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QUARK MODEL WITHOUT SCALING PROPERTY

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ABSTRACT

We propose a simple field theoretic quark model in which the quarks are composite. This quark bootstrap is the new feature of the model. Applications to spectroscopy and deep inelastic processes are considered. A number of testable predictions are given. In particular, the ratio $R = \sigma(e^+e^- \to \text{hadrons})/\sigma(e^+e^- \to \mu^+\mu^-)$ is predicted to grow linearly with Q^2 at least until $\sqrt{Q^2} \sim 6$ GeV, where R is about 7, and drop thereafter towards a value of about one.

1. INTRODUCTION

The existing models for hadrons and their interactions do not at present give a consistent picture of the hadron world. To be more specific we mention the following difficulties, closely related to the quark model. Resonance spectroscopy and scaling phenomena can be qualitatively described by the constituent and current quark models but the connection between the two models is not fully understood. Thus a synthesis of both is needed. Secondly, the bootstrap principle is rather well supported by experimental data but it is not in accordance with the original quark idea . Again, a synthesis of both is necessary. Thirdly, experimental data often prove "reliable" predictions wrong. For example, in the inclusive reaction $e^+e^- \to$ hadrons several ideas predict $\frac{1}{Q^2}$ behavior for this cross section [1] whereas the recent data indicate an approximately constant cross section with Q^2 between 10 to 25 GeV^2 . Further, it is generally accepted that a composite structure for hadrons is at present obviously necessary to explain data on form factors, scaling, Regge trajectories, etc. However, no constituents have ever been found in the laboratory. This makes the construction of such models difficult.

Consider now the following two hadron models: the bound state model of T.D.Lee [2] and the quark model with nine colored quarks [3] . In the first model the nucleon consists of the elementary fields $\psi(x)$ and $\pi(x)$, with $J^P = 1/2^-$ and 0^- , respectively. These particles are bound together by a scalar particle $\phi(x)$ and the interaction Lagrangian density is

$$L_I = f_1\bar{\psi}\psi\phi + if_2\,\bar{\psi}\gamma_5\psi\pi + f_3\,\pi^4 + f_4\,\pi^2\phi^2 + k\,\pi^2\phi + ie\,\bar{\psi}\gamma_\mu\psi\,A_\mu \qquad (1)$$

where A_μ is the electromagnetic field and the couplings satisfy $k > f_i$ $(i = 1...4)$ and $f_1^2/4\pi < 0.1$. With this model one can obtain $(\frac{1}{Q^2})^2$ behavior for nucleon electromagnetic form factors and also approximate scaling results in deep inelastic ep scattering . At the same time resonances have large widths in agreement with experiment. But the model does not include successful rules for determining the hadron quantum numbers.

The quark model, on the other hand, explains the multiplets discovered in the laboratory and gives appr. scaling asymptotically ; here we quote optimistically the recent results from models with asymptotic freedom and infrared slavery (or hot cloud or vacuum polarization current neutralization) [4] . Now, however, scaling in e^+e^- seems to be badly violated experimentally [5] and therefore any quark model is in serious trouble. To have insight in these problems we propose a model which is a synthesis of the color quark model and the Lee model [6] .

2. COMPOSITE QUARKS

Let us unify the color quark model with the Lee model and see whether we then could, at least in principle, explain the difficulties of the conventional quark model. Suppose that the quarks are not really elementary fields but rather composite systems, a little like the hadrons themselves. We construct the quarks of the following basic fields: a charged spin 1/2 field $\psi(x)$, a neutral pseudoscalar field $\pi(x)$ and a neutral scalar field $\phi(x)$ [7] . These meson fields are, of course, ultimately $q\bar{q}$ systems, i.e. color singlets while the $\psi(x)$ field carries the color quantum number. Clearly, the quarks may now have also excited states q^{*} or $q^{(n)}$ $(n \geq 1)$, with suitable couplings and masses in the Hamiltonian, in addition to the $s_{1/2}$ ground state $q = q^{(0)} = (\psi\pi)$. The lowest excited state is a $p_{1/2}$ $(\psi\phi)$ state. Higher excitations are either $(\psi\pi)$ or $(\psi\phi)$ states with $\ell > 1$. These quark excitations certainly need a firm motivation and in this paper we give arguments for them. Before going into the applications let us note that our earlier analysis I indicates that the couplings in (1), when applied to quarks, should satisfy $k \sim 1 \gtrsim f_4 > f_1 >> f_2$. We use these estimates in the following .

2.a. Spectroscopy

A hadronic model à la Bohr could be the following. The quarks that make up the hadrons behave like soft spheres of finite size bound together by the 1/r type color potential. Because of the finite size of the quarks and the free field behavior of the interaction at short distances the one particle potential does not have singular form at the origin but is rather flat bottomed as required by the data [8] . Thus we have the L=0 baryon and meson SU(3) multiplets and also the corresponding L- excited states. However, states like $N^*(1470)$ and $\rho'(1600)$ deserve additional care. Calculations with radially excited harmonic oscillator wave functions contradict data in the N^* (1470) electroproduction [9] and in the ρ' decay [10] . To avoid these difficulties we have suggested in I that the N^* (1470) and $\rho'(1600)$ have the following quark content

$$\epsilon_{ijk}\, q_\ell^{*i}\, q_m^j\, q_n^k \qquad \text{and} \qquad q_\ell^{*i}\, \bar{q}_m^{*i} \quad ,$$

respectively, where ℓ , m , n are the usual SU(3) indices and i, j , k are $SU(3)_{color}$ indices. Otherwise these particles are like their mother particles N(940) and $\rho(770)$, eg. L = 0 for both and the non-relativistic approximations to the wave functions are nodeless. Such non-relativistic approximations would be possible for high quark masses and strong binding [8] .

Any unstable baryon decays by emitting a quark. Both this emitted quark and the two quark system are neutralized into color singlet hadron states by the vacuum polarization current [4] thus yielding two, or more, hadrons in the final state. Alternatively, if the decaying hadron contains q^*'s , the decay takes place by π (or ϕ) emission, $q^* \rightarrow q + \pi\ (\phi)$.

The $N^*(1470)$ electroproduction goes via the graph of fig. 1 and the ratio $|A(p\gamma N^*)/A(p\gamma\Delta^+)|$ is proportional to f_2/f_1, which is roughly 1/6 experimentally [9]. Now $f_2/f_1 \lesssim 0.1$ and hence the above ratio should also be of the order 0.1. This holds for the real photodecay too. Similarly, the observed ρ' decays are described by the graphs in fig. 2. We can estimate that $|A(\rho'\to\rho\pi\pi)/A(\rho'\to\pi\pi)| \sim f_1 f_4/k\, f_2$. This is bigger than one since $f_2/f_1 \lesssim 0.1$ and f_4 is not much smaller than k. This was called a "radial" selection rule in I. From these arguments it follows further that, for example, in $e^+e^- \to \pi^+\pi^-$ (or 4π) there should only be a weakly coupled ρ' state at $m \simeq 1.270$ GeV since the corresponding graph in fig. 3.a. is proportional to ef_2. The first strong ρ' should occur at $M \simeq 1.600$ GeV corresponding to the graph in fig. 3.b. which is proportional to $e \gg ef_2$. Thus the pion form factor should, in the vector dominance model, be ρ dominated up to $\sqrt{Q^2} \simeq m_{\rho'} \simeq 1.600$ GeV. These conclusions agree well with data [11].

In high energy πp and pp collisions diffractively produced high mass pion and nucleon excitations have been observed [12], [13]. The masses of these states are at least 6 GeV and 10 GeV, respectively. This fact combined with the observed increase of the corresponding inelastic cross section suggests that new degrees of freedom are being excited in these collisions. In I we have suggested that the production of hadrons containing excited quark states would lead to the type of phenomena mentioned above. In addition, we argue that in e^+e^- annihilation into hadrons such excited quarks also occur (see section 2.b) forming heavy vector mesons. Thus we are lead to conclude that, for example, inclusive pion distributions in e^+e^- and hadron-hadron collisions should have similar behaviour in particular at large p_T values. The large p_T events would be due to the decays $q^* \to q + \pi$, where q^* and q mass difference can be large. Therefore these pions behave differently from these produced by the proposed vacuum polarization current or hot cloud.

2.b. Deep inelastic phenomena

A crucial test of the present model is the calculation of $\sigma(e^+e^- \to \text{hadrons})$ or its ratio to $\sigma(e^+e^- \to \mu^+\mu^-)$ denoted usually by R . Experimentally this ratio is growing still linearly with Q^2 at $Q^2 \sim 25\ \text{GeV}^2$ [5] while the quark model and some other arguments give [1]

$$R = \sum_i e_i^2 \qquad \text{(or constant)} \qquad (2)$$

where the e_i are the quark charges. With nine colored quarks one has $R = 2$. In the present model there are more than nine fermions and each quark has a form factor. Consequently, (2) should now be replaced by

$$\sum_{n=0}^{N} \sum_i e_i^2 \mid F^i_{q(n)} (Q^2)|^2 \qquad (2')$$

where $F^i_{q(n)} = F_{q(n)}$ is the n'th quark multiplet form factor. We must next calculate this electromagnetic form factor. Inparticular, we wish to understand when scaling is expected to hold and when not.

The general quark current operator is

$$j_\mu = i\, e_i\, \bar{u}_q\, (p') \left[\gamma_\mu F_1(q^2) + \frac{g-1}{2m_q} q_\nu\, \sigma_{\mu\nu}\, F_2(q^2) \right] u_q(p) \qquad (3)$$

where $e_i(g-1)/2m_q$ is the anomalous magnetic moment and m_q the mass of the quark. We assume the anomalous magnetic moment to be very small and neglect it [8] . The electromagnetic field couples to the $\psi(x)$ field with a piece $ie\bar{\psi}\gamma_\mu\psi A_\mu$ in the interaction Lagrangian (1) . The form factor F_1 in (3) is thus in the present model due to the composite structure of the quark. Such form factors can be studied in the ladder approximation with the Bethe-Salpeter equation with a proper kernel. Such an analysis gives $(\frac{1}{Q^2})^2$ behavior for $F(Q^2)$ for large $-Q^2$ values [14] . Here we must use a more phenomenological approach in the following.

We set two requirements for the quark form factor F_q. First, high energy deep inelastic lepton nucleon data [15] indicate that the F_q is a very slowly varying function of Q^2 at large $-Q^2$ values and it does not seem to approach zero. Secondly, the form factor F_q, if some kind of perturbation expansion is possible, should contain the effects of forces between the quark-antiquark pair. We assume that when Q^2 approaches infinity these forces may be neglected and scaling is obtained for pointlike quarks [16] . In our model the quarks have internal structure and correspondingly the $q\bar{q}$ interaction is more complicated. Phenomenologically we expect in F_q vector meson poles to show up at $\sqrt{Q^2} \sim 1$ GeV. The $q\bar{q}$ state has the quantum numbers of the ρ^o. We therefore assume that the F_q has a pole at $Q^2 = m_\rho{}^2$. A solution for these conditions for the F_q, normalized as $F_q(0) = 1$, is

$$F_q(Q^2) = 1 + \frac{CQ^2}{M^2{}_\rho - Q^2 - iM_\rho\Gamma_\rho} \tag{4}$$

At large Q^2 values this approaches the value 1 - C. Approximate lepton nucleon scaling now implies $0 \leq C < 1$. We estimate from ∿15 % scale breaking at $-Q^2 \sim 10\ \mathrm{GeV}^2$ that $C \sim 0.1$.

For the excited quark form factors $F_{q^{(n)}}$, $n = 1...N$, we make the assumption that they approach zero asymptotically. This we motivate by non-relativistic arguments for hydrongen like bound states, namely

$$F_{hydrogen}(Q^2) = 2\pi \int e^{-iQ\cdot x}\, r^{2l} \exp(-ar)\, d^3x$$
$$\simeq Q^{-2(l+1)} \qquad (5)$$

Thus the form factors of higher orbital angular momentum states approach zero faster than the ground state form factor. Secondly, we have identified the $q^{(n)}\bar{q}^{(n)}$, $J^P = 1^-$, states with heavy vector mesons, the daughters in the Veneziano model. Of these mesons the $\rho'(1600)$ is known to exist and we assume more mesons to exist. We parametrize their masses M_n as follows $M_n^2 = M_\rho^2 + 2n\ \mathrm{GeV}^2$, $n=1...N$, fitting the $\rho(770)$ and the $\rho'(1600)$ and neglecting the weakly coupled $\rho'(1270)$. Since in our model the $q^{(n)}\bar{q}^{(n)}$ state has the quantum numbers of a heavy vector meson we postulate in the form factor $F_{q^{(n)}}$ the existence of a pole at $Q^2 = M^2_n$, $n=1...N$. A solution now for the $F_{q^{(n)}}$ is

$$F_{q^{(n)}}(Q^2) = \frac{M_n^{\ 2}}{M_n^{\ 2} - Q^2 - iM_n\Gamma} \qquad (6)$$

where for the $\rho'(1600)$ one has $\Gamma \sim 0.35$ GeV and we assume this value also for all heavier states.

With the energy $\sqrt{Q^2}$ in the region $\sim$ 5 GeV we have several terms (6) contributing in the annihilation $e^+e^- \rightarrow$ hadrons. The corresponding cross section is

$$\sigma(e^+e^- \rightarrow \text{hadrons}) = \frac{4\pi\alpha^2}{3Q^2}\, 2 \left[|F_q(Q^2)|^2 + \sum |aF_{q^{(n)}}(Q^2)|^2 \right] \quad (7)$$

where a is the relative strength of q and q^* contributions and the ratio R is

$$R = 2\left[|F_q(Q^2)|^2 + \sum_{n=1}^{N} |a\, F_{q^{(n)}}(Q^2)|^2 \right] \qquad (8)$$

Assuming (6) to be a reasonable approximation we can plot R in (8) with a given N (we expect N to be finite) because of the finite range of the ϕ exchange potential). This is seen in fig. 4 . The present inaccurate data from CEA and SLAC indicate that N in (8) is at least 12. A lower limit for R can be obtained as follows. In πp collisions at high energy diffraction dissociation of pions into states of mass about 6 GeV has been reported, as mentioned in section 2.a. We have suggested to identify these heavy mesons as states containing quark excitations with spin > 1/2 to reach the high mass. Assuming that these states have spin one daughters, due to quark spin vector coupling, we would expect to see vector mesons with a similar mass. Thus N in (8) should be as high as N $\sim$ 18 and consequently R would bend down at the earliest at $Q^2 \sim 36\ \text{GeV}^2$. Asymptotically the contributions from the second term of (8) with finite N vanish and we are left with the result

$$R \rightarrow 2(1-C)^2 \sim 1 \qquad \text{as} \quad Q^2 \rightarrow \infty \qquad (9)$$

Interesting enough the measurements at SLAC and DESY next year will extend in energy up to 9 GeV and thus will tell whether there is some fundamental mass value in this region corresponding to the bending down of R.

The vacuum polarization current, proposed to neutralize the quark quantum numbers, may lead to an interesting result concerning hadron electric charges in the e^+e^- annihilation final states. The time of neutralization of the quark pair, into which the photon dissociates, increases linearly with $\sqrt{Q^2}$ [17]. Thus the strong color force is getting softer with increasing energy. This means that the electromagnetic forces between the quarks which ultimately combine to form hadrons in the final state may begin to play a role beyond certain energy. The electromagnetic force is attractive between p and $\bar{p}$, n and $\bar{n}$, n and $\bar{\lambda}$, λ and $\bar{\lambda}$, and λ and $\bar{n}$. Taking into account also the color potential we see that the strongest attraction is for quark systems which are electrically neutral color singlets. This would lead to an excess of neutral mesons in the final state with the lightest mesons being produced most abundantly. This prediction seems to be supported by the preliminary SLAC-LBL data [18].

We have seen above how scale breaking may take place in the time like region. In the spacelike region the inelastic form factor $q\gamma\bar{q}^{(n)}$ would decrease more rapidly than the elastic quark form factor F_q but it is proportional to f_2, fig. 1, which in turn is small. Thus scaling holds approximately in lepton-nucleon deep in elastic scattering in a limited Q^2 region and is smoothly broken with increasing $-Q^2$. μN experiments at NAL will soon show whether this holds.

Finally, let us emphasize that we do not claim to make accurate predictions or to have an exact model. Instead we have made an attempt to understand qualitatively what kind of model could be theoretically satisfactory and would describe the data in a natural way. In other words, we have tried to indicate what are the relevant degrees of freedom in the processes discussed above.

3. CONCLUSIONS

In summary, we have shown that our composite quark model, as applied phenomenologically above, gives

(i) $R = \sigma(e^+e^- \to \text{hadrons})/\sigma(e^+e^- \to \mu^+\mu^-)$ growing with Q^2 when $Q^2 \gtrsim 9\ \text{GeV}^2$; thus naive scaling is broken although there are point like photon couplings. Using the results from πp diffraction data we predict R to increase at least until $\sqrt{Q^2} \sim 6$ GeV where R is about 7 . In the limit $Q^2 \to \infty$ R should approach a constant of about one , eq. (9). An excess of neutral particles is predicted,

(ii) approximate scaling in deep inelastic lepton-hadron scattering which becomes broken smoothly when $-Q^2$ increases because of the smoothly decreasing quark form factor,

(iii) increasing $\sigma_{\text{inelastic}}$ in hadron-hadron scattering at the energy where the quark excitations begin to play a role,

(iv) similar inclusive pion distributions in e^+e^- and hadron-hadron collisions at high energy, and

(v) natural scheme for the $\rho'(1600)$, $N^*(1470)$ etc. and for the diffractively produced high mass states.

On the more theoretical side it goes without saying that any detailed quark model meets with difficulties: the model contains directly unmeasurable quantities related to quark properties. When proposing this type of model one must pay great attention to the economy of the model. Any new or unconventional feature of the model must explain as many previously unexplainable problems as possible. The present model shows some promise in this respect: (i) it combines the constituent and current quark models, (ii) it indicates how quark and bootstrap theories may be married and (iii) it helps to explain important discrepancies between data and the traditional quark model. On the other hand, the model is speculative at present and many questions are yet to be solved.

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FIGURE CAPTIONS

Fig. 1. The $q\gamma q^*$ vertex.

Fig. 2. Graphs indicating the ρ' decay into a) $\rho\pi\pi$ and b) $\pi\pi$.

Fig. 3. Quark contributions to the annihilation $e^+e^- \to \gamma \to$ a) $\bar{q}^*q$ and b) $\bar{q}^*q^*$.

Fig. 4. Our calculation and experimental points for $R = \sigma(e^+e^- \to \text{hadrons})/\sigma(e^+e^- \to \mu^+\mu^-)$. It is argued in the text that R should bend down at $\sqrt{Q^2} \sim 6$ GeV or later at some finite $\sqrt{Q^2}$ value. It is quite possible that the decrease of R, if confirmed experimentally, is slower than shown in the figure. The curve has been drawn with $a^{-2} = 130$, $C = 0$ and $N = 18$ in eq. (8).

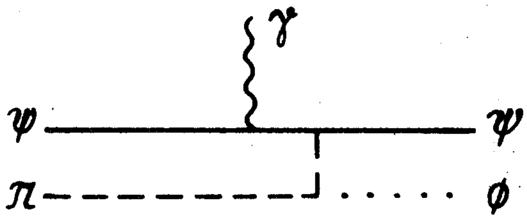

Fig.1.

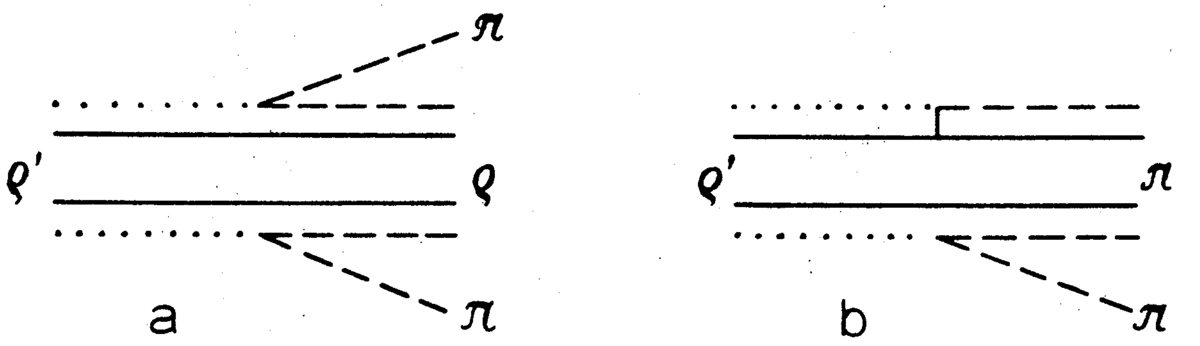

Fig.2.

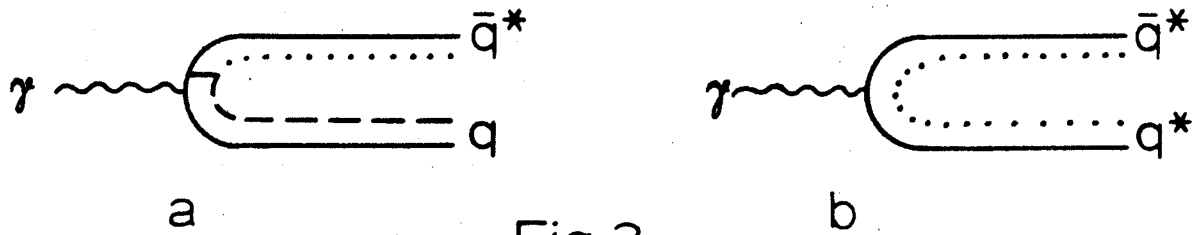

Fig.3.

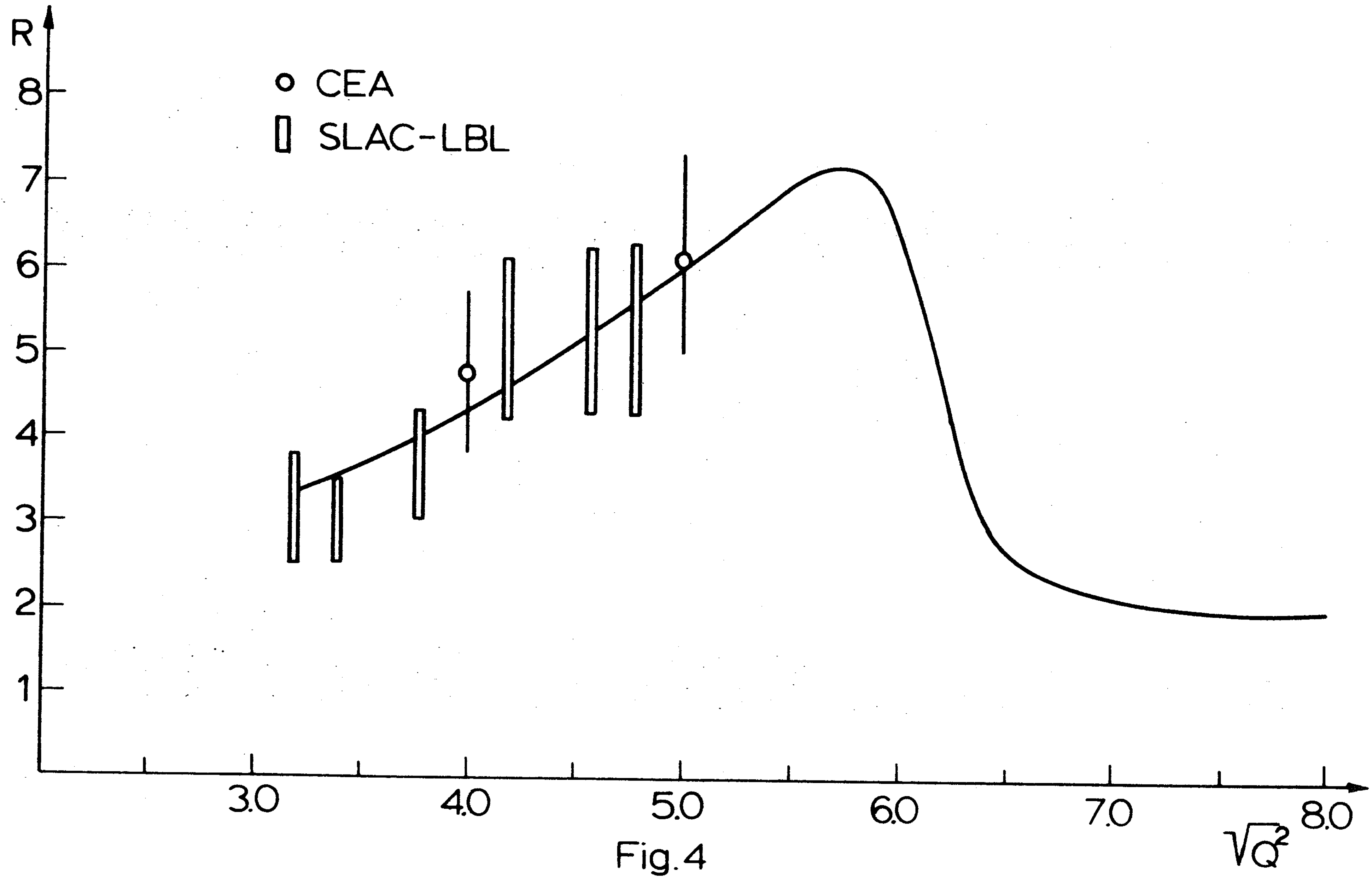

Fig. 4