First Draft

Weyl Symmetry from the Minimal Fractal Manifold

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Abstract

A number of recent studies hint that Weyl symmetry plays a key role in the gauge theory of gravitation, as well as in the phenomenology of cosmic inflation and Dark Matter. Here we point out that Weyl symmetry follows naturally from the minimal fractality of spacetime near or above the electroweak scale. In particular, Weyl symmetry provides a field theoretic framework for the interpretation of Dark Matter as Cantor Dust.

Key words: Weyl symmetry, local scale invariance, Minimal Fractal Manifold, cosmic inflation, Dark Matter, Cantor Dust.

1. Introduction

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2. Weyl symmetry and its implications in Particle Physics and Cosmology

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Let $M^{3,1}$ denote a pseudo-Riemannian space-time endowed with metric $g_{\alpha\beta}$ having the signature (+, - , - , -). Let $\Omega(x) = s^{\nu_x(x)}$ be a strictly positive function on $M^{3,1}$ which has an inverse. The local conformal transformation in $M^{3,1}$ is defined through the following change of metric,

$$g_{\mu\nu}(x) \to \overline{g}_{\mu\nu}(x) = \Omega^2(x) g_{\mu\nu}(x) \tag{1}$$

The set of all local conformal transformations forms the multiplicative abelian infinite-dimensional group *C*. The effect of a *local conformal transformation* is to re-label the length scale according to

$$dl(x) = \sqrt{-g_{ij} dx^i dx^j} \rightarrow d\bar{l}(x) = \Omega(x) dl(x) \tag{2}$$

The meaning of the symmetry associated with group *C* is that the structure of physical laws must be independent from the units chosen to measure length, time and mass. In general, a field theory that depends on *dimensionful* rather than *dimensionless* variables not only fails to comply with local conformal invariance, but typically breaks requirements of gauge symmetry and renormalizability. It also leads to anomalies when classical theory is quantized and radiative corrections are accounted for [ ].

Let Φ be a local tensor or spinor field of arbitrary spin and consider the map

$$\Omega(x) \rightarrow U(\Omega(x)) \tag{3}$$

whose operation is described by

$$\overline{\Phi}(x) = U(\Omega(x))\Phi(x) = s^{\nu_\Phi(x)}\Phi(x) \tag{4}$$

The number

$$\kappa(x) = \frac{\nu_x(x)}{\nu_\Phi(x)} \tag{5}$$

is called the *conformal weight* of Φ and map (3, 4) defines the representation of *C* in field space. Using (5) in a global rather than local sense, it can be shown that the Maxwell, Yang-Mills tensors and Dirac field in four dimensional space-time (D = 4) have conformal weights $\kappa = 0$ and $\kappa = -3/2$, respectively [ ].

As with the standard arguments for the existence of gauge fields, demanding that the theory stays invariant to local conformal transformations (1), (2) and (4) implies that there is a gauge field S (referred to as *Weyl boson)* and a corresponding covariant derivative defined through [ ]

$$\partial_\mu \Phi(x) \to [\partial_\mu + \kappa(x) S_\mu] \Phi(x) \tag{6}$$

$$S(x) \to S_\mu(x) - \partial_\mu \kappa(x) \tag{7}$$

The field tensor associated with S is given by

$$H_{\mu\nu} = \partial_\mu S_\nu - \partial_\nu S_\mu \tag{8}$$

The scalar component of S is called the *Weyl scalar* (σ). It has conformal weight -1 in D = 4 and satisfies $H_{\mu\nu} = 0$. Since $\kappa = \kappa(x(\varepsilon))$ on account of [ ], the constraint of local conformal transformation induced by (6) shows that space-time dimension takes on the role of a *local gauge coupling*. This conclusion is consistent with the content of [ ], where the entire flavor structure of SM emerges from the properties of RG flow in continuous dimension $D = 4 - \varepsilon$.

Weyl boson has several remarkable features, namely [ ]:

a) it does not couple to either one of SM particles. It can only form a Bose-Einstein condensate under the effect of classical gravity and can be thus interpreted as a likely candidate for *dark matter*.

b) Weyl scalar represents the Goldstone boson arisen from breaking of local scale invariance and turns S into a massive vector particle.

c) Weyl scalar is related to classical gravity and its vacuum is linked to Newton's constant.

d) Higgs scenario of electroweak symmetry breaking represents a particular embodiment of the Weyl boson model.

e) Weyl boson can supply a mechanism for cosmic inflation.

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3. Minimal Fractal Manifold as natural geometry of effective field theories

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4. Weyl Symmetry as field theoretic description of Cantor Dust

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5. Concluding remarks

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References

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