

On the Ramanujan's Mock theta functions of tenth order: new possible mathematical developments and mathematical connections with some sectors of Particle Physics and Black Hole physics II

Michele Nardelli¹, Antonio Nardelli

Abstract

In the present research thesis, we have obtained various and interesting new possible mathematical developments concerning some Ramanujan's Mock theta functions of tenth order and mathematical connections with some sectors of Particle Physics and Black Hole physics

¹ M.Nardelli have studied by Dipartimento di Scienze della Terra Università degli Studi di Napoli Federico II, Largo S. Marcellino, 10 - 80138 Napoli, Dipartimento di Matematica ed Applicazioni "R. Caccioppoli" - Università degli Studi di Napoli "Federico II" – Polo delle Scienze e delle Tecnologie Monte S. Angelo, Via Cintia (Fuorigrotta), 80126 Napoli, Italy

The Ramanujan's mathematical paradise

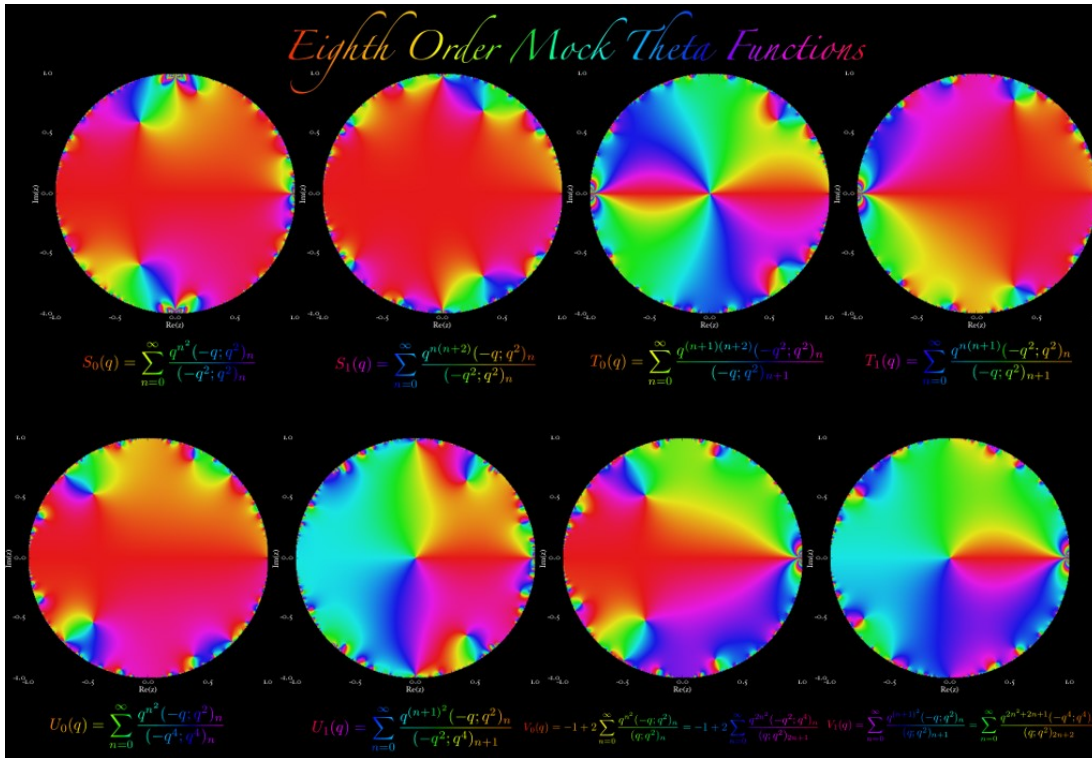


From:

**Ramanujan Institute for Advanced Study in Mathematics
University of Madras, Chennai, India.**

From:

<http://owen.maresh.info/mocktheta.html>



We want to highlight that the formulas and the very relevant results obtained, are based on our personal and original interpretation, also if with values corresponding to the indications provided by the Notebooks consulted. (for example for $|q| < 1$, for $x \neq 0$, for each non-negative integer n , (for $n \geq 2$), we take $q = 0,5$ $n = 2$, and $x = 0.625, 1, 2$ and 3).

From:

Ramanujan's Lost Notebook Part V
Tenth Order Mock Theta Functions: Part IV

For $|q| < 1$, for $x \neq 0$, for each non-negative integer n , (for $n \geq 2$):

Now,

$$\begin{aligned}
 & \sum_{s=-\infty}^{\infty} \frac{(1 - q^{4s+1})q^{4s^2+2s}}{(1 - xq^{2s})(1 - q^{2s+1}/x)} \\
 &= \sum_{s=-\infty}^{\infty} \frac{(1 - q^{2s+1}/x + (q^{2s+1}/x)(1 - xq^{2s}))q^{4s^2+2s}}{(1 - xq^{2s})(1 - q^{2s+1}/x)} \\
 &= \sum_{s=-\infty}^{\infty} \frac{q^{4s^2+2s}}{1 - xq^{2s}} + \frac{1}{x} \sum_{s=-\infty}^{\infty} \frac{q^{4s^2+4s+1}}{1 - q^{2s+1}/x} \\
 &= \sum_{s=-\infty}^{\infty} \frac{q^{(2s)^2+2s}}{1 - xq^{2s}} - \sum_{s=-\infty}^{\infty} \frac{q^{(2s+1)^2+2s}}{1 - xq^{2s+1}} \\
 &= \sum_{n=-\infty}^{\infty} \frac{(-1)^n q^{n^2+n}}{1 - xq^n}, \tag{11.3.15}
 \end{aligned}$$

For $q = 0.5$, $n = 2$ and $x = 0.625, 1, 2$ and 3 , we have calculated the inverse:

$$1 / (((0.5)^6 / (1 - 1 * 0.5^2)))$$

Input:

$$\frac{1}{\frac{0.5^6}{1 - 1 \cdot 0.5^2}}$$
[Open code](#)

Enlarge Data Customize A [Plaintext](#) [Interactive](#)

Result:
48

$$1 / (((0.5)^6 / (1 - 2 * 0.5^2)))$$

Input:

$$\frac{1}{\frac{0.5^6}{1 - 2 \cdot 0.5^2}}$$
[Open code](#)

Enlarge Data Customize A [Plaintext](#) [Interactive](#)

Result:
32

$$1 / (((0.5)^6 / (1 - 3 * 0.5^2)))$$

Input:

$$\frac{1}{\frac{0.5^6}{1-3 \times 0.5^2}}$$

[Open code](#)

Enlarge Data Customize A [Plaintext](#) [Interactive](#)

Result:

16

$$1 / (((0.5)^6 / (1 - 0.625 \times 0.5^2)))$$

Input:

$$\frac{1}{\frac{0.5^6}{1-0.625 \times 0.5^2}}$$

[Open code](#)

Enlarge Data Customize A [Plaintext](#) [Interactive](#)

Result:

54

$$[1 / (((0.5)^6 / (1 - 2 \times 0.5^2)))] * [1 / (((0.5)^6 / (1 - 0.625 \times 0.5^2)))]$$

Input:

$$\frac{1}{\frac{0.5^6}{1-2 \times 0.5^2}} \times \frac{1}{\frac{0.5^6}{1-0.625 \times 0.5^2}}$$

[Open code](#)

Enlarge Data Customize A [Plaintext](#) [Interactive](#)

Result:

1728

This result is very near to the mass of candidate glueball $f_0(1710)$ meson. Furthermore, 1728 occurs in the algebraic formula for the j -invariant of an elliptic curve. As a consequence, it is sometimes called a Zagier as a pun on the Gross–Zagier theorem. The number 1728 is one less than the Hardy–Ramanujan number 1729

$$((((1 / (((0.5)^6 / (1 - 2 \times 0.5^2)))] * [1 / (((0.5)^6 / (1 - 0.625 \times 0.5^2)))])))))^{1/3}$$

Input:

$$\sqrt[3]{\frac{1}{\frac{0.5^6}{1-2 \times 0.5^2}} \times \frac{1}{\frac{0.5^6}{1-0.625 \times 0.5^2}}}$$

[Open code](#)

Enlarge Data Customize A [Plaintext](#) [Interactive](#)

Result:

12

This result a very good approximation to the value of Black Hole entropy 12,19

$$2 * (((([1/ (((0.5)^6 / (1- 2*0.5^2)))] * [1/ (((0.5)^6 / (1- 0.625*0.5^2)))])))))^1/3$$

Input:

$$2 \sqrt[3]{\frac{1}{\frac{0.5^6}{1-2 \times 0.5^2}} \times \frac{1}{\frac{0.5^6}{1-0.625 \times 0.5^2}}}$$

[Open code](#)

Enlarge Data Customize A [Plaintext](#) [Interactive](#)

Result:

24

This value is linked to the "Ramanujan function" (an elliptic modular function that satisfies the need for "conformal symmetry") that has 24 "modes" corresponding to the physical vibrations of a bosonic string.

$$((((([1/ (((0.5)^6 / (1- 2*0.5^2)))] * [1/ (((0.5)^6 / (1- 0.625*0.5^2)))])))))^1/15$$

Input:

$$\sqrt[15]{\frac{1}{\frac{0.5^6}{1-2 \times 0.5^2}} \times \frac{1}{\frac{0.5^6}{1-0.625 \times 0.5^2}}}$$

[Open code](#)

Enlarge Data Customize A [Plaintext](#) [Interactive](#)

Result:

More digits

1.64375...

$$1.64375 \approx \zeta(2) = \frac{\pi^2}{6} = 1.6449$$

$$1/3 * (((([1/ (((0.5)^6 / (1- 2*0.5^2)))] * [1/ (((0.5)^6 / (1- 0.625*0.5^2)))])))))^1/3$$

Input:

$$\frac{1}{3} \sqrt[3]{\frac{1}{\frac{0.5^6}{1-2 \times 0.5^2}} \times \frac{1}{\frac{0.5^6}{1-0.625 \times 0.5^2}}}$$

[Open code](#)

Enlarge Data Customize A [Plaintext](#) [Interactive](#)

Result:

4

This result is the minimal possible value of the mass of hypothetical DM particles

$$24 + \frac{1}{6} * (((((((([1/ (((0.5)^6 / (1- 2*0.5^2))]) * [1/ (((0.5)^6 / (1- 0.625*0.5^2))])])))))))^1/15))))^16$$

Input:

$$24 + \frac{1}{6} \sqrt[15]{\frac{1}{\frac{0.5^6}{1-2 \times 0.5^2}} \times \frac{1}{\frac{0.5^6}{1-0.625 \times 0.5^2}}}^{16}$$

[Open code](#)

Enlarge Data Customize A [Plaintext](#) [Interactive](#)

Result:

• More digits

497.401...

This result is practically equal to the rest mass of Kaon meson 497,614

$$11 + \frac{1}{3} * (((((((([1/ (((0.5)^6 / (1- 2*0.5^2))]) * [1/ (((0.5)^6 / (1- 0.625*0.5^2))])])))))))^1/15))))^16$$

Input:

$$11 + \frac{1}{3} \sqrt[15]{\frac{1}{\frac{0.5^6}{1-2 \times 0.5^2}} \times \frac{1}{\frac{0.5^6}{1-0.625 \times 0.5^2}}}^{16}$$

[Open code](#)

Enlarge Data Customize A [Plaintext](#) [Interactive](#)

Result:

• More digits

957.801...

This result is practically equal to the rest mass of Eta prime meson 957,66

Note that:

$$-64 + 8 + \frac{1}{3} * (((((((([1 / (((0.5)^6 / (1 - 2*0.5^2)))] * [1 / (((0.5)^6 / (1 - 0.625*0.5^2)))])))))^1/15))))^16 + 497.401$$

Input interpretation:

$$-64 + 8 + \frac{1}{3} \sqrt[15]{\frac{1}{\frac{0.5^6}{1-2 \times 0.5^2}} \times \frac{1}{\frac{0.5^6}{1-0.625 \times 0.5^2}}}^{16} + 497.401$$

[Open code](#)

Result:

More digits

1388.20...

This value is very near to the rest mass of Sigma baryon 1387,2

$$(-48 - 108 - 64 + 8) + \frac{1}{3} * (((((((([1 / (((0.5)^6 / (1 - 2*0.5^2)))] * [1 / (((0.5)^6 / (1 - 0.625*0.5^2)))])))))^1/15))))^16 + 497.401$$

Input interpretation:

$$(-48 - 108 - 64 + 8) + \frac{1}{3} \sqrt[15]{\frac{1}{\frac{0.5^6}{1-2 \times 0.5^2}} \times \frac{1}{\frac{0.5^6}{1-0.625 \times 0.5^2}}}^{16} + 497.401$$

[Open code](#)

Result:

More digits

1232.20...

This result is practically equal to the rest mass of Delta baryon 1232.

Now, we have:

$$6 \log \left(- \left(- \frac{0.5^{15} \times 3^3}{\frac{1-0.5^{10}}{3^2}} - \frac{0.5^{15} \times 3^5}{\frac{1-0.5^{12}}{3^2}} \right) \right)$$

[Open code](#)

- $\log(x)$ is the natural logarithm

Enlarge Data Customize A [Plaintext](#) [Interactive](#)

Result:

More digits

-15.6075...

Series representations:

More

$$6 \log \left(- \left(- \frac{0.5^{15} \times 3^3}{\frac{1-0.5^{10}}{3^2}} - \frac{0.5^{15} \times 3^5}{\frac{1-0.5^{12}}{3^2}} \right) \right) = -6 \sum_{k=1}^{\infty} \frac{(-1)^k (-0.925819)^k}{k}$$

[Open code](#)

Enlarge Data Customize A [Plaintext](#) [Interactive](#)

$$6 \log \left(- \left(- \frac{0.5^{15} \times 3^3}{\frac{1-0.5^{10}}{3^2}} - \frac{0.5^{15} \times 3^5}{\frac{1-0.5^{12}}{3^2}} \right) \right) = 12 i \pi \left[\frac{\arg(0.0741813 - x)}{2 \pi} \right] + 6 \log(x) - 6 \sum_{k=1}^{\infty} \frac{(-1)^k (0.0741813 - x)^k x^{-k}}{k} \text{ for } x < 0$$

[Open code](#)

$$6 \log \left(- \left(- \frac{0.5^{15} \times 3^3}{\frac{1-0.5^{10}}{3^2}} - \frac{0.5^{15} \times 3^5}{\frac{1-0.5^{12}}{3^2}} \right) \right) = 6 \left[\frac{\arg(0.0741813 - z_0)}{2 \pi} \right] \log \left(\frac{1}{z_0} \right) + 6 \log(z_0) + 6 \left[\frac{\arg(0.0741813 - z_0)}{2 \pi} \right] \log(z_0) - 6 \sum_{k=1}^{\infty} \frac{(-1)^k (0.0741813 - z_0)^k z_0^{-k}}{k}$$

[Open code](#)

Integral representation:

$$6 \log \left(- \left(- \frac{0.5^{15} \times 3^3}{\frac{1-0.5^{10}}{3^2}} - \frac{0.5^{15} \times 3^5}{\frac{1-0.5^{12}}{3^2}} \right) \right) = 6 \int_1^{0.0741813} \frac{1}{t} dt$$

[Open code](#)

This result -15,6075 is very near to the value of black hole entropy 15,6730 with minus sign

$$\text{sqrt}(\frac{\text{colog}(-\frac{0.5^{15} \times 3^3}{(1-0.5^{10})/3^2} - \frac{0.5^{15} \times 3^5}{(1-0.5^{12})/3^2})}{(1-0.5^{10})/3^2} - \frac{0.5^{15} \times 3^5}{(1-0.5^{12})/3^2})$$

Input:

$$\sqrt{-\log\left(-\left(\frac{0.5^{15} \times 3^3}{1-0.5^{10}} - \frac{0.5^{15} \times 3^5}{1-0.5^{12}}\right)\right)}$$

[Open code](#)

Enlarge Data Customize A Plaintext Interactive

Result:

More digits

1.61284...

Series representations:

More

$$\sqrt{-\log\left(-\left(\frac{0.5^{15} \times 3^3}{1-0.5^{10}} - \frac{0.5^{15} \times 3^5}{1-0.5^{12}}\right)\right)} = \sqrt{\sum_{k=1}^{\infty} \frac{(-1)^k (-0.925819)^k}{k}}$$

[Open code](#)

Enlarge Data Customize A Plaintext Interactive

$$\sqrt{-\log\left(-\left(\frac{0.5^{15} \times 3^3}{1-0.5^{10}} - \frac{0.5^{15} \times 3^5}{1-0.5^{12}}\right)\right)} = \sqrt{-1 - \log(0.0741813)} \sum_{k=0}^{\infty} \binom{\frac{1}{2}}{k} (-1 - \log(0.0741813))^{-k}$$

[Open code](#)

$$\sqrt{-\log\left(-\left(\frac{0.5^{15} \times 3^3}{1-0.5^{10}} - \frac{0.5^{15} \times 3^5}{1-0.5^{12}}\right)\right)} = \sqrt{-1 - \log(0.0741813)} \sum_{k=0}^{\infty} \frac{(-1)^k (-1 - \log(0.0741813))^{-k} \left(-\frac{1}{2}\right)_k}{k!}$$

[Open code](#)

- $\log(x)$ is the natural logarithm

- $\binom{n}{m}$ is the binomial coefficient
 - $n!$ is the factorial function
- $(\alpha)_n$ is the Pochhammer symbol (rising factorial)

- [More information](#)

Integral representation:

$$\sqrt{-\log\left(-\left(\frac{0.5^{15} \times 3^3}{1-0.5^{10}} - \frac{0.5^{15} \times 3^5}{1-0.5^{12}}\right)\right)} = \sqrt{-\int_1^{0.0741813} \frac{1}{t} dt}$$

This value 1,61284 is very near to the golden ratio

$$\text{sqrt}(\text{colog}(-\left(\frac{0.5^{15} \times 3^3}{1-0.5^{10}} - \frac{0.5^{15} \times 3^5}{1-0.5^{12}}\right)))^{1.6548 \times 3 \times \text{Pi}}$$

where 1,6548 is very near to the 14th root of the following Ramanujan's class

$$\text{invariant } Q = (G_{505}/G_{101/5})^3 = 1164,2696 \text{ i.e. } 1,65578\dots$$

Input interpretation:

$$\sqrt{-\log\left(-\left(\frac{0.5^{15} \times 3^3}{1-0.5^{10}} - \frac{0.5^{15} \times 3^5}{1-0.5^{12}}\right)\right)}^{1.6548 \times 3 \times \pi}$$

[Open code](#)

- $\log(x)$ is the natural logarithm

Enlarge Data Customize A [Plaintext](#) Interactive

Result:

More digits

1728.25...

This result is very near to the mass of candidate glueball $f_0(1710)$ meson.

Furthermore, 1728 occurs in the algebraic formula for the j-invariant of an elliptic curve. As a consequence, it is sometimes called a Zagier as a pun on the Gross–Zagier theorem. The number 1728 is one less than the Hardy–Ramanujan number 1729

$$\text{sqrt}(\text{colog}(-\left(\frac{0.5^{15} \times 3^3}{1-0.5^{10}} - \frac{0.5^{15} \times 3^5}{1-0.5^{12}}\right)))^{1.6548 \times 3 \times \text{Pi}}$$

Input interpretation:

Series representations:

More

$$\frac{1}{3} \sqrt[3]{\sqrt{-\log\left(-\frac{0.5^{15} \times 3^3}{3^2} - \frac{0.5^{15} \times 3^5}{3^2}\right)}}^{1.6548 \times 3 \pi} = \frac{1}{3} \sqrt[3]{\sqrt{\sum_{k=1}^{\infty} \frac{(-1)^k (-0.925819)^k}{k}}^{4.9644 \pi}}$$

Open code

Enlarge Data Customize A Plaintext Interactive

$$\frac{1}{3} \sqrt[3]{\sqrt{-\log\left(-\frac{0.5^{15} \times 3^3}{3^2} - \frac{0.5^{15} \times 3^5}{3^2}\right)}}^{1.6548 \times 3 \pi} = \frac{1}{3} \sqrt[3]{\left(\sqrt{-1 - \log(0.0741813)} \sum_{k=0}^{\infty} \binom{\frac{1}{2}}{k} (-1 - \log(0.0741813))^{-k}\right)^{4.9644 \pi}}$$

Open code

$$\frac{1}{3} \sqrt[3]{\sqrt{-\log\left(-\frac{0.5^{15} \times 3^3}{3^2} - \frac{0.5^{15} \times 3^5}{3^2}\right)}}^{1.6548 \times 3 \pi} = \frac{1}{3} \sqrt[3]{\left(\sqrt{-1 - \log(0.0741813)} \sum_{k=0}^{\infty} \frac{(-1)^k (-1 - \log(0.0741813))^{-k} \left(-\frac{1}{2}\right)_k}{k!}\right)^{4.9644 \pi}}$$

Open code

Integral representation:

$$\frac{1}{3} \sqrt[3]{\sqrt{-\log\left(-\frac{0.5^{15} \times 3^3}{3^2} - \frac{0.5^{15} \times 3^5}{3^2}\right)}}^{1.6548 \times 3 \pi} = \frac{1}{3} \sqrt[3]{\sqrt{-\int_1^{0.0741813} \frac{1}{t} dt}}^{4.9644 \pi}$$

This result 4,00019 is the minimal possible value of the mass of hypothetical DM particles

$$\log \left(\frac{1}{\frac{0.5^{15} \times 0.625^3}{1-0.5^{10}} - \frac{0.5^{15} \times 0.625^5}{1-0.5^{12}}} \right) = \log(246890.) - \sum_{k=1}^{\infty} \frac{(-1)^k e^{-12.4167k}}{k}$$

[Open code](#)

Enlarge Data Customize A Plaintext Interactive

$$\log \left(\frac{1}{\frac{0.5^{15} \times 0.625^3}{1-0.5^{10}} - \frac{0.5^{15} \times 0.625^5}{1-0.5^{12}}} \right) = 2i\pi \left[\frac{\arg(246891. - x)}{2\pi} \right] + \log(x) - \sum_{k=1}^{\infty} \frac{(-1)^k (246891. - x)^k x^{-k}}{k} \text{ for } x < 0$$

[Open code](#)

$$\log \left(\frac{1}{\frac{0.5^{15} \times 0.625^3}{1-0.5^{10}} - \frac{0.5^{15} \times 0.625^5}{1-0.5^{12}}} \right) = \left[\frac{\arg(246891. - z_0)}{2\pi} \right] \log \left(\frac{1}{z_0} \right) + \log(z_0) + \left[\frac{\arg(246891. - z_0)}{2\pi} \right] \log(z_0) - \sum_{k=1}^{\infty} \frac{(-1)^k (246891. - z_0)^k z_0^{-k}}{k}$$

[Open code](#)

Integral representations:

$$\log \left(\frac{1}{\frac{0.5^{15} \times 0.625^3}{1-0.5^{10}} - \frac{0.5^{15} \times 0.625^5}{1-0.5^{12}}} \right) = \int_1^{246891.} \frac{1}{t} dt$$

[Open code](#)

Enlarge Data Customize A Plaintext Interactive

$$\log \left(\frac{1}{\frac{0.5^{15} \times 0.625^3}{1-0.5^{10}} - \frac{0.5^{15} \times 0.625^5}{1-0.5^{12}}} \right) = \frac{1}{2i\pi} \int_{-i\infty+\gamma}^{i\infty+\gamma} \frac{e^{-12.4167s} \Gamma(-s)^2 \Gamma(1+s)}{\Gamma(1-s)} ds$$

for $-1 < \gamma < 0$

This result 12,4167 is very near to the value of black hole entropy 12,57

For $x = 3$, we obtain:

$$4\pi * \text{sqrt}(\text{((((((colog -(((([-(((0.5)^15 * 3^3)) / (((1- 0.5^10))/3^2))) - [(((0.5)^15 * 3^5)) / (((1- 0.5^12))/3^2))])])])])])])])])$$

Input:

$$4\pi \sqrt{-\log\left(-\left(-\frac{0.5^{15} \times 3^3}{\frac{1-0.5^{10}}{3^2}} - \frac{0.5^{15} \times 3^5}{\frac{1-0.5^{12}}{3^2}}\right)\right)}$$

[Open code](#)

- $\log(x)$ is the natural logarithm
- [Units »](#)

Enlarge Data Customize A [Plaintext](#) [Interactive](#)

Result:

More digits

20.2675...

Series representations:

More

$$4\pi \sqrt{-\log\left(-\left(-\frac{0.5^{15} \times 3^3}{\frac{1-0.5^{10}}{3^2}} - \frac{0.5^{15} \times 3^5}{\frac{1-0.5^{12}}{3^2}}\right)\right)} = 4\pi \sqrt{\sum_{k=1}^{\infty} \frac{(-1)^k (-0.925819)^k}{k}}$$

[Open code](#)

Enlarge Data Customize A [Plaintext](#) [Interactive](#)

$$4\pi \sqrt{-\log\left(-\left(-\frac{0.5^{15} \times 3^3}{\frac{1-0.5^{10}}{3^2}} - \frac{0.5^{15} \times 3^5}{\frac{1-0.5^{12}}{3^2}}\right)\right)} = 4\pi \sqrt{-1 - \log(0.0741813)} \sum_{k=0}^{\infty} \binom{\frac{1}{2}}{k} (-1 - \log(0.0741813))^{-k}$$

[Open code](#)

$$4\pi \sqrt{-\log\left(-\left(-\frac{0.5^{15} \times 3^3}{\frac{1-0.5^{10}}{3^2}} - \frac{0.5^{15} \times 3^5}{\frac{1-0.5^{12}}{3^2}}\right)\right)} = 4\pi \sqrt{-1 - \log(0.0741813)} \sum_{k=0}^{\infty} \frac{(-1)^k (-1 - \log(0.0741813))^{-k} \left(-\frac{1}{2}\right)_k}{k!}$$

Open code

- $\binom{n}{m}$ is the binomial coefficient
- $n!$ is the factorial function
- $(a)_n$ is the Pochhammer symbol (rising factorial)
 - Units »
- [More information](#)

Integral representation:

$$4\pi \sqrt{-\log\left(-\left(\frac{0.5^{15} \times 3^3}{1-0.5^{10}} - \frac{0.5^{15} \times 3^5}{1-0.5^{12}}\right)\right)} = 4\pi \sqrt{-\int_1^{0.0741813} \frac{1}{t} dt}$$

This result 20,2675 is very near to the value of black hole entropy 20,5520

$$11 * \text{sqrt}(\text{colog}(-(((1-0.5^{10})/3^2) - [((0.5^{15} * 3^3)) / ((1-0.5^{10})/3^2)])) - [(((0.5^{15} * 3^5)) / ((1-0.5^{12})/3^2)]))$$

Input:

$$11 \sqrt{-\log\left(-\left(\frac{0.5^{15} \times 3^3}{1-0.5^{10}} - \frac{0.5^{15} \times 3^5}{1-0.5^{12}}\right)\right)}$$

Open code

- $\log(x)$ is the natural logarithm

Enlarge Data Customize A [Plaintext](#) Interactive

Result:

More digits

17.7412...

Series representations:

More

$$11 \sqrt{-\log\left(-\left(\frac{0.5^{15} \times 3^3}{1-0.5^{10}} - \frac{0.5^{15} \times 3^5}{1-0.5^{12}}\right)\right)} = 11 \sqrt{\sum_{k=1}^{\infty} \frac{(-1)^k (-0.925819)^k}{k}}$$

Open code

Enlarge Data Customize A [Plaintext](#) Interactive

$$11 \sqrt{-\log\left(-\left(\frac{0.5^{15} \times 3^3}{1-0.5^{10}} - \frac{0.5^{15} \times 3^5}{1-0.5^{12}}\right)\right)} = 11 \sqrt{-1 - \log(0.0741813)} \sum_{k=0}^{\infty} \binom{\frac{1}{2}}{k} (-1 - \log(0.0741813))^{-k}$$

Open code

$$11 \sqrt{-\log\left(-\left(\frac{0.5^{15} \times 3^3}{1-0.5^{10}} - \frac{0.5^{15} \times 3^5}{1-0.5^{12}}\right)\right)} = 11 \sqrt{-1 - \log(0.0741813)} \sum_{k=0}^{\infty} \frac{(-1)^k (-1 - \log(0.0741813))^{-k} \left(-\frac{1}{2}\right)_k}{k!}$$

Open code

- $\binom{n}{m}$ is the binomial coefficient
- $n!$ is the factorial function
- $(a)_n$ is the Pochhammer symbol (rising factorial)
- [More information](#)

Integral representation:

$$11 \sqrt{-\log\left(-\left(\frac{0.5^{15} \times 3^3}{1-0.5^{10}} - \frac{0.5^{15} \times 3^5}{1-0.5^{12}}\right)\right)} = 11 \sqrt{-\int_1^{0.0741813} \frac{1}{t} dt}$$

This result 17,7412 is practically equal to the value of black hole entropy 17,7715

$$19 * \text{sqrt}(\text{colog}(-(((1-0.5^{10})/3^2) - (((0.5^{15} * 3^3) / ((1-0.5^{12})/3^2)))))) - (((0.5^{15} * 3^5)) / ((1-0.5^{12})/3^2))))))$$

Input:

$$19 \sqrt{-\log\left(-\left(\frac{0.5^{15} \times 3^3}{1-0.5^{10}} - \frac{0.5^{15} \times 3^5}{1-0.5^{12}}\right)\right)}$$

Open code

- $\log(x)$ is the natural logarithm

Enlarge Data Customize A Plaintext Interactive

Result:

More digits

30.6439...

Series representations:

More

$$19 \sqrt{-\log\left(-\left(\frac{0.5^{15} \times 3^3}{1-0.5^{10}} - \frac{0.5^{15} \times 3^5}{1-0.5^{12}}\right)\right)} = 19 \sqrt{\sum_{k=1}^{\infty} \frac{(-1)^k (-0.925819)^k}{k}}$$

Open code

$$19 \sqrt{-\log\left(-\left(\frac{0.5^{15} \times 3^3}{1-0.5^{10}} - \frac{0.5^{15} \times 3^5}{1-0.5^{12}}\right)\right)} = 19 \sqrt{-1 - \log(0.0741813)} \sum_{k=0}^{\infty} \binom{\frac{1}{2}}{k} (-1 - \log(0.0741813))^{-k}$$

Open code

$$19 \sqrt{-\log\left(-\left(\frac{0.5^{15} \times 3^3}{1-0.5^{10}} - \frac{0.5^{15} \times 3^5}{1-0.5^{12}}\right)\right)} = 19 \sqrt{-1 - \log(0.0741813)} \sum_{k=0}^{\infty} \frac{(-1)^k (-1 - \log(0.0741813))^{-k} \left(-\frac{1}{2}\right)_k}{k!}$$

Open code

- $\binom{n}{m}$ is the binomial coefficient
- $n!$ is the factorial function
- $(a)_n$ is the Pochhammer symbol (rising factorial)
- [More information](#)

Integral representation:

$$19 \sqrt{-\log\left(-\left(\frac{0.5^{15} \times 3^3}{1-0.5^{10}} - \frac{0.5^{15} \times 3^5}{1-0.5^{12}}\right)\right)} = 19 \sqrt{-\int_1^{0.0741813} \frac{1}{t} dt}$$

This result 30,6439 is very near to the value of black hole entropy 30,5963

Now, from the reciprocal of the formulas, for $x = 3$, we have obtained the following results:

$$1 / ((([-(((0.5)^{15} * 3^3)) / ((1 - 0.5^{10})/3^2))]) - [(((0.5)^{15} * 3^5)) / ((1 - 0.5^{12})/3^2)]))$$

Input:

$$\frac{1}{-\frac{0.5^{15} \times 3^3}{1 - 0.5^{10}} - \frac{0.5^{15} \times 3^5}{1 - 0.5^{12}}}$$

[Open code](#)

Enlarge Data Customize A [Plaintext](#) [Interactive](#)

Result:

More digits

-13.4804931755301962764758747457963057992381355771136578995...

[Open code](#)

$$((((3 + (2 * 64) / -(((0.5)^{15} * 3^3)) / ((1 - 0.5^{10})/3^2))]) - [(((0.5)^{15} * 3^5)) / ((1 - 0.5^{12})/3^2)]))$$

Input:

$$3 + \frac{2 \times 64}{-\frac{0.5^{15} \times 3^3}{1 - 0.5^{10}} - \frac{0.5^{15} \times 3^5}{1 - 0.5^{12}}}$$

1728.503126467865123388911967461927142302481353870548211139...

[Open code](#)

Continued fraction:

Linear form

$$1728 + \frac{1}{1 + \frac{1}{1 + \frac{1}{79 + \frac{1}{2 + \frac{1}{6 + \frac{1}{6 + \frac{1}{2 + \frac{1}{2 + \frac{1}{5 + \frac{1}{\dots}}}}}}}}}}}}$$

Possible closed forms:

More

$$\frac{10753063 + 4834\pi^2}{1989\pi} \approx 1728.503126467865123364831$$

Enlarge Data Customize A [Plaintext](#) [Interactive](#)

$$\frac{1}{7} (73 e^\pi + 3114\pi - 6457 \log(\pi) + 4337 \log(2\pi) + 38 \tan^{-1}(\pi)) \approx$$

1728.50312646786512338860696

This value is linked to the "Ramanujan function" (an elliptic modular function that satisfies the need for "conformal symmetry") that has 24 "modes" corresponding to the physical vibrations of a bosonic string.

$$\frac{1}{3} * ((((((3+(2*64)/ -(((([-(((0.5)^{15} * 3^3)) / (((1- 0.5^{10}))/3^2)))) - [(((0.5)^{15} * 3^5)) / (((1- 0.5^{12}))/3^2))])))))))))))^{1/3}$$

Input:

$$\frac{1}{3} \sqrt[3]{3 + \frac{2 \times 64}{-\frac{0.5^{15} \times 3^3}{1-0.5^{10}} - \frac{0.5^{15} \times 3^5}{1-0.5^{12}}}}$$

[Open code](#)

Enlarge Data Customize A [Plaintext](#) [Interactive](#)

Result:

More digits

4.00039...

This result 4,00039 is the minimal possible value of the mass of hypothetical DM particles

Now, we have:

Lemma 10.2.1. *Recall that $f(a, b)$ denotes Ramanujan's ubiquitous general theta function defined in (9.2.1), and that $\varphi(q) = f(q, q)$ and $\psi(q) = f(q, q^3)$. Then*

$$\begin{aligned} & -b_1(q)qf(-q^2, q^3) + b_2(q)q^2f(q, -q^4) \\ & - \frac{2q(q^{10}; q^{10})_{\infty}^3 \psi(-q^5)\varphi(q^5)}{\psi(q^5)\varphi(-q^5)f(1, q^{10})} \end{aligned} \quad (10.2.1)$$

and

$$\begin{aligned} & 2b_1(q)f(-q^4, -q^{16}) - 2b_2(q)f(-q^8, -q^{12}) \\ & = -\frac{2q(q^{10}; q^{10})_{\infty}^3 \varphi(-q^{10})f(1, q^{10})}{\psi(q^5)\varphi(-q^5)\varphi(q^5)}, \end{aligned} \quad (10.2.2)$$

where

$$b_1(q) := -\frac{(q^5; q^5)_{\infty}(q^{10}; q^{10})_{\infty}f(-q^2, -q^3)}{f(-q^2, -q^8)f(-q, -q^4)} \quad (10.2.3)$$

and

$$b_2(q) := -\frac{(q^5; q^5)_{\infty}(q^{10}; q^{10})_{\infty}f(-q, -q^4)}{f(-q^4, -q^6)f(-q^2, -q^3)}. \quad (10.2.4)$$

And in conclusion:

$$\begin{aligned} & \frac{G(q)H(q^4)}{H(q^2)H(q)} - \frac{H(q)G(q^4)}{G(q^2)G(q)} \\ & - \frac{G(q)}{H(q)} \left(\frac{H(q)}{G(-q)} \frac{(q; q^2)_{\infty}}{(q^5; q^{10})_{\infty}} \right) - \frac{H(q)}{G(q)} \left(\frac{G(q)}{H(-q)} \frac{(q; q^2)_{\infty}}{(q^5; q^{10})_{\infty}} \right) \\ & = \frac{(q; q^2)_{\infty}}{(q^5; q^{10})_{\infty}} \frac{(G(q)H(-q) - H(q)G(-q))}{G(-q)H(-q)} \\ & - \frac{(q; q^2)_{\infty}}{(q^5; q^{10})_{\infty}} \frac{2q\psi(q^{10})}{(q^2; q^2)_{\infty}} \frac{(q; q)_{\infty}}{(-q^5; -q^5)_{\infty}}, \end{aligned} \quad (10.2.9)$$

We have calculated:

$$\frac{(q; q^2)_{\infty}}{(q^5; q^{10})_{\infty}} \frac{2q\psi(q^{10})}{(q^2; q^2)_{\infty}} \frac{(-q; -q)_{\infty}}{(-q^5; -q^5)_{\infty}},$$

From:

$$f_3(q) := \sum_{n=0}^{\infty} \frac{q^{n^2}}{(-q; q)_n^2}, \quad (2.1.1)$$

$$\phi_3(q) := \sum_{n=0}^{\infty} \frac{q^{n^2}}{(-q^2; q^2)_n}, \quad (2.1.2)$$

$$\psi_3(q) := \sum_{n=1}^{\infty} \frac{q^{n^2}}{(q; q^2)_n}, \quad (2.1.3)$$

and

$$\chi_3(q) := \sum_{n=0}^{\infty} \frac{q^{n^2}}{\prod_{j=1}^n (1 - q^j + q^{2j})}. \quad (2.1.4)$$

For $q = 0.5$ we have interpreted and calculated 10.2.9 as follows:

$$[((0.5)^1) / (((1 - (0.5)^1)))] + [(((0.5)^4)) / (((1 - (0.5)^1)) ((1 - (0.5)^3)))]$$

Input:

$$\frac{0.5^1}{1 - 0.5^1} + \frac{0.5^4}{(1 - 0.5^1)(1 - 0.5^3)}$$

[Open code](#)

Enlarge Data Customize A [Plaintext](#) [Interactive](#)

Result:

More digits

1.142857142857142857142857142857142857142857142857142857142...

$$1 / ((((((0.5)^5) / (((1 - (0.5)^5)))) + [(((0.5)^10)) / (((1 - (0.5)^5)) ((1 - (0.5)^7)))]))))))$$

Input:

$$\frac{1}{\frac{0.5^5}{1 - 0.5^5} + \frac{0.5^{10}}{(1 - 0.5^5)(1 - 0.5^7)}}$$

[Open code](#)

Enlarge Data Customize A [Plaintext](#) [Interactive](#)

Result:

More digits

30.05343511450381679389312977099236641221374045801526717557...

$$1 / ((((((1 + [(0.5)^1]) / (((1 + (0.5)^2)))) + [(((0.5)^4)) / (((1 + (0.5)^2)) ((1 + (0.5)^4)))]))))))$$

32.94238260369664048307564143951681560106322721438711917732

$$\left(24 \times \frac{0.461538^{(24+1) \times (24+2)/2}}{0.461538^{(24+1) \times (24+2)/2+1}} \right)$$

[Open code](#)

Result:

More digits

1713.005608397833702953636308491182902438190253338383535604...

[Open code](#)

With the value of $q = 0.5$, we obtain:

$$32.94238260369664048307564143951681560106322721438711917732 * [26 * (((0.5^{((26+1)(26+2)/2)})) / (((0.5^{((26+1)(26+2)/2)+1})))))]$$

Input interpretation:

32.94238260369664048307564143951681560106322721438711917732

$$\left(26 \times \frac{0.5^{(26+1) \times (26+2)/2}}{0.5^{(26+1) \times (26+2)/2+1}} \right)$$

[Open code](#)

Enlarge Data Customize A [Plaintext](#) [Interactive](#)

Result:

More digits

1713.003895392225305119933354854874411255287815148130197220...

The results 1713.005 and 1713.003 are very near to the mass of candidate glueball $f_0(1710)$ meson.

$$(((32.94238260369664048307564143951681560106322721438711917732 * 26 * (((0.5^{((26+1)(26+2)/2)})) / (((0.5^{((26+1)(26+2)/2)+1})))))]))^{1/3}$$

Input interpretation:

$$\left(32.94238260369664048307564143951681560106322721438711917732 \times 26 \times \frac{0.5^{(26+1) \times (26+2)/2}}{0.5^{(26+1) \times (26+2)/2+1}} \right)^{(1/3)}$$

[Open code](#)

Enlarge Data Customize A [Plaintext](#) [Interactive](#)

Result:

More digits

11.9652...

This result 11.9652 is very near to the value of black hole entropy 12.19

$$2 * (((((32.94238260369664 * 26 * (((0.5^{((26+1)(26+2)/2)})) / (((((0.5^{((26+1)(26+2)/2}) + 1))))))))))^{1/3}$$

Input interpretation:

$$2 \sqrt[3]{32.94238260369664 \times 26 \times \frac{0.5^{(26+1) \times (26+2)/2}}{0.5^{(26+1) \times (26+2)/2+1}}}$$

[Open code](#)

Result:

[More digits](#)

23.9304...

This value is very near to the value of black hole entropy 23,9078 and is linked to the "Ramanujan function" (an elliptic modular function that satisfies the need for "conformal symmetry") that has 24 "modes" corresponding to the physical vibrations of a bosonic string.

$$(((32.94238260369664 * 26 * (((0.5^{((26+1)(26+2)/2)})) / (((((0.5^{((26+1)(26+2)/2}) + 1))))))))^{1/15}$$

Input interpretation:

$$15 \sqrt{32.94238260369664 \times 26 \times \frac{0.5^{(26+1) \times (26+2)/2}}{0.5^{(26+1) \times (26+2)/2+1}}}$$

[Open code](#)

Enlarge Data Customize A [Plaintext](#) [Interactive](#)

Result:

[Fewer digits](#)

[More digits](#)

1.642796958176285274704146343873475170578066208848244657427...

1.6427969581762852747041463438734751705780662088482446557427...

Or:

Input interpretation:

$$15 \sqrt{32.9423826 \times 26 \times \frac{0.5^{(26+1) \times (26+2)/2}}{0.5^{(26+1) \times (26+2)/2+1}}}$$

[Open code](#)

Enlarge Data Customize A [Plaintext](#) [Interactive](#)

Result:

[Fewer digits](#)

[More digits](#)

1.642796958163995475631124152505272382573180674921959447666...

1.6427969581639954756311241525052723825731806749219594

$$1.64279 \approx \zeta(2) = \frac{\pi^2}{6} = 1.6449$$

Continued fraction:

Linear form

$$\begin{array}{c}
 1 \\
 1 + \frac{1}{\text{---}} \\
 1 + \frac{1}{1 + \frac{1}{\text{---}}} \\
 1 + \frac{1}{1 + \frac{1}{1 + \frac{1}{\text{---}}}} \\
 1 + \frac{1}{3 + \frac{1}{\text{---}}} \\
 1 + \frac{1}{83 + \frac{1}{\text{---}}} \\
 1 + \frac{1}{78 + \frac{1}{\text{---}}} \\
 21 + \frac{1}{\text{---}} \\
 2 + \frac{1}{\text{---}} \\
 2 + \frac{1}{\text{---}} \\
 18 + \frac{1}{\text{---}} \\
 1 + \frac{1}{\text{---}} \\
 26 + \frac{1}{\text{---}} \\
 3 + \frac{1}{\text{---}} \\
 1 + \frac{1}{\text{---}} \\
 17 + \frac{1}{\text{---}} \\
 3 + \frac{1}{\text{---}} \\
 2 + \frac{1}{\text{---}} \\
 \dots
 \end{array}$$

[Open code](#)

Possible closed forms:

More

$$\frac{30\,094\,383}{5\,831\,116\pi} \approx 1.64279695816399551322$$

Enlarge Data Customize A Plaintext Interactive

$$\frac{154 e^{\pi} - 59\pi - 25 \log(\pi) + 97 \log(2\pi) - 1428 \tan^{-1}(\pi)}{1050} \approx$$

$$\frac{1.642796958163995475625037}{2\,244\,149\,014\pi} \approx 1.642796958163995475613276$$

We have that:

Lemma 11.3.7. For $q \neq 0$,

$$xk(x, q) = m(-qx^4, q^4, -q^{-1}x^{-2}) + q^{-1}x^2m(-q^{-1}x^4, q^4, -q^{-1}x^{-2}). \quad (11.3.19)$$

Proof. In Theorem 10.6.1 of Chapter 10, set $z = q/x^2$. Now, $B(q/x^2, x, q) = 0$ by (10.6.1). Hence,

$$\begin{aligned} j(-qx^2; q^4)(k(x, q) - 1/x) &= -P(q/x^2) \\ &= -\frac{1}{2} \sum_{n=-\infty}^{\infty} \frac{q^{2n^2+3n+1}x^{2n-1}}{1 - q^{4n+2}/x^2} - \frac{1}{2} \sum_{n=-\infty}^{\infty} \frac{q^{2n^2+3n+1}x^{2n+1}}{1 - q^{4n+4}/x^2} \\ &\quad + \frac{1}{2} \sum_{n=-\infty}^{\infty} \frac{q^{2n^2+5n}x^{-2n+1}}{1 - q^{4n}x^2} + \frac{1}{2} \sum_{n=-\infty}^{\infty} \frac{q^{2n^2+5n+2}x^{-2n-1}}{1 - q^{4n+2}x^2} \\ &= -\sum_{n=-\infty}^{\infty} \frac{q^{2n^2+3n+1}x^{2n-1}}{1 - q^{4n+2}/x^2} - \sum_{n=-\infty}^{\infty} \frac{q^{2n^2+3n+1}x^{2n+1}}{1 - q^{4n+4}/x^2}, \end{aligned}$$

For $q = 0,5$ $x = 0,8$ $n = 2$, we obtain:

$$-(0.5^{15} * 0.8^3)/(((1-0.5^{10})/0.8^2))) - (0.5^{15} * 0.8^5)/(((1-0.5^{12})/0.8^2)))$$

Input:

$$-\frac{0.5^{15} \times 0.8^3}{\frac{1-0.5^{10}}{0.8^2}} - \frac{0.5^{15} \times 0.8^5}{\frac{1-0.5^{12}}{0.8^2}}$$

[Open code](#)

Enlarge Data Customize A [Plaintext](#) Interactive

Result:

More digits

-0.00001641133805262837520902037031069289133805262837520902...

[Open code](#)

$$10^5 * -[-(0.5^{15} * 0.8^3)/(((1-0.5^{10})/0.8^2)))-(0.5^{15} * 0.8^5)/(((1-0.5^{12})/0.8^2)))]$$

Input:

$$10^5 \times (-1) \left(-\frac{0.5^{15} \times 0.8^3}{\frac{1-0.5^{10}}{0.8^2}} - \frac{0.5^{15} \times 0.8^5}{\frac{1-0.5^{12}}{0.8^2}} \right)$$

[Open code](#)

Enlarge Data Customize A [Plaintext](#) Interactive

Result:

More digits

1.641133805262837520902037031069289133805262837520902037031...

1.641133805262837520902037031069289133805262837520902037031

$$1.64113 \approx \zeta(2) = \frac{\pi^2}{6} = 1.6449$$

colog $-\left[-\frac{0.5^{15} \times 0.8^3}{\frac{1-0.5^{10}}{0.8^2}} - \frac{0.5^{15} \times 0.8^5}{\frac{1-0.5^{12}}{0.8^2}}\right]$

Input:

$$-\log\left(-\left(\frac{0.5^{15} \times 0.8^3}{\frac{1-0.5^{10}}{0.8^2}} - \frac{0.5^{15} \times 0.8^5}{\frac{1-0.5^{12}}{0.8^2}}\right)\right)$$

[Open code](#)

- $\log(x)$ is the natural logarithm

Enlarge Data Customize A [Plaintext](#) [Interactive](#)

Result:

More digits

11.0175...

Series representations:

More

$$-\log\left(-\left(\frac{0.5^{15} \times 0.8^3}{\frac{1-0.5^{10}}{0.8^2}} - \frac{0.5^{15} \times 0.8^5}{\frac{1-0.5^{12}}{0.8^2}}\right)\right) = \sum_{k=1}^{\infty} \frac{(-1)^k (-0.999984)^k}{k}$$

[Open code](#)

Enlarge Data Customize A [Plaintext](#) [Interactive](#)

$$-\log\left(-\left(\frac{0.5^{15} \times 0.8^3}{\frac{1-0.5^{10}}{0.8^2}} - \frac{0.5^{15} \times 0.8^5}{\frac{1-0.5^{12}}{0.8^2}}\right)\right) = -2i\pi \left[\frac{\arg(0.0000164113 - x)}{2\pi} \right] - \log(x) + \sum_{k=1}^{\infty} \frac{(-1)^k (0.0000164113 - x)^k x^{-k}}{k} \text{ for } x < 0$$

[Open code](#)

$$-\log\left(-\left(\frac{0.5^{15} \times 0.8^3}{\frac{1-0.5^{10}}{0.8^2}} - \frac{0.5^{15} \times 0.8^5}{\frac{1-0.5^{12}}{0.8^2}}\right)\right) = -\left[\frac{\arg(0.0000164113 - z_0)}{2\pi} \right] \log\left(\frac{1}{z_0}\right) - \log(z_0) + \left[\frac{\arg(0.0000164113 - z_0)}{2\pi} \right] \log(z_0) + \sum_{k=1}^{\infty} \frac{(-1)^k (0.0000164113 - z_0)^k z_0^{-k}}{k}$$

[Open code](#)

Integral representation:

$$-\log\left(-\left(-\frac{0.5^{15} \times 0.8^3}{\frac{1-0.5^{10}}{0.8^2}} - \frac{0.5^{15} \times 0.8^5}{\frac{1-0.5^{12}}{0.8^2}}\right)\right) = -\int_1^{0.0000164113} \frac{1}{t} dt$$

$$\left(\frac{\sqrt{5}+1}{2}\right) \cdot \log\left[-\left(-\frac{0.5^{15} \times 0.8^3}{\frac{1-0.5^{10}}{0.8^2}}\right) - \left(-\frac{0.5^{15} \times 0.8^5}{\frac{1-0.5^{12}}{0.8^2}}\right)\right]$$

Input:

$$\left(\frac{1}{2}(\sqrt{5} + 1)\right) \left(-\log\left(-\left(-\frac{0.5^{15} \times 0.8^3}{\frac{1-0.5^{10}}{0.8^2}} - \frac{0.5^{15} \times 0.8^5}{\frac{1-0.5^{12}}{0.8^2}}\right)\right)\right)$$

[Open code](#)

- $\log(x)$ is the natural logarithm

Enlarge Data Customize A [Plaintext](#) [Interactive](#)

Result:

More digits

17.8268...

Series representations:

More

$$\frac{1}{2} \left(-\log\left(-\left(-\frac{0.5^{15} \times 0.8^3}{\frac{1-0.5^{10}}{0.8^2}} - \frac{0.5^{15} \times 0.8^5}{\frac{1-0.5^{12}}{0.8^2}}\right)\right)\right) (\sqrt{5} + 1) =$$

$$\frac{1}{2} \left(\sum_{k=1}^{\infty} \frac{(-1)^k (-0.9999984)^k}{k}\right) \left(1 + \sqrt{4} \sum_{k=0}^{\infty} 4^{-k} \binom{\frac{1}{2}}{k}\right)$$

[Open code](#)

Enlarge Data Customize A [Plaintext](#) [Interactive](#)

$$\frac{1}{2} \left(-\log\left(-\left(-\frac{0.5^{15} \times 0.8^3}{\frac{1-0.5^{10}}{0.8^2}} - \frac{0.5^{15} \times 0.8^5}{\frac{1-0.5^{12}}{0.8^2}}\right)\right)\right) (\sqrt{5} + 1) =$$

$$\frac{1}{2} \left(\sum_{k=1}^{\infty} \frac{(-1)^k (-0.9999984)^k}{k}\right) \left(1 + \sqrt{4} \sum_{k=0}^{\infty} \frac{\left(-\frac{1}{4}\right)^k \left(-\frac{1}{2}\right)_k}{k!}\right)$$

[Open code](#)

$$\frac{1}{2} \left(-\log \left(-\left(-\frac{0.5^{15} \times 0.8^3}{\frac{1-0.5^{10}}{0.8^2}} - \frac{0.5^{15} \times 0.8^5}{\frac{1-0.5^{12}}{0.8^2}} \right) \right) \right) (\sqrt{5} + 1) =$$

$$-\frac{\log(0.0000164113)}{2} - \frac{1}{2} \exp\left(i\pi \left\lfloor \frac{\arg(5-x)}{2\pi} \right\rfloor\right) \log(0.0000164113)$$

$$\sqrt{x} \sum_{k=0}^{\infty} \frac{(-1)^k (5-x)^k x^{-k} \left(-\frac{1}{2}\right)_k}{k!} \text{ for } (x \in \mathbb{R} \text{ and } x < 0)$$

Integral representation:

$$\frac{1}{2} \left(-\log \left(-\left(-\frac{0.5^{15} \times 0.8^3}{\frac{1-0.5^{10}}{0.8^2}} - \frac{0.5^{15} \times 0.8^5}{\frac{1-0.5^{12}}{0.8^2}} \right) \right) \right) (\sqrt{5} + 1) = -\frac{1}{2} (1 + \sqrt{5}) \int_1^{0.0000164113} \frac{1}{t} dt$$

Open code

This result 17,8268 is very near to the value of black hole entropy 17,7715

$$10^2 * 1.5849 * \text{colog} \left[-\left(\frac{0.5^{15} * 0.8^3}{\left(\frac{1-0.5^{10}}{0.8^2} \right)} \right) - \left(\frac{0.5^{15} * 0.8^5}{\left(\frac{1-0.5^{12}}{0.8^2} \right)} \right) \right]$$

Input interpretation:

$$10^2 \times 1.5849 \left(-\log \left(-\left(-\frac{0.5^{15} \times 0.8^3}{\frac{1-0.5^{10}}{0.8^2}} - \frac{0.5^{15} \times 0.8^5}{\frac{1-0.5^{12}}{0.8^2}} \right) \right) \right)$$

Open code

- $\log(x)$ is the natural logarithm

Enlarge Data Customize A [Plaintext](#) [Interactive](#)

Result:

More digits

1746.17...

Series representations:

More

$$(10^2 \times 1.5849) (-1) \log \left(-\left(-\frac{0.5^{15} \times 0.8^3}{\frac{1-0.5^{10}}{0.8^2}} - \frac{0.5^{15} \times 0.8^5}{\frac{1-0.5^{12}}{0.8^2}} \right) \right) =$$

$$158.49 \sum_{k=1}^{\infty} \frac{(-1)^k (-0.999984)^k}{k}$$

Open code

Enlarge Data Customize A [Plaintext](#) [Interactive](#)

$$(10^2 \times 1.5849)(-1) \log \left(- \left(- \frac{0.5^{15} \times 0.8^3}{\frac{1-0.5^{10}}{0.8^2}} - \frac{0.5^{15} \times 0.8^5}{\frac{1-0.5^{12}}{0.8^2}} \right) \right) =$$

$$-316.98 i \pi \left[\frac{\arg(0.0000164113 - x)}{2 \pi} \right] - 158.49 \log(x) +$$

$$158.49 \sum_{k=1}^{\infty} \frac{(-1)^k (0.0000164113 - x)^k x^{-k}}{k} \quad \text{for } x < 0$$

Open code

$$(10^2 \times 1.5849)(-1) \log \left(- \left(- \frac{0.5^{15} \times 0.8^3}{\frac{1-0.5^{10}}{0.8^2}} - \frac{0.5^{15} \times 0.8^5}{\frac{1-0.5^{12}}{0.8^2}} \right) \right) =$$

$$-158.49 \left[\frac{\arg(0.0000164113 - z_0)}{2 \pi} \right] \log \left(\frac{1}{z_0} \right) -$$

$$158.49 \log(z_0) - 158.49 \left[\frac{\arg(0.0000164113 - z_0)}{2 \pi} \right] \log(z_0) +$$

$$158.49 \sum_{k=1}^{\infty} \frac{(-1)^k (0.0000164113 - z_0)^k z_0^{-k}}{k}$$

Integral representation:

$$(10^2 \times 1.5849)(-1) \log \left(- \left(- \frac{0.5^{15} \times 0.8^3}{\frac{1-0.5^{10}}{0.8^2}} - \frac{0.5^{15} \times 0.8^5}{\frac{1-0.5^{12}}{0.8^2}} \right) \right) = -158.49 \int_1^{0.0000164113} \frac{1}{t} dt$$

This result 1746,17 is in the range of the mass of pseudo-scalar meson Eta (1760) 1751±15; 1744±10±15 J/ψ → γωω. Indeed: 1747,5 is the mean of values. Furthermore this result is also very near to the mass of candidate glueball f₀(1710) meson.

$$10^2 * (\pi/2) * \operatorname{colog} \left[- \left(- \frac{0.5^{15} * 0.8^3}{((1-0.5^{10})/0.8^2)} \right) - \left(\frac{0.5^{15} * 0.8^5}{((1-0.5^{12})/0.8^2)} \right) \right]$$

Input:

$$10^2 \times \frac{\pi}{2} \left(- \log \left(- \left(- \frac{0.5^{15} \times 0.8^3}{\frac{1-0.5^{10}}{0.8^2}} - \frac{0.5^{15} \times 0.8^5}{\frac{1-0.5^{12}}{0.8^2}} \right) \right) \right)$$

Open code

- log(x) is the natural logarithm

Enlarge Data Customize A Plaintext Interactive

Result:

More digits

1730.63...

Series representations:

More

$$\frac{1}{2} \left(10^2 \left(-\log \left(-\left(-\frac{0.5^{15} \times 0.8^3}{\frac{1-0.5^{10}}{0.8^2}} - \frac{0.5^{15} \times 0.8^5}{\frac{1-0.5^{12}}{0.8^2}} \right) \right) \right) \right) \pi = 50 \pi \sum_{k=1}^{\infty} \frac{(-1)^k (-0.999984)^k}{k}$$

Open code

Enlarge Data Customize A Plaintext Interactive

$$\begin{aligned} & \frac{1}{2} \left(10^2 \left(-\log \left(-\left(-\frac{0.5^{15} \times 0.8^3}{\frac{1-0.5^{10}}{0.8^2}} - \frac{0.5^{15} \times 0.8^5}{\frac{1-0.5^{12}}{0.8^2}} \right) \right) \right) \right) \pi = \\ & -100 i \pi^2 \left[\frac{\arg(0.0000164113 - x)}{2 \pi} \right] - 50 \pi \log(x) + \\ & 50 \pi \sum_{k=1}^{\infty} \frac{(-1)^k (0.0000164113 - x)^k x^{-k}}{k} \quad \text{for } x < 0 \end{aligned}$$

Open code

$$\begin{aligned} & \frac{1}{2} \left(10^2 \left(-\log \left(-\left(-\frac{0.5^{15} \times 0.8^3}{\frac{1-0.5^{10}}{0.8^2}} - \frac{0.5^{15} \times 0.8^5}{\frac{1-0.5^{12}}{0.8^2}} \right) \right) \right) \right) \pi = \\ & -100 i \pi^2 \left[\frac{-\pi + \arg\left(\frac{0.0000164113}{z_0}\right) + \arg(z_0)}{2 \pi} \right] - \\ & 50 \pi \log(z_0) + 50 \pi \sum_{k=1}^{\infty} \frac{(-1)^k (0.0000164113 - z_0)^k z_0^{-k}}{k} \end{aligned}$$

Integral representation:

$$\frac{1}{2} \left(10^2 \left(-\log \left(-\left(-\frac{0.5^{15} \times 0.8^3}{\frac{1-0.5^{10}}{0.8^2}} - \frac{0.5^{15} \times 0.8^5}{\frac{1-0.5^{12}}{0.8^2}} \right) \right) \right) \right) \pi = -50 \pi \int_1^{0.0000164113} \frac{1}{t} dt$$

This result 1730,63 is very near to the mass of candidate glueball $f_0(1710)$ meson

$$\left(\left(\left(\left(\left(10^2 * (\pi/2) * \operatorname{colog} \left[-\left(\frac{0.5^{15} * 0.8^3}{\left(\frac{1-0.5^{10}}{0.8^2} \right)} - \left(\frac{0.5^{15} * 0.8^5}{\left(\frac{1-0.5^{12}}{0.8^2} \right)} \right) \right] \right) \right) \right) \right) \right)^{1/15}$$

Input:

$$\sqrt[15]{10^2 \times \frac{\pi}{2} \left(-\log \left(-\left(-\frac{0.5^{15} \times 0.8^3}{\frac{1-0.5^{10}}{0.8^2}} - \frac{0.5^{15} \times 0.8^5}{\frac{1-0.5^{12}}{0.8^2}} \right) \right) \right)}$$

[Open code](#)

- $\log(x)$ is the natural logarithm

Enlarge Data Customize A [Plaintext](#) [Interactive](#)

Result:

More digits

1.643919...

This result is also very near to the Hausdorff dimension 1,6402. Furthermore:

$$1.643919 \approx \zeta(2) = \frac{\pi^2}{6} = 1.6449$$

$$-8 + \frac{\pi^4}{89} * ((((((10^2 * \text{colog} -[-(0.5^{15} * 0.8^3)/((1-0.5^{10})/0.8^2)]) - (0.5^{15} * 0.8^5)/((1-0.5^{12})/0.8^2)))))))))$$

Input:

$$-8 + \frac{\pi^4}{89} \left(10^2 \left(-\log \left(-\left(-\frac{0.5^{15} \times 0.8^3}{\frac{1-0.5^{10}}{0.8^2}} - \frac{0.5^{15} \times 0.8^5}{\frac{1-0.5^{12}}{0.8^2}} \right) \right) \right) \right)$$

[Open code](#)

- $\log(x)$ is the natural logarithm

Enlarge Data Customize A [Plaintext](#) [Interactive](#)

Result:

More digits

1197.85...

Series representations:

More

$$-8 + \frac{1}{89} \left(10^2 \left(-\log \left(-\left(-\frac{0.5^{15} \times 0.8^3}{\frac{1-0.5^{10}}{0.8^2}} - \frac{0.5^{15} \times 0.8^5}{\frac{1-0.5^{12}}{0.8^2}} \right) \right) \right) \right) \pi^4 =$$

$$-8 + \frac{100}{89} \pi^4 \sum_{k=1}^{\infty} \frac{(-1)^k (-0.999984)^k}{k}$$

[Open code](#)

Enlarge Data Customize A [Plaintext](#) [Interactive](#)

Entry 11.1.1. *In the notation above,*

$$\phi_{10}(q) - q^{-1}\psi_{10}(-q^4) + q^{-2}\chi_{10}(q^8) = \frac{\varphi(q)h(-q^2)}{\psi(-q)}, \quad (11.1.1)$$

where $h(q)$ is defined by

$$h(q) := \sum_{n=-\infty}^{\infty} (-1)^n q^{n(5n+3)/2}. \quad (11.1.2)$$

Entry 11.1.2. *In the notation above,*

$$\psi_{10}(q) + q\phi_{10}(-q^4) + q^{-2}X_{10}(q^8) = \frac{\varphi(q)g(-q^2)}{\psi(-q)}, \quad (11.1.3)$$

where $g(q)$ is defined by

$$g(q) := \sum_{n=-\infty}^{\infty} (-1)^n q^{n(5n+1)/2}. \quad (11.1.4)$$

For $\varphi(-q) = (-1)^n q^{n^2} = 0.5^4 = 0.0625$; $\varphi(q) = 0.0625 = 1/16$

$\psi(q) = q^{n(n+1)/2} = 0.5^3 = 0.125 = 1/8$; $\psi(-q) = -q^{n(n+1)/2} = -0.5^3 = -0.125$;

$h(q) = q^{n(5n+3)/2} = 0.5^{13} = 0,0001220703125 = 1/8192$

$g(q) = q^{n(5n+1)/2} = 0.5^{11} = 0,00048828125 = 1/2048$

We note that $2048 = 4096/2$ and $8192 = 4096*2$ where $4096 = 64^2$

We first address [Entry 11.1.2](#).

Proof. By Lemmas 11.4.1–11.4.3,

$$\begin{aligned}\psi_{10}(q) + q\phi_{10}(-q^4) + X_{10}(q^8) &= -m(q^3, q^{10}, q) - m(q^3, q^{10}, q^3) \\ &\quad + q(q^{-4}m(-q^4, q^{40}, -q^4) + q^{-4}m(-q^4, q^{40}, q^8)) \\ &\quad + m(-q^{16}, q^{40}, q^8) + m(-q^{16}, q^{40}, q^{32}).\end{aligned}\tag{11.4.16}$$

By (11.3.3) and (11.3.1),

$$m(q^3, q^{10}, q) = m(q^3, q^{10}, q^{-4}) = m(q^3, q^{10}, q^6).\tag{11.4.17}$$

By (11.3.3) and (11.3.1),

$$m(q^3, q^{10}, q^3) = m(q^3, q^{10}, q^{-6}) = m(q^3, q^{10}, q^4).\tag{11.4.18}$$

By (11.3.1),

$$m(-q^{16}, q^{40}, q^{-8}) = m(-q^{16}, q^{40}, q^{32}).\tag{11.4.19}$$

By (11.3.2),

$$q^{-3}m(-q^4, q^{40}, -q^4) = -q^{-7}m(-q^{-4}, q^{40}, -q^{-4})\tag{11.4.20}$$

and

$$q^{-3}m(-q^4, q^{40}, q^8) = -q^{-7}m(-q^4, q^{40}, q^{-8}).\tag{11.4.21}$$

Now, in (11.4.16), we do not alter $m(-q^{16}, q^{40}, q^8)$, but we do replace each of the five remaining m -functions by the expressions given in (11.4.17)–(11.4.21). Accordingly, we deduce that

$$\begin{aligned}\psi_{10}(q) + q\phi_{10}(-q^4) + X_{10}(q^8) &= (-m(q^3, q^{10}, q^6) + m(-q^{16}, q^{40}, q^{-8}) - q^{-7}m(-q^{-4}, q^{40}, q^{-8})) \\ &\quad + (-m(q^3, q^{10}, q^4) + m(-q^{16}, q^{40}, q^8) - q^{-7}m(-q^{-4}, q^{40}, q^8)) \\ &= -D(q^3, q^{10}, q^6, q^{-8}) - D(q^3, q^{10}, q^4, q^8),\end{aligned}\tag{11.4.22}$$

by (11.3.31). We now apply Lemma 11.3.11 and the Jacobi triple product identity (11.1.6) many times to find that

$$\begin{aligned}\psi_{10}(q) + q\phi_{10}(-q^4) + X_{10}(q^8) &= \frac{(q^{20}; q^{20})_{\infty}^3 j(-q^{14}; q^{20}) j(q^{20}; q^{40})}{j(q; q^{10}) j(q^8; q^{40}) j(-q^8; q^{20}) j(q^6; q^{20})} \\ &\quad + \frac{q(q^{20}; q^{20})_{\infty}^3 j(-q^{18}; q^{20}) j(q^{20}; q^{40})}{j(q^7; q^{10}) j(q^8; q^{40}) j(-q^4; q^{20}) j(q^6; q^{20})} \\ &= \frac{(q^{20}; q^{20})_{\infty}^3 j(q^{20}; q^{40})}{j(q^8; q^{40}) j(q^6; q^{20}) j(q; q^{10}) j(q^7; q^{10}) j(-q^4; q^{20}) j(-q^8; q^{20})} \\ &\quad \times (j(-q^{14}; q^{20}) j(q^7; q^{10}) j(-q^4; q^{20}) + q j(-q^{18}; q^{20}) j(q; q^{10}) j(-q^8; q^{20})) \\ &= \frac{(q^{20}; q^{20})_{\infty}^5 j(q^{20}; q^{40})}{(q^{10}; q^{10})_{\infty} j(q^8; q^{40}) j(q^6; q^{20}) j(q; q^{10}) j(q^7; q^{10}) j(-q^4; q^{20}) j(-q^8; q^{20})}\end{aligned}$$

$$\begin{aligned} & \times (j(-q^4; q^{10})j(q^7; q^{10}) + qj(-q^8; q^{10})j(q; q^{10})) \\ &= \frac{(q^{20}; q^{20})_{\infty}^5 j(q^{20}; q^{40})j(-q; -q^5)j(-q^3; -q^5)}{(q^{10}; q^{10})_{\infty} j(q^8; q^{40})j(q^6; q^{20})j(q; q^{10})j(q^7; q^{10})j(-q^4; q^{20})j(-q^8; q^{20})}, \end{aligned} \quad (11.4.23)$$

where we applied Lemma 6.3.2 with $x = -q$, $y = -q^3$, and q replaced by $-q^5$. We are now faced with the laborious task of applying the Jacobi triple product identity (11.1.6) to each of the j -functions on the far right side of (11.4.23) and simplifying. Using (11.1.4) and the familiar product representations

$$\varphi(q) = (-q; q^2)_{\infty}^2 (q^2; q^2)_{\infty} \quad \text{and} \quad \psi(q) = \frac{(q^2; q^2)_{\infty}}{(q; q^2)_{\infty}}, \quad (11.4.24)$$

respectively, from (3.1.14) (or (5.1.1)) and (3.1.15), we eventually reduce the right-hand side of (11.4.23) to

$$\frac{\varphi(q)g(-q^2)}{\psi(-q)},$$

and so the proof of Entry 11.1.2 is complete. \square

Now:

$$\frac{\varphi(q)g(-q^2)}{\psi(-q)},$$

From the previous expressions:

$$g(q) = q^{n(5n+1)/2} = 0.5^{11} = 0,00048828125;$$

$$\varphi(-q) = (-1)^n q^{n^2} = 0.5^4 = 0.0625; \quad \varphi(q) = 0.0625$$

$$\psi(q) = q^{n(n+1)/2} = 0.5^3 = 0.125 = 1/8; \quad \psi(-q) = -q^{n(n+1)/2} = -0.5^3 = -0.125;$$

we obtain:

$$((0.0625 * (-0.00048828125)^2)) / (-0.125)$$

$$((0.0625 * (-0.00048828125)^2)) / (-0.125)$$

Input interpretation:

$$\frac{0.0625 (-0.00048828125)^2}{-0.125}$$

[Open code](#)

Enlarge Data Customize A Plaintext Interactive

Result:

$$-1.1920928955078125 \times 10^{-7}$$

Rational form:

$$-\frac{1}{8388608}$$

Where $8388608 = 64^3 * 32$

Indeed:

Input interpretation:

$$\frac{1}{\frac{0.0625(-0.00048828125)^2}{0.125}}$$

[Open code](#)

Enlarge Data Customize A Plaintext Interactive

Result:

$$-8388608$$

Now:

$$\ln -[1/((((0.0625 * (-0.00048828125)^2)) / (-0.125)))]$$

Input interpretation:

$$\log\left(-\frac{1}{\frac{0.0625(-0.00048828125)^2}{0.125}}\right)$$

[Open code](#)

- $\log(x)$ is the natural logarithm

Enlarge Data Customize A Plaintext Interactive

Result:

More digits

$$15.94239\dots$$

Series representations:

More

$$\log\left(-\frac{1}{\frac{0.0625(-0.000488281)^2}{0.125}}\right) = \log(8.38861 \times 10^6) - \sum_{k=1}^{\infty} \frac{(-1)^k e^{-15.9424k}}{k}$$

[Open code](#)

Enlarge Data Customize A Plaintext Interactive

$$\log\left(-\frac{1}{\frac{0.0625(-0.000488281)^2}{0.125}}\right) = 2i\pi \left[\frac{\arg(8.38861 \times 10^6 - x)}{2\pi} \right] + \log(x) - \sum_{k=1}^{\infty} \frac{(-1)^k (8.38861 \times 10^6 - x)^k x^{-k}}{k} \quad \text{for } x < 0$$

Open code

$$\log\left(-\frac{1}{\frac{0.0625(-0.000488281)^2}{0.125}}\right) = \left[\frac{\arg(8.38861 \times 10^6 - z_0)}{2\pi} \right] \log\left(\frac{1}{z_0}\right) + \log(z_0) + \left[\frac{\arg(8.38861 \times 10^6 - z_0)}{2\pi} \right] \log(z_0) - \sum_{k=1}^{\infty} \frac{(-1)^k (8.38861 \times 10^6 - z_0)^k z_0^{-k}}{k}$$

Integral representations:

$$\log\left(-\frac{1}{\frac{0.0625(-0.000488281)^2}{0.125}}\right) = \int_1^{8.38861 \times 10^6} \frac{1}{t} dt$$

Open code

Enlarge Data Customize A Plaintext Interactive

$$\log\left(-\frac{1}{\frac{0.0625(-0.000488281)^2}{0.125}}\right) = \frac{1}{2i\pi} \int_{-i\infty+\gamma}^{i\infty+\gamma} \frac{e^{-15.9424s} \Gamma(-s)^2 \Gamma(1+s)}{\Gamma(1-s)} ds \quad \text{for } -1 < \gamma < 0$$

Open code

$$27 * 4 \ln -[1/(((0.0625 * (-0.00048828125)^2) / (-0.125)))]$$

Input interpretation:

$$27 \times 4 \log\left(-\frac{1}{\frac{0.0625(-0.00048828125)^2}{0.125}}\right)$$

Open code

- $\log(x)$ is the natural logarithm

Enlarge Data Customize A Plaintext Interactive

Result:

More digits

1721.778...

This result is very near to the mass of candidate glueball $f_0(1710)$ meson.

Series representations:

More

- $\log(x)$ is the natural logarithm

Enlarge Data Customize A Plaintext Interactive

Result:

More digits

11.98558...

This result 11,985 is very near to the values of black hole entropies 11,8458 and 12,1904

$$2 * ((((((27 * 4 \ln -[1/ (((0.0625 * (-0.00048828125)^2)) / (-0.125)))])))))^1/3$$

Input interpretation:

$$2 \sqrt[3]{27 \times 4 \log \left(-\frac{1}{\frac{0.0625 (-0.00048828125)^2}{0.125}} \right)}$$

[Open code](#)

- $\log(x)$ is the natural logarithm

Enlarge Data Customize A Plaintext Interactive

Result:

More digits

23.97116...

Series representations:

More

$$2 \sqrt[3]{27 \times 4 \log \left(-\frac{1}{\frac{0.0625 (-0.000488281)^2}{0.125}} \right)} = 6 \times 2^{2/3} \sqrt[3]{\log(8.38861 \times 10^6) - \sum_{k=1}^{\infty} \frac{(-1)^k e^{-15.9424 k}}{k}}$$

[Open code](#)

Enlarge Data Customize A Plaintext Interactive

$$2 \sqrt[3]{27 \times 4 \log \left(-\frac{1}{\frac{0.0625 (-0.000488281)^2}{0.125}} \right)} = 6 \times 2^{2/3} \sqrt[3]{2 i \pi \left[\frac{\arg(8.38861 \times 10^6 - x)}{2 \pi} \right] + \log(x) - \sum_{k=1}^{\infty} \frac{(-1)^k (8.38861 \times 10^6 - x)^k x^{-k}}{k}} \text{ for } x < 0$$

[Open code](#)

$$2 \sqrt[3]{27 \times 4 \log \left(-\frac{1}{\frac{0.0625 (-0.000488281)^2}{0.125}} \right)} =$$

$$6 \times 2^{2/3} \left(\log(z_0) + \left[\frac{\arg(8.38861 \times 10^6 - z_0)}{2\pi} \right] \left(\log\left(\frac{1}{z_0}\right) + \log(z_0) \right) - \sum_{k=1}^{\infty} \frac{(-1)^k (8.38861 \times 10^6 - z_0)^k z_0^{-k}}{k} \right)^{1/3}$$

Integral representations:

$$2 \sqrt[3]{27 \times 4 \log \left(-\frac{1}{\frac{0.0625 (-0.000488281)^2}{0.125}} \right)} = 6 \times 2^{2/3} \sqrt[3]{\int_1^{8.38861 \times 10^6} \frac{1}{t} dt}$$

[Open code](#)

Enlarge Data Customize A Plaintext Interactive

$$2 \sqrt[3]{27 \times 4 \log \left(-\frac{1}{\frac{0.0625 (-0.000488281)^2}{0.125}} \right)} =$$

$$6 \sqrt[3]{2} \sqrt[3]{\frac{1}{i\pi} \int_{-i\infty+\gamma}^{i\infty+\gamma} \frac{e^{-15.9424 s} \Gamma(-s)^2 \Gamma(1+s)}{\Gamma(1-s)} ds} \quad \text{for } -1 < \gamma < 0$$

This value is linked to the "Ramanujan function" (an elliptic modular function that satisfies the need for "conformal symmetry") that has 24 "modes" corresponding to the physical vibrations of a bosonic string and to the value of black hole entropy 23,9078

Now:

We next turn to the proof of Entry 11.1.1.

Proof. By Lemmas 11.4.1, 11.4.2, and 11.4.5,

$$\begin{aligned} \phi_{10}(q) - q^{-1}\psi_{10}(-q^4) + q^{-2}\chi_{10}(q^8) &= -q^{-1}m(q, q^{10}, q) - q^{-1}m(q, q^{10}, q^2) \\ &\quad - q^{-1}(-m(-q^{12}, q^{40}, -q^4) - m(-q^{12}, q^{40}, -q^{12})) \\ &\quad + q^{-2}(m(-q^8, q^{40}, q^{16}) + m(-q^8, q^{40}, q^{24})). \end{aligned} \quad (11.4.25)$$

By (11.3.3) and (11.3.1),

$$m(q, q^{10}, q) = m(q, q^{10}, q^{-2}) = m(q, q^{10}, q^8). \quad (11.4.26)$$

By (11.3.3),

$$m(-q^{12}, q^{40}, -q^{12}) = m(-q^{12}, q^{40}, q^{-24}), \quad (11.4.27)$$

and

$$m(-q^{12}, q^{40}, -q^4) = m(-q^{12}, q^{40}, q^{-16}). \quad (11.4.28)$$

By (11.3.2),

$$q^{-2}m(-q^8, q^{40}, q^{16}) - -q^{-10}m(-q^{-8}, q^{40}, q^{-16}) \quad (11.4.29)$$

and

$$q^{-2}m(-q^8, q^{40}, q^{24}) - -q^{-10}m(-q^{-8}, q^{40}, q^{-24}). \quad (11.4.30)$$

Now in (11.4.25) we do not alter $m(q, q^{10}, q^2)$, but we do replace the other five m -functions with (11.4.26)–(11.4.30). We thus obtain

$$\begin{aligned}
 & \phi_{10}(q) - q^{-1}\psi_{10}(-q^4) + q^{-2}\chi_{10}(q^8) \\
 & - q^{-1}m(q, q^{10}, q^8) + q^{-1}m(-q^{12}, q^{40}, q^{-24}) - q^{-10}m(-q^{-8}, q^{40}, q^{-24}) \\
 & - q^{-1}m(q, q^{10}, q^2) + q^{-1}m(-q^{12}, q^{40}, q^{-16}) - q^{-10}m(-q^{-8}, q^{40}, q^{-16}) \\
 & = -q^{-1}D(q, q^{10}, q^8, q^{-24}) - q^{-1}D(q, q^{10}, q^2, q^{-16}), \tag{11.4.31}
 \end{aligned}$$

by (11.3.31). We next apply Lemma 11.3.12 to deduce that

$$\begin{aligned}
 & -q^{-1}D(q, q^{10}, q^8, q^{-24}) - q^{-1}D(q, q^{10}, q^2, q^{-16}) \\
 & - \frac{(q^{20}; q^{20})_{\infty}^3 j(-q^2; q^{20}) j(q^{20}; q^{40})}{j(q^9; q^{10}) j(q^{24}; q^{40}) j(q^{12}; q^{20}) j(q^{18}; q^{20})} \\
 & + \frac{q(q^{20}; q^{20})_{\infty}^3 j(-q^6; q^{20}) j(q^{20}; q^{40})}{j(q^3; q^{10}) j(q^{16}; q^{40}) j(-q^4; q^{20}) j(q^2; q^{20})} \\
 & - \frac{(q^{20}; q^{20})_{\infty}^3 j(q^{20}; q^{40})}{j(q^2; q^{20}) j(q^{16}; q^{40}) j(q^9; q^{10}) j(q^{12}; q^{20}) j(q^3; q^{10}) j(q^4; q^{20})} \\
 & \times (j(q^3; q^{10}) j(-q^6; q^{20}) j(-q^4; q^{20}) + qj(q^9; q^{10}) j(-q^{12}; q^{20}) j(-q^2; q^{20})) \\
 & = \frac{(q^{20}; q^{20})_{\infty}^5 j(q^{20}; q^{40})}{(q^{10}; q^{10})_{\infty} j(q^2; q^{20}) j(q^{16}; q^{40}) j(q^9; q^{10}) j(-q^{12}; q^{20}) j(q^3; q^{10}) j(-q^4; q^{20})} \\
 & \times (j(q^3; q^{10}) j(-q^4; q^{10}) + qj(q^9; q^{10}) j(-q^2; q^{10})) \\
 & = \frac{(q^{20}; q^{20})_{\infty}^5 j(q^{20}; q^{40}) j(-q; q^5) j(-q^3; -q^5)}{(q^{10}; q^{10})_{\infty} j(q^2; q^{20}) j(q^{16}; q^{40}) j(q^9; q^{10}) j(-q^{12}; q^{20}) j(q^3; q^{10}) j(-q^4; q^{20})}, \tag{11.4.32}
 \end{aligned}$$

where we made exactly the same application of Lemma 6.3.2 as before, i.e., with $x = -q$, $y = -q^3$, and q replaced by $-q^5$. We are now faced with the laborious task of applying the Jacobi triple product identity (11.1.6) to each of the j -functions on the far right side of (11.4.32) and simplifying. Using (11.1.2) and the product representations (11.4.24), we find that the far right side of (11.4.32) reduces to

$$\frac{\varphi(q)h(-q^2)}{\psi(-q)},$$

and so the proof of Entry 11.1.1 is complete. □

From:

$$g(q) = q^{n(5n+1)/2} = 0.5^{11} = 0,00048828125;$$

$$\varphi(-q) = (-1)^n q^{n^2} = 0.5^4 = 0.0625; \quad \varphi(q) = 0.0625$$

$$\psi(q) = q^{n(n+1)/2} = 0.5^3 = 0.125 = 1/8; \quad \psi(-q) = -q^{n(n+1)/2} = -0.5^3 = -0.125;$$

$$h(q) = q^{n(5n+3)/2} = 0.5^{13} = 0,0001220703125 = 1/8192$$

$$\frac{\varphi(q)h(-q^2)}{\psi(-q)},$$

We obtain:

$$(((0.0625*(-0.0001220703125)^2)) / (-0.125))$$

$$(((0.0625*(-0.0001220703125)^2)) / (-0.125))$$

Input interpretation:

$$\frac{0.0625(-0.0001220703125)^2}{-0.125}$$

[Open code](#)

Enlarge Data Customize A [Plaintext](#) [Interactive](#)

Result:

$$-7.450580596923828125 \times 10^{-9}$$

Rational form:

$$-\frac{1}{134217728}$$

Where $134217728 = 64^4 * 8$

Indeed:

Input interpretation:

$$\frac{1}{\frac{0.0625(-0.0001220703125)^2}{-0.125}}$$

[Open code](#)

Enlarge Data Customize A [Plaintext](#) [Interactive](#)

Result:

$$-134217728$$

$$\ln -[1/((((((0.0625*(-0.0001220703125)^2)) / (-0.125))))))]]$$

Input interpretation:

$$\log\left(-\frac{1}{\frac{0.0625(-0.0001220703125)^2}{0.125}}\right)$$

[Open code](#)

- $\log(x)$ is the natural logarithm

Enlarge Data Customize A [Plaintext](#) [Interactive](#)

Result:

More digits

18.71497...

Series representations:

More

$$\log\left(-\frac{1}{\frac{0.0625(-0.00012207)^2}{0.125}}\right) = \log(1.34218 \times 10^8) - \sum_{k=1}^{\infty} \frac{(-1)^k e^{-18.715 k}}{k}$$

[Open code](#)

Enlarge Data Customize A [Plaintext](#) [Interactive](#)

$$\log\left(-\frac{1}{\frac{0.0625(-0.00012207)^2}{0.125}}\right) = 2i\pi \left[\frac{\arg(1.34218 \times 10^8 - x)}{2\pi} \right] + \log(x) - \sum_{k=1}^{\infty} \frac{(-1)^k (1.34218 \times 10^8 - x)^k x^{-k}}{k} \quad \text{for } x < 0$$

[Open code](#)

$$\log\left(-\frac{1}{\frac{0.0625(-0.00012207)^2}{0.125}}\right) = \left[\frac{\arg(1.34218 \times 10^8 - z_0)}{2\pi} \right] \log\left(\frac{1}{z_0}\right) + \log(z_0) + \left[\frac{\arg(1.34218 \times 10^8 - z_0)}{2\pi} \right] \log(z_0) - \sum_{k=1}^{\infty} \frac{(-1)^k (1.34218 \times 10^8 - z_0)^k z_0^{-k}}{k}$$

[Open code](#)

Integral representations:

$$\log\left(-\frac{1}{\frac{0.0625(-0.00012207)^2}{0.125}}\right) = \int_1^{1.34218 \times 10^8} \frac{1}{t} dt$$

[Open code](#)

Enlarge Data Customize A [Plaintext](#) [Interactive](#)

$$\log\left(-\frac{1}{\frac{0.0625(-0.00012207)^2}{0.125}}\right) = \frac{1}{2i\pi} \int_{-i\infty+\gamma}^{i\infty+\gamma} \frac{e^{-18.715 s} \Gamma(-s)^2 \Gamma(1+s)}{\Gamma(1-s)} ds \quad \text{for } -1 < \gamma < 0$$

[Open code](#)

$$(108 - 16) * \ln -[1/(((((((0.0625*(-0.0001220703125)^2)) / (-0.125)))))))]$$

Input interpretation:

$$(108 - 16) \log \left(-\frac{1}{\frac{0.0625 (-0.0001220703125)^2}{0.125}} \right)$$

Open code

- $\log(x)$ is the natural logarithm

Enlarge Data Customize A [Plaintext](#) [Interactive](#)

Result:

More digits

1721.778...

This result is very near to the mass of candidate glueball $f_0(1710)$ meson.

Series representations:

More

$$(108 - 16) \log \left(-\frac{1}{\frac{0.0625 (-0.00012207)^2}{0.125}} \right) = 92 \log(1.34218 \times 10^8) - 92 \sum_{k=1}^{\infty} \frac{(-1)^k e^{-18.715 k}}{k}$$

Open code

Enlarge Data Customize A [Plaintext](#) [Interactive](#)

$$(108 - 16) \log \left(-\frac{1}{\frac{0.0625 (-0.00012207)^2}{0.125}} \right) = 184 i \pi \left[\frac{\arg(1.34218 \times 10^8 - x)}{2 \pi} \right] + 92 \log(x) - 92 \sum_{k=1}^{\infty} \frac{(-1)^k (1.34218 \times 10^8 - x)^k x^{-k}}{k} \text{ for } x < 0$$

Open code

$$(108 - 16) \log \left(-\frac{1}{\frac{0.0625 (-0.00012207)^2}{0.125}} \right) = 92 \left[\frac{\arg(1.34218 \times 10^8 - z_0)}{2 \pi} \right] \log \left(\frac{1}{z_0} \right) + 92 \log(z_0) + 92 \left[\frac{\arg(1.34218 \times 10^8 - z_0)}{2 \pi} \right] \log(z_0) - 92 \sum_{k=1}^{\infty} \frac{(-1)^k (1.34218 \times 10^8 - z_0)^k z_0^{-k}}{k}$$

Integral representations:

$$(108 - 16) \log \left(-\frac{1}{\frac{0.0625 (-0.00012207)^2}{0.125}} \right) = 92 \int_1^{1.34218 \times 10^8} \frac{1}{t} dt$$

Open code

the physical vibrations of a bosonic string and to the value of black hole entropy 23,9078

Now, we have the following expressions:

$$\left(\left(\left(\left(27 * 4 \ln \left[-\frac{1}{\left(\left(\left(0.0625 * (-0.00048828125)^2\right) / (-0.125)\right)\right]\right)\right]\right)\right)\right)\right)^{1/15}$$

Input interpretation:

$$\sqrt[15]{27 \times 4 \log \left(-\frac{1}{\frac{0.0625 (-0.00048828125)^2}{0.125}} \right)}$$

Open code

- $\log(x)$ is the natural logarithm

Enlarge Data Customize A [Plaintext](#) [Interactive](#)

Result:

More digits

1.6433566...

And:

$$\left(\left(\left(\left(108 - 16\right) * \ln \left[-\frac{1}{\left(\left(\left(0.0625 * (-0.0001220703125)^2\right) / (-0.125)\right)\right]\right)\right]\right)\right)\right)^{1/15}$$

Input interpretation:

$$\sqrt[15]{(108 - 16) \log \left(-\frac{1}{\frac{0.0625 (-0.0001220703125)^2}{0.125}} \right)}$$

Open code

- $\log(x)$ is the natural logarithm

Enlarge Data Customize A [Plaintext](#) [Interactive](#)

Result:

More digits

1.6433566...

$$1.6433566... \approx \zeta(2) = \frac{\pi^2}{6} = 1.6449$$

Thence, we have the following identity:

$$\frac{\varphi(q)g(-q^2)}{\psi(-q)} = \frac{\varphi(q)h(-q^2)}{\psi(-q)}$$

We note this interesting new physical connection with the above expression concerning the tenth order mock theta functions:

$$1/10^{16} * 4 * \left(\left(\left(\left(27 * 4 \ln \left[-\frac{1}{\left(\left(\left(0.0625 * (-0.00048828125)^2\right) / (-0.125)\right)\right]\right)\right]\right)\right)\right)\right)^{1/15}$$

Input interpretation:

$$\frac{1}{10^{16}} \times 4 \sqrt[15]{27 \times 4 \log \left(-\frac{1}{\frac{0.0625 (-0.00048828125)^2}{0.125}} \right)}$$

[Open code](#)

- $\log(x)$ is the natural logarithm

Enlarge Data Customize A Plaintext Interactive

Result:

More digits

$$6.5734263... \times 10^{-16}$$

Series representations:

More

$$\frac{4 \sqrt[15]{27 \times 4 \log \left(-\frac{1}{\frac{0.0625 (-0.000488281)^2}{0.125}} \right)}}{10^{16}} = \frac{\sqrt[5]{3} \sqrt[15]{\log(8.38861 \times 10^6) - \sum_{k=1}^{\infty} \frac{(-1)^k e^{-15.9424 k}}{k}}}{1250000000000000 \times 2^{13/15}}$$

[Open code](#)

Enlarge Data Customize A Plaintext Interactive

$$\frac{4 \sqrt[15]{27 \times 4 \log \left(-\frac{1}{\frac{0.0625 (-0.000488281)^2}{0.125}} \right)}}{10^{16}} = \frac{\sqrt[5]{3} \sqrt[15]{2 i \pi \left[\frac{\arg(8.38861 \times 10^6 - x)}{2 \pi} \right] + \log(x) - \sum_{k=1}^{\infty} \frac{(-1)^k (8.38861 \times 10^6 - x)^k x^{-k}}{k}}{1250000000000000 \times 2^{13/15}} \quad \text{for } x < 0$$

[Open code](#)

$$\frac{4 \sqrt[15]{27 \times 4 \log \left(-\frac{1}{\frac{0.0625 (-0.000488281)^2}{0.125}} \right)}}{10^{16}} = \frac{\left(\sqrt[5]{3} \left(\log(z_0) + \left[\frac{\arg(8.38861 \times 10^6 - z_0)}{2 \pi} \right] \left(\log\left(\frac{1}{z_0}\right) + \log(z_0) \right) - \sum_{k=1}^{\infty} \frac{(-1)^k (8.38861 \times 10^6 - z_0)^k z_0^{-k}}{k} \right) \right)^{\wedge}}{(1/15)} \Big/ (1250000000000000 \times 2^{13/15})$$

Integral representations:

$$\frac{4 \sqrt[15]{27 \times 4 \log\left(-\frac{1}{\frac{0.0625(-0.000488281)^2}{0.125}}\right)}}{10^{16}} = \frac{\sqrt[5]{3} \sqrt[15]{\int_1^{8.38861 \times 10^6} \frac{1}{t} dt}}{1250000000000000 \times 2^{13/15}}$$

[Open code](#)

Enlarge Data Customize A [Plaintext](#) Interactive

$$\frac{4 \sqrt[15]{27 \times 4 \log\left(-\frac{1}{\frac{0.0625(-0.000488281)^2}{0.125}}\right)}}{10^{16}} = \frac{\sqrt[5]{3} \sqrt[15]{\frac{1}{i\pi} \int_{-i\infty+\gamma}^{i\infty+\gamma} \frac{e^{-15.9424s} \Gamma(-s)^2 \Gamma(1+s)}{\Gamma(1-s)} ds}}{1250000000000000 \times 2^{14/15}}$$

for $-1 < \gamma < 0$

This result $6.5734263 \times 10^{-16}$ is practically equal to the very fundamental physical value $6.582119514(40) \times 10^{-16} \text{ eV} \cdot \text{s}$, that is the reduced Planck constant.

In conclusion, we have obtained another interesting physical connection:

$$2.529 * (((((27 * 4 \ln -[1/ (((0.0625 * (-0.00048828125)^2)) / (-0.125)))])))))^1/15$$

Where 2.529 is a following Hausdorff dimension:

$$\frac{\log\left(\frac{\sqrt{7}}{6} - \frac{1}{3}\right)}{\log(\sqrt{2} - 1)}$$

$$(((\ln(\sqrt{7}/6 - 1/3)) / ((\ln(\sqrt{2} - 1)))))) * (((((27 * 4 \ln -[1/ (((0.0625 * (-0.00048828125)^2)) / (-0.125)))])))))^1/15$$

Input interpretation:

$$\frac{\log\left(\frac{\sqrt{7}}{6} - \frac{1}{3}\right)}{\log(\sqrt{2} - 1)} \sqrt[15]{27 \times 4 \log\left(-\frac{1}{\frac{0.0625(-0.00048828125)^2}{0.125}}\right)}$$

[Open code](#)

- $\log(x)$ is the natural logarithm

Enlarge Data Customize A [Plaintext](#) Interactive

Result:

[More digits](#)

4.1562473...

Series representations:

More

$$\frac{15 \sqrt[15]{27 \times 4 \log\left(-\frac{1}{\frac{0.0625(-0.000488281)^2}{0.125}}\right) \log\left(\frac{\sqrt{7}}{6} - \frac{1}{3}\right)}}{\log(\sqrt{2} - 1)} = \left(2^{2/15} \sqrt[5]{3} \sqrt[15]{\log(8.38861 \times 10^6)} \right. \\ \left. \log\left(\frac{1}{6} \left(-2 + \exp\left(i\pi \left[\frac{\arg(7-x)}{2\pi}\right]\right)\sqrt{x} \sum_{k=0}^{\infty} \frac{(-1)^k (7-x)^k x^{-k} \left(-\frac{1}{2}\right)_k}{k!}\right)\right)\right) / \\ \log\left(-1 + \exp\left(i\pi \left[\frac{\arg(2-x)}{2\pi}\right]\right)\sqrt{x} \sum_{k=0}^{\infty} \frac{(-1)^k (2-x)^k x^{-k} \left(-\frac{1}{2}\right)_k}{k!}\right)$$

for $(x \in \mathbb{R}$ and $x < 0)$

[Open code](#)

Enlarge Data Customize A Plaintext Interactive

$$\frac{15 \sqrt[15]{27 \times 4 \log\left(-\frac{1}{\frac{0.0625(-0.000488281)^2}{0.125}}\right) \log\left(\frac{\sqrt{7}}{6} - \frac{1}{3}\right)}}{\log(\sqrt{2} - 1)} = \left(2^{2/15} \sqrt[5]{3} \right. \\ \left. \sqrt[15]{2i\pi \left[\frac{\arg(8.38861 \times 10^6 - x)}{2\pi}\right] + \log(x) - \sum_{k=1}^{\infty} \frac{(-1)^k (8.38861 \times 10^6 - x)^k x^{-k}}{k}} \right. \\ \left. \left(2i\pi \left[\frac{\arg\left(\frac{1}{6}(-2 - 6x + \sqrt{7})\right)}{2\pi}\right] + \log(x) - \sum_{k=1}^{\infty} \frac{\left(-\frac{1}{6}\right)^k x^{-k} (-2 - 6x + \sqrt{7})^k}{k}\right)\right) / \\ \left(2i\pi \left[\frac{\arg(-1 - x + \sqrt{2})}{2\pi}\right] + \log(x) - \sum_{k=1}^{\infty} \frac{(-1)^k x^{-k} (-1 - x + \sqrt{2})^k}{k}\right) \text{ for } x < 0$$

[Open code](#)

$$\begin{aligned}
& \frac{\sqrt[15]{27 \times 4 \log\left(-\frac{1}{\frac{0.0625(-0.000488281)^2}}{0.125}}\right) \log\left(\frac{\sqrt{7}}{6} - \frac{1}{3}\right)}{\log(\sqrt{2} - 1)} = \left(2^{2/15} \sqrt[5]{3} \right. \\
& \left. \sqrt[15]{2i\pi \left[\frac{\arg(8.38861 \times 10^6 - x)}{2\pi} \right] + \log(x) - \sum_{k=1}^{\infty} \frac{(-1)^k (8.38861 \times 10^6 - x)^k x^{-k}}{k} \right. \\
& \left. \left(2i\pi \left[\frac{\arg\left(-\frac{1}{3} - x + \frac{\sqrt{7}}{6}\right)}{2\pi} \right] + \log(x) - \sum_{k=1}^{\infty} \frac{\left(-\frac{1}{6}\right)^k x^{-k} (-2 - 6x + \sqrt{7})^k}{k} \right) \right) / \\
& \left(2i\pi \left[\frac{\arg(-1 - x + \sqrt{2})}{2\pi} \right] + \log(x) - \sum_{k=1}^{\infty} \frac{(-1)^k x^{-k} (-1 - x + \sqrt{2})^k}{k} \right) \text{ for } x < 0
\end{aligned}$$

Integral representation:

$$\begin{aligned}
& \frac{\sqrt[15]{27 \times 4 \log\left(-\frac{1}{\frac{0.0625(-0.000488281)^2}}{0.125}}\right) \log\left(\frac{\sqrt{7}}{6} - \frac{1}{3}\right)}{\log(\sqrt{2} - 1)} = \\
& \frac{2^{2/15} \sqrt[5]{3} \sqrt[15]{\int_1^{8.38861 \times 10^6} \frac{1}{t} dt} \int_1^{\frac{1}{6}(-2+\sqrt{7})} \frac{1}{t} dt}{\int_1^{-1+\sqrt{2}} \frac{1}{t} dt}
\end{aligned}$$

This result 4,1562473 is in the range of the mass of DM particle that is between 4 – 4.2 eV

Now, we have:

Lemma 12.2.12. For $z \in \mathcal{H}$,

$$\begin{aligned}
& \eta_{50}^{18} \eta_{10,2}^4 \eta_{10,3} \eta_{20,2} \eta_{20,4} \eta_{100,50} \\
& \times \left(-\eta_{50,5} \eta_{50,10}^2 \eta_{50,15} \eta_{50,25} \eta_{100,40} + \eta_{50,5} \eta_{50,10} \eta_{50,15} \eta_{50,20} \eta_{50,25} \eta_{100,40} \right. \\
& \quad - \eta_{50,15}^2 \eta_{50,20}^2 \eta_{50,25} \eta_{100,20} + 2\eta_{50,5} \eta_{50,15} \eta_{50,20}^2 \eta_{50,25} \eta_{100,40} \\
& \quad \left. - 2\eta_{50,5}^2 \eta_{50,20} \eta_{50,25} \eta_{100,30} \eta_{100,40}^2 + \eta_{50,5}^2 \eta_{50,15} \eta_{50,20} \eta_{100,40}^2 \eta_{100,50} \right) \\
& = \eta_{10}^2 \eta_{100}^{16} \eta_{10,1}^2 \eta_{10,2}^4 \eta_{20,8}^2 \eta_{50,10} \eta_{50,15} \eta_{50,20}^2 \eta_{50,25} \eta_{100,50}^9. \tag{12.2.25}
\end{aligned}$$

By Lemma 12.2.11 and a straightforward calculation, each g_j^1 is a modular form of weight 0 on $\Gamma_1(300)$ with the multiplier system I . Therefore, each $f_j^1 g_j^1$ is a modular form of weight 9 on $\Gamma_1(300)$ with multiplier system v_1 . By Lemma 12.2.10, $[\Gamma(1) : \Gamma_1(300)] = 57600$. Let F_1 denote the difference of the left- and right-hand sides of (12.2.25). Applying Lemma 12.2.8 for a fundamental region R for $\Gamma_1(300)$, we deduce that

$$\sum_{z \in R} \text{Ord}_{\Gamma_1(300)}(F_1; z) = \frac{9 \cdot 57600}{12} = 43200 \geq \text{ord}(F_1; \infty), \quad (12.2.26)$$

Lemma 12.2.13. *For $z \in \mathcal{H}$,*

$$\begin{aligned} & \eta_{50}^{18} \eta_{10,1} \eta_{10,2}^{16} \eta_{20,6} \eta_{20,8} \eta_{100,50} \\ & \times \left(\eta_{50,5} \eta_{50,10} \eta_{50,15} \eta_{50,20} \eta_{50,25} \eta_{100,20} + \eta_{50,5} \eta_{50,15} \eta_{50,20}^2 \eta_{50,25} \eta_{100,20} \right. \\ & \quad \eta_{50,5}^2 \eta_{50,10}^2 \eta_{50,25} \eta_{100,40} \quad 2\eta_{50,5} \eta_{50,10}^2 \eta_{50,15} \eta_{50,25} \eta_{100,20} \\ & \quad \left. - 2\eta_{50,10} \eta_{50,15}^2 \eta_{50,25} \eta_{100,10} \eta_{100,20}^2 + \eta_{50,5} \eta_{50,10} \eta_{50,15}^2 \eta_{100,20}^2 \eta_{100,50} \right) \\ & = \eta_{10}^2 \eta_{100}^{16} \eta_{10,2}^{16} \eta_{10,3}^2 \eta_{20,4} \eta_{50,5} \eta_{50,10}^2 \eta_{50,20} \eta_{50,25} \eta_{100,50}^9. \end{aligned} \quad (12.2.27)$$

By Lemma 12.2.11 and a straightforward calculation, each g_j^2 is a modular form of weight 0 on $\Gamma_1(300)$ with the multiplier system I . Hence, each $f_j^2 g_j^2$ is a modular form of weight 9 on $\Gamma_1(300)$ with multiplier system v_2 . Let F_2 denote the difference of the left- and right-hand sides of (12.2.27). Applying Lemma 12.2.8 for a fundamental region R for $\Gamma_1(300)$, we deduce that

$$\sum_{z \in R} \text{Ord}_{\Gamma_1(300)}(F_2; z) = \frac{9 \cdot 57600}{12} = 43200 \geq \text{ord}(F_2; \infty), \quad (12.2.28)$$

Lemma 12.2.14. *For $z \in \mathcal{H}$,*

$$\begin{aligned} & \eta_5^2 \eta_{100}^2 \eta_{5,1}^{10} \eta_{6,1}^3 \\ & \times \left(\eta_{50,5} \eta_{50,10}^3 \eta_{100,40} \eta_{100,50} + \eta_{50,15} \eta_{50,20}^3 \eta_{100,20} \eta_{100,50} - 3\eta_{50,10}^2 \eta_{50,20}^2 \eta_{100,25}^2 \right) \\ & = \eta_{20}^2 \eta_{25}^2 \eta_{5,1}^{10} \eta_{6,1}^3 \eta_{20,5}^2 \eta_{25,5}^2 \eta_{25,10}^2 \eta_{50,10}^2 \eta_{50,20}^2. \end{aligned} \quad (12.2.29)$$

By Lemma 12.2.11 and a straightforward calculation, each g_j^3 is a modular form of weight 0 on $\Gamma_1(300)$ with the multiplier system I . Hence, each $f_j^3 g_j^3$ is a modular form of weight 2 on $\Gamma_1(300)$ with multiplier system v_3 . Let F_3 denote the difference of the left- and right-hand sides of (12.2.29). Applying Lemma 12.2.8 for a fundamental region R for $\Gamma_1(300)$, we deduce that

$$\sum_{z \in R} \text{Ord}_{\Gamma_1(300)}(F_3; z) - \frac{2 \cdot 57600}{12} - 9600 \geq \text{ord}(F_3; \infty), \quad (12.2.30)$$

Lemma 12.2.15. *For $z \in \mathcal{H}$,*

$$\begin{aligned} & \eta_4^2 \eta_{50}^2 \eta_{10,1}^2 \eta_{10,2}^3 \eta_{10,3} \eta_{20,2} \\ & \quad \times (\eta_{50,5} \eta_{50,15} \eta_{50,25} \eta_{100,50} + \eta_{50,5} \eta_{50,15} \eta_{50,25} \eta_{100,30} \\ & \quad - \eta_{50,5} \eta_{50,15} \eta_{50,25} \eta_{100,10} - \eta_{50,5}^2 \eta_{50,15} \eta_{100,50} - \eta_{50,5} \eta_{50,25}^2 \eta_{100,30} \\ & \quad + 2\eta_{50,5}^2 \eta_{50,25} \eta_{100,30} - \eta_{50,15}^2 \eta_{50,25} \eta_{100,10}) \\ & = \eta_{10}^2 \eta_{20}^2 \eta_{10,1}^3 \eta_{10,2}^3 \eta_{20,4}^3 \eta_{20,8}^3 \eta_{50,5} \eta_{50,15} \eta_{50,25}. \end{aligned} \quad (12.2.31)$$

By Lemma 12.2.11 and a straightforward calculation, each g_j^4 is a modular form of weight 0 on $\Gamma_1(300)$ with the multiplier system I . Hence, each $f_j^4 g_j^4$ is a modular form of weight 2 on $\Gamma_1(300)$ with multiplier system v_4 . Let F_4 denote the difference of the left- and right-hand sides of (12.2.31). Applying Lemma 12.2.8 for a fundamental region R for $\Gamma_1(300)$, we deduce that

$$\sum_{z \in R} \text{Ord}_{\Gamma_1(300)}(F_4; z) = \frac{2 \cdot 57600}{12} = 9600 \geq \text{ord}(F_4; \infty), \quad (12.2.32)$$

Lemma 12.2.16. For $z \in \mathcal{H}$,

$$\begin{aligned}
& \eta_{10}^8 \eta_{20}^3 \eta_{50} \eta_{10,1} \eta_{10,4} \eta_{20,4} \eta_{20,8}^2 \eta_{50,5} \eta_{50,15} \eta_{50,25} \eta_{200,50} \eta_{200,100} \\
& \quad \times (\eta_{10,1} \eta_{20,10} \eta_{50,10} \eta_{100,40} - 4\eta_{10,3} \eta_{20,2} \eta_{50,10} \eta_{100,40} \\
& \quad + \eta_{10,1} \eta_{20,10} \eta_{50,20} \eta_{100,20} + 6\eta_{10,3} \eta_{20,2} \eta_{50,20} \eta_{100,20}) \\
& \quad - 4\eta_{10,1} \eta_{10,4} \eta_{20,4} \eta_{20,8}^2 \eta_{40,10} \eta_{50,5} \eta_{50,10} \eta_{50,15} \eta_{50,20} \eta_{50,25} \eta_{200,100} \\
& \quad \times (\eta_{10}^7 \eta_{20}^2 \eta_{50}^2 \eta_{100} \eta_{10,1} \eta_{50,10}^2 \eta_{50,20}^2 + \eta_{10} \eta_{20}^{10} \eta_{100} \eta_{10,3} \eta_{20,2} \eta_{20,10}^3) \\
& - \eta_{10}^8 \eta_{20}^3 \eta_{50} \eta_{50,5} \eta_{50,10} \eta_{50,15} \eta_{50,20} \eta_{100,50} \eta_{200,50} \eta_{200,100} \\
& \quad \times (\eta_{10,1}^3 \eta_{20,6} \eta_{20,8}^2 \eta_{20,10} - 4\eta_{10,1} \eta_{10,3} \eta_{10,4} \eta_{20,2} \eta_{20,5}^2 \eta_{20,8} \\
& \quad - \eta_{10,3}^3 \eta_{20,2} \eta_{20,4}^2 \eta_{20,10}) \\
& \quad + \eta_{10}^8 \eta_{20}^3 \eta_{50} \eta_{50,5} \eta_{50,10} \eta_{50,20} \eta_{50,25} \eta_{100,30} \eta_{200,50} \eta_{200,100} \\
& \quad \times (2\eta_{10,1}^3 \eta_{20,6} \eta_{20,8}^2 \eta_{20,10} + 2\eta_{10,1} \eta_{10,3} \eta_{10,4} \eta_{20,2} \eta_{20,5}^2 \eta_{20,8} \\
& \quad + 3\eta_{10,3}^3 \eta_{20,2} \eta_{20,4}^2 \eta_{20,10}) \\
& \quad - \eta_{10}^8 \eta_{20}^3 \eta_{50} \eta_{50,10} \eta_{50,15} \eta_{50,20} \eta_{50,25} \eta_{100,10} \eta_{200,50} \eta_{200,100} \\
& \quad \times (3\eta_{10,1}^3 \eta_{20,6} \eta_{20,8}^2 \eta_{20,10} - 2\eta_{10,1} \eta_{10,3} \eta_{10,4} \eta_{20,2} \eta_{20,5}^2 \eta_{20,8} \\
& \quad + 2\eta_{10,3}^3 \eta_{20,2} \eta_{20,4}^2 \eta_{20,10}). \tag{12.2.33}
\end{aligned}$$

By Lemma 12.2.11 and a straightforward calculation, each g_j^5 is a modular form of weight 0 on $\Gamma_1(200)$ with the multiplier system I . Hence, each $f_j^5 g_j^5$ is a modular form of weight 6 on $\Gamma_1(200)$ with multiplier system v_5 . By Lemma 12.2.10, $[I(1) : \Gamma_1(200)] = 28800$. Let F_5 denote the difference of the left and right sides of (12.2.33). Applying Lemma 12.2.8 for a fundamental region R for $\Gamma_1(200)$, we deduce that

$$\sum_{z \in R} \text{Ord}_{\Gamma_1(200)}(F_5; z) = \frac{6 \cdot 28800}{12} = 14400 \geq \text{ord}(F_5; \infty), \tag{12.2.34}$$

Lemma 12.2.17. For $z \in \mathcal{H}$,

$$\begin{aligned}
& \eta_{10}^8 \eta_{20}^3 \eta_{50}^3 \eta_{10,1} \eta_{20,4}^2 \eta_{20,6} \eta_{20,8}^2 \eta_{50,5} \eta_{50,15} \eta_{50,25} \eta_{200,50} \eta_{200,100} \\
& \quad \times (\eta_{10,1} \eta_{20,10} \eta_{50,10} \eta_{100,40} - \eta_{10,1} \eta_{20,10} \eta_{50,20} \eta_{100,20} \\
& \quad - 2\eta_{10,3} \eta_{20,2} \eta_{50,20} \eta_{100,20}) + 4\eta_{10} \eta_{20}^{10} \eta_{50}^2 \eta_{100} \eta_{10,1} \eta_{10,3} \eta_{20,2} \eta_{20,4}^2 \eta_{20,6} \\
& \quad \times \eta_{20,8}^2 \eta_{20,10}^3 \eta_{40,10} \eta_{50,5} \eta_{50,10} \eta_{50,15} \eta_{50,20} \eta_{100,25}^2 \eta_{200,100} \\
& = \eta_{10}^8 \eta_{20}^3 \eta_{50}^3 \eta_{50,10} \eta_{50,20} \eta_{200,50} \eta_{200,100} \\
& \quad \times (\eta_{10,1}^3 \eta_{20,6} \eta_{20,8}^2 \eta_{20,10} \eta_{50,5} \eta_{50,15} \eta_{100,50} \\
& \quad + \eta_{10,3}^3 \eta_{20,2} \eta_{20,4}^2 \eta_{20,10} \eta_{50,5} \eta_{50,15} \eta_{100,50} \\
& \quad - 2\eta_{10,1} \eta_{10,3} \eta_{20,2} \eta_{20,4}^2 \eta_{20,5}^2 \eta_{20,6} \eta_{20,8} \eta_{50,5} \eta_{50,25} \eta_{100,30} \\
& \quad - \eta_{10,3}^3 \eta_{20,2} \eta_{20,4}^2 \eta_{20,10} \eta_{50,5} \eta_{50,25} \eta_{100,30} \\
& \quad - \eta_{10,1}^3 \eta_{20,6} \eta_{20,8}^2 \eta_{20,10} \eta_{50,15} \eta_{50,25} \eta_{100,10} \\
& \quad + 2\eta_{10,1} \eta_{10,3} \eta_{20,2} \eta_{20,4}^2 \eta_{20,5}^2 \eta_{20,6} \eta_{20,8} \eta_{50,15} \eta_{50,25} \eta_{100,10}). \tag{12.2.35}
\end{aligned}$$

By Lemma 12.2.11 and a straightforward calculation, each g_j^6 is a modular form of weight 0 on $\Gamma_1(200)$ with the multiplier system I . Hence, each $f_j^6 g_j^6$ is a modular form of weight 7 on $\Gamma_1(200)$ with multiplier system v_6 . Let F_6 denote the difference of the left and right sides of (12.2.35). Applying Lemma 12.2.8 for a fundamental region R for $\Gamma_1(200)$, we deduce that

$$\sum_{z \in R} \text{Ord}_{\Gamma_1(200)}(F_6; z) = \frac{7 \cdot 28800}{12} = 16800 \geq \text{ord}(F_6; \infty), \tag{12.2.36}$$

Our next task is to take each of the six eta function identities from Lemmas 12.2.12–12.2.17 and rewrite them in terms of Ramanujan's theta functions $f(a, b)$.

Lemma 12.2.18. *If $f(a, b)$ is defined by (12.1.5), then*

$$\begin{aligned}
& - \frac{f(-q^5, -q^{45})f(-q^{40}, -q^{60})f(-q^{50}, -q^{50})}{f^2(-q^{20}, -q^{30})} \\
& \times (q^6 f(-q^{10}, -q^{40}) - q^4 f(-q^{20}, -q^{30})) \\
& \quad \frac{f(-q^{50}, -q^{50})}{f(-q^{10}, -q^{40})} \\
& \times (q^3 f(-q^{15}, -q^{35})f(-q^{20}, -q^{80}) - 2q^2 f(-q^5, -q^{45})f(-q^{40}, -q^{60})) \\
& - 2q \frac{f^2(-q^5, -q^{45})f(-q^{40}, -q^{60})(q^{50}; q^{50})_\infty (q^{100}; q^{100})_\infty}{f(-q^{15}, -q^{35})f(-q^{10}, -q^{90})f(-q^{20}, -q^{80})} \\
& + \frac{f^2(-q^5, -q^{45})f(-q^{40}, -q^{60})(q^{50}; q^{50})_\infty^3}{f(-q^{25}, -q^{25})f(-q^{10}, -q^{90})f(-q^{20}, -q^{80})f(-q^{30}, -q^{70})} \\
& = \frac{f^2(-q, -q^9)f(-q^8, -q^{12})(q^{10}; q^{10})_\infty (q^{20}; q^{20})_\infty}{f(-q^3, -q^7)f(-q^2, -q^{18})f(-q^4, -q^{16})}. \tag{12.2.37}
\end{aligned}$$

Proof. Simplify slightly the right-hand side of (12.2.25) by using the identity $\eta_{100,50}\eta_{100}^2 = \eta_{50}^2$. Then divide both sides of (12.2.25) by

$$q^{451/30}\eta_{50}^{18}\eta_{10,2}^4\eta_{10,3}\eta_{20,2}\eta_{20,4}\eta_{50,10}\eta_{50,15}\eta_{50,20}^2\eta_{50,25}/\eta_{100}^2.$$

Using (12.2.23) and (12.2.24) to convert the resulting identity, we deduce (12.2.37). \square

Lemma 12.2.19. *We have*

$$\begin{aligned}
& \frac{f(-q^{15}, -q^{35})f(-q^{20}, -q^{80})f(-q^{50}, -q^{50})}{f^2(-q^{10}, -q^{40})} \\
& \times (q^2 f(-q^{10}, -q^{40}) + f(-q^{20}, -q^{30})) \\
& - \frac{f(-q^{50}, -q^{50})}{f(-q^{20}, -q^{30})} \\
& \times (q^3 f(-q^5, -q^{45})f(-q^{40}, -q^{60}) + 2q^4 f(-q^{15}, -q^{35})f(-q^{20}, -q^{80})) \\
& - 2q^5 \frac{f^2(-q^{15}, -q^{35})f(-q^{20}, -q^{80})(q^{50}; q^{50})_\infty (q^{100}; q^{100})_\infty}{f(-q^5, -q^{45})f(-q^{30}, -q^{70})f(-q^{40}, -q^{60})} \\
& + q \frac{f^2(-q^{15}, -q^{35})f(-q^{20}, -q^{80})(q^{50}; q^{50})_\infty^3}{f(-q^{25}, -q^{25})f(-q^{10}, -q^{90})f(-q^{30}, -q^{70})f(-q^{40}, -q^{60})} \\
& = \frac{f^2(-q^3, -q^7)f(-q^4, -q^{16})(q^{10}; q^{10})_\infty (q^{20}; q^{20})_\infty}{f(-q, -q^9)f(-q^6, -q^{14})f(-q^8, -q^{12})}. \tag{12.2.38}
\end{aligned}$$

Proof. Simplify the right-hand side of (12.2.27) with the identity $\eta_{100,50}\eta_{100}^2 = \eta_{50}^2$. Then divide both sides of (12.2.25) by

$$q^{589/30}\eta_{50}^{18}\eta_{10,1}\eta_{10,2}^{16}\eta_{20,6}\eta_{20,8}\eta_{50,5}\eta_{50,10}^2\eta_{50,20}\eta_{50,25}/\eta_{100}^2.$$

Using (12.2.23) and (12.2.24) to convert the resulting identity to a q -series identity, we deduce (12.2.38). \square

Lemma 12.2.20. *We have*

$$\begin{aligned}
& q^5 \frac{f(-q^5, -q^{45})f(-q^{10}, -q^{40})f(-q^{40}, -q^{60})f(-q^{50}, -q^{50})}{f^2(-q^{20}, -q^{30})} \\
& + \frac{f(-q^{15}, -q^{35})f(-q^{20}, -q^{30})f(-q^{20}, -q^{80})f(-q^{50}, -q^{50})}{f^2(-q^{10}, -q^{40})} \\
& - 3q^5 f^2(-q^{25}, -q^{75}) = f^2(-q^5, -q^{15}). \tag{12.2.39}
\end{aligned}$$

Proof. Employ the identity $\eta_{25,5}\eta_{25,10}\eta_{25} = \eta_5$ on the right-hand side of (12.2.29) in Lemma 12.2.14, divide both sides of (12.2.29) by

$$q^{25/4}\eta_5^2\eta_{5,1}^{10}\eta_{6,1}^3\eta_{50,10}^2\eta_{50,20}^2,$$

apply (12.2.23) and (12.2.24) to express the identity in terms of theta functions, and finally deduce (12.2.39). \square

Lemma 12.2.21. *We have*

$$\begin{aligned}
& f(q^{25}, q^{25})f(-q^{25}, -q^{25}) + q^2 f(q^{15}, q^{35})f(-q^{15}, -q^{35}) \\
& - q^8 f(q^5, q^{45})f(-q^5, -q^{45}) - q^4 f(q^{25}, q^{25})f(-q^5, -q^{45}) \\
& - qf(q^{15}, q^{35})f(-q^{25}, -q^{25}) + 2q^5 f(q^{15}, q^{35})f(-q^5, -q^{45}) \\
& - q^5 f(q^5, q^{45})f(-q^{15}, -q^{35}) = \frac{f(-q, -q^9)(q^4; q^4)_\infty (q^{10}; q^{10})_\infty^2}{f(-q^3, -q^7)f(-q^2, -q^{18})}. \tag{12.2.40}
\end{aligned}$$

Proof. Using the Jacobi triple product identity (12.1.6), we can readily show that, for each integer n ,

$$\frac{(q^{50}; q^{50})_\infty^2 f(-q^{2n}, -q^{100-2n})}{(q^{100}; q^{100})_\infty f(-q^n, -q^{50-n})} = f(q^n, q^{50-n}). \tag{12.2.41}$$

Now use the identity $\eta_{20}\eta_{20,4}\eta_{20,8} = \eta_4$ on the right side of (12.2.31), divide both sides of (12.2.31) by

$$q^{329/30}\eta_4^2\eta_{10,1}^2\eta_{10,2}^3\eta_{10,3}\eta_{20,2}\eta_{50,5}\eta_{50,15}\eta_{50,25},$$

apply (12.2.23), (12.2.24), and (12.2.41) with $n = 5, 15,$ and $25,$ respectively, and thus complete the proof of Lemma 12.2.21. \square

Lemma 12.2.22. *We have*

$$\begin{aligned}
& f(q^{20}, q^{30}) \left(\frac{f(-q, -q^9)(q^4; q^4)_\infty (q^{10}; q^{10})_\infty^2}{f(-q^3, -q^7)f(-q^2, -q^{18})} - 4qf(-q^4, -q^{16})f(-q^8, -q^{12}) \right) \\
& + q^2 f(q^{10}, q^{40}) \left(\frac{f(-q, -q^9)(q^4; q^4)_\infty (q^{10}; q^{10})_\infty^2}{f(-q^3, -q^7)f(-q^2, -q^{18})} \right. \\
& \quad \left. + 6qf(-q^4, -q^{16})f(-q^8, -q^{12}) \right)
\end{aligned}$$

$$\begin{aligned}
& -4q^6 f(q^{50}, q^{150}) \left(\frac{f(-q, -q^9)(q^4; q^4)_\infty (q^{10}; q^{10})_\infty^2}{f(-q^3, -q^7)f(-q^2, -q^{18})} \right. \\
& \quad \left. + qf(-q^4, -q^{16})f(-q^8, -q^{12}) \right) \\
& = f(q^{25}, q^{25}) \left(\frac{f^2(-q, -q^9)f(-q^8, -q^{12})(q^{10}; q^{10})_\infty (q^{20}; q^{20})_\infty}{f(-q^3, -q^7)f(-q^2, -q^{18})f(-q^4, -q^{16})} \right. \\
& \quad -4qf^2(-q^5, -q^{15}) - q \frac{f^2(-q^3, -q^7)f(-q^4, -q^{16})(q^{10}; q^{10})_\infty (q^{20}; q^{20})_\infty}{f(-q, -q^9)f(-q^6, -q^{14})f(-q^8, -q^{12})} \left. \right) \\
& \quad + qf(q^{15}, q^{35}) \left(2 \frac{f^2(-q, -q^9)f(-q^8, -q^{12})(q^{10}; q^{10})_\infty (q^{20}; q^{20})_\infty}{f(-q^3, -q^7)f(-q^2, -q^{18})f(-q^4, -q^{16})} \right. \\
& \quad \left. +2qf^2(-q^5, -q^{15}) + 3q \frac{f^2(-q^3, -q^7)f(-q^4, -q^{16})(q^{10}; q^{10})_\infty (q^{20}; q^{20})_\infty}{f(-q, -q^9)f(-q^6, -q^{14})f(-q^8, -q^{12})} \right) \\
& \quad - q^4 f(q^{25}, q^{25}) \left(3 \frac{f^2(-q, -q^9)f(-q^8, -q^{12})(q^{10}; q^{10})_\infty (q^{20}; q^{20})_\infty}{f(-q^3, -q^7)f(-q^2, -q^{18})f(-q^4, -q^{16})} \right. \\
& \quad \left. -2qf^2(-q^5, -q^{15}) + 2q \frac{f^2(-q^3, -q^7)f(-q^4, -q^{16})(q^{10}; q^{10})_\infty (q^{20}; q^{20})_\infty}{f(-q, -q^9)f(-q^6, -q^{14})f(-q^8, -q^{12})} \right). \tag{12.2.42}
\end{aligned}$$

Proof. First, employ the identities

$$\begin{aligned}
\eta_{20}^2 \eta_{20,10} &= \eta_{10}^2, \\
\eta_{20} \eta_{40,10} &= \eta_{10}, \\
\eta_{20} \eta_{20,4} \eta_{20,8} &= \eta_4, \\
\eta_{50} \eta_{50,10} \eta_{50,20} &= \eta_{10}
\end{aligned}$$

in (12.2.33). Second, divide both sides of (12.2.33) by

$$\begin{aligned}
& q^{233/6} \eta_{10}^8 \eta_{20} \eta_{10,1} \eta_{10,3} \eta_{20,2} \eta_{20,4} \eta_{20,6} \eta_{20,8} \\
& \times \eta_{50,5} \eta_{50,10} \eta_{50,15} \eta_{50,20} \eta_{50,25} \eta_{200,50} \eta_{200,100}.
\end{aligned}$$

Third, use the identities (12.2.23), (12.2.24), and (12.2.41) with n replaced by 5, 10, 15, 20, and 25, respectively, and with q replaced by q^4 and q^{50} , respectively. The identity (12.2.33) then assumes the form (12.2.42). \square

Lemma 12.2.23. *We have*

$$\begin{aligned}
& f(q^{20}, q^{30}) \frac{f(-q, -q^9)(q^4; q^4)_\infty (q^{10}; q^{10})_\infty^2}{f(-q^3, -q^7)f(-q^2, -q^{18})} \\
& - q^2 f(q^{10}, q^{40}) \left(\frac{f(-q, -q^9)(q^4; q^4)_\infty (q^{10}; q^{10})_\infty^2}{f(-q^3, -q^7)f(-q^2, -q^{18})} \right. \\
& \quad \left. + 2q f(-q^4, -q^{16})f(-q^8, -q^{12}) \right) \\
& + 4q^7 f(q^{50}, q^{150}) f(-q^4, -q^{16}) f(-q^8, -q^{12})
\end{aligned}$$

$$\begin{aligned}
&= f(q^{25}, q^{25}) \left(\frac{f^2(-q, -q^9)f(-q^8, -q^{12})(q^{10}; q^{10})_\infty (q^{20}; q^{20})_\infty}{f(-q^3, -q^7)f(-q^2, -q^{18})f(-q^4, -q^{16})} \right. \\
&\quad \left. + q \frac{f^2(-q^3, -q^7)f(-q^4, -q^{16})(q^{10}; q^{10})_\infty (q^{20}; q^{20})_\infty}{f(-q, -q^9)f(-q^6, -q^{14})f(-q^8, -q^{12})} \right) \\
&\quad - q^2 f(q^{15}, q^{35}) \left(2f^2(-q^5, -q^{15}) \right. \\
&\quad \left. + 3q \frac{f^2(-q^3, -q^7)f(-q^4, -q^{16})(q^{10}; q^{10})_\infty (q^{20}; q^{20})_\infty}{f(-q, -q^9)f(-q^6, -q^{14})f(-q^8, -q^{12})} \right) \\
&\quad - q^4 f(q^5, q^{45}) \left(\frac{f^2(-q, -q^9)f(-q^8, -q^{12})(q^{10}; q^{10})_\infty (q^{20}; q^{20})_\infty}{f(-q^3, -q^7)f(-q^2, -q^{18})f(-q^4, -q^{16})} \right. \\
&\quad \left. - 2q f^2(-q^5, -q^{15}) \right). \tag{12.2.43}
\end{aligned}$$

Proof. First, simplify (12.2.35) by using the identities

$$\begin{aligned}
\eta_{20}^2 \eta_{20,10} &= \eta_{10}^2, \\
\eta_{20} \eta_{40,10} &= \eta_{10}, \\
\eta_{20} \eta_{20,4} \eta_{20,8} &= \eta_4, \\
\eta_{50} \eta_{100,25} &= \eta_{25}.
\end{aligned}$$

Second, divide both sides of (12.2.35) by

$$\begin{aligned}
&q^{233/6} \eta_{10}^8 \eta_{20} \eta_{50}^2 \eta_{10,1} \eta_{10,3} \eta_{20,2} \eta_{20,4} \eta_{20,6} \eta_{20,8} \\
&\quad \times \eta_{50,5} \eta_{50,10} \eta_{50,15} \eta_{50,20} \eta_{50,25} \eta_{200,50} \eta_{200,100}.
\end{aligned}$$

Third, apply (12.2.23), (12.2.24), and (12.2.41) with n replaced by 5, 10, 15, 20, and 25, respectively, and with q replaced by q^4 and q^{50} , respectively. Then (12.2.43) follows. \square

We have that:

Let F_1 denote the difference of the left- and right-hand sides of (12.2.25), and divide both sides of (12.2.25) by

$$q^{451/30} \eta_{50}^{18} \eta_{10,2}^4 \eta_{10,3} \eta_{20,2} \eta_{20,4} \eta_{50,10} \eta_{50,15} \eta_{50,20}^2 \eta_{50,25} / \eta_{100}^2,$$

We obtain the following mathematical connection with the theta function identity:

$$\frac{f^2(-q, -q^9)f(-q^8, -q^{12})(q^{10}; q^{10})_\infty (q^{20}; q^{20})_\infty}{f(-q^3, -q^7)f(-q^2, -q^{18})f(-q^4, -q^{16})}$$

Thence:

$$\begin{aligned}
& \eta_{50}^{18} \eta_{10,2}^4 \eta_{10,3} \eta_{20,2} \eta_{20,4} \eta_{100,50} \\
& \times \left(-\eta_{50,5}^2 \eta_{50,10}^2 \eta_{50,15} \eta_{50,25} \eta_{100,40} + \eta_{50,5} \eta_{50,10} \eta_{50,15} \eta_{50,20} \eta_{50,25} \eta_{100,40} \right. \\
& \quad - \eta_{50,15}^2 \eta_{50,20}^2 \eta_{50,25} \eta_{100,20} + 2\eta_{50,5} \eta_{50,15} \eta_{50,20}^2 \eta_{50,25} \eta_{100,40} \\
& \quad \left. - 2\eta_{50,5}^2 \eta_{50,20} \eta_{50,25} \eta_{100,30} \eta_{100,40}^2 + \eta_{50,5}^2 \eta_{50,15} \eta_{50,20}^2 \eta_{100,40} \eta_{100,50} \right) / \\
& \left(q^{451/30} \eta_{50}^{18} \eta_{10,2}^4 \eta_{10,3} \eta_{20,2} \eta_{20,4} \eta_{50,10} \eta_{50,15} \eta_{50,20}^2 \eta_{50,25} / \eta_{100}^2 \right) - \\
& \left(\eta_{10}^2 \eta_{100}^{16} \eta_{10,1}^2 \eta_{10,2}^4 \eta_{20,8}^2 \eta_{50,10} \eta_{50,15} \eta_{50,20}^2 \eta_{50,25} \eta_{100,50}^9 / \right. \\
& \left. q^{451/30} \eta_{50}^{18} \eta_{10,2}^4 \eta_{10,3} \eta_{20,2} \eta_{20,4} \eta_{50,10} \eta_{50,15} \eta_{50,20}^2 \eta_{50,25} / \eta_{100}^2 \right) = \\
& = 43200 / q^{451/30} \eta_{50}^{18} \eta_{10,2}^4 \eta_{10,3} \eta_{20,2} \eta_{20,4} \eta_{50,10} \eta_{50,15} \eta_{50,20}^2 \eta_{50,25} / \eta_{100}^2 \\
& = \frac{f^2(-q, -q^9) f(-q^8, -q^{12}) (q^{10}; q^{10})_{\infty} (q^{20}; q^{20})_{\infty}}{f(-q^3, -q^7) f(-q^2, -q^{18}) f(-q^4, -q^{16})} \quad (\text{a})
\end{aligned}$$

$$\begin{aligned}
& \eta_{50}^{18} \eta_{10,2}^4 \eta_{10,3} \eta_{20,2} \eta_{20,4} \eta_{100,50} \\
& \times \left(-\eta_{50,5}^2 \eta_{50,10}^2 \eta_{50,15} \eta_{50,25} \eta_{100,40} + \eta_{50,5} \eta_{50,10} \eta_{50,15} \eta_{50,20} \eta_{50,25} \eta_{100,40} \right. \\
& \quad - \eta_{50,15}^2 \eta_{50,20}^2 \eta_{50,25} \eta_{100,20} + 2\eta_{50,5} \eta_{50,15} \eta_{50,20}^2 \eta_{50,25} \eta_{100,40} \\
& \quad \left. - 2\eta_{50,5}^2 \eta_{50,20} \eta_{50,25} \eta_{100,30} \eta_{100,40}^2 + \eta_{50,5}^2 \eta_{50,15} \eta_{50,20}^2 \eta_{100,40} \eta_{100,50} \right) / \\
& \left(q^{589/30} \eta_{50}^{18} \eta_{10,1}^2 \eta_{10,2}^4 \eta_{20,6} \eta_{20,8} \eta_{50,5} \eta_{50,10}^2 \eta_{50,20} \eta_{50,25} / \eta_{100}^2 \right) - \\
& \left(\eta_{10}^2 \eta_{100}^{16} \eta_{10,1}^2 \eta_{10,2}^4 \eta_{20,8}^2 \eta_{50,10} \eta_{50,15} \eta_{50,20}^2 \eta_{50,25} \eta_{100,50}^9 / \right. \\
& \left. q^{589/30} \eta_{50}^{18} \eta_{10,1}^2 \eta_{10,2}^4 \eta_{20,6} \eta_{20,8} \eta_{50,5} \eta_{50,10}^2 \eta_{50,20} \eta_{50,25} / \eta_{100}^2 \right) = \\
& = 43200 / q^{589/30} \eta_{50}^{18} \eta_{10,1}^2 \eta_{10,2}^4 \eta_{20,6} \eta_{20,8} \eta_{50,5} \eta_{50,10}^2 \eta_{50,20} \eta_{50,25} / \eta_{100}^2 = \\
& = \frac{f^2(-q^3, -q^7) f(-q^4, -q^{16}) (q^{10}; q^{10})_{\infty} (q^{20}; q^{20})_{\infty}}{f(-q, -q^9) f(-q^6, -q^{14}) f(-q^8, -q^{12})} \quad (\text{b})
\end{aligned}$$

$$\begin{aligned}
& \eta_5^2 \eta_{100}^2 \eta_{5,1}^{10} \eta_{6,1}^3 \\
& \times (\eta_{50,5} \eta_{50,10}^3 \eta_{100,40} \eta_{100,50} + \eta_{50,15} \eta_{50,20}^3 \eta_{100,20} \eta_{100,50} - 3\eta_{50,10}^2 \eta_{50,20}^2 \eta_{100,25}^2) / \\
& q^{25/4} \eta_5^2 \eta_{5,1}^{10} \eta_{6,1}^3 \eta_{50,10}^2 \eta_{50,20}^2) - (\eta_{20}^2 \eta_{25}^2 \eta_{5,1}^{10} \eta_{6,1}^3 \eta_{20,5}^2 \eta_{25,5}^2 \eta_{25,10}^2 \eta_{50,10}^2 \eta_{50,20}^2 / \\
& q^{25/4} \eta_5^2 \eta_{5,1}^{10} \eta_{6,1}^3 \eta_{50,10}^2 \eta_{50,20}^2) = 9600 / q^{25/4} \eta_5^2 \eta_{5,1}^{10} \eta_{6,1}^3 \eta_{50,10}^2 \eta_{50,20}^2 = f^2(-q^5, -q^{15}) \quad (c)
\end{aligned}$$

$$\begin{aligned}
& \eta_4^2 \eta_{50}^2 \eta_{10,1}^2 \eta_{10,2}^3 \eta_{10,3} \eta_{20,2} \\
& \times (\eta_{50,5} \eta_{50,15} \eta_{50,25} \eta_{100,50} + \eta_{50,5} \eta_{50,15} \eta_{50,25} \eta_{100,30} \\
& - \eta_{50,5} \eta_{50,15} \eta_{50,25} \eta_{100,10} - \eta_{50,5}^2 \eta_{50,15} \eta_{100,50} - \eta_{50,5} \eta_{50,25}^2 \eta_{100,30} \\
& + 2\eta_{50,5}^2 \eta_{50,25} \eta_{100,30} - \eta_{50,15}^2 \eta_{50,25} \eta_{100,10}) / \\
& q^{329/30} \eta_4^2 \eta_{10,1}^2 \eta_{10,2}^3 \eta_{10,3} \eta_{20,2} \eta_{50,5} \eta_{50,15} \eta_{50,25}) - (\eta_{10}^2 \eta_{20}^2 \eta_{10,1}^3 \eta_{10,2}^3 \eta_{20,4}^3 \eta_{20,8}^3 \eta_{50,5} \eta_{50,15} \eta_{50,25} / \\
& q^{329/30} \eta_4^2 \eta_{10,1}^2 \eta_{10,2}^3 \eta_{10,3} \eta_{20,2} \eta_{50,5} \eta_{50,15} \eta_{50,25}) = 9600 / q^{329/30} \eta_4^2 \eta_{10,1}^2 \eta_{10,2}^3 \eta_{10,3} \eta_{20,2} \eta_{50,5} \eta_{50,15} \eta_{50,25} \\
& = \frac{f(-q, -q^9)(q^4; q^4)_\infty (q^{10}; q^{10})_\infty^2}{f(-q^3, -q^7)f(-q^2, -q^{18})} \quad (d)
\end{aligned}$$

$$\begin{aligned}
& \eta_{10}^8 \eta_{20}^3 \eta_{50}^3 \eta_{10,1} \eta_{10,4} \eta_{20,4} \eta_{20,8}^2 \eta_{50,5} \eta_{50,15} \eta_{50,25} \eta_{200,50} \eta_{200,100} \\
& \times (\eta_{10,1} \eta_{20,10} \eta_{50,10} \eta_{100,40} - 4\eta_{10,3} \eta_{20,2} \eta_{50,10} \eta_{100,40} \\
& + \eta_{10,1} \eta_{20,10} \eta_{50,20} \eta_{100,20} + 6\eta_{10,3} \eta_{20,2} \eta_{50,20} \eta_{100,20}) \\
& - 4\eta_{10,1} \eta_{10,4} \eta_{20,4} \eta_{20,8}^2 \eta_{40,10} \eta_{50,5} \eta_{50,10} \eta_{50,15} \eta_{50,20} \eta_{50,25} \eta_{200,100} \\
& \times (\eta_{10}^7 \eta_{20}^2 \eta_{50}^2 \eta_{100} \eta_{10,1} \eta_{50,10}^2 \eta_{50,20}^2 + \eta_{10} \eta_{20}^{10} \eta_{100} \eta_{10,3} \eta_{20,2} \eta_{20,10}^3) /
\end{aligned}$$

$$\begin{aligned}
& q^{233/6} \eta_{10}^8 \eta_{20} \eta_{10,1} \eta_{10,3} \eta_{20,2} \eta_{20,4} \eta_{20,6} \eta_{20,8} \\
& \times \eta_{50,5} \eta_{50,10} \eta_{50,15} \eta_{50,20} \eta_{50,25} \eta_{200,50} \eta_{200,100}) - (\\
& \eta_{10}^8 \eta_{20}^3 \eta_{50} \eta_{50,5} \eta_{50,10} \eta_{50,15} \eta_{50,20} \eta_{100,50} \eta_{200,50} \eta_{200,100} \\
& \times (\eta_{10,1}^3 \eta_{20,6} \eta_{20,8}^2 \eta_{20,10} - 4\eta_{10,1} \eta_{10,3} \eta_{10,4} \eta_{20,2} \eta_{20,5}^2 \eta_{20,8} \\
& - \eta_{10,3}^3 \eta_{20,2} \eta_{20,4}^2 \eta_{20,10}) \\
& + \eta_{10}^8 \eta_{20}^3 \eta_{50} \eta_{50,5} \eta_{50,10} \eta_{50,20} \eta_{50,25} \eta_{100,30} \eta_{200,50} \eta_{200,100} \\
& \times (2\eta_{10,1}^3 \eta_{20,6} \eta_{20,8}^2 \eta_{20,10} + 2\eta_{10,1} \eta_{10,3} \eta_{10,4} \eta_{20,2} \eta_{20,5}^2 \eta_{20,8} \\
& + 3\eta_{10,3}^3 \eta_{20,2} \eta_{20,4}^2 \eta_{20,10}) \\
& - \eta_{10}^8 \eta_{20}^3 \eta_{50} \eta_{50,10} \eta_{50,15} \eta_{50,20} \eta_{50,25} \eta_{100,10} \eta_{200,50} \eta_{200,100} \\
& \times (3\eta_{10,1}^3 \eta_{20,6} \eta_{20,8}^2 \eta_{20,10} - 2\eta_{10,1} \eta_{10,3} \eta_{10,4} \eta_{20,2} \eta_{20,5}^2 \eta_{20,8} \\
& + 2\eta_{10,3}^3 \eta_{20,2} \eta_{20,4}^2 \eta_{20,10}) \cdot
\end{aligned}$$

$$\begin{aligned}
& q^{233/6} \eta_{10}^8 \eta_{20} \eta_{10,1} \eta_{10,3} \eta_{20,2} \eta_{20,4} \eta_{20,6} \eta_{20,8} \\
& \times \eta_{50,5} \eta_{50,10} \eta_{50,15} \eta_{50,20} \eta_{50,25} \eta_{200,50} \eta_{200,100})
\end{aligned}$$

$$= 14400 / \begin{aligned} & q^{233/6} \eta_{10}^8 \eta_{20} \eta_{10,1} \eta_{10,3} \eta_{20,2} \eta_{20,4} \eta_{20,6} \eta_{20,8} \\ & \times \eta_{50,5} \eta_{50,10} \eta_{50,15} \eta_{50,20} \eta_{50,25} \eta_{200,50} \eta_{200,100} \end{aligned} =$$

$$\begin{aligned}
& f(q^{25}, q^{25}) \left(\frac{f^2(-q, -q^9) f(-q^8, -q^{12}) (q^{10}; q^{10})_\infty (q^{20}; q^{20})_\infty}{f(-q^3, -q^7) f(-q^2, -q^{18}) f(-q^4, -q^{16})} \right. \\
& \quad \left. 4q f^2(q^5, q^{15}) \frac{f^2(-q^3, -q^7) f(-q^4, -q^{16}) (q^{10}; q^{10})_\infty (q^{20}; q^{20})_\infty}{f(-q, -q^9) f(-q^6, -q^{14}) f(-q^8, -q^{12})} \right) \\
& + q f(q^{15}, q^{35}) \left(2 \frac{f^2(-q, -q^9) f(-q^8, -q^{12}) (q^{10}; q^{10})_\infty (q^{20}; q^{20})_\infty}{f(-q^3, -q^7) f(-q^2, -q^{18}) f(-q^4, -q^{16})} \right. \\
& \quad \left. + 2q f^2(-q^5, -q^{15}) + 3q \frac{f^2(-q^3, -q^7) f(-q^4, -q^{16}) (q^{10}; q^{10})_\infty (q^{20}; q^{20})_\infty}{f(-q, -q^9) f(-q^6, -q^{14}) f(-q^8, -q^{12})} \right) \\
& - q^4 f(q^{25}, q^{25}) \left(3 \frac{f^2(-q, -q^9) f(-q^8, -q^{12}) (q^{10}; q^{10})_\infty (q^{20}; q^{20})_\infty}{f(-q^3, -q^7) f(-q^2, -q^{18}) f(-q^4, -q^{16})} \right. \\
& \quad \left. - 2q f^2(-q^5, -q^{15}) + 2q \frac{f^2(-q^3, -q^7) f(-q^4, -q^{16}) (q^{10}; q^{10})_\infty (q^{20}; q^{20})_\infty}{f(-q, -q^9) f(-q^6, -q^{14}) f(-q^8, -q^{12})} \right). \quad (e)
\end{aligned}$$

$$\begin{aligned}
& \eta_{10}^8 \eta_{20}^3 \eta_{50} \eta_{10,1} \eta_{20,4} \eta_{20,6} \eta_{20,8}^2 \eta_{50,5} \eta_{50,15} \eta_{50,25} \eta_{200,50} \eta_{200,100} \\
& \times (\eta_{10,1} \eta_{20,10} \eta_{50,10} \eta_{100,40} - \eta_{10,1} \eta_{20,10} \eta_{50,20} \eta_{100,20} \\
& - 2\eta_{10,3} \eta_{20,2} \eta_{50,20} \eta_{100,20}) + 4\eta_{10} \eta_{20}^{10} \eta_{50}^2 \eta_{100} \eta_{10,1} \eta_{10,3} \eta_{20,2} \eta_{20,4}^2 \eta_{20,6} \\
& \times \eta_{20,8}^2 \eta_{20,10}^3 \eta_{40,10} \eta_{50,5} \eta_{50,10} \eta_{50,15} \eta_{50,20} \eta_{100,25} \eta_{200,100}
\end{aligned}$$

$$\begin{aligned}
& q^{233/6} \eta_{10}^8 \eta_{20}^2 \eta_{50}^2 \eta_{10,1} \eta_{10,3} \eta_{20,2} \eta_{20,4} \eta_{20,6} \eta_{20,8} \\
& \times \eta_{50,5} \eta_{50,10} \eta_{50,15} \eta_{50,20} \eta_{50,25} \eta_{200,50} \eta_{200,100} - (\\
& \eta_{10}^8 \eta_{20}^3 \eta_{50}^3 \eta_{50,10} \eta_{50,20} \eta_{200,50} \eta_{200,100} \\
& \times (\eta_{10,1}^3 \eta_{20,6} \eta_{20,8}^2 \eta_{20,10} \eta_{50,5} \eta_{50,15} \eta_{100,50} \\
& + \eta_{10,3}^3 \eta_{20,2} \eta_{20,4}^2 \eta_{20,10} \eta_{50,5} \eta_{50,15} \eta_{100,50} \\
& - 2\eta_{10,1} \eta_{10,3} \eta_{20,2} \eta_{20,4} \eta_{20,5}^2 \eta_{20,6} \eta_{20,8} \eta_{50,5} \eta_{50,25} \eta_{100,30} \\
& \eta_{10,3}^3 \eta_{20,2} \eta_{20,4}^2 \eta_{20,10} \eta_{50,5} \eta_{50,25} \eta_{100,30} \\
& - \eta_{10,1}^3 \eta_{20,6} \eta_{20,8}^2 \eta_{20,10} \eta_{50,15} \eta_{50,25} \eta_{100,10} \\
& | 2\eta_{10,1} \eta_{10,3} \eta_{20,2} \eta_{20,4} \eta_{20,5}^2 \eta_{20,6} \eta_{20,8} \eta_{50,15} \eta_{50,25} \eta_{100,10}) / \times q^{233/6} \eta_{10}^8 \eta_{20}^2 \eta_{50}^2 \eta_{10,1} \eta_{10,3} \eta_{20,2} \eta_{20,4} \eta_{20,6} \eta_{20,8} \\
& \times \eta_{50,5} \eta_{50,10} \eta_{50,15} \eta_{50,20} \eta_{50,25} \eta_{200,50} \eta_{200,100}) = \\
& = 16800 / \times q^{233/6} \eta_{10}^8 \eta_{20}^2 \eta_{50}^2 \eta_{10,1} \eta_{10,3} \eta_{20,2} \eta_{20,4} \eta_{20,6} \eta_{20,8} \\
& \times \eta_{50,5} \eta_{50,10} \eta_{50,15} \eta_{50,20} \eta_{50,25} \eta_{200,50} \eta_{200,100} =
\end{aligned}$$

$$\begin{aligned}
& f(q^{25}, q^{25}) \left(\frac{f^2(-q, -q^9) f(-q^8, -q^{12}) (q^{10}; q^{10})_{\infty} (q^{20}; q^{20})_{\infty}}{f(-q^3, -q^7) f(-q^2, -q^{18}) f(-q^4, -q^{16})} \right. \\
& + q \frac{f^2(-q^3, -q^7) f(-q^4, -q^{16}) (q^{10}; q^{10})_{\infty} (q^{20}; q^{20})_{\infty}}{f(-q, -q^9) f(-q^6, -q^{14}) f(-q^8, -q^{12})} \left. \right) \\
& - q^2 f(q^{15}, q^{35}) \left(2f^2(-q^5, -q^{15}) \right. \\
& + 3q \frac{f^2(-q^3, -q^7) f(-q^4, -q^{16}) (q^{10}; q^{10})_{\infty} (q^{20}; q^{20})_{\infty}}{f(-q, -q^9) f(-q^6, -q^{14}) f(-q^8, -q^{12})} \left. \right) \\
& - q^4 f(q^5, q^{45}) \left(\frac{f^2(-q, -q^9) f(-q^8, -q^{12}) (q^{10}; q^{10})_{\infty} (q^{20}; q^{20})_{\infty}}{f(-q^3, -q^7) f(-q^2, -q^{18}) f(-q^4, -q^{16})} \right. \\
& \left. - 2q f^2(-q^5, -q^{15}) \right). \tag{12.2.43} \tag{f}
\end{aligned}$$

We have:

$$(F_1; z) = \frac{9 \cdot 57600}{12} = 43200$$

$$(F_2; z) = \frac{9 \cdot 57600}{12} = 43200$$

$$(F_3; z) = \frac{2 \cdot 57600}{12} = 9600$$

$$(F_4; z) = \frac{2 \cdot 57600}{12} = 9600$$

$$(F_5; z) = \frac{6 \cdot 28800}{12} = 14400$$

$$(F_6; z) = \frac{7 \cdot 28800}{12} = 16800$$

And obtain:

$$(((1/15 * \text{sqrt}(12 * (4800*9))))^{1/8}$$

Input:

$$\sqrt[8]{\frac{1}{15} \sqrt{12 (4800 \times 9)}}$$

[Open code](#)

Enlarge Data Customize A [Plaintext](#) [Interactive](#)

Result:

- Approximate form
- Step-by-step solution

$$\sqrt{2} \sqrt[8]{3}$$

Decimal approximation:

- More digits

1.622389603610977569320981470004102150530979016760707997198...

1.622389603610977569320981470004102150530979016760707997198

This value 1,62238... is in the range of the golden ratio value. It can be defined a golden number. We define golden numbers those in the range that goes from 1.6 to 1,675, i.e. an interval of 1.61803398

We note that:

$$(((1/15 * \text{sqrt}(12 * (4800*9))))^{1/8} * (1.7848+0.88137))$$

Input interpretation:

$$\sqrt[8]{\frac{1}{15} \sqrt{12 (4800 \times 9)}} (1.7848 + 0.88137)$$

[Open code](#)

Enlarge Data Customize A [Plaintext](#) [Interactive](#)

Result:

- Fewer digits
- More digits

4.325566489459480065996521165880837030681180325116896840890...

This result 4,32556...is a good approximation to the value of Cosmological Constant $4.33 \times 10^{-66} \text{ eV}^2$ in natural units

$$\left(\left(\left(\frac{1}{12} * \text{sqrt}(12 * (4800*2))\right)\right)\right)^{1/7}$$

Input:

$$\sqrt[7]{\frac{1}{12} \sqrt{12 (4800 \times 2)}}$$

[Open code](#)

Enlarge Data Customize A [Plaintext](#) [Interactive](#)

Result:

- Approximate form
- Step-by-step solution

$$2^{5/14} \sqrt[7]{5}$$

Decimal approximation:

- More digits

1.611994554959420107391315321121497878691528310633308658891...

[Open code](#)

1.611994554959420107391315321121497878691528310633308658891

This value 1,61199... is in the range of the golden ratio value. It can be defined a golden number. We define golden numbers those in the range that goes from 1.6 to 1,675, i.e. an interval of 1.61803398

Note that:

$$\left(\left(\left(\frac{1}{12} * \text{sqrt}(12 * (4800*2))\right)\right)\right)^{1/7} * ((2.7268-0.69897)*(1.328))$$

Input interpretation:

$$\sqrt[7]{\frac{1}{12} \sqrt{12 (4800 \times 2)}} \quad ((2.7268 - 0.69897) \times 1.328)$$

[Open code](#)

Enlarge Data Customize A [Plaintext](#) [Interactive](#)

Result:

- Fewer digits
- More digits

4.341034019613103243821127498452383753564871582313248171425...

This result 4,3410...is a good approximation to the value of Cosmological Constant $4.33 \times 10^{-66} \text{ eV}^2$ in natural units

$$\left(\left(\left(\frac{1}{12} * \text{sqrt}(12 * (4800*3))\right)\right)\right)^{1/7}$$

Input:

$$\sqrt[7]{\frac{1}{12} \sqrt{12(4800 \times 3)}}$$

[Open code](#)

Enlarge Data Customize A [Plaintext](#) [Interactive](#)

Result:

- Approximate form
- Step-by-step solution

$$2^{2/7} \sqrt[14]{3} \sqrt[7]{5}$$

Decimal approximation:

- More digits

1.659363441249059998468894975666886829865439502419041316767...

1.659363441249059998468894975666886829865439502419041316767

where 1,6593634... is very near to the 14th root of the following Ramanujan's class invariant $Q = (G_{505}/G_{101/5})^3 = 1164,2696$ i.e. 1,65578...

Note that

$$(((1/12 * \text{sqrt}(12 * (4800*3))))^{1/7} * 2 * 1.3057$$

where 1.3057 is a Hausdorff dimension

Input interpretation:

$$\sqrt[7]{\frac{1}{12} \sqrt{12(4800 \times 3)}} \times 2 \times 1.3057$$

[Open code](#)

Enlarge Data Customize A [Plaintext](#) [Interactive](#)

Result:

- Fewer digits
- More digits

4.333261690477795280001672339456508267510608716617084494607...

This result 4,33326...is practically equal to the value of Cosmological Constant $4.33 \times 10^{-66} \text{ eV}^2$ in natural units.

$$(((1/10 * \text{sqrt}(12 * (2400*7))))^{1/8}$$

Input:

$$\sqrt[8]{\frac{1}{10} \sqrt{12(2400 \times 7)}}$$

[Open code](#)

Enlarge Data Customize A Plaintext Interactive

Result:

$$2^{5/16} \sqrt[8]{3} \sqrt[16]{7}$$

Decimal approximation:

More digits

1.608905950768348836938596240641201705808003330039853405286...

[Open code](#)

1.608905950768348836938596240641201705808003330039853405286

This value 1,6089059... is in the range of the golden ratio value. It can be defined a golden number. We define golden numbers those in the range that goes from 1.6 to 1,675, thus an interval of 1.61803398

Note that:

$$(((1/10 * \text{sqrt}(12 * (2400*7))))^{1/8} * ((2.7268-0.69897)*(1.328)))$$

Where 0.69897, 1.328 and 2.7268 are Hausdorff dimensions

Input interpretation:

$$\sqrt[8]{\frac{1}{10} \sqrt{12 (2400 \times 7)}} ((2.7268 - 0.69897) \times 1.328)$$

[Open code](#)

Result:

Fewer digits

More digits

4.332716537506659331628209120267747017157718425578262756158...

4.3327165375066593316282091202677470171577184255782627

Continued fraction:

Linear form

$$\begin{array}{r}
4 + \frac{1}{\dots} \\
3 + \frac{1}{\dots} \\
179 + \frac{1}{\dots} \\
1 + \frac{1}{\dots} \\
4 + \frac{1}{\dots} \\
4 + \frac{1}{\dots} \\
5 + \frac{1}{\dots} \\
3 + \frac{1}{\dots} \\
14 + \frac{1}{\dots} \\
3 + \frac{1}{\dots} \\
1 + \frac{1}{\dots} \\
7 + \frac{1}{\dots} \\
5 + \frac{1}{\dots} \\
2 + \frac{1}{\dots} \\
3 + \frac{1}{\dots} \\
1 + \frac{1}{\dots} \\
2 + \frac{1}{\dots} \\
9 + \frac{1}{\dots} \\
63 + \frac{1}{\dots} \\
3 + \frac{1}{\dots} \\
4 + \frac{1}{\dots} \\
\dots
\end{array}$$

Possible closed forms:

- More

$$\frac{355\,822\,487\pi}{258\,001\,949} \approx 4.33271653750665933118808$$

Enlarge Data Customize A Plaintext Interactive

$$\begin{array}{l}
\text{root of } 40x^5 + 44x^4 - 785x^3 - 385x^2 - 1132x - 600 \text{ near } x = 4.33272 \approx \\
4.33271653750665933150753 \\
\frac{333\,548\pi^2 - 76\,103}{236\,260\pi} \approx 4.33271653750665933135824
\end{array}$$

This result 4,3327... is practically equal to the value of Cosmological Constant $4.33 \times 10^{-66} \text{ eV}^2$ in natural units.

Now, we have:

$$16 * (((((2400*18))/900))))^{1/8}$$

Input:

$$16 \sqrt[8]{\frac{2400 \times 18}{900}}$$

[Open code](#)

Enlarge Data Customize A Plaintext Interactive

Result:

- Approximate form
- Step-by-step solution

$$16 \sqrt{2} \sqrt[8]{3}$$

Decimal approximation:

- More digits

$$\begin{array}{l}
25.95823365777564110913570352006563440849566426817132795517\dots \\
25.95823365777564110913570352006563440849566426817132795517
\end{array}$$

$$16 * (((4800*3))/300))^{1/8}$$

Input:

$$16 \sqrt[8]{\frac{4800 \times 3}{300}}$$

[Open code](#)

Enlarge Data Customize A Plaintext Interactive

Result:

- Approximate form
- Step-by-step solution

$$16 \sqrt{2} \sqrt[8]{3}$$

Decimal approximation:

- More digits
25.95823365777564110913570352006563440849566426817132795517...

$$16 * (((2400*7))/350))^{1/8}$$

Input:

$$16 \sqrt[8]{\frac{2400 \times 7}{350}}$$

[Open code](#)

Enlarge Data Customize A Plaintext Interactive

Result:

- Approximate form
- Step-by-step solution

$$16 \sqrt{2} \sqrt[8]{3}$$

Decimal approximation:

- More digits
25.95823365777564110913570352006563440849566426817132795517...

[Open code](#)

$$16 * (((2400*4))/200))^{1/8}$$

Input:

$$16 \sqrt[8]{\frac{2400 \times 4}{200}}$$

[Open code](#)

Enlarge Data Customize A Plaintext Interactive

Result:

- Approximate form
- Step-by-step solution

$$16 \sqrt{2} \sqrt[8]{3}$$

Decimal approximation:

- More digits
25.95823365777564110913570352006563440849566426817132795517...

[Open code](#)

Enlarge Data Customize A Plaintext Interactive

Result:

- Approximate form
- Step-by-step solution

$$1056 \sqrt{2} \sqrt[8]{3}$$

Decimal approximation:

- More digits

1713.243421413192313202956432324331870960713841699307645041...

[Open code](#)

1713.243421413192313202956432324331870960713841699307645041

$$(27*2+12) * 16 * (((4800*3))/300))^{1/8}$$

Input:

$$(27 \times 2 + 12) \times 16 \sqrt[8]{\frac{4800 \times 3}{300}}$$

[Open code](#)

Enlarge Data Customize A Plaintext Interactive

Result:

- Approximate form
- Step-by-step solution

$$1056 \sqrt{2} \sqrt[8]{3}$$

Decimal approximation:

- More digits

1713.243421413192313202956432324331870960713841699307645041...

$$(27*2+12) * 16 * (((2400*7))/350))^{1/8}$$

Input:

$$(27 \times 2 + 12) \times 16 \sqrt[8]{\frac{2400 \times 7}{350}}$$

[Open code](#)

Enlarge Data Customize A Plaintext Interactive

Result:

- Approximate form
- Step-by-step solution

$$1056 \sqrt{2} \sqrt[8]{3}$$

Decimal approximation:

- More digits

1713.243421413192313202956432324331870960713841699307645041...

[Open code](#)

$$(27*2+12) * 16 * (((2400*4))/200))^{1/8}$$

Input:

$$(27 \times 2 + 12) \times 16 \sqrt[8]{\frac{2400 \times 4}{200}}$$

[Open code](#)

Enlarge Data Customize A Plaintext Interactive

Result:

- Approximate form
- Step-by-step solution

$$1056\sqrt{2}\sqrt[8]{3}$$

Decimal approximation:

- More digits

1713.243421413192313202956432324331870960713841699307645041...

[Open code](#)

This result is very near to the mass of candidate glueball $f_0(1710)$ meson.

Continued fraction:

Linear form

$$1713 + \frac{1}{4 + \frac{1}{9 + \frac{1}{3 + \frac{1}{1 + \frac{1}{119 + \frac{1}{3 + \frac{1}{2 + \frac{1}{663 + \frac{1}{3 + \frac{1}{2 + \frac{1}{5 + \frac{1}{20 + \frac{1}{11 + \frac{1}{3 + \frac{1}{1 + \frac{1}{5 + \frac{1}{2 + \frac{1}{10 + \frac{1}{\dots}}}}}}}}}}}}}}}}}}}}}}}}}}$$

Possible closed forms:

More

$$1056\sqrt{2}\sqrt[8]{3} \approx$$

1713.24342141319231320295643232433187096071384169930764504164995

Enlarge Data Customize A Plaintext Interactive

$$\frac{1091422\pi}{979} - \frac{5502619}{979\pi} \approx 1713.243421413192313261385$$

$\pi \sqrt[3]{\text{root of } 4x^5 - 2180x^4 - 744x^3 - 1486x^2 + 558x + 2287 \text{ near } x = 545.342} \approx$

1713.243421413192313222933

Now:

$(1713.2434214131923132029564323243318709607138416993076)^{1/3}$

Input interpretation:

$$\sqrt[3]{1713.2434214131923132029564323243318709607138416993076}$$

[Open code](#)

Enlarge Data Customize A [Plaintext](#) [Interactive](#)

Result:

More digits

11.965743554321136571146417712828078608206842272098138...

This result 11,9657 is very near to the values of black hole entropies 11,8458 and 12,1904

$$(2\pi)/((8(\sqrt{5}))) * (1713.2434214131923132029564323243318709607138416993076)^{1/3}$$

Input interpretation:

$$\frac{2\pi}{8\sqrt{5}} \sqrt[3]{1713.2434214131923132029564323243318709607138416993076}$$

[Open code](#)

Enlarge Data Customize A [Plaintext](#) [Interactive](#)

Result:

More digits

4.2028565794125370233026911521072716605184884943405582...

This result 4,2028 is in the range of the mass of DM particle that is between 4 – 4.2 eV

$$(1713.2434214131923132029564323243318709607138416993076)^{1/15}$$

Input interpretation:

$$\sqrt[15]{1713.2434214131923132029564323243318709607138416993076}$$

[Open code](#)

Enlarge Data Customize A [Plaintext](#) [Interactive](#)

Result:

More digits

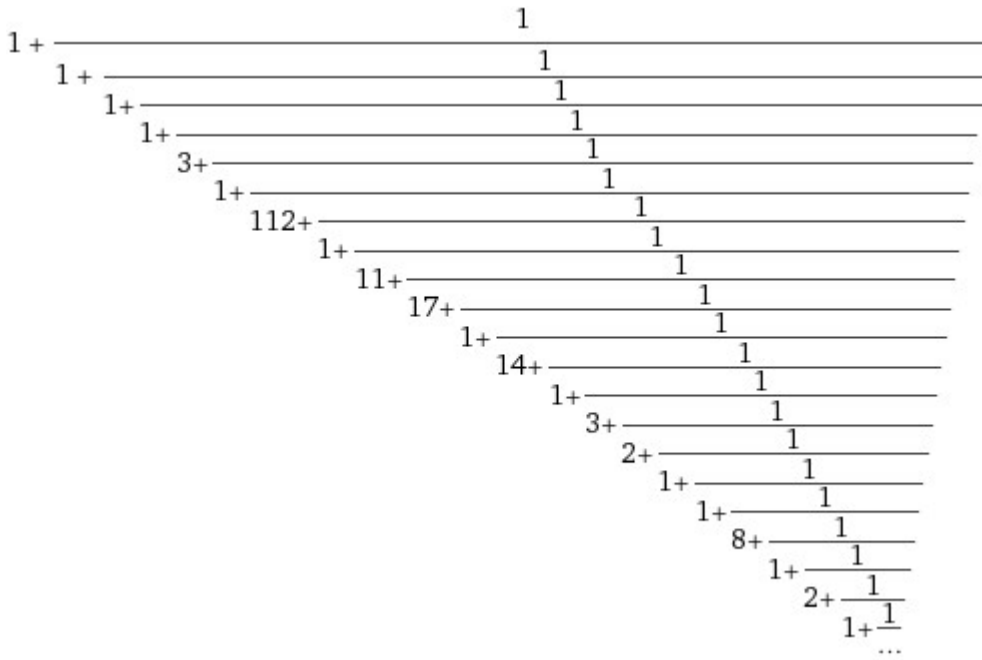
1.64281227111847641604539420823322219271934307659689796...

1.64281227111847641604539420823322219271934307659689796

$$1.642812... \approx \zeta(2) = \frac{\pi^2}{6} = 1.6449$$

Continued fraction:

Linear form



Possible closed forms:

More

$$\frac{1}{744} (-120 e^\pi + 846 \pi - 10 \log(\pi) + 47 \log(2 \pi) + 1003 \tan^{-1}(\pi)) \approx 1.642812271118476416038276$$

Enlarge Data Customize A [Plaintext](#) [Interactive](#)

$$\frac{695760768\pi}{1330521421} \approx 1.64281227111847641613729$$

root of $30x^5 - 184x^4 + 147x^3 + 920x^2 - 1155x - 256$ near $x = 1.64281$	\approx	1.64281227111847641622559
---	-----------	---------------------------

Now:

$$1.64281227111847641604539420823322219271934307659689796 * (1.3057 + 1.328)$$

Where 1,3057 and 1,328 are a Hausdorff dimensions

Input interpretation:

$$1.64281227111847641604539420823322219271934307659689796 (1.3057 + 1.328)$$

[Open code](#)

Enlarge Data Customize A [Plaintext](#) [Interactive](#)

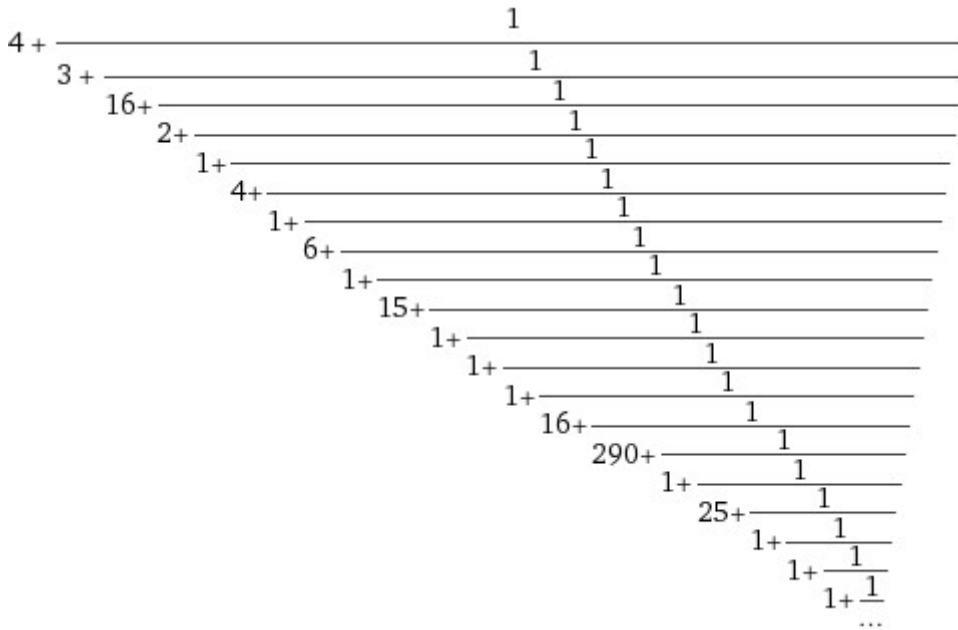
Result:

More digits

$$4.326674678444731336938754726223837288964933860833250157252... \\ 4.326674678444731336938754726223837288964933860833250157252$$

Continued fraction:

Linear form



Possible closed forms:

More

$$\frac{680\,299\,059\pi}{493\,964\,230} \approx 4.32667467844473133637773$$

Enlarge Data Customize A Plaintext Interactive

$$\sqrt[4]{\text{root of } 7x^4 - 1904x^3 + 6234x^2 + 7790x + 1357 \text{ near } x = 4.32667} \approx 4.3266746784447313371020$$

$$\sqrt[4]{\text{root of } 1357x^4 + 7790x^3 + 6234x^2 - 1904x + 7 \text{ near } x = 0.231124} \approx 4.3266746784447313371020$$

Note that:

$$(4.32667 * 493964230)^{1/4} * 8$$

Input interpretation:

$$\sqrt[4]{4.32667 \times 493964230 \times 8}$$

[Open code](#)

Enlarge Data Customize A Plaintext Interactive

Result:

More digits

1720.094...
 1720.094...

This result is very near to the mass of candidate glueball $f_0(1710)$ meson.

From this result, we can to obtain a good approximation to Pi:

$$\ln(1720.094) * 1/(0.538*2.3296*1.8928)$$

where 0.538, 1.8928 and 2.3296 are Hausdorff dimensions

Input interpretation:

$$\log(1720.094) \times \frac{1}{0.538 \times 2.3296 \times 1.8928}$$

[Open code](#)

- $\log(x)$ is the natural logarithm

Enlarge Data Customize A [Plaintext](#) [Interactive](#)

Result:

- Fewer digits
- More digits

3.140477862417781310014896368569899207372512819308474946284...

Alternative representations:

More

$$\frac{\log(1720.09)}{0.538 \times 2.3296 \times 1.8928} = \frac{\log_e(1720.09)}{2.37229}$$

[Open code](#)

Enlarge Data Customize A [Plaintext](#) [Interactive](#)

$$\frac{\log(1720.09)}{0.538 \times 2.3296 \times 1.8928} = \frac{\log(a) \log_a(1720.09)}{2.37229}$$

[Open code](#)

$$\frac{\log(1720.09)}{0.538 \times 2.3296 \times 1.8928} = -\frac{\text{Li}_1(-1719.09)}{2.37229}$$

Series representations:

More

$$\frac{\log(1720.09)}{0.538 \times 2.3296 \times 1.8928} = 0.421533 \log(1719.09) - 0.421533 \sum_{k=1}^{\infty} \frac{(-1)^k e^{-7.44955 k}}{k}$$

[Open code](#)

Enlarge Data Customize A [Plaintext](#) [Interactive](#)

$$\frac{\log(1720.09)}{0.538 \times 2.3296 \times 1.8928} = 0.843066 i \pi \left[\frac{\arg(1720.09 - x)}{2 \pi} \right]_+ + 0.421533 \log(x) - 0.421533 \sum_{k=1}^{\infty} \frac{(-1)^k (1720.09 - x)^k x^{-k}}{k} \text{ for } x < 0$$

[Open code](#)

$$\frac{\log(1720.09)}{0.538 \times 2.3296 \times 1.8928} = 0.421533 \left[\frac{\arg(1720.09 - z_0)}{2\pi} \right] \log\left(\frac{1}{z_0}\right) + 0.421533 \log(z_0) + 0.421533 \left[\frac{\arg(1720.09 - z_0)}{2\pi} \right] \log(z_0) - 0.421533 \sum_{k=1}^{\infty} \frac{(-1)^k (1720.09 - z_0)^k z_0^{-k}}{k}$$

Open code

- $\arg(z)$ is the complex argument
 - $\lfloor x \rfloor$ is the floor function
 - i is the imaginary unit
 - [More information](#)

Integral representations:

$$\frac{\log(1720.09)}{0.538 \times 2.3296 \times 1.8928} = 0.421533 \int_1^{1720.09} \frac{1}{t} dt$$

Open code

$$\frac{\log(1720.09)}{0.538 \times 2.3296 \times 1.8928} = \frac{0.210767}{i\pi} \int_{-i\infty+\gamma}^{i\infty+\gamma} \frac{e^{-7.44955s} \Gamma(-s)^2 \Gamma(1+s)}{\Gamma(1-s)} ds$$

for $-1 < \gamma < 0$

And:

$$2 * \ln(1720.094) * 1/(0.538*2.3296*1.8928)$$

Input interpretation:

$$2 \log(1720.094) \times \frac{1}{0.538 \times 2.3296 \times 1.8928}$$

Open code

- $\log(x)$ is the natural logarithm

Result:

- Fewer digits
- More digits

6.280955724835562620029792737139798414745025638616949892569...

6.2809557... $\approx 2\pi$ that is the length of a circle of radius equal to 1

Alternative representations:

More

$$\frac{2 \log(1720.09)}{0.538 \times 2.3296 \times 1.8928} = \frac{2 \log_e(1720.09)}{2.37229}$$

Open code

Enlarge Data Customize A Plaintext Interactive

$$\frac{2 \log(1720.09)}{0.538 \times 2.3296 \times 1.8928} = \frac{2 \log(a) \log_a(1720.09)}{2.37229}$$

Open code

$$\frac{2 \log(1720.09)}{0.538 \times 2.3296 \times 1.8928} = -\frac{2 \operatorname{Li}_1(-1719.09)}{2.37229}$$

Series representations:

More

$$\frac{2 \log(1720.09)}{0.538 \times 2.3296 \times 1.8928} = 0.843066 \log(1719.09) - 0.843066 \sum_{k=1}^{\infty} \frac{(-1)^k e^{-7.44955 k}}{k}$$

Open code

$$\frac{2 \log(1720.09)}{0.538 \times 2.3296 \times 1.8928} = 1.68613 i \pi \left\lfloor \frac{\arg(1720.09 - x)}{2 \pi} \right\rfloor + 0.843066 \log(x) - 0.843066 \sum_{k=1}^{\infty} \frac{(-1)^k (1720.09 - x)^k x^{-k}}{k} \quad \text{for } x < 0$$

Open code

$$\frac{2 \log(1720.09)}{0.538 \times 2.3296 \times 1.8928} = 0.843066 \left\lfloor \frac{\arg(1720.09 - z_0)}{2 \pi} \right\rfloor \log\left(\frac{1}{z_0}\right) + 0.843066 \log(z_0) + 0.843066 \left\lfloor \frac{\arg(1720.09 - z_0)}{2 \pi} \right\rfloor \log(z_0) - 0.843066 \sum_{k=1}^{\infty} \frac{(-1)^k (1720.09 - z_0)^k z_0^{-k}}{k}$$

Open code

- $\arg(z)$ is the complex argument
 - $\lfloor x \rfloor$ is the floor function
 - i is the imaginary unit
 - [More information](#)

Integral representations:

$$\frac{2 \log(1720.09)}{0.538 \times 2.3296 \times 1.8928} = 0.843066 \int_1^{1720.09} \frac{1}{t} dt$$

Open code

Enlarge Data Customize A [Plaintext](#) [Interactive](#)

$$\frac{2 \log(1720.09)}{0.538 \times 2.3296 \times 1.8928} = \frac{0.421533}{i \pi} \int_{-i \infty + \gamma}^{i \infty + \gamma} \frac{e^{-7.44955 s} \Gamma(-s)^2 \Gamma(1+s)}{\Gamma(1-s)} ds$$

for $-1 < \gamma < 0$

And from the closed form:

$$\frac{680299059 \pi}{493964230} \approx 4.32667467844473133637773$$

We obtain:

$$(((4.32667 * 493964230)^{1/4} * 8))^{1/3}$$

Input interpretation:

$$\sqrt[3]{\sqrt[4]{4.32667 \times 493964230} \times 8}$$

[Open code](#)

Enlarge Data Customize A [Plaintext](#) [Interactive](#)

Result:

More digits

11.98167...

This result 11,98 is very near to the values of black hole entropies 11,8458 and 12,1904

$$2 * (((4.32667 * 493964230)^{1/4} * 8))^{1/3}$$

Input interpretation:

$$2 \sqrt[3]{\sqrt[4]{4.32667 \times 493964230} \times 8}$$

[Open code](#)

Enlarge Data Customize A [Plaintext](#) [Interactive](#)

Result:

More digits

23.96334...

23.963344472599296136826318934698537192209073618745205

Continued fraction:

Linear form

$$\begin{aligned}
& q^{451/30} \eta_{50}^{18} \eta_{10,2}^4 \eta_{10,3} \eta_{20,2} \eta_{20,4} \eta_{50,10} \eta_{50,15} \eta_{50,20}^2 \eta_{50,25} / \eta_{100}^2 = \\
& = 43200 / q^{451/30} \eta_{50}^{18} \eta_{10,2}^4 \eta_{10,3} \eta_{20,2} \eta_{20,4} \eta_{50,10} \eta_{50,15} \eta_{50,20}^2 \eta_{50,25} / \eta_{100}^2 \\
& = \frac{f^2(-q, -q^9) f(-q^8, -q^{12}) (q^{10}; q^{10})_{\infty} (q^{20}; q^{20})_{\infty}}{f(-q^3, -q^7) f(-q^2, -q^{18}) f(-q^4, -q^{16})} \quad (a)
\end{aligned}$$

$$((2400*18))/900 = 48$$

Input:

$$\frac{2400 \times 18}{900}$$
[Open code](#)

Enlarge Data Customize A Plaintext Interactive

Result:

• Step-by-step solution

48

Thence:

$$\left(q^{451/30} \eta_{50}^{18} \eta_{10,2}^4 \eta_{10,3} \eta_{20,2} \eta_{20,4} \eta_{50,10} \eta_{50,15} \eta_{50,20}^2 \eta_{50,25} / \eta_{100}^2 \right) = 900 \quad \text{and}$$

$$\left(\frac{f^2(-q, -q^9) f(-q^8, -q^{12}) (q^{10}; q^{10})_{\infty} (q^{20}; q^{20})_{\infty}}{f(-q^3, -q^7) f(-q^2, -q^{18}) f(-q^4, -q^{16})} \right) = 48$$

The expression is:

$$\begin{aligned}
& 43200 / \left(q^{451/30} \eta_{50}^{18} \eta_{10,2}^4 \eta_{10,3} \eta_{20,2} \eta_{20,4} \eta_{50,10} \eta_{50,15} \eta_{50,20}^2 \eta_{50,25} / \eta_{100}^2 \right) = \\
& = \left(\frac{f^2(-q, -q^9) f(-q^8, -q^{12}) (q^{10}; q^{10})_{\infty} (q^{20}; q^{20})_{\infty}}{f(-q^3, -q^7) f(-q^2, -q^{18}) f(-q^4, -q^{16})} \right) = 48
\end{aligned}$$

Note that $2400 * 18 = 43200$; $43200 / 900 = 48$ and $43200 / 1200 = 36$.

Thence:

$$(((2400*18))/1200))*48$$

Input:

$$\frac{2400 \times 18}{1200} \times 48$$

Open code

Enlarge Data Customize A Plaintext Interactive

Result:
 Step-by-step solution
 1728

That is: $36 * 48 = 1728$.

Now:

$$[(36*48)]^{1/3}$$

Input:

$$\sqrt[3]{36 \times 48}$$

Open code

Enlarge Data Customize A Plaintext Interactive

Exact result:
 12

$$2 * [(36*48)]^{1/3}$$

Input:

$$2 \sqrt[3]{36 \times 48}$$

Open code

Enlarge Data Customize A Plaintext Interactive

Exact result:
 Step-by-step solution
 24

And that:

$$1 / [(36*48)]^{1/3}$$

Input:

$$\frac{1}{\sqrt[3]{36 \times 48}}$$

Open code

Enlarge Data Customize A Plaintext Interactive

Exact result:
 Step-by-step solution

$$36 * \left(\frac{f^2(-q, -q^9)f(-q^8, -q^{12})(q^{10}; q^{10})_{\infty}(q^{20}; q^{20})_{\infty}}{f(-q^3, -q^7)f(-q^2, -q^{18})f(-q^4, -q^{16})} \right) = 1728$$

$$2 \times \sqrt[3]{36 * \left(\frac{f^2(-q, -q^9)f(-q^8, -q^{12})(q^{10}; q^{10})_{\infty}(q^{20}; q^{20})_{\infty}}{f(-q^3, -q^7)f(-q^2, -q^{18})f(-q^4, -q^{16})} \right)} = 24$$

$$\sqrt[15]{36 * \left(\frac{f^2(-q, -q^9)f(-q^8, -q^{12})(q^{10}; q^{10})_{\infty}(q^{20}; q^{20})_{\infty}}{f(-q^3, -q^7)f(-q^2, -q^{18})f(-q^4, -q^{16})} \right)} = 1.6437518 \dots$$

1.643751829517225762308497936230979517383492589945475200411

We note that:

The result 1728 is very near to the mass of candidate glueball $f_0(1710)$ meson. Furthermore, 1728 occurs in the algebraic formula for the j -invariant of an elliptic curve. As a consequence, it is sometimes called a Zagier as a pun on the Gross–Zagier theorem. The number 1728 is one less than the Hardy–Ramanujan number 1729

The value 24 is linked to the "Ramanujan function" (an elliptic modular function that satisfies the need for "conformal symmetry") that has 24 "modes" corresponding to the physical vibrations of a bosonic string.

$$1.64375 \approx \zeta(2) = \frac{\pi^2}{6} = 1.6449$$

Continued fraction:
Linear form

Alternate forms:

$$\frac{3^{3/16}}{2^{5/8}} + \frac{\sqrt[5]{3}}{2^{3/5}}$$

[Open code](#)

$$\frac{3^{3/16} \left(1 + \sqrt[40]{2} \sqrt[80]{3}\right)}{2^{5/8}}$$

```
root of 1 125 899 906 842 624 x80 - 6 755 399 441 055 744 x75 +
18 999 560 927 969 280 x70 - 33 249 231 623 946 240 x65 -
148 434 069 749 760 x64 + 40 522 501 041 684 480 x60 -
862 105 077 106 606 080 x59 - 36 470 250 937 516 032 x55 -
63 055 126 806 354 984 960 x54 + 25 073 297 519 542 272 x50 -
819 369 563 241 893 068 800 x49 + 7 827 577 896 960 x48 -
13 432 123 671 183 360 x45 - 3 374 567 517 567 791 923 200 x44 -
162 923 206 347 325 440 x43 + 5 666 677 173 780 480 x40 -
5 580 889 105 880 926 126 080 x39 + 32 199 776 876 321 832 960 x38 -
1 888 892 391 260 160 x35 - 4 140 497 702 729 117 859 840 x34 -
540 848 566 163 252 183 040 x33 - 206 391 214 080 x32 +
495 834 252 705 792 x30 - 1 440 211 855 454 219 796 480 x29 +
1 502 565 913 250 900 213 760 x28 - 3 051 287 708 958 720 x27 -
101 420 642 598 912 x25 - 233 772 701 611 504 435 200 x24 -
883 163 826 121 628 712 960 x23 - 224 679 926 543 155 200 x22 +
15 846 975 406 080 x20 - 16 755 940 501 114 060 800 x19 +
115 227 272 119 319 101 440 x18 - 587 549 251 716 710 400 x17 +
2 720 977 920 x16 - 1 828 497 162 240 x15 - 466 134 667 451 228 160 x14 -
2 969 021 418 242 088 960 x13 - 97 492 325 961 139 200 x12 -
4 456 961 832 960 x11 + 146 932 807 680 x10 - 3 812 563 516 078 080 x9 +
10 475 755 128 455 040 x8 - 797 636 990 891 520 x7 + 3 064 161 260 160 x6 -
7 346 640 384 x5 - 4 449 309 082 560 x4 - 1 717 277 189 760 x3 -
112 495 430 880 x2 - 2 295 825 120 x + 157 837 977 near x = 1.61862
```

Minimal polynomial:

$$\begin{aligned}
& 1\ 125\ 899\ 906\ 842\ 624\ x^{80} - 6\ 755\ 399\ 441\ 055\ 744\ x^{75} + \\
& 18\ 999\ 560\ 927\ 969\ 280\ x^{70} - 33\ 249\ 231\ 623\ 946\ 240\ x^{65} - \\
& 148\ 434\ 069\ 749\ 760\ x^{64} + 40\ 522\ 501\ 041\ 684\ 480\ x^{60} - \\
& 862\ 105\ 077\ 106\ 606\ 080\ x^{59} - 36\ 470\ 250\ 937\ 516\ 032\ x^{55} - \\
& 63\ 055\ 126\ 806\ 354\ 984\ 960\ x^{54} + 25\ 073\ 297\ 519\ 542\ 272\ x^{50} - \\
& 819\ 369\ 563\ 241\ 893\ 068\ 800\ x^{49} + 7\ 827\ 577\ 896\ 960\ x^{48} - \\
& 13\ 432\ 123\ 671\ 183\ 360\ x^{45} - 3\ 374\ 567\ 517\ 567\ 791\ 923\ 200\ x^{44} - \\
& 162\ 923\ 206\ 347\ 325\ 440\ x^{43} + 5\ 666\ 677\ 173\ 780\ 480\ x^{40} - \\
& 5\ 580\ 889\ 105\ 880\ 926\ 126\ 080\ x^{39} + 32\ 199\ 776\ 876\ 321\ 832\ 960\ x^{38} - \\
& 1\ 888\ 892\ 391\ 260\ 160\ x^{35} - 4\ 140\ 497\ 702\ 729\ 117\ 859\ 840\ x^{34} - \\
& 540\ 848\ 566\ 163\ 252\ 183\ 040\ x^{33} - 206\ 391\ 214\ 080\ x^{32} + 495\ 834\ 252\ 705\ 792\ x^{30} - \\
& 1\ 440\ 211\ 855\ 454\ 219\ 796\ 480\ x^{29} + 1\ 502\ 565\ 913\ 250\ 900\ 213\ 760\ x^{28} - \\
& 3\ 051\ 287\ 708\ 958\ 720\ x^{27} - 101\ 420\ 642\ 598\ 912\ x^{25} - \\
& 233\ 772\ 701\ 611\ 504\ 435\ 200\ x^{24} - 883\ 163\ 826\ 121\ 628\ 712\ 960\ x^{23} - \\
& 224\ 679\ 926\ 543\ 155\ 200\ x^{22} + 15\ 846\ 975\ 406\ 080\ x^{20} - \\
& 16\ 755\ 940\ 501\ 114\ 060\ 800\ x^{19} + 115\ 227\ 272\ 119\ 319\ 101\ 440\ x^{18} - \\
& 587\ 549\ 251\ 716\ 710\ 400\ x^{17} + 2\ 720\ 977\ 920\ x^{16} - 1\ 828\ 497\ 162\ 240\ x^{15} - \\
& 466\ 134\ 667\ 451\ 228\ 160\ x^{14} - 2\ 969\ 021\ 418\ 242\ 088\ 960\ x^{13} - \\
& 97\ 492\ 325\ 961\ 139\ 200\ x^{12} - 4\ 456\ 961\ 832\ 960\ x^{11} + 146\ 932\ 807\ 680\ x^{10} - \\
& 3\ 812\ 563\ 516\ 078\ 080\ x^9 + 10\ 475\ 755\ 128\ 455\ 040\ x^8 - 797\ 636\ 990\ 891\ 520\ x^7 + \\
& 3\ 064\ 161\ 260\ 160\ x^6 - 7\ 346\ 640\ 384\ x^5 - 4\ 449\ 309\ 082\ 560\ x^4 - \\
& 1\ 717\ 277\ 189\ 760\ x^3 - 112\ 495\ 430\ 880\ x^2 - 2\ 295\ 825\ 120\ x + 157\ 837\ 977
\end{aligned}$$

This result 1.618615670181102435516227417163352011810811958816290019893 is a very good approximation to the golden ratio!

Note that;

$$(1.2683+1.3057) * (36*48)^{1/15}$$

Input interpretation:

$$(1.2683 + 1.3057) \sqrt[15]{36 \times 48}$$

[Open code](#)

• [Units »](#)

Enlarge Data Customize A Plaintext Interactive

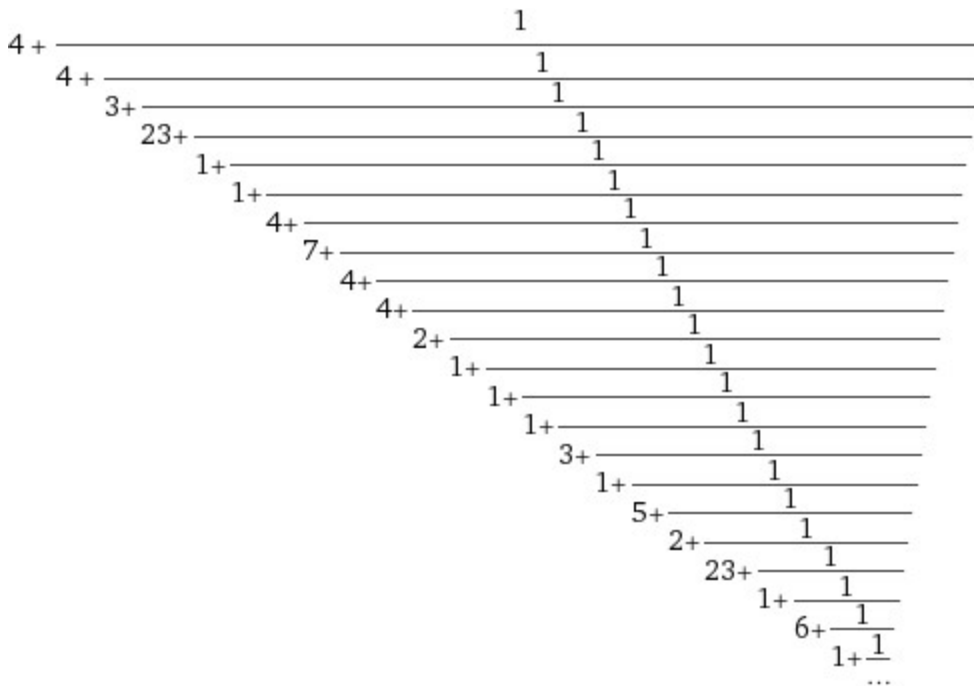
Result:

- Fewer digits
- More digits

4.231017209177339112182073687858541277745109926519653165857...

Continued fraction:

- Linear form



[Open code](#)

Enlarge Data Customize A Plaintext Interactive

Possible closed forms:

More

- $$\frac{7}{12} \pi \cot^2\left(\frac{6353638}{10915619}\right) \approx 4.23101720917733911265645$$

$$\frac{1163 + 800e + 823e^2}{4(45 + 161e + 10e^2)} \approx 4.23101720917733911234656$$

$$\frac{3249590579\pi}{2412868912} \approx 4.23101720917733911201642$$

This result 4.2310172091773391121820736878585412777451099265196531 is in the range of the mass of DM particle that is between 4 – 4.2 eV

We have also that:

$$\left(\left(\left(36 \cdot 48\right)^{1/16}\right)\right)^{\left(\left(2 \cdot 0.6309\right) / 2 / \left(2 \cdot 1.4649\right)\right)}$$

where 0.6309 and 1.4649 are Hausdorff dimensions

Input interpretation:

$$16\sqrt[36 \times 48]{\left(\frac{2 \cdot 0.6309}{2}\right) / (2 \cdot 1.4649)}$$

[Open code](#)

Enlarge Data Customize A Plaintext Interactive

Result:

Fewer digits
More digits

1.105536474062154875583860972932148075123111997869765141149...

1.1055364740621548755838609729321480751231119978697651...

Continued fraction:

Linear form

$$1 + \frac{1}{9 + \frac{1}{2 + \frac{1}{9 + \frac{1}{1 + \frac{1}{1 + \frac{1}{20 + \frac{1}{2 + \frac{1}{3 + \frac{1}{1 + \frac{1}{2 + \frac{1}{1 + \frac{1}{8 + \frac{1}{10 + \frac{1}{1 + \frac{1}{1 + \frac{1}{3 + \frac{1}{70 + \frac{1}{3 + \frac{1}{8 + \frac{1}{\dots}}$$

[Open code](#)

Enlarge Data Customize A [Plaintext](#) [Interactive](#)

Possible closed forms:

More

$$\frac{37978575}{3480694\pi^2} \approx 1.10553647406215491166$$
$$\frac{2(126 - 8e + 51e^2)}{331 - 859e + 389e^2} \approx 1.1055364740621548746068$$
$$\frac{2091103816\pi}{5942270147} \approx 1.105536474062154875571703$$

This result 1.105536474062... is practically (as absolute value) equal to the value of the Cosmological Constant $1.1056 \cdot 10^{-52}$

Analyzing the minimal polynomial

If we take a random coefficient, for example 883163826121628712960 and divide it by 1728, we obtain:

$$883163826121628712960 / 1728 = 511090177153720320;$$

$$511090177153720320 / 1728 = 295769778445440;$$

$$295769778445440 / 1728 = 171163066230;$$

And:

$$32199776876321832960 / 1728 = 18634130136760320;$$

$$18634130136760320 / 1728 = 10783640125440;$$

$$10783640125440 / 1728 = 6240532480;$$

And:

$$16755940501114060800 / 1728 = 9696724827033600;$$

$$9696724827033600 / 1728 = 5611530571200;$$

$$5611530571200 / 1728 = 3247413525$$

And:

$$1502565913250900213760 / 1728 = 869540459057233920;$$

$$869540459057233920 / 1728 = 503206284176640;$$

$$503206284176640 / 1728 = 291207340380$$

And so on...

We note that each coefficient is divisible three times by 1728. What is the meaning behind this sort of division performed three times for the same number, in this case 1728?

If we take the results of above divisions and multiply them, and then divide twice by 1728, we obtain:

$$(171163066230 \times 6240532480 \times 3247413525 \times 291207340380) / 1728 / 1728$$

Input:

$$\frac{171163066230 \times 6240532480 \times 3247413525 \times 291207340380}{1728}$$

$$1728$$

[Open code](#)

Enlarge Data Customize A Plaintext Interactive

Result:

338 286 091 664 464 205 403 826 008 962 137 500

Scientific notation:

$3.382860916644642054038260089621375 \times 10^{35}$

338286091664464205403826008962137500

If we divide them two by two and multiply them together, dividing them by two, we obtain:

$$1/2*(291207340380/171163066230)*(6240532480/3247413525)$$

Input:

$$\frac{1}{2} \times \frac{291\,207\,340\,380}{171\,163\,066\,230} \times \frac{6\,240\,532\,480}{3\,247\,413\,525}$$

[Open code](#)

Enlarge Data Customize A Plaintext Interactive

Exact result:

Step-by-step solution

$$\frac{465\,971\,504\,116\,873\,216}{285\,044\,746\,797\,832\,185}$$

Decimal approximation:

More digits

1.634731070653139328564984223149181366613791398608107505038...

[Open code](#)

1.634731070653139328564984223149181366613791398608107505038

$$1.63473 \approx \zeta(2) = \frac{\pi^2}{6} = 1.6449$$

Continued fraction:

Linear form

$$1 + \frac{1}{1 + \frac{1}{1 + \frac{1}{1 + \frac{1}{2 + \frac{1}{1 + \frac{1}{4 + \frac{1}{2 + \frac{1}{1 + \frac{1}{66 + \frac{1}{1 + \frac{1}{2 + \frac{1}{1 + \frac{1}{11 + \frac{1}{4 + \frac{1}{2 + \frac{1}{1 + \frac{1}{2 + \frac{1}{7 + \frac{1}{3 + \frac{1}{1 + \frac{1}{5 + \frac{1}{\dots}}}}}}}}}}}}}}}}}}}}}}}}}}}}}}$$

Possible closed forms:

More

$$\frac{5(800 C_{MFB} + 1801)}{97 C_{MFB} + 5950} \approx 1.6347310706531393243735$$

Enlarge Data Customize A Plaintext Interactive

$$\left(\frac{20\,059\,855}{10\,416\,761}\right)^{3/4} \approx 1.634731070653139364884$$

$$\frac{2\,391\,052\,016 \pi}{4595\,074\,739} \approx 1.634731070653139328589552$$

Note that:

$$1.634731070653139328564984223149181366613791398608107505038 * 2 * 1.328$$

Where 1.328 is a Hausdorff dimension

Input interpretation:

$$1.634731070653139328564984223149181366613791398608107505038 \times 2 \times 1.328$$

[Open code](#)

Enlarge Data Customize A Plaintext Interactive

Result:

More digits

4.341845723654738056668598096684225709726229954703133533380...

This result is very near to the value of Cosmological Constant 4.33×10^{-66} eV² in natural units

Furthermore:

$$(171163066230 * 6240532480 * 3247413525 * 291207340380) =$$

$$= 1010116857132623485908538001544799180800000$$

We have:

$$1/8 * \ln(171163066230 * 6240532480 * 3247413525 * 291207340380)$$

Input:

$$\frac{1}{8} \log(171\,163\,066\,230 \times 6\,240\,532\,480 \times 3\,247\,413\,525 \times 291\,207\,340\,380)$$

[Open code](#)

• $\log(x)$ is the natural logarithm

Enlarge Data Customize A Plaintext Interactive

Exact result:

$$\frac{\log(1\,010\,116\,857\,132\,623\,485\,908\,538\,001\,544\,799\,180\,800\,000)}{8}$$

Decimal approximation:

More digits

12.08982999125520665379144464145768376445038282072930206319...

Property:

$\frac{\log(1\,010\,116\,857\,132\,623\,485\,908\,538\,001\,544\,799\,180\,800\,000)}{8}$
is a transcendental number

Alternate forms:

$$\frac{7 \log(2)}{4} + \frac{7 \log(3)}{8} + \frac{5 \log(5)}{8} + \frac{\log(221)}{4} + \frac{\log(184\,700\,609\,004\,437\,094\,738\,068\,977)}{8}$$

Enlarge Data Customize A Plaintext Interactive

$$\frac{1}{8} (14 \log(2) + 7 \log(3) + 5 \log(5) + \log(7) + \log(11) + 2 \log(13) + 2 \log(17) + \log(31) + \log(37) + \log(179) + \log(181) + \log(443) + \log(330\,233) + \log(441\,223\,243)) \\ \frac{7 \log(2)}{4} + \frac{7 \log(3)}{8} + \frac{5 \log(5)}{8} + \frac{\log(7)}{8} + \frac{\log(11)}{8} + \frac{\log(13)}{4} + \frac{\log(17)}{4} + \frac{\log(31)}{8} + \frac{\log(37)}{8} + \frac{\log(179)}{8} + \frac{\log(181)}{8} + \frac{\log(443)}{8} + \frac{\log(330\,233)}{8} + \frac{\log(441\,223\,243)}{8}$$

Continued fraction:

Linear form

•

$$12 + \cfrac{1}{11 + \cfrac{1}{7 + \cfrac{1}{1 + \cfrac{1}{1 + \cfrac{1}{3 + \cfrac{1}{5 + \cfrac{1}{3 + \cfrac{1}{1 + \cfrac{1}{2 + \cfrac{1}{4 + \cfrac{1}{1 + \cfrac{1}{6 + \cfrac{1}{7 + \cfrac{1}{1 + \cfrac{1}{1 + \cfrac{1}{9 + \cfrac{1}{1 + \cfrac{1}{2 + \cfrac{1}{2 + \cfrac{1}{2 + \cfrac{1}{\dots}}}}}}}}}}}}}}}}}}}}}}}}}}}}$$

Series representations:

More

•

$$\frac{1}{8} \log(171\,163\,066\,230 \times 6\,240\,532\,480 \times 3\,247\,413\,525 \times 291\,207\,340\,380) = \frac{\log(1\,010\,116\,857\,132\,623\,485\,908\,538\,001\,544\,799\,180\,799\,999)}{8}$$

$$\frac{1}{8} \sum_{k=1}^{\infty} \left(\frac{1}{1010116857132623485908538001544799180799999} \right)^k$$

[Open code](#)

Enlarge Data Customize A Plaintext Interactive

$$\frac{1}{8} \log(171\,163\,066\,230 \times 6\,240\,532\,480 \times 3\,247\,413\,525 \times 291\,207\,340\,380) =$$

$$\frac{1}{4} i \pi \left[\frac{\arg(1\,010\,116\,857\,132\,623\,485\,908\,538\,001\,544\,799\,180\,800\,000 - x)}{2 \pi} \right] +$$

$$\frac{\log(x)}{8} - \frac{1}{8} \sum_{k=1}^{\infty} \frac{(-1)^k (1\,010\,116\,857\,132\,623\,485\,908\,538\,001\,544\,799\,180\,800\,000 - x)^k x^{-k}}{k} \quad \text{for } x < 0$$

[Open code](#)

$$\frac{1}{8} \log(171\,163\,066\,230 \times 6\,240\,532\,480 \times 3\,247\,413\,525 \times 291\,207\,340\,380) =$$

$$\frac{1}{4} i \pi \left[\frac{\pi - \arg\left(\frac{1}{z_0}\right) - \arg(z_0)}{2 \pi} \right] + \frac{\log(z_0)}{8} -$$

$$\frac{1}{8} \sum_{k=1}^{\infty} \frac{(-1)^k (1\,010\,116\,857\,132\,623\,485\,908\,538\,001\,544\,799\,180\,800\,000 - z_0)^k z_0^{-k}}{k}$$

Integral representations:

$$\frac{1}{8} \log(171\,163\,066\,230 \times 6\,240\,532\,480 \times 3\,247\,413\,525 \times 291\,207\,340\,380) =$$

$$\frac{1}{8} \int_1^{1\,010\,116\,857\,132\,623\,485\,908\,538\,001\,544\,799\,180\,800\,000} \frac{1}{t} dt$$

[Open code](#)

Enlarge Data Customize A [Plaintext](#) [Interactive](#)

$$\frac{1}{8} \log(171\,163\,066\,230 \times 6\,240\,532\,480 \times 3\,247\,413\,525 \times 291\,207\,340\,380) =$$

$$-\frac{i}{16 \pi} \int_{-i \infty + \gamma}^{i \infty + \gamma} \frac{1\,010\,116\,857\,132\,623\,485\,908\,538\,001\,544\,799\,180\,799\,999^{-s} \Gamma(-s)^2 \Gamma(1+s)}{\Gamma(1-s)} ds \quad \text{for } -1 < \gamma < 0$$

This value 12,0898 is very near to the value of black hole entropy 12.1904

And:

$$1/\pi * \ln(171163066230*6240532480*3247413525*291207340380)$$

Input:

$$\frac{1}{\pi} \log(171\,163\,066\,230 \times 6\,240\,532\,480 \times 3\,247\,413\,525 \times 291\,207\,340\,380)$$

[Open code](#)

- $\log(x)$ is the natural logarithm

Enlarge Data Customize A [Plaintext](#) [Interactive](#)

Exact result:

$$\frac{\log(1\,010\,116\,857\,132\,623\,485\,908\,538\,001\,544\,799\,180\,800\,000)}{\pi}$$

Decimal approximation:

More digits

30.78649926798259039796026198325508181617838655241639992338...

Decimal approximation:

More digits

30.78649926798259039796026198325508181617838655241639992338...

[Open code](#)

Enlarge Data Customize A Plaintext Interactive

Alternate forms:

$$\frac{14 \log(2) + 7 \log(3) + 5 \log(5) + 2 \log(221) + \log(184\,700\,609\,004\,437\,094\,738\,068\,977)}{\pi}$$

$$\frac{1}{\pi} (14 \log(2) + 7 \log(3) + 5 \log(5) + \log(7) + \log(11) + 2 \log(13) + 2 \log(17) + \log(31) + \log(37) + \log(179) + \log(181) + \log(443) + \log(330\,233) + \log(441\,223\,243))$$

$$\frac{14 \log(2)}{\pi} + \frac{7 \log(3)}{\pi} + \frac{5 \log(5)}{\pi} + \frac{\log(7)}{\pi} + \frac{\log(11)}{\pi} + \frac{2 \log(13)}{\pi} + \frac{2 \log(17)}{\pi} + \frac{\log(31)}{\pi} + \frac{\log(37)}{\pi} + \frac{\log(179)}{\pi} + \frac{\log(181)}{\pi} + \frac{\log(443)}{\pi} + \frac{\log(330\,233)}{\pi} + \frac{\log(441\,223\,243)}{\pi}$$

Continued fraction:

Linear form

$$30 + \frac{1}{1 + \frac{1}{3 + \frac{1}{1 + \frac{1}{2 + \frac{1}{6 + \frac{1}{7 + \frac{1}{46 + \frac{1}{2 + \frac{1}{3 + \frac{1}{2 + \frac{1}{5 + \frac{1}{2 + \frac{1}{23 + \frac{1}{1 + \frac{1}{16 + \frac{1}{5 + \frac{1}{1 + \frac{1}{3 + \frac{1}{17 + \frac{1}{2 + \frac{1}{\dots}}}}}}}}}}}}}}}}}}}}}}}}}}}}}}}}}}}}}}$$

Series representations:

More

$$\frac{\log(171\,163\,066\,230 \times 6\,240\,532\,480 \times 3\,247\,413\,525 \times 291\,207\,340\,380)}{\log(1\,010\,116\,857\,132\,623\,485\,908\,538\,001\,544\,799\,180\,799\,999)} =$$

$$\frac{\sum_{k=1}^{\infty} \left(\frac{1}{1\,010\,116\,857\,132\,623\,485\,908\,538\,001\,544\,799\,180\,799\,999} \right)^k}{k} \pi$$

Open code

Enlarge Data Customize A [Plaintext](#) Interactive

$$\frac{\log(171\,163\,066\,230 \times 6\,240\,532\,480 \times 3\,247\,413\,525 \times 291\,207\,340\,380)}{\log(x)} =$$

$$2i \left[\frac{\arg(1\,010\,116\,857\,132\,623\,485\,908\,538\,001\,544\,799\,180\,800\,000 - x)}{2\pi} \right] +$$

$$\frac{\sum_{k=1}^{\infty} \frac{(-1)^k (1\,010\,116\,857\,132\,623\,485\,908\,538\,001\,544\,799\,180\,800\,000 - x)^k x^{-k}}{k}}{\pi} \quad \text{for } x < 0$$

Open code

$$\frac{\log(171\,163\,066\,230 \times 6\,240\,532\,480 \times 3\,247\,413\,525 \times 291\,207\,340\,380)}{\log(z_0)} =$$

$$2i \left[\frac{\pi - \arg\left(\frac{1}{z_0}\right) - \arg(z_0)}{2\pi} \right] + \frac{\log(z_0)}{\pi} -$$

$$\frac{\sum_{k=1}^{\infty} \frac{(-1)^k (1010\,116\,857\,132\,623\,485\,908\,538\,001\,544\,799\,180\,800\,000 - z_0)^k z_0^{-k}}{k}}{\pi}$$

Integral representations:

$$\frac{\log(171\,163\,066\,230 \times 6\,240\,532\,480 \times 3\,247\,413\,525 \times 291\,207\,340\,380)}{\log(z_0)} =$$

$$\frac{1}{\pi} \int_1^{1010\,116\,857\,132\,623\,485\,908\,538\,001\,544\,799\,180\,800\,000} \frac{1}{t} dt$$

Open code

Enlarge Data Customize A [Plaintext](#) Interactive

$$\frac{\log(171\,163\,066\,230 \times 6\,240\,532\,480 \times 3\,247\,413\,525 \times 291\,207\,340\,380)}{\log(z_0)} =$$

$$-\frac{i}{2\pi^2} \int_{-i\infty+\gamma}^{i\infty+\gamma} \frac{1010\,116\,857\,132\,623\,485\,908\,538\,001\,544\,799\,180\,799\,999^{-s} \Gamma(-s)^2 \Gamma(1+s)}{\Gamma(1-s)} ds \quad \text{for } -1 < \gamma < 0$$

This result 30,7864 is very near to the values of black hole entropies 30,5963 and 30,7812

We have, from the sum of the coefficients:

(883163826121628712960+32199776876321832960+16755940501114060800+1502565913250900213760)

Input:

883 163 826 121 628 712 960 + 32 199 776 876 321 832 960 +
16 755 940 501 114 060 800 + 1502565 913 250 900 213 760

[Open code](#)

Enlarge Data Customize A [Plaintext](#) Interactive

Result:

2434685456749964820480

Scientific notation:

$2.43468545674996482048 \times 10^{21}$

Now:

$\frac{1}{4} \ln$

(883163826121628712960+32199776876321832960+16755940501114060800+1502565913250900213760)

Input:

$\frac{1}{4} \log(883 163 826 121 628 712 960 + 32 199 776 876 321 832 960 +$
 $16 755 940 501 114 060 800 + 1502565 913 250 900 213 760)$

[Open code](#)

- $\log(x)$ is the natural logarithm

Enlarge Data Customize A [Plaintext](#) Interactive

Exact result:

$\frac{\log(2434685456749964820480)}{4}$

Decimal approximation:

More digits

12.31102613129822162100974729252428678673073532220090668761...

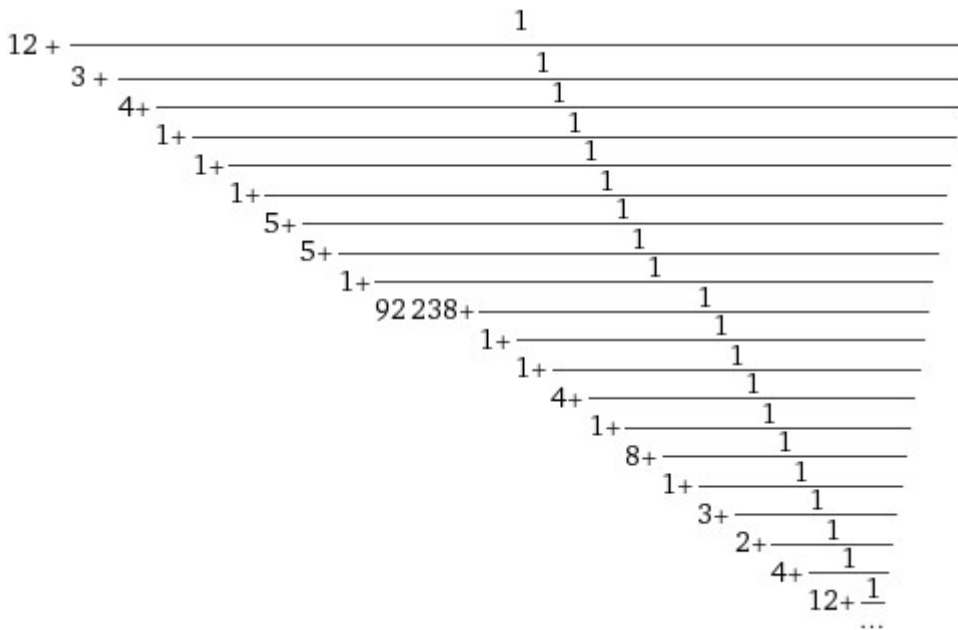
[Open code](#)

Property:

$\frac{\log(2434685456749964820480)}{4}$ is a transcendental number

Continued fraction:

Linear form



Series representations:

More

$$\frac{1}{4} \log(883\,163\,826\,121\,628\,712\,960 + 32\,199\,776\,876\,321\,832\,960 + 16\,755\,940\,501\,114\,060\,800 + 1\,502\,565\,913\,250\,900\,213\,760) = \frac{\log(2\,434\,685\,456\,749\,964\,820\,479)}{4} - \frac{1}{4} \sum_{k=1}^{\infty} \frac{\left(-\frac{1}{2\,434\,685\,456\,749\,964\,820\,479}\right)^k}{k}$$

[Open code](#)

Enlarge Data Customize A [Plaintext](#) [Interactive](#)

$$\frac{1}{4} \log(883\,163\,826\,121\,628\,712\,960 + 32\,199\,776\,876\,321\,832\,960 + 16\,755\,940\,501\,114\,060\,800 + 1\,502\,565\,913\,250\,900\,213\,760) = \frac{1}{2} i \pi \left[\frac{\arg(2\,434\,685\,456\,749\,964\,820\,480 - x)}{2 \pi} \right] + \frac{\log(x)}{4} - \frac{1}{4} \sum_{k=1}^{\infty} \frac{(-1)^k (2\,434\,685\,456\,749\,964\,820\,480 - x)^k x^{-k}}{k} \text{ for } x < 0$$

[Open code](#)

$$\frac{1}{4} \log(883\,163\,826\,121\,628\,712\,960 + 32\,199\,776\,876\,321\,832\,960 + 16\,755\,940\,501\,114\,060\,800 + 1\,502\,565\,913\,250\,900\,213\,760) = \frac{1}{2} i \pi \left[\frac{\pi - \arg\left(\frac{1}{z_0}\right) - \arg(z_0)}{2 \pi} \right] + \frac{\log(z_0)}{4} - \frac{1}{4} \sum_{k=1}^{\infty} \frac{(-1)^k (2\,434\,685\,456\,749\,964\,820\,480 - z_0)^k z_0^{-k}}{k}$$

Integral representations:

$$\frac{1}{4} \log(883\,163\,826\,121\,628\,712\,960 + 32\,199\,776\,876\,321\,832\,960 + 16\,755\,940\,501\,114\,060\,800 + 1\,502\,565\,913\,250\,900\,213\,760) = \frac{1}{4} \int_1^{2\,434\,685\,456\,749\,964\,820\,480} \frac{1}{t} dt$$

[Open code](#)

Enlarge Data Customize A [Plaintext](#) [Interactive](#)

$$\frac{1}{4} \log(883\,163\,826\,121\,628\,712\,960 + 32\,199\,776\,876\,321\,832\,960 + 16\,755\,940\,501\,114\,060\,800 + 1\,502\,565\,913\,250\,900\,213\,760) = -\frac{i}{8\pi} \int_{-i\infty+\gamma}^{i\infty+\gamma} \frac{2\,434\,685\,456\,749\,964\,820\,479^{-s} \Gamma(-s)^2 \Gamma(1+s)}{\Gamma(1-s)} ds \text{ for } -1 < \gamma < 0$$

This result 12,31 is very near to the values of black hole entropies 12,1904 and 12,5664

$$\frac{1}{((\sqrt{5}+1)/2)} \ln(883163826121628712960+32199776876321832960+16755940501114060800+1502565913250900213760)$$

Input:

$$\frac{1}{\frac{1}{2}(\sqrt{5}+1)} \log(883\,163\,826\,121\,628\,712\,960 + 32\,199\,776\,876\,321\,832\,960 + 16\,755\,940\,501\,114\,060\,800 + 1\,502\,565\,913\,250\,900\,213\,760)$$

[Open code](#)

- $\log(x)$ is the natural logarithm

Enlarge Data Customize A [Plaintext](#) [Interactive](#)

Exact result:

$$\frac{2 \log(2\,434\,685\,456\,749\,964\,820\,480)}{1 + \sqrt{5}}$$

Decimal approximation:

More digits

$$30.43453034212170639085224206089342998092386858188475076236\dots$$

[Open code](#)

Property:

$$\frac{2 \log(2\,434\,685\,456\,749\,964\,820\,480)}{1 + \sqrt{5}} \text{ is a transcendental number}$$

Series representations:

More

$$\frac{1}{\frac{1}{2}(\sqrt{5} + 1)} \log(883\,163\,826\,121\,628\,712\,960 + 32\,199\,776\,876\,321\,832\,960 + 16\,755\,940\,501\,114\,060\,800 + 1\,502\,565\,913\,250\,900\,213\,760) = \frac{2 \log(2\,434\,685\,456\,749\,964\,820\,479)}{1 + \sqrt{5}} - \frac{2 \sum_{k=1}^{\infty} \frac{\left(-\frac{1}{2\,434\,685\,456\,749\,964\,820\,479}\right)^k}{k}}{1 + \sqrt{5}}$$

[Open code](#)

Enlarge Data Customize A [Plaintext](#) [Interactive](#)

$$\frac{1}{\frac{1}{2}(\sqrt{5} + 1)} \log(883\,163\,826\,121\,628\,712\,960 + 32\,199\,776\,876\,321\,832\,960 + 16\,755\,940\,501\,114\,060\,800 + 1\,502\,565\,913\,250\,900\,213\,760) = \frac{4i\pi \left[\frac{\arg(2\,434\,685\,456\,749\,964\,820\,480 - x)}{2\pi} \right]}{1 + \sqrt{5}} + \frac{2 \log(x)}{1 + \sqrt{5}} - \frac{2 \sum_{k=1}^{\infty} \frac{(-1)^k (2\,434\,685\,456\,749\,964\,820\,480 - x)^k x^{-k}}{k}}{1 + \sqrt{5}} \quad \text{for } x < 0$$

[Open code](#)

$$\frac{1}{\frac{1}{2}(\sqrt{5} + 1)} \log(883\,163\,826\,121\,628\,712\,960 + 32\,199\,776\,876\,321\,832\,960 + 16\,755\,940\,501\,114\,060\,800 + 1\,502\,565\,913\,250\,900\,213\,760) = \frac{1}{1 + \sqrt{5}} 2 \left(\log(z_0) + \left[\frac{\arg(2\,434\,685\,456\,749\,964\,820\,480 - z_0)}{2\pi} \right] \left(\log\left(\frac{1}{z_0}\right) + \log(z_0) \right) - \sum_{k=1}^{\infty} \frac{(-1)^k (2\,434\,685\,456\,749\,964\,820\,480 - z_0)^k z_0^{-k}}{k} \right)$$

Integral representations:

$$\frac{1}{\frac{1}{2}(\sqrt{5} + 1)} \log(883\,163\,826\,121\,628\,712\,960 + 32\,199\,776\,876\,321\,832\,960 + 16\,755\,940\,501\,114\,060\,800 + 1\,502\,565\,913\,250\,900\,213\,760) = \frac{2}{1 + \sqrt{5}} \int_1^{2\,434\,685\,456\,749\,964\,820\,480} \frac{1}{t} dt$$

[Open code](#)

Enlarge Data Customize A [Plaintext](#) [Interactive](#)

$$\frac{1}{\frac{1}{2}(\sqrt{5} + 1)} \log(883\,163\,826\,121\,628\,712\,960 + 32\,199\,776\,876\,321\,832\,960 + 16\,755\,940\,501\,114\,060\,800 + 150\,256\,591\,325\,090\,213\,760) =$$

$$-\frac{i}{\pi + \sqrt{5}} \int_{-i\infty + \gamma}^{i\infty + \gamma} \frac{2\,434\,685\,456\,749\,964\,820\,479^{-s} \Gamma(-s)^2 \Gamma(1+s)}{\Gamma(1-s)} ds \quad \text{for } -1 < \gamma < 0$$

[Open code](#)

This result 30,4345 is very near to the value of black hole entropy 30,4615

$$\frac{1}{\left(\frac{\sqrt{5}+1}{2}\right)^2} \ln(883163826121628712960+32199776876321832960+16755940501114060800+1502565913250900213760)$$

Input:

$$\frac{1}{\left(\frac{1}{2}(\sqrt{5} + 1)\right)^2} \log(883\,163\,826\,121\,628\,712\,960 + 32\,199\,776\,876\,321\,832\,960 + 16\,755\,940\,501\,114\,060\,800 + 150\,256\,591\,325\,090\,213\,760)$$

[Open code](#)

- $\log(x)$ is the natural logarithm

Enlarge Data Customize A [Plaintext](#) [Interactive](#)

Exact result:

$$\frac{4 \log(2\,434\,685\,456\,749\,964\,820\,480)}{(1 + \sqrt{5})^2}$$

Decimal approximation:

More digits

18.80957418307118009318674710920371716599907270691887598809...

[Open code](#)

Property:

$\frac{4 \log(2\,434\,685\,456\,749\,964\,820\,480)}{(1 + \sqrt{5})^2}$ is a transcendental number

Series representations:

More

$$\frac{1}{\left(\frac{1}{2}(\sqrt{5} + 1)\right)^2} \log(883\,163\,826\,121\,628\,712\,960 + 32\,199\,776\,876\,321\,832\,960 + 16\,755\,940\,501\,114\,060\,800 + 150\,256\,591\,325\,090\,213\,760) =$$

$$\frac{2 \log(2\,434\,685\,456\,749\,964\,820\,479)}{3 + \sqrt{5}} - \frac{2 \sum_{k=1}^{\infty} \left(\frac{1}{2\,434\,685\,456\,749\,964\,820\,479}\right)^k}{3 + \sqrt{5}}$$

[Open code](#)

Enlarge Data Customize A Plaintext Interactive

$$\frac{1}{\left(\frac{1}{2}(\sqrt{5} + 1)\right)^2} \log(883\,163\,826\,121\,628\,712\,960 + 32\,199\,776\,876\,321\,832\,960 + 16\,755\,940\,501\,114\,060\,800 + 1\,502\,565\,913\,250\,900\,213\,760) =$$

$$\frac{4i\pi \left[\frac{\arg(2\,434\,685\,456\,749\,964\,820\,480 - x)}{2\pi} \right] + \frac{2 \log(x)}{3 + \sqrt{5}} - 2 \sum_{k=1}^{\infty} \frac{(-1)^k (2\,434\,685\,456\,749\,964\,820\,480 - x)^k x^{-k}}{k(3 + \sqrt{5})}}{3 + \sqrt{5}} \quad \text{for } x < 0$$

[Open code](#)

$$\frac{1}{\left(\frac{1}{2}(\sqrt{5} + 1)\right)^2} \log(883\,163\,826\,121\,628\,712\,960 + 32\,199\,776\,876\,321\,832\,960 + 16\,755\,940\,501\,114\,060\,800 + 1\,502\,565\,913\,250\,900\,213\,760) = \frac{1}{(1 + \sqrt{5})^2}$$

$$4 \left(\log(z_0) + \left[\frac{\arg(2\,434\,685\,456\,749\,964\,820\,480 - z_0)}{2\pi} \right] \left(\log\left(\frac{1}{z_0}\right) + \log(z_0) \right) - \sum_{k=1}^{\infty} \frac{(-1)^k (2\,434\,685\,456\,749\,964\,820\,480 - z_0)^k z_0^{-k}}{k} \right)$$

Integral representations:

$$\frac{1}{\left(\frac{1}{2}(\sqrt{5} + 1)\right)^2} \log(883\,163\,826\,121\,628\,712\,960 + 32\,199\,776\,876\,321\,832\,960 + 16\,755\,940\,501\,114\,060\,800 + 1\,502\,565\,913\,250\,900\,213\,760) =$$

$$\frac{2}{3 + \sqrt{5}} \int_1^{2\,434\,685\,456\,749\,964\,820\,480} \frac{1}{t} dt$$

[Open code](#)

Enlarge Data Customize A Plaintext Interactive

$$\frac{1}{\left(\frac{1}{2}(\sqrt{5} + 1)\right)^2} \log(883\,163\,826\,121\,628\,712\,960 + 32\,199\,776\,876\,321\,832\,960 + 16\,755\,940\,501\,114\,060\,800 + 1\,502\,565\,913\,250\,900\,213\,760) =$$

$$-\frac{i}{3\pi + \sqrt{5}} \int_{-i\infty + \gamma}^{i\infty + \gamma} \frac{2\,434\,685\,456\,749\,964\,820\,479^{-s} \Gamma(-s)^2 \Gamma(1 + s)}{\Gamma(1 - s)} ds \quad \text{for } -1 < \gamma < 0$$

This result 18,809 is very near to the value of black hole entropy 18,7328

From the product of the coefficients, we have:

(883163826121628712960*32199776876321832960*16755940501114060800*1502565913250900213760)

Input:

883 163 826 121 628 712 960 × 32 199 776 876 321 832 960 ×
16 755 940 501 114 060 800 × 1502 565 913 250 900 213 760

[Open code](#)

Enlarge Data Customize A Plaintext Interactive

Result:

715 972 722 289 651 672 191 964 120 401 066 036 877 700 897 580 713 653 423 988
346 007 419 931 852 800 000

Decimal approximation:

More digits

7.1597272228965167219196412040106603687770089758071365... × 10⁸⁰
7.1597272228965167219196412040106603687770089758071365 × 10⁸⁰

Note that:

$(7.1597272228965167219196412040106603687770089758071365 \times 10^{80})^{(1.2619) \cdot 12}$

Where 1,2619 is a Hausdorff dimension

Input interpretation:

$(7.1597272228965167219196412040106603687770089758071365 \times 10^{80})^{1.2619 \cdot 12}$

[Open code](#)

Enlarge Data Customize A Plaintext Interactive

Result:

More digits

1.28818... × 10¹⁰³

This result is in the range of SMBHs entropy contained within the cosmic event horizon $1,2 \cdot 10^{103}$

And:

$(7.1597272228965167219196412040106603687770089758071365 \times 10^{80}) \cdot (4.92906 \cdot 10^6) \cdot (0.081816)^2 \cdot (1.08753)$

Where $4,92906 \cdot 10^6$, $0,081816$ and $1,08753$ are results of Ramanujan mock theta functions (see previous our papers)

Input interpretation:

$7.1597272228965167219196412040106603687770089758071365 \times 10^{80} \times 4.92906 \times 10^6 \times 0.081816 (2 \times 1.08753)$

[Open code](#)

Enlarge Data Customize A Plaintext Interactive

Result:

628 015 070 798 111 317 317 306 604 060 620 207 918 155 286 213 607 529 609 853 ∙
910 374 240 000 000 000 000 000 000

Scientific notation:

6.28015070798111317317306604060620207918155286213607529609853910374 ∙
 24×10^{86}

$6.2801507079811131731730660406062020791815528621360752 \times 10^{86}$

This result is practically equal to the Dark Matter entropy contained within the cosmic event horizon 6×10^{86} . Furthermore, this is a multiple of length of a circle with radius equal to 1: 2π

$1/(5\pi) \cdot \ln$

$(883163826121628712960 \cdot 32199776876321832960 \cdot 16755940501114060800 \cdot 1502565913250900213760)$

Input:

$\frac{1}{5\pi} \log(883\ 163\ 826\ 121\ 628\ 712\ 960 \times 32\ 199\ 776\ 876\ 321\ 832\ 960 \times$
 $16\ 755\ 940\ 501\ 114\ 060\ 800 \times 1\ 502\ 565\ 913\ 250\ 900\ 213\ 760)$

[Open code](#)

- $\log(x)$ is the natural logarithm

Enlarge Data Customize A Plaintext Interactive

Exact result:

$\frac{1}{5\pi} \log($
 $715\ 972\ 722\ 289\ 651\ 672\ 191\ 964\ 120\ 401\ 066\ 036\ 877\ 700\ 897\ 580\ 713\ 653\ 423\ 988 \cdot$
 $346\ 007\ 419\ 931\ 852\ 800\ 000)$

Decimal approximation:

- More digits
11.85228639427033142608281825040505417357628967543038281643...

Series representations:

- More

$$\frac{1}{5\pi} \log(883\,163\,826\,121\,628\,712\,960 \times 32\,199\,776\,876\,321\,832\,960 \times 16\,755\,940\,501\,114\,060\,800 \times 1\,502\,565\,913\,250\,900\,213\,760) = \frac{2}{5} i \left[\frac{1}{2\pi} \arg(715\,972\,722\,289\,651\,672\,191\,964\,120\,401\,066\,036\,877\,700\,897\,580\,713 \cdot 653\,423\,988\,346\,007\,419\,931\,852\,800\,000 - x) \right] + \frac{\log(x)}{5\pi} - \frac{1}{5\pi} \left(\sum_{k=1}^{\infty} \frac{1}{k} (-1)^k (715\,972\,722\,289\,651\,672\,191\,964\,120\,401\,066\,036\,877\,700\,897\,580\,713 \cdot 653\,423\,988\,346\,007\,419\,931\,852\,800\,000 - x)^k x^{-k} \right) \text{ for } x < 0$$

[Open code](#)

Enlarge Data Customize A Plaintext Interactive

$$\frac{1}{5\pi} \log(883\,163\,826\,121\,628\,712\,960 \times 32\,199\,776\,876\,321\,832\,960 \times 16\,755\,940\,501\,114\,060\,800 \times 1\,502\,565\,913\,250\,900\,213\,760) = \frac{1}{5\pi} \left(\log(z_0) + \left[\frac{1}{2\pi} \arg(715\,972\,722\,289\,651\,672\,191\,964\,120\,401\,066\,036\,877\,700\,897\,580\,713 \cdot 653\,423\,988\,346\,007\,419\,931\,852\,800\,000 - z_0) \right] \left(\log\left(\frac{1}{z_0}\right) + \log(z_0) \right) - \sum_{k=1}^{\infty} \frac{1}{k} (-1)^k (715\,972\,722\,289\,651\,672\,191\,964\,120\,401\,066\,036\,877\,700\,897\,580\,713 \cdot 653\,423\,988\,346\,007\,419\,931\,852\,800\,000 - z_0)^k z_0^{-k} \right)$$

[Open code](#)

$$\frac{1}{5\pi} \log(883\,163\,826\,121\,628\,712\,960 \times 32\,199\,776\,876\,321\,832\,960 \times 16\,755\,940\,501\,114\,060\,800 \times 1\,502\,565\,913\,250\,900\,213\,760) = \frac{1}{5\pi} \log(715\,972\,722\,289\,651\,672\,191\,964\,120\,401\,066\,036\,877\,700\,897\,580\,713\,653\,423\,988\,346\,007\,419\,931\,852\,799\,999) - \frac{1}{5\pi} \left(\sum_{k=1}^{\infty} \frac{1}{k} (-1)^k (715\,972\,722\,289\,651\,672\,191\,964\,120\,401\,066\,036\,877\,700\,897\,580\,713\,653\,423\,988\,346\,007\,419\,931\,852\,799\,999)^k \right)$$

Integral representations:

$$\frac{1}{5\pi} \log(883\,163\,826\,121\,628\,712\,960 \times 32\,199\,776\,876\,321\,832\,960 \times 16\,755\,940\,501\,114\,060\,800 \times 1\,502\,565\,913\,250\,900\,213\,760) = \frac{1}{5\pi} \int_1^{715\,972\,722\,289\,651\,672\,191\,964\,120\,401\,066\,036\,877\,700\,897\,580\,713\,653\,423\,988\,346\,007\,419\,931\,852\,800\,000} \frac{1}{t} dt$$

[Open code](#)

Enlarge Data Customize A [Plaintext](#) [Interactive](#)

$$\frac{1}{5\pi} \log(883\,163\,826\,121\,628\,712\,960 \times 32\,199\,776\,876\,321\,832\,960 \times 16\,755\,940\,501\,114\,060\,800 \times 1\,502\,565\,913\,250\,900\,213\,760) = -\frac{i}{10\pi^2} \int_{-i\infty+\gamma}^{i\infty+\gamma} \frac{1}{\Gamma(1-s)} \cdot 715\,972\,722\,289\,651\,672\,191\,964\,120\,401\,066\,036\,877\,700\,897\,580\,713\,653\,423\,988\,346\,007\,419\,931\,852\,799\,999^{-s} \Gamma(-s)^2 \Gamma(1+s) ds$$

for $-1 < \gamma < 0$

This result 11,8522 is very near to the value of black hole entropy 11,8458

1/8 ln

(883163826121628712960*32199776876321832960*16755940501114060800*1502565913250900213760)

Input:

$$\frac{1}{8} \log(883\,163\,826\,121\,628\,712\,960 \times 32\,199\,776\,876\,321\,832\,960 \times 16\,755\,940\,501\,114\,060\,800 \times 1\,502\,565\,913\,250\,900\,213\,760)$$

[Open code](#)

- $\log(x)$ is the natural logarithm

Enlarge Data Customize A [Plaintext](#) [Interactive](#)

Exact result:

$$\frac{1}{8} \log(715\,972\,722\,289\,651\,672\,191\,964\,120\,401\,066\,036\,877\,700\,897\,580\,713\,653\,423\,988\,346\,007\,419\,931\,852\,800\,000)$$

Decimal approximation:

More digits

23.27190991530120804982513730073263854804359154017397188308...

[Open code](#)

Property:

$\frac{1}{8} \log(715\,972\,722\,289\,651\,672\,191\,964\,120\,401\,066\,036\,877\,700\,897\,580\,713\,653\,423 \cdot 988\,346\,007\,419\,931\,852\,800\,000)$ is a transcendental number

Series representations:

More

$$\frac{1}{8} \log(883\,163\,826\,121\,628\,712\,960 \times 32\,199\,776\,876\,321\,832\,960 \times 16\,755\,940\,501\,114\,060\,800 \times 1502\,565\,913\,250\,900\,213\,760) = \frac{1}{8} \log(715\,972\,722\,289\,651\,672\,191\,964\,120\,401\,066\,036\,877\,700\,897\,580\,713\,653\,423 \cdot 988\,346\,007\,419\,931\,852\,799\,999) - \frac{1}{8} \sum_{k=1}^{\infty} \frac{1}{k} (-1/715\,972\,722\,289\,651\,672\,191\,964\,120\,401\,066\,036\,877\,700\,897 \cdot 580\,713\,653\,423\,988\,346\,007\,419\,931\,852\,799\,999)^k$$

[Open code](#)

Enlarge Data Customize A [Plaintext](#) [Interactive](#)

$$\frac{1}{8} \log(883\,163\,826\,121\,628\,712\,960 \times 32\,199\,776\,876\,321\,832\,960 \times 16\,755\,940\,501\,114\,060\,800 \times 1502\,565\,913\,250\,900\,213\,760) = \frac{1}{4} i \pi \left[\frac{1}{2\pi} \arg(715\,972\,722\,289\,651\,672\,191\,964\,120\,401\,066\,036\,877\,700\,897\,580\,713 \cdot 653\,423\,988\,346\,007\,419\,931\,852\,800\,000 - x) \right] + \frac{\log(x)}{8} - \frac{1}{8} \sum_{k=1}^{\infty} \frac{1}{k} (-1)^k (715\,972\,722\,289\,651\,672\,191\,964\,120\,401\,066\,036\,877\,700\,897\,580\,713 \cdot 653\,423\,988\,346\,007\,419\,931\,852\,800\,000 - x)^k x^{-k} \text{ for } x < 0$$

[Open code](#)

$$\frac{1}{8} \log(883\,163\,826\,121\,628\,712\,960 \times 32\,199\,776\,876\,321\,832\,960 \times 16\,755\,940\,501\,114\,060\,800 \times 1502\,565\,913\,250\,900\,213\,760) = \frac{1}{4} i \pi \left[\frac{\pi - \arg\left(\frac{1}{z_0}\right) - \arg(z_0)}{2\pi} \right] + \frac{\log(z_0)}{8} - \frac{1}{8} \sum_{k=1}^{\infty} \frac{1}{k} (-1)^k (715\,972\,722\,289\,651\,672\,191\,964\,120\,401\,066\,036\,877\,700\,897\,580\,713 \cdot 653\,423\,988\,346\,007\,419\,931\,852\,800\,000 - z_0)^k z_0^{-k}$$

Integral representations:

$$\frac{1}{8} \log(883\,163\,826\,121\,628\,712\,960 \times 32\,199\,776\,876\,321\,832\,960 \times 16\,755\,940\,501\,114\,060\,800 \times 1\,502\,565\,913\,250\,900\,213\,760) = \frac{1}{8} \int_1^{715972\,722289\,651672\,191964\,120401066036\,877700\,897580\,713653\,423988\,346007419931\,852800000} \frac{1}{t} dt$$

[Open code](#)

Enlarge Data Customize A [Plaintext](#) [Interactive](#)

$$\frac{1}{8} \log(883\,163\,826\,121\,628\,712\,960 \times 32\,199\,776\,876\,321\,832\,960 \times 16\,755\,940\,501\,114\,060\,800 \times 1\,502\,565\,913\,250\,900\,213\,760) = -\frac{i}{16\pi} \int_{-i\infty+\gamma}^{i\infty+\gamma} \frac{1}{\Gamma(1-s)} \cdot 715\,972\,722\,289\,651\,672\,191\,964\,120\,401\,066\,036\,877\,700\,897\,580\,713\,653 \cdot 423\,988\,346\,007\,419\,931\,852\,799\,999^{-s} \Gamma(-s)^2 \Gamma(1+s) ds$$

for $-1 < \gamma < 0$

This result 23,2719 is very near to the value of black hole entropy 23,3621

Now, from the sum of the coefficients:

$$(883163826121628712960+32199776876321832960+16755940501114060800+1502565913250900213760)$$

we obtain also:

$$(6121628712960+32199776876321832960+16755940501114060800+1502565913250900213760)^{1/16}$$

Input:

$$(883\,163\,826\,121\,628\,712\,960 + 32\,199\,776\,876\,321\,832\,960 + 16\,755\,940\,501\,114\,060\,800 + 1\,502\,565\,913\,250\,900\,213\,760)^{(1/16)}$$

[Open code](#)

Enlarge Data Customize A [Plaintext](#) [Interactive](#)

Result:

- Approximate form
- Step-by-step solution

$$2 \sqrt[8]{2} 3^{9/16} \sqrt[16]{471\,858\,352\,615}$$

Decimal approximation:

- More digits

$$21.70964285030260014312574140194735046228431183536223817690\dots$$

This result 21,7096 is very near to the value of black hole entropy 21,7656

Now, we have calculated good approximations to golden ratio:

$$\frac{1}{2} * (((((2434685456749964820480)^{1/101} + (2434685456749964820480)^{1/103}))))$$

Where 101 and 103 are twin prime numbers

Input:

$$\frac{1}{2} \left(\sqrt[101]{2434685456749964820480} + \sqrt[103]{2434685456749964820480} \right)$$

[Open code](#)

Enlarge Data Customize A [Plaintext](#) [Interactive](#)

Result:

Approximate form

Step-by-step solution

$$\frac{1}{2} \left(2^{18/103} \times 3^{9/103} \sqrt[103]{471858352615} + 2^{18/101} \times 3^{9/101} \sqrt[101]{471858352615} \right)$$

Decimal approximation:

More digits

1.620675358633664283243745225518245534726135348078671427781...

[Open code](#)

Alternate forms:

$$\frac{3^{9/103} \sqrt[103]{471858352615}}{2^{85/103}} + \frac{3^{9/101} \sqrt[101]{471858352615}}{2^{83/101}}$$

[Open code](#)

Enlarge Data Customize A [Plaintext](#) [Interactive](#)

$$\frac{3^{9/103} \sqrt[103]{471858352615} \left(1 + 2^{36/10403} \times 3^{18/10403} \times 471858352615^{2/10403} \right)}{2^{85/103}}$$

Continued fraction:

Linear form

From the two results:

1.620675358633664283243745225518245534726135348078671427781

1.617809499074550678031612164949056918541467923599060657841

We obtain another more precise approximation to golden ratio:

$(1.620675358633664283243745225518245534726135348078671427781 + 1.617809499074550678031612164949056918541467923599060657841) / ((\sqrt{(1.2683 + 0.69897 + 1.2108 + 1.1056) / 2}) + 0.538 + (1/16)^{(6 \times 10^{86})})$

Where 1.2683, 0,69897, 1,2108 and 0,538 are a Hausdorff dimensions, while 1.1056 is the value of Cosmological Constant and $6 * 10^{86}$ is the value of Dark Matter entropy contained within cosmic event horizon

Input interpretation:

$$\frac{(1.620675358633664283243745225518245534726135348078671427781 + 1.617809499074550678031612164949056918541467923599060657841)}{\left(\sqrt{\frac{1}{2}(1.2683 + 0.69897 + 1.2108 + 1.1056)} + 0.538 + \left(\frac{1}{16}\right)^{6 \times 10^{86}}\right)}$$

[Open code](#)

Enlarge Data Customize A Plaintext Interactive

Result:

- Fewer digits
- More digits

1.618028147651811091138026448546081563286125751484548779631...

1.6180281476518110911380264485460815632861257514845487

Continued fraction:

- Linear form

Enlarge Data Customize A Plaintext Interactive

Result:

- Fewer digits
- More digits

26.00030844115454316651447216654454350926729125584468808193...

Series representations:

- More

$$0.538 + \frac{1}{1.934} \log(883\,163\,826\,121\,628\,712\,960 + 32\,199\,776\,876\,321\,832\,960 + 16\,755\,940\,501\,114\,060\,800 + 1\,502\,565\,913\,250\,900\,213\,760) =$$

$$0.538 + 0.517063 \log(2\,434\,685\,456\,749\,964\,820\,479) -$$

$$0.517063 \sum_{k=1}^{\infty} \frac{\left(-\frac{1}{2\,434\,685\,456\,749\,964\,820\,479}\right)^k}{k}$$

[Open code](#)

Enlarge Data Customize A Plaintext Interactive

$$0.538 + \frac{1}{1.934} \log(883\,163\,826\,121\,628\,712\,960 + 32\,199\,776\,876\,321\,832\,960 + 16\,755\,940\,501\,114\,060\,800 + 1\,502\,565\,913\,250\,900\,213\,760) =$$

$$0.538 + 1.03413 i \pi \left\lfloor \frac{\arg(2\,434\,685\,456\,749\,964\,820\,480 - x)}{2\pi} \right\rfloor + 0.517063 \log(x) -$$

$$0.517063 \sum_{k=1}^{\infty} \frac{(-1)^k (2\,434\,685\,456\,749\,964\,820\,480 - x)^k x^{-k}}{k} \quad \text{for } x < 0$$

[Open code](#)

$$0.538 + \frac{1}{1.934} \log(883\,163\,826\,121\,628\,712\,960 + 32\,199\,776\,876\,321\,832\,960 + 16\,755\,940\,501\,114\,060\,800 + 1\,502\,565\,913\,250\,900\,213\,760) =$$

$$0.538 + 0.517063 \left\lfloor \frac{\arg(2\,434\,685\,456\,749\,964\,820\,480 - z_0)}{2\pi} \right\rfloor \log\left(\frac{1}{z_0}\right) +$$

$$0.517063 \log(z_0) + 0.517063 \left\lfloor \frac{\arg(2\,434\,685\,456\,749\,964\,820\,480 - z_0)}{2\pi} \right\rfloor \log(z_0) -$$

$$0.517063 \sum_{k=1}^{\infty} \frac{(-1)^k (2\,434\,685\,456\,749\,964\,820\,480 - z_0)^k z_0^{-k}}{k}$$

Integral representations:

$$0.538 + \frac{1}{1.934} \log(883\,163\,826\,121\,628\,712\,960 + 32\,199\,776\,876\,321\,832\,960 + 16\,755\,940\,501\,114\,060\,800 + 1\,502\,565\,913\,250\,900\,213\,760) =$$

$$0.538 + 0.517063 \int_1^{2\,434\,685\,456\,749\,964\,820\,480} \frac{1}{t} dt$$

[Open code](#)

Enlarge Data Customize A Plaintext Interactive

and:

$$(1.2083+1.5236)+1/8*$$

$$\ln(((883163826121628712960*32199776876321832960*16755940501114060800*1502565913250900213760)))$$

Where 1.2083 and 1.5236 are Hausdorff dimensions:

$$3 \frac{\log(\varphi)}{\log\left(\frac{3 + \sqrt{13}}{2}\right)} \log_2 \left(\frac{1 + \sqrt[3]{73 - 6\sqrt{87}} + \sqrt[3]{73 + 6\sqrt{87}}}{3} \right)$$

Input interpretation:

$$(1.2083 + 1.5236) + \frac{1}{8} \log(883\,163\,826\,121\,628\,712\,960 \times 32\,199\,776\,876\,321\,832\,960 \times 16\,755\,940\,501\,114\,060\,800 \times 1502565\,913\,250\,900\,213\,760)$$

[Open code](#)

- $\log(x)$ is the natural logarithm

Enlarge Data Customize A [Plaintext](#) Interactive

Result:

- Fewer digits
- More digits

26.00380991530120804982513730073263854804359154017397188308...

Series representations:

More

$$(1.2083 + 1.5236) + \frac{1}{8} \log(883\,163\,826\,121\,628\,712\,960 \times 32\,199\,776\,876\,321\,832\,960 \times 16\,755\,940\,501\,114\,060\,800 \times 1502565\,913\,250\,900\,213\,760) = 2.7319 + 0.125 \log(715\,972\,722\,289\,651\,672\,191\,964\,120\,401\,066\,036\,877\,700\,897\,580\,713\,653 \cdot 423\,988\,346\,007\,419\,931\,852\,799\,999) - 0.125 \sum_{k=1}^{\infty} \frac{1}{k} (-1/715\,972\,722\,289\,651\,672\,191\,964\,120\,401\,066\,036\,877\,700\,897 \cdot 580\,713\,653\,423\,988\,346\,007\,419\,931\,852\,799\,999)^k$$

[Open code](#)

Enlarge Data Customize A [Plaintext](#) Interactive

$$\begin{aligned}
& (1.2083 + 1.5236) + \frac{1}{8} \log(883\,163\,826\,121\,628\,712\,960 \times \\
& \quad 32\,199\,776\,876\,321\,832\,960 \times 16\,755\,940\,501\,114\,060\,800 \times \\
& \quad 1502565\,913\,250\,900\,213\,760) = 2.7319 + \frac{1}{4} i\pi \left[\frac{1}{2\pi} \arg(\right. \\
& \quad 715\,972\,722\,289\,651\,672\,191\,964\,120\,401\,066\,036\,877\,700\,897\,580\,713 \cdot \\
& \quad \left. 653\,423\,988\,346\,007\,419\,931\,852\,800\,000 - x) \right] + \\
& \quad \frac{\log(x)}{8} - \frac{1}{8} \sum_{k=1}^{\infty} \frac{1}{k} (-1)^k \\
& \quad (715\,972\,722\,289\,651\,672\,191\,964\,120\,401\,066\,036\,877\,700\,897\,580\,713 \cdot \\
& \quad 653\,423\,988\,346\,007\,419\,931\,852\,800\,000 - x)^k \\
& \quad x^{-k} \text{ for } x < 0
\end{aligned}$$

Open code

$$\begin{aligned}
& (1.2083 + 1.5236) + \frac{1}{8} \log(883\,163\,826\,121\,628\,712\,960 \times 32\,199\,776\,876\,321\,832\,960 \times \\
& \quad 16\,755\,940\,501\,114\,060\,800 \times 1502565\,913\,250\,900\,213\,760) = \\
& \quad 2.7319 + \frac{1}{4} i\pi \left[-\frac{1}{2\pi} (-\pi + \arg(\right. \\
& \quad 715\,972\,722\,289\,651\,672\,191\,964\,120\,401\,066\,036\,877\,700\,897\,580 \cdot \\
& \quad 713\,653\,423\,988\,346\,007\,419\,931\,852\,800\,000 / z_0) + \\
& \quad \left. \arg(z_0) \right] + \frac{\log(z_0)}{8} - \frac{1}{8} \sum_{k=1}^{\infty} \frac{1}{k} (-1)^k \\
& \quad (715\,972\,722\,289\,651\,672\,191\,964\,120\,401\,066\,036\,877\,700\,897\,580\,713 \cdot \\
& \quad 653\,423\,988\,346\,007\,419\,931\,852\,800\,000 - z_0)^k z_0^{-k}
\end{aligned}$$

Integral representations:

$$\begin{aligned}
& (1.2083 + 1.5236) + \frac{1}{8} \log(883\,163\,826\,121\,628\,712\,960 \times \\
& \quad 32\,199\,776\,876\,321\,832\,960 \times 16\,755\,940\,501\,114\,060\,800 \times \\
& \quad 1502565\,913\,250\,900\,213\,760) = 2.7319 + 0.125 \\
& \quad \int_1^{715\,972\,722\,289\,651\,672\,191\,964\,120\,401\,066\,036\,877\,700\,897\,580\,713 \cdot 653\,423\,988\,346\,007\,419\,931\,852\,800\,000} \\
& \quad \frac{1}{t} dt
\end{aligned}$$

Open code

Enlarge Data Customize A [Plaintext](#) Interactive

$$\begin{aligned}
& (1.2083 + 1.5236) + \frac{1}{8} \log(883\,163\,826\,121\,628\,712\,960 \times 32\,199\,776\,876\,321\,832\,960 \times \\
& \quad 16\,755\,940\,501\,114\,060\,800 \times 1502565\,913\,250\,900\,213\,760) = \\
& \quad 2.7319 + \frac{1}{16 i\pi} \int_{-i\infty+\gamma}^{i\infty+\gamma} \frac{1}{\Gamma(1-s)} \\
& \quad 715\,972\,722\,289\,651\,672\,191\,964\,120\,401\,066\,036\,877\,700\,897\,580\,713 \cdot \\
& \quad 653\,423\,988\,346\,007\,419\,931\,852\,799\,999^{-s} \Gamma(-s)^2 \Gamma(1+s) ds \\
& \quad \text{for } -1 < \gamma < 0
\end{aligned}$$

Continued fraction:

Linear form

$$26 + \frac{1}{262 + \frac{1}{2 + \frac{1}{8 + \frac{1}{1 + \frac{1}{3 + \frac{1}{3 + \frac{1}{6 + \frac{1}{1 + \frac{1}{1 + \frac{1}{2 + \frac{1}{2 + \frac{1}{1 + \frac{1}{1 + \frac{1}{2 + \frac{1}{25 + \frac{1}{7 + \frac{1}{5 + \frac{1}{2 + \frac{1}{3 + \frac{1}{9 + \frac{1}{\dots}}$$

[Open code](#)

Enlarge Data Customize A Plaintext **Interactive**

Possible closed forms:

More

$$\frac{10}{9} \pi \operatorname{csch}^2\left(\frac{13627782}{37997993}\right) \approx 26.0038099153012080483702$$

$$\pi \left[\text{root of } 164x^4 - 821x^3 - 5219x^2 + 6455x - 94 \text{ near } x = 8.27727 \right] \approx$$

$$26.003809915301208052894$$

$$\frac{21092636833\pi}{2548260164} \approx 26.0038099153012080498274838$$

•

The result 26.003809915301208049825137300732638548043591540173971 in the context of string theory is a very good approximation to the value of the *critical dimension* that is 26 for the bosonic string theory.

Appendix A

From:

Three-dimensional AdS gravity and extremal CFTs at $c = 8m$

Spyros D. Avramis, Alex Kehagias and Constantina Mattheopoulou

Received: September 7, 2007 - Accepted: October 28, 2007 - Published: November 9, 2007

m	L_0	d	S	S_{BH}
3	1	196883	12.1904	12.5664
	2	21296876	16.8741	17.7715
	3	842609326	20.5520	21.7656
4	2/3	139503	11.8458	11.8477
	5/3	69193488	18.0524	18.7328
	8/3	6928824200	22.6589	23.6954
5	1/3	20619	9.9340	9.3664
	4/3	86645620	18.2773	18.7328
	7/3	24157197490	23.9078	24.7812
6	1	42987519	17.5764	17.7715
	2	40448921875	24.4233	25.1327
	3	8463511703277	29.7668	30.7812
7	2/3	7402775	15.8174	15.6730
	5/3	33934039437	24.2477	24.7812
	8/3	16953652012291	30.4615	31.3460
8	1/3	278511	12.5372	11.8477
	4/3	13996384631	23.3621	23.6954
	7/3	19400406113385	30.5963	31.3460

Table 1: Degeneracies, microscopic entropies and semiclassical entropies for the first few values of m and L_0 .

Phenomenological consequences of superfluid dark matter with baryon-phonon coupling

Lasha Berezhiani - Max-Planck-Institut für Physik, Fohringer Ring 6, 80805 München, Germany

Benoit Famaey - Université de Strasbourg, CNRS UMR 7550, Observatoire astronomique de Strasbourg, - 11 rue de l'Université, F-67000 Strasbourg, France

Justin Khoury - Center for Particle Cosmology, Department of Physics and Astronomy, University of Pennsylvania, Philadelphia PA 19104, USA - (Dated: November 17, 2017)

Using (22) this translates to an upper bound on the mass of the DM particle:

$$m \lesssim 4.2 \left(\frac{\sigma/m}{\text{cm}^2/\text{g}} \right)^{1/4} \text{ eV}. \quad (24)$$

Smaller and less massive galaxies result in a somewhat weaker bound.

The bound (24) on the DM particle mass is the main result of this Section. It shows that for values of σ/m satisfying the merging-cluster bound $\sim 1 \text{ cm}^2/\text{g}$ [85–88], m must be somewhat below 4 eV. The dependence on the

The value of the mass of DM particle is between 4 – 4.2 eV

Appendix B

From Wikipedia:

Order 10

Ramanujan (1988, p. 9) listed four order-10 mock theta functions in his lost notebook, and stated some relations between them, which were proved by Choi (1999, 2000, 2002, 2007).

$$\phi(q) = \sum_{n \geq 0} \frac{q^{n(n+1)/2}}{(q; q^2)_{n+1}} \quad \psi(q) = \sum_{n \geq 0} \frac{q^{(n+1)(n+2)/2}}{(q; q^2)_{n+1}} \quad X(q) = \sum_{n \geq 0} \frac{(-1)^n q^{n^2}}{(-q; q)_{2n}}$$

$$\chi(q) = \sum_{n \geq 0} \frac{(-1)^n q^{(n+1)^2}}{(-q; q)_{2n+1}}$$

For the second mock theta function, we have take the sequence A053282 in the OEIS:

A053282 Coefficients of the '10th order' mock theta function $\psi(q)$.

0, 1, 1, 2, 2, 2, 4, 4, 4, 6, 7, 8, 10, 11, 12, 16, 18, 20, 24, 26, 30, 36, 40, 44, 52, 58, 64, 74, 82, 91, 104, 116, 128, 144, 159, 176, 198, 218, 240, 268, 294, 324, 360, 394, 432, 478, 524, 572, 630, 688, 752, 826, 900, 980, 1072, 1168, 1270, 1386, 1505, 1634 ([list](#); [graph](#); [refs](#); [listen](#); [history](#); [text](#); [internal format](#))

OFFSET 0, 4

COMMENTS Number of partitions (d_1, d_2, \dots, d_m) of n such that $0 < d_1/1 \leq d_2/2 \leq \dots \leq d_m/m$. - [Seiichi Manyama](#), Mar 17 2018

REFERENCES Srinivasa Ramanujan, *The Lost Notebook and Other Unpublished Papers*, Narosa Publishing House, New Delhi, 1988, p. 9

LINKS Vaclav Kotesovec, [Table of \$n, a\(n\)\$ for \$n = 0..10000\$](#) (terms 0..1000 from Seiichi Manyama)
 Youn-Seo Choi, [Tenth order mock theta functions in Ramanujan's lost notebook](#), *Inventiones Mathematicae*, 136 (1999) p. 497-569.

FORMULA G.f.: $\psi(q) = \sum_{n \geq 0} q^{(n+1)(n+2)/2} / ((1-q)(1-q^3)\dots(1-q^{2n+1}))$.
 $a(n) \sim \exp(\pi \sqrt{n/5}) / (2^5 \cdot 5^{1/4} \sqrt{\phi n})$, where $\phi = \frac{1+\sqrt{5}}{2}$ is the golden ratio. - [Vaclav Kotesovec](#), Jun 12 2019

We take the second function:

$$\psi(q) = \sum_{n \geq 0} \frac{q^{(n+1)(n+2)/2}}{(q; q^2)_{n+1}}$$

We have that:

$$-1/q + \left[\sum_{n=0}^{\infty} \frac{q^{(24+1)(24+2)/2}}{q^{(24+1)(24+2)/2+1}} \right], n=0 \dots n$$

$$-\frac{1}{q} + \sum_{n=0}^{\infty} \frac{q^{(24+1)(24+2)/2}}{q^{(24+1)(24+2)/2+1}}$$

Result:

$$\frac{n+1}{q} - \frac{1}{q}$$

And

$$-1/0.461538 + \left[\sum_{n=0}^{\infty} \frac{0.461538^{(24+1)(24+2)/2}}{0.461538^{(24+1)(24+2)/2+1}} \right], n=0 \dots n$$

Input interpretation:

$$-\frac{1}{0.461538} + \sum_{n=0}^{\infty} \frac{0.461538^{(24+1)(24+2)/2}}{0.461538^{(24+1)(24+2)/2+1}}$$

Enlarge Data Customize A Plaintext Interactive

Result:

$$2.16667(n+1) - 2.16667$$

Values:

Less

n	$2.16667(n + 1) - 2.16667$
1	2.16667
2	4.33334
3	6.50001
4	8.66668
5	10.83333
6	13.
7	15.1667
8	17.3334
9	19.5
10	21.6667
11	23.8334
12	26.
13	28.1667
14	30.3334
15	32.5

For $n = 24$, $q = 0,461538$ we have $24 * 2.16667 = 52,00008$ (note that for $n = 12$, the result is 26).

The expression is:

$$[24 * (((0.461538^{((24+1)(24+2)/2)})) / ((((((0.461538^{((24+1)(24+2)/2)+1)})))))))]$$

Input interpretation:

$$24 \times \frac{0.461538^{(24+1) \times (24+2) / 2}}{0.461538^{(24+1) \times (24+2) / 2 + 1}}$$

[Open code](#)

Result:

More digits

52.00005200005200005200005200005200005200005200005200005200005200...

[Open code](#)

Enlarge Data Customize A Plaintext Interactive

Repeating decimal:

52.000052 (period 6)

For $n = 60$, $q = 0,0337$ we have:

$$[60 * (((0.0337^{((60+1)(60+2)/2)})) / (((0.0337^{(((60+1)(60+2)/2)+1)})))]$$

Input:

$$60 \times \frac{0.0337^{(60+1) \times (60+2)/2}}{0.0337^{(60+1) \times (60+2)/2+1}}$$

[Open code](#)

Enlarge Data Customize A [Plaintext](#) [Interactive](#)

Result:

More digits

1780.415430267062314540059347181008902077151335311572700296...

Thence:

$$[(((60 * (((0.0337^{((60+1)(60+2)/2)})) / (((0.0337^{(((60+1)(60+2)/2)+1)})))])) - (((24 * (((0.461538^{((24+1)(24+2)/2)})) / (((0.461538^{(((24+1)(24+2)/2)+1)})))])))]$$

Input interpretation:

$$60 \times \frac{0.0337^{(60+1) \times (60+2)/2}}{0.0337^{(60+1) \times (60+2)/2+1}} - 24 \times \frac{0.461538^{(24+1) \times (24+2)/2}}{0.461538^{(24+1) \times (24+2)/2+1}}$$

[Open code](#)

Enlarge Data Customize A [Plaintext](#) [Interactive](#)

Result:

More digits

1728.415378267010314488059295180956902025151283311520700244...

This result is very near to the mass of candidate glueball $f_0(1710)$ meson. Furthermore, 1728 occurs in the algebraic formula for the j -invariant of an elliptic curve. As a consequence, it is sometimes called a Zagier as a pun on the Gross–Zagier theorem. The number 1728 is one less than the Hardy–Ramanujan number 1729

$$2((((((((([(((60 * (((0.0337^{((60+1)(60+2)/2)})) / (((0.0337^{(((60+1)(60+2)/2)+1)})))] - (((24 * (((0.461538^{((24+1)(24+2)/2)})) / (((0.461538^{(((24+1)(24+2)/2)+1)})))])))))^1/3$$

Input interpretation:

$$2 \sqrt[3]{60 \times \frac{0.0337^{(60+1) \times (60+2)/2}}{0.0337^{(60+1) \times (60+2)/2+1}} - 24 \times \frac{0.461538^{(24+1) \times (24+2)/2}}{0.461538^{(24+1) \times (24+2)/2+1}}}$$

[Open code](#)

Enlarge Data Customize A [Plaintext](#) [Interactive](#)

Result:

More digits

24.0019...

This value is linked to the "Ramanujan function" (an elliptic modular function that satisfies the need for "conformal symmetry") that has 24 "modes" corresponding to the physical vibrations of a bosonic string.

For $n = 18$ and $q = 0.75$, we obtain:

$$[18 * (((0.75^{((18+1)(18+2)/2)})) / ((((((0.75^{((18+1)(18+2)/2)+1)})))))))]$$

Input:

$$18 \times \frac{0.75^{(18+1) \times (18+2)/2}}{0.75^{(18+1) \times (18+2)/2+1}}$$

[Open code](#)

Enlarge Data Customize A [Plaintext](#) [Interactive](#)

Result:

24

For $n = 27$ and $q = 0.3648649$, we obtain:

$$[27 * (((0.3648649^{((27+1)(27+2)/2)})) / ((((((0.3648649^{((27+1)(27+2)/2)+1)})))))))]$$

Input interpretation:

$$27 \times \frac{0.3648649^{(27+1) \times (27+2)/2}}{0.3648649^{(27+1) \times (27+2)/2+1}}$$

[Open code](#)

Enlarge Data Customize A [Plaintext](#) [Interactive](#)

Result:

More digits

73.99999287407476027428234395799650774848443903483179664582...

Note that:

$$[27 * (((0.3648649^{((27+1)(27+2)/2)})) / ((((((0.3648649^{((27+1)(27+2)/2)+1)}))))))] * ((([18 * (((0.75^{((18+1)(18+2)/2)})) / ((((((0.75^{((18+1)(18+2)/2)+1)}))))]))]$$

Input interpretation:

$$27 \times \frac{0.3648649^{(27+1) \times (27+2)/2}}{0.3648649^{(27+1) \times (27+2)/2+1}} \left(18 \times \frac{0.75^{(18+1) \times (18+2)/2}}{0.75^{(18+1) \times (18+2)/2+1}} \right)$$

[Open code](#)

Enlarge Data Customize A [Plaintext](#) [Interactive](#)

Result:

• More digits

1775.999828977794246582776254991916185963626536835963119499...

And

$$\left[\left(\left(27 * \left(\left(0.3648649^{((27+1)(27+2)/2)} \right) \right) / \left(\left(0.3648649^{(((27+1)(27+2)/2+1)} \right) \right) \right) \right)^{1/3} * \left[\left(\left(\left(18 * \left(\left(0.75^{((18+1)(18+2)/2)} \right) \right) / \left(\left(0.75^{(((18+1)(18+2)/2+1)} \right) \right) \right) \right)^{1/3} \right] \right)^{1/3}$$

Input interpretation:

$$\sqrt[3]{27 \times \frac{0.3648649^{(27+1) \times (27+2)/2}}{0.3648649^{(27+1) \times (27+2)/2+1}}} \sqrt[3]{18 \times \frac{0.75^{(18+1) \times (18+2)/2}}{0.75^{(18+1) \times (18+2)/2+1}}}$$

[Open code](#)

Enlarge Data Customize A [Plaintext](#) [Interactive](#)

Result:

• More digits

12.1101...

And

Input interpretation:

$$2 \left(\sqrt[3]{27 \times \frac{0.3648649^{(27+1) \times (27+2)/2}}{0.3648649^{(27+1) \times (27+2)/2+1}}} \sqrt[3]{18 \times \frac{0.75^{(18+1) \times (18+2)/2}}{0.75^{(18+1) \times (18+2)/2+1}}} \right)$$

[Open code](#)

Enlarge Data Customize A [Plaintext](#) [Interactive](#)

Result:

• More digits

24.2202...

Where 12.1101 is very near to the value of black hole entropy 12,19 and 24,2202 is a value that is linked to the "Ramanujan function" (an elliptic modular function that satisfies the need for "conformal symmetry") that has 24 "modes" corresponding to the physical vibrations of a bosonic string.

References

Ramanujan's Lost Notebook, Part I, by George Andrews and Bruce C. Berndt (Springer, 2005, ISBN 0-387-25529-X)^[13]

Ramanujan's Lost Notebook, Part II, George E. Andrews, Bruce C. Berndt (Springer, 2008, ISBN 978-0-387-77765-8)

Ramanujan's Lost Notebook: Part III, George E. Andrews, Bruce C. Berndt (Springer, 2012, ISBN 978-1-4614-3809-0)

Ramanujan's Lost Notebook: Part IV, George E. Andrews, Bruce C. Berndt (Springer, 2013, ISBN 978-1-4614-4080-2)

Collected Papers of Srinivasa Ramanujan – 15 mar 2000 di Srinivasa Ramanujan (Autore), G. H. Hardy (a cura di), P. V. Seshu Aiyar (a cura di), B. M. Wilson (a cura di), Bruce Berndt (Collaboratore)