Intuitionistic fuzzy decision-making in the framework of Dempster-Shafer structures

Liguo Fei*

***Abstract*—The main emphasis of this paper is placed on the problem of multi-criteria decision making (MCDM) in intuitionistic fuzzy environments. Some limitations in the existing literature that explains Atanassov' intuitionistic fuzzy sets (A-IFS) from the perspective of Dempster-Shafer theory (DST) of evidence have been analyzed. To address the issues of using Dempster's rule to aggregate intuitionistic fuzzy values (IFVs), a novel aggregation operator named OWA-based MOS is proposed based on ordered weighted averaging (OWA) aggregation operator, which allows the expression of decision makers' subjectivity by introducing the attitudinal character. The effectiveness of the developed OWA-based MOS approach in aggregating IFVs is demonstrated by the known example of MCDM problem. To compare different IFVs obtained from the OWA-based MOS approach, the golden rule representative value for IFVs comparison is introduced, which can get over the shortcomings of score functions. The hierarchical structure of the proposed decision approach is presented based on the above researches, which allow us to solve MCDM problem without intermediate defuzzification when not only criteria, but their weights are represented by IFVs. The proposed OWA-based MOS approach is illustrated as a more flexible decision-making method, which can better solve the problem of intuitionistic fuzzy multi-criteria decision making in the framework of DST.**

***Index Terms*—Intuitionistic fuzzy set, Dempster-Shafer evidence theory, Multi-criteria decision making, Intuitionistic/DST approach, Ordered weighted average, Golden rule representative values.**

I. Introduction

As a crucial component in many fields associated with engineering, technology, economics, management, military, etc., multi-criteria decision making technology has received considerable attention in both theory and practice [1], [2]. In the process of multi-criteria decision making, the evaluation information derived from different experts or other sources is usually imperfect, i.e., ambiguous, uncertain and even conflicting. To express the information more effectively, quiet a few fuzziness theories, such as fuzzy sets (FSs) [3], intuitionistic fuzzy sets (IFSs) [4], [5], hesitant fuzzy sets (HFSs) [6] and pythagorean fuzzy sets (PFSs) [7] have been developed. Among these fuzziness theories, IFS has been widely employed in MCDM problems because of its flexibility in representing and managing fuzzy information. Intuitionistic fuzzy sets theory was proposed by Atanassov [8], which is a feasible extension of fuzzy sets theory and appears to be significant and available in a multitude of applications. Note that, in the current paper, the Atanassov's intuitionistic fuzzy sets is denoted as A-IFS. The degree of membership μ and non-membership ν are simultaneous taken into account in the definition of A-IFS with the requirement that $0 \leq \mu + \nu \leq 1$. The most paramount applications of A-IFS are the decision making problems [9], [10]. In the framework of A-IFS, a decision making problem can be described as follows.

Let $A = \{x_1, x_2, \ldots, x_m\}$ be a set of alternatives, $C = \{c_1, c_2, \ldots, x_n\}$ be a set of criteria, whose weights are denoted as $W = \{w_1, w_2, \ldots, w_n\}$. Let μ_{ij} be the degree to which x_i satisfies criterion c_j and ν_{ij} be the degree to which x_i does not satisfy criteria c_j, and $0 \leq \mu+\nu \leq 1$, consequently alternative x_i can be represented as $x_i = \{(w_1, <\mu_{i1}, \nu_{i1}>), (w_2, <\mu_{i2}, \nu_{i2}>), \ldots, (w_n, <\mu_{in}, \nu_{in}>)\}$.

The decision problem in intuitionistic fuzzy environments is mainly divided into two facets, including developing effective aggregation operators and determining appropriate score functions. In the decision process, different criteria need to be aggregated to obtain the final evaluation of an alternative, accordingly how to define reasonable aggregation operators has captured scholars' attention. Xu [11] developed some aggregation operators, such as the intuitionistic fuzzy weighted averaging operator, intuitionistic fuzzy ordered weighted averaging operator, and intuitionistic fuzzy hybrid aggregation operator. And some geometric aggregation operators, such as intuitionistic fuzzy weighted geometric operator, the intuitionistic fuzzy ordered weighted geometric operator, and intuitionistic fuzzy hybrid geometric operator were presented by Xu and Yager in [12]. Based on which, the induced intuitionistic fuzzy ordered weighted geometric operator was proposed by Wei [13]. The intuitionistic fuzzy archimedean heronian aggregation operator and intuitionistic fuzzy weight archimedean heronian aggregation operator were introduced by Liu and Chen [14]. A series of generalized geometric interaction averaging aggregation operators are developed by Garg [15], which includes the weighted, ordered weighted and hybrid weighted averaging operators. And a multitude of other aggregation operators were presented from different perspectives [16], [17], [18], [19], [20], [21].

It is worth noting that A-IFS does not exist independently, but could be combined with other uncertainty modeling theories. As an example, a dynamic weight Determination approach was developed based on the intuitionistic fuzzy Bayesian network and was applied to emergency decision making in [22]. In addition, combined with A-IFS and rough sets theory, a pair of lower and upper intuitionistic fuzzy rough approximation operators induced from an arbitrary intuitionistic fuzzy relation are defined by Zhou and Wu [23]. And recently an interpretation of intuitionistic fuzzy sets in terms of Dempster-shafer theory was introduced for decision

Liguo Fei is with the School of Management, Harbin Institute of Technology, Harbin 150001, China.
*Correspondence: feiliguohit@163.com (L. Fei).

making by Dymova and Sevastjanov [24]. Based on which, in the current paper, we focus on the shortcomings of using DST to represent A-IFS, and propose some improved algorithms. Details are discussed in later sections. To manifest why the interpretation of A-IFS in terms of DST are indispensable, we shall first briefly introduce some limitations of traditional intuitionistic fuzzy decision. Generally, the final evaluation results of alternatives associated with A-IFS decision are represented by IFVs, accordingly score functions are needed for decision making. However, imprecise conclusions may be obtained in this process, as any defuzzification operation inevitably leads to loss of information. To address this issue, a multitude of aggregation operators have been proposed as described above, which have the common limitation that the weights $w_i, i = 1, \ldots, n$, should be real values, and they can also be expressed by IFVs in quiet a few cases. In [24], to break this limitation, a novel approach to aggregate criteria was presented based on DST. But there are still some shortcomings of applying DST to the operation of A-IFS, the details are discussed in Sec. III. In response to these shortcomings, in this study, we propose the improved DST approach for intuitionistic fuzzy decision.

In the first place, a local aggregation algorithm for IFVs is introduced based on the idea in [25] in terms of DST, which essentially employs the knowledge of average aggregation, i.e. first averages all criteria and then combines the average $p-1$ times using the Dempster's rule (the number of criteria is p). Some unreasonable points of this algorithm are observed by a numerical example, as an improvement, we further develop the OWA-based mean orthogonal sum (MOS) algorithm inspired by the idea of soft likelihood functions in [26]. In OWA-based MOS, the attitudinal character α is introduced to express the attitude of decision makers, which can be reflected by OWA weights. If α is closer to 1, it means that the weight assigned to w (note that w here refers to the weight vector associated with OWA operator, not the criteria weight) has a smaller index, i.e. the OWA-based MOS is determined with more optimistic attitude by decision makers, consequently the MOS is larger, and vice versa. In OWA-based MOS, all the IFVs to be aggregated need to be sorted in descending order based on an index function determined by golden rule representative values (it will be introduced later), then the step-by-step combination of ordered IFVs will be performed based on the defined local aggregation algorithm. Finally, the results of step-by-step combination will be aggregated based on the OWA operator. To optimize the proposed OWA-based MOS algorithm, the index function γ is redefined by considering reliability and compatibility, respectively. The reliability is measured from the perspective of entropy (i.e. uncertainty) of IFVs, and the compatibility is determined by the extent to which an IFV is supported by other IFVs. Some analysis is provided to demonstrate the effectiveness of the improved OWA-based MOS algorithm.

The aggregation results of the OWA-based MOS algorithm is in the form of IFVs, accordingly the problem of comparison of IFVs arises. This is another facet of the issue that intuitionistic fuzzy decision-making needs to address. The classical score function was proposed by Chen and Tan [27] as $S(x_j) = \mu(x_j) - \nu(x_j)$. It is obvious that if $S(x_i) > S(x_j)$ then x_i should be more superior than x_j, but $S(x_i) = S(x_j)$ does not invariably imply that x_i is equal to x_j. Consequently, accuracy function was developed by Hong and Choi [28] as $H(x_j) = \mu(x_j) + \nu(x_j)$. So a pair of IFVs can be compared based on [29] as:

if $S(x_i) > S(x_j)$, then x_i is better than x_j,
if $S(x_i) > S(x_j)$, then
  if $H(x_i) = H(x_j)$, then x_i is equal to x_j,
  if $H(x_i) < H(x_j)$, then x_j is better than x_i.

A number of other comparison methods were followed. A new score and accuracy functions were introduced by Wu and Chiclana [30] considering decision makers' attitudinal character. The novel score function and accuracy function were proposed by Wang and Chen [31] using the linear programming methodology. And many other score functions have also been developed [28], [29], [32]. Dymova and Sevastjanov [24] noted that "The method for IFVs comparison based on the functions S and H seems to be intuitively obvious and this is its undeniable merit. On the other hand, as two different functions S and H are needed to compare IFVs, this method generally does not provide an appropriate technique for the estimation of an extent to which one IFV is grater/smaller than the other, whereas such information is usually important for a decision maker." To overcome the shortcomings of the existent methods for IFVs comparison and interval comparison, some associated scalar values are employed in this study, which are referred to as representative values. Details are introduced in Sec. IV.

In summary, this paper analyzes the shortcomings in existing literature using DST to solve the intuitionistic fuzzy decision problems. For these limitations, we propose the OWA-based mean orthogonal sum algorithm, which can effectively address these issues and avoid the loss of information caused by the process of defuzzification. In addition, based on the theoretical research of this paper, the hierarchical structure is provided to solve the MCDM problem in the framework of intuitionistic/DST approach. Some numerical examples and analysis demonstrate the effectiveness of the developed decision approach.

The rest of this paper proceeds as follow. Sec. II devotes to the basic introduction of the DST and A-IFS. Sec. III discusses the limitations of applying DST to the operation of IFVs. Sec. IV introduces the representative values and golden rule for A-IFSs. Sec. V provides the concept of ordered weighted averaging (OWA) aggregation operator. Sec. VI presents the improved DST approach for intuitionistic fuzzy decision. Sec. VII illustrates the decision process of MCDM problems in the framework of intuitionistic/DST approach. Sec. VIII concludes this paper and gives some future research perspectives.

II. Preliminaries

A. Dempster-Shafer Theory

Dempster-Shafer theory (DST) is called evidence theory or belief function theory, and its origins can be traced back to Dempster's work [33], where a system of upper and lower probabilities was developed. Following this work, DST was further refined in the book "A Mathematical Theory of Evidence" by Shafer [34] in 1976. DST is mostly used for uncertain information modeling [35], [36], [37], [38], [39], [40], [40]. The basic concepts of DST are introduced as follows.

Let us consider a finite discrete set $\Theta = \{\theta_1, \theta_2, \ldots, \theta_n\}$ including n mutually exclusive and exhaustive hypotheses. And Θ is the frame of discernment (FoD) including the problems under consideration. Let us denote 2^Θ as the power set of Θ. So for $\Theta = \{\theta_1, \theta_2, \ldots, \theta_n\}$, $2^\Theta = \{\phi, \theta_1, \theta_2, \ldots, \theta_n, \theta_1 \cup \theta_2, \theta_1 \cup \theta_3, \ldots, \Theta\}$. Each subset (e.g. $\theta_i, \theta_i \cup \theta_j, \ldots$) in 2^Θ is called a proposition which characterizes the solution to a decision problem in this paper. If the proposition is represented by a single subset (e.g. $\theta_i \in \Theta$), it indicates that the solution contains only one alternative. But for the case of multiple subsets, such as $\{\theta_i, \theta_j\}$, it denotes the proposition "there are two alternatives θ_i and θ_j in the solution, but which one is better is unknown".

A basic probability assignment (BPA) is defined as a function $m : 2^\Theta \to [0, 1]$ satisfying

$$m(\phi) = 0, \sum_{A \in 2^\Theta} m(A) = 1 \tag{1}$$

A denotes one of the propositions in 2^Θ and is called focal element if $m(A) > 0$. The cardinality of focal element is defined as the number of elements in it.

In [34], some measures associated with BPA were given by Shafer. The belief measure is defined by a mapping $Bel : 2^\Theta \to [0, 1]$ which satisfies:

$$Bel(A) = \sum_{B \in 2^\Theta | B \subseteq A} m(B), \forall A \subseteq \Theta \tag{2}$$

The plausibility measure is also defined in [34] by a mapping $Pl : 2^\Theta \to [0, 1]$ as:

$$Pl(A) = \sum_{B \in 2^\Theta | A \cap B \neq \phi} m(B), \forall A \subseteq \Theta \tag{3}$$

Note that $Bel(\cdot)$ and $Pl(\cdot)$ are the lower and upper bounds of probability associated with a BPA, respectively, which can be proved to satisfy $Bel(A) \leq Pl(A)$ for all $A \subseteq \Theta$. In DST, the interval $[Bel(A), Pl(A)]$ is used to measure the degree of imprecision for proposition A, which is called the belief interval (BI).

The DST is most commonly used in multi-source information fusion, where the Dempster's rules of combination [33] will be used. The rule assumes that the sources of evidence are independent, and n pieces of evidence can be combined by orthogonal sum: $m = m_1 \oplus m_2 \oplus \ldots \oplus m_n$, where $\oplus$ denotes the operator of fusion. The Dempster's rule of combination for two pieces of evidence can be defined as follows.

Definition II.1. *Let m_1 and m_2 be two BPAs, the Dempster's rule denoted by $m = m_1 \oplus m_2$ is defined as:*

$$m(A) = \frac{\sum_{B \cap C = A} m_1(B) m_2(C)}{1 - K} \tag{4}$$

with

$$K = \sum_{B \cap C = \phi} m_1(B) m_2(C) \tag{5}$$

Note that the Dempster's rule can only be employed for m_1 and m_2 when $K < 1$.

B. Interpretation of intuitionistic fuzzy sets in the framework of DST

In [24], a new interpretation of intuitionistic fuzzy sets in the framework of DST was presented. Based on which, the relationship between intuitionistic fuzzy sets and Dempster-Shafer theory will be reviewed briefly as follows. Intuitionistic fuzzy sets proposed by Atanassov [8] is referred to as $A{-}IFS$, and its basic concepts will be introduced next.

Definition II.2. *Let $X = \{x_1, x_2, \ldots, x_n\}$ be a finite universal set. An intuitionistic fuzzy set A in X can be expressed by the form: $A = \{< x_j, \mu_A(x_j), \nu_A(x_j) > | x_j \in X\}$, where functions $\mu_A : X \to [0, 1]$, $x_j \in X \to \mu_A(x_j) \in [0, 1]$ and $\nu_A : X \to [0, 1]$, $x_j \in X \to \nu_A(x_j) \in [0, 1]$ represent the degree of membership and degree of non-membership of element $x_j \in X$ to the set $A \subseteq X$, respectively, and satisfy $\mu_A(x_j) + \nu_A(x_j) \in [0, 1]$ for each $x_j \in X$.*

Note that $\pi_A(x_j) = 1 - \mu_A(x_j) - \nu_A(x_j)$ is called hesitation degree (or intuitionistic index) of element x_j in set A. Obviously, we can get $\pi_A(x_j) \in [0, 1]$ for each $x_j \in X$.

The interval representation of intuitionistic fuzzy sets was proposed in [28] for MCDM problems, specifically, the intuitionistic fuzzy set $A =< \mu_A(x_j), \nu_A(x_j) >$ in X can be represented by interval $[\mu_A(x_j), 1 - \nu_A(x_j)]$. Two characteristics and advantages were summarized as follows. (1) the interval expression $[\mu_A(x_j), 1 - \nu_A(x_j)]$ is a regular interval because it is obvious that $\mu_A(x_j) \leq 1 - \nu_A(x_j)$, which can be regard as the interval valued fuzzy sets interpretation of A-IFS; (2) the basic operators of A-IFS can be redefined in terms of DST. To represent A-IFS with DST, the element $\mu_A(x_j)$ and $\nu_A(x_j)$ constitute the FoD in DST as $\Theta = \{\mu_A(x_j), \nu_A(x_j)\}$. For any situation in context of A-IFS, there are three general hypotheses: $x_j \in A$, $x_j \notin A$ and the case where both the hypotheses $x_j \in A$ and $x_j \notin A$ cannot be distinguished (hesitation). These three hypotheses can be denoted in the environment of DST as $\{Yes\}(x_j \in A)$, $\{No\}(x_j \notin A)$ and $\{Yes, No\}(x_j \in A$ and $x_j \in A)$.

In this context, $\mu_A(x_j)$ can be regard as the belief degree of $x_j \in A$ satisfying $m(\{Yes\}) = \mu_A(x_j)$. Similarly, we have $m(\{No\}) = \nu_A(x_j)$. For hesitation degree $\pi_A(x_j)$, we have $m\{Yes, No\} = \pi_A(x_j)$. According to Def. II.2, we have $\mu_A(x_j) + \nu_A(x_j) + \pi_A(x_j) = 1$, so $m(\{Yes\}) = \mu_A(x_j), m(\{No\}) = \nu_A(x_j), m\{Yes, No\} = \pi_A(x_j)$ can satisfy a standard BPA. Based on Eqs. (2) and (3), we have $Bel_A(x_j) = m(\{Yes\}) = \mu_A(x_j)$ and $Pl_A(x_j) = m(\{Yes\}) + m(\{Yes, No\}) = \mu_A(x_j) + \pi_A(x_j) = 1 - \nu_A(x_j)$. The interval representation of the A-IFSs in DST can be defined as follows.

Definition II.3. *Let $X = \{x_1, x_2, \ldots, x_n\}$ be a finite universal set and x_j is an object in X whose degree of membership and degree of non-membership are represented by functions $\mu_A(x_j)$ and $\nu_A(x_j)$, an interval representation of IFS can be denoted as: $A = \{< x_j, BI_A(x_j) > | x_j \in X\}$ where*

$BI_A(x_j) = [Bel_A(x_j), Pl_A(x_j)] = [\mu_A(x_j), 1-\nu_A(x_j)]$ *is called belief interval.*

Dymova and Sevastjanov [24] pointed out that DST semantics can enhance the performance of A-IFS in MCDM problems. What's more, the IFVs information represented by belief interval can be aggregated based on Dempster's rule of combination, then an approach can be developed without defuzzification when assessment information and their weights are expressed by IFVs. The advantages of using interval representation of A-IFS in DST are very obvious. However, in some cases, the limitations of aggregating A-IFSs based on Dempster's rule can be found in context of MCDM problems. Several critical examples are given to illustrate this problem in the next section.

III. The limitations of applying DST to the operation of IFVs

Some limitations of using Dempster's rule of combination for the aggregation operator of IFVs have been analysed in [24]. In this section, the additional drawbacks are discussed. To make our consideration more clearly, the example in [41], [42] is used, whose background is described as follows. Consider the problem to select an air-condition system, the alternatives are denoted by $X = \{x_1, x_2, x_3\}$. And there are three criteria: economical (c_1), function (c_2) and being operative (c_3), which are denoted by $C = \{c_1, c_2, c_3\}$. The degrees μ_{ij} of membership and the degrees ν_{ij} of non-membership for the alternative $x_j \in X$ with respect to the criterion $c_i \in C$ to the fuzzy evaluation "excellent" can be obtained by using some statistical approaches as follows.

$$(<\mu_{ij}, \nu_{ij}>)_{3\times 3} = \begin{bmatrix} & c_1 & c_2 & c_3 \\ x_1 & <0.75, 0.1> & <0.60, 0.25> & <0.80, 0.2> \\ x_2 & <0.8, 0.15> & <0.68, 0.20> & <0.45, 0.5> \\ x_3 & <0.4, 0.45> & <0.75, 0.05> & <0.60, 0.3> \end{bmatrix} \quad (6)$$

Then the degrees ρ_i of membership and the degrees τ_i of non-membership for criterion $c_i \in C$ to the fuzzy evaluation "importance" can be obtained as follows.

$$(<\rho_i, \tau_i>)_{1\times 3} = \begin{bmatrix} c_1 & c_2 & c_3 \\ <0.25, 0.25> & <0.35, 0.4> & <0.3, 0.65> \end{bmatrix} \quad (7)$$

Dymova and Sevastjanov [24] considered C as a set of local criteria, so μ_{ij} and ν_{ij} are the degrees to which x_j satisfies and dissatisfies the local criterion $c_i \in C$. That is, the air-condition system selection problem can be reformulated in terms MCDM. DST interpretation of A-IFS was employed in [24] to obtain the final alternatives' evaluations (FAE). Based on Sec. II-B, for the pair (x_i, c_j), its BPA representation can be denoted as $m^e_{ij}(\{Yes\}) = \mu_{ij}$, $m^e_{ij}(\{No\}) = \nu_{ij}$, $m^e_{ij}(\{Yes, No\}) = 1-\mu_{ij}-\nu_{ij} = \pi_{ij}$; for the relative weight of criteria, its BPA representation can be denoted as $m^w_j(\{Yes\}) = \rho_j$, $m^w_j(\{No\}) = \tau_j$, $m^w_j(\{Yes, No\}) = 1-\rho_{ij}-\tau_{ij}$. To select the optimal air-condition system, for each alternative x_i, the weighted local criteria values will be obtained by aggregated the evaluation information and its corresponding weight based on Dempster's rule. To get the $FAE(x_i)$, the weighted evaluation values will be aggregated by using Dempster's rule. The final BPA of x_i can be denoted as $m_i(\{Yes\})$, $m_i(\{No\})$ and $m_i(\{Yes, No\})$, so the corresponding belief interval is $[Bel(x_i), Pl(x_i)] = [m_i(\{Yes\}), m_i(\{Yes\}) + m_i(\{Yes, No\})]$. Based on the above process, the following results for the air-condition system selection problem can be obtained:

$$\begin{aligned} FAE(x_1) &= [Bel(x_1), Pl(x_1)] = [.9257, .9257], \\ FAE(x_2) &= [Bel(x_2), Pl(x_2)] = [.8167, .8168], \\ FAE(x_3) &= [Bel(x_3), Pl(x_3)] = [.7377, .7379]. \end{aligned} \quad (8)$$

A limitation can be found from the result that $FAE(x_1) = [Bel(x_1), Pl(x_1)] = [.9257, .9257]$ has degenerated into a real number, which means that $m_1(\{Yes, No\}) = 0$. This result can be explained from the calculation process. The weighting process of the pair (x_1, c_3) can be expressed as:

$$m^w_{1,3}(\{Yes, No\}) = \frac{(1-\mu_{13}-\nu_{13})(1-\rho_3-\tau_3)}{1-K} \quad (9)$$

as $1-\mu_{13}-\nu_{13} = 1-0.8-0.2 = 0$ in this case, so $m^w_{1,3} = 0$ and the final result can be obtained as:

$$\begin{aligned} & m_1(\{Yes, No\}) = \\ & \frac{m^w_{1,1}(\{Yes, No\}) m^w_{1,2}(\{Yes, No\}) m^w_{1,3}(\{Yes, No\})}{1-K} \\ & = 0 \end{aligned} \quad (10)$$

In terms of A-IFS, the above result means that the final evaluation of x_1 can be expressed by the intuitionistic fuzzy value $<m_1, 1-m_1>$ which cannot represent hesitancy degree of "x_1 is excellent". So the limitation of using the interpretation of A-IFS in the framework of DST can be summarized as: if any IFV representation of criteria values has no hesitation, then the aggregation result of using Dempster's rule will has no hesitation. The belief interval representation of A-IFS was employed in [24], and some aggregation methods were introduced, which can solve this problem to some extent. But this means the advantages of DST in terms of combination have been abandoned. A more reasonable solution will be proposed in this study.

To further demonstrate the limitation of using Dempster's rule as an aggregation operator for A-IFS, a more extreme example is given as follows. In the above example, to explain the problem more clearly, we re-present the degrees $\bar{\rho}_i$ of membership and the degrees $\bar{\tau}_i$ of non-membership for criterion $c_i \in C$ to the fuzzy evaluation "importance" as:

$$(<\bar{\rho}_i, \bar{\tau}_i>)_{1\times 3} = \begin{bmatrix} c_1 & c_2 & c_3 \\ <0, 1> & <0.35, 0.4> & <0.3, 0.65> \end{bmatrix} \quad (11)$$

This case can be interpreted as the decision maker holds that criterion c_1 is not important at all, that is, the air-condition system can be selected without considering economic issues. The decision process can be given as in the previous example. For each pair (x_i, c_j), the combined BPAs can be calculated as in Eq. (9). So the weighted evaluation values of each alternative x_i can be presented by the structure M_i as follows:

$$M_1 = \begin{bmatrix} \overline{m}_{11}(\{Yes\}) = 0 & \overline{m}_{11}(\{No\}) = 1 & \overline{m}_{11}(\{Yes, No\}) = 0 \\ \overline{m}_{12}(\{Yes\}) = 0.61 & \overline{m}_{12}(\{No\}) = 0.33 & \overline{m}_{12}(\{Yes, No\}) = 0.06 \\ \overline{m}_{13}(\{Yes\}) = 0.67 & \overline{m}_{13}(\{No\}) = 0.33 & \overline{m}_{13}(\{Yes, No\}) = 0 \end{bmatrix} \quad (12)$$

$$M_2 = \begin{bmatrix} \overline{m}_{21}(\{Yes\}) = 0 & \overline{m}_{21}(\{No\}) = 1 & \overline{m}_{21}(\{Yes, No\}) = 0 \\ \overline{m}_{22}(\{Yes\}) = 0.68 & \overline{m}_{22}(\{No\}) = 0.27 & \overline{m}_{22}(\{Yes, No\}) = 0.05 \\ \overline{m}_{23}(\{Yes\}) = 0.31 & \overline{m}_{23}(\{No\}) = 0.68 & \overline{m}_{23}(\{Yes, No\}) = 0.01 \end{bmatrix} \quad (13)$$

$$M_3 = \begin{bmatrix} \overline{m}_{31}(\{Yes\}) = 0 & \overline{m}_{31}(\{No\}) = 1 & \overline{m}_{31}(\{Yes, No\}) = 0 \\ \overline{m}_{32}(\{Yes\}) = 0.76 & \overline{m}_{32}(\{No\}) = 0.16 & \overline{m}_{32}(\{Yes, No\}) = 0.08 \\ \overline{m}_{33}(\{Yes\}) = 0.46 & \overline{m}_{33}(\{No\}) = 0.52 & \overline{m}_{33}(\{Yes, No\}) = 0.02 \end{bmatrix} \quad (14)$$

The final BPAs of each alternative x_i can be obtained as in Eq. (10):

- $\overline{m}_1(\{Yes\}) = 0, \overline{m}_1(\{No\}) = 1, \overline{m}_1(\{Yes, No\}) = 0,$
- $\overline{m}_2(\{Yes\}) = 0, \overline{m}_2(\{No\}) = 1, \overline{m}_2(\{Yes, No\}) = 0,$
- $\overline{m}_3(\{Yes\}) = 0, \overline{m}_3(\{No\}) = 1, \overline{m}_3(\{Yes, No\}) = 0.$

So the final alternatives' evaluations can be obtained as:

$$\begin{aligned} \overline{FAE}(x_1) &= [\overline{Bel}(x_1), \overline{Pl}(x_1)] = [0, 1], \\ \overline{FAE}(x_2) &= [\overline{Bel}(x_2), \overline{Pl}(x_2)] = [0, 1], \\ \overline{FAE}(x_3) &= [\overline{Bel}(x_3), \overline{Pl}(x_3)] = [0, 1]. \end{aligned} \quad (15)$$

It is obvious that the results are unreasonable. It means that using Dempster's rule of combination to aggregate IFVs in decision-making problems can lead to counterintuitive results. This result illustrates an important feature of the Dempsters rule of combination: for multi-source evidence, if one of the pieces of evidence fully supports a proposition, the fusion result will still fully distribute the belief to this proposition regardless of how other pieces of evidence are distributed. Therefore, in intuitionistic fuzzy decision-making environments, even if x is "excellent" under most criteria, if x performs poorly under a certain criterion, e.g. $< \mu_j, \nu_j >=< 0, 1 >$, the aggregation result will tell that x is worthless. This is equivalent to denying everything of an alternative because of one of its shortcomings, which is obviously unreasonable. A more efficient approach will be proposed to solve this problem in this study.

In this section, some limitations of using rules in terms of DST to A-IFS are summarized by several examples. To address these issues, one of the aims of this paper is to develop such frameworks, based on the novel definition of fusion rule in DST, which make it possible to obtain the set of operations on A-IFV free of the limitations illustrated by above examples.

IV. Representative values and golden rule for A-IFSs

In this paper, decision-making is carried out in intuitionistic fuzzy environments, and belief interval is involved in the study, so appropriate approaches are needed to compare different IFVs and BIs for decision-making. In this section, to overcome the shortcomings of the existent methods for IFVs comparison and interval comparison, some associated scalar values are employed in this study, which are referred to as representative values. Particularly, the golden rule representative values is mainly used here as a notable representative value.

A. Golden rule representative value for belief interval

A scalar value was defined in [43] to address the problem ranking alternatives in MCDM problems uncer intuitionistic fuzzy environments, which is called representative value. Based on representative values, different intuitionistic fuzzy values for alternatives can be compared, and the bigger the representative value the more better the alternative. The subjectivity was considered by Yager in MCDM problem from the dimension of optimism/pessimism, where optimism means that this representative value prefers to assign weights to support for membership and pessimism the contrary. To include the decision makers' preference, golden rule representative value was developed by Yager [44], [45] to compare Atanassov type intuitionistic fuzzy values. In this section, the golden rule for belief interval will be presented firstly, and then intuitionistic fuzzy values' golden rule will be introduced next.

Consider the situations in which the criteria are belief intervals, and the interval for the overall degree of satisfaction of an alternative x_i to the criteria can be denoted as $BI(x_i) = [Bel_i, Pl_i]$. It is obvious that $Bel_i \leq Pl_i$, so $BI(x_i) \subseteq [0, 1]$. To compare different belief intervals, we indicate the belief interval of another alternative x_j: $BI(x_j) = [Bel_j, Pl_j]$. Obviously, if $Bel_j \geq Pl_i$ then the conclusion can be drawn that x_j is preferred to x_i, which is denoted as $[Bel_j, Pl_j] \succ [Bel_i, Pl_i]$. To be the optimal alternative in $\{x_1, x_2, \ldots, x_n\}$, x_k should satisfy $[Bel_k, Pl_k] \succ [Bel_i, Pl_i]$, $\forall i \neq k$, which is very difficult to meet generally. To address this issue, a representative scalar value $Rep(x_i)$ was assigned to each belief interval $BI(x_i)$ so that alternatives can be compared by their representative value. In this case, if $Rep(x_j) \geq Rep(x_i)$ we have x_j is preferred to x_i. In addition, it is clear that if $[Bel_j, Pl_j] \succ [Bel_i, Pl_i]$ then $Rep(x_j) \geq Rep(x_i)$. In employing the representative values, the subjective preferences of decision makers are used implicitly by Yager [44]. So we need to determine an appropriate function Rep to compare different belief intervals considering with the preferences and attitude of decision makers. Allowing some subjective space is important to select the appropriate function Rep, especially when we have some intuitive understanding of the attitude in MCDM problem. One common Rep function is the mid-point:

$$Rep_{BI}(x_i) = m_i = \frac{Bel_i + Pl_i}{2} \quad (16)$$

Although it is simple and effective, it will lead to unreasonable conclusions in some cases. Consider the case in Fig. 2, where alternatives x_i and x_j have the same mid-point value, but ranges (denoted as $r_. = Pl_. - Bel_.$) for their possible degrees of satisfactions are different. Some decision makers may prefer larger range (variability), where x_j is preferred to x_i; in other cases, decision makers may prefer smaller variability, where x_i is better.

So the more general formulation for the Rep function of belief intervals which includes the mid-point $m_.$, the range $r_.$ and a subjective parameter $\lambda \in [-1, 1]$ reflecting the attitude of decision makers is given as:

$$Rep_{BI}(x_.) = m_. + \lambda \frac{r_.}{2} \quad (17)$$

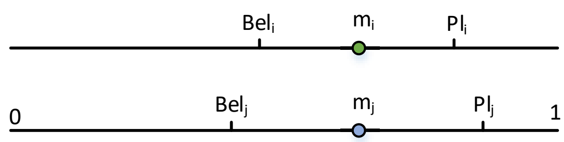

Fig. 1. Example of two belief intervals.

where parameter $\lambda > 0$ stands for decision maker's optimistic attitude toward the uncertainty; $\lambda < 0$ expresses the pessimistic attitude toward the uncertainty associated with the satisfaction by an alternative; $\lambda = 0$ indicates the neutral attitude of the decision maker. Some extreme cases are analyzed as follows:

- if $\lambda = 0$, $Rep_{BI}(x_.) = m_. = \frac{1}{2}(Pl_. + Bel_.)$;
- if $\lambda = 1$, $Rep_{BI}(x_.) = Pl_.$;
- if $\lambda = -1$, $Rep_{BI}(x_.) = Bel_.$.

the conclusion can be drawn that optimistic decision makers prefer wider r value, while pessimistic decision makers prefer narrower r values. So for two alternatives with the same $m_.$, if $\lambda > 0$ the one with the larger r will be selected, and if $\lambda < 0$ the one with the smaller r will be chosen.

A rule-based specification of Rep function was suggested in [46] and implemented by Takagi-Sugeno method [47], [48], which can be illustrated by four rules reflecting the preference of decision makers as follows:

- if $m_.$ is large and $r_.$ is small, $Rep_{BI}(x_.) = 1$;
- if $m_.$ is large and $r_.$ is large, $Rep_{BI}(x_.) = 0.5$;
- if $m_.$ is small and $r_.$ is large, $Rep_{BI}(x_.) = 0.5$;
- if $m_.$ is small and $r_.$ is small, $Rep_{BI}(x_.) = 0$.

These rules can be explained by decision makers: if the mean of the belief interval is large I am satisfied and I prefer no variability, but if the mean is small I want a large variability. According to [47], [48], the Rep function of alternative $x_.$: $BI(x_.) = [Bel_., Pl_.]$ can be defined as:

$$
\begin{aligned}
Rep_{BI}(x_.) &= \frac{m_.(1-r_.) + 0.5m_.r_. + 0.5(1-m_.)r_. + 0(1-m_.)(1-r_.)}{m_.(1-r_.) + m_.r_. + (1-m_.)r_. + (1-m_.)(1-r_.)} \\
&= m_. + (\frac{1}{2} - m_.)r_.
\end{aligned} \tag{18}
$$

where $m_. = \frac{Bel_. + Pl_.}{2}$, $r_. = Pl_. - Bel_.$. This function Rep is called golden rule representative value which can be used to compare different belief intervals.

B. Golden rule representative value for A-IFS

The golden rule representative value for belief intervals has been introduced in the lase section, based on the relationship between belief interval and A-IFS, the golden rule representative value for A-IFS can be deduced directly as follows.

Consider the situations in which the criteria are intuitionistic fuzzy sets, we assume the IFS representation of alternative x_i is denoted as $< \mu(x_i), \nu(x_i) >$. Based on Def. II.3, its corresponding belief interval can be expressed as $BI(x_i) = [Bel(x_i), Pl(x_i)] = [\mu(x_i), 1 - \nu(x_i)]$. Now we define $Hes(x_i) = \pi(x_i) = 1 - (\mu(x_i) + \nu(x_i))$ as the hesitancy degree of x_i, and $Bias(x_i) = \mu(x_i) - \nu(x_i)$ as the bias degree of x_i. Let $Rep_{A-IFS}(x_i)$ denote the representative value of $< \mu(x_i), \nu(x_i) >$ and the bigger the representative value the more preferred the alternative. According to Eq. (17), we have:

$$
\begin{aligned}
Rep_{BI}(x_i) &= m_i + \lambda \frac{r_i}{2} \\
&= \frac{Bel(x_i)(1-\lambda) + Pl(x_i)(1+\lambda)}{2} \\
&= \frac{\mu(x_i)(1-\lambda) + (1-\nu(x_i))(1+\lambda)}{2} \\
&= \frac{1 + Bias(x_i) + \lambda Hes(x_i)}{2}
\end{aligned} \tag{19}
$$

so the Rep function under A-IFS environment of x_i, which includes mid-point m_i, range r_i and subjective parameter λ can be defined as:

$$
Rep_{A-IFS}(x_i) = \frac{1 + Bias(x_i) + \lambda Hes(x_i)}{2} \tag{20}
$$

where $\lambda \in [-1, 1]$, and if $\lambda > 0$ the hesitancy degree is added to the representative value and if $\lambda < 0$ it is subtracted. several extreme cases are also analyzed as follows:

- if $\lambda = 0$, $Rep_{A-IFS}(x_i) = \frac{1+Bias(x_i)}{2}$;
- if $\lambda = 1$, $Rep_{A-IFS}(x_i) = \frac{1+Bias(x_i)+Hes(x_i)}{2}$;
- if $\lambda = -1$, $Rep_{A-IFS}(x_i) = \frac{1+Bias(x_i)-Hes(x_i)}{2}$.

It is obvious that λ is a kind of description of attitude of decision makers. And $\lambda > 0$ represents optimistic with the larger λ the more optimistic. On the other hand $\lambda < 0$ indicates pessimistic with the smaller λ the more pessimistic. Further, the golden rule representative value for A-IFS based on Takagi-Sugeno model [47], [48] can be obtained as:

$$
\begin{aligned}
Rep_{A-IFS}(x_i) &= m_i + (\frac{1}{2} - m_i)r_i \\
&= \frac{Bel(x_i) + Pl(x_i)}{2} + \\
&\quad (\frac{1 - Bel(x_i) - Pl(x_i)}{2})(Pl(x_i) - Bel(x_i)) \\
&= \frac{1}{2} + \frac{\mu(x_i) - \nu(x_i)}{2}(\mu(x_i) + \nu(x_i)) \\
&= \frac{1}{2} + \frac{1}{2} Bias(x_i)(1 - Hes(x_i))
\end{aligned} \tag{21}
$$

This function Rep is called golden rule representative value for A-IFS which can be used to compare different IFVs.

V. Ordered Weighted Averaging Aggregation Operator

The OWA operator was originally proposed by Yager [49]. An n dimension OWA operator is a mapping: $F : \Re^n \to \Re$, which has an associated weighting vector W: $W = (w_1, w_2, \ldots, w_n)^T$, such that $\sum_j w_j = 1$ and $w_j \in [0, 1]$. Then $F(a_1, \ldots, a_n) = w_1 b_1 + w_2 b_2 + \cdots + w_n b_n$, where b_j is the jth largest element of $\{a_1, a_2, \ldots, a_n\}$.

W denotes the OWA weighting vector and its elements, w_j, represent the OWA weights. Many methods [50], [51], [52], [53] have been proposed to determine OWA weights. But these methods are often highly dependent on the application environment. Some illustrative examples of different types of aggregation were provided by Yager [49].

Let λ be an index function, so $\lambda(j)$ is the index of jth largest value, then we have

$$OWA(a_1,\ldots,a_n)=\sum_{j=1}^{n} w_j a_{\lambda(j)} \tag{22}$$

Some special cases are given to demonstrate the effect of different weight distributions on the final aggregation result.

1) W^*: $w_1 = 1$ and $w_j = 0$ $(j \neq 1)$. So we have $OWA(a_1,\ldots,a_n) = a_{\lambda(1)} = max_i(a_i)$.
2) W_*: $w_n = 1$ and $w_j = 0$ $(j \neq n)$. So we have $OWA(a_1,\ldots,a_n) = a_{\lambda(n)} = min_i(a_i)$.
3) W_n: $w_j = 1/n$ $(j = 1,\ldots,n)$. So we have $OWA(a_1,\ldots,a_n) = (1/n)\sum_{i=1}^{n} a_i$.
4) W_k: $w_k = 1$ and $w_j = 0$ $(j \neq k)$. So we have $OWA(a_1,\ldots,a_n) = a_{\lambda(k)}$.

It can be seen that if more weight with small index is assigned to w_j, the aggregated result will be larger. Conversely, if more weight with large index is assigned to w_j, the aggregated result will be smaller. From another perspective, the weighting vectors can be regard as the the attitudes of decision makers, optimism or pessimism. The attitudinal character [49], [52] extended from OWA operator can be defined as:

$$AC(W)=\sum_{j=1}^{n}\frac{n-j}{n-1}w_j \tag{23}$$

We can have $AC(W)\in[0,1]$, $AC(W^*)=1$, $AC(W_*)=0$, $AC(W_N)=0.5$ and $AC(W_k)=\frac{n-k}{n-1}$. That is, the more of the weight is assigned to the smaller index, the larger the $AC(W)$. So the greater the attitudinal character, the larger the aggregated result, namely, the more optimism the decision makers, the larger the aggregated result. Attitudinal character can express risk or benefits in decision problems.

Here we introduce a function method to obtain the OWA weights. Assume function f: $[0,1]\to[0,1]$ is monotonic, namely, if $x>y$, then $f(x)>f(y)$, with $f(0)=0$ and $f(1)=1$. Accordingly, the weight can be calculated by the following formula

$$w_j = f(\frac{j}{n}) - f(\frac{j-1}{n}) \tag{24}$$

It is obvious that $w_j\in[0,1]$, $\sum_{j=1}^{n} w_j=1$, and w_j has all the properties of the OWA weights [52], [54]. Yager [52] pointed out that the degree of optimism can be measured from the given function f, and the attitudinal character was defined as:

$$Opt(f)=\int_0^1 f(x)dx \tag{25}$$

A significant function is $f(x)=x^m$ for $m\geq 0$, which is useful to determine the optimism degree α as:

$$\alpha=\int_0^1 x^m dx=\frac{x^{m+1}}{m+1}\bigg|_0^1=\frac{1}{m+1} \tag{26}$$

that is $m=\frac{1-\alpha}{\alpha}$. From the above definition, we have $\alpha\in[0,1]$, and the larger α the more optimism. By the above functional form, the OWA weights can be obtained as:

$$w_j = f(\frac{j}{n}) - f(\frac{j-1}{n}) = (\frac{j}{n})^m - (\frac{j-1}{n})^m \tag{27}$$

for a given α, we have

$$w_j = (\frac{j}{n})^{\frac{1-\alpha}{\alpha}} - (\frac{j-1}{n})^{\frac{1-\alpha}{\alpha}} \tag{28}$$

VI. Improved DST approach for intuitionistic fuzzy decision

Consider the MCDM problem in which the criteria are intuitionistic fuzzy values. Let the set X consisting of the elements x_i, for $i=1$ to m, be the set of alternatives. And n criteria are denoted as $C=\{c_j|j=1,\ldots,n\}$. Let I_{ij} be the intuitionistic fuzzy representation that the optimal solution is x_i under evidence c_j.

For a set of intuitionistic fuzzy values $I=\{I_j|j=1,\ldots,n\}$, to aggregate them to obtain the final result, the novel combination algorithm based on Dempster's rule is proposed, which can improve the shortcomings of the traditional method. The detailed steps of the new rule are given as follows.

As analyzed in Sec. III, in two cases, the Dempster's rule of combination cannot obtain satisfactory results for fusing the evidence representation of IFVs. (1) If there exist one IFV without hesitancy degree in the IFVs to be aggregated, then the combination result obtained by Dempster's rule is not hesitant; (2) If one of the IFVs satisfies that all the weights are assigned to the membership degree or the non-membership degree, i.e. $<1,0>$ or $<0,1>$, the aggregation result will also be $<1,0>$ or $<0,1>$. These two issues are fatal in MCDM problem because they can produce erroneous decision results.

The golden rule representative value of each IFV I_j will be calculated based on Eq. (21) and denoted as $Rep_{A-IFS}(I_j)$, which is abbreviated as $\Re(I_j)$. The ordering of different IFVs will be determined based on $\Re$, and let γ be an index function so that $\gamma(k)$ is the index of the kth largest IFV. So $I_{\gamma(k)}$ is the kth largest IFV of all the IFVs to be aggregated, specifically, $I_{\gamma(1)}$ is the largest one. A local aggregation algorithm for IFVs will be introduced based on the literature [25] in terms of DST as follows.

Definition VI.1. *Let $I_{loc}=\{I_l|l=1,\ldots,p\}$ be a set of IFVs from I, which is called local IFVs to be aggregated. For IFV $I_i=<\mu_i,\nu_i>$, its DST representation can be denoted as $m_i(\{Yes\})=\mu_i$, $m_i(\{No\})=\nu_i$, $m_i(\{Yes,No\})=1-\mu_i-\nu_i$. The average of membership degree, non-membership degree and hesitation degree in terms of A-IFS can be calculated by DST representation as follows.*

$$m(\{Yes\})=\frac{1}{p}\sum_{i=1}^{p} m_i(\{Yes\})=\frac{1}{p}\sum_{i=1}^{p}\mu_i \tag{29}$$

$$m(\{No\})=\frac{1}{p}\sum_{i=1}^{p} m_i(\{No\})=\frac{1}{p}\sum_{i=1}^{p}\nu_i \tag{30}$$

$$m(\{Yes,No\})=\frac{1}{p}\sum_{i=1}^{p} m_i(\{Yes,No\})=\frac{1}{p}\sum_{i=1}^{p}(1-\mu_i-\nu_i) \tag{31}$$

It is easy to prove $m(\{Yes\})+m(\{No\})+m(\{Yes,No\})=0$, so m is still a BPA in DST. To obtain the final aggregation *result, m will be combined $p-1$ times based on Dempster's rule.*

$$m_f = \overline{\oplus}(m_1, m_2, \ldots, m_p) = \underbrace{m \oplus m \oplus \ldots \oplus m}_{p-1} \tag{32}$$

Note that the BPA representation of the final aggregation result can be expressed in terms of A-IFS theory as $\mu_f = m_f(\{Yes\})$, $\nu_f = m_f(\{No\})$ and $\pi_f = m_f(\{Yes, No\})$.

More generally, the definition of local aggregation algorithm of IFVs can be used to the overall IFVs to be aggregated. However, some particular cases can be found in the above process, where the counterintuitive results will appear. To illustrate this issue, a numerical example is given as follows.

Example VI.1. *Here a fictitious case is constructed for a decision problem. To select the best employees, 5 experts are invited by the company to evaluate the candidates, and the evaluation results are expressed as IFVs. We mainly focus on the results of candidate x: $I_1 =< 0.1, 0.6 >$, $I_2 =< 0.5, 0.4 >$, $I_3 =< 0.5, 0.4 >$, $I_4 =< 0.5, 0.4 >$ and $I_5 =< 0.5, 0.4 >$, where the degree of membership indicates that the candidate is qualified for the job, while the degree of non-membership indicates that the candidate is not qualified for the job. According to the evaluation information, the first expert believes that x is competent for the job, but the last four experts consider he could not. As common sense, this candidate will be selected by the company. To determine if candidate x should be hired, the local aggregation algorithm proposed in Def. VI.1 is employed to assist decision making. First the BPA representation of five evaluation results will be obtained as: $m_1 = \{'Yes', 0.1; 'No', 0.6; 'YesNo', 0.3\}$, $m_2 = \{'Yes', 0.5; 'No', 0.4; 'YesNo', 0.1\}$, $m_3 = \{'Yes', 0.5; 'No', 0.4; 'YesNo', 0.1\}$, $m_4 = \{'Yes', 0.5; 'No', 0.4; 'YesNo', 0.1\}$ and $m_5 = \{'Yes', 0.5; 'No', 0.4; 'YesNo', 0.1\}$. Then the average BPA representation of $m_1 - m_5$ can be calculated based on Eqs. (29-31) as: $m = \{'Yes', 0.42; 'No', 0.44; 'YesNo', 0.14\}$, and the final result can be calculated based on Eq. (32) as $m_f = \{'Yes', 0.4560; 'No', 0.5435; 'YesNo', 0.0005\}$. It can be expressed in terms of A-IFS theory as $I_f =< 0.4560, 0.5435 >$, based on which the decision can be made as: candidate x will not be selected. This is obviously contrary to common sense. So it is not appropriate to apply this algorithm directly to the global environment, which is the reason why it is defined as the local aggregation method.*

To make the decision more reasonable, the following approach is proposed based on the defined local aggregation algorithm and OWA operator. Based on the index function $\gamma(k)$, the mean orthogonal sum (MOS) of the j largest IFVs associated with $\Re$.

Definition VI.2. *For a set of IFVs $I = \{I_j | j = 1, \ldots, n\}$, the golden rule representative value of I_j is $\Re(I_j)$, so the mean orthogonal sum (MOS) of the j largest IFVs can be defined as:*

$$MOS(j) = \overline{\oplus}_{k=1}^{j} I_{\gamma(k)} \tag{33}$$

where $I_{\gamma(k)}$ represents the kth largest IFV, and $\overline{\oplus}$ is the operation of mean orthogonal sum defined in Def. VI.1. Specifically, $MOS(n)$ indicates the orthogonal sum of n IFVs, which can be denoted as $MOS(n) = \overline{\oplus}_{j=1}^{n} I_j$. The ordering of the IFVs in this case does not need to be considered.

Inspired by the idea of soft likelihood functions in [26], a class of softer orthogonal sum function is defined based on $MOS(j)$ by using the OWA aggregation operator as follows.

Definition VI.3. *Let $MOS(j)$ be the mean orthogonal sum of the j largest IFVs in $I = \{I_j | j = 1, \ldots, n\}$, and the weighting vector be w, the OWA-based mean orthogonal sum (OWA-based MOS) can be defined as:*

$$MOS_w = \sum_{j=1}^{n} w_j MOS(j) \tag{34}$$

where weighting vector w satisfies $w_j \in [0, 1]$ and $\sum_{j=1}^{n} w_j = 1$.

The specific form of the OWA-based mean orthogonal sum, is determined by the weighting vector w. Several special cases of the OWA-based MOS will be discussed based on OWA operator introduced in Sec. V.

Case 1: W_*: $w_n = 1$ and $w_j = 0$ $(j \neq n)$. The OWA-based mean orthogonal sum can be expressed as:

$$MOS_w = MOS(n) = \overline{\oplus}_{j=1}^{n} I_j \tag{35}$$

In this case, the OWA-based MOS degenerates into the special case analyzed in Def. VI.2. This is the most pessimistic case to determine the orthogonal sum of a set of IFVs.

Case 2: W^*: $w_1 = 1$ and $w_j = 0$ $(j \neq 1)$. The OWA-based mean orthogonal sum can be expressed as:

$$MOS_w = MOS(1) = I_{\gamma(1)} \tag{36}$$

In this case, the OWA-based mean orthogonal sum is equal to the largest IFV associated with $\Re$. It is obvious the largest IFV and the most optimistic form of MOS_w.

Case 3: W_n: $w_j = 1/n$ $(j = 1, \ldots, n)$. The OWA-based mean orthogonal sum can be expressed as:

$$MOS_w = \frac{1}{n}\sum_{j=1}^{n} MOS(j) = \frac{1}{n}\sum_{j=1}^{n}(\overline{\oplus}_{k=1}^{j} I_{\gamma(k)}) \tag{37}$$

The form in this case can be considered as a kind of simple average of the $MOS(j)$.

Generally, if the weight assigned to w_j has a smaller index, it indicates the OWA-based MOS is determined with more optimistic attitude by decision makers, and vice versa. So the form of MOS_w depends on the weighing vector w, which can be mapped to a attitudinal character α. If α is closer to 1, then the attitude is optimistic, and MOS_w is larger, while if α is closer to 0, then the attitude is pessimistic, and MOS_w is smaller.

Case 4: Based on Eq. (24), the weighing vector can be obtained based on function $f(x) = x^m$. And the factor of attitude α can be calculated by $m = \frac{1-\alpha}{\alpha}$. Then the OWA-based mean orthogonal sum can be expressed as:

$$MOS_\alpha = \sum_{j=1}^{n}((\frac{j}{n})^{\frac{1-\alpha}{\alpha}} - (\frac{j-1}{n})^{\frac{1-\alpha}{\alpha}})\overline{\oplus}_{k=1}^{j} I_{\gamma(k)} \tag{38}$$

Example VI.1 is used again to illustrate the computation process and show the effectiveness of the proposed OWA-based MOS.

Example VI.2. *(Continued Example VI.1) To make more reasonable decisions on the issue of human resource selection presented in Example VI.1, the OWA-based MOS is employed as follows. Step 1: the golden rule representative values of evaluation information of candidate x will be calculated based on Eq. (21); Step 2: the index of the kth largest IFV $\gamma(k)$ can be obtained based on index function γ; Step 3: the kth largest IFV of all the IFVs to be aggregated can be obtained next; Step 4: the mean orthogonal sum $MOS(j)$ can be calculated based on Def. VI.2. The results of the above four steps are shown in Table I. Step 5 is to aggregate the mean orthogonal sum $MOS(1) - MOS(5)$ using the OWA operator. A special case is provided to illustrate the process of calculating the OWA-based mean orthogonal sum MOS_α, in which $\alpha = 0.5$. We have $w_j = (\frac{j}{5}) - (\frac{j-1}{5}) = 1/5$, so $MOS_\alpha = MOS_{0.5} = \frac{1}{5}\sum_{j=1}^{5} \overline{\bigoplus}_{k=1}^{j} I_{\gamma(k)} =< 0.6114, 0.3645 >$. From this result, the conclusion can be drawn that the candidate x should be selected, which is consistent with the common sense and can demonstrate the effectiveness of the (OWA-based) mean orthogonal sum. In addition, the sensitivity analysis of attitude factor α to aggregation results is performed. As shown in Fig. 2, when the value of α changes from 0 to 1, with regard to the aggregation result, the degree of membership μ_f is always greater than the degree of non-membership ν_f, so it is always possible to conclude that candidate x should be selected, which is consistent with common sense.*

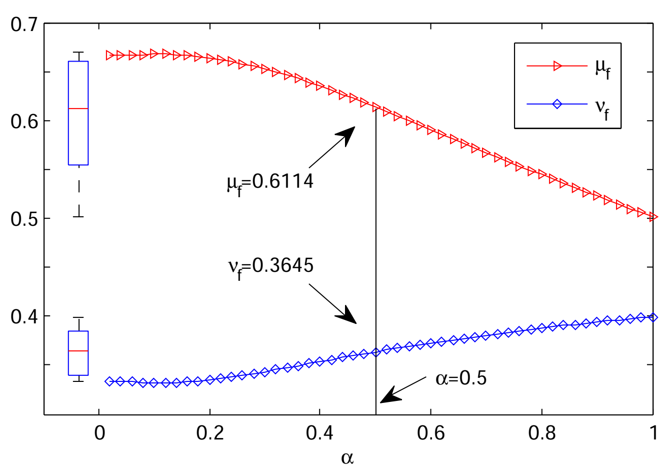

Fig. 2. Illustration of the sensibility of α to the aggregation result by using the OWA-based mean orthogonal sum.

According to Example VI.2, the OWA-based mean orthogonal sum has proven to be effective in aggregating IFVs in terms of DST. To further discuss the performance of the OWA-based MOS, the following example is conducted, which is based on Example VI.1.

Example VI.3. *(Continued Example VI.1) In this example, we used the decision-making context consistent with Example VI.1, but we re-evaluate the performance of candidate x as: $I_1 =< 0.6, 0.1 >$, $I_2 =< 0.4, 0.5 >$, $I_3 =< 0.4, 0.5 >$, $I_4 =< 0.4, 0.5 >$ and $I_5 =< 0.4, 0.5 >$. It is easy to observe that we have exchanged the degree of membership and the degree of non-membership of all evaluation information. It is obvious, in this case, the company should not hire candidate x based on common sense. Then the OWA-based MOS is employed to aggregate all the IFVs and the result is $MOS_\alpha = MOS_{0.5} = \frac{1}{5}\sum_{j=1}^{5} \overline{\bigoplus}_{k=1}^{j} I_{\gamma(k)} =< 0.7471, 0.1720 >$. The result shows that the company should hire candidate x, which is contrary to common sense. The cause of this phenomenon is analyzed as follows.*

In OWA-based MOS, the first step is to determine the index function γ of all the IFVs, which is based on the golden rule representative value $\Re$. In our opinion, the index function γ is an essential part of the OWA-based MOS algorithm. The ordering of IFVs will affect the final aggregation results according to the characteristics of Dempster's rule. The index function γ thereby needs to be refined, and more factors need to be taken into account to determine the more reasonable ordering of IFVs. In this part, the other two aspects are also considered as the factors to determine the index function γ, which are denoted as reliability and compatibility respectively. With regard to reliability, we consider it from the perspective of entropy of IFVs, i.e. uncertainty, which is an important indicator to measure the quality of IFVs. To define the reliability of an IFV, the distance and entropy measures of IFVs will be given as preliminary knowledge as follows.

Definition VI.4. *Let $A =< \mu_A, \nu_A >$ and $B =< \mu_B, \nu_B >$ be two IFVs, the normalized Hamming distance of them can be defined as:*

$$D_1(A, B) = \frac{1}{2}(|\mu_A - \mu_B| + |\nu_A - \nu_B|) \tag{39}$$

the normalized Euclidean distance of them can be defined as:

$$D_2(A, B) = \sqrt{\frac{1}{2}((\mu_A - \mu_B)^2 + (\nu_A - \nu_B)^2)} \tag{40}$$

Based on the Jousselme distance [55] of evidence, we re-define the distance of IFVs in terms of DST as follows. Two vectors $I_A = [\mu_A, \nu_A, 1 - \mu_A - \nu_A]$ and $I_B = [\mu_B, \nu_B, 1 - \mu_B - \nu_B]$ and a matrix M are provided as intermediate variables.

$$M = \begin{bmatrix} 1 & 0 & 1/2 \\ 0 & 1 & 1/2 \\ 1/2 & 1/2 & 1 \end{bmatrix} \tag{41}$$

the Jousselme distance of IFVs can be defined as:

$$D_3(A, B) = \sqrt{\frac{1}{2}(I_A - I_B)M(I_A - I_B)^T} \tag{42}$$

According to [10], there exists a one-to-one correlation between similarity and distance measures for IFVs such that the distance D and its corresponding similarity measure S satisfy $S + D = 1$. Based on the distance measure of an IFV, its entropy measure can be defined as follows.

Definition VI.5. *Let $A =< \mu_A, \nu_A >$ be an IFV and D_i, i=1,2,3 be the aforementioned three distance measures, then A's entropy function can be defined as:*

$$E_i(A) = 1 - 2D_i(A, < 1/2, 1/2 >), i = 1, 2, 3 \tag{43}$$

TABLE I
THE INTERMEDIATE RESULTS OF THE MEAN ORTHOGONAL SUM FOR CANDIDATE x.

I_j	Golden rule value	$\gamma(k)$	$I_{\gamma(k)}$	MOS_j
$I_1 =< 0.1, 0.6 >$	$\Re(I_1) = 0.325$	$\gamma(1) = 2$	$I_{\gamma(1)} = I_5 =< 0.5, 0.4 >$	$MOS_1 = I_{\gamma(1)} =< 0.5, 0.4 >$
$I_2 =< 0.5, 0.4 >$	$\Re(I_2) = 0.545$	$\gamma(2) = 3$	$I_{\gamma(2)} = I_1 =< 0.5, 0.4 >$	$MOS_2 = \overline{\oplus}(MOS_1, I_{\gamma(2)}) =< 0.5833, 0.4000 >$
$I_3 =< 0.5, 0.4 >$	$\Re(I_3) = 0.545$	$\gamma(3) = 4$	$I_{\gamma(3)} = I_2 =< 0.5, 0.4 >$	$MOS_3 = \overline{\oplus}(MOS_2, I_{\gamma(3)}) =< 0.6324, 0.3647 >$
$I_4 =< 0.5, 0.4 >$	$\Re(I_4) = 0.545$	$\gamma(4) = 5$	$I_{\gamma(4)} = I_3 =< 0.5, 0.4 >$	$MOS_4 = \overline{\oplus}(MOS_3, I_{\gamma(4)}) =< 0.6745, 0.3250 >$
$I_5 =< 0.5, 0.4 >$	$\Re(I_5) = 0.545$	$\gamma(5) = 1$	$I_{\gamma(5)} = I_4 =< 0.1, 0.6 >$	$MOS_5 = \overline{\oplus}(MOS_4, I_{\gamma(5)}) =< 0.6669, 0.3330 >$

specifically,

$$E_1(A) = 1 - (|\mu_A - 1/2| + |\nu_A - 1/2|) \quad (44)$$

$$E_2(A) = 1 - \sqrt{2((\mu_A - 1/2)^2 + (\nu_A - 1/2)^2)} \quad (45)$$

$$E_3(A) = 1 - \sqrt{(\mu_A - \nu_A)^2 + (1 - \mu_A - \nu_A)^2} \quad (46)$$

It is easy to prove that $D_i \in [0, 1]$ and $E_i \in [0, 1]$, i = 1,2,3. The entropy function of IFVs is a kind of uncertainty measure, while in practice, the reliability measure is more commonly employed. Therefore, the reliability of IFVs is developed based on the definition of entropy as follows.

Definition VI.6. *Let $A =< \mu_A, \nu_A >$ be an IFV and E_i, i=1,2,3 be the three proposed entropy measures of IFVs, the reliability of A can be defined as:*

$$\boldsymbol{R}(A) = -\frac{2}{1 + exp(-\delta E_i(A))} + 2, i = 1, 2, 3 \quad (47)$$

where δ is a tuning parameter, the function image under different δ value has been given in Fig. 3, to completely map E to $R \in [0, 1]$, we set $\delta = 6$ in this study.

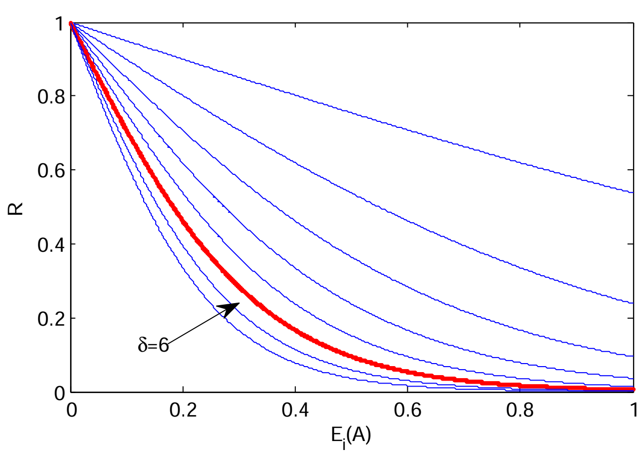

Fig. 3. Function image of **R** under different δ values.

With regard to a set of IFVs to be aggregated, compatibility is mainly used to describe the extent to which an IFV is supported by other IFVs. If there are more IFVs similar to it, the more support degree it holds. So the compatibility of an IFV is defined based on its similarity with other IFVs, and the specific definition is as follows.

Definition VI.7. *Let $I =< I_1, I_2, \ldots, I_n >$ be a set of IFVs need to be aggregated, the similarity measure matrix (SMM) can be constructed firstly, which represents the agreement among different IFVs.*

$$SMM = \begin{bmatrix} 1 & S_{12} & \cdots & S_{1n} \\ S_{21} & 1 & \cdots & S_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ S_{n1} & S_{n2} & \cdots & 1 \end{bmatrix}, \quad (48)$$

where S_{ij} indicates the similarity of IFVs I_i and I_j introduced in Def. VI.4, then the support degree of IFV I_i can be calculated as:

$$Sup(I_i) = \sum_{j=1, j\neq i} S_{ij} = 1 - D_\varrho(I_i, I_j), \varrho = 1, 2, 3 \quad (49)$$

where D is one of the distance measures introduced in Def. VI.4. Further, the credibility degree of I_i can be calculated as:

$$Crd_i = \frac{Sup(I_i)}{\sum_{i=1}^{n} Sup(I_i)} \quad (50)$$

So the compatibility of the IVFN representation of A_i under criterion C_j can be obtained as:

$$\boldsymbol{C}(I_i) = (Crd_i - \min_{1\leq j\leq n} Crd_j)/(\max_{1\leq j\leq n} Crd_j - \min_{1\leq j\leq n} Crd_j) \quad (51)$$

The reliability and compatibility of IFVs have been defined above, now we also consider the representative values of IFVs as a component of the index function, so function $\overline{\gamma}$ consists of three parts, which are reliability, compatibility, and representation, respectively.

$$\overline{\gamma}(i) = \nabla_{i=1}^{n}[\phi * \mathbf{R}(I_i) + \psi * \mathbf{C}(I_i) + \xi * \Re(I_i)] \quad (52)$$

where $\phi, \psi, \xi \in [0, 1]$ are three parameters that satisfy $\phi + \psi + \xi = 1$, and '∇' represents the operation: sorting a set of numbers in descending order and taking indices.

In this part, the index function is redefined by increasing more factors in the sorting process, which overcomes the shortcomings of using the representative value only. The following aggregation process is consistent with the OWA-based mean orthogonal sum. To illustrate the performance of the improved OWA-based MOS, the decision problem in Example VI.3 is resolved as follows. Four cases are constructed to demonstrate this issue. In case 1, parameters ϕ,ψ and ξ take all the values that satisfy the condition in Eq. (52), the degree of non-membership (i.e. candidate x should not be hired, which is consistent with common sense) of the aggregation result based on the improved OWA-based MOS are shown in Fig. 4. The degree of non-membership of the aggregation results in different perspectives are shown in Fig. 4a, b and c,

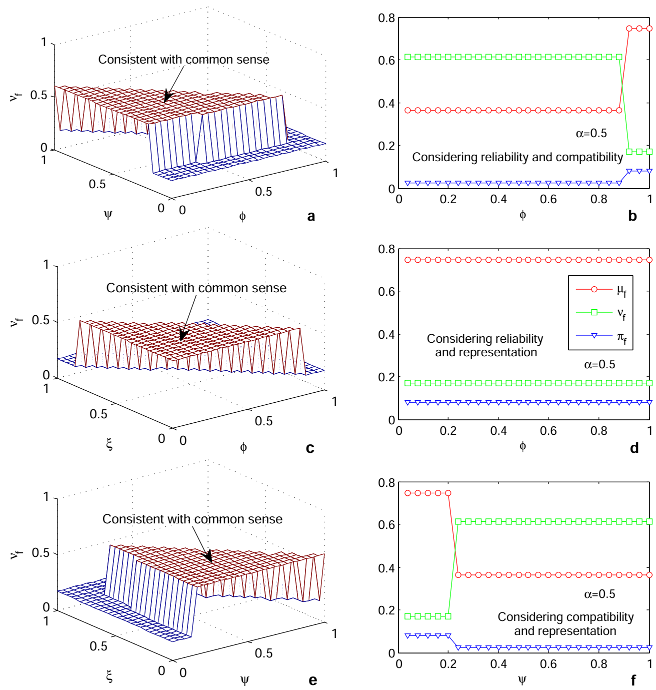

Fig. 4. **Illustration of the effectiveness of the improved OWA-based mean orthogonal sum. a**,**c**,**e**, the degree of non-membership (consistent with common sense) of the aggregation result when considering three factors, reliability, compatibility and representation. **b**, the aggregation result only when considering reliability and compatibility. **d**, the aggregation result only when considering reliability and representation. **f**, the aggregation result only when considering compatibility and representation.

respectively. In case 2, only reliability and compatibility are considered (i.e. $\xi = 0$), and parameters ϕ and ψ take all the values that satisfy the condition in Eq. (52), the corresponding aggregation results are shown in Fig. 4b. In case 3, only reliability and representation are considered (i.e. $\psi = 0$), and parameters ϕ and ξ take all the values that satisfy the condition in Eq. (52), the corresponding aggregation results are shown in Fig. 4c. In case 4, only compatibility and representation are considered (i.e. $\phi = 0$), and parameters ψ and ξ take all the values that satisfy the condition in Eq. (52), the corresponding aggregation results are shown in Fig. 4d. The results of the four cases are analyzed as follows.

With regard to case 1, as a whole, the aggregation result consistent with common sense exist only in certain areas based on the improved OWA-based MOS. The essential cause is that the different values of ϕ, ψ and ξ affect the index function (i.e. the ordering of IFVs), thereby affecting the final aggregated result further. It can be found from Fig. 4a that the smaller the ϕ (i.e. reliability) is, the better the aggregation result is, and the larger the ψ (i.e. compatibility) is, the better the aggregation result is. It can be found from Fig. 4c that the smaller the ϕ (i.e. reliability) is, the better the aggregation result is, and the smaller the ξ (i.e. representation) is, the better the aggregation result is. It can be found from Fig. 4e that the larger the ψ (i.e. compatibility) is, the better the aggregation result is, and the smaller the ξ (i.e. representation) is, the better the aggregation result is. In addition, it can be observed from Fig. 4d and f that considering representation will reduces the utility of our method, especially in Fig. 4d. Therefore, the general conclusion can be drawn that the representation does not apply to the OWA-based MOS. This conclusion will be explained from the perspective of evaluation information as follows.

Now let us review this decision problem in Example VI.3. The evaluation information of candidate x is represented by IFVs as $I_1 =< 0.6, 0.1 >$, $I_2 =< 0.4, 0.5 >$, $I_3 =< 0.4, 0.5 >$, $I_4 =< 0.4, 0.5 >$ and $I_5 =< 0.4, 0.5 >$. It is easy to observe that I_2 to I_5 are the negative evaluation, and I_1 is the positive evaluation, so I_1's compatibility with others should be more smaller. In addition, it can be found that for I_2 to I_5, their degree of membership and non-membership are very similar, i.e. their entropy should be more larger, so the reliability of them should be more smaller than I_1. In summary, compatibility and reliability of IFVs should be two very significant factors for index function γ. In essence, the representative value of an IFV is similar to a score function that is used to IFV comparison, and based on the above analysis we will no longer use the representation in OWA-based MOS algorithm, but apply it to IFV comparison as a score function. So the index function can be redefined based on reliability and compatibility of IFVs as:

$$\gamma(i) = \nabla_{i=1}^{n}[\phi * \mathbf{R}(I_i) + \psi * \mathbf{C}(I_i)] \tag{53}$$

where $\phi, \psi \in [0, 1]$ and $\phi + \psi = 1$, and '∇' stands for the descending operation.

Without considering the representation of IFVs, the aggregation results are shown in Fig. 4b. It can be seen that the counterintuitive result only appears when ϕ is greater than 0.9, which means that the aggregation result is correct more often. In fact, the compatibility should have a greater proportion in this case, so the experimental results are consistent with the facts, which indicates the effectiveness of the proposed OWA-based MOS algorithm. In practical applications, parameters ϕ and ψ can be determined by training in order to obtain optimal decision results.

VII. MCDM PROBLEM IN THE FRAMEWORK OF INTUITIONISTIC/DST APPROACH

A. The proposed decision approach

An extended Dempster's rule of combination (OWA-based MOS) for multi-criteria decision making based on intuitionistic fuzzy values consists of the following four main steps presented in Fig. 5.

1) Problem description: including determining the MCDM problem and selecting all the potential alternatives and important criteria. Consider a MCDM problem in intuitionistic fuzzy environments, assume there are n alternatives, denoted as A_i $(i = 1, \ldots, m)$, and n criteria, indicated by C_j $(j = 1, \ldots, n)$, where the weight of each criterion is expressed as w_j $(j = 1, \ldots, n)$ by using intuitionistic fuzzy value. The assessment results of alternative A_i under criterion C_j can be

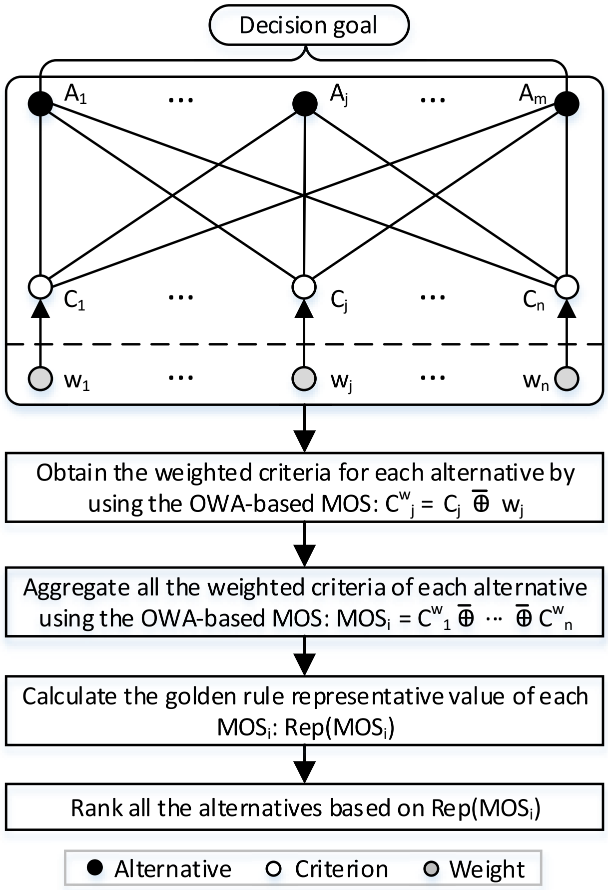

Fig. 5. Hierarchical structure of the proposed decision method based on the developed MOC algorithm.

represented by IFVs as I_{ij}, then the decision matrix can be given as:

$$DM = \begin{array}{c} \\ A_1 \\ A_2 \\ \vdots \\ A_m \end{array} \begin{array}{c} \begin{array}{cccc} C_1 & C_2 & \cdots & C_n \end{array} \\ \left[\begin{array}{cccc} I_{11} & I_{12} & \cdots & I_{1n} \\ I_{21} & I_{22} & \cdots & I_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ I_{m1} & I_{m2} & \cdots & I_{mn} \end{array} \right] \end{array} \tag{54}$$

where $I_{ij} =< \mu_{ij}, \nu_{ij} >$, μ_{ij} and ν_{ij} indicate the degree of membership and non-membership associated with the satisfaction by alternative A_i. Note here that for the decision environment of our approach we are assuming all the criteria are independent. The hierarchical structure of the decision problem is shown in the top half of Fig. 5. The goal of the decision problem is to select the best alternative with the greatest satisfaction for all criteria based on the decision matrix provided.

2) Criterion weight: calculating all the weighted criteria of each alternative. In this study, the criteria values and their weights are provided by intuitionistic fuzzy values. To obtain weighted criteria, the proposed OWA-based mean orthogonal sum is employed to aggregate criteria and weights associated with each alternative. The weighted IFV representation I_{ij}^w for alternative A_i under criterion C_j can be calculated based on Eq. (34) as: $I_{ij}^w = I_{ij} \overline{\oplus} w_j$. As an extension of the criterion weighting, the other two ways are taken into account in expressing and processing weight information. Several special cases are introduced below.

Case 1: When the criterion weight is expressed in positive integer, it should satisfy $w_j \in [0, 1]$, $\sum_{j=1}^{n} w_j = 1$. In [56], De et al. defined the IFV ϖI for any positive integer ϖ as: $\varpi I =< \mu_{\varpi I}, \nu_{\varpi I} >$, where $\mu_{\varpi I} = 1 - (1 - \mu_I)^{\varpi}$ and $\mu_{\varpi I} = [\nu_I]^{\varpi}$, based on which the weighted criteria I_{ij}^w in this case can be obtained as: $I_{ij}^w = I_{ij} w_j =< \mu_{w_j I_{ij}}, \nu_{w_j I_{ij}} >$, where $\mu_{w_j I_{ij}} = 1 - (1 - \mu_{I_{ij}})^{w_j}$ and $\nu_{w_j I_{ij}} = [\nu_{I_{ij}}]^{w_j}$.

Case 2: When criterion weights are represented by fuzzy numbers $w_j = \mu_{w_j}$, it can be considered as a special IFV $w_j =< \mu_{w_j}, 0 >$ with the degree of non-membership 0. The following calculation process is consistent with the case of IFV expression.

Case 3: When criterion weights are represented by intuitionistic fuzzy values, it can also be converted to the weight vector in the form of positive integers. The Rep value of each weight will be calculated firstly based on Eq. (21) as $\Re(w_j)$, then they will be normalized as $\widetilde{w}_j = \Re(w_j) / \sum_{j=1}^{n} \Re w_j$. The following calculation process is consistent with in case 1.

In this study we calculate the weighted criteria by way of aggregation based on the proposed OWA-based MOS algorithm.

3) Criterion aggregation: obtaining all the aggregated criteria of each alternative. To obtain the final IFV representation of each alternative, its multiple evaluation information under different criteria will be aggregated based on the proposed OWA-based MOS algorithm as $I_i = \overline{\oplus}_{j=1}^{n} I_{ij}, i = 1, \ldots, m$. When using the OWA-based MOS algorithm to aggregate IFVs, it will involve the selection of parameters, including the tuning parameters (i.e. ϕ and ψ) to determine the index function defined in Eq. (53) and the attitudinal character α introduced in Sec. V. The determination of the parameters needs to be implemented according to the actual application environment. For example, when determining the index function, if the reliability is more important, ϕ should be larger, and if the compatibility is more important, ψ should be larger. When determining the attitudinal character, if decision makers hold a more optimistic attitude, then α is greater, and vice versa. The ranking of alternatives will be obtained based on their IFV representation I_i in the next section.

4) Alternative selection: including computing the golden rule representative value for each alternative, ranking them and selecting the optimal one. To determine the ordering of all the alternatives, their Rep values need to be calculated as $\Re(A_i)$. According to the descending values of $\Re(A_i)$, all alternatives $A_i (i = 1, \ldots, m)$ are rank ordered and the best one can be selected.

B. Numerical example

The air-condition system selection problem [41], [42] introduced in Sec. III is employed again in this section. This decision problem will be solved using the OWA-based MOS algorithm proposed in this paper. The framework of air-condition system selection problem is provided in Fig. 6.

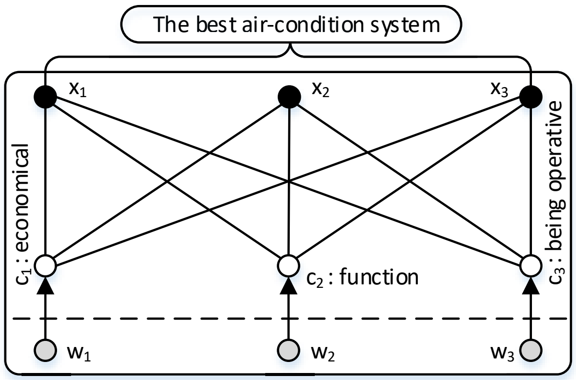

Fig. 6. The framework of air-condition system selection problem.

The decision matrix can be seen in Eq. (6). Then, the weighted criteria can be calculated based on Eq. (34). The weighted decision matrix can be obtained as:

$$I_{ij}^w = (<\mu_{ij}^w, \nu_{ij}^w>)_{3\times 3} =$$
$$\begin{bmatrix} & c_1 & c_2 & c_3 \\ x_1 & <0.798, 0.097> & <0.647, 0.256> & <0.818, 0.182> \\ x_2 & <0.837, 0.128> & <0.728, 0.196> & <0.275, 0.697> \\ x_3 & <0.320, 0.332> & <0.805, 0.067> & <0.315, 0.654> \end{bmatrix} \tag{55}$$

Note that in this numerical example, we let $\alpha = 0.5$. The different criteria of each alternative will be aggregated based on Eq. (34) in the following steps. To use the proposed OWA-based MOS algorithm, the first step is to determine the index function based on Eq. (53). The different values of the parameters can affect the ordering of the IFVs to be aggregated. The orderings of different criteria with the changing parameter ϕ for alternatives x_1, x_2 and x_3 are shown in Fig. 7. The specific analysis is as follows. For alternative x_1, when $\phi \in [0, 0.27]$, the ordering of criteria is $c_1 \succ c_3 \succ c_2$ and the *Rep* value of aggregation result is $\Re(x_1) = 0.9148$, while the ordering is $c_3 \succ c_1 \succ c_2$ and $\Re(x_1) = 0.9188$ when $\phi \in [0.27, 1]$. For alternative x_2, when $\phi \in [0, 0.49]$, the ordering of criteria is $c_1 \succ c_2 \succ c_3$ and the *Rep* value of aggregation result is $\Re(x_2) = 0.9222$, while the ordering is $c_2 \succ c_1 \succ c_3$ and $\Re(x_2) = 0.8693$ when $\phi \in [0.49, 1]$. For alternative x_3, when $\phi \in [0, 0.24]$, the ordering of criteria is $c_2 \succ c_1 \succ c_3$ and the *Rep* value of aggregation result is $\Re(x_3) = 0.8772$, when $\phi \in [0.24, 0.39]$, the ordering is $c_1 \succ c_2 \succ c_3$ and $\Re(x_3) = 0.5979$, while the ordering is $c_1 \succ c_3 \succ c_2$ and $\Re(x_3) = 0.4678$ when $\phi \in [0.39, 1]$. In this example, we let $\phi = 0.5$, so we have $\Re(x_1) = 0.9188$, $\Re(x_2) = 0.8693$ and $\Re(x_3) = 0.4678$. Further, the ordering of alternatives is $x_1 \succ x_2 \succ x_3$, so the best one is x_1.

C. Some discussions

According to Eq. (53), the parameter ϕ reflects the proportion of the IFVs' reliability in determining index functions. The larger the value of ϕ, the more important the reliability is. The smaller the value of ϕ, the more important the compatibility. In Fig. 7, it can be found that the different values of ϕ can affect the index function, which in turn affects the aggregation process and ultimately leads to different ordering

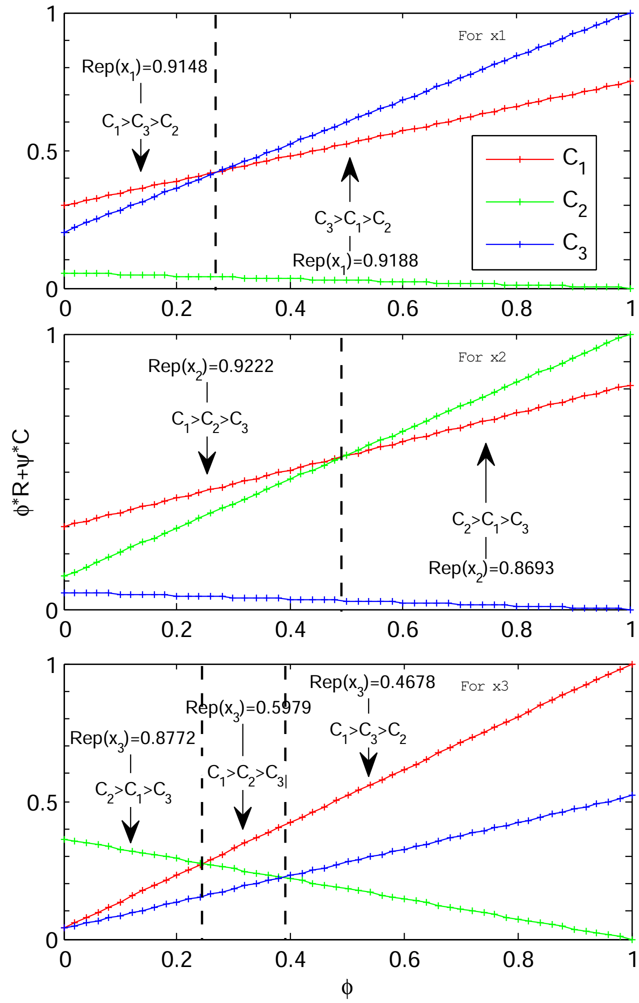

Fig. 7. Illustration of index functions and *Rep* values of alternatives under different ϕ values.

of alternatives. This feature can be considered as an advantage of the method proposed in this paper, as long as the appropriate parameters are selected according to the actual application environments.

In addition, the evaluation results of most intuitionistic fuzzy decision approaches are represented by IFVs, which requires real valued score functions to rank all alternatives, and this process inevitably leads to loss of information due to the intermediate defuzzification process. The merit of the developed OWA-based MOS algorithm based on the interpretation on A-IFS in terms of DST is that it allows intuitionistic fuzzy decision making without defuzzification when criteria and their weights are represented by IFVs.

What's more, some limitations of using Dempster's rule of combination for the aggregation operator of IFVs have been analysed and resolved in this study by improving the combination rule and considering the reliability of the source of information.

VIII. Conclusion

Dymova and Sevastjanov [24] pointed out that "DST can serve as an effective methodological base for interpretation of

A-IFS", and then some following studies have been carried out to solve the problems of A-IFS with the methods in terms of DST. The shortcomings of existing methods are firstly analyzed in this paper, including the information loss caused by defuzzification and the deviation of decision results caused by traditional score functions. To remedy the limitations of Dempster's rule of combination in aggregating intuitionistic fuzzy values, a new aggregation operator called OWA-based mean orthogonal sum (MOS) is proposed based on ordered weighted averaging (OWA) operator, which can not only effectively aggregate IFVs, but also take into account the subjective attitude of decision makers. In addition, to sort alternatives according to the results of aggregating criteria based on OWA-based MOS approach, the golden rule representative value for IFVs comparison is introduced as an alternative of score function. The multi-criteria decision-making (MCDM) framework is constructed in intuitionistic fuzzy environments based on the proposed OWA-based MOS method. The proposed decision framework is employed in the well-known example of MCDM problem and its usefulness can be demonstrated by detailed analysis. Note that there still remains some problems to be solved in future research. Below are summaries of several significant points.

Although the proposed OWA-based MOS approach has better aggregation effects than the classical Dempster's rule, it cannot satisfy some good properties (e.g. associativity) of Dempster's rules, which is determined by the unique aggregation characteristics of the approach. An optimized alternative can be developed in the future study to meet the corresponding property. Then, OWA formulations of aggregation operator are employed in the proposed OWA-based MOS approach, in which the parameter α is variable but crucial for the aggregation results. So a reasonable formula is indispensable to determine the value of α in the future study. In addition, as an extension of the OWA-based MOS approach, the OWA-based aggregation operator on belief intervals can be studied in future research.

In short, although the current version of OWA-based MOS approach has some shortcomings, it is still a flexible and effective multi-criteria decision-making approach in intuitionistic fuzzy environments. It provides a more reasonable interpretation of A-IFS in terms of DST. In future research, the framework of the intuitionistic fuzzy decision-making with Dempster-Shafer structures will continue to be studied and improved further.

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