A comparison of Bell's Theorem and Malus's Law: action-at-a-distance is not required in order to explain results of Bell's Theorem experiments

Austin J. Fearnley
Manchester
England
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Abstract

This paper shows that, using counterfactual definiteness, there is an enforceable duality between results of Malus Law experiments and the results from Bell experiments. The results are shown here to be equivalent in the two experiments subject to extending the Malus experiment by doubling it to match the structure of the results table of a Bell experiment. The Malus intensities also need to be converted into counterfactual correlations in order to enable results in both experiments to be compared using a common statistic. It is therefore possible to use the duality to explain the more esoteric Bell results via the simpler Malus results. As Malus results involve singleton particles rather than matched pairs of particles then there is no requirement for action at a distance nor entanglement to feature in an explanation of Malus results and therefore, using the duality, neither in Bell results. The 'magic' in Bell's Theorem results is not eliminated as it still exists contained within Malus results, and that 'magic' [of somehow exceeding the Bell Inequalities] remains unexplained by this paper, except it is shown that the 'magic' does not involve action-at-a-distance nor entanglement.

Malus's Law and Bell's Theorem

In 1802 Malus showed empirically that the intensity of plane polarised light after being passed through a polarising filter at an angle of θ to the plane polarised light had an intensity proportional to $\cos^2 \theta$. It is assumed here that the intensity of the filtered light is proportional to the number of photons passing through the filter.

In Quantum Mechanics the correlation (Bell correlation) between the results of researchers Alice and Bob are correlated by the amount $-\cos \theta$ where θ is the angle between the settings of the detectors of Alice and Bob in an experiment testing Bell's Theorem.

In a classical simulation of the Bell correlation, using local hidden variables, the sawtooth pattern of results is given by a classical correlation = $2*\theta/\pi -1$. These three functions are shown in Figure A for values of θ between 0 and 90^o.

Figure A **Plots of $\cos^2\theta$, $2*\theta/\pi$ -1 (i.e. sawtooth, where θ is in radians) and -cos θ: for θ between 0° and 90°**

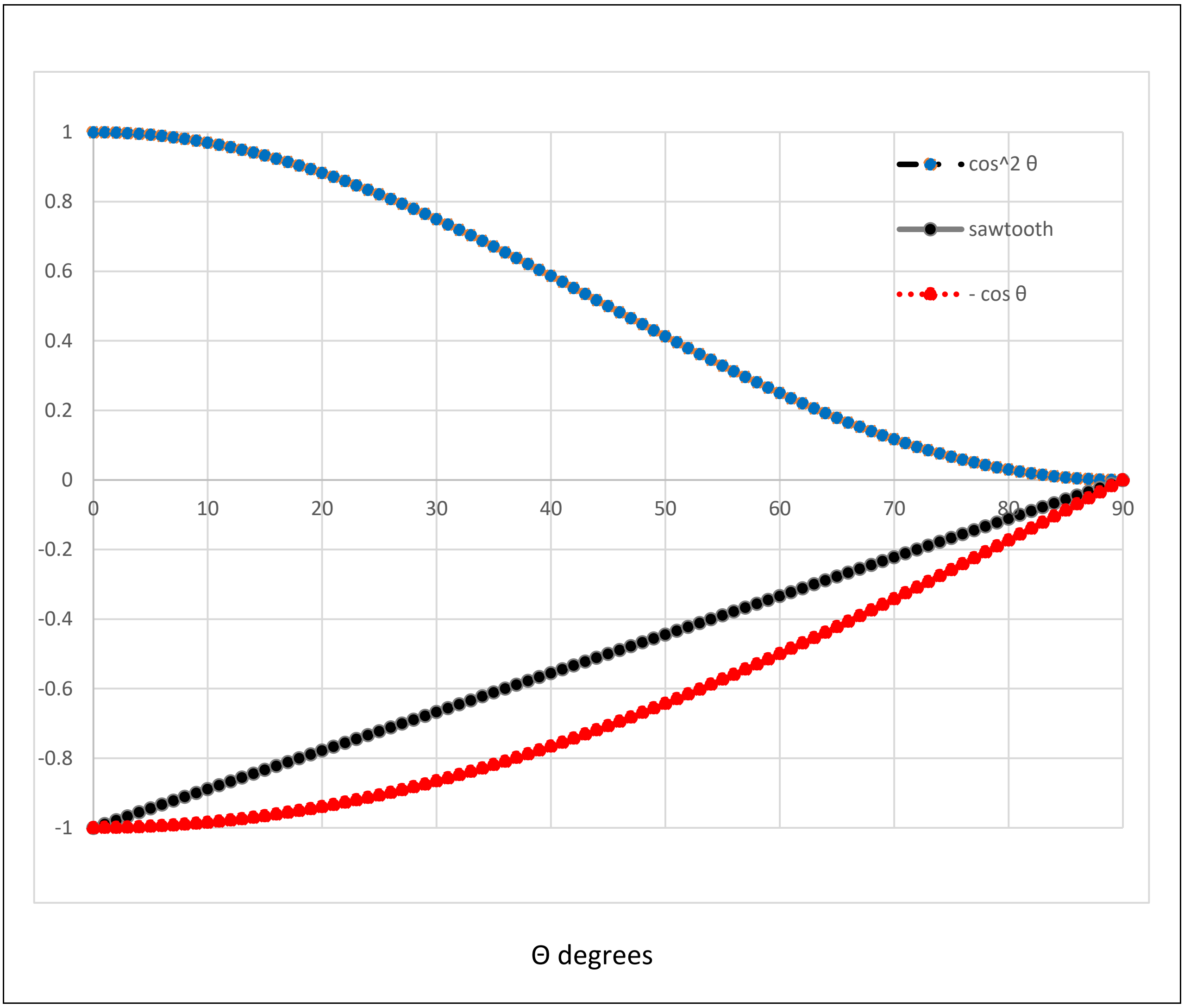


Figure A is included as it shows some interesting aspects of the functions. The sawtooth curve (which is merely a straight line between θ values of 0° and 90°) is regular in that the classical Bell correlations for θ = 0°, 30°, 60° and 90° are evenly spaced out at -1, -2/3, -1/3 and 0. The -cos θ curve represents the value of the Quantum Mechanics Bell correlation and is greater (in its absolute value) than the sawtooth value for any given θ. The $\cos^2\theta$ curve at the top of Figure A also has a regular pattern, but this time for Malus intensities for θ = 0°, 30°, 45°, 60° and 90° with the intensities being 1, 3/4, 1/2 and ¼ and 0, respectively.

Comparison of results in Malus and Bell experiments

Malus and Bell results should not strictly be compared as Malus results are for intensities only, which I interpret as proportions derived from numbers of photons. Malus results do not use correlations nor do they use entangled states of pairs of photons and this is simply because Malus results are based on singleton particles.

The theory of Bell's experiments makes use of counterfactual definiteness: what would you expect the results to be if 'this or that' were to be done. And that is when one cannot actually do the 'this or that' in the experiment as one can only measure the state of a particle once: which is because a particle is no longer in the same state after measurement as it was before the measurement.

A way is needed to compare Malus with Bell results despite their apparent differences. To make comparisons, a formula is needed to convert correlations into intensities and, conversely, the same formula can supply a hypothetical correlation for a given intensity. At present simply note that the justification in physics for the latter will be shown later in the Results and Discussion sections, while the paper at this point deals with the mathematics of the comparisons.

In an analysis of Bell results, with a negative correlation found, there is an anti-symmetrical table with four cells, for example: (A, B) = (1, 1), (1, 0), (0, 1) and (0, 0). The proportion of particles' results found in each cell is p++, p+-, p-+ and p--, respectively. When the correlation coefficient between A and B is negative then the value of p++ will contain the smaller proportion and p+- will contain the larger proportion. Knowing the value of p++ (= a measure of intensity) completely determines the value of correlation due to there being only one degree of freedom in the table of results.

It is easy to show that p++ = (1 + correlation coefficient)/4 and, conversely,

correlation coefficient = 4 * p++ - 1

A complication in making comparisons is that Bell correlations can be based on electrons or photons whereas the Malus Law is only for use with photons. Electrons have spin ½ whereas photons have spin 1. The effect of spin is that a Bell result on electrons for an angle of θ is equivalent or dual to a Bell result on photons for an angle of θ/2.

Results

Result I Malus results for photons with θ = 22.5°

Malus's Law shows that if a beam of plane polarised light is used as a source, and then a polarising filter at an angle of 22.5° is applied, then the intensity of the beam passing through the filter is $\cos^2 22.5^o = 0.8536$. Next, this result needs to be specified closer to the way in which Bell results can be displayed. Let Alice control the initial filter which produces a plane polarised beam out of an unpolarised beam. Her filter rejects particles which could be measured as A = 0 and allows through her filter particles with A = 1. So in Table 1, the Row for A = 1 represents all the particles used in the Malus experiment. The equal number of particles for which A = 0 are rejected from the Malus experiment. Bob controls the second filter which is at an angle of 22.5° to Alice's first filter. At an angle of 22.5° the bulk of the beam passes through Bob's filter and this bulk corresponds to the intensity of 0.8536. In displaying this result in Table 1 it should be noted that 0.8536 is a proportion of particles for which A = 1, which is only half the particles in the whole table. So this result must be halved to give 0.427 and placed in the cell (A, B) = (1, 1).

Table 1 Malus results for photons with θ = 22.5°

Results as proportions	B = 1	B = 0	Total
A = 1	P++ = 0.427	0.073	0.5
A = 0	0.073	0.427	0.5
Total	0.5	0.5	1.0

The discarded particles, which never entered the Malus experiment, are equivalent to the second row where A = 0. These particles could be used in a thought experiment is a second Malus experiment where Bob measures them, again at an angle of 22.5°. Using counterfactual definiteness, the result would again be an intensity of 0.8536 halved to 0.427 and placed in the (0, 0) cell of Table 1.

Still ignoring the physical propriety of this thought experiment, these proportions convert as follows to a hypothetical correlation:

correlation coefficient = 4 * p++ - 1

= 4 * 0.427 - 1

= 0.707

Result II **Bell results for electrons with θ = 45°**

The Bell result for electrons at θ = 45° is dual to the Bell result for photons at θ = 22.5° and should be comparable with the Malus result in Result I above.

The Quantum Mechanics correlation is cos 45° = - 0.707.

So using p++ = (1 + correlation coefficient)/4,

Then intensity = p++ = (1 − 0.707)/4 = 0.073.

Table 2 **Bell results for electrons with θ = 45°**

Results as proportions	B = 1	B = 0	Total
A = 1	P++ = 0.073	0.427	0.5
A = 0	0.427	0.073	0.5
Total	0.5	0.5	1.0

Table 2 exactly matches the results of Table 1 except that Table 1 is symmetric while Table 2 is anti-symmetric. This matching verifies the duality of the Malus result in Table 1 and the Bell results I Table 2. The reason that Table 1 is symmetric is that if Alice's Malus filter passes 50% of the photons, most of those plane polarised particles will also pass through Bob's filter when θ = 22.5°. However, when Alice detects 50 per cent of electrons in the Bell experiment, then most of the anti-parallel positrons paired with those electrons will *not* be detected by Bob when his setting is θ = 45°. So the Table for Bell is anti-symmetric and the correlation is negative.

Next, this matching will be replicated for a different angle.

**Result III		Malus results for photons with $\theta = 30^o$**

Malus's Law for a polarising angle of 30^o, as used in Bob's filter, results in an intensity of $\cos^2 30^o = 0.75$. The proportion 0.75/2 = 0.375 is then put in cell (1, 1) of Table 3.

**Table 3		Malus results for photons with $\theta = 30^o$**

Results as proportions	B = 1	B = 0	Total
A = 1	P++ = 0.375	0.125	0.5
A = 0	0.125	0.375	0.5
Total	0.5	0.5	1.0

The intensity of 0.75 is seen from 0.375/0.5 from the top row of Table 1. The top row can be equated with the plane polarised particles used in a Malus experiment. The discarded particles, which never entered the Malus experiment, are equivalent to the second row where A = 0.

These proportions convert as follows to a hypothetical correlation:

correlation coefficient = 4 * p++ - 1

= 4 * 0.75/2 - 1

= 0.5

**Result IV		Bell results for electrons with $\theta = 60^o$**

The Bell result for electrons at $\theta = 60$ is dual to the Bell result for photons at $\theta = 30^o$ and should be comparable with the Malus result in Result III above.

The Quantum Mechanics correlation is $- \cos 60^o = - 0.5$.

So using p++ = (1 + correlation coefficient)/4,

Then p++ = (1 – 0.5)/4 = 0.125.

Table 4 Bell results for electrons with θ = 60°

Results as proportions	B = 1	B = 0	Total
A = 1	P++ = 0.125	0.375	0.5
A = 0	0.375	0.125	0.5
Total	0.5	0.5	1.0

Table 4 matches Table 3 which verifies the duality of the Malus result in Table 3 and the Bell results in Table 4.

Discussion

How can a singleton particle experiment be compared with a matched pair experiment?

The comparison needs the use of counterfactual definiteness to supply hypothetical data for row 2 of the Tables 1 and 3, as Bell experiments use four cells whereas Malus experiments use only two cells. The counterfactual definite data do not appear on the surface to be matched data but, however, experience with simulating Bell experiments, particularly using generation of random vectors on a 3D unit sphere, shows a way to compare Malus with Bell results despite the apparent differences. In generating matched data for a Bell computer simulation, it is necessary to generate pairs of particles with vectors pointing in exactly opposite directions in 3D. This is done for Bell (local hidden variable) simulations, but cannot be done overtly on a Malus experiment measuring intensities. If a sufficiently large number of singleton vectors (for row 1 of the table) is generated randomly on the surface of a sphere then the whole surface of the sphere is uniformly sampled. This can be repeated to obtain data for row 2. Although particles in these two data sets are not overtly paired, they could be paired, if one bothered to do so, as the whole surfaces are uniformly sampled. But it is not worth pairing them as only the intensities are being measured in the modified/extended Malus thought experiment. The more data that are generated the better the matching of the pairs.

Matching the pairs as opposite vectors, enforces the conservation of linear and angular momentum in the Bell experiments. It is only necessary to see that pairs could be matched in the extended Malus experiment so as to be able to claim parity of results with the Bell experiments. Overt matching of pairs for the extended Malus experiment is not required, but for sufficiently large sets of generated particle vectors, approximate matching is implicitly guaranteed. A real experiment on extended Malus data, however, could not hope to guarantee matching unless a very large number of photons were to be used

The Bell experiment is confounded by entanglement and notions of action-at–a-distance. Instead the focus should be on the Malus experiment. The results in Table 1 for the extended Malus thought experiment contains within it, in the first row, the results which could be obtainable from real Malus experiments where Alice's detector represents the first polarising filter which selects a beam of polarised light, labelled A = 1: that is the top row of Table 1. This beam is then passed through a second polarising filter, at an angle of 22.5 degrees to that of Alice, which is controlled by Bob and the intensity of the beam passing through Bob's filter is represented by p++/0.5 in the top row. That is, intensity = 0.427/0.5 = 0.8536 = $\cos^2 22.5^o$ which is in accordance with Malus's Law.

Results in Table 1 for polarised photons are claimed in this paper to be dual to the Bell results in Table 2 for electrons. Table 2 gives the Quantum Mechanics Bell correlation between A and B of -cos 45^o = -0.707. This correlation is shown to equate, using counterfactual definiteness, to a Malus intensity of 0.8536 as found in Table 1 for the equivalent Malus experiment.

Appendix A of Ref.4 (Fearnley, 2017) gives a computer program to simulate an event-by-event Bell experiment for electrons where $\theta = 45^o$. The software allows one to use integer values of measurements A and B, where the result is a correlation between A and B of 0.5. This is a result which would lie on the classical sawtooth line (see Figure A above), were the sign of the correlation coefficient to be reversed. The design of the simulation deliberately targets a positive correlation as it was deemed to be less confusing to deal with positive values. To do that, the simulation let Alice and Bob measure identical vectors of particles rather than vectors with opposite directions. In a real experiment two successive measurements cannot be achieved because the first measurement destroys the original state of the particle. But this can be achieved in a simulation where the original state of a particle is recoverable for its re-use. In effect the target result is changed from an anti-symmetric table such as Table 2, above, to a symmetric target similar to Table 1. The simulation in Ref. 4 does not achieve its target correlation because Bell's Inequalities were not broken in that attempt. Generating local hidden vectors to simulate individual particles has never been shown to produce a correlation of +/- cos 45^o in an event-by-event (that is, particle-at-a-time) simulation of a Bell experiment for electrons.

The sawtooth classical correlation of 0.5 is given by the set of (A, B) measurements in Table 5.

p++ = (1 + correlation coefficient)/4, hence

p++ = (1 + 0.5)/4 = 0.375.

This is equivalent to an intensity of 0.375*2 = 0.75 in a simulated Malus experiment for photons for θ = 22.5° using local hidden variables. In a real Malus experiment the intensity would be 0.8536 as shown in Table 1.

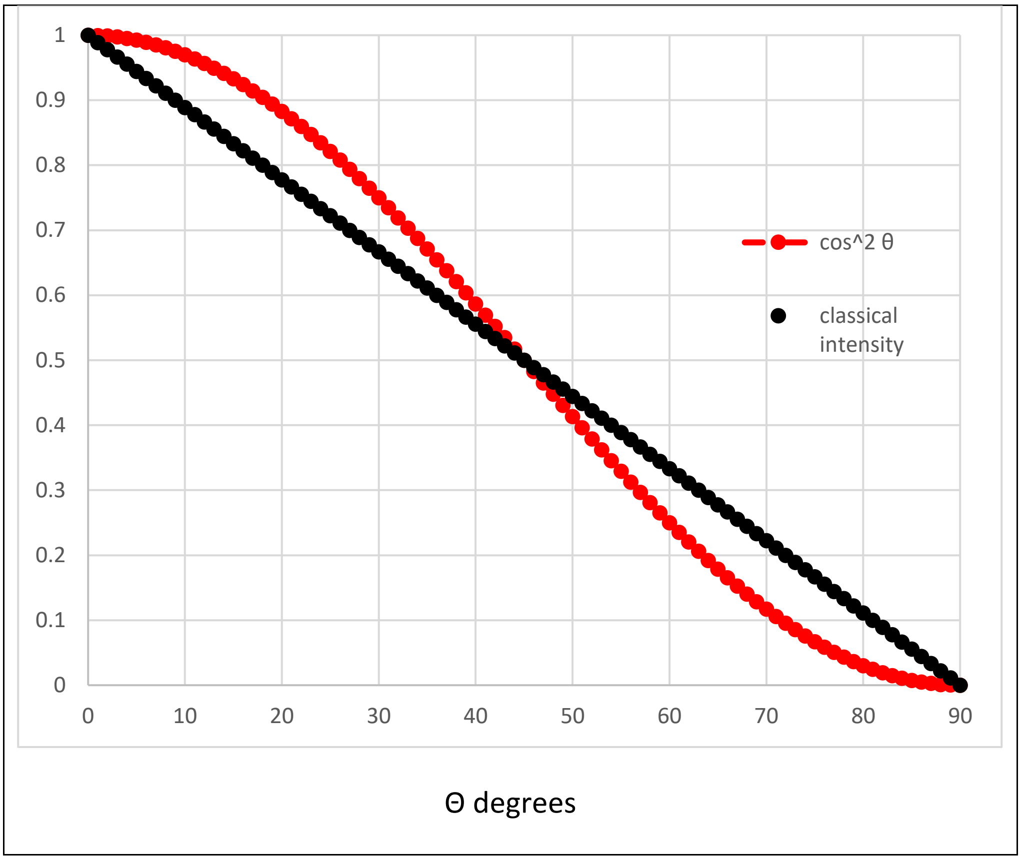
Table 5 Results for a Bell experiment *simulation* for electrons with θ = 45° (targeted at a positive correlation of 0.707 but only obtaining 0.5)

Results as proportions	B = 1	B = 0	Total
A = 1	P++ = 0.375	0.125	0.5
A = 0	0.125	0.375	0.5
Total	0.5	0.5	1.0

There is a duality between obtaining an intensity of 0.75 in a local hidden variables simulation for Malus at 22.5° (instead of the real experiment result of 0.8536) and of obtaining a correlation of -0.5 in a local hidden variables simulation for Bell (instead of the real experiment result of -0.707) at 45° using electrons.

The 'magic' of the real Bell experiment is in having a higher Bell correlation (absolute value) than achievable by an event-by-event simulation using local hidden variables. In the real experiment the Bell Inequalities are apparently broken. The same 'magic' is present in the Malus intensities in real experiments exceeding the intensities found by such simulations. As the 'magic' is present in real experiments for both Bell and Malus, it is simpler to look for the explanation of the 'magic' using the Malus Intensities rather than the Bell correlations. There is no need to be concerned with entanglement in the Malus experiments and therefore, because of the duality, there is no need to be concerned with entanglement in the explanation of the common 'magic' in the two experiments. This paper concludes that entanglement and the associated action-at-a-distance are irrelevant to the 'magic' found in real Bell and Malus experiments. Instead the simpler focus should be concentrated on the identical 'magic' that is seen in Figure B, which is that the Malus Intensity is different from the intensity found with simulations using local hidden variables.

Figure B **Plots of $\cos^2 \theta$ (= the Malus Intensity) and $1 - 2*\theta/\pi$ (= the classical intensity using local hidden variables, with θ in radians) for θ between 0° and 90°**



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Manchester
England