign of integration consists of the sum of three squares, and is therefore essentially positive or zero. Hence the result of integration can only be positive or zero.

Let us suppose the function V known, and let us find what values of u, v, w will make Q a minimum.

If we assume that at every point $u = 0$, $v = 0$, and $w = 0$, these values fulfil the solenoidal conditions, and the second term of Q is zero, and Q is then a minimum as regards the variation of u, v, w.

For if any of these quantities had at any point values differing from zero, the second term of Q would have a positive value, and Q would be greater than in the case which we have assumed.

But if $u = 0$, $v = 0$, and $w = 0$, then

$$(11) \quad a = K\frac{dV}{dx}, \quad b = K\frac{dV}{dy}, \quad c = K\frac{dV}{dz}.$$

Hence these values of a, b, c make Q a minimum.

But the values of a, b, c, as expressed in equations (12), are perfectly general, and include all values of these quantities consistent with the conditions of the theorem. Hence, no other values of a, b, c can make Q a minimum.

Again, Q is a quantity essentially positive, and therefore Q is always capable of a minimum value by the variation of a, b, c. Hence the values of a, b, c which make Q a minimum must have a real existence. It does not follow that our mathematical methods are sufficiently powerful to determine them.

But if $u = 0$, $v = 0$, and $w = 0$, then

$$(11) \quad a = K\frac{dV}{dx}, \quad b = K\frac{dV}{dy}, \quad c = K\frac{dV}{dz}.$$

Hence these values of a, b, c make Q a minimum.

Green's Theorem shews that the quantity Q, when it has its minimum value corresponding to a given distribution of electricity, represents the potential energy of that distribution of electricity. See Art. 100, equation (11).

From:

ELETTROMAGNETISMO
PARTE II - POTENZIALE ELETTRICO
ESERCIZI SVOLTI DAL PROF. GIANLUIGI TRIVIA

Exercise 14. Trovare il potenziale che raggiunge una sfera conduttrice isolata di raggio $16.0\,cm$ con una carica di $1.50 \cdot 10^{-8}\,C$ con $V = 0$ all'infinito.

Soluzione: Il potenziale di una sfera conduttrice isolata è espresso da

$$V = k_0\frac{q}{r} = \frac{8.99 \cdot 10^9\,\frac{Nm^2}{C^2} \cdot 1.50 \cdot 10^{-8}\,C}{0.16\,m} = 843\,V$$

From the example above, considering the charge of an electron, then q = 1.6 * 10^{-19} and the radius of the sphere of 1cm, remembering the coulomb constant, whose value is:

$$k = \frac{1}{4\pi\varepsilon_0} = \frac{c_0^2\mu_0}{4\pi} = c_0^2 \cdot 10^{-7}\mathrm{H\,m^{-1}} = 8.987\,551\,787\,368\,176 \cdot 10^9\ \mathrm{N\,m^2\,C^{-2}}$$

from the formula of the potential energy electric

$$U_{\mathrm{E}}(r) = k\frac{qQ}{r}$$

that in our case is: $V = k_0\frac{q}{r}$, we have:

$$V = k_0\frac{q}{r} = \frac{8.99 \cdot 10^9 Nm^2C^{-2} \cdot 1.602 \cdot 10^{-19}C}{0.01m} = 1{,}440198 \cdot 10^{-7}$$

Now we take the equation (9):

$$8\pi Q = \iiint \frac{1}{K}(a^2 + b^2 + c^2)\, dx\, dy\, dz,$$

where Q represents the potential energy of a certain distribution of electricity. We have, for $Q = 1{,}440198 \cdot 10^{-7}$, $8\pi Q = 3{,}61961236 * 10^{-6}$.

We proceed to carry out the following double integration on the value obtained. We have:

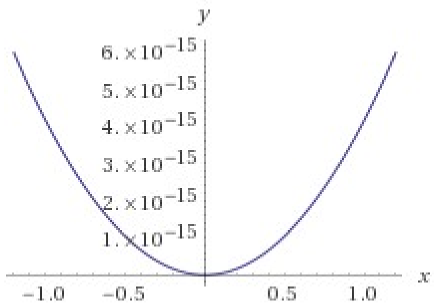
1/(10^9) * ((0.61803398/(0.66482^3)) * 1.08643 * integrate integrate [(3.61961236) *(10^-6)]

$$\frac{1}{10^9}\left(\frac{0.61803398}{0.66482^3} \times 1.08643 \int\left(\int \frac{3.61961236}{10^6}\, dx\right) dx\right)$$

Result:

$4.13556 \times 10^{-15}\, x^2$

Plot:



(x from -1.2 to 1.2)

Now we take the following example:

Exercise 28. Due cariche $q = +.20\,\mu C$ sono fisse nello spazio a una distanza $d = 2.0\,cm$, come mostrato in figura. Con $V = 0$ all'infinito, trovare il potenziale elettrico nel punto C. Una terza carica identica alle precedenti viene portata lentamente dall'infinito nel punto C. Trovare il lavoro necessario. Trovare, infine, l'energia potenziale della configurazione quando anche la terza carica è al suo posto.

Soluzione: Il potenziale elettrico nel punto C si calcola applicando il principio di sovrapposizione, sommando cioè il potenziale in C relativo alle due cariche considerate sole. La distanza di ogni carica dal punto C può essere calcolata osservando che tale distanza è la diagonale di un quadrato di lato $d/2$, cioè $\frac{d}{2}\sqrt{2}$

$$V_C = \frac{1}{4\pi\varepsilon_0}\left(\frac{q}{\frac{d}{2}\sqrt{2}} + \frac{q}{\frac{d}{2}\sqrt{2}}\right) = \frac{1}{4\pi\varepsilon_0}\frac{4q}{d\sqrt{2}} = \frac{2q\sqrt{2}}{4\pi\varepsilon_0 d} = 8.99\cdot 10^9\,\frac{Nm^2}{C^2} \times \frac{2\sqrt{2}\times 2.0\cdot 10^{-6}\,C}{0.02\,m} = 2.54\cdot 10^6\,V$$

Se una terza carica q_3, uguale alle precedenti, viene portata nel punto C contro le forze del campo elettrico si compirà un lavoro positivo

$$L = U = qV = 2.0\cdot 10^{-6}\,C \times 2.54\cdot 10^6\,V = 5.1\,J$$

Calcoliamo ora l'energia potenziale della configurazione con le tre cariche

$$U = U_{q_q q_2} + U_{q_1 q_3} + U_{q_2 q_3} = \frac{1}{4\pi\varepsilon_0}\left(\frac{q_1 q_2}{d} + \frac{q_1 q_3}{\frac{d}{\sqrt{2}}} + \frac{q_2 q_3}{\frac{d}{\sqrt{2}}}\right)$$

le cariche sono tutte uguali, per cui

$$U = \frac{1}{4\pi\varepsilon_0}\left(\frac{q^2}{d} + \frac{2\sqrt{2}q^2}{d}\right) = \frac{q^2}{4\pi\varepsilon_0 d}\left(1+2\sqrt{2}\right) = 8.99\cdot 10^9\,\frac{Nm^2}{C^2} \times \frac{(1+2\sqrt{2})\times(2.0\cdot 10^{-6}\,C)^2}{0.02\,m} = 6.9\,J$$

Now, we calculate the following double integration, for Q = 2.54 * 10^6 V. We have:

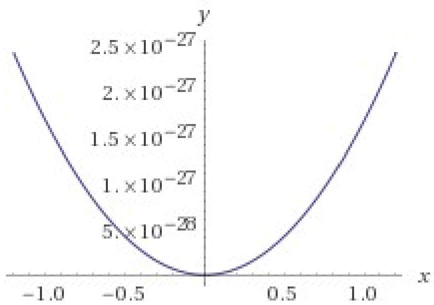
1/(10^35) * (1.6156351)^10 * 1.08643 * integrate integrate [(2.54) *(10^6)]

$$\frac{1}{10^{35}} \times 1.6156351^{10} \times 1.08643 \int\left(\int 2.54\times 10^6\,dx\right)dx$$

Result:

$$1.67201\times 10^{-27}\,x^2$$

Plot:



(x from -1.2 to 1.2)

result that is very near to the value of the mass of the proton.

or:

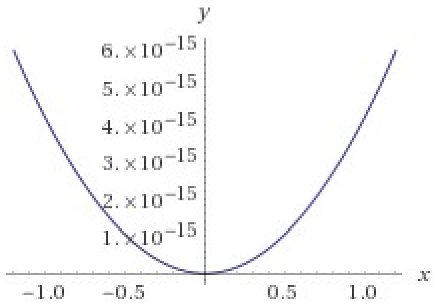
1/(10^21) * (3) * 1.08643 * integrate integrate [(2.54) *(10^6)]

$$\frac{1}{10^{21}} \times 3 \times 1.08643 \int \left(\int 2.54 \times 10^6 \, dx \right) dx$$

Result:

$4.1393 \times 10^{-15} x^2$

Plot:



(x from -1.2 to 1.2)

Finally, taking into account also 8Pigreco, we have:

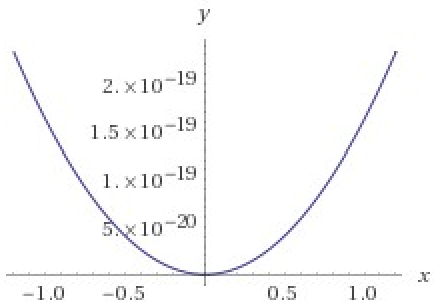
1/(10^53) * (1.6156351)^128 * 1.08643 * integrate integrate [8*Pi*(2.54) *(10^6)]

$$\frac{1}{10^{53}} \times 1.6156351^{128} \times 1.08643 \int \left(\int 8\,\pi \times 2.54 \times 10^6 \, dx \right) dx$$

Result:

$1.6143 \times 10^{-19} x^2$

Plot:



(x from -1.2 to 1.2)

result that is very near to the value of the electric charge of the positron.

Now:

$$Q = \frac{1}{8\,\pi} \iiint K \left(\overline{\frac{dV_0}{dx}}\Big|^2 + \&c. \right) dx\,dy\,dz + \frac{1}{8\,\pi} \iiint K \left(\overline{\frac{dU}{dx}}\Big|^2 + \&c. \right) dx\,dy\,dz. \quad (7)$$

We note that $\frac{1}{8\pi} + \frac{1}{8\pi} = \frac{2}{8\pi} = 0{,}079577471545947$. Now we calculate the following double integral:

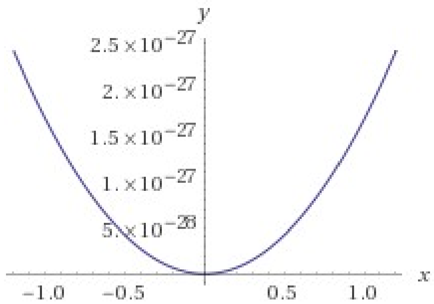
1/(10^26) * (1.645)^e) * 1.08643 * integrate integrate [(1/(8Pi)+(1/(8Pi)]

$$\frac{1}{10^{26}} \times 1.645^{e} \times 1.08643 \int \left(\int \left(\frac{1}{8\pi} + \frac{1}{8\pi} \right) dx \right) dx$$

Result:

$1.67248 \times 10^{-27} x^2$

Plot:



(x from −1.2 to 1.2)

result that is very near to the value of the mass of the proton.

and:

$$Q = \frac{1}{8\pi} \iiint \frac{1}{K} (a^2 + b^2 + c^2)\, dx\, dy\, dz \qquad (14)$$

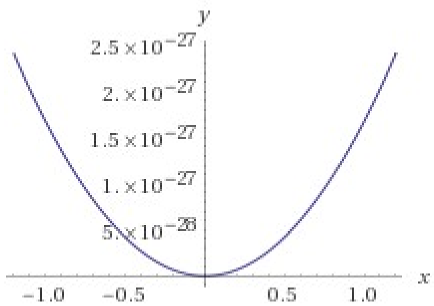
We have the following double integral:

1/(10^26) * (1.63358)^4) * 1.08643^2 * integrate integrate [(1/(8Pi)]

$$\frac{1}{10^{26}} \times 1.63358^{4} \times 1.08643^{2} \int \left(\int \frac{1}{8\pi}\, dx \right) dx$$

Result:

$1.67223 \times 10^{-27} x^2$



(x from −1.2 to 1.2)

result that is very near to the value of the mass of the proton.

or:

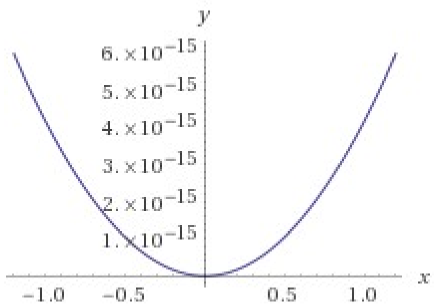
1/(10^13) * (0.6359)^(1/-0.8) * 1.08643^2 * integrate integrate [(1/(8Pi)]

$$\frac{1}{10^{13}} \times 0.6359^{-1/0.8} \times 1.08643^2 \int\left(\int \frac{1}{8\pi}\,dx\right)dx$$

Result:

$4.13521 \times 10^{-15}\, x^2$

Plot:



(x from -1.2 to 1.2)

Now:

If σ_1, σ_2 are the surface-densities on the opposed surfaces of a solid sphere of radius a, and a spherical hollow of radius b, then

$$\sigma_1 = \frac{1}{4\pi a^2}\frac{A-B}{a^{-1}-b^{-1}}, \quad \sigma_2 = \frac{1}{4\pi b^2}\frac{B-A}{a^{-1}-b^{-1}}.$$

If E_1 and E_2 be the whole charges of electricity on these surfaces,

$$E_1 = 4\pi a^2 \sigma_1 = \frac{A-B}{a^{-1}-b^{-1}} = -E_2.$$

The capacity of the enclosed sphere is therefore $\frac{ab}{b-a}$.

If the outer surface of the shell be also spherical and of radius c, then, if there are no other conductors in the neighbourhood, the charge on the outer surface is

$$E_3 = Bc.$$

Hence the whole charge on the inner sphere is

$$E_1 = \frac{ab}{b-a}(A-B),$$

and that of the outer

$$E_2 + E_3 = \frac{ab}{b-a}(B-A) + Bc.$$

If we put $b = \infty$, we have the case of a sphere in an infinite space. The electric capacity of such a sphere is a, or it is numerically equal to its radius.

The electric tension on the inner sphere per unit of area is

$$p = \frac{1}{8\pi}\frac{b^2}{a^2}\frac{(A-B)^2}{(b-a)^2}.$$

The resultant of this tension over a hemisphere is $\pi a^2 p = F$ normal to the base of the hemisphere, and if this is balanced by a surface tension exerted across the circular boundary of the hemisphere, the tension on unit of length being T, we have

$$F = 2\pi a T.$$

Hence
$$F = \frac{b^2}{8}\frac{(A-B)^2}{(b-a)^2} = \frac{E_1^2}{8a^2},$$

$$T = \frac{b^2}{16\pi a}\frac{(A-B)^2}{(b-a)^2}.$$

If a spherical soap bubble is electrified to a potential A, then, if its radius is a, the charge will be Aa, and the surface-density will be

$$\sigma = \frac{1}{4\pi}\frac{A}{a}.$$

The resultant electrical force just outside the surface will be $4\pi\sigma$, and inside the bubble it is zero, so that by Art. 79 the electrical force on unit of area of the surface will be $2\pi\sigma^2$, acting outwards. Hence the electrification will diminish the pressure of the air within the bubble by $2\pi\sigma^2$, or

$$\frac{1}{8\pi}\frac{A^2}{a^2}.$$

But it may be shewn that if T is the tension which the liquid film exerts across a line of unit length, then the pressure from within required to keep the bubble from collapsing is $2\frac{T}{a}$. If the electrical force is just sufficient to keep the bubble in equilibrium when the air within and without is at the same pressure

$$A^2 = 16\pi a T.$$

Now, we analyze

$$T = \frac{b^2}{16\pi a}\frac{(A-B)^2}{(b-a)^2}$$

Where Aa = 1.602 * 10^{-19} for a = 1 and B = -1.602 * 10^{-19} where A and B are the potentials, a = 1 and b = 2 are the radii of a solid sphere and a hollow sphere respectively. We have: T = 8.16911766 * 10^{-39} which represents the electric tension. We calculate the following double integration:

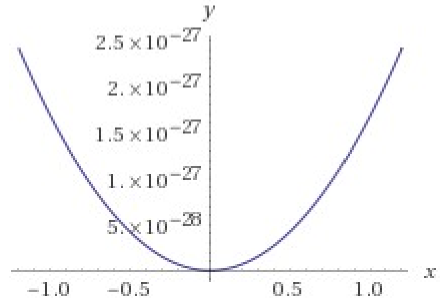
(10^11) * ((2*1.6305*(Pi)/(e)) * 1.08643 * integrate integrate [(8.16911765)*(10^-39)]

$$10^{11}\left(\left(2\times 1.6305\times\frac{\pi}{e}\right)\times 1.08643\int\left(\int\frac{8.16911765}{10^{39}}\,dx\right)dx\right)$$

Result:

$1.67245\times 10^{-27}\,x^2$

Plot:



(x from −1.2 to 1.2)

result that is very near to the value of the mass of the proton.

Now, for:

$$A^2 = 16\,\pi\,a\,T.$$

We have $A^2 = 4{,}1062464 * 10^{-37}$; $A = 6{,}408 * 10^{-19}$. We calculate the following double integral:

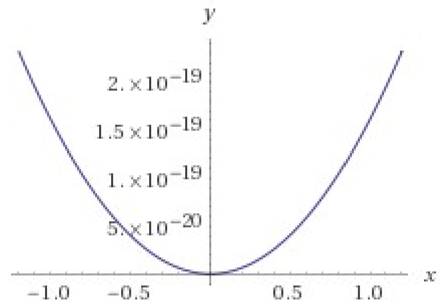
1/10 * ((4Pi)/(e)) * 1.08643 * integrate integrate [(6.408)*(10^-19)]

$$\frac{1}{10} \times \frac{4\pi}{e} \times 1.08643 \int \left(\int \frac{6.408}{10^{19}}\, dx \right) dx$$

Result:

$1.6092 \times 10^{-19}\, x^2$

Plot:



(x from −1.2 to 1.2)

result that is very near to the value of the electric charge of the positron.

Now, let's take the potential of the previous example $Q = 2.54 * 10^6$ V

The quantity of electricity on a planetary ellipsoid maintained at potential V in an infinite field, is

$$Q = c\frac{V}{\frac{\pi}{2}-\gamma}, \qquad (37)$$

where $c \sec\gamma$ is the equatorial radius, and $c \tan\gamma$ is the polar radius.

If $\gamma = 0$, the figure is a circular disk of radius c, and

$$\sigma = \frac{V}{\pi^2\sqrt{c^2-r^2}}, \qquad (38)$$

$$Q = c\frac{V}{\frac{\pi}{2}}. \qquad (39)$$

From (39), we obtain, putting c = 1: Q = 2.54 * 10^6 / 1,5707963… = 1617014,22181

We calculate the following double integral:

(1/((10^2)* (24*e*26*Pi)) * 1.08643 * integrate integrate [1617014.22181]

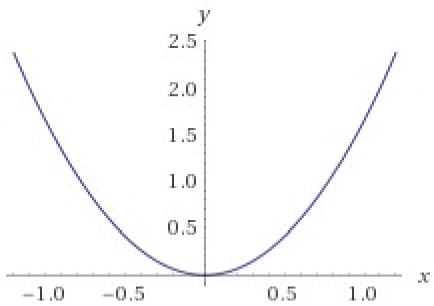
$$\frac{1}{10^2\,(24\,e\times 26\,\pi)}\times 1.08643\int\left(\int 1.61701422181\times 10^6\,dx\right)dx$$

$$\frac{1}{10^2}\times\frac{1}{24\,e\times 26\,\pi}\left(1.08643\int\left(\int 1.61701422181\times 10^6\,dx\right)dx\right)$$

Result:

$1.64838\,x^2$

Plot:



(x from −1.2 to 1.2)

result that is very near to the value of the mass of the proton.

From (38) for c = 5 ed r = 3, we have: σ = 2.54 * 10^6 / 9.8696044 * 4 = 64338,95 and Q = 8085071,109

We calculate the following double integrals:

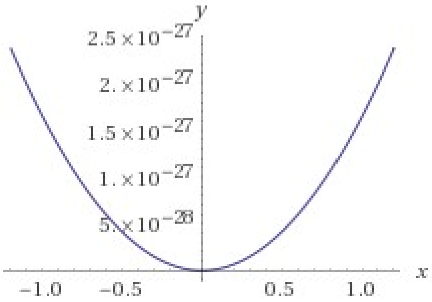
1/(10^34)* (5*e*11*Pi)) * 1.08643 * integrate integrate [64338.95]

$$\frac{1}{10^{34}} (5\, e \times 11\, \pi) \times 1.08643 \int \left(\int 64338.95\, dx \right) dx$$

Result:

$1.64154 \times 10^{-27}\, x^2$

Plot:



(x from −1.2 to 1.2)

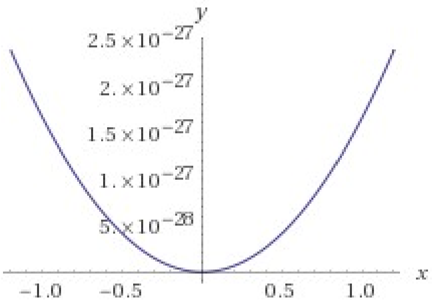
1/(10^36)* (4*e*11*Pi)) * 1.08643 * integrate integrate [8085071.109]

$$\frac{1}{10^{36}} (4\, e \times 11\, \pi) \times 1.08643 \int \left(\int 8.085071109 \times 10^{6}\, dx \right) dx$$

Result:

$1.65026 \times 10^{-27}\, x^2$

Plot:



(x from −1.2 to 1.2)

result that is very near to the value of the mass of the proton.

The quantity of electricity on an ovary ellipsoid maintained at a potential V in an infinite field is

$$Q = c\frac{V}{\gamma}. \qquad (44)$$

If the polar radius is $A = c \cot h\, \gamma$, and the equatorial radius is $B = c \operatorname{cosec} h\, \gamma$,

$$\gamma = \log \frac{A + \sqrt{A^2 - B^2}}{2B}. \qquad (45)$$

We recall that from the mathematical point of view, a reference ellipsoid is usually an oblate (flattened) spheroid whose semi-axes are defined: equatorial radius (the major semi-axis a) and polar radius (the minor semi-axis b).

For A = 3 and B = 5, we have:

Ln((3+sqrt(-16))/10)

$$\log\left(\frac{1}{10}\left(3+\sqrt{-16}\right)\right)$$

$$-0.69314718055994530941723212145817656807550013436025525\ldots + 0.92729521800161223242851246292242880405707410857224052\ldots i$$

$$(0.92729521800\, i)^2$$

$$-0.859876421325667524$$

γ = -1,5530236 and for c =1, V = 2.54 * 10^6

Q = 1635519,25418261512574567443791 58 = 1635519,25

We calculate the following double integral:

(1.7323726*10^-6) * 1.08643^2 * integrate integrate [1635519.25]

(sqrt(3)*10^-6) * 1.08643^2 * integrate integrate [1635519.25]

$$1.7323726 \times 10^{-6} \times 1.08643^2 \int\left(\int 1.63551925 \times 10^6\, dx\right) dx$$ *

$$\frac{\sqrt{3}}{10^6} \times 1.08643^2 \int\left(\int 1.63551925 \times 10^6\, dx\right) dx$$

Result:

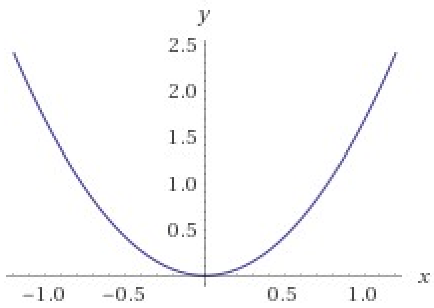
$1.67213\, x^2$

Result:

$1.67182\, x^2$

result that is very near to the value of the mass of the proton.

Plot: *



(x from -1.2 to 1.2)

We now analyze, always for V = 2.54 * 10^6 and a = 1:

If the sphere instead of being at potential zero is at potential V, we must superpose a distribution of electricity on its outer surface having the uniform surface-density

$$\sigma = \frac{V}{4\pi a}.$$

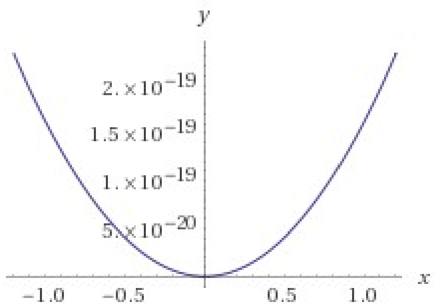
We have that the uniform surface-density", is: σ = (2.54 * 10^6) / 4π = 202126,77.
We calculate the following double integral:

(1/10^24) * 4/e * 1.08643 * integrate integrate [202126.77]

$$\frac{1}{10^{24}} \times \frac{4}{e} \times 1.08643 \int \left(\int 202\,126.77\, dx \right) dx$$

Result:
$1.6157 \times 10^{-19} x^2$

Plot:



(x from -1.2 to 1.2)

result that is very near to the value of the electric charge of the positron.

Distribution of Electricity on Three Spherical Surfaces which Intersect at Right Angles.

169.] Let the radii of the spheres be α, β, γ, then

$$BC = \sqrt{\beta^2+\gamma^2}, \quad CA = \sqrt{\gamma^2+\alpha^2}, \quad AB = \sqrt{\alpha^2+\beta^2}.$$

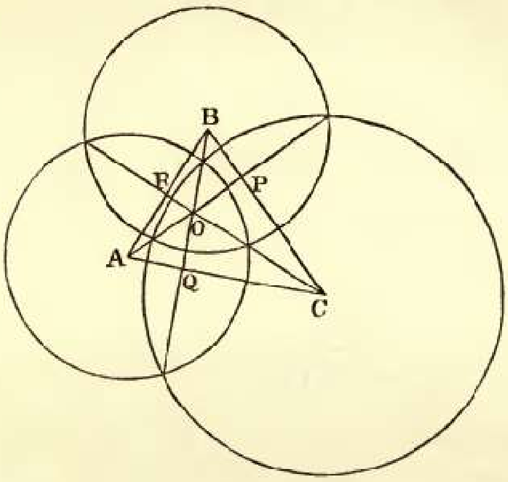


Fig. 13.

Let PQR, Fig. 13, be the feet of the perpendiculars from ABC on the opposite sides of the triangle, and let O be the intersection of perpendiculars.

Then P is the image of B in the sphere γ, and also the image of C in the sphere β. Also O is the image of P in the sphere α.

Let charges α, β, and γ be placed at A, B, and C.

Then the charge to be placed at P is

$$-\frac{\beta\gamma}{\sqrt{\beta^2+\gamma^2}} = -\frac{1}{\sqrt{\frac{1}{\beta^2}+\frac{1}{\gamma^2}}}.$$

Also $AP = \dfrac{\sqrt{\beta^2\gamma^2+\gamma^2\alpha^2+\alpha^2\beta^2}}{\sqrt{\beta^2+\gamma^2}}$, so that the charge at O, considered as the image of P, is

$$\frac{\alpha\beta\gamma}{\sqrt{\beta^2\gamma^2+\gamma^2\alpha^2+\alpha^2\beta^2}} = \frac{1}{\sqrt{\frac{1}{\alpha^2}+\frac{1}{\beta^2}+\frac{1}{\gamma^2}}}.$$

In the same way we may find the system of images which are electrically equivalent to four spherical surfaces at potential unity intersecting at right angles.

If the radius of the fourth sphere is δ, and if we make the charge at the centre of this sphere $= \delta$, then the charge at the intersection of the line of centres of any two spheres, say α and β, with their plane of intersection, is

$$-\frac{1}{\sqrt{\frac{1}{\alpha^2}+\frac{1}{\beta^2}}}.$$

The charge at the intersection of the plane of any three centres ABC with the perpendicular from D is

$$+\frac{1}{\sqrt{\frac{1}{\alpha^2}+\frac{1}{\beta^2}+\frac{1}{\gamma^2}}},$$

Now we analyze the two equations of the electric charges putting α = 1, β = 2 and γ = 3:

$$-\frac{1}{\sqrt{\frac{1}{\alpha^2}+\frac{1}{\beta^2}}}\cdot$$

$$+\frac{1}{\sqrt{\frac{1}{\alpha^2}+\frac{1}{\beta^2}+\frac{1}{\gamma^2}}},$$

We have the following two double integrals:

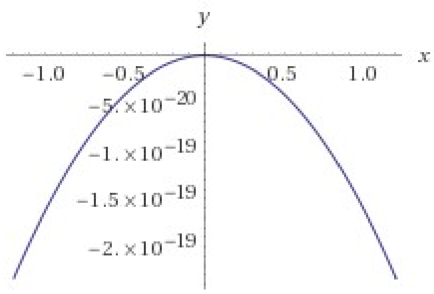
(1/(3*10^18)) * 1.08643 * integrate integrate [-1/(sqrt(1+(1/4))]

$$\frac{1}{3\times 10^{18}}\times 1.08643\int\left(\int -\frac{1}{\sqrt{1+\frac{1}{4}}}\,dx\right)dx$$

Result:

$-1.61955\times 10^{-19}\,x^2$

Plot:



(x from −1.2 to 1.2)

result that is very near to the value of the electric charge of the electron.

(1/(2.826419*10^26)) * 1.08643 * integrate integrate [1/(sqrt(1+(1/4)+(1/9))]

where 2.826 419 is the "Murata's constant" that is obtained by the following asymptotic formula:

$$\pi(x)^{-1}\sum_{\substack{p\leqslant x\\ p:\,\text{prime}}}\frac{p-1}{\varphi(p-1)}=C+O\left(\frac{\log\log x}{\log x}\right),$$

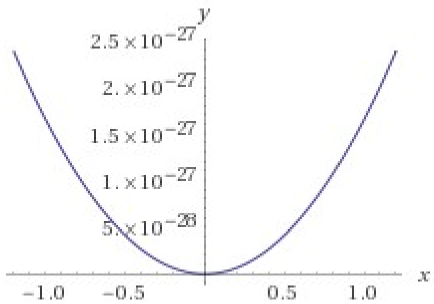
$$C=\prod_{p:\,\text{prime}}\left(1+\frac{1}{(p-1)^2}\right)\doteqdot 2.827.$$

$$\frac{1}{2.826419 \times 10^{26}} \times 1.08643 \int \left(\int \frac{1}{\sqrt{1+\frac{1}{4}+\frac{1}{9}}} \, dx \right) dx$$

Result:

$1.64736 \times 10^{-27} x^2$

Plot:



(x from −1.2 to 1.2)

result that is very near to the value of the mass of the proton.

In conclusion:

and the charge at the intersection of the four perpendiculars is

$$-\frac{1}{\sqrt{\frac{1}{\alpha^2}+\frac{1}{\beta^2}+\frac{1}{\gamma^2}+\frac{1}{\delta^2}}}.$$

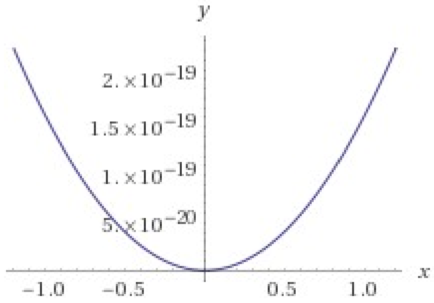
(1/(2.826419*10^18)) * 1.08643 * integrate integrate [1/(sqrt(1+(1/4)+(1/9)+(1/16))]

$$\frac{1}{2.826419 \times 10^{18}} \times 1.08643 \int \left(\int \frac{1}{\sqrt{1+\frac{1}{4}+\frac{1}{9}+\frac{1}{16}}} \, dx \right) dx$$

Result:

$1.61079 \times 10^{-19} x^2$

Plot:



(x from -1.2 to 1.2)

For δ = 5, we obtain:

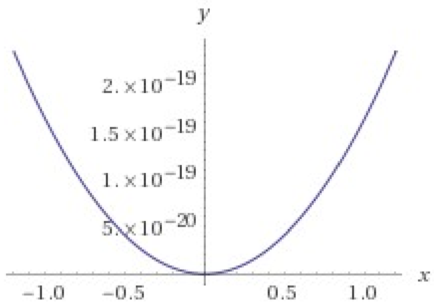
(1/(2.826419*10^18)) * 1.08643 * integrate integrate [1/(sqrt(1+(1/4)+(1/9)+(1/25))]

$$\frac{1}{2.826419\times 10^{18}}\times 1.08643 \int\left(\int \frac{1}{\sqrt{1+\frac{1}{4}+\frac{1}{9}+\frac{1}{25}}}\,dx\right)dx$$

Result:

$1.62367\times 10^{-19}\,x^2$

Plot:



(x from -1.2 to 1.2)

result that is very near to the value of the electric charge of the positron.

Now:

When the spheres are equal the charge of each for potential unity is

$$E_a = a \sum_{s=1}^{s=\infty} \frac{1}{2s(2s-1)},$$

$$= a\left(1-\tfrac{1}{2}+\tfrac{1}{3}-\tfrac{1}{4}+\&c.\right),$$

$$= a \log_e 2 = 1.0986\, a.$$

When the sphere A is very small compared with the sphere B the charge on A is

$$E_a = \frac{a^2}{b} \sum_{s=1}^{s=\infty} \frac{1}{s^2} \text{ approximately;}$$

or $$E_a = \frac{\pi^2}{6}\frac{a^2}{b}.$$

The charge on B is nearly the same as if A were removed, or

$$E_b = b.$$

The mean density on each sphere is found by dividing the charge by the surface. In this way we get

Recall that in electromagnetism the density of electric current is the vector whose flow through a surface represents the electric current that passes through that surface λ

$$\sigma_a = \frac{E_a}{4\pi a^2} = \frac{\pi}{24b},$$

$$\sigma_b = \frac{E_b}{4\pi b^2} = \frac{1}{4\pi b},$$

$$\sigma_a = \frac{\pi^2}{6}\sigma_b.$$

Hence, if a very small sphere is made to touch a very large one, the mean density on the small sphere is equal to that on the large sphere multiplied by $\frac{\pi^2}{6}$, or 1.644936.

We have that: for a = 1 and b = 2, σ_a = 0,065449 σ_b = 0,03978873 ed E_a = 0,822468

We calculate the following double integral on E_a = 0,822468

(16Pi)/(5e) * 1.08643 * integrate integrate [0.822468]

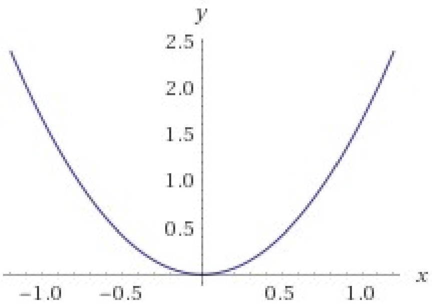
Input interpretation:

$$\frac{16\pi}{5e} \times 1.08643 \int\left(\int 0.822468\, dx\right) dx$$

Result:

$$1.65233\, x^2$$

Plot:



(x from −1.2 to 1.2)

and then on σ_a = 0,065449:

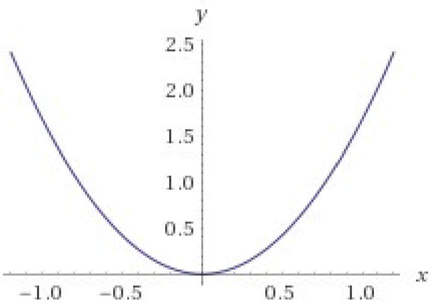
5*(3Pi) 1.084 * integrate integrate [0.065449]

$$5(3\pi)\left(1.084 \int\left(\int 0.065449\, dx\right) dx\right)$$

Result:

$$1.67164\, x^2$$

Plot:



(x from −1.2 to 1.2)

and on σ_b = 0,03978873

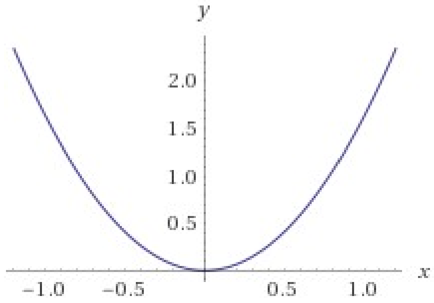
(24Pi) 1.08643 * integrate integrate [0.03978873]

$$(24\pi) \times 1.08643 \int\left(\int 0.03978873\, dx\right) dx$$

Result:

$$1.62964\, x^2$$

Plot:



(x from -1.2 to 1.2)

result that is very near to the value of the mass of the proton and to the electric charge of the positron.

Now:

Now let us consider the sphere as divided into two parts, one of which, the spherical segment on which we have determined the electric distribution, we shall call the *bowl*, and the other the remainder, or unoccupied part of the sphere on which the influencing point Q is placed.

180.] We have now only to suppose $\rho+\rho'=0$, and we get the case of the bowl maintained at potential V and free from external influence.

If σ is the density on either surface of the bowl at a given point when the bowl is at potential zero, and is influenced by the rest of the sphere electrified to density ρ, then, when the bowl is maintained at potential V, we must increase the density on the outside of the bowl by ρ', the density on the supposed enveloping sphere.

The result of this investigation is that if f is the diameter of the sphere, a the chord of the radius of the bowl, and r the chord of the distance of P from the pole of the bowl, then the surface-density σ on the *inside* of the bowl is

$$\sigma = \frac{V}{2\pi^2 f}\left\{\sqrt{\frac{f^2-a^2}{a^2-r^2}} - \tan^{-1}\sqrt{\frac{f^2-a^2}{a^2-r^2}}\right\},$$

and the surface-density on the outside of the bowl at the same point is

$$\sigma + \frac{V}{2\pi f}.$$

Thence, for f = 8, a = 3, r = 2 and for V = 2.54 * 10^6 volt , we have:

σ = 1124399,10468

$(2.54 * 10^6)/2*8\pi^2$ [√(64 – 9) / (9 – 4) - $\tan^{-1}$ √(64 – 9) / (9 – 4)] =

= (2540000/157,91367) * [3,31662479 – $\tan^{-1}$ (3,31662479)] =

= 16084,7379457 * (3,31662479 – 73,22134511) =

=16084,7379457 * (– 69,90472032) = – 1124399,107514

Now we calculate the following double integral:

(1/(5*262*34e*Pi)) * 1.08643 * integrate integrate [– 1124399.107514]

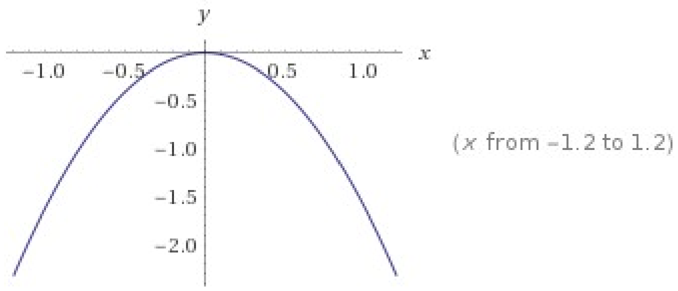
where 262 = 233 + 21 + 8,   5 e 34 are Fibonacci's numbers

$$\frac{1}{5\times 262\times 34\,(e\,\pi)}\times 1.08643\int\left(\int -1.124399107514\times 10^{6}\,dx\right)dx$$

Result:

$-1.60582\,x^2$

Plot:



result that is very near to the electric charge of the electron.

From:

Maxwell, James Clerk (1873), ***A treatise on electricity and magnetism Vol II***, Oxford : Clarendon Press

When X is less than L, $\quad I = \frac{2}{3} M \frac{X}{D}$.

When X is equal to L, $\quad I = \frac{2}{3} M \frac{L}{D}$.

When X is between L and D,

$$I = M \left\{ \frac{2}{3}\frac{X}{D} + \left(1 - \frac{L^2}{X^2}\right)\left[\sqrt{1 - \frac{L^2}{D^2}} - \frac{2}{3}\sqrt{\frac{X^2}{D^2} - \frac{L^2}{D^2}}\right]\right\}.$$

When X is equal to D,

$$I = M \left\{ \frac{2}{3} + \frac{1}{3}\left(1 - \frac{L^2}{D^2}\right)^{\frac{3}{2}} \right\}.$$

When X is greater than D,

$$I = M \left\{ \frac{1}{3}\frac{X}{D} + \frac{1}{2} - \frac{1}{6}\frac{D}{X} + \frac{(D^2 - L^2)^{\frac{3}{2}}}{6\,X^2 D} - \frac{\sqrt{X^2 - L^2}}{6\,X^2 D}\left(2\,X^2 - 3\,XD + L^2\right) \right\}.$$

When X is infinite, $\quad I = M$.

When X is less than L the magnetization follows the former law, and is proportional to the magnetizing force. As soon as X exceeds L the magnetization assumes a more rapid rate of increase on account of the molecules beginning to be transferred from the one cone to the other. This rapid increase, however, soon comes to an end as the number of molecules forming the negative cone diminishes, and at last the magnetization reaches the limiting value M.

If we were to assume that the values of L and of D are different for different molecules, we should obtain a result in which the different stages of magnetization are not so distinctly marked.

The residual magnetization, I', produced by the magnetizing force X, and observed after the force has been removed, is as follows:

When X is less than L, $\quad$ No residual magnetization.

When X is between L and D,

$$I' = M\left(1 - \frac{L^2}{D^2}\right)\left(1 - \frac{L^2}{X^2}\right).$$

When X is equal to D,

$$I' = M\left(1 - \frac{L^2}{D^2}\right)^2.$$

When X is greater than D,

$$I' = \frac{1}{4}M\left\{1 - \frac{L^2}{XD} + \sqrt{1-\frac{L^2}{D^2}}\sqrt{1-\frac{L^2}{X^2}}\right\}^2.$$

When X is infinite,

$$I' = \frac{1}{4}M\left\{1 + \sqrt{1-\frac{L^2}{D^2}}\right\}^2.$$

If we make

$$M = 1000, \qquad L = 3, \qquad D = 5,$$

we find the following values of the temporary and the residual magnetization :—

Magnetizing Force. X	Temporary Magnetization. I	Residual Magnetization. I'
0	0	0
1	133	0
2	267	0
3	400	0
4	729	280
5	837	410
6	864	485
7	882	537
8	897	575
∞	1000	810

For X = 5, M = 1000, L = 3 e D = 5, we have I’ = 409.6 and I = 837 (here are all magnetic forces and magnetizing force, concerning the induced magnetism)

We calculate the following double integral:

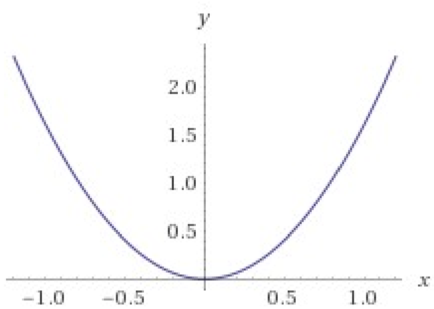
(0.6629)^12 * 1.08643 * integrate integrate [409.6]

$$0.6629^{12} \times 1.08643 \int\left(\int 409.6\, dx\right) dx$$

Result:

$1.60217 x^2$

Plot:



(x from −1.2 to 1.2)

and:

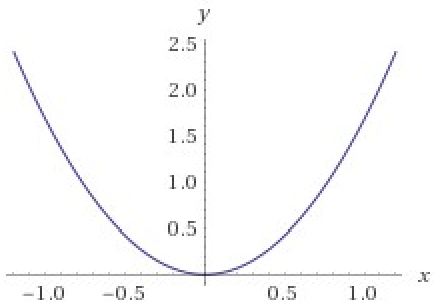
((0.6453)^8)* 1/4 * 1.08643 * integrate integrate [409.6]

$$0.6453^8 \times \frac{1}{4} \times 1.08643 \int \left(\int 409.6 \, dx \right) dx$$

Result:

$$1.6725\, x^2$$

Plot:



(x from −1.2 to 1.2)

result that is very near to the value of the mass of the proton and to the electric charge of the positron.

For I = 837, we have:

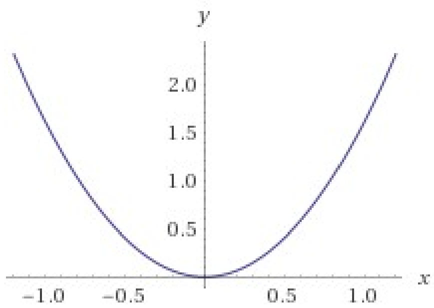
(0.62458)^12 * 1.08643 * integrate integrate [837]

$$0.62458^{12} \times 1.08643 \int \left(\int 837 \, dx \right) dx$$

Result:

$$1.60234\, x^2$$

Plot:



(x from −1.2 to 1.2)

and:

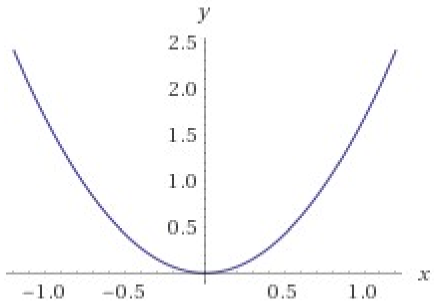
(0.6497312)^13 * 1.08643 * integrate integrate [837]

$$0.6497312^{13} \times 1.08643 \int \left(\int 837\, dx \right) dx$$

Result:

$$1.672\, x^2$$

Plot:



(x from −1.2 to 1.2)

result that is very near to the value of the mass of the proton and to the electric charge of the positron.

In the case of air, the electrical permittivity is $\varepsilon_r = 1{,}000\ 59$, approximated to 1 which is the value assigned to the relative dielectric constant in vacuum. Air is the only physical medium that is in fact assimilated to empty space. Normally the electrical permittivity is indicated with the symbol ε, and its value is usually written as a product $\varepsilon = \varepsilon_r\, \varepsilon_0$ of the relative permittivity ε_r of the permittivity of the void ε_0 also called dielectric constant of the vacuum. The dielectric constant of the vacuum or electrical permittivity of the vacuum is the characteristic electric permittivity of the vacuum, in which the electrical susceptibility is null and there is no polarization phenomenon. Its value is:

$$\varepsilon_0 = 8{,}854\,187\,817\,62 \cdot 10^{-12}\ \mathrm{F/m}$$

	Symbol.	Dimensions in Electrostatic System.	Dimensions in Electromagnetic System.
Quantity of electricity	e	$[L^{\frac{3}{2}} M^{\frac{1}{2}} T^{-1}]$	$[L^{\frac{1}{2}} M^{\frac{1}{2}}]$.
Line-integral of electro-motive intensity . .	E	$[L^{\frac{1}{2}} M^{\frac{1}{2}} T^{-1}]$	$[L^{\frac{3}{2}} M^{\frac{1}{2}} T^{-2}]$.
Quantity of magnetism; Electrokinetic momentum of a circuit .	m, p	$[L^{\frac{1}{2}} M^{\frac{1}{2}}]$	$[L^{\frac{3}{2}} M^{\frac{1}{2}} T^{-1}]$.
Electric current; Magnetic potential	C, Ω	$[L^{\frac{3}{2}} M^{\frac{1}{2}} T^{-2}]$	$[L^{\frac{1}{2}} M^{\frac{1}{2}} T^{-1}]$.
Electric displacement; Surface-density	$\mathfrak{D}$	$[L^{-\frac{1}{2}} M^{\frac{1}{2}} T^{-1}]$	$[L^{-\frac{3}{2}} M^{\frac{1}{2}}]$.
Electromotive intensity	$\mathfrak{E}$	$[L^{-\frac{1}{2}} M^{\frac{1}{2}} T^{-1}]$	$[L^{\frac{1}{2}} M^{\frac{1}{2}} T^{-2}]$.
Magnetic induction	$\mathfrak{B}$	$[L^{-\frac{3}{2}} M^{\frac{1}{2}}]$	$[L^{-\frac{1}{2}} M^{\frac{1}{2}} T^{-1}]$.
Magnetic force	$\mathfrak{H}$	$[L^{\frac{1}{2}} M^{\frac{1}{2}} T^{-2}]$	$[L^{-\frac{1}{2}} M^{\frac{1}{2}} T^{-1}]$.
Strength of current at a point	$\mathfrak{C}$	$[L^{-\frac{1}{2}} M^{\frac{1}{2}} T^{-2}]$	$[L^{-\frac{3}{2}} M^{\frac{1}{2}} T^{-1}]$.
Vector potential	$\mathfrak{A}$	$[L^{-\frac{1}{2}} M^{\frac{1}{2}}]$	$[L^{\frac{1}{2}} M^{\frac{1}{2}} T^{-1}]$.

627.] We have already considered the products of the pairs of these quantities in the order in which they stand. Their ratios are in certain cases of scientific importance. Thus

	Symbol.	Electrostatic System.	Electromagnetic System.
$\frac{e}{E}$ = capacity of an accumulator . .	q	$[L]$	$\left[\frac{T^2}{L}\right]$.
$\frac{p}{C}$ = coefficient of self-induction of a circuit, or electro-magnetic capacity .	L	$\left[\frac{T^2}{L}\right]$	$[L]$.
$\frac{\mathfrak{D}}{\mathfrak{E}}$ = specific inductive capacity of dielectric .	K	$[0]$	$\left[\frac{T^2}{L^2}\right]$.
$\frac{\mathfrak{B}}{\mathfrak{H}}$ = magnetic inductive capacity . .	μ	$\left[\frac{T^2}{L^2}\right]$	$[0]$.
$\frac{E}{C}$ = resistance of a conductor	R	$\left[\frac{T}{L}\right]$	$\left[\frac{L}{T}\right]$.
$\frac{\mathfrak{E}}{\mathfrak{C}}$ = specific resistance of a substance . . .	r	$[T]$	$\left[\frac{L^2}{T}\right]$.

Esercizio 8

Si osserva che una particella di carica q=+e con velocità $v=3\cdot 10^5$ m/s descrive una traiettoria di raggio R=0,3 m in un campo magnetico B=0,01 T ortogonale alla traiettoria della particella. Calcola la massa della particella.

Esercizio 8:soluzione

Una particella di carica q e velocità $\vec{v}$ in presenza di un campo magnetico $\vec{B}$ è soggetta ad una forza

$$\overline{F} = q\overline{v} \times \overline{B}$$

Il moto della particella avverrà conformemente al secondo principio della dinamica

$$\overline{F} = m\overline{a}$$

Nel nostro caso essendo $\vec{B}$ uniforme ed ortogonale a $\vec{v}$, il moto della particella è circolare uniforme, Si ha quindi con riferimento all'intensità dei vettori

$$m a = q v B \qquad ed \qquad a = \frac{v^2}{R} \qquad da\ cui$$

$$m = \frac{RqB}{v} = \frac{1{,}6 \cdot 10^{-19} \cdot 3 \cdot 10^{-1} \cdot 10^{-2}}{3 \cdot 10^5} = 1{,}6 \cdot 10^{-27}\ kg$$

chiaramente, si tratta di un protone.

Magnetic Energy.

†632.] We may treat the energy due to magnetization in a way similar to that pursued in the case of electrification, Art. 85. If A, B, C are the components of magnetization and α, β, γ the components of magnetic force, the potential energy of the system of magnets is then, by Art. 389,

$$-\tfrac{1}{2}\iiint(A\alpha + B\beta + C\gamma)\,dx\,dy\,dz, \qquad (6)$$

the integration being extended over the space occupied by magnetized matter. This part of the energy, however, will be included in the kinetic energy in the form in which we shall presently obtain it.

633.] We may transform this expression when there are no electric currents by the following method.

We know that
$$\frac{da}{dx} + \frac{db}{dy} + \frac{dc}{dz} = 0. \qquad (7)$$

Hence, by Art. 97, if

$$\alpha = -\frac{d\Omega}{dx}, \quad \beta = -\frac{d\Omega}{dy}, \quad \gamma = -\frac{d\Omega}{dz}, \tag{8}$$

as is always the case in magnetic phenomena where there are no currents,

$$\iiint (a\alpha + b\beta + c\gamma)\, dx\, dy\, dz = 0, \tag{9}$$

the integral being extended throughout all space, or

$$\iiint \{(\alpha + 4\pi A)\,\alpha + (\beta + 4\pi B)\,\beta + (\gamma + 4\pi C)\gamma\}\, dx\, dy\, dz = 0. \tag{10}$$

Hence, the energy due to a magnetic system

$$-\tfrac{1}{2}\iiint (A\alpha + B\beta + C\gamma)\, dx dy dz = \frac{1}{8\pi}\iiint (\alpha^2 + \beta^2 + \gamma^2)\, dx dy dz,$$
$$= \frac{1}{8\pi}\iiint \mathfrak{H}^2\, dx\, dy\, dz. \tag{11}$$

We have the following value for the magnetic force: B = 10^{-2} T = 0,01 . Thence, for the normal triple integral, we obtain:

$$\frac{1}{8\pi}\int\int\int 0.0001\, dx\, dy\, dz$$

Result:

$$3.97887 \times 10^{-6}\, x\, y\, z$$

We calculated the following double integral of the result 3,97887 * 10^{-6} :

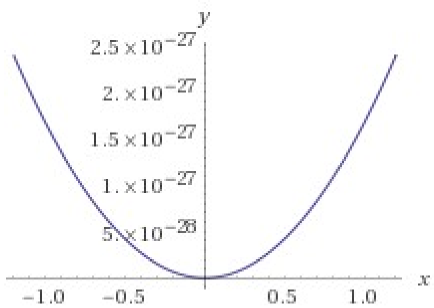
(1/10^21)(0.768225) * 1.08643 * integrate integrate [3.97887*10^-6]

$$\frac{1}{10^{21}} \times 0.768225 \times 1.08643 \int\left(\int 3.97887 \times 10^{-6}\, dx\right) dx$$

Result:

$$1.66043 \times 10^{-27}\, x^2$$

Plot:



(x from −1.2 to 1.2)

result that is a good approximation to the value of the mass of the proton.

We note that 0,768225 is $\Gamma(1/4)/(2\pi^{3/4})$; one of four values found by Ramanujan! Indeed, we have that (from: "Theta-function identities and the explicit formulas for theta-function and their applications" - Jinhee Yi - https://core.ac.uk/download/pdf/82662862.pdf)

$$h_{4,4} = \frac{2^{3/4}}{\sqrt[4]{2}+1},$$

that is equal to $\frac{2^{3/4}}{\sqrt[4]{2}+1} = \frac{1,681792830\ldots}{1,189207115\ldots+1} = 0,76822\ldots$

We can write the expression also as follow:

(1/10^21)((2^(3/4))/((2^(1/4))+1)) * 1.08643 * integrate integrate [3.97887*10^-6]

$$\frac{1}{10^{21}} \times \frac{2^{3/4}}{\sqrt[4]{2}+1} \times 1.08643 \int\left(\int 3.97887 \times 10^{-6}\, dx\right) dx$$

Result:

$1.66042 \times 10^{-27}\, x^2$

From the same integral, multiplied 10^{-21}, the constant 1,08643 and 0.61803398^2 i.e. the square of the golden ratio conjugate, we obtain:

(1/(10^21)) * 1.08643 * (0.61803398)^2 * 1/(8Pi) * integrate [0.0001] dx dy dz

$$\frac{1}{10^{21}} \times 0.61803398^2 \times 1.08643 \times \frac{1}{8\pi} \int\int\int 0.0001\, dx\, dy\, dz$$

Result:

$1.65115 \times 10^{-27}\, x\, y\, z$

A result very near to the mass of proton.

The equations

$$\left.\begin{aligned}\frac{dH}{dy}-\frac{dG}{dz}&=-\frac{d\Omega}{dx},\\ \frac{dF}{dz}-\frac{dH}{dx}&=-\frac{d\Omega}{dy},\\ \frac{dG}{dx}-\frac{dF}{dy}&=-\frac{d\Omega}{dz},\end{aligned}\right\} \tag{8}$$

which connect the components F, G, H of the vector-potential due to the current-sheet with the scalar potential Ω, are satisfied if we make

$$F=\frac{dP}{dy}, \qquad G=-\frac{dP}{dx}, \qquad H=0. \tag{9}$$

We may also obtain these values by direct integration, thus for F {we have by Art. 616 if μ is everywhere equal to unity},

$$F=\iint\frac{u}{r}dx'\,dy'=\iint\frac{1}{r}\frac{d\phi}{dy'}dx'\,dy',$$

$$=\int\frac{\phi}{r}dx'-\iint\phi\frac{d}{dy'}\frac{1}{r}dx'\,dy'.$$

Since the integration is to be estimated over the infinite plane sheet, and since the first term vanishes at infinity, the expression is reduced to the second term; and by substituting

$$\frac{d}{dy}\frac{1}{r}\text{ for }-\frac{d}{dy'}\frac{1}{r},$$

and remembering that ϕ depends on x' and y', and not on x, y, z, we obtain

$$F=\frac{d}{dy}\iint\frac{\phi}{r}dx'\,dy',$$

$$=\frac{dP}{dy}, \text{ by (1).}$$

If Ω' is the magnetic potential due to any magnetic or electric system external to the sheet, we may write

$$P'=-\int\Omega'\,dz, \tag{10}$$

and we shall then have

$$F'=\frac{dP'}{dy}, \qquad G'=-\frac{dP'}{dx}, \qquad H'=0, \tag{11}$$

for the components of the vector-potential due to this system.

Now, we have the following potential Ω' = 2.54 * 10^6 volt

Now, we calculate the following integral from 0 to π, multiplied 10^{-26} and the constant $(1,08643)^3$:

1/(10^26) * 1.08643^3 * - integrate [2540000] z,[0, Pi]

$$\frac{1}{10^{26}} \times 1.08643^3 \times (-1) \int_0^{\pi} 2540000\, z\, dz$$

Result:

$$-1.60734 \times 10^{-19}$$

And also with the following double integration, we obtain:

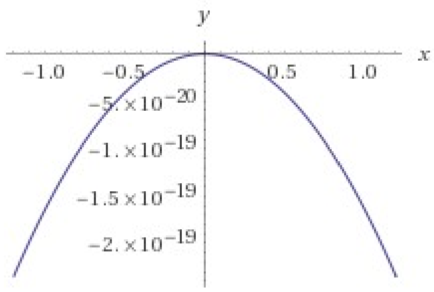
1/(10^25) * 1.08643^3 * -(integrate integrate [2540000])

$$\frac{1}{10^{25}} \times 1.08643^3 \times (-1) \int \left(\int 2540000\, dx \right) dx$$

Result:

$$-1.62858 \times 10^{-19}\, x^2$$

Plot:



(x from −1.2 to 1.2)

result that is a good approximation to the value of the electric charge of the electron.

If Ω_2 is the magnetic potential due to external magnets, and if we write

$$P' = -\int \Omega_2 dz, \tag{21}$$

the component of the magnetic force normal to the disk due to the magnets will be

$$\gamma_2 = \frac{d^2 P'}{dz^2}. \tag{22}$$

We may now write equation (18), remembering that

$$\gamma = \gamma_1 + \gamma_2,$$

$$\frac{\sigma}{2\pi}\frac{d^3 Q}{dz^3} - \omega\frac{d^3 Q}{d\theta\, dz^2} = \omega\frac{d^2 P'}{dz^2}. \tag{23}$$

Integrating twice with respect to z, and writing R for $\frac{\sigma}{2\pi}$,

$$\left(R\frac{d}{dz} - \omega\frac{d}{d\theta}\right) Q = \omega P'. \tag{24}$$

If the values of P and Q are expressed in terms of r, the distance from the axis of the disk, and of ξ and ζ two new variables such that

$$2\xi = z + \frac{R}{\omega}\theta, \qquad 2\zeta = z - \frac{R}{\omega}\theta, \tag{25}$$

equation (24) becomes, by integration with respect to ζ,

$$Q = \int \frac{\omega}{R} P'\, d\zeta. \tag{26}$$

669.] The form of this expression taken in conjunction with the method of Art. 662 shews that the magnetic action of the currents in the disk is equivalent to that of a trail of images of the magnetic system in the form of a helix.

We put that the disk have ω = 0,3456 rad/s , R = σ/2π = 0,1591549 for σ = 1 and that P' is equal to:

-integrate [2540000] x, [0, Pi]

$$-\int_0^{\pi} 2540000\, x\, dx = -1270000\,\pi^2 \approx -1.2534 \times 10^7$$

Indefinite integral:

$$-\int 2540000\,x\,dx = -1270000\,x^2 + \text{constant}$$

(we take the value of the indefinite integral)

Thence, Q is equal to:

integrate [((0.3456* (-1270000))/(0.1591549)]

$$\int \frac{0.3456\,(-1270000)}{0.1591549}\,dx = -2.75777\times10^{6}\,x + \text{constant}$$

Now, we calculate the following integral double:

1.08643^2 * integrate integrate [((0.3456* (-1270000))/(0.1591549)]

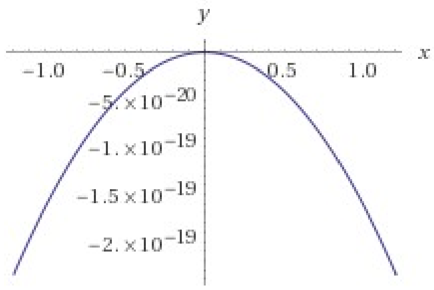
(1/10^25) * 1.08643^2 * integrate integrate [((0.3456* (-1270000))/(0.1591549)]

$$\frac{1}{10^{25}}\times1.08643^2 \int\left(\int \frac{0.3456\times(-1270000)}{0.1591549}\,dx\right)dx$$

Result:

$$-1.62754\times10^{-19}\,x^2$$

Plot:



(x from -1.2 to 1.2)

It is very significant the result that is a value very near to the charge of the electron.

Now:

If the whole number of windings is N, the number between the pole and the polar distance θ is $\frac{1}{2}N\sin^2\theta$.

The windings are closest at latitude 45°. At the equator the direction of winding changes, and in the other hemisphere the windings are in the contrary direction.

Let γ be the strength of the current in the wire, then within the shell

$$\Omega = -\frac{4\pi}{5}N\gamma\frac{r^2}{a^2}\left(\tfrac{3}{2}\cos^2\theta - \tfrac{1}{2}\right).$$

Let us now consider a conductor in the form of a plane closed curve placed anywhere within the shell with its plane perpendicular to the axis. To determine its coefficient of induction we have to find the surface-integral of $-\frac{d\Omega}{dz}$ over the plane bounded by the curve, putting $\gamma = 1$.

Now
$$\Omega = -\frac{4\pi}{5a^2}N\{z^2 - \tfrac{1}{2}(x^2+y^2)\},$$

and
$$-\frac{d\Omega}{dz} = \frac{8\pi}{5a^2}Nz.$$

Hence, if S is the area of the closed curve, its coefficient of induction is

$$M = \frac{8\pi}{5a^2}NSz.$$

If the current in this conductor is γ', there will be, by Art. 583, a force Z, urging it in the direction of z, where

$$Z = \gamma\gamma'\frac{dM}{dz} = \frac{8\pi}{5a^2}NS\gamma\gamma',$$

and, since this is independent of x, y, z, the force is the same in whatever part of the shell the circuit is placed.

The radius of the sphere is a , N is the whole number of windings, S is the area of the closed curve and γ' is the current. If $\gamma' = 0{,}5$ $N = 496$ $S = \pi * 5^2 = 25\pi$ $a = 8$, we have:

$$Z = \frac{8\pi}{5\cdot 64}\cdot 496\cdot 25\pi\cdot 0{,}5 = 1529{,}788$$

Now, calculate the following integral:

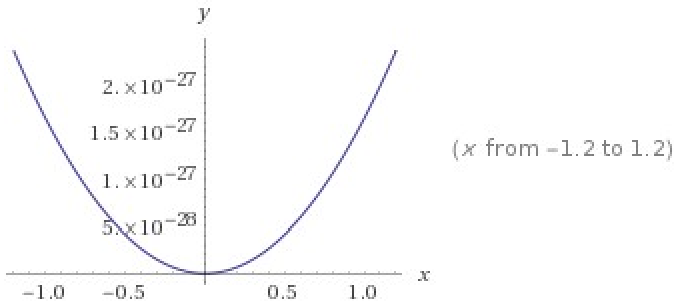
(1/10^29)*13/(21Pi) * 1.08643 * integrate integrate [1529.788]

$$\frac{1}{10^{29}} \times \frac{13}{21\pi} \times 1.08643 \int \left(\int 1529.788\, dx \right) dx$$

Result:

$1.63748 \times 10^{-27} x^2$

Plot:



result that is a good approximation to the value of the mass of the proton.

Now:

683.] If the current is a function of r, the distance from the axis of z, and if we write

$$x = r\cos\theta, \quad \text{and} \quad y = r\sin\theta, \tag{4}$$

and β for the magnetic force, in the direction in which θ is measured perpendicular to the plane through the axis of z, we have

$$4\pi w = \frac{d\beta}{dr} + \frac{1}{r}\beta = \frac{1}{r}\frac{d}{dr}(\beta r). \tag{5}$$

If C is the whole current flowing through a section bounded by a circle in the plane xy, whose centre is the origin and whose radius is r,

$$C = \int_0^r 2\pi r w\, dr = \tfrac{1}{2}\beta r. \tag{6}$$

If magnetic force B = 0,03T r = 5 , we have that C = 0,075. We calculate the following double integral:

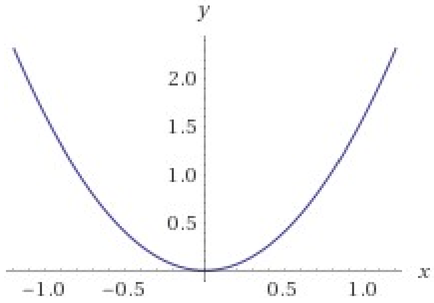
4Pi^2 * 1.08643 * integrate integrate [0.075]

$$4\pi^2 \times 1.08643 \int \left(\int 0.075\, dx \right) dx$$

Result:

$1.6084\, x^2$

Plot:



(x from -1.2 to 1.2)

For B = 3 and r = 5 , we have C = 7,5 and

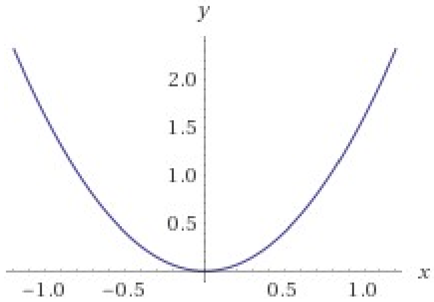
(1/10^2) * (4Pi^2) * 1.08643 * integrate integrate [7.5]

$$\frac{1}{10^2}(4\pi^2)\times 1.08643\int\left(\int 7.5\,dx\right)dx$$

Result:

$1.6084\,x^2$

Plot:



(x from -1.2 to 1.2)

result that is a good approximation to the value of the electric charge of the positron.

Now:

The vector-potential H, the density of the current w, and the electromotive intensity at any point, must be considered as functions of the time and of the distance from the axis of the wire.

The total current, C, through the section of the wire, and the total electromotive force, E, acting round the circuit, are to be regarded as the variables, the relation between which we have to find.

Let us assume as the value of H,

$$H = S + T_0 + T_1 r^2 + \&c. + T_n r^{2n} + \ldots, \tag{1}$$

where S, T_0, T_1, &c. are functions of the time.

Then, from the equation

$$\frac{d^2H}{dr^2} + \frac{1}{r}\frac{dH}{dr} = -4\pi w, \tag{2}$$

we find $$-\pi w = T_1 + \&c. + n^2 T_n r^{2n-2} + \ldots. \tag{3}$$

If ρ denotes the specific resistance of the substance per unit of volume, the electromotive intensity at any point is ρw, and this may be expressed in terms of the electric potential and the vector-potential H by equations (B), Art. 598,

$$\rho w = -\frac{d\Psi}{dz} - \frac{dH}{dt}, \tag{4}$$

or $$-\rho w = \frac{d\Psi}{dz} + \frac{dS}{dt} + \frac{dT_0}{dt} + \frac{dT_1}{dt} r^2 + \&c. + \frac{dT_n}{dt} r^{2n} + \ldots. \tag{5}$$

Comparing the coefficients of like powers of r in equations (3) and (5),

$$T_1 = \frac{\pi}{\rho}\left(\frac{d\Psi}{dz} + \frac{dS}{dt} + \frac{dT_0}{dt}\right), \tag{6}$$

$$T_2 = \frac{\pi}{\rho}\frac{1}{2^2}\frac{dT_1}{dt}, \tag{7}$$

$$T_n = \frac{\pi}{\rho}\frac{1}{n^2}\frac{dT_{n-1}}{dt}. \tag{8}$$

Hence we may write $$\frac{dS}{dt} = -\frac{d\Psi}{dz}, \tag{9}$$

$$T_0 = T, \qquad T_1 = \frac{\pi}{\rho}\frac{dT}{dt}, \ldots \qquad T_n = \frac{\pi^n}{\rho^n}\frac{1}{(n!)^2}\frac{d^nT}{dt^n}. \tag{10}$$

690.] To find the total current C, we must integrate w over the section of the wire whose radius is a,

$$C = 2\pi \int_0^a wr\,dr. \qquad (11)$$

Substituting the value of πw from equation (3), we obtain

$$C = -(T_1 a^2 + \&c. + nT_n a^{2n} + \ldots). \qquad (12)$$

The value of H at any point outside the wire depends only on the total current C, and not on the mode in which it is distributed within the wire. Hence we may assume that the value of H at the surface of the wire is AC, where A is a constant to be determined by calculation from the general form of the circuit. Putting $H = AC$ when $r = a$, we obtain

$$AC = S + T_0 + T_1 a^2 + \&c. + T_n a^{2n} + \ldots. \qquad (13)$$

If the density of the current is $2 * 10^6$ A/m^2 , the radius a = 3, we obtain:

2Pi * integrate [2*10^6] x, [0, 3]

$$2\pi \int_0^3 2\,000\,000\, x\, dx = 18\,000\,000\,\pi \approx 5.6549 \times 10^7$$

We calculate now the following double integral:

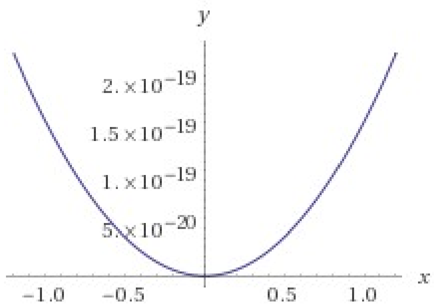
(1/10^32)(1/48(e^13*256)) * 1.08643 * (2Pi) * integrate integrate [2*10^6]

$$\frac{1}{10^{32}}\left(\frac{1}{48}\left(e^{13} \times 256\right)\right) \times 1.08643\,(2\pi) \int\left(\int 2 \times 10^6\, dx\right) dx$$

Result:

$1.61068 \times 10^{-19}\, x^2$

Plot:



(x from -1.2 to 1.2)

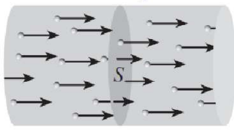
result that is a good approximation to the value of the electric charge of the positron.

We have take the following example:

$$J = \frac{I}{S} = \frac{200\ A}{10^{-4}\ m^2} = 2\times10^6\ A/m^2$$

3.1 Corrente elettrica e densità di corrente

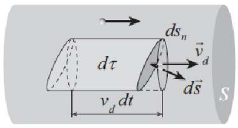
Consideriamo il moto non accelerato e con velocità piccole rispetto a quella della luce nel vuoto di un insieme di particelle dotate di carica elettrica. In tali condizioni possono ritenersi valide le leggi dell'elettrostatica. Supponiamo, per semplicità, che il moto avvenga attraverso un conduttore filiforme; esaminando una sezione S di tale conduttore, osserveremo che in un tempo dt una quantità di carica dq attraversa la sezione considerata. Si definisce pertanto l'*intensità di corrente I* come:



$$I = \frac{dq}{dt},$$

e si misura in *ampere* (A), dove $1A \equiv 1C/1s$. Se con un opportuno dispositivo si stabilisce ai capi del conduttore una differenza di potenziale costante nel tempo, a regime si osserva che il conduttore è sede di una corrente costante che prende il nome di *corrente stazionaria*.

Una descrizione del moto delle cariche attraverso l'uso della sola intensità di corrente risulta incompleta poiché tale grandezza non fornisce alcuna informazione riguardo la direzione ed il verso del flusso delle cariche. Allo scopo di completare questa descrizione consideriamo un conduttore di sezione S all'interno del quale il numero di portatori liberi di carica q per unità di volume sia n. Sia $\vec{v}_d$ la velocità media di tali cariche (*velocità di deriva*). Per stabilire la quantità di carica dq che durante l'intervallo di tempo dt attraversa una sezione $d\vec{s}$, consideriamo un volume $d\tau$ di base ds_n e altezza $v_d\,dt$, dove ds_n, pari a $ds\cos\vartheta$, è la proiezione della sezione $d\vec{s}$ perpendicolarmente alla direzione di $\vec{v}_d$ e ϑ è l'angolo tra $d\vec{s}$ e $\vec{v}_d$. La quantità di carica che attraversa la sezione $d\vec{s}$ nel tempo dt è pari alla carica contenuta tra un tempo t e il tempo $t+dt$ nel volume $d\tau$, cioè:



$$dq = nq\,d\tau = nq\,v_d\,dt\,ds_n = nq\,\vec{v}_d\cdot d\vec{s}\,dt\,. \tag{3.1}$$

Sia:

$$\vec{J} \equiv nq\vec{v}_d\,, \tag{3.2}$$

allora, dalla (3.1) segue:

$$dI = d\left(\frac{dq}{dt}\right) = \vec{J}\cdot d\vec{s}\,;$$

quindi, integrando sulla sezione S dell'intero conduttore, si ha:

$$I = \int_S \vec{J}\cdot d\vec{s}\,.$$

Pertanto, il flusso del vettore $\vec{J}$ attraverso la sezione S fornisce il valore dell'intensità della corrente attraverso la superficie considerata; il vettore $\vec{J}$ prende il nome di *densità di corrente*.

Nei metalli le cariche associate alla corrente sono gli elettroni, così la carica che compare nella (3.1) è pari a $-e$:

$$\vec{J} = -en\vec{v}_d \tag{3.3}$$

e in questo caso i vettori $\vec{J}$ e $\vec{v}_d$ sono antiparalleli. Ne segue che, qualora in un conduttore metallico la corrente scorre in una certa direzione, il moto dei corrispondenti elettroni si esplica nella direzione opposta.

Esempio: Consideriamo un conduttore di rame di sezione uniforme S pari a $1\,cm^2$, percorso da una corrente di $200\,A$; stabiliamo la velocità media degli elettroni nell'ipotesi che partecipino alla conduzione due elettroni per atomo di rame. Siccome una quantità di rame pari al suo peso atomico A_{Cu}, 63.5, espresso in grammi, contiene un numero di atomi pari al *numero di Avogadro* N_A, 6.022×10^{23}, il numero di atomi di rame per unità di volume è dato dalla relazione:

$$n_A \approx \frac{N_A}{A_{Cu}\times 1\,g}\rho = \frac{6.02\times10^{23}\ atomi}{63.5\ g} 8.94\times10^6\ g/m^3 = 8.47\times10^{28}\ atomi/m^3,$$

in cui ρ indica la densità del rame, così la concentrazione di elettroni di conduzione vale:

$$n = 2n_A = 1.69\times10^{29}\ elettroni/m^3.$$

La densità di corrente attraverso il conduttore è:

$$J = \frac{I}{S} = \frac{200\ A}{10^{-4}\ m^2} = 2\times10^6\ A/m^2,$$

così, dalla relazione (3.3) segue:

$$v_d = \frac{J}{ne} = \frac{2\times10^6\ A/m^2}{\left(1.69\times10^{29}\ elettroni/m^3\right)\times\left(1.60\times10^{-19}\ C\right)} = 7.36\times10^{-5}\ m/s,$$

On the Potential Energy of two Circular Currents.

696.] Let us begin by supposing the two magnetic shells which are equivalent to the currents to be portions of two concentric spheres, their radii being c_1 and c_2, of which c_1 is the greater (Fig. 47). Let us also suppose that the axes of the two shells coincide, and that a_1 is the angle subtended by the radius of the first shell, and a_2 the angle subtended by the radius of the second shell at the centre C.

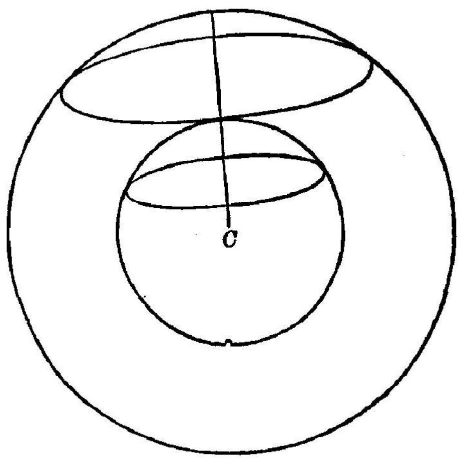


Fig. 47.

Let ω_1 be the potential due to the first shell at any point within it, then the work required to carry the second shell to an infinite distance is the value of the surface-integral

$$M = -\iint \frac{d\omega_1}{dr}\,dS$$

To find M by Elliptic Integrals.

701.] When the distance of the circumferences of the two circles is moderate as compared with the radius of the smaller, the series already given do not converge rapidly. In every case, however, we may find the value of M for two parallel circles by elliptic integrals.

For let b be the length of the line joining the centres of the circles, and let this line be perpendicular to the planes of the two circles, and let A and a be the radii of the circles, then

$$M = \iint \frac{\cos \epsilon}{r} ds ds',$$

the integration being extended round both curves.

In this case,

$$r^2 = A^2 + a^2 + b^2 - 2Aa \cos(\phi - \phi'),$$

$$\epsilon = \phi - \phi', \qquad ds = a d\phi, \qquad ds' = A d\phi',$$

$$M = \int_0^{2\pi} \int_0^{2\pi} \frac{Aa \cos(\phi - \phi') \, d\phi \, d\phi'}{\sqrt{A^2 + a^2 + b^2 - 2Aa \cos(\phi - \phi')}}$$

$$= -4\pi \sqrt{Aa} \left\{ \left(c - \frac{2}{c} \right) F + \frac{2}{c} E \right\},$$

where $$c = \frac{2\sqrt{Aa}}{\sqrt{(A+a)^2 + b^2}},$$

and F and E are complete elliptic integrals to modulus c.

Thence, we have for A = 8, a = 2 and b = 5 that c = 0,715541758... The value of the potential M is

-4π√Aa ((0,7155418 – 2,79508478)F+2,79508478E) =

-50,2654824(-2,07954298F+2,79508478E) = 104,529231F – 140,4962848E

Putting E and F equal to 1, we have the following result: M = – 35,967

Now, we calculate the following double integral:

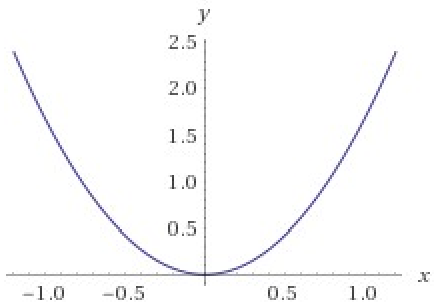
-((0.61803398*3)* (1/26 * 1.08643^Pi)) * (integrate integrate [-35.967])

$$-\left((0.61803398\times 3)\left(\frac{1}{26}\times 1.08643^{\pi}\right)\right)\int\left(\int -35.967\,dx\right)dx$$

Result:

$1.66394\,x^2$

Plot:



(x from −1.2 to 1.2)

result that is a good approximation to the value of the mass of the proton.

Coefficient of Induction of Two Parallel Circles when the Distance between the Arcs is small compared with the Radius of either Circle.

704.] We might deduce the value of M in this case from the expansion of the elliptic integrals already given when their modulus is nearly unity. The following method, however, is a more direct application of electrical principles.

First Approximation.

Let a and $a+c$ be the radii of the circles and b the distance between their planes, then the shortest distance between their circumferences is given by

$$r = \sqrt{c^2+b^2}.$$

We have to find the magnetic induction through the one circle due to a unit current in the other.

We shall begin by supposing the two circles to be in one plane. Consider a small element δs of the circle whose radius is $a+c$. At a point in the plane of the circle, distant ρ from the centre of δs, measured in a direction making an angle θ with the direction of δs, the magnetic force due to δs is perpendicular to th plane and equal to

$$\frac{1}{\rho^2}\sin\theta\,\delta s.$$

To calculate the surface integral of this force over the space which lies within the circle of radius a we must find the value of the integral

$$2\,\delta s\int_{\theta_1}^{\frac{1}{2}\pi}\int_{r_2}^{r_1}\frac{\sin\theta}{\rho}\,d\theta\,d\rho,$$

where r_1, r_2 are the roots of the equation

$$r^2 - 2(a+c)\sin\theta\, r + c^2 + 2ac = 0,$$

viz.
$$r_1 = (a+c)\sin\theta + \sqrt{(a+c)^2\sin^2\theta - c^2 - 2ac},$$
$$r_2 = (a+c)\sin\theta - \sqrt{(a+c)^2\sin^2\theta - c^2 - 2ac},$$

and
$$\sin^2\theta_1 = \frac{c^2+2ac}{(c+a)^2}.$$

When c is small compared to a we may put

$$r_1 = 2a\sin\theta,$$
$$r_2 = c/\sin\theta.$$

Integrating with regard to ρ we have

$$2\delta s\int_{\theta_1}^{\frac{1}{2}\pi}\log\left(\frac{2a}{c}\sin^2\theta\right).\sin\theta d\theta =$$

$$2\delta s\left[\cos\theta\left\{2-\log\left(\frac{2a}{c}\sin^2\theta\right)\right\}+2\log\tan\frac{\theta}{2}\right]_{\theta_1}^{\frac{\pi}{2}}$$

$$= 2\delta s\left(\log_e\frac{8a}{c}-2\right), \text{ nearly.}$$

We thus find for the whole induction

$$M_{ac} = 4\pi a\left(\log_e\frac{8a}{c}-2\right).$$

Since the magnetic force at any point, the distance of which from a curved wire is small compared with the radius of curvature, is nearly the same as if the wire had been straight, we can (Art. 684) calculate the difference between the induction through the circle whose radius is $a-c$ and the circle A by the formula

$$M_{aA} - M_{ac} = 4\pi a\,\{\log_e c - \log_e r\}.$$

Hence we find the value of the induction between A and a to be

$$M_{Aa} = 4\pi a\,(\log_e 8a - \log_e r - 2)$$

approximately, provided r the shortest distance between the circles is small compared with a.

From:

$$M_{ac} = 4\pi a\left(\log_e \frac{8a}{c} - 2\right).$$

For a = 5, c = 2, we have that the potential M_{ac} is 62,5637

Now, we calculate the following integral:

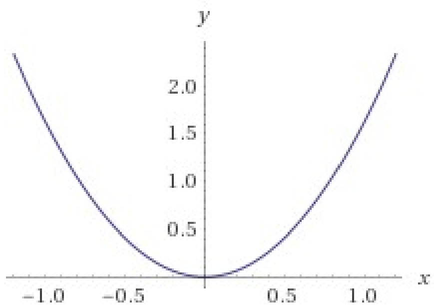
((0.61803398*2)* (1/26 * 1.08643)) * (integrate integrate [62.56370])

$$(0.61803398 \times 2)\left(\frac{1}{26} \times 1.08643\right)\int\left(\int 62.56370\, dx\right) dx$$

Result:

$1.61571\, x^2$

Plot:



(x from −1.2 to 1.2)

result that is a good approximation to the value of the mass of the proton.

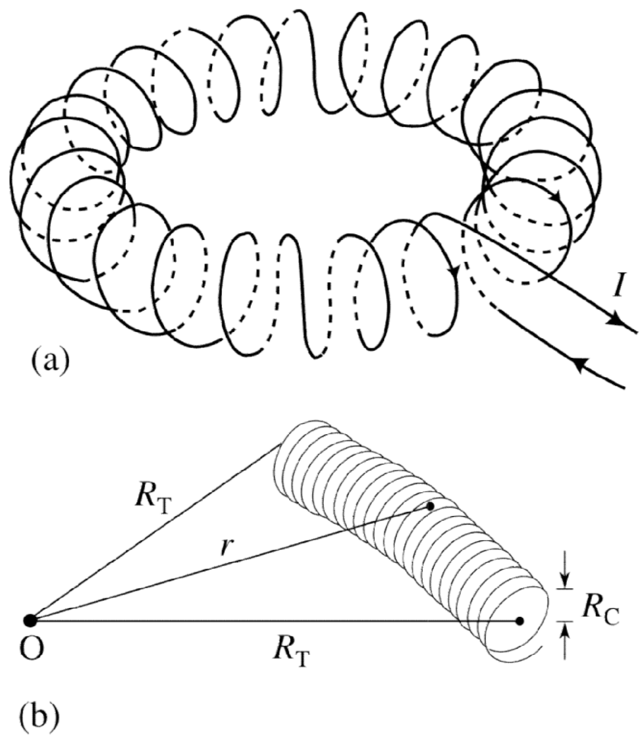


Figure 20 (a) A toroidal solenoid. (b) A section of a toroidal solenoid, having radius R_T which is much larger than the coil turns radius R_C.

To find the form of a coil for which the coefficient of self-induction is a maximum, the total length and thickness of the wire being given.

706.] Omitting the corrections of Art. 705, we find by Art. 693

$$L = 4\pi n^2 a \left(\log\frac{8a}{R} - 2\right),$$

where n is the number of windings of the wire, a is the mean radius of the coil, and R is the geometrical mean distance of the transverse section of the coil from itself. See Art. 691. If this section is always similar to itself, R is proportional to its linear dimensions, and n varies as R^2.

Since the total length of the wire is $2\pi an$, a varies inversely as n. Hence

$$\frac{dn}{n} = 2\frac{dR}{R}, \text{ and } \frac{da}{a} = -2\frac{dR}{R},$$

and we find the condition that L may be a maximum

$$\log\frac{8a}{R} = \tfrac{7}{2}.$$

If the transverse section of the channel of the coil is circular, of radius c, then, by Art. 692,

$$\log\frac{R}{c} = -\tfrac{1}{4},$$

$$\text{and} \quad \log\frac{8a}{c} = \tfrac{13}{4},$$

whence $$a = 3{\cdot}22\,c;$$

Thence, for the radius that is R = 2 and c = 8, a = 3,22c = 25,76 n = 233, we obtain:

$$L = 4\pi n^2 a \left(\log\frac{8a}{R} - 2\right)$$

L = 21946658,87

Now calculate the following double integral:

(1/10^34) * 10/(Pi*2.279585) * 1.08643 * integrate integrate [21946658.87]

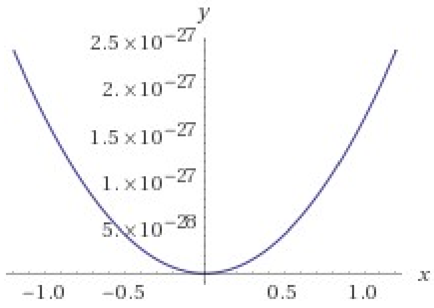
with the Bessel functions of the first kind 2,279585:

$$\frac{1}{10^{34}} \times \frac{10}{\pi \times 2.279585} \times 1.08643 \int \left(\int 2.194665887 \times 10^7 \, dx \right) dx$$

Result:

$1.66469 \times 10^{-27} x^2$

Plot:



(x from −1.2 to 1.2)

result that is a good approximation to the value of the mass of the proton.

and:

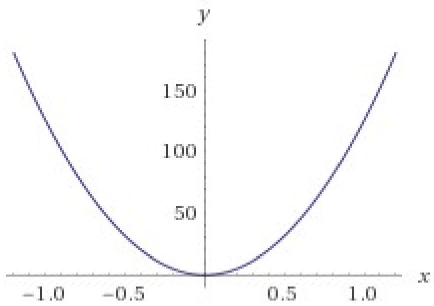
1/(10^5) * 1.08643 *((1.205932)^0.25)) * integrate integrate [2.194665887*10^7]

$$\frac{1}{10^5} \times 1.08643 \times 1.205932^{0.25} \int \left(\int 2.194665887 \times 10^7 \, dx \right) dx$$

Result:

$124.931 x^2$

Plot:



(x from −1.2 to 1.2)

result that is a good approximation to the value of the mass of the Higgs boson.

Where 1.205932 is the "Polar angle of circumscribed cone", acos(1/(Φ√3) that is

$\cos^{-1}(0.35682209956264315256606828789977)$

1.2059324882020269021467177266218...

(result in radians)

Now return to:

$$M = \int_0^{2\pi}\int_0^{2\pi} \frac{Aa\cos(\phi-\phi')\,d\phi\,d\phi'}{\sqrt{A^2+a^2+b^2-2Aa\cos(\phi-\phi')}}$$

$$= -4\pi\sqrt{Aa}\left\{\left(c-\frac{2}{c}\right)F+\frac{2}{c}E\right\},$$

where
$$c = \frac{2\sqrt{Aa}}{\sqrt{(A+a)^2+b^2}},$$

and F and E are complete elliptic integrals to modulus c.

From this, remembering that

$$\frac{dF}{dc} = \frac{1}{c(1-c^2)}\{E-(1-c^2)F\}, \qquad \frac{dE}{dc} = \frac{1}{c}(E-F),$$

and that c is a function of b, we find

$$\frac{dM}{db} = \frac{\pi}{\sqrt{Aa}}\frac{bc}{1-c^2}\{(2-c^2)E-2(1-c^2)F\}.$$

If r_1 and r_2 denote the greatest and least values of r,

$$r_1^2 = (A+a)^2+b^2; \qquad r_2^2 = (A-a)^2+b^2,$$

and if an angle γ be taken such that $\cos\gamma = \frac{r_2}{r_1}$,

$$\frac{dM}{db} = -\pi\frac{b\sin\gamma}{\sqrt{Aa}}\{2F_\gamma-(1+\sec^2\gamma)E_\gamma\},$$

where F_γ and E_γ denote the complete elliptic integrals of the first and second kind whose modulus is $\sin\gamma$.

If $A = a$, $\cot\gamma = \frac{b}{2a}$, and

$$\frac{dM}{db} = -2\pi\cos\gamma\,\{2F_\gamma-(1+\sec^2\gamma)E_\gamma\}.$$

The quantity $-\frac{dM}{db}$ represents the attraction between two parallel circular circuits, the current in each being unity.

On account of the importance of the quantity M in electromagnetic calculations the values of $\log(M/4\pi\sqrt{Aa})$, which is a function of c and therefore of γ only, have been tabulated for intervals of 6′ in the value of the angle γ between 60 and 90 degrees. The table will be found in an appendix to this chapter.

We analyze some values of the following Table, where M is the potential energy:

Table of the values of $\log \frac{M}{4\pi\sqrt{Aa}}$

63° 30′	$\bar{1}$·5963782
36′	$\bar{1}$·5991329
42′	$\bar{1}$·6018871
48′	$\bar{1}$·6046408
54′	$\bar{1}$·6073942

For the values 1,5963782 and 1,6018871 we calculate the following integrals:

integrate [1.5963782] x, [0, 4Pi]

$$\int_0^{4\pi} 1.5963782\, x\, dx = 126.045$$

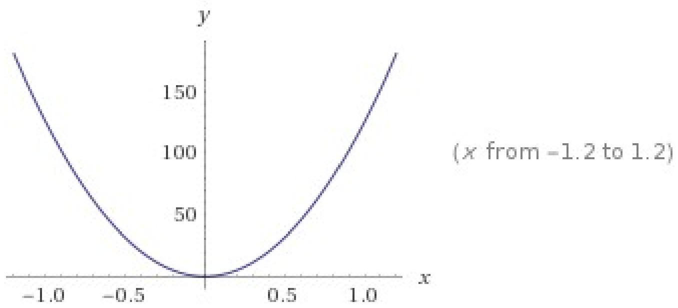
46Pi * 1.08643 * integrate integrate [1.5963782]

$$46\pi \times 1.08643 \int\left(\int 1.5963782\, dx\right) dx$$

Result:

$125.319\, x^2$

Plot:



integrate [1.6018871] x, [0, 4Pi]

$$\int_0^{4\pi} 1.6018871\, x\, dx = 126.48$$

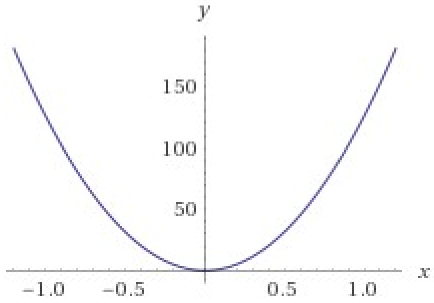
46Pi * 1.08643 * integrate integrate [1.6018871]

$$46\pi \times 1.08643 \int\left(\int 1.6018871\, dx\right) dx$$

Result:

$125.751\, x^2$

Plot:



(x from −1.2 to 1.2)

result that is a good approximation to the value of the mass of the Higgs boson.

66° 0′	$\bar{1}$·6651732
6′	$\bar{1}$·6679250
12′	$\bar{1}$·6706772
18′	$\bar{1}$·6734296
24′	$\bar{1}$·6761824
30′	$\bar{1}$·6789356
36′	$\bar{1}$·6816891
42′	$\bar{1}$·6844431
48′	$\bar{1}$·6871976
54′	$\bar{1}$·6899526

We calculate the following integral:

integrate [1.6734296] x, [e, 4Pi]

$$\int_e^{4\pi} 1.6734296\, x\, dx = 125.946$$

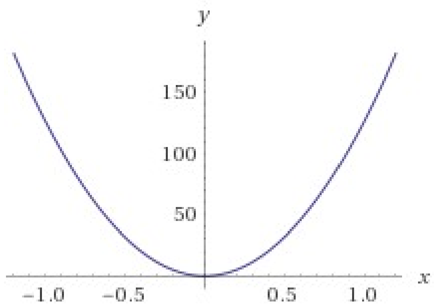
44Pi * 1.08643 * integrate integrate [1.6734296]

$$44\pi \times 1.08643 \int\left(\int 1.6734296\, dx\right) dx$$

Result:

$125.656\, x^2$

Plot:



(x from −1.2 to 1.2)

result that is a good approximation to the value of the mass of the Higgs boson.

789.] The only dielectric of which the capacity has been hitherto determined with sufficient accuracy is paraffin, for which in the solid form MM. Gibson and Barclay found *

$$K = 1{\cdot}975. \qquad (12)$$

792.] The electrostatic energy per unit of volume at any point of the wave in a non-conducting medium is

$$\tfrac{1}{2} f P = \frac{K}{8\pi} P^2 = \frac{K}{8\pi} \overline{\frac{dF}{dt}}\Big|^2. \qquad (22)$$

793.] Thus, if in strong sunlight the energy of the light which falls on one square foot is 83·4 foot pounds per second, the mean energy in one cubic foot of sunlight is about 0·0000000882 of a foot pound, and the mean pressure on a square foot is 0·0000000882 of a pound weight. A flat body exposed to sunlight would experience this pressure on its illuminated side only, and would therefore be repelled from the side on which the light falls. It is probable that a much greater energy of radiation might be obtained by means of the concentrated rays of the electric lamp. Such rays falling on a thin metallic disk, delicately suspended in a vacuum, might perhaps produce an observable mechanical effect. When a disturbance of any kind consists of terms involving sines or cosines of angles which vary with the time, the maximum energy is double of the mean energy. Hence, if P is the maximum electromotive intensity and β the maximum magnetic force which are called into play during the propagation of light,

$$\frac{K}{8\pi} P^2 = \frac{\mu}{8\pi} \beta^2 = \text{mean energy in unit of volume.} \qquad (24)$$

With Pouillet's data for the energy of sunlight, as quoted by Thomson, *Trans. R. S. E.*, 1854, this gives in electromagnetic measure

$P = 60000000$, or about 600 Daniell's cells per mètre ;*

$\beta = 0{\cdot}193$, or rather more than a tenth of the horizontal magnetic force in Britain †.

$$\frac{K}{8\pi} P^2$$

We have for K = 1,975 P = 60000000 the following mean energy in unit of volume:

$(1{,}975 * 60000000^2) / 8\pi = 282897911345844{,}02582716260490228$

We calculate the following double integral:

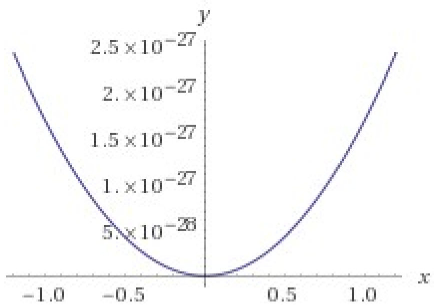
1/(10^41) * 1.08643^2 * integrate integrate [282897911345844.02582716260490228]

$$\frac{1}{10^{41}} \times 1.08643^2 \int\left(\int 2.8289791134584402582716260490228 \times 10^{14}\, dx\right) dx$$

Result:

$1.66956 \times 10^{-27}\, x^2$

Plot:



(x from -1.2 to 1.2)

result that is a good approximation to the value of the mass of the proton.

The natural logarithm is 33,276107211196598901862161176809

1.08643 * integrate [33.276107211196598901862161176809] x, [0, 60Pi/72]

$$1.08643 \times \int_0^{60 \times \frac{\pi}{72}} 33.276107211196598901862161176809\, x\, dx$$

Result:

123.892

1.08643 * integrate [33.276107211196598901862161176809] x, [0, 1.618^2]

$$1.08643 \int_0^{2.61792} 33.276107211196598901862161176809\, x\, dx = 123.885$$

result that is a good approximation to the value of the mass of the Higgs boson.

Now:

849.] Whatever hypothesis we adopt, there can be no doubt that the total transfer of electricity, reckoned algebraically, along the first circuit, is represented by

$$ve + v_1 e_1 = c i ds,$$

where c is the number of units of statical electricity which are transmitted by the unit electric current in the unit of time; so that we may write equation (9)

$$\Sigma(vv'ee') = c^2 ii' ds ds'. \qquad (11)$$

Hence, if we assume for the repulsion of the two particles either of the modified expressions

$$\frac{ee'}{r^2}\left[1 + \frac{1}{c^2}\left(u^2 - \tfrac{3}{2}\left(\frac{\partial r}{\partial t}\right)^2\right)\right], \qquad (18)$$

or

$$\frac{ee'}{r^2}\left[1 + \frac{1}{c^2}\left(r\frac{\partial^2 r}{\partial t^2} - \tfrac{1}{2}\left(\frac{\partial r}{\partial t}\right)^2\right)\right],^* \qquad (19)$$

we may deduce from them both the ordinary electrostatic forces, and the forces acting between currents as determined by Ampère.

853.] The formula of Gauss is inconsistent with this principle, and must therefore be abandoned, as it leads to the conclusion that energy might be indefinitely generated in a finite system by physical means. This objection does not apply to the formula of Weber, for he has shewn * that if we assume as the potential energy of a system consisting of two electric particles,

$$\psi = \frac{ee'}{r}\left[1 - \frac{1}{2c^2}\left(\frac{\partial r}{\partial t}\right)^2\right], \qquad (20)$$

the repulsion between them, which is found by differentiating this quantity with respect to r, and changing the sign, is that given by the formula (19).

Helmholtz † has therefore stated a case in which the distances are not too small, nor the velocities too great, for experimental verification. A fixed non-conducting spherical surface, of radius a, is uniformly charged with electricity to the surface-density σ. A particle, of mass m and carrying a charge e of electricity, moves within the sphere with velocity v. The electrodynamic potential calculated from the formula (20) is

$$4\pi a\sigma e\left(1-\frac{v^2}{6c^2}\right), \tag{21}$$

Now, if $a = 0.08$m; $\sigma = 1 * 10^{-4}$ C/m^3 $m = 9.109 * 10^{-31}$ kg, $e = -1{,}602176 * 10^{-19}$ C; $v = 2100000$ m/s; $c = 0.017 * 10^{-3}$J

We have:

$(-1{,}6106829924345217829206797200798 * 10^{-19}) *$

$(1 – 2.543.252.595.155.709.342.560{,}553633218) =$

$= (-1{,}6106829924345217829206797200798 * 10^{-19}) *$

$(-2.543.252.595.155.709.342.559{,}553633218) =$

$= 409{,}63737004822612818208792625391$ V^2/s^2

that is the electrodynamic potential.

We have that: $(409{,}63737004822612818208792625391 \text{ V}^2/\text{s}^2)^{1/2} =$

$= 20{,}239500242057018937979878680609$ V/s

Now, we calculate the following double integral:

1/(sqrt(Pi)) * 1.08643 * integrate integrate [409.63737004822612818208792625391]

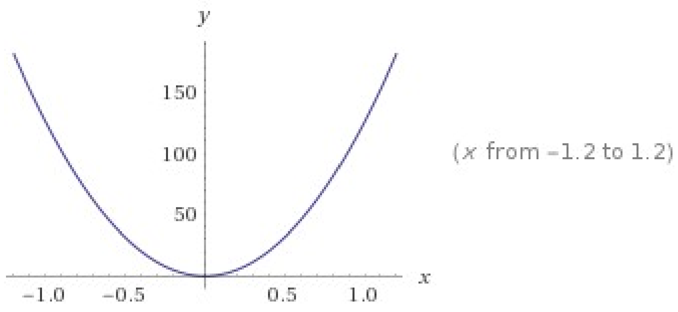
$$\frac{1}{\sqrt{\pi}} \times 1.08643 \int\left(\int 409.63737004822612818208792625391\, dx\right) dx$$

Result:

$125.544\, x^2$

Plot:

result that is a good approximation to the value of the mass of the Higgs boson.



We have that ln (409,637370048226) = 6,0152723050328974854292293082325

Now, we calculate the following integral:

1.08643 * integrate [6.0152723050328974854292293082325] x,[0, 11/(5Pi)]

$$1.08643 \int_0^{\frac{11}{5\pi}} 6.0152723050328974854292293082325\, x\, dx$$

Result:

1.60241

Or:

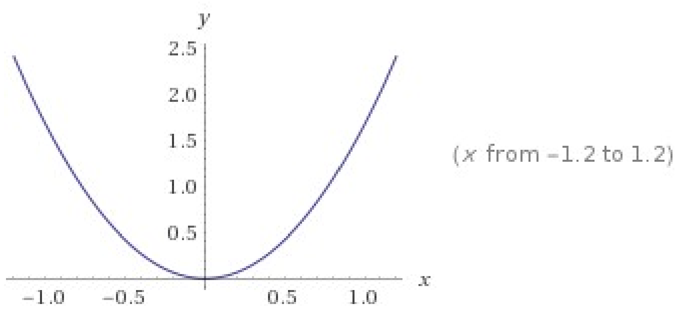
11/(2.5176*e*Pi) * 1.08643 * integrate integrate [6.0152723050328974854292293082325]

$$\frac{11}{2.5176\, e\, \pi} \times 1.08643 \int\left(\int 6.0152723050328974854292293082325\, dx\right) dx$$

Result:

$1.67182\, x^2$

Plot:



result that is a good approximation to the value of the electric charge of the positron and of the mass of the proton.

For the electric potential

$V = Q / (4\pi\varepsilon r)$ where

$\varepsilon_0 = 8,854\,187\,82 \cdot 10^{-12}\ \mathrm{C^2 m^{-2} N^{-1}}$

and the radius of the electron:

$$r_e = \frac{1}{4\pi\varepsilon_0}\frac{e^2}{m_e c^2} = 2,8179403267(27) \times 10^{-15}\mathrm{m}$$

We have:

$V = (-1.602176 * 10^{-19}) / (4\pi * 8,85418782 * 10^{-12} * 2,8179403267 * 10^{-15}) =$

$= (-1.602176 * 10^{-19}) / (3,1353783306364123139027145 1864 * 10^{-25}) =$

$= -510999,2578390990473918498838 2171$ V

Now:

Helmholtz † has therefore stated a case in which the distances are not too small, nor the velocities too great, for experimental verification. A fixed non-conducting spherical surface, of radius a, is uniformly charged with electricity to the surface-density σ. A particle, of mass m and carrying a charge e of electricity, moves within the sphere with velocity v. The electrodynamic potential calculated from the formula (20) is

$$4\pi a\sigma e\left(1 - \frac{v^2}{6c^2}\right), \qquad (21)$$

and is independent of the position of the particle within the sphere. Adding to this V, the remainder of the potential energy arising from the action of other forces, and $\frac{1}{2}mv^2$, the kinetic energy of the particle, we find as the equation of energy

$$\frac{1}{2}\left(m - \frac{4}{3}\frac{\pi a\sigma e}{c^2}\right)v^2 + 4\pi a\sigma e + V = \text{const.} \qquad (22)$$

From eq. (22), we have, for a mass $9,10938356 \times 10^{-31}$ kg and $a = 0.08$m; $\sigma = 1 * 10^{-4}$ C/m^3, $e = -1,602176 * 10^{-19}$ C; $v = 2100000$ m/s; $c = 0.017 * 10^{-3}$J :

$0.5\ (9,10938356 \times 10^{-31} - (1,3333 * \pi\ 0.08 * 1 * 10^{-4} * -1.602176 * 10^{-19}) /$

$(0.017 * 10^{-3})^2) * (2100000)^2 + 4\pi\ 0.08 * 1 * 10^{-4} * -1.602176 * 10^{-19} -$

$510999,257839$;

- [0.5 (9,10938356 × 10^{-31} – (-5,3689433081 * 10^{-24}) / (0,000000000289)) * (4410000000000) + (-1,610682992434521 * 10^{-23}) -510999,257839] =

= 510999,21687526 that is a const. and is defined as the equation of energy.

Now, we calculate the following double integral:

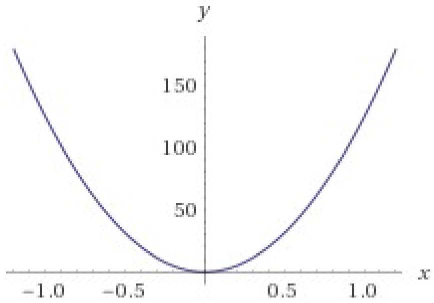
1/(10^3) * 1/(sqrt(5)) * 1.08643 * integrate integrate [510999.21687526]

$$\frac{1}{10^3} \times \frac{1}{\sqrt{5}} \times 1.08643 \int \left(\int 510\,999.21687526 \, dx \right) dx$$

Result:

$124.139 \, x^2$

Plot:



(x from −1.2 to 1.2)

result that is a good approximation to the value of the mass of the Higgs boson.

From: "SQUARE SERIES GENERATING FUNCTION TRANSFORMATIONS" MAXIE D. SCHMIDT - https://arxiv.org/abs/1609.02803v2

Corollary 4.7 (Special Values of Ramanujan's φ-Function). *For any $k \in \mathbb{R}^+$, the variant of the Ramanujan φ-function, $\varphi\left(e^{-k\pi}\right) \equiv \vartheta_3\left(e^{-k\pi}\right)$, has the integral representation*

$$\varphi\left(e^{-k\pi}\right) = 1 + \int_0^\infty \frac{e^{-t^2/2}}{\sqrt{2\pi}} \left[\frac{4e^{k\pi}\left(e^{2k\pi} - \cos\left(\sqrt{2\pi k}t\right)\right)}{e^{4k\pi} - 2e^{2k\pi}\cos\left(\sqrt{2\pi k}t\right) + 1}\right] dt. \tag{33}$$

Moreover, the special values of this function corresponding to the particular cases of $k \in \{1, 2, 3, 5\}$ in (33) have the respective integral representations

$$\frac{\pi^{1/4}}{\Gamma\left(\frac{3}{4}\right)} = 1 + \int_0^\infty \frac{e^{-t^2/2}}{\sqrt{2\pi}} \left[\frac{4e^{\pi}\left(e^{2\pi} - \cos\left(\sqrt{2\pi}t\right)\right)}{e^{4\pi} - 2e^{2\pi}\cos\left(\sqrt{2\pi}t\right) + 1}\right] dt \tag{34}$$

$$\frac{\pi^{1/4}}{\Gamma\left(\frac{3}{4}\right)} \cdot \frac{\sqrt{\sqrt{2}+2}}{2} = 1 + \int_0^\infty \frac{e^{-t^2/2}}{\sqrt{2\pi}} \left[\frac{4e^{2\pi}\left(e^{4\pi} - \cos\left(2\sqrt{\pi}t\right)\right)}{e^{8\pi} - 2e^{4\pi}\cos\left(2\sqrt{\pi}t\right) + 1}\right] dt$$

$$\frac{\pi^{1/4}}{\Gamma\left(\frac{3}{4}\right)} \cdot \frac{\sqrt{\sqrt{3}+1}}{2^{1/4}3^{3/8}} = 1 + \int_0^\infty \frac{e^{-t^2/2}}{\sqrt{2\pi}} \left[\frac{4e^{3\pi}\left(e^{6\pi} - \cos\left(\sqrt{6\pi}t\right)\right)}{e^{12\pi} - 2e^{6\pi}\cos\left(\sqrt{6\pi}t\right) + 1}\right] dt$$

$$\frac{\pi^{1/4}}{\Gamma\left(\frac{3}{4}\right)} \cdot \frac{\sqrt{5+2\sqrt{5}}}{5^{3/4}} = 1 + \int_0^\infty \frac{e^{-t^2/2}}{\sqrt{2\pi}} \left[\frac{4e^{5\pi}\left(e^{10\pi} - \cos\left(\sqrt{10\pi}t\right)\right)}{e^{20\pi} - 2e^{10\pi}\cos\left(\sqrt{10\pi}t\right) + 1}\right] dt.$$

We know that, from the first of (34):

$$\frac{\pi^{1/4}}{\Gamma\left(\frac{3}{4}\right)} = 1 + \int_0^\infty \frac{e^{-t^2/2}}{\sqrt{2\pi}} \left[\frac{4e^{\pi}\left(e^{2\pi} - \cos\left(\sqrt{2\pi}t\right)\right)}{e^{4\pi} - 2e^{2\pi}\cos\left(\sqrt{2\pi}t\right) + 1}\right] dt$$

we have:

$$\Gamma\left(\frac{3}{4}\right) = \frac{\pi\sqrt{2}}{\Gamma\left(\frac{1}{4}\right)} = \frac{4{,}44288293815}{3{,}625609908} = 1{,}2254167025$$

$$\frac{\pi^{1/4}}{\Gamma\left(\frac{3}{4}\right)} = \frac{1{,}3313353638}{1{,}2254167025} = 1{,}08643481\ldots$$

With regard the integral, from 0 to 0,58438 for t = 2, where (2.71828^2)/(sqrt6.283185307) = 2,94780 for t=2, we have:

integrate (2.94780)[4e^3.14159265 * (e^6.283185307 - cos((sqrt6.283185307)2))]/[e^12.56637 - 2e^6.283185307 (cos(sqrt6.283185307)2))+1] x, [0,0.58438]

$$\int_0^{0.58438} \frac{2.94780\left(4\, e^{3.14159265}\left(e^{6.283185307} - \cos\left(\sqrt{6.283185307}\ 2\right)\right)\right) x}{e^{12.56637} - \left(2\, e^{6.283185307}\right)\left(\cos\left(\sqrt{6.283185307}\right) 2\right) + 1}\, dx =$$

0.0864364

Thence, 1 + 0,0864364 = 1,0864364; 1,08643481 ≅ 1,0864364.

In this paper, we have used 1,08643 as a new "Ramanujan's constant" and we can see as this constant is fundamental for the results that we have obtained in all the various equations that we have analyzed and developed.

In conclusion, we observe the new mathematical connection that we have obtained between the fundamental Maxwell's equation and the particles, i.e. fermions as electrons, positrons and protons (that are open strings), by the use of this "Ramanujan's constant".

Indeed, from eqs. (130-135) of paper **"On Physical Lines of Force"**: (from the Philosophical Magazine, Vol. xxi.] - XXIII. J.C.Maxwell), we have:

1/(10^36) * (sqrt(6)/4)^8 * 1.08643 * integrate integrate [155370000140.18299865]

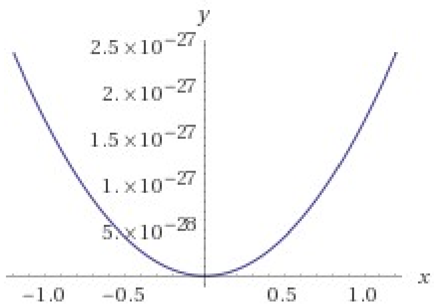
dove "(√6)/4 = Circumradius sphere, congruent with vertices (Tetrahedron)"

$$\frac{1}{10^{36}}\left(\frac{\sqrt{6}}{4}\right)^8 \times 1.08643 \int\left(\int 1.5537000014018299865\times 10^{11}\, dx\right) dx$$

Result:

$1.66903\times 10^{-27}\, x^2$

Plot:



(x from -1.2 to 1.2)

result that is a good approximation to the value of the mass of the proton.

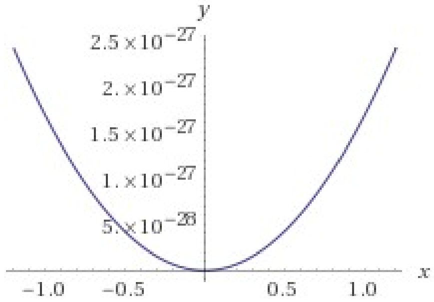
1/2 * 1/(10^36) * (sqrt(6)/4)^8 * 1.08643 * integrate integrate [310740000280.3659973]

$$\frac{1}{2} \times \frac{1}{10^{36}} \left(\frac{\sqrt{6}}{4}\right)^8 \times 1.08643 \int \left(\int 3.107400002803659973 \times 10^{11} \, dx \right) dx$$

Result:

$$1.66903 \times 10^{-27} x^2$$

Plot:



(x from -1.2 to 1.2)

result that is a good approximation to the value of the mass of the proton.

Furthermore, from:

Electrical Displacements (f, g, h).

(55) Electrical displacement consists in the opposite electrification of the sides of a molecule or particle of a body which may or may not be accompanied with transmission through the body. Let the quantity of electricity which would appear on the faces $dy.dz$ of an element dx, dy, dz cut from the body be $f.dy.dz$, then f is the component of electric displacement parallel to x. We shall use f, g, h to denote the electric displacements parallel to x, y, z respectively.

The variations of the electrical displacement must be added to the currents p, q, r to get the total motion of electricity, which we may call p', q', r', so that

$$\left.\begin{aligned} p' &= p + \frac{df}{dt}, \\ q' &= q + \frac{dg}{dt}, \\ r' &= r + \frac{dh}{dt}, \end{aligned}\right\} \quad \ldots \ldots \ldots \ldots \text{(A)}$$

Coefficient of Magnetic Induction (μ).

(60) Let μ be the ratio of the magnetic induction in a given medium to that in air under an equal magnetizing force, then the number of lines of force in unit of area perpendicular to x will be $\mu\alpha$ (μ is a quantity depending on the nature of the medium, its temperature, the amount of magnetization already produced, and in crystalline bodies varying with the direction).

(61) Expressing the electric momentum of small circuits perpendicular to the three axes in this notation, we obtain the following

Equations of Magnetic Force.

$$\left.\begin{aligned} \mu\alpha &= \frac{dH}{dy}-\frac{dG}{dz}, \\ \mu\beta &= \frac{dF}{dz}-\frac{dH}{dx}, \\ \mu\gamma &= \frac{dG}{dx}-\frac{dF}{dy}. \end{aligned}\right\} \quad \ldots\ldots\ldots \quad \text{(B)}$$

Equations of Currents.

(62) It is known from experiment that the motion of a magnetic pole in the electromagnetic field in a closed circuit cannot generate work unless the circuit which the pole describes passes round an electric current. Hence, except in the space occupied by the electric currents,

$$\alpha dx+\beta dy+\gamma dz=d\phi \quad \ldots\ldots\ldots \quad (31)$$

a complete differential of ϕ, the magnetic potential.

The quantity ϕ may be susceptible of an indefinite number of distinct values, according to the number of times that the exploring point passes round electric currents in its course, the difference between successive values of ϕ corresponding to a passage completely round a current of strength c being $4\pi c$.

Hence if there is no electric current,

$$\frac{}{dy}-\frac{d\beta}{dz}=0;$$

but if there is a current p',

$$\frac{d\gamma}{dy}-\frac{d\beta}{dz}=4\pi p'.$$

Similarly,

$$\left.\begin{aligned} \frac{d\gamma}{dy}-\frac{d\beta}{dz}&=4\pi p'. \\ \frac{d\alpha}{dz}-\frac{d\gamma}{dx}&=4\pi q', \\ \frac{d\beta}{dx}-\frac{d\alpha}{dy}&=4\pi r'. \end{aligned}\right\} \quad \ldots\ldots\ldots \quad \text{(C)}$$

We may call these the Equations of Currents.

(65) The complete equations of electromotive force on a moving conductor may now be written as follows:—

Equations of Electromotive Force.

$$\left.\begin{aligned} P&=\mu\left(\gamma\frac{dy}{dt}-\beta\frac{dz}{dt}\right)-\frac{dF}{dt}-\frac{d\Psi}{dx}, \\ Q&=\mu\left(\alpha\frac{dz}{dt}-\gamma\frac{dx}{dt}\right)-\frac{dG}{dt}-\frac{d\Psi}{dy}, \\ R&=\mu\left(\beta\frac{dx}{dt}-\alpha\frac{dy}{dt}\right)-\frac{dH}{dt}-\frac{d\Psi}{dz}. \end{aligned}\right\} \quad \text{(D)}$$

The first term on the right-hand side of each equation represents the electromotive force arising from the motion of the conductor itself. This electromotive force is perpendicular to the direction of motion and to the lines of magnetic force; and if a parallelogram be drawn whose sides represent in direction and magnitude the velocity of the conductor and the magnetic induction at that point of the field, then the area of the parallelogram will represent the electromotive force due to the motion of the conductor, and the direction of the force is perpendicular to the plane of the parallelogram.

The second term in each equation indicates the effect of changes in the position or strength of magnets or currents in the field.

The third term shows the effect of the electric potential Ψ. It has no effect in causing a circulating current in a closed circuit. It indicates the existence of a force urging the electricity to or from certain definite points in the field.

Equations of Electric Elasticity,

$$\left.\begin{aligned} P&=kf, \\ Q&=kg, \\ R&=kh. \end{aligned}\right\} \quad \text{(E)}$$

(68) Let e represent the quantity of free positive electricity contained in unit of volume at any part of the field, then, since this arises from the electrification of the different parts of the field not neutralizing each other, we may write the

Equation of Free Electricity,

$$e+\frac{df}{dx}+\frac{dg}{dy}+\frac{dh}{dz}=0. \quad \text{(G)}$$

(79) If there is no motion or change of strength of currents or magnets in the field, the electromotive force is entirely due to variation of electric potential, and we shall have (§ 65)

$$P=-\frac{d\Psi}{dx}, \quad Q=-\frac{d\Psi}{dy}, \quad R=-\frac{d\Psi}{dz}.$$

Integrating by parts the expression (I) for the energy due to electric displacement, and remembering that P, Q, R vanish at an infinite distance, it becomes

$$\tfrac{1}{2}\Sigma\left\{\Psi\left(\frac{df}{dx}+\frac{dg}{dy}+\frac{dh}{dz}\right)\right\}dV,$$

or by the equation of Free Electricity (G), p. 485,

$$-\tfrac{1}{2}\Sigma(\Psi e)dV.$$

By the same demonstration as was used in the case of the mechanical action on a magnet, it may be shown that the mechanical force on a small body containing a quantity e_2 of free electricity placed in a field whose potential arising from other electrified bodies is Ψ_1, has for components

$$\left.\begin{aligned} X&=e_2\frac{d\Psi_1}{dx}=-P_1e_2, \\ Y&=e_2\frac{d\Psi_1}{dy}=-Q_1e_2, \\ Z&=e_3\frac{d\Psi_1}{dz}=-R_1e_2. \end{aligned}\right\} \quad \text{(D)}$$

So that an electrified body is urged in the direction of the electromotive force with a force equal to the product of the quantity of free electricity and the electromotive force.

If the electrification of the field arises from the presence of a small electrified body containing e_1 of free electrity, the only solution of Ψ_1 is

$$\Psi_1=\frac{k}{4\pi}\frac{e_1}{r}, \quad \text{(43)}$$

where r is the distance from the electrified body.

The repulsion between two electrified bodies e_1, e_2 is therefore

$$e_2\frac{d\Psi_1}{dr}=\frac{k}{4\pi}\frac{e_1e_2}{r^2}. \quad \text{(44)}$$

(80) The quantities with which we have had to do have been hitherto expressed in terms of the Electromagnetic System of measurement, which is founded on the mechanical action between currents. The electrostatic system of measurement is founded on the mechanical action between electrified bodies, and is independent of, and incompatible with, the electromagnetic system; so that the units of the different kinds of quantity have different values according to the system we adopt, and to pass from the one system to the other, a reduction of all the quantities is required.

According to the electrostatic system, the repulsion between two small bodies charged with quantities η_1, η_2 of electricity is

$$\frac{\eta_1\eta_2}{r^2},$$

where r is the distance between them.

Let the relation of the two systems be such that one electromagnetic unit of electricity contains v electrostatic units; then $\eta_1=ve_1$ and $\eta_2=ve_2$, and this repulsion becomes

$$v^2\frac{e_1e_2}{r^2}=\frac{k}{4\pi}\frac{e_1e_2}{r^2} \text{ by equation (44)}, \quad \dots \quad (45)$$

whence k, the coefficient of "electric elasticity" in the medium in which the experiments are made, *i. e.* common air, is related to v, the number of electrostatic units in one electromagnetic unit, by the equation

$$k=4\pi v^2. \quad \dots \quad (46)$$

The quantity v may be determined by experiment in several ways. According to the experiments of MM. WEBER and KOHLRAUSCH,

$$v=310{,}740{,}000 \text{ metres per second.}$$

(81) It appears from this investigation, that if we assume that the medium which constitutes the electromagnetic field is, when dielectric, capable of receiving in every part of it an electric polarization, in which the opposite sides of every element into which we may conceive the medium divided are oppositely electrified, and if we also assume that this polarization or electric displacement is proportional to the electromotive force which produces or maintains it, then we can show that electrified bodies in a dielectric medium will act on one another with forces obeying the same laws as are established by experiment.

The energy, by the expenditure of which electrical attractions and repulsions are produced, we suppose to be stored up in the dielectric medium which surrounds the electrified bodies, and not on the surface of those bodies themselves, which on our theory are merely the bounding surfaces of the air or other dielectric in which the true springs of action are to be sought.

(91) At the commencement of this paper we made use of the optical hypothesis of an elastic medium through which the vibrations of light are propagated, in order to show that we have warrantable grounds for seeking, in the same medium, the cause of other phenomena as well as those of light. We then examined electromagnetic phenomena, seeking for their explanation in the properties of the field which surrounds the electrified or magnetic bodies. In this way we arrived at certain equations expressing certain properties of the electromagnetic field. We now proceed to investigate whether these properties of that which constitutes the electromagnetic field, deduced from electromagnetic phenomena alone, are sufficient to explain the propagation of light through the same substance.

(92) Let us suppose that a plane wave whose direction cosines are l, m, n is propagated through the field with a velocity V. Then all the electromagnetic functions will be functions of

$$w=lx+my+nz-\mathrm{V}t.$$

The equations of Magnetic Force (B), p. 482, will become

$$\mu\alpha=m\frac{d\mathrm{H}}{dw}-n\frac{d\mathrm{G}}{dw},$$

$$\mu\beta=n\frac{d\mathrm{F}}{dw}-l\frac{d\mathrm{H}}{dw},$$

$$\mu\gamma=l\frac{d\mathrm{G}}{dw}-m\frac{d\mathrm{F}}{dw}.$$

If we multiply these equations respectively by l, m, n, and add, we find

$$l\mu\alpha+m\mu\beta+n\mu\gamma=0, \quad \ldots \quad (62)$$

which shows that the direction of the magnetization must be in the plane of the wave.

(93) If we combine the equations of Magnetic Force (B) with those of Electric Currents (C), and put for brevity

$$\frac{d\mathrm{F}}{dx}+\frac{d\mathrm{G}}{dy}+\frac{d\mathrm{H}}{dz}=\mathrm{J}, \text{ and } \frac{d^2}{dx^2}+\frac{d^2}{dy^2}+\frac{d^2}{dz^2}=\nabla^2, \quad \ldots \quad (63)$$

$$\left.\begin{aligned} 4\pi\mu p' &= \frac{d\mathrm{J}}{dx}-\nabla^2\mathrm{F},\\ 4\pi\mu q' &= \frac{d\mathrm{J}}{dy}-\nabla^2\mathrm{G},\\ 4\pi\mu r' &= \frac{d\mathrm{J}}{dz}-\nabla^2\mathrm{H}. \end{aligned}\right\} \quad \ldots \quad (64)$$

If the medium in the field is a perfect dielectric there is no true conduction, and the currents p', q', r' are only variations in the electric displacement, or, by the equations of Total Currents (A),

$$p'=\frac{df}{dt}, \qquad q'=\frac{dg}{dt}, \qquad r'=\frac{dh}{dt}. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (65)$$

But these electric displacements are caused by electromotive forces, and by the equations of Electric Elasticity (E),

$$P=kf, \qquad Q=kg, \qquad R=kh. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (66)$$

These electromotive forces are due to the variations either of the electromagnetic or the electrostatic functions, as there is no motion of conductors in the field; so that the equations of electromotive force (D) are

$$\left.\begin{aligned} P&=-\frac{dF}{dt}-\frac{d\Psi}{dx}, \\ Q&=-\frac{dG}{dt}-\frac{d\Psi}{dy}, \\ R&=-\frac{dH}{dt}-\frac{d\Psi}{dz}. \end{aligned}\right\} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (67)$$

(94) Combining these equations, we obtain the following:—

$$\left.\begin{aligned} k\left(\frac{dJ}{dx}-\nabla^2F\right)+4\pi\mu\left(\frac{d^2F}{dt^2}+\frac{d^2\Psi}{dxdt}\right)=0, \\ k\left(\frac{dJ}{dy}-\nabla^2G\right)+4\pi\mu\left(\frac{d^2G}{dt^2}+\frac{d^2\Psi}{dydt}\right)=0, \\ k\left(\frac{dJ}{dz}-\nabla^2H\right)+4\pi\mu\left(\frac{d^2H}{dt^2}+\frac{d^2\Psi}{dzdt}\right)=0. \end{aligned}\right\} \quad . \quad . \quad . \quad . \quad . \quad . \quad (68)$$

If we differentiate the third of these equations with respect to y, and the second with respect to z, and subtract, J and Ψ disappear, and by remembering the equations (B) of magnetic force, the results may be written

$$\left.\begin{aligned} k\nabla^2\mu\alpha&=4\pi\mu\frac{d^2}{dt^2}\mu\alpha, \\ k\nabla^2\mu\beta&=4\pi\mu\frac{d^2}{dt^2}\mu\beta, \\ k\nabla^2\mu\gamma&=4\pi\mu\frac{d^2}{d^2t}\mu\gamma. \end{aligned}\right\} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (69)$$

(95) If we assume that α, β, γ are functions of $lx+my+nz-Vt=w$, the first equation becomes

$$k\mu\frac{d^2\alpha}{dw^2}=4\pi\mu^2V^2\frac{d^2\alpha}{dw^2}, \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (70)$$

or

$$V=\pm\sqrt{\frac{k}{4\pi\mu}}. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (71)$$

The other equations give the same value for V, so that the wave is propagated in either direction with a velocity V.

This wave consists entirely of magnetic disturbances, the direction of magnetization being in the plane of the wave. No magnetic disturbance whose direction of magnetization is not in the plane of the wave can be propagated as a plane wave at all.

Hence magnetic disturbances propagated through the electromagnetic field agree with light in this, that the disturbance at any point is transverse to the direction of propagation, and such waves may have all the properties of polarized light.

(96) The only medium in which experiments have been made to determine the value of k is air, in which $\mu=1$, and therefore, by equation (46),

$$V=v. \qquad (72)$$

By the electromagnetic experiments of MM. WEBER and KOHLRAUSCH *,

$$v=310{,}740{,}000 \text{ metres per second}$$

is the number of electrostatic units in one electromagnetic unit of electricity, and this, according to our result, should be equal to the velocity of light in air or vacuum.

The velocity of light in air, by M. FIZEAU'S † experiments, is

$$V=314{,}858{,}000;$$

according to the more accurate experiments of M. FOUCAULT ‡,

$$V=298{,}000{,}000.$$

The velocity of light in the space surrounding the earth, deduced from the coefficient of aberration and the received value of the radius of the earth's orbit, is

$$V=308{,}000{,}000.$$

(97) Hence the velocity of light deduced from experiment agrees sufficiently well with the value of v deduced from the only set of experiments we as yet possess. The value of v was determined by measuring the electromotive force with which a condenser of known capacity was charged, and then discharging the condenser through a galvanometer, so as to measure the quantity of electricity in it in electromagnetic measure. The only use made of light in the experiment was to see the instruments. The value of V found by M. FOUCAULT was obtained by determining the angle through which a revolving mirror turned, while the light reflected from it went and returned along a measured course. No use whatever was made of electricity or magnetism.

The agreement of the results seems to show that light and magnetism are affections of the same substance, and that light is an electromagnetic disturbance propagated through the field according to electromagnetic laws.

(98) Let us now go back upon the equations in (94), in which the quantities J and Ψ occur, to see whether any other kind of disturbance can be propagated through the medium depending on these quantities which disappeared from the final equations.

If we determine χ from the equation

$$\nabla^2\chi = \frac{d^2\chi}{dx^2} + \frac{d^2\chi}{dy^2} + \frac{d^2\chi}{dz^2} = J, \quad \ldots \quad (73)$$

and F', G', H' from the equations

$$F' = F - \frac{d\chi}{dx}, \quad G' = G - \frac{d\chi}{dy}, \quad H' = H - \frac{d\chi}{dz}, \quad \ldots \quad (74)$$

then

$$\frac{dF'}{dx} + \frac{dG'}{dy} + \frac{dH'}{dz} = 0, \quad \ldots \quad (75)$$

and the equations in (94) become of the form

$$k\nabla^2 F' = 4\pi\mu\left(\frac{a^2F'}{dt^2} + \frac{d}{dxdt}\left(\Psi + \frac{d\chi}{dt}\right)\right). \quad \ldots \quad (76)$$

Differentiating the three equations with respect to x, y, and z, and adding, we find that

$$\Psi = -\frac{d\chi}{dt} + \phi(x, y, z), \quad \ldots \quad (77)$$

and that

$$\left.\begin{aligned} k\nabla^2 F' &= 4\pi\mu\frac{d^2F'}{dt^2}, \\ k\nabla^2 G' &= 4\pi\mu\frac{d^2G'}{dt^2}, \\ k\nabla^2 H' &= 4\pi\mu\frac{d^2H'}{dt^2}. \end{aligned}\right\} \quad \ldots \quad (78)$$

Hence the disturbances indicated by F', G', H' are propagated with the velocity $V = \sqrt{\frac{k}{4\pi\mu}}$ through the field; and since

$$\frac{dF'}{dx} + \frac{dG'}{dy} + \frac{dH'}{dx} = 0,$$

the resultant of these disturbances is in the plane of the wave.

Now, from (45), we have that:

$$v^2 \frac{e_1e_2}{r^2} = \frac{k}{4\pi}\frac{e_1e_2}{r^2}$$

$$k = 4\pi v^2.$$

For V = v = 310.740.000, r = 1 and putting e_1 and e_2 = -1,602176 * 10^{-19} , we obtain:

$v^2 e_1e_2 = (310740000)^2 (-1,602176*10^{-19})^2 =$

= (96559347600000000)(2,566967934976 * 10^{-38})

= 2,4786474911140178165 76 * 10^{-21}

Then, we have that, for r = 1: $(v^2 e_1e_2)/ v_4 = (v^2 e_1e_2)/ v_4 = (e_1e_2)/ v_2 = (e_1e_2)/ v_2$.

Now, $(e_1e_2)/ v_2$ = (2,566967934976 * 10^{-38}) / (96559347600000000) =

= 2,6584354583771027881302710872914 * 10^{-55}

If we calculate the natural logarithm of this number, (the natural logarithm can be defined for any positive real number a as the area under the curve y = 1/x from 1 to a) we obtain:

Ln (2,6584354583771027881302710872914 * 10^{-55}) = – 125,664...

a value practically equal to the boson Higgs mass that is (125,09 ± 0,24) GeV/c^2 .

We have that:

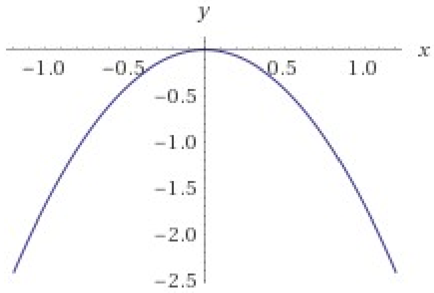
(0.6289858)^8 * 1.08643 * integrate [-125.664] x

$$0.6289858^8 \times 1.08643 \int -125.664\, x\, dx$$

Result:

$$-1.67228\, x^2$$

Plot:



(x from -1.2 to 1.2)

Now, we calculate the following double integral for this value:

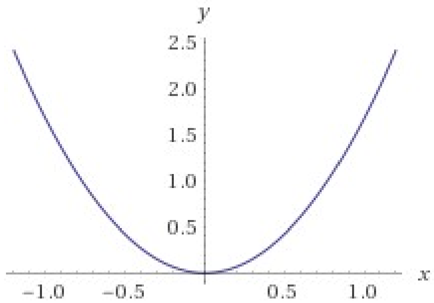
(0.6289858)^8 * 1.08643 * integrate integrate [125.664]

$$0.6289858^8 \times 1.08643 \int \left(\int 125.664\, dx \right) dx$$

Result:

$$1.67228\, x^2$$

Plot:



(x from -1.2 to 1.2)

A value practically equal to the proton and anti-proton mass.

From (21)

$$4\pi a \sigma e\left(1-\frac{v^2}{6c^2}\right)$$

for a = 21m; σ = 4.51* 10^{-8} C/m^3 m = 9.109 * 10^{-31} kg, e = -1,602176 * 10^{-19} C; v = 2189000 m/s; c = 0.017 *10^{-3}J

We obtain: - 5.26936 * 10^{-7}

We calculate the following double integral:

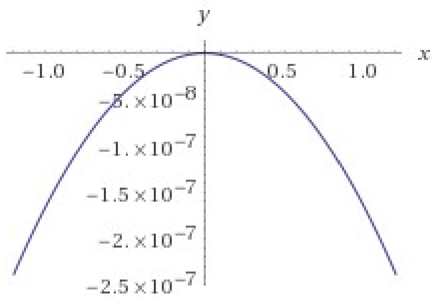
Pi/(2e) 1.08643 * integrate integrate [-5.26936*10^-7]

$$\frac{\pi}{2e} \times 1.08643 \int\left(\int -5.26936 \times 10^{-7}\, dx\right) dx$$

Result:

$-1.65407 \times 10^{-7} x^2$

Plot:



(x from -1.2 to 1.2)

result that is a good approximation to the value of the mass of the anti-proton.

From:

RAMANUJAN'S THEORY OF THETA FUNCTIONS
Bruce Berndt - University of Illinois at Urbana-Champaign
π, 2009

$$a(q) := \sum_{m,n=-\infty}^{\infty} q^{m^2+mn+n^2}$$

If $\chi_0(n)$ denotes the principal character modulo 3, then

$$a^2(q) = 1 + 12\sum_{n=1}^{\infty} \chi_0(n)\frac{nq^n}{1-q^n}.$$

If $\left(\frac{n}{3}\right)$ denotes the Legendre symbol, then

$$a^3(q) = 1 - 9\sum_{n=1}^{\infty} \left(\frac{n}{3}\right)\frac{n^2q^n}{1-q^n} + 27\sum_{n=1}^{\infty} \frac{n^2q^n}{1+q^n+q^{2n}}.$$

Note that 1-9 = -8; 1728 / -8 = -216 and 1728 / 27 = 64

Examples of class invariants

$$G_{117} = \left(\frac{3+\sqrt{13}}{2}\right)^{1/4}\left(2\sqrt{3}+\sqrt{13}\right)^{1/6} \times \left(3^{1/4}+\sqrt{4+\sqrt{3}}\right)$$

is equal to 6,9292879. We note that 1729 / 7 = 247; 1729 / 6,9292879 = 249,52058

We calculate the following double integral:

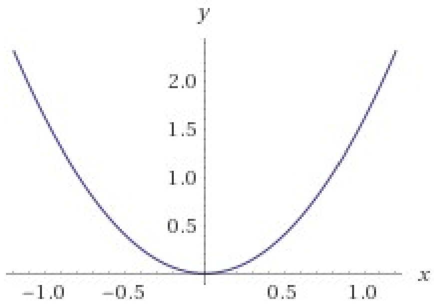
1.08643^2 * Pi/8 * integrate integrate [6.9292879]

$$1.08643^2 \times \frac{\pi}{8} \int\left(\int 6.9292879\, dx\right) dx$$

Result:

$$1.60591\, x^2$$

Plot:



result that is a good approximation to the value of the electric charge of the positron.

Now:

$$G_{1353} = (3+\sqrt{11})^{1/4}(5+3\sqrt{3})^{1/4} \times \left(\frac{11+\sqrt{123}}{2}\right)^{1/4}\left(\frac{6817+321\sqrt{451}}{4}\right)^{1/12} \times \left(\sqrt{\frac{17+3\sqrt{33}}{8}}+\sqrt{\frac{25+3\sqrt{33}}{8}}\right)^{1/2} \times \left(\sqrt{\frac{561+99\sqrt{33}}{8}}+\sqrt{\frac{569+99\sqrt{33}}{8}}\right)^{1/2}$$

The result is G_{1353} = 103,7118167...

We note that:

$$\left(\sqrt{\frac{561+99\sqrt{33}}{8}}+\sqrt{\frac{569+99\sqrt{33}}{8}}\right)^{1/2}$$

is equal to $(23,808697708)^{1/2}$ and that 23,80869... is about 24 and 24 *72 = 1728

We calculate the following double integral:

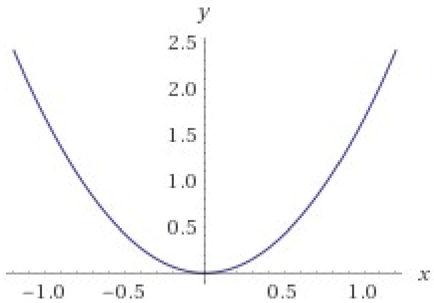
1.08643 * (0.6442)^8 * integrate integrate [103.7118167]

$$1.08643 \times 0.6442^8 \int \left(\int 103.7118167 \, dx \right) dx$$

Result:

$1.67096 \, x^2$

Plot:



(x from −1.2 to 1.2)

result that is a good approximation to the value of the mass of the proton.

Now:

Let $a = 60^{1/4}$, $b = 2 - \sqrt{3} + \sqrt{5}$. If

$$2c = \frac{a+b}{a-b}\sqrt{5} + 1,$$

then

$$R(e^{-6\pi}) = \sqrt{c^2 + 1} - c.$$

We have that 2c = 43.353157239 and c = 21,6765786195

$e^{-6\pi}$ = 6,512494708 * 10^{-9} ; √(469.874 + 1) – 21.6765786195 = 0,02305411120225;

Thence R = 0,02305411120225 / 6,512494708 * 10^{-9} = 3539981,564

Note that: $(3539981{,}564)^{1/32}$ = 1,60197701749786

Now, we calculate the following double integral:

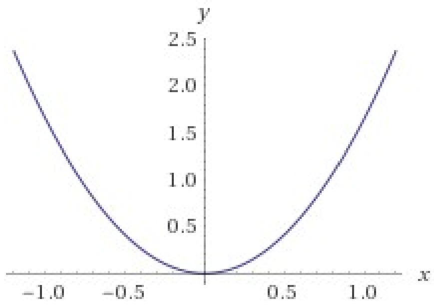
1.08643 * (Pi/((26*10)^e)) * integrate integrate [3539981.564]

$$1.08643 \times \frac{\pi}{(26 \times 10)^e} \int \left(\int 3.539981564 \times 10^6 \, dx \right) dx$$

Result:

$1.64643 \, x^2$

Plot:



(x from -1.2 to 1.2)

result that is a good approximation to the value of the mass of the proton.

Now, from Ramanujan's cubic class invariant:

$$\lambda_n = \frac{1}{3\sqrt{3}} \frac{f^6(q)}{\sqrt{q} f^6(q^3)} = \frac{1}{3\sqrt{3}} \left(\frac{\eta\left(\frac{1+i\sqrt{n/3}}{2}\right)}{\eta\left(\frac{1+i\sqrt{3n}}{2}\right)} \right)^6,$$

where $q = e^{-\pi\sqrt{n/3}}$.

$$\lambda_9 = 3,$$
$$\lambda_{17} - 4 + \sqrt{17},$$
$$\lambda_{73} = \left(\sqrt{\frac{11+\sqrt{73}}{8}} + \sqrt{\frac{3+\sqrt{73}}{8}} \right)^6.$$

We have that λ_{73} = 446,1418559374 where $(446{,}1418559374)^{1/12}$ = 1,6626062706...

Now, we calculate the following double integral:

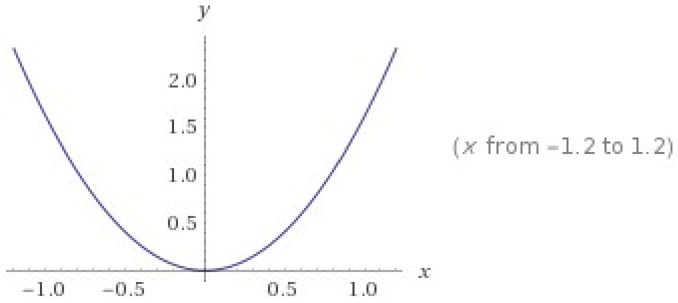
1.08643^2 * Pi/(512) * integrate integrate [446.1418559374]

$$1.08643^2 \times \frac{\pi}{512} \int \left(\int 446.1418559374 \, dx \right) dx$$

Result:

$$1.61557 \, x^2$$

Plot:



result that is a good approximation to the value of the electric charge of the positron.

Mathematical connections

We know that (see pg.142):

$$\frac{\pi^{1/4}}{\Gamma\left(\frac{3}{4}\right)} = 1 + \int_0^\infty \frac{e^{-t^2/2}}{\sqrt{2\pi}} \left[\frac{4e^{\pi}\left(e^{2\pi} - \cos\left(\sqrt{2\pi}t\right)\right)}{e^{4\pi} - 2e^{2\pi}\cos\left(\sqrt{2\pi}t\right) + 1}\right] dt$$

we have:

$$\Gamma\left(\frac{3}{4}\right) = \frac{\pi\sqrt{2}}{\Gamma\left(\frac{1}{4}\right)} = \frac{4{,}44288293815}{3{,}625609908} = 1{,}2254167025$$

$$\frac{\pi^{1/4}}{\Gamma\left(\frac{3}{4}\right)} = \frac{1{,}3313353638}{1{,}2254167025} = 1{,}08643481\ldots$$

With regard the integral, from 0 to 0,58438 for t = 2, where (2.71828^2)/(sqrt6.283185307) = 2,94780 for t=2, we have:

integrate (2.94780)[4e^3.14159265 * (e^6.283185307 - cos((sqrt6.283185307)2))]/[e^12.56637 - 2e^6.283185307 (cos(sqrt6.283185307)2))+1] x, [0,0.58438]

$$\int_0^{0.58438} \frac{2.94780\left(4\, e^{3.14159265}\left(e^{6.283185307} - \cos\left(\sqrt{6.283185307}\ 2\right)\right)\right) x}{e^{12.56637} - \left(2\, e^{6.283185307}\right)\left(\cos\left(\sqrt{6.283185307}\right) 2\right) + 1}\, dx =$$

0.0864364

Thence, 1 + 0,0864364 = 1,0864364; 1,08643481 ≅ 1,0864364.

With regard the Maxwell's equations (see pg. 79-81), we have calculate the following double integrals for 1,53678975*10^{23} , for 1.591549430918954 , for 155370000000, and for 310740000000

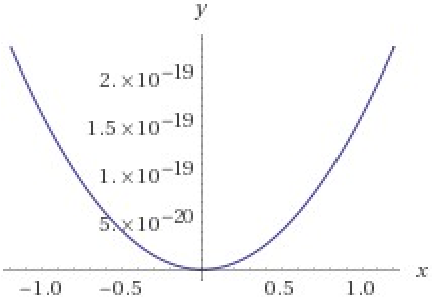
1/(2*10^40) * 1/(26) * 1.08643 * integrate integrate [1.53678975*10^23]

$$\frac{1}{2\times 10^{40}} \times \frac{1}{26} \times 1.08643 \int\left(\int 1.53678975 \times 10^{23}\, dx\right) dx$$

Result:

$1.6054 \times 10^{-19}\, x^2$

Plot:



(x from −1.2 to 1.2)

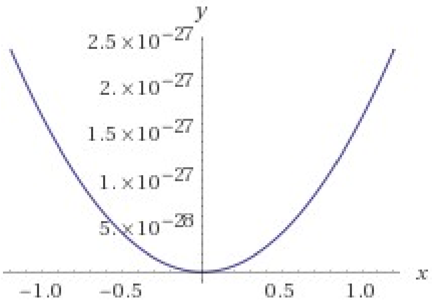
1/(2*10^25) * 1/(26) * 1.08643 * integrate integrate [1.591549430918954]

$$\frac{1}{2\times 10^{25}} \times \frac{1}{26} \times 1.08643 \int\left(\int 1.591549430918954\, dx\right) dx$$

Result:

$1.6626 \times 10^{-27}\, x^2$

Plot:



(x from −1.2 to 1.2)

result that is a good approximation to the value of the electric charge of the positron and to the mass of the proton.

1/(10^36) * (sqrt(6)/4)^8 * 1.08643 * integrate integrate [155370000140.18299865]

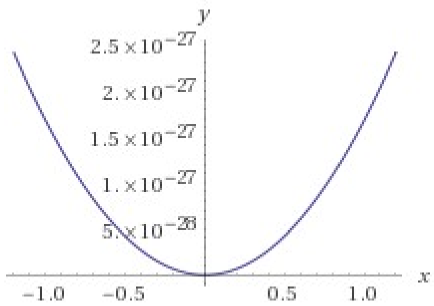
dove "(√6)/4 = Circumradius sphere, congruent with vertices (Tetrahedron)"

$$\frac{1}{10^{36}}\left(\frac{\sqrt{6}}{4}\right)^8 \times 1.08643 \int\left(\int 1.5537000014018299865\times 10^{11}\,dx\right)dx$$

Result:

$1.66903\times 10^{-27}\,x^2$

Plot:



(x from -1.2 to 1.2)

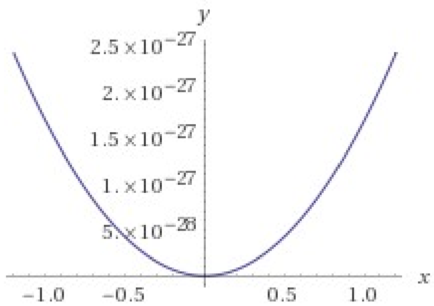
1/2 * 1/(10^36) * (sqrt(6)/4)^8 * 1.08643 * integrate integrate [310740000280.3659973]

$$\frac{1}{2}\times\frac{1}{10^{36}}\left(\frac{\sqrt{6}}{4}\right)^8 \times 1.08643 \int\left(\int 3.107400002803659973\times 10^{11}\,dx\right)dx$$

Result:

$1.66903\times 10^{-27}\,x^2$

Plot:



(x from -1.2 to 1.2)

result that is a good approximation to the value of the mass of the proton.

The above result of double integral, can be related to the following Ramanujan's equation concerning the invariant class of theta function:

$$G_{1353} = (3+\sqrt{11})^{1/4}(5+3\sqrt{3})^{1/4} \times \left(\frac{11+\sqrt{123}}{2}\right)^{1/4}\left(\frac{6817+321\sqrt{451}}{4}\right)^{1/12} \times \left(\sqrt{\frac{17+3\sqrt{33}}{8}}+\sqrt{\frac{25+3\sqrt{33}}{8}}\right)^{1/2} \times \left(\sqrt{\frac{561+99\sqrt{33}}{8}}+\sqrt{\frac{569+99\sqrt{33}}{8}}\right)^{1/2}$$

The result is G_{1353} = 103,7118167...

We have calculate the following double integral:

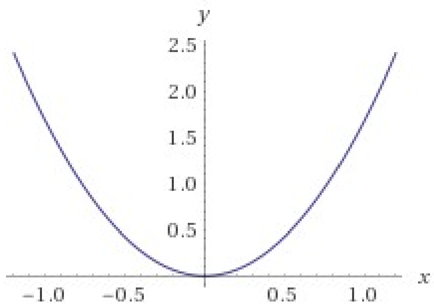
1.08643 * (0.6442)^8 * integrate integrate [103.7118167]

$$1.08643 \times 0.6442^8 \int\left(\int 103.7118167\, dx\right) dx$$

Result:

$1.67096\, x^2$

Plot:



(x from −1.2 to 1.2)

result that is a good approximation to the value of the mass of the proton.

Thence, mathematical connection between the fundamental Maxwell's equation:

$$\frac{1}{2} \times \frac{1}{10^{36}} \left(\frac{\sqrt{6}}{4}\right)^8 \times 1.08643 \int\left(\int 3.107400002803659973 \times 10^{11}\, dx\right) dx$$

$1.66903 \times 10^{-27}\, x^2$

and Ramanujan's equation concerning the invariant class of theta function:

$$1.08643 \times 0.6442^8 \int \left(\int 103.7118167 \, dx \right) dx$$

$$1.67096 \, x^2$$

We have also calculate the integral of the wave function of the Universe of Hawking regarding the “no-boundary proposal”, that is (see pg.21):

$$\psi_0(a_0) = 2 \cos \left[\frac{(H^2 a_0^2 - 1)^{3/2}}{3H^2} - \frac{\pi}{4} \right]$$

$\psi_0 a_0 = -3{,}357714479.$

1.08643 * integrate [-3.357714479] x,[0, -Pi^2/10.53] where 0,937284 is

$0{,}6354749^{1/7}$

$$1.08643 \int_0^{-0.937284} -3.35771 \, x \, dx = -1.60235$$

After, we have calculate the following double integral:

1.08643 * integrate integrate [-3.357714479] [0, -Pi^2/11]

where – Pi^2/11 = – 0.89723676 that is

$0{,}6480794355^{1/4}$ (i^2) = 0.89723676 * – 1 = – 0.89723676

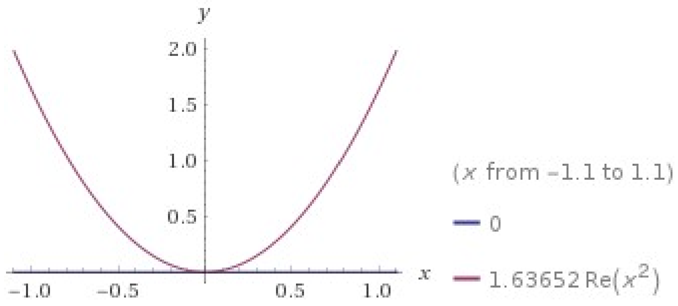
$$1.08643 \int \left(\int -3.357714479 \left\{ 0, -\frac{\pi^2}{11} \right\} dx \right) dx$$

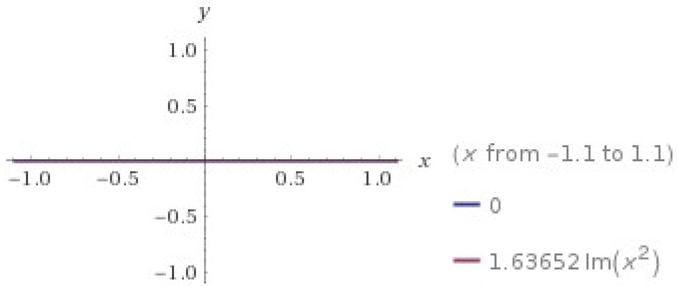
Result:

$$\{0, 1.63652 \, x^2\}$$

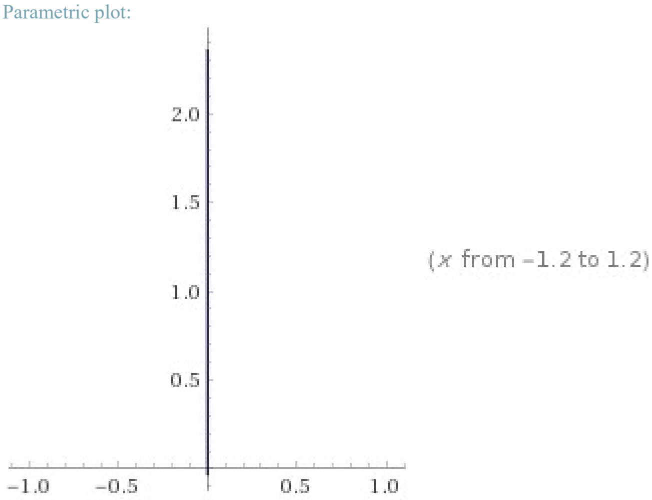
result that is a good approximation to the value of the electric charge of the electron and of the mass of the proton.

Plot:





Parametric plot:



From the eq. (28), (see pg.36) we have obtained:

$$Q = \frac{\Xi q}{4\pi l\sqrt{1+f'(R_0)}}.$$

Q = (2.25 * 1) / 4π*1*√(1+1) = 0,12660698195959304103119988623532;

We have calculate the following double integral for Q:

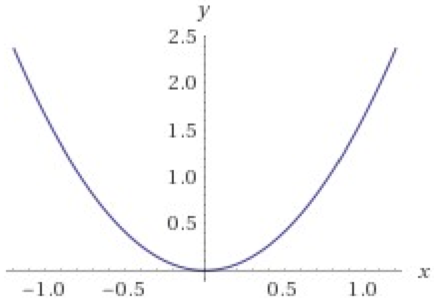
7Pi * 1.08643^2 * integrate integrate [0.12660698195959304103119988623532]

$$7\pi \times 1.08643^2 \int\left(\int 0.12660698195959304103119988623532\,dx\right)dx$$

Result:

$1.64316\,x^2$

Plot:



(x from −1.2 to 1.2)

Or:

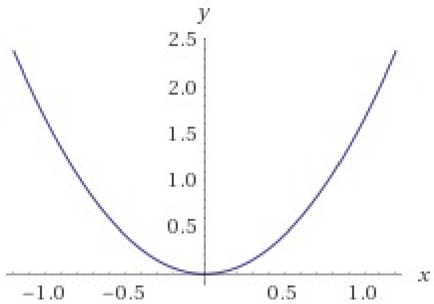
(24) * 1.08643 * integrate integrate [0.12660698195959304103119988623532]

$$24\times 1.08643 \int\left(\int 0.12660698195959304103119988623532\, dx\right) dx$$

Result:

$1.6506\, x^2$

Plot:



(x from −1.2 to 1.2)

values that are a good approximations to the value of the mass of the proton.

And:

(53Pi*11) * 1.08643 * integrate integrate [0.12660698195959304103119988623532]

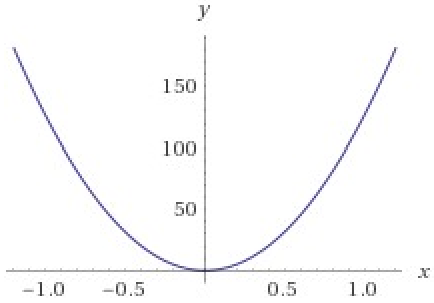
$$(53\,\pi\times 11)\times 1.08643 \int\left(\int 0.12660698195959304103119988623532\, dx\right) dx$$

Result:

$125.964\, x^2$

result that is practically equal to the value of the mass of the Higgs boson.

Plot:



(x from -1.2 to 1.2)

Where 53 is a prime number and is the sum of five prime numbers

53 = 5 + 7 + 11 + 13 + 17.

We have obtained (see pg.40-41), evident mathematical connections between this Maxwell's equation and the charge of black strings (charged rotating black holes).

From the eq. (30) concerning the electric potential:

$$U - \frac{q}{\Xi r_+}\sqrt{1 + f'(R_0)}.$$

U = 1 / (1.25 * 0.7) √3 =

1,1428571428571428571428571428571 * 1,7320508075688772935274463415059 =

= 1,9794866372215740497456529617 21

We have calculate the following integral:

(Pi*(ln1.606695)) * 1.08643 * integrate integrate
[1.97948663722157404974565296172 1]

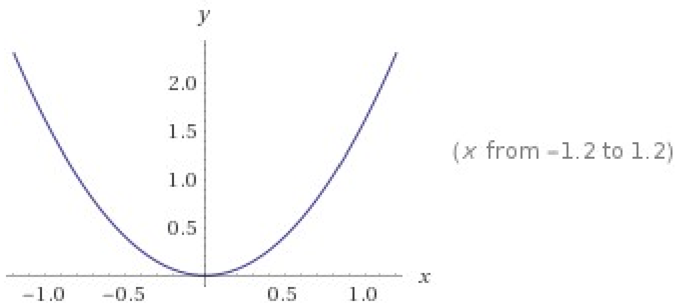
where 1.606 695 is the "Erdös - Borwein constant"

$$(\pi \log(1.606695)) \times 1.08643 \int\left(\int 1.9794866372215740497456529617 21\, dx\right) dx$$

Result:

$1.60183\, x^2$

Plot:



We have calculate the following double integral:

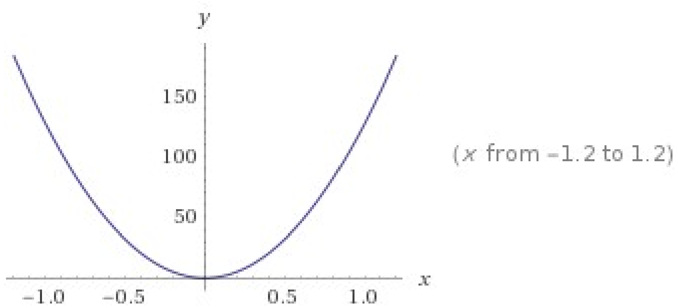
(11Pi^2) * 1.08643^2 * integrate integrate [1.979486637221574049745652961721]

$$(11\pi^2)\times 1.08643^2 \int\left(\int 1.979486637221574049745652961721\,dx\right)dx$$

Result:

$126.829\,x^2$

Plot:



Results that are good approximation to the values of the electric charge of the positron and of the mass of the Higgs boson,

$$\nabla\cdot\mathbf{E}=\frac{\rho}{\epsilon_0}$$

$$\int_S \mathbf{E}\cdot d\mathbf{a}=\frac{1}{\epsilon_0}\int \rho\, dV$$

are the Maxwell's equation concerning the Gauss' law in differential and integral form.

With regard the mathematical connections with the eleven-dimensional supergravity, we have that (see pg.41-42)

The supergravity multiplet consists of the metric g, the gravitino $\psi_{I\alpha}$, and a three-form C (with field strength G, normalized as in a previous footnote). The supergravity Lagrangian, up to terms quartic in the gravitino (which we will not need), is [8]

$$L_S = \frac{1}{\kappa^2}\int_{M^{11}} d^{11}x\sqrt{g}\left(-\frac{1}{2}R - \frac{1}{2}\overline{\psi}_I\Gamma^{IJK}D_J\psi_K - \frac{1}{48}G_{IJKL}G^{IJKL} - \frac{\sqrt{2}}{192}\left(\overline{\psi}_I\Gamma^{IJKLMN}\psi_N + 12\overline{\psi}^J\Gamma^{KL}\psi^M\right)G_{JKLM} - \frac{\sqrt{2}}{3456}\epsilon^{I_1I_2\ldots I_{11}}C_{I_1I_2I_3}G_{I_4\ldots I_7}G_{I_8\ldots I_{11}}\right). \quad (2.1)$$

Note that: 3456 = 1728 * 2 and that $\frac{\sqrt{2}}{3456} = \frac{1}{1728\sqrt{2}}$

We have that:

$$\left(-\frac{1}{2}R - \frac{1}{2}\overline{\psi}_I\Gamma^{IJK}D_J\psi_K - \frac{1}{48}G_{IJKL}G^{IJKL} - \frac{\sqrt{2}}{192}\left(\overline{\psi}_I\Gamma^{IJKLMN}\psi_N + 12\overline{\psi}^J\Gamma^{KL}\psi^M\right)G_{JKLM} - \frac{\sqrt{2}}{3456}\epsilon^{I_1I_2\ldots I_{11}}C_{I_1I_2I_3}G_{I_4\ldots I_7}G_{I_8\ldots I_{11}}\right).$$

$$-\frac{1}{2} - \frac{1}{2} - \frac{1}{48} - \frac{\sqrt{2}}{192} - \frac{12\sqrt{2}}{192} - \frac{\sqrt{2}}{3456} =$$

$$= \frac{-1728 - 1728 - 72 - 18\sqrt{2} - 216\sqrt{2} - \sqrt{2}}{3456} =$$

$$= \frac{1728}{3456} - \frac{1728}{3456} - \frac{72}{3456} - \frac{18\sqrt{2}}{3456} - \frac{216\sqrt{2}}{3456} - \frac{\sqrt{2}}{3456} =$$

-3860,34018656 / 3456 = – 1,11699658

We have calculate the following integral (see pg.43-44):

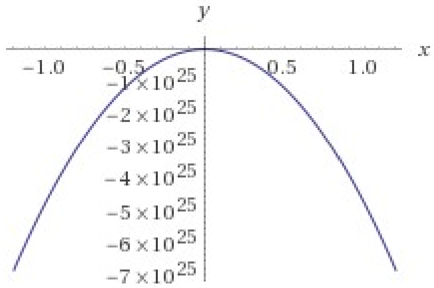
5.9049 * (10^36) integrate [(1.44853201 * 10^-11)*(-1.11699658)] x

$$5.9049 \times 10^{36} \int\left(\frac{1.44853201}{10^{11}} \times (-1.11699658)\right) x\, dx$$

Result:

$-4.77708 \times 10^{25}\, x^2$

Plot:



(x from -1.2 to 1.2)

Indefinite integral assuming all variables are real:

$-1.59236\times10^{25}\,x^3$ + constant

and:

(1/10^54) * 1.08643^2 * 5.9049 * (10^36) integrate [(1.44853201 * 10^-11)*(-1.11699658)] x, [0, 34/(2Pi)]

$$\frac{1}{10^{54}}\times1.08643^2\times5.9049\times10^{36}\int_0^{\frac{34}{2\pi}}\left(\frac{1.44853201}{10^{11}}\times(-1.11699658)\right)x\,dx$$

Result:

-1.65106×10^{-27}

result very near to the value of the mass of the anti-proton.

We have calculate the following double integral:

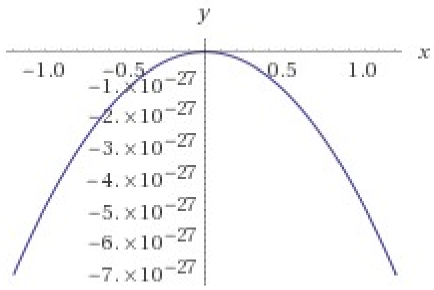
(1*10^-52) * (2*0.618)^3 * 1.08643 integrate integrate [-4.77708*10^25]

$$1\times10^{-52}\,(2\times0.618)^3\times1.08643\int\left(\int-4.77708\times10^{25}\,dx\right)dx$$

Result:

$-4.89993\times10^{-27}\,x^2$

Plot:



(x from -1.2 to 1.2)

Indefinite integral assuming all variables are real:

$-1.63331 \times 10^{-27} x^3 + \text{constant}$

result very near to the value of the mass of the anti-proton.

From pg.46 and 49-50

1728 = 13744512 / 7954

We calculate the following integral:

integrate [13744512] x, [0, 1/(1.644934^13*Pi)]

$$\int_0^{\frac{1}{1.644934^{13}\pi}} 13744512\, x\, dx = 1.67086$$

where 1,644934 = $\zeta(2) = \pi^2/6$

and the following double integral:

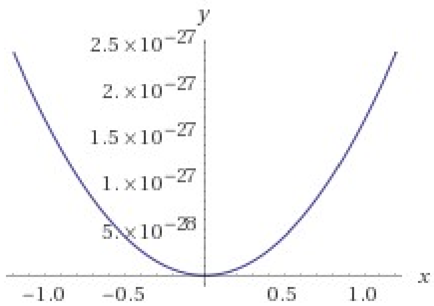
1/(10^34) * 1.08643 * (Pi/sqrt(2)) integrate integrate [13744512]

$$\frac{1}{10^{34}} \times 1.08643 \times \frac{\pi}{\sqrt{2}} \int\left(\int 13744512\, dx\right) dx$$

Result:

$1.65858 \times 10^{-27} x^2$

Plot:



(x from -1.2 to 1.2)

result very near to the value of the mass of the proton.

From pg. 49-50

We calculate the following double integral:

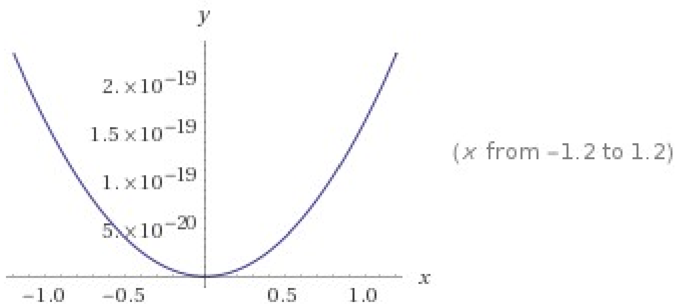
1/(10^22) * 1.08643^2 * (Pi/2) integrate integrate [1728]

$$\frac{1}{10^{22}} \times 1.08643^2 \times \frac{\pi}{2} \int \left(\int 1728 \, dx \right) dx$$

Result:

$$1.60191 \times 10^{-19} \, x^2$$

Plot:



and:

1.08643 integrate [1728] x, [0, Pi/((1.618)^9)]

Definite integral:

$$1.08643 \int_0^{0.0413374} 1728 \, x \, dx = 1.60399$$

results very near to the value of the electric charge of the positron.

From Wikipedia

The **Dirac sea** is a theoretical model of the vacuum as an infinite sea of particles with negative energy. It was first postulated by the British physicist Paul Dirac in 1930[1] to explain the anomalous negative-energy quantum states predicted by the Dirac equation for relativistic electrons.[2] The positron, the antimatter counterpart of the electron, was originally conceived of as a hole in the Dirac sea, well before its experimental discovery in 1932.[nb 1]

Upon solving the free Dirac equation,

$$i\hbar \frac{\partial \Psi}{\partial t} = (c\hat{\boldsymbol{\alpha}} \cdot \hat{\boldsymbol{p}} + mc^2 \hat{\beta})\Psi,$$

one finds

$$\Psi_{\mathbf{p}\lambda} = N \begin{pmatrix} U \\ \frac{(c\hat{\sigma}\cdot p)}{mc^2+\lambda E_p} U \end{pmatrix} \frac{\exp[i(\mathbf{p}\cdot\mathbf{x} - \varepsilon t)/\hbar]}{\sqrt{2\pi\hbar}^3},$$

where

$$\varepsilon = \pm E_p, \quad E_p = +c\sqrt{\mathbf{p}^2 + m^2c^2}, \quad \lambda = \operatorname{sgn}\varepsilon$$

for plane wave solutions with 3-momentum **p**. This is a direct consequence of the relativistic energy-momentum relation

$$E^2 = p^2c^2 + m^2c^4$$

upon which the Dirac equation is built. The quantity U is a constant 2 × 1 column vector and N is a normalization constant. The quantity ε is called the *time evolution factor*, and its interpretation in similar roles in, for example, the plane wave solutions of the Schrödinger equation, is the energy of the wave (particle). This interpretation is not immediately available here since it may acquire negative values. A similar situation prevails for the Klein–Gordon equation. In that case, the *absolute value* of ε can be interpreted as the energy of the wave since in the canonical formalism, waves with negative ε actually have *positive* energy E_p. But this is not the case with the Dirac equation. The energy in the canonical formalism associated with negative ε is $-E_p$. In hole theory, the solutions with negative time evolution factors are reinterpreted as representing the positron, discovered by Carl Anderson. The interpretation of this result requires a Dirac sea, showing that the Dirac equation is not merely a combination of special relativity and quantum mechanics, but it also implies that the number of particles cannot be conserved.

Generalization of the Dirac's Equation and Sea
H. Javadi, F. Forouzbakhsh and H.Daei Kasmaei - 14 June 2016

$$\beta mc^2 \rightarrow \begin{bmatrix} mc^2 & 0 & 0 & 0 \\ 0 & mc^2 & 0 & 0 \\ 0 & 0 & -mc^2 & 0 \\ 0 & 0 & 0 & -mc^2 \end{bmatrix} \qquad (16)$$

For eigenvalues and considering $p = 0$ (in equation (4)), we will have[9]:

$$E_+ = mc^2, \ E_- = -mc^2 \qquad (17)$$

From the Dirac equation to the photon structure

In pair production of "electron-positron", one photon with spin 1 and at least energy $E = 1.022\ MeV$ is converted to two fermions, electron and positron with spin $\frac{1}{2}$, each of them with context of energy 0.511 MeV in vicinity of a heavy nucleus so that we have the following relation:

$$\gamma \rightarrow e^- + e^+ \qquad (18)$$

Relation (18) is justifiable according to Dirac equation by relations (16) and (17), (Figure 1.A). In pair decay, an electron is combined with a positron and is produced two photons (Figure 1.B).

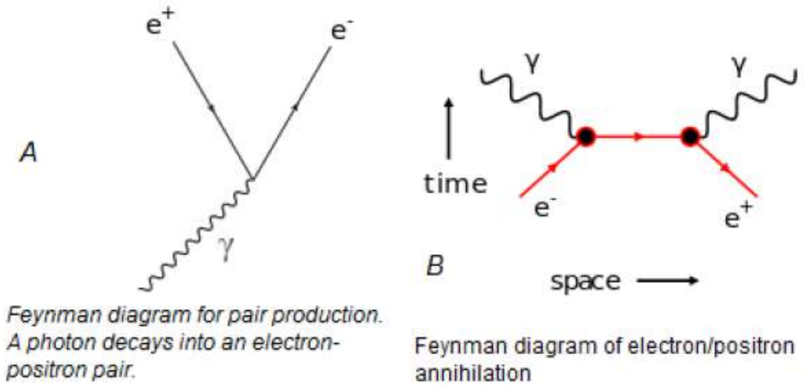


Fig1: Production and decay of pair "electron-positron"

In pair decay, reverse of relation (18) takes place and we will have:

$$e^- + e^+ \rightarrow 2\gamma \qquad (19)$$

In all physical processes including pair production and decay, it must be held the following conservation laws:

1- Electric charge conservation law, pure charge before and after the process must be equal.

2- Linear momentum and total energy conservation laws: These rules has made forbidden production of just one photon (Gamma ray). As it is seen in Figure (2), two photon with the same energy move but in two opposite directions. Angular momentum conservation law must be held too. In fact, in the process of "electron-positron" decay, these following relations hold:

$$e^- + e^+ \rightarrow 2\gamma$$

$$E_{2\gamma} = 2m_0c^2 + E_{e^-} + E_{e^+}$$

$$m_0c^2 = 0.511\ MeV$$

In which m_0c^2 is zero rest mass of electron (also positron) and E_{e^-} , E_{e^+} are kinetic energy of electron and positron that are converted to energy of photons ($E_{2\gamma}$) at the time of pair decay.

We take the value E = 1.022 MeV and calculate the following integral:

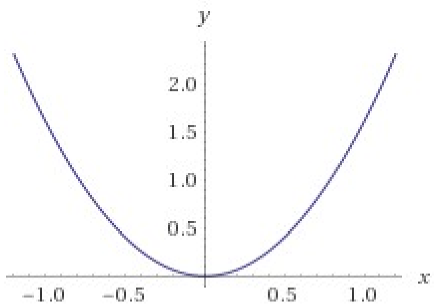
1.08643 * [(2*0.61803398)^5] integrate integrate [1.022]

$$1.08643\,(2 \times 0.61803398)^5 \int \left(\int 1.022\, dx \right) dx$$

Result:

$1.6019\, x^2$

Plot:



(x from −1.2 to 1.2)

result that is practically equal to the electric charge of the positron!

Conclusion

In conclusion, the reason why performing the double integrals of the various equations of Maxwell's electromagnetic theory are lepton-like solutions (protons, neutrons, electrons, positrons) and plots involving open curves, it allows to deduce that in the toroidal infinite-dimensional Hilbert space , as for branes, infinite open strings are anchored, therefore fermionics. We know that the interactions between open strings can always result in closed strings, therefore from the collision of two open strings, as happens for the annihilation between electron and positron, energy is emitted, which in the case of our model, generates a world toroidal brane, therefore a closed 3d string, also composed of bosons, just like the photons that make up the electromagnetic field (light). Practically the ends of two open strings, which come off the Hilbert space, when the two strings are annihilated, in reality what for us is the emission of bosons, is the creation from the two open strings, of a closed string that expanding becomes our D3-brane. The solutions of the integral equations that instead identify with the mass of the Higgs boson, are the further confirmation that it gives the mass to the fermions of the Standard Model.

"From the gravitational equations (4) follow the four independent linear combinations (27) of the basic electrodynamics equations (5) and their first derivatives. This is the exact mathematical expression of the aforementioned statement generally expressed on the character of electrodynamics as a consequence of gravitation" (from David Hilbert Paper).

In our case the Maxwell's equations are a consequence of the 11D supergravity and therefore connected to it. This is further confirmation that gravity and electromagnetism at the Planck scale are part of a single superforce. This explains the reason of the resulting particle-type solutions (fermions) from which, by annihilation, are produced bosons like photons, the quanta of the electromagnetic field

Fig.1

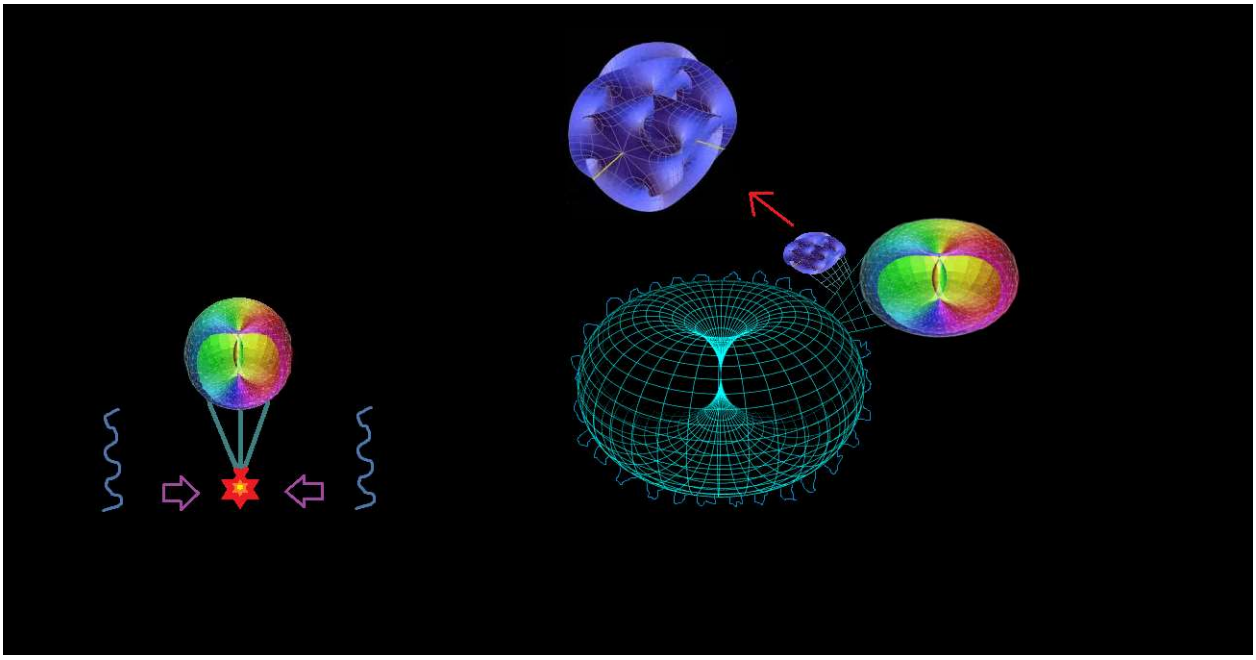


Fig.1 shows the infinite-dimensional torus (Hilbert space), a purely geometric entity belonging to the informal phase. On this space lie the ends of open strings (massive bosons and / or fermions) and of them a very large number of pairs annihilate each other, giving rise to torus-spheroidal universes (bubbles). They constitute the finite multiverse of an eternal inflationary cycle (in each cycle a non-infinite multiverse is born, but with a well-defined number of bubble-universes), of a succession of phases of singularity / expansion.

(The model proposed here therefore also contemplates the "no-boundary proposal" and the consequent limited multiverse model of S. W. Hawking, developed by Hertog.)

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