Naturalness begets Naturalness: An Emergent Definition

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We offer a model based upon three 'assumptions'. The first is geometric, that the vacuum wavefunction is comprised of Euclid's fundamental geometric objects of space - point, line, plane, and volume elements - components of the geometric representation of Clifford algebra. The second is electromagnetic, that physical manifestation follows from introducing the dimensionless coupling constant α. The third takes the electron mass to define the scale of space. Such a model is arguably maximally 'natural'. Wavefunction interactions are modeled by the geometric product of Clifford algebra. What emerges is more naturalness. We offer an emergent definition.

0. Introduction

"...naturalness seems to be one of the best-kept secrets of physicists from the general public, a secret weapon for evaluating and motivating theories of the world on its deepest levels"[1].

Motivated then by investigations into quadratic mass divergence of scalar fields[2, 3], and ever more true now.

Physics has its roots in natural philosophy. It was only in the 1800s, when the pace of scientific knowledge exceeded the capacity of any one individual, that the separate disciplines of physics, chemistry, and biology emerged. With that emergence and the technology it spawned, naturalness became less direct, more instrumental, not so easily intuited and defined, and even now still lingers unsettled in foundations of the standard models of physics and cosmology[4–8].

To understand what is emergent in nature one must first understand what is fundamental in nature. One must understand the wavefunction. It's all about the enigmatic unobservable wavefunction[9–11]. At the most fundamental level, naturalness must be present in the wavefunction and wavefunction interactions.

1. Naturalness - Part I

This section focuses on properties of naturalness relevant to the physics at hand - those assumptions essential to define a natural quantum mechanics, and their relation to the phenomenon of emergence.

- **Simplicity** - whereas naturalness evaluates the type of assumptions, simplicity looks at their number. The most efficient route to simplicity has minimal assumptions input by hand. Simplicity can be further quantified, *"... measured by the length of a computer program that executes a calculation."*[12].
- **Small Dimensionless Numbers** - 'Smallness' is a requirement imposed by effective field theories, that the coupling constant be of order unity at the cutoffs. The 'dimensionless' requirement avoids model-dependent scaling inherent in any system of units. This property is sometimes called Dirac naturalness.
- **Absence of Fine-tuning** - Fine-tuning may be defined in terms of the 'small dimensionless numbers' requirement as *"...a measure of how much a model departs from naturalness."*[13]
- **Small Numbers and Broken Symmetry** - An observed coupling constant can be much smaller than one, if a symmetry is restored when set to zero. This property is called technical or 't Hooft naturalness[3]. The **Higgs** mass scalar is the only standard model parameter not protected by a broken symmetry, not 't Hooft natural. By the criterion of Dirac naturalness, almost every standard model parameter is unnatural.
- **Scale Invariance** - *"... theory at low resolution should not sensitively depend on theory at high resolution... "* Renormalization group flow is a fine example[12].
- **Robustness** - One seeks a model that is robust, seeks the absence of *"...sensitivity of some parameter in the theory to variations of the other parameters."*[13] Robustness expands the scope of sensitivity, from scale invariance to all parameters of a model.
- **Quantum Interpretation** - One seeks a model that resolves the paradoxes driving ongoing proliferation of quantum interpretations[9–11], a model properly anchored in philosophy of the naive realist[14].

2. Geometric Wavefunctions and their Interactions - Part I

The model has three assumptions - vacuum wavefunction, electromagnetic coupling constant, and electron mass.

- **vacuum wavefunction** - while the Dirac matrix formalism in flat Minkowski spacetime is the foundation of particle physicists' representations of Clifford algebra [15], geometric representation is more useful for present purposes - less abstract, more intuitive, more natural.[16–21]. Geometric Algebra takes Pauli matrices to be the basis vectors of real physical space, Dirac matrices those of real physical spacetime.

  In GA the vacuum wavefunction is comprised of one scalar point, three vector line elements (three orientational degrees-of-freedom), three bivector area elements, and one trivector volume element. This eight-component Pauli wavefunction in physical 3D space can be identified with the string theory octonion vacuum wavefunction[22, 23].

  Wavefunction interactions are modeled by the geometric product (figure 1). As in the two-component positron and electron spinors of the Dirac wavefunction, the product of two octonion wavefunctions yields observables - the particle physicist's S-matrix (figure 5)[24–27]. This requires that fields be associated with the geometry.

- **coupling constant** - four fundamental constants define the dimensionless electromagnetic coupling constant:

  $$\alpha = \frac{1}{4\pi\epsilon_0}\frac{e^2}{\hbar c} \approx 0.0073 \qquad 1/\alpha \approx 137 \tag{1}$$

  These four permit assignment of topologically appropriate quantized electric and magnetic fields to the eight-component vacuum wavefunction, and calculation of quantized impedance networks of wavefunction interactions in powers of α (figures 6-9). Given that wavefunction fields are quantized in quantum field theory, it is unavoidable that impedances of wavefunction interactions will likewise be quantized..

  this is important: Impedance matching governs amplitude and phase of energy transmission, governs amplitude and phase of the flow of information in quantum mechanics.

- **electron mass** - quantum excitation requires a 'mass gap', requires a particle with both rest mass and electromagnetic fields to couple to the photon of QED[28]. In the GWI model the lightest stable rest mass particle defines the scale of space via the electron Compton wavelength octonion wavefunction[29].

Vacuum wavefunction is the same at all scales. Field quantization yields scale-dependent physics (figures 6-9).

3. Naturalness - Part II

With this brief outline of the GWI model, it is possible to begin addressing requirements of naturalness.

- **Simplicity** - Only three assumptions are required. The model is arguably maximally natural. Network calculations are precise, 10^{-2} or better in almost all instances, with some at the part-per-billion limit of experimental accuracy, can be accomplished real-time interactive on a workstation-class laptop.[23, 29–65].
- **Small Dimensionless Numbers** - The electromagnetic coupling constant $\alpha \sim .0073$ is the only dimensionless number in the model. How small is too small? GA has Hamilton's essential property of invertibility. It would seem that $1/\alpha \sim 137$ is just too big.
- **Absence of Fine-tuning** - there is no fine-tuning in the GWI model, just the three defining assumptions.
- **Small Numbers and Broken Symmetry** - The model defines a magnetic coupling constant by the Dirac relation $eg = \hbar$, where g is magnetic charge, the spin 1 topological dual of electric charge[66]. For $\alpha_e \approx 0.0073$ we have $\alpha_g \approx 137$. Their product is Dirac naturalness = 1. The broken symmetry is topological, discussed on the next page. Restoring it sets the coupling not to zero, but to one. The model is 'effective' at all scales.
- **Scale Invariance** - The GWI model is effective at all scales, the same model from Planck length to boundary of the observable universe. Vacuum wavefunction is the same at all scales. Interactions of quantized wavefunction fields yields different physics at different confinement scales, at different Compton wavelengths, at the Planck length, deBroglie wavelength (which finds its origin in the Doppler shift of Compton frequency[67]),....
- **Robustness** - The only parameter which is varied in the GWI model is the length scale, the scales to which wavefunctions are assigned, the wavelengths of interacting wavefunctions.
- **Quantum Interpretation** - The vacuum wavefunction model can be visualized in physical 3D space, as can the electric and magnetic fields that make possible physical manifestation. This intuitively natural perspective permits successfully addressing interpretational paradoxes of the unobservable wavefunction[10, 11]

4. Topological Symmetry Breaking

Wavefunction interactions are modeled by the geometric product, which multiplies not numbers or symbols but geometric objects, changing their dimensionality, their grade[68], making GA unique in the ability to handle geometric and topological dynamics in all dimensions.

Topologically, the *"...problem is that even though we can transform the line continuously into a point, we cannot undo this transformation and have a function from the point back onto the line..."* [69].

Grade increasing operations break topological symmetry, with the exception of wedge products of scalars. Scalars are point objects having no spatial dimensionality, cannot raise grades, don't break topological symmetry. This is of particular interest for the **Higgs**.

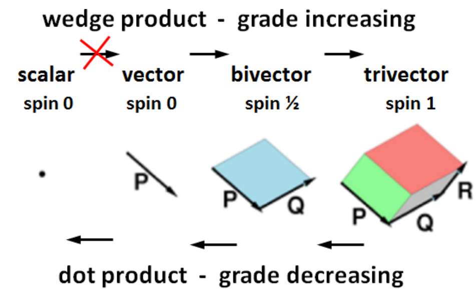

FIG. 1: 3D Pauli algebra of space[70]

Given two vectors a and b, the geometric product ab changes grades. In the product $ab = a \cdot b + a \wedge b$, two grade 1 vector bosons are transformed into grade 0 scalar boson and grade 2 bivector fermion. The product turns fermions into bosons, bosons into fermions. Dynamic supersymmetry emerges naturally from interactions, as does time in the grade increase from space to spacetime generated by the wedge product.

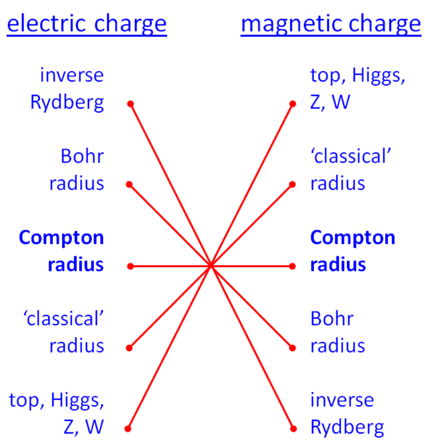

FIG. 2: Inversion of fundamental lengths by magnetic charge

The photon is our fiducial in measurements of the properties of space. Topological duality[66] arises from the difference in coupling to the photon of magnetic and electric charge. If we take magnetic charge g to be defined by the Dirac relation $eg = \hbar$ and the electromagnetic coupling constant to be $\alpha = e^2/4\pi\epsilon_0\hbar c$, then e is proportional to $\sqrt{\alpha}$ whereas g varies as $1/\sqrt{\alpha}$. The characteristic coherence lengths of figure 2, precisely spaced in powers of α, are inverted for magnetic charge[71]. The Compton wavelength $\lambda = h/mc$ is independent of charge.

With electric charge, fundamental lengths correspond to specific physical mechanisms of photon emission or absorption, matched in both quantized impedance and energy. Inversion results in mismatches in both.

Magnetic charge g is 'dark', cannot couple to the photon, not despite its great strength, but rather because of it. The α-spaced lengths of figure 2 correspond to specific physical mechanisms of photon absorption and emission. Bohr radius cannot be inside Compton wavelength in the basic photon-charge coupling of QED, Rydberg cannot be inside Bohr,... Specific physical mechanisms of photon emission and absorption no longer work.

The GWI model has two mechanisms of topological symmetry breaking. The first arises from the grade increasing property of geometric products, the second from inverse coupling strengths of electric and magnetic charge. This second symmetry breaking offers the possibility of protecting the **Higgs**.

An aside - there is possibility for confusion in terminology of GA. The highest grade element of an algebra is pseudoscalar of that algebra. In the Dirac algebra bivector is interposed between vector and pseudovector of the Pauli algebra wavefunction (figure 3), opening possibilities for endless confusion. Employing the GA grade nomenclature in the middle row of the figure reduces the confusion.

It remains that there is useful information in the terms 'pseudovector' and 'pseudoscalar'. Scalars and vectors - poles and dipoles - contain singularities. Pseudoscalars and pseudovectors (often called 3D axial vectors, despite being 2D elements in 3D space) do not.

terminology difference between Pauli and Dirac algebras					
3D Pauli	scalar	vector	pseudovector	pseudoscalar	_
	e	**d_E, ϕ_B**	**ϕ_E, μ_B**	**g**	_
GA grade	0 - scalar	1 - vector	2 - bivector	3 - trivector	4 - quadvector
4D Dirac	**e**	**d_E, ϕ_B**	**ϕ_E, μ_B**	**g**	**I**
	scalar	vector	bivector	pseudovector	pseudoscalar

FIG. 3: Bivector and trivector are pseudovector and pseudoscalar of the Pauli algebra. Trivector and quadvector are pseudovector and pseudoscalar of the Dirac algebra.

5. Geometric Wavefunction Interactions - Part II

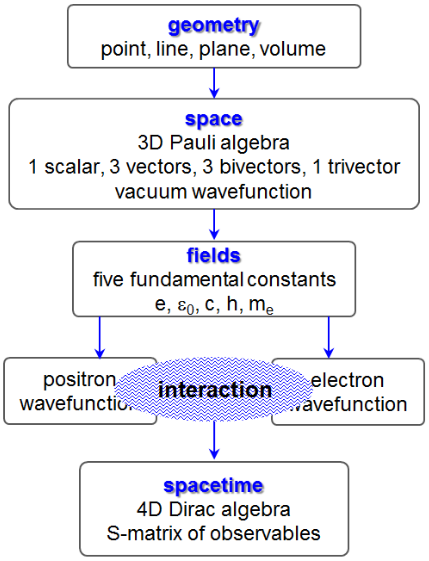

FIG. 4: Emergence of spacetime

Figure 4 shows the theoretical minimum for grasping the point of this essay. It's all about wavefunctions and their interactions, the emergent manifestation of spacetime and observables.

In figure 5, the impedance representation of the S-matrix is generated by the geometric product of two eight-component octonion wavefunction at top and left[72–74]. The wavefunctions are minimally complete representations of the incomplete two-component (scalar electric charge and bivector magnetic moment) electron and positron spinors of the Dirac equation. They counter-rotate in phase space at the Compton frequency. The two photon polarizations are indicated by red and blue γ's just off the main diagonal, in the yellow-highlighted transition modes of what we suggest are perhaps color SU3 matrices. Energy of a photon of Compton wavelength is rest mass of the particle, one of the possible modes of the S-matrix.

The role of spin in figure 5 appears interesting. If angular momentum is conserved, then interaction of two spin 1 magnetic charge trivectors (the pseudoscalars in the octonion algebra) yields the product at lower right in the figure, a spin 0 scalar boson and a spin 2 topological symmetry-breaking object. It would seem most appropriate if **Higgs** and graviton both naturally emerge from the same interaction, from the coupling of two pseudoscalar magnetic charges.

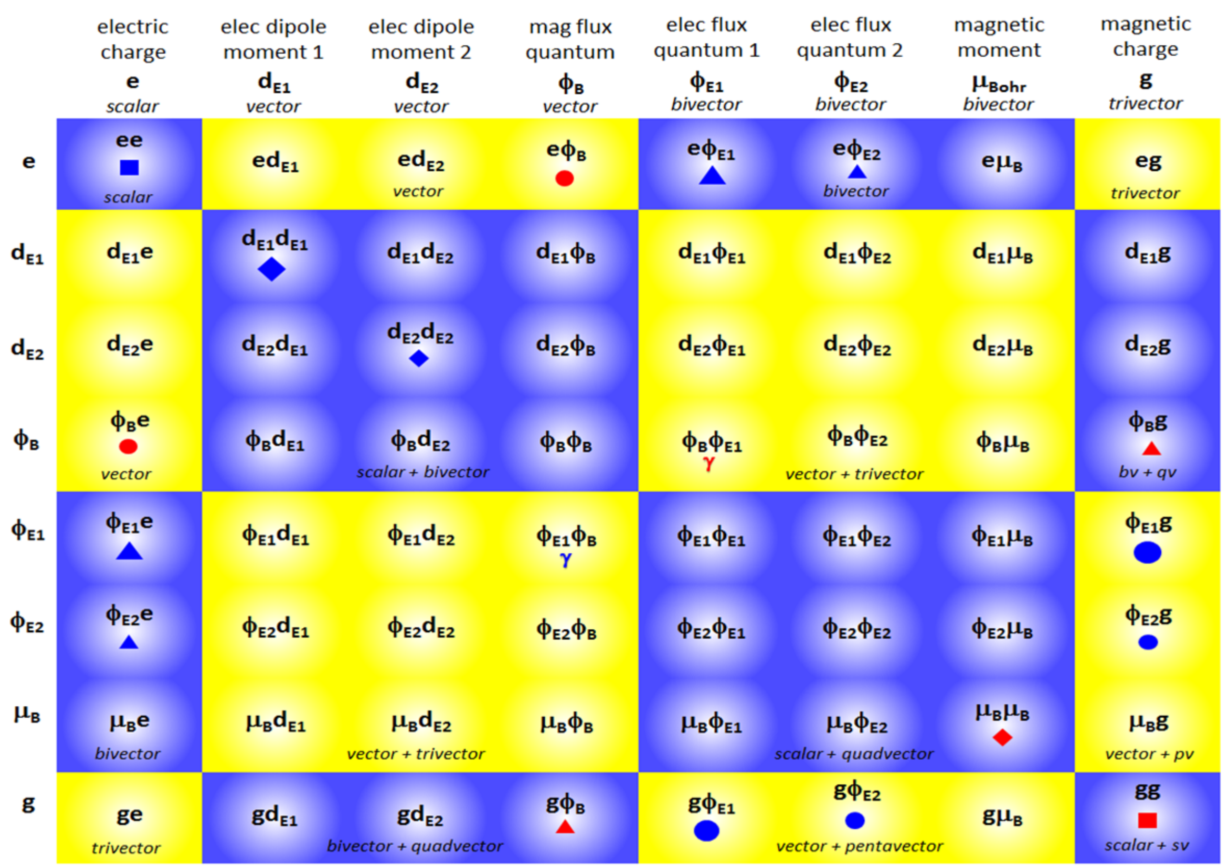

FIG. 5: Impedance representation of the S-matrix, arranged in even flavor eigenmodes (blue) of Dirac algebra and odd color transition modes (yellow) by geometric grade[48]. Modes indicated by colored symbols are plotted in figure 7. Transformation between impedance and scattering representations mixes the grades.

6. Impedance Quantization

Given that wavefunction fields are quantized in quantum field theory, it is unavoidable that impedances of wavefunction interactions are likewise quantized.

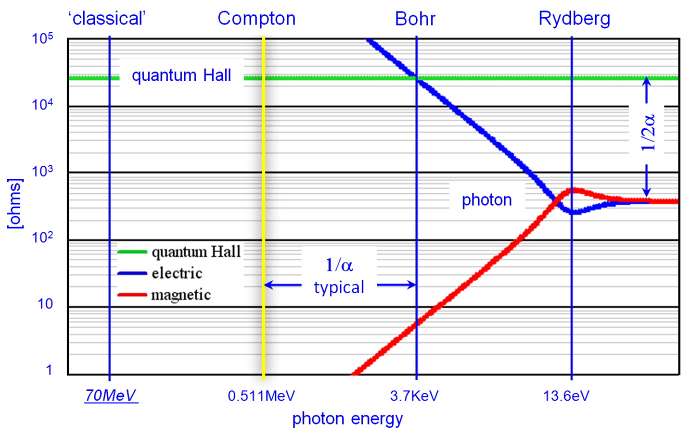

FIG. 6: Photon impedance match to a free electron[31].

Impedance may be defined as the amplitude and phase of opposition to the flow of energy, as opposition to information transmission. In classical electromagnetic theory there are three kinds of impedances - inductive, resistive, and capacitive. Resistive impedance, resistance, is the most familiar of the three. It is dissipative, turns coherent information into incoherent heat. Ideal inductors and capacitors have no resistance. Their only effect is to either retard (capacitive) or advance (inductive) phase of oscillations[75, 76]. Quantum systems are coherent - there is no dissipative resistance, only the phase shifts of quantum impedances, the origin of decoherence, of wavefunction collapse.

With fields of the GWI wavefunction quantized by the fundamental constants that define α, it is unavoidable that impedance networks of wavefunction interactions are organized in powers of α.

Figure 6 shows four of the five fundamental lengths of figure 2 and corresponding energies of same wavelength photons, quantum Hall impedance, and impedance of a 13.6 eV photon entering from right. When separated by the inverse Rydberg, near-field electron dipole impedance (large blue diamonds in fig.7, where photon energy is not 13.6eV but .511MeV) shifts relative phase of electric and magnetic flux quanta comprising the photon, decoupling them. Energy flows via dipole impedance to the Bohr radius, coupling via quantum Hall (circles) to capacitive Coulomb (squares) and scalar Lorentz (triangles) modes, forming a three-component C-L-C oscillator with mode-coupling assistance from topological phase shifts (quantum Hall, centrifugal,..).

Here the mainstream community is immediately lost, at the outset of exploring the Rosetta stone of atomic physics, the Hydrogen atom. Neither photon[77] nor electron[29] near-field impedances can be found in textbooks, curricula, or journals of the physics community, are absent from physicists' education and practice. What governs the flow of energy in the basic photon-electron interaction of QED was lost in physics[43].

There are two essential points. First, what matters are not absolute impedances, but rather their relative values, whether impedances are matched. In this they are like the energy whose transmission they govern. Absolute values of energy are said to be (singularities complicate the question) of no consequence, only differences.

The second point distinguishes between scale-dependent and invariant impedances. *Scale dependent impedances* are geometric, include Coulomb, scalar Lorentz, and dipole-dipole, with $1/r$ and $1/r^3$ potentials. They are causal and local, communicate both amplitude and phase, correspond to translation gauge fields. Logarithmic dependence renders them parametric[78, 79], permitting essential noiseless frequency domain translation of energy.

Invariant impedances are topological, include vector Lorentz of quantum Hall and Aharonov-Bohm effects, centrifugal, chiral, Coriolis, and three body. Associated potentials are inverse square, the forces often termed 'fictitious'[80]. Resulting motion is perpendicular to applied force. They cannot do work, cannot communicate information, communicate only phase, not a single measurement observable. Like gravity, they cannot be shielded. They are the acausal channels of non-local entanglement, correspond to rotation gauge fields of gauge theory gravity[45, 46, 81–83].

7. Emergence

"Emergence refers to the existence or formation of collective behaviors - what parts of a system do together that they would not do alone. In describing collective behaviors, emergence refers to how collective properties arise from the properties of parts, how behavior at a larger scale arises from the detailed structure, behavior and relationships at a finer scale."[84]

In what follows we present four examples of *"...how collective properties arise from the properties of parts..."*, the parts being wavefunctions and what *"...they would not do alone..."* being their interactions, at three scales - Compton wavelength of the electron, Planck length, and radius of the observable universe - illustrating how *"...behavior at a larger scale arises from the detailed structure, behavior and relationships at a finer scale"*

To calculate quantized impedance networks of GWI wavefunction interactions using Maxwell's equations is a formidably daunting prospect. However, every massive particle has mechanical impedance[85]. Fundamental quantum logic of the background independence essential for quantum gravity[86] requires application of Mach's principle to the two-body problem[30], yielding a simple calculation of mechanical impedances, easily converted to electromagnetic via electromechanical oscillators[29]. It is this simple solution to the problem of calculating quantum impedances that permits phenomenology of the following four examples of emergence from the three assumptions.

7.1 Compton Wavelength - the 'Mass Gap' and the Unstable Particle Spectrum

In the GWI model the unstable particle spectrum is generated by excitation of the mass gap at the electron Compton wavelength, as shown in figure 7. Vacuum wavefunction impedances excited by dark modes differ from those of stable particles (γ, e, p). Mixed modes decohere, a consequence of the resulting differential phase shifts. Selecting visible wavefunction components only (electric charge, magnetic moment, magnetic flux quantum) in fig.5 opens possible calculation of nucleon structure and spin [49, 50, 61, 62].

Looking *briefly ahead*, the scale-dependent quantized impedance of the magnetic Coulomb **Higgs** mode indicated by the red square in figure 5 is plotted in figure 7. It intersects the ~377 ohm photon far-field impedance at the dominant ~10 GeV bottomonium decay channel of the family of coupled modes of the superheavies, of particular interest when examining the **Higgs** mode of the octonion wavefunction at the Planck length of figure 8.

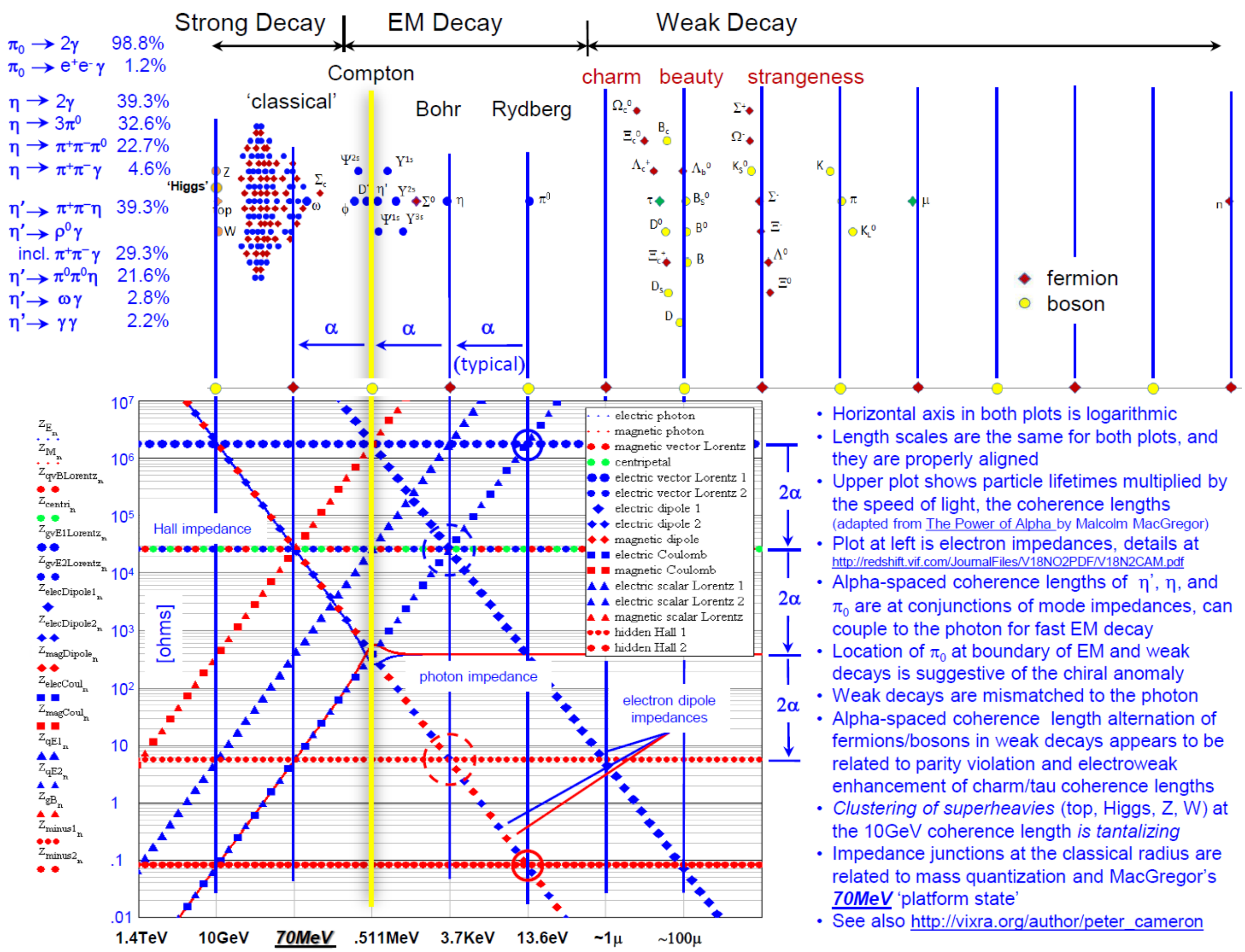

FIG. 7: Modes indicated by colored symbols in figure 5 are plotted in the impedance network at lower left. Phase correlation of unstable particle causal lifetimes/light cone coherence lengths [87–89] with impedance nodes of the network follows from the fact that impedances must be matched for the energy transmission essential in decay[42].

7.2 Planck Length - Gravitation in the near-field

Not all are in agreement that Einstein whole-heartedly endorsed curved space interpretations. He expressed this quite clearly in politically correct private communication:

"It is wrong to think that 'geometrization' is something essential. It is only a kind of crutch for finding of numerical laws. Whether one links 'geometrical' intuitions with a theory is a ... private matter."[90, 91]

Riemann's curvature tensor preceded general relativity by six decades[92]. Absent Clifford's geometric interpretation [15, 43], Einstein's adoption of Riemann's formalism led inevitably to dominance of curved space interpretations. However, the equivalence of flat Minkowski spacetime gauge theory gravity with curved space general relativity was introduced by the Cambridge group and Professor Hestenes, and elaborated upon by them over the course of following decades.[81, 82, 93–97]. In that context, impedance quantization offers immediate possibilities for quantizing gravity at the Planck length[36, 38, 45]. Of importance in general relativity is not geometrization, but rather the equivalence of gravitational and inertial mass, the equivalence principle[58].

Impedance mismatches between Compton and Planck wavefunctions reveal an identity. Gravitational force between the two wavefunctions equals mismatch-attenuated electromagnetic force they share, at the ppb accuracy of the five fundamental constants input by hand. Newton's big G, by many orders of magnitude the least accurate of the fundamental constants, cancels out in the ratio of ratios establishing the identity[36]. This is perhaps an example of emergence in what Einstein called the strength of equation systems[98, 99], from the GWI perspective in terms of simple algebra rather than differential equations.

Flat spacetime phase shifts of electromagnetic wavefunction interactions are the gauge theory gravity equivalent of spatial curvature of general relativity. While strong classical arguments have been advanced against electromagnetic models of gravitation[100], preliminary examination suggests such arguments fail point-by-point when full consequences of geometric wavefunction interaction impedances enter gauge theory gravity[46].

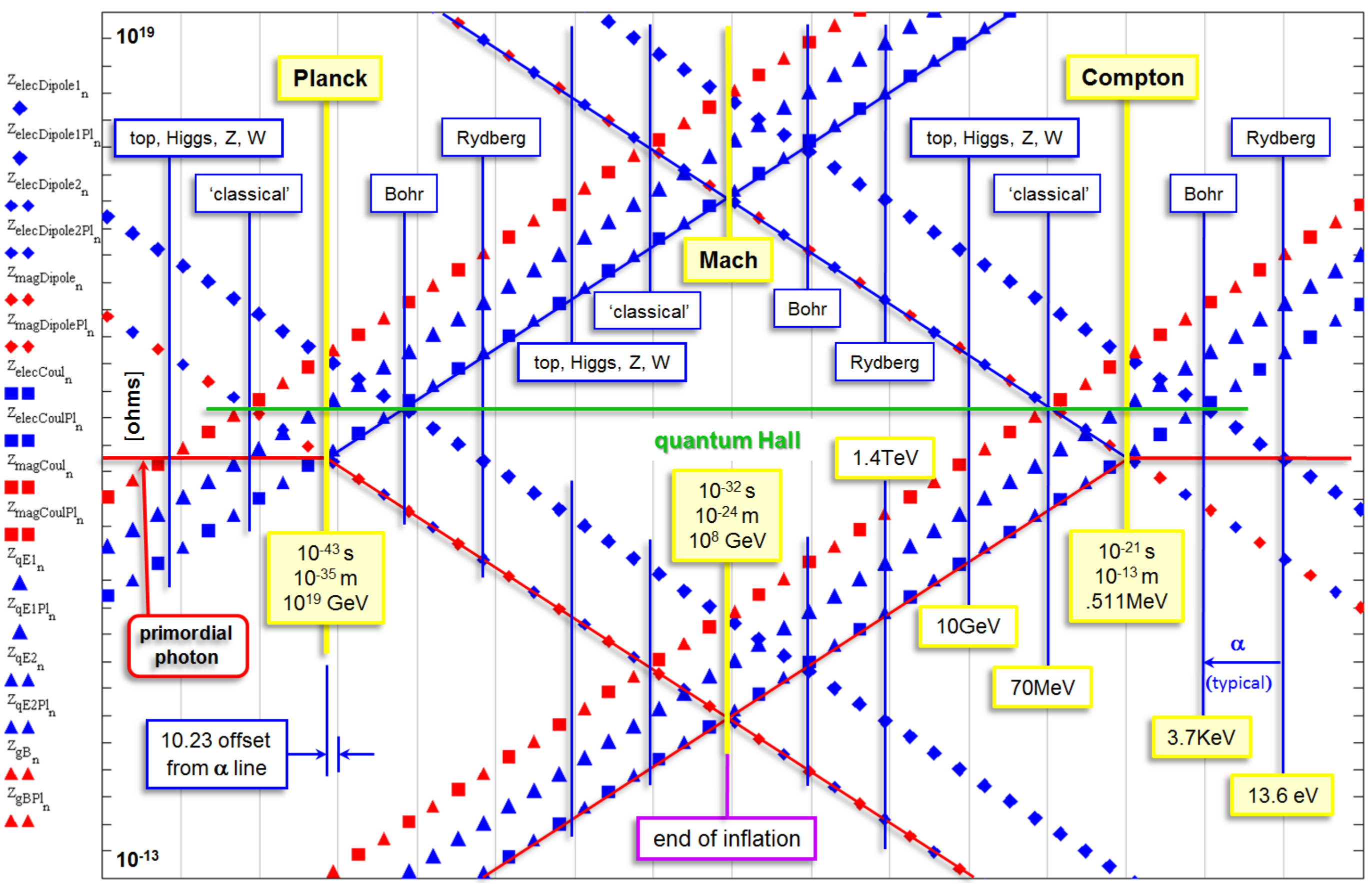

FIG. 8: A subset of interaction impedance networks of octonion wavefunctions defined at Compton and Planck lengths, showing a .511 Mev photon entering from right and 'primordial photon' from left. [36, 38, 39, 44]

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Coming back to the equivalence principle, in the GWI model inertial mass finds its origin in the Compton wavefunction, gravitational mass in the Planck wavefunction. Origin of inertial mass is in electric and magnetic fields of the wavefunction at the Compton wavelength. Origin of gravitational mass is in the mismatch to the Planck length. They are equal. However to say they are equivalent (or not) requires a more refined understanding of 'equivalence'.

The earth-moon system $1/r$ monopole potential of gravitation is associated with a local scale-dependent impedance, transmits both amplitude and phase, is causal. The $1/r^2$ centrifugal potential of inertia is associated with a non-local scale-invariant topological impedance, is acausal, communicates only phase, not single measurement observable. Unlike potentials associated with scale-dependent impedances, potentials associated with invariant impedances cannot be shielded (for example, the vector Lorentz potential of quantum Hall and Aharonov-Bohm effects). This distinction is potentially problematic for the equivalence principle, and discussed in the next subsection.

Of immediate interest, the **Higgs** enters again here. The geometric octonion vacuum wavefunction is the same at Planck and Compton scales. Taking the GWI model seriously at all scales, in the big bang (or bounce) the first mode excited by the primordial photon of figure 8 is the fermionic top, breaking chiral symmetry at the outset, the vacuum wavefunction not yet fully physically manifested. The next mode is the **Higgs** scalar, the first essential gauge, which then couples to Z and W, transmitting energy from primordial photon and top, 'giving mass' so to speak.

7.3 Observable Universe - Gravitation in the macroscale

Planck wavefunction event horizon is unstable, almost instantaneously radiates its energy as a Hawking photon. Impedance mismatches (figures 8 and 9) reflect back all but an almost infinitesimal fraction, yielding the continuously increasing Hawking photon wavelength of figure 9, where the horizontal scale is logarithmic and vertical the ratio of field energy to mass gap.

Hawking photon energy (gravitational mass) at the electron Compton wavelength[36] precisely equals electromagnetic self-energy of electron wavefunction fields (inertial mass)[29, 33, 49]. The progressively attenuated Hawking photon resonates correspondingly smaller mass gaps at impedance nodes of successively greater wavelengths.

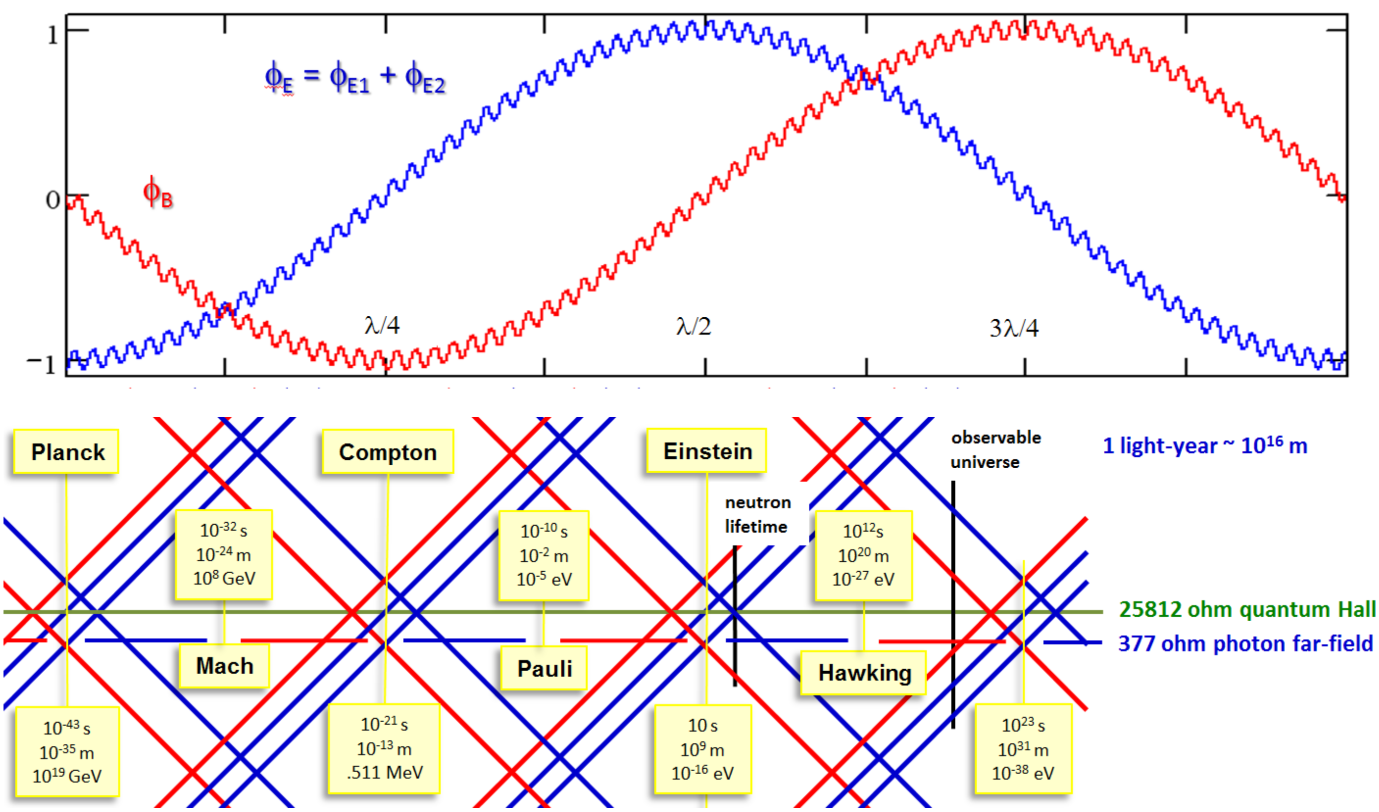

FIG. 9: Amplitude and phase correlation of Hawking photon radiated from Planck length with α-spaced nodes of the impedance network generated by octonion vacuum wavefunction mass gap excitation.
Planck and Compton wavefunctions comprise a quarter-wave resonator.

Coming back again to the equivalence principle - potentials associated with the scale-invariant centrifugal impedance cannot be shielded, Aharonov-Bohm effect being the prime example. This is not true for the scale-dependent monopole impedance. The problematic character of causal gravitation vs acausal inertia, shieldable or not, can be understood in terms of quantum phase. With scale-invariant topological impedances, phase shifts are quantized, discrete, unchanging with scale. Quantum Hall effect, and particularly fractional quantum Hall, are good examples. With scale-dependent geometric impedances the phase shifts are smooth, continuous, vary with scale. For the Hawking photon, the variation of phase with scale on the scale of experimental observables of physics is extremely slow. The Hawking photon near-field of figure 9 extends beyond the limit of the observable universe. Gravitational potential as communicated by the near-field Hawking photon is *almost* scale invariant, *almost* unshieldable, just *barely* causal.

Interpretation of figure 9 is highly speculative, dependent upon both initial phase of the Hawking photon and relative phases of E and B flux quanta (handedness) that comprise the photon. Initial and relative phases of the figure were quasi-randomly selected - pure electric field at the Planck length, left-handed. It suggests gravitation was repulsive in the first quarter-wave of the first zeptosecond, peaked in attractive strength on solar system scale, and again becomes repulsive at a time far beyond present age of the observable universe.

Timescale at Hawking nodes and beyond is potentially interesting for astrophysics, question among others being whether 'dark energy' can be attributed to weakening attraction. In any case, one would expect the scenario outlined here to have observable effects for astrophysics, and to provide a first step towards guidance in the search.

The duality of impedance nodes, both extreme high and extreme low impedances, found the Hawking scale and at Mach and Pauli scales as well is curious, suggesting things are happening that are extremely difficult for us to observe due to extreme impedance mismatches of the nodes at those scales. In particular, the 10^{-5} eV Pauli scale brings to mind difficultes of axion searches.

Timescale between Pauli and Einstein nodes is potentially interesting for CERN antimatter experiments[101]. Model presented here suggests antimatter phase shift is opposite of matter, so anti-gravitation will be repulsive. Antimatter falls up. There is a second effect - acceleration is time-dependent for brand-new accelerator antimatter, of interest in the first minute or so whether repulsive or attractive.

Timescale between Planck and Compton nodes is of interest to those who favor inflationary models, particularly in the context of the quarter-wave resonator as shown in figure 9. Last of the three GWI model assumptions took the electron mass to define the scale of space. Implicit in that is the existence of the point, of the singularity, approximated with excellent accuracy by the event horizon at the Planck length in the model.

Not one but two wavefunctions are required to define the scale of space, one explicit in the assumptions, the other implicit. That Newton's big G cancels out in the impedance match to the Planck length, erasing that additional assumed fundamental constant input by hand, seems in hindsight quite remarkable. Equally remarkable is the appearance of nodes between Planck and Compton scales, at impedance extremes of the Mach scale. In context of the big bang bounce, the appearance of this second wavefunction, where energy is equally shared between electric and magnetic fields, defined the scale of space, marked the 10^{-32} second end of inflation. This seems somehow recursively emergent, that the need for the electron mass to define the scale of space is lost when viewed from the perspective of the primordial photon, that we are down to two assumptions - vacuum wavefunction and coupling constant.

The observable universe is within the extreme near-field first cycle of transition Hawking photons radiated from Planck lengths of every massive particle in the universe.

The consequent phase shift is what we call gravity.

7.4 The Anomalous Chiral Anomaly

Having taken a quick look at what emerges from the three assumptions at three scales - electron Compton wavelength, Planck length, and boundary of observable universe - it seems timely to return to the network at the electron Compton wavelength in greater detail for impedance-based branching ratio calculations relevant to the chiral anomaly.

Anomalies are *"...breakings of classical symmetries by quantum corrections, which arise when the regularizations needed to evaluate small fermion loop Feynman diagrams conflict with a classical symmetry of the theory."*[102]

Chiral symmetry appears to be broken only by weak interactions. The presence of the anomaly in strong and electromagnetic quantum field theory calculations[102–105] seems to be an inevitable result of the regularization needed to remove infinities before mass and charge renormalizations can be accomplished. In the presence of the anomaly *either* chiral symmetry *or* gauge invariance must be broken.

The GWI model is naturally gauge invariant and finite, free of regularization and renormalization. It appears there is no manifest chiral anomaly in the model, no need for chiral symmetry breaking here.

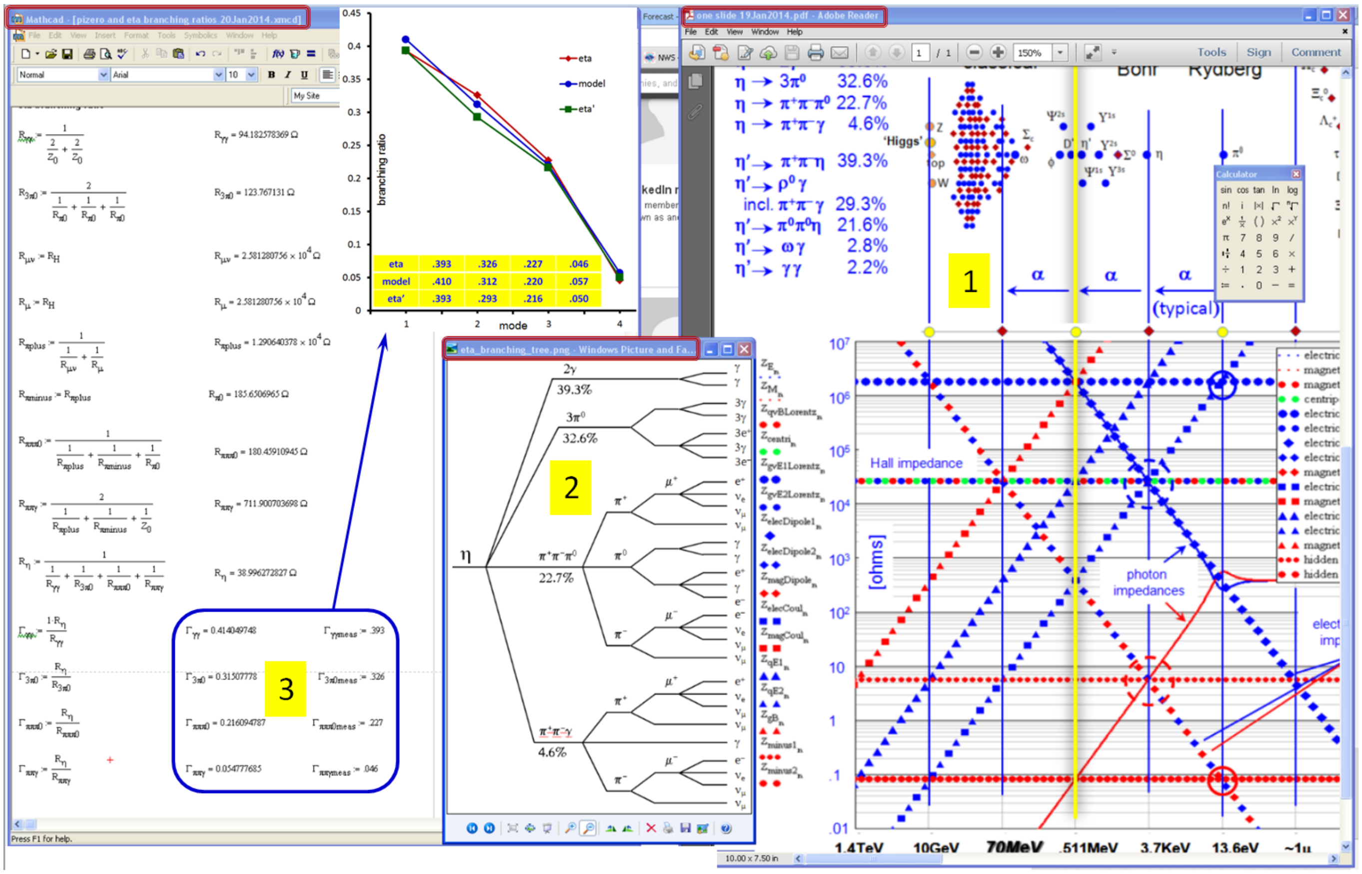

FIG. 10: Impedance-based branching ratio calculations of the η.

What then of the anomaly? QFT requires it for calculation of π^0 branching ratios.

Figure 10 is a composite flowchart. Window 1 shows nodes of the impedance network at coherence lengths of the π_0, η, and η' used in the calculation[42]. Window 2 shows the logic tree used to calculate mismatches for the η, and the third window a portion of the mathcad calculation[41]. Comparison with data is shown in the table and graph.

The much less complex π_0 calculation agrees with experimental data at the $\frac{\alpha}{\pi} \sim .002$ level, suggesting that including higher-order α-spaced impedance nodes in the calculation would improve the agreement.

While it remains on good authority that *"nobody understands chirality"*[106], the GWI model gives accurate answers for branching ratios of not only π_0, but η and η' as well. The anomaly causes QFT to fail for the π_0, inverts the ratio, and says nothing about η and η'.

The chiral impedance is inverse square, topological, scale invariant. The chiral current comprises quantum phase and only quantum phase, not a single-measurement observable. With proper inclusion of chiral phase, or more generally with inclusion of all appropriate invariant impedances, conflict between chiral symmetry and gauge invariance might possibly be removed. The phase information is contained in correlated α-spacing of impedance nodes and particle coherence lengths. The vacuum wavefunction is in some sense a virtual chirp to the unstable particle spectrum, generating a transformation from amplitude to phase. Amplitudes are calculated from impedance matches. The phase information (absence of which required the unsuccessful analytic continuations of the 60s era S-matrix bootstrap[26, 27]) presents itself 'for free' in the correlation between impedance network nodes and particle lifetimes.

This suggests that one or more inverse-square potential terms are missing from the Lagrangian, that these would remove the anomaly, and that in their presence the correct π^0 branching ratios would be found.

And it remains that nobody understands chirality.

The model presented here deals exclusively with two-body interactions, with interactions of two wavefunction components. This is integral in the philosophical foundation of the model[30]. Like the chiral impedance, the three-body impedance is scale invariant. It seems that to understand chirality in the GWI model it might be necessary to apply it to the three component neutrino, a work in progress.

8. Claims of Geometric Algebra and the GWI Model

Having illustrated how *"...behavior at a larger scale arises from the detailed structure, behavior and relationships at a finer scale"*[84] via the four examples of the previous section, the following claims offer more detail on how maximally natural wavefunctions simplify our efforts to understand quantum mechanics.

8.1 Claims of Geometric Algebra

The preface to the 50th anniversary second edition[18] of Professor Hestenes' original text makes four claims for innovation in SpaceTime Algebra, summarized here:

- STA enables a co-ordinate free formulation for all of relativistic physics, is **background independent**.
- Pauli and Dirac matrices are **basis vectors** in space and spacetime, no connection to spin.
- STA reveals that the **unit imaginary** in quantum mechanics has its origin in spacetime geometry.
- STA reduces the mathematical divide between classical, quantum, and relativistic physics.

These are some of the many properties of GA that make it the natural mathematical language of physics, especially background independence and the basis.

8.2 Claims of the GWI Model

To these we add the following claims of the Geometric Wavefunction Interaction model[46, 55]:

- **photon-electron interaction** - Dirac spoke to the core of the model in asserting that

  "Until we have a really satisfactory explanation of how electrons and photons interact, it will hardly be possible to go on and explain the other particles."[107]

  It appears the two perspectives offered by team GWI/QED is adequate to claim a satisfactory explanation.
- **gauge invariance** - Impedances shift phases, provide a coherent alternative formulation of the effect of the covariant derivative. GWI is *naturally gauge invariant.*
- **finiteness** - Impedance mismatches provide natural QED cutoffs. Both singularity and the boundary at infinity are decoupled by the infinite quantum impedance mismatches. No renormalization. GWI is *naturally finite.*
- **confinement** - Confinement is the flip side of finiteness. Energy is reflected from mismatches, back to matched impedance nodes at the wavefunction wavelength, be it Planck, Compton, deBroglie,... GWI contains the strong and weak nuclear forces, is *naturally confined.*
- **asymptotic freedom** follows from exact matching at wavefunction impedance network nodes.
- **background independence** - In STA, motion is described with respect to the object in question. Similarly, in the two body problem motion is with respect to one of the two. There is no background. GWI is naturally background independent, a requirement for calculating impedances from Mach's principle[30]
- **gravitation** - Matching quantized impedances at the Planck scale reveals an exact identity between electromagnetism and gravity[46].
- **all scales** - The model is effective at all scales. Mis-interpretation of the measured running of α results from overlooking impedance quantization, from conflating running and mismatching.[108]
- **heirarchy** - Absence of renormalization and presence of inert vacuum wavefunction in flat space of Pauli and Dirac algebras resolve the heirarchy problems of both Higgs mass and cosmological constant.
- **string theory** - Assignment of E and B fields to the octonion vacuum wavefunction yields a representation of ten 'dimensional' string theory in the ten degrees of freedom of the GWI model in flat 4D Minkowski spacetime.
- **quantum interpretation** - GWI wavefunctions exist as electromagnetic fields configured as geometric objects in 3D space, interacting via Maxwell's equations in a network of quantized impedances.

  Wavefunctions and their interactions can be visualized. This permits resolution of many if not all paradoxes found in proliferating worlds of quantum interpretations[10, 11, 50, 62, 64].
- **naturalness** - By the criteria presented in the first page of this paper, the model is arguably maximally natural.

9. Summary

After a brief introduction, the first section of this paper offered short descriptions of properties considered useful in defining what we mean by naturalness in physics.

The second section introduced the reader to the three assumptions needed to construct the GWI model - vacuum wavefunction, electromagnetic coupling constant, and electron mass (to set the scale of space).

With that brief description of the model, in the third section it was possible to begin addressing the desired properties of naturalness presented in the first section.

The fourth section explored details of topological symmetry breaking inherent in both the geometric representation of Clifford algebra and the inversion of fundamental lengths - Rydberg, Bohr, Compton, classical, and **Higgs** - by the strength of magnetic charge.

Emergence of the impedance representation of the S-matrix from geometric wavefunction interactions was introduced in section five. The possibility was mentioned that **Higgs** and graviton both naturally emerge from the same interaction, from the coupling of two pseudoscalar magnetic charges - the equivalence principle at a quantum level.

To prepare the reader for the detailed emergence at all scales presented in the phenomenology of what followed, what governs amplitude and phase of energy transmission, of the flow of information, was discussed in section six. Given that wavefunction fields are quantized in any quantum field theory, it is unavoidable that impedances of wavefunction interactions are likewise quantized.

Section seven presented emergence of beyond Standard Model analyses of four examples - lifetimes of the unstable particle spectrum from excitation of the mass gap, quantum gravity in the near field from impedance matching to the Planck wavefunction, quantum gravity on the macroscale from mismatch attenuation of the near field Hawking photon, and the chiral anomaly.

And finally, the previous section presented claims of Professor Hestenes for the geometric representation of Clifford algebra, followed by a selection of claims for the GWI model.

Here we offer two additional claims. The first goes back to antiquity, is buried deep in our intuitive experience of the world, has not yet been mentioned in the present work. Structural naturalness is well exemplified by the contrast between unnatural retrograde epicycles of geocentric solar system models versus smooth natural ellipses of the heliocentric model. The GWI model, with nothing more than its three natural assumptions, is arguably maximally structurally natural.

The second 'claim' reflects the title of this paper. We suggest that natural models are maximally emergent, that manifest emergence must be included in a list of desirable properties of a natural model.

10. Conclusion

"The concept of causality requires that macroscopic phenomena follow from microscopic equations." [3]

Feynman expresses the passion of the physicist:

"...people who have studied (physics) far enough to begin to understand a little of how things work are fascinated by it, and this fascination drives them on in this investigation that the race is making into its own environment."[110]

Just as an intuitive understanding of electron and photon interactions is the foundation of a proper understanding of quantum mechanics, and a proper understanding of quantum mechanics is the foundation of this investigation we are making into our own environment, our philosophical insight is the foundation of it all.

No one says this better than Einstein himself:

"A knowledge of the historic and philosophical background gives that kind of independence from prejudices of his generation from which most scientists are suffering. This independence created by philosophical insight is - in my opinion - the mark of distinction between a mere artisan or specialist and a real seeker after truth."[90].

FIG. 11: Duality and collapse of the wavefunction [109]

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