

**On the new developments concerning the Mock theta functions of various order.  
Further mathematical connections with some sectors of Particle Physics and  
Black Hole Physics.**

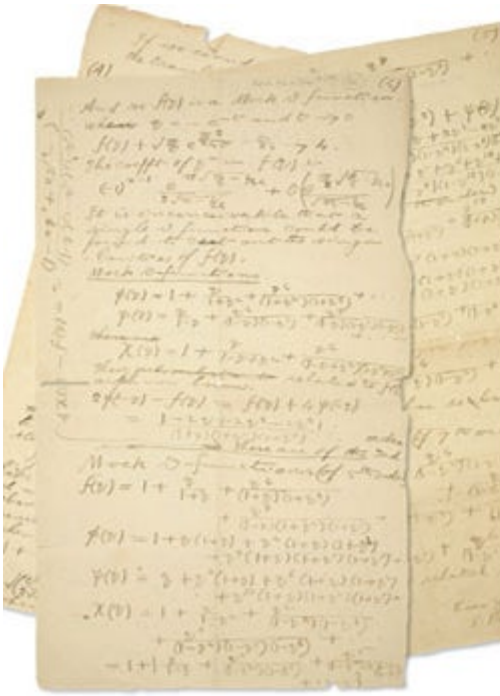
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**Abstract**

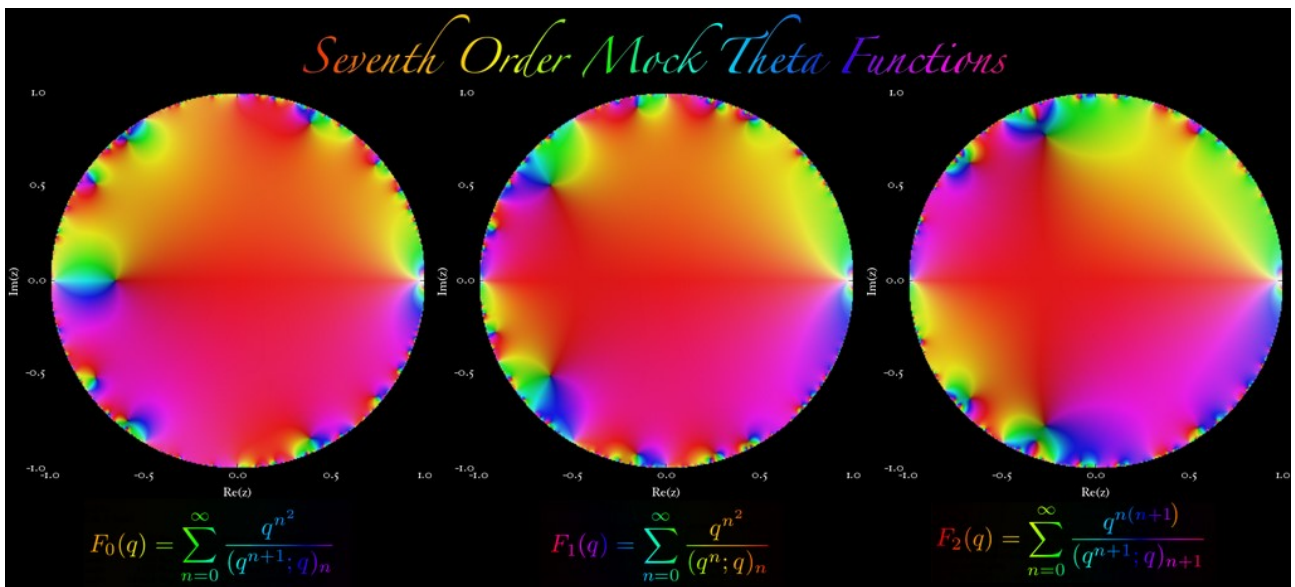
*In the present research thesis, we have obtained further interesting mathematical connections with various Ramanujan's Mock theta functions of order 8, order 7, order 6, order 2 and some sectors of Particle Physics and Black Hole Physics.*

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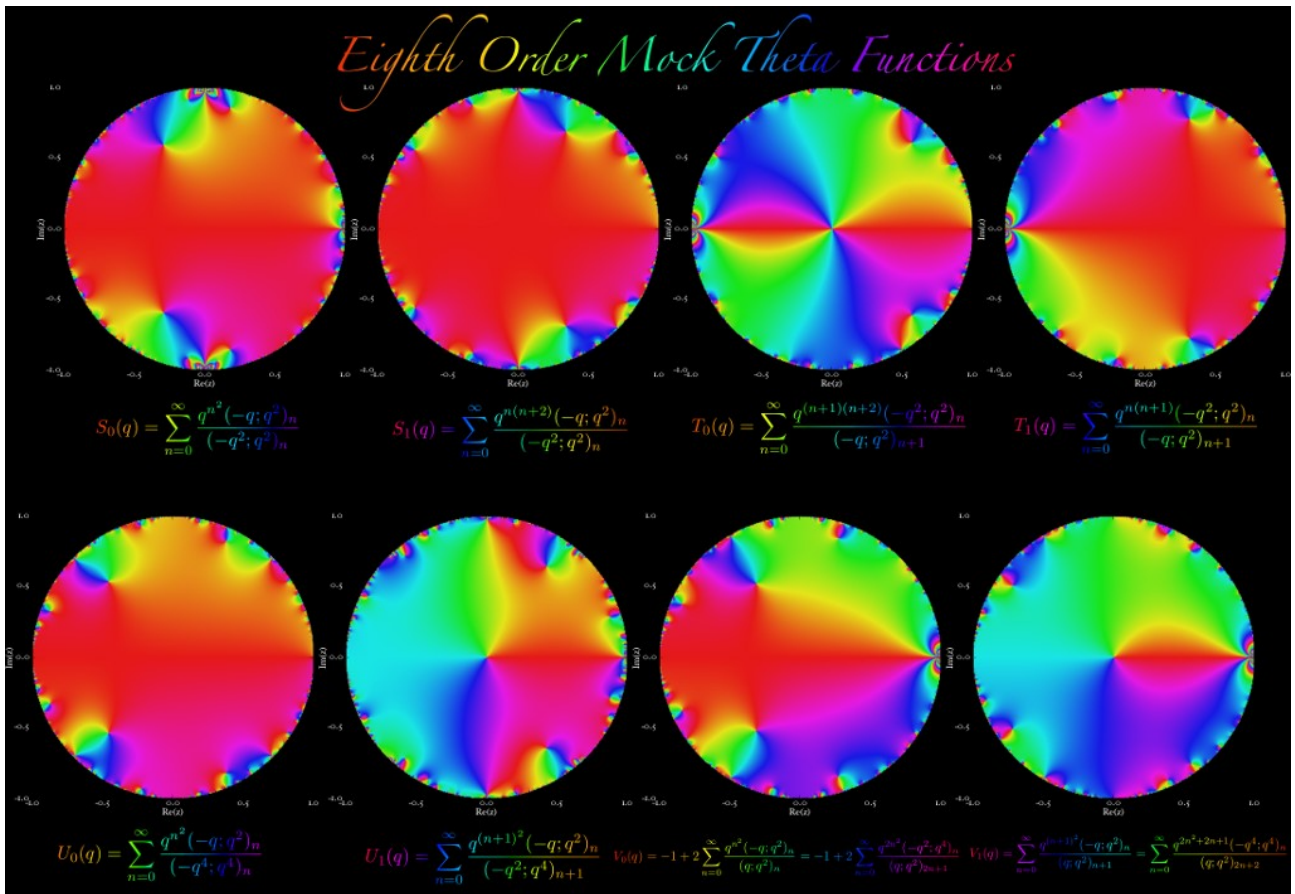
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<https://www.scientificamerican.com/article/one-of-srinivasa-ramanujans-neglected-manuscripts-has-helped-solve-long-standing-mathematical-mysteries/?redirect=1>



<http://owen.maresh.info/mocktheta.html>



<http://owen.maresh.info/mocktheta.html>

Gordon & McIntosh (2000) found eight mock theta functions of order 8. They found five linear relations involving them, and expressed four of the functions as Appell–Lerch sums, and described their transformations under the modular group. **The two functions  $V_1$  and  $U_0$  were found earlier by Ramanujan (1988, p. 8, eqn 1; p. 29 eqn 6) in his lost notebook.**

### Order 6

Ramanujan (1988) wrote down seven mock theta functions of order 6 in his lost notebook, and stated 11 identities between them, which were proved in (Andrews & Hickerson 1991). Two of Ramanujan's identities relate  $\phi$  and  $\psi$  at various arguments, four of them express  $\phi$  and  $\psi$  in terms of Appell–Lerch series, and the last five identities express the remaining five sixth-order mock theta functions in terms of  $\phi$  and  $\psi$ . Berndt & Chan (2007) discovered two more sixth order functions. The order 6 mock theta functions are:

$$\begin{aligned} \phi(q) &= \sum_{n \geq 0} \frac{(-1)^n q^{n^2} (q; q^2)_n}{(-q; q)_{2n}} \quad (\text{sequence A053268 in the OEIS}) \\ \psi(q) &= \sum_{n \geq 0} \frac{(-1)^n q^{(n+1)^2} (q; q^2)_n}{(-q; q)_{2n+1}} \quad (\text{sequence A053269 in the OEIS}) \\ \rho(q) &= \sum_{n \geq 0} \frac{q^{n(n+1)/2} (-q; q)_n}{(q; q^2)_{n+1}} \quad (\text{sequence A053270 in the OEIS}) \\ \sigma(q) &= \sum_{n \geq 0} \frac{q^{(n+1)(n+2)/2} (-q; q)_n}{(q; q^2)_{n+1}} \quad (\text{sequence A053271 in the OEIS}) \\ \lambda(q) &= \sum_{n \geq 0} \frac{(-1)^n q^n (q; q^2)_n}{(-q; q)_n} \quad (\text{sequence A053272 in the OEIS}) \\ 2\mu(q) &= \sum_{n \geq 0} \frac{(-1)^n q^{n+1} (1 + q^n) (q; q^2)_n}{(-q; q)_{n+1}} \quad (\text{sequence A053273 in the OEIS}) \\ \gamma(q) &= \sum_{n \geq 0} \frac{q^{n^2} (q; q)_n}{(q^3; q^3)_n} \quad (\text{sequence A053274 in the OEIS}) \\ \phi_-(q) &= \sum_{n \geq 1} \frac{q^n (-q; q)_{2n-1}}{(q; q^2)_n} \quad (\text{sequence A153251 in the OEIS}) \\ \psi_-(q) &= \sum_{n \geq 1} \frac{q^n (-q; q)_{2n-2}}{(q; q^2)_n} \quad (\text{sequence A153252 in the OEIS}) \end{aligned}$$

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**Relations Connecting Eighth Order Mock Theta Functions - *Roselin Antony***

Mock theta functions of order 8:

Gordon and McIntosh[8] found the following eight mock theta functions of order 8;

$$S_0(q) = \sum_{n=0}^{\infty} \frac{q^{n^2} (-q; q^2)_n}{(-q^2; q^2)_n} \quad (1.3)$$

$$S_1(q) = \sum_{n=0}^{\infty} \frac{q^{n(n+2)} (-q; q^2)_n}{(-q^2; q^2)_n} \quad (1.4)$$

$$T_0(q) = \sum_{n=0}^{\infty} \frac{q^{(n+1)(n+2)} (-q^2; q^2)_n}{(-q; q^2)_{n+1}} \quad (1.5)$$

$$T_1(q) = \sum_{n=0}^{\infty} \frac{q^{n(n+1)} (-q^2; q^2)_n}{(-q; q^2)_{n+1}} \quad (1.6)$$

$$U_0(q) = \sum_{n=0}^{\infty} \frac{q^{n^2} (-q; q^2)_n}{(-q^4; q^4)_n} \quad (1.7)$$

$$U_1(q) = \sum_{n=0}^{\infty} \frac{q^{(n+1)^2} (-q; q^2)_n}{(-q^2; q^4)_{n+1}} \quad (1.8)$$

$$V_0(q) = -1 + 2 \sum_{n=0}^{\infty} \frac{q^{n^2} (-q; q^2)_n}{(q; q^2)_n} \quad (1.9)$$

$$= -1 + 2 \sum_{n=0}^{\infty} \frac{q^{2n^2} (-q^2; q^4)_n}{(q; q^2)_{2n+1}}$$

$$V_1(q) = \sum_{n=0}^{\infty} \frac{q^{(n+1)^2} (-q; q^2)_n}{(q; q^2)_{n+1}} \quad (1.10)$$

$$= \sum_{n=0}^{\infty} \frac{q^{2n^2+2n+1} (-q^4; q^4)_n}{(q; q^2)_{2n+2}}$$

Now, we analyze some mock theta functions of order 8.

From Wikipedia:

$$V_1(q) = \sum_{n \geq 0} \frac{q^{(n+1)^2} (-q; q^2)_n}{(q; q^2)_{n+1}} = \sum_{n \geq 0} \frac{q^{2n^2+2n+1} (-q^4; q^4)_n}{(q; q^2)_{2n+2}}$$

$$V_0(q) = -1 + 2 \sum_{n \geq 0} \frac{q^{n^2} (-q; q^2)_n}{(q; q^2)_n} = -1 + 2 \sum_{n \geq 0} \frac{q^{2n^2} (-q^2; q^4)_n}{(q; q^2)_{2n+1}}$$

$$U_1(q) = \sum_{n \geq 0} \frac{q^{(n+1)^2} (-q; q^2)_n}{(-q^2; q^4)_{n+1}}$$

$$U_0(q) = \sum_{n \geq 0} \frac{q^{n^2} (-q; q^2)_n}{(-q^4; q^4)_n}$$

We have from OEIS, that

$$V_1(q) = \sum_{n \geq 0} \frac{q^{(n+1)^2} (-q; q^2)_n}{(q; q^2)_{n+1}} = \sum_{n \geq 0} \frac{q^{2n^2+2n+1} (-q^4; q^4)_n}{(q; q^2)_{2n+2}}$$

Is:  $\sum_{n \geq 0} q^{((n+1)^2)} (1+q)(1+q^3) \dots (1+q^{(2n-1)}) / ((1-q)(1-q^3) \dots (1-q^{(2n+1)}))$

sum  $0.5^{((n+1)^2)} (1+0.5)(1+0.5^3) (1+0.5^{(2n-1)}) / ((1-0.5)(1-0.5^3) (1-0.5^{(2n+1)}))$ ,  
 $n = 0$  to  $5$

Sum:

$$\sum_{n=0}^5 \frac{(1+0.5)(1+0.5^3) 0.5^{(n+1)^2} (0.5^{2n-1} + 1)}{(1-0.5)(1-0.5^3) (1-0.5^{2n+1})} = \frac{5939749430500584097989}{495247224344833687552}$$

[Open code](#)

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[Decimal approximation:](#)

[More digits](#)

11.99350372605989606003952886769686016385896304671547803396...

[Open code](#)

$$\left( \left( \left( \sum_{n=0}^5 0.5^{(n+1)^2} (1+0.5)(1+0.5^3)(1+0.5^{2n-1}) / ((1-0.5)(1-0.5^3)(1-0.5^{2n+1})) \right) \right) \right)^3$$

Input interpretation:

$$\left( \sum_{n=0}^5 0.5^{(n+1)^2} (1+0.5)(1+0.5^3) \times \frac{1+0.5^{2n-1}}{(1-0.5)(1-0.5^3)(1-0.5^{2n+1})} \right)^3$$

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Result:

1725.2

$$2 * \sum_{n=0}^5 0.5^{(n+1)^2} (1+0.5)(1+0.5^3)(1+0.5^{2n-1}) / ((1-0.5)(1-0.5^3)(1-0.5^{2n+1})), n = 0 \text{ to } 5$$

Input interpretation:

$$2 \sum_{n=0}^5 0.5^{(n+1)^2} (1+0.5)(1+0.5^3) \times \frac{1+0.5^{2n-1}}{(1-0.5)(1-0.5^3)(1-0.5^{2n+1})}$$

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Result:

23.987

$$\left[ \left( \left( \left( \sum_{n=0}^5 0.5^{(n+1)^2} (1+0.5)(1+0.5^3)(1+0.5^{2n-1}) / ((1-0.5)(1-0.5^3)(1-0.5^{2n+1})) \right) \right) \right)^3 \right]^{1/15}$$

Input interpretation:

$$15 \sqrt{\left( \sum_{n=0}^5 0.5^{(n+1)^2} (1+0.5)(1+0.5^3) \times \frac{1+0.5^{2n-1}}{(1-0.5)(1-0.5^3)(1-0.5^{2n+1})} \right)^3}$$

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Result:

1.64357

1.64357 very near to the value of  $\zeta(2) = \pi^2/6 = 1.6449\dots$

Then:

$$\left( \left( \left( \sum_{n=0}^5 q^{(n+1)^2} (1+q)(1+q^3)(1+q^{2n-1}) / ((1-q)(1-q^3)(1-q^{2n+1})) \right) \right) \right)$$

Input:

$$\frac{q^{(n+1)^2} (1+q)(1+q^3)(1+q^{2n-1})}{(1-q)(1-q^3)(1-q^{2n+1})}$$

[Open code](#)

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Alternate form:

$$\frac{(q+1)^2 ((q-1)q+1) q^{n(n+2)} (q^{2n}+q)}{(q-1)^2 (q^2+q+1)(q^{2n+1}-1)}$$

Expanded form:

Step-by-step solution

$$\frac{q^{(n+1)^2}}{(1-q)(1-q^3)(1-q^{2n+1})} + \frac{q^{(n+1)^2+1}}{(1-q)(1-q^3)(1-q^{2n+1})} +$$

$$\frac{q^{(n+1)^2+3}}{(1-q)(1-q^3)(1-q^{2n+1})} + \frac{q^{(n+1)^2+4}}{(1-q)(1-q^3)(1-q^{2n+1})} + \frac{q^{(n+1)^2+2n-1}}{(1-q)(1-q^3)(1-q^{2n+1})} +$$

$$\frac{q^{(n+1)^2+2n}}{(1-q)(1-q^3)(1-q^{2n+1})} + \frac{q^{(n+1)^2+2n+2}}{(1-q)(1-q^3)(1-q^{2n+1})} + \frac{q^{(n+1)^2+2n+3}}{(1-q)(1-q^3)(1-q^{2n+1})}$$

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Series expansion at  $q = 0$ :

$$\frac{1}{q^{2n+1}-1} q^{n(n+2)} \left( q^{2n} (-1 - 2q - 2q^2 - 4q^3 - 6q^4 - 6q^5 + O(q^6)) + \right.$$

$$\left. (-q - 2q^2 - 2q^3 - 4q^4 - 6q^5 + O(q^6)) \right)$$

[Open code](#)

Series expansion at  $q = \infty$ :

$$\frac{1}{q^{2n+1}-1} q^{n(n+2)} \left( q^{2n} \left( -1 - \frac{2}{q} - \frac{2}{q^2} - \frac{4}{q^3} - \frac{6}{q^4} - \frac{6}{q^5} - \frac{8}{q^6} - \frac{10}{q^7} - \frac{10}{q^8} + O\left(\left(\frac{1}{q}\right)^9\right) \right) + \right.$$

$$\left. \left( -q - 2 - \frac{2}{q} - \frac{4}{q^2} - \frac{6}{q^3} - \frac{6}{q^4} - \frac{8}{q^5} - \frac{10}{q^6} - \frac{10}{q^7} + O\left(\left(\frac{1}{q}\right)^8\right) \right) \right)$$

Derivative:

Step-by-step solution

$$\frac{\partial}{\partial q} \left( \frac{q^{(n+1)^2} (1+q)(1+q^3)(1+q^{2n-1})}{(1-q)(1-q^3)(1-q^{2n+1})} \right) =$$

$$-\left( (1+q)q^{(n+1)^2-2} \left( -n^2 q^{2n+2} + n^2 q^{2n+8} - (n^2 + 2n-1)q^{4n+1} + (n^2 + 2n-1)q^{4n+7} + \right. \right.$$

$$2q^{2n+1} + 6q^{2n+3} - 6q^{2n+5} - n(n+4)q^{2n+6} - 2q^{2n+7} - 2q^{4n+2} -$$

$$8q^{4n+4} - 2q^{4n+6} + n(n+4)q^{2n} - (n+1)^2 q^7 + (n+1)^2 q +$$

$$\left. \left. 2q^6 + 8q^4 + 2q^2 \right) \right) / \left( (q-1)^3 (q^2 + (1+q))^2 (1-q^{2n+1})^2 \right)$$

For  $q = 0.5$  and  $n = 2$ , we obtain:

$$\left( \frac{(0.5^{(2+1)^2} (1+0.5)(1+0.5^3)(1+0.5^{2*2-1}))}{(1-0.5)(1-0.5^3)(1-0.5^{2*2+1})} \right)$$

Input:

$$\frac{0.5^{(2+1)^2} (1+0.5)(1+0.5^3)(1+0.5^{2*2-1})}{(1-0.5)(1-0.5^3)(1-0.5^{2*2+1})}$$

[Open code](#)

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Result:

More digits

0.008748559907834101382488479262672811059907834101382488479...

[Open code](#)

Repeating decimal:



0.0087485599078341013824884792626728110 (period 30)

And

$$(0.923910279+0.924340867) * [(((0.5^{(2+1)^2}(1+0.5)(1+0.5^3)(1+0.5^{(2*2-1)})) / ((1-0.5)(1-0.5^3)(1-0.5^{(2*2+1)})))] * 10^2$$

where 0.923910279 and 0.924340867 are results of some Ramanujan mock theta functions (see our previous papers)

Input interpretation:

$$(0.923910279 + 0.924340867) \left( \frac{0.5^{(2+1)^2} (1 + 0.5) (1 + 0.5^3) (1 + 0.5^{2 \times 2 - 1})}{(1 - 0.5) (1 - 0.5^3) (1 - 0.5^{2 \times 2 + 1})} \times 10^2 \right)$$

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Result:

More digits

1.616953587550403225806451612903225806451612903225806451612...

[Open code](#)

Repeating decimal:

1.6169535875504032258064516129 (period 15)

1.616953587550403225806451612903225806451612903225806451612

This result is a golden number, near to the value of golden ratio

Continued fraction:

Linear form

•

$$1 + \frac{1}{1 + \frac{1}{1 + \frac{1}{1 + \frac{1}{1 + \frac{1}{1 + \frac{1}{3 + \frac{1}{6 + \frac{1}{2 + \frac{1}{2 + \frac{1}{3 + \frac{1}{6 + \frac{1}{2 + \frac{1}{1 + \frac{1}{3 + \frac{1}{5 + \frac{1}{8 + \frac{1}{5 + \frac{1}{1 + \frac{1}{1 + \frac{1}{\dots}}}}}}}}}}}}}}}}}}}}}}}}}}}}}}}}}}$$

[Open code](#)

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Possible closed forms:

More

- $-\csc\left(\tan\left(\frac{159430603}{19353094}\right)\right) \approx 1.6169535875504032246440$

$$\begin{aligned} & \boxed{\text{root of } 11310x^3 - 17874x^2 - 4598x + 6353 \text{ near } x = 1.61695} \approx \\ & \frac{1.6169535875504032258058173}{2804211324\pi} \approx 1.61695358755040322579991 \\ & \frac{5448325643}{5448325643} \approx 1.61695358755040322579991 \end{aligned}$$

$$1,616953587550403225 * 5448325643 \approx 2804211324\pi$$

$$\begin{aligned} \pi &= (8809689694,591707445757398675 / 2804211324) = \\ &= 3,1415926535897932369084886681671 \end{aligned}$$

Now:

$$V_0(q) = -1 + 2 \sum_{n \geq 0} \frac{q^{n^2} (-q; q^2)_n}{(q; q^2)_n} = -1 + 2 \sum_{n \geq 0} \frac{q^{2n^2} (-q^2; q^4)_n}{(q; q^2)_{2n+1}}$$

$$V_0(q) = -1 + 2 \sum_{n \geq 0} q^{n^2} (1+q)(1+q^3) \dots (1+q^{2n-1}) / ((1-q)(1-q^3) \dots (1-q^{2n-1}))$$

Input interpretation:

$$-1 + 2 \sum_{n=0}^5 q^{n^2} (1+q)(1+q^3) \times \frac{1+q^{2n-1}}{(1-q)(1-q^3)(1-q^{2n-1})}$$

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Result:

$$\begin{aligned} & 2 \left( \frac{(q+1)^2 (q^3+1)q}{(1-q)^2 (1-q^3)} + \frac{\left(\frac{1}{q}+1\right)(q+1)(q^3+1)}{\left(1-\frac{1}{q}\right)(1-q)(1-q^3)} + \right. \\ & \quad \frac{(q+1)(q^3+1)^2 q^4}{(1-q)(1-q^3)^2} + \frac{(q+1)(q^3+1)(q^9+1)q^{25}}{(1-q)(1-q^3)(1-q^9)} + \\ & \quad \left. \frac{(q+1)(q^3+1)(q^7+1)q^{16}}{(1-q)(1-q^3)(1-q^7)} + \frac{(q+1)(q^3+1)(q^5+1)q^9}{(1-q)(1-q^3)(1-q^5)} \right) - 1 \end{aligned}$$

$$-1 + 2 \sum_{n=0}^5 0.5^{n^2} (1+0.5)(1+0.5^3)(1+0.5^{2n-1}) / ((1-0.5)(1-0.5^3)(1-0.5^{2n-1}))$$

Input interpretation:

$$-1 + 2 \sum_{n=0}^5 0.5^{n^2} (1+0.5)(1+0.5^3) \times \frac{1+0.5^{2n-1}}{(1-0.5)(1-0.5^3)(1-0.5^{2n-1})}$$

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Result:

-11.9354

$-27 + \left[ \left( \left( \left( -1 + 2 \sum_{n=0}^5 0.5^{n^2} (1+0.5)(1+0.5^3)(1+0.5^{2n-1}) \right) / \left( (1-0.5)(1-0.5^3)(1-0.5^{2n-1}) \right) \right) \right)^3 \right]$

Input interpretation:

$$-27 + \left( -1 + 2 \sum_{n=0}^5 0.5^{n^2} (1+0.5)(1+0.5^3) \times \frac{1+0.5^{2n-1}}{(1-0.5)(1-0.5^3)(1-0.5^{2n-1})} \right)^3$$

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Result:

-1727.23

1727.23

This result is very near to the mass of candidate glueball  $f_0(1710)$  meson.

$2 \left[ \left( \left( \left( -1 + 2 \sum_{n=0}^5 0.5^{n^2} (1+0.5)(1+0.5^3)(1+0.5^{2n-1}) \right) / \left( (1-0.5)(1-0.5^3)(1-0.5^{2n-1}) \right) \right) \right)^3 \right]$

Input interpretation:

$$2 \left( -1 + 2 \sum_{n=0}^5 0.5^{n^2} (1+0.5)(1+0.5^3) \times \frac{1+0.5^{2n-1}}{(1-0.5)(1-0.5^3)(1-0.5^{2n-1})} \right)$$

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Result:

-23.8707

23.8707 This result is very near to the value of black hole entropy 23.90

$\left( \left( \left( -(-27 + \left[ \left( \left( -1 + 2 \sum_{n=0}^5 0.5^{n^2} (1+0.5)(1+0.5^3)(1+0.5^{2n-1}) \right) / \left( (1-0.5)(1-0.5^3)(1-0.5^{2n-1}) \right) \right) \right)^3 \right) \right) \right)^{1/15}$

Input interpretation:

$$15 \sqrt{-27 + \left( -1 + 2 \sum_{n=0}^5 0.5^{n^2} (1+0.5)(1+0.5^3) \times \frac{1+0.5^{2n-1}}{(1-0.5)(1-0.5^3)(1-0.5^{2n-1})} \right)^3}$$

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Result:

1.6437

1.6437 very near to the value of  $\zeta(2) = \pi^2/6 = 1.6449...$

Then:

$-1 + 2 * q^{(n^2)}(1+q)(1+q^3)...(1+q^{(2n-1)}) / ((1-q)(1-q^3)...(1-q^{(2n-1)}))$

$$-1 + 2 * (((q^{n^2})(1+q)(1+q^3)(1+q^{2n-1}))/(((1-q)(1-q^3)(1-q^{2n-1})))$$

Input:

$$-1 + 2 \times \frac{q^{n^2} (1+q)(1+q^3)(1+q^{2n-1})}{(1-q)(1-q^3)(1-q^{2n-1})}$$

[Open code](#)

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Alternate forms:

$$-\frac{2(q+1)^2((q-1)q+1)(q^{2n}+q)q^{n^2}}{(q-1)^2(q^2+q+1)(q^{2n}-q)} - 1$$

[Open code](#)

$$-\left(\frac{2q^{n^2+1} + 2q^{n^2+2} + 2q^{n^2+4} + 2q^{n^2+5} + 2q^{n^2+2n} + 2q^{n^2+2n+1} + 2q^{n^2+2n+3} + 2q^{n^2+2n+4} - q^{2n+1} - q^{2n+3} + q^{2n+4} + q^{2n} - q^5 + q^4 + q^2 - q}{(q-1)^2(q^2+q+1)(q^{2n}-q)}\right)$$

[Open code](#)

Expanded form:

Step-by-step solution

$$\frac{2q^{n^2}}{(1-q)(1-q^3)(1-q^{2n-1})} + \frac{2q^{n^2+1}}{(1-q)(1-q^3)(1-q^{2n-1})} + \frac{2q^{n^2+3}}{(1-q)(1-q^3)(1-q^{2n-1})} + \frac{2q^{n^2+4}}{(1-q)(1-q^3)(1-q^{2n-1})} + \frac{2q^{n^2+2n-1}}{(1-q)(1-q^3)(1-q^{2n-1})} + \frac{2q^{n^2+2n}}{(1-q)(1-q^3)(1-q^{2n-1})} + \frac{2q^{n^2+2n+2}}{(1-q)(1-q^3)(1-q^{2n-1})} + \frac{2q^{n^2+2n+3}}{(1-q)(1-q^3)(1-q^{2n-1})} - 1$$

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Series expansion at q = 0:

$$-\frac{1}{(-q + O(q^6)) + q^{2n}} \left( q^{n^2} (2q + 4q^2 + 4q^3 + 8q^4 + 12q^5 + O(q^6)) + q^{n(n+2)} (2 + 4q + 4q^2 + 8q^3 + 12q^4 + 12q^5 + O(q^6)) + (-q + O(q^6)) + q^{2n} \right)$$

Derivative:

Step-by-step solution

$$\frac{\partial}{\partial q} \left( -1 + \frac{2(q^{n^2}(1+q)(1+q^3)(1+q^{2n-1}))}{(1-q)(1-q^3)(1-q^{2n-1})} \right) = -\left(\frac{2(1+q)q^{n^2-1}(n^2(q^6-1)(q^{4n}-q^2) - 4n(q^6-1)q^{2n+1} + 2q(q^{2n+6} - 4q^{4n+2} - q^{4n+4} - q^{4n} - q^{2n} + q^6 + 4q^4 + q^2))}{((q-1)^3(q^2+(1+q))^2(q-q^{2n})^2)}\right)$$

For  $q = 0,5$  and  $n = 2$ , we obtain:

$$-1 + 2 * (((0.5^{(2^2)})(1+0.5)(1+0.5^3)(1+0.5^{(2*2-1)})))/(((1-0.5)(1-0.5^3)(1-0.5^{(2*2-1))}))$$

Input:

$$-1 + 2 \times \frac{0.5^{2^2} (1 + 0.5) (1 + 0.5^3) (1 + 0.5^{2 \times 2 - 1})}{(1 - 0.5) (1 - 0.5^3) (1 - 0.5^{2 \times 2 - 1})}$$

[Open code](#)

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Result:

More digits

•  $-0.38010204081632653061224489795918367346938775510204081632\dots$

[Open code](#)

$$\text{sqrt}(\frac{-1}{-1 + 2 * (((0.5^{(2^2)})(1+0.5)(1+0.5^3)(1+0.5^{(2*2-1)})))/(((1-0.5)(1-0.5^3)(1-0.5^{(2*2-1))}))})$$

Input:

$$\sqrt{\frac{1}{-1 + 2 \times \frac{0.5^{2^2} (1 + 0.5) (1 + 0.5^3) (1 + 0.5^{2 \times 2 - 1})}{(1 - 0.5) (1 - 0.5^3) (1 - 0.5^{2 \times 2 - 1})}}}$$

[Open code](#)

Enlarge Data Customize A Plaintext Interactive

Result:

Fewer digits

More digits

•  $1.621996449817778272583121046910102734468496179612698348604\dots$

$1.6219964498177782725831210469101027344684961796126983$

$1.62199644\dots$  is a golden number

Continued fraction:

• Linear form



$$U_1(q) = \sum_{n \geq 0} \frac{q^{(n+1)^2} (-q; q^2)_n}{(-q^2; q^4)_{n+1}}$$

Sum\_{n >= 0}

$$q^{((n+1)^2)}(1+q)(1+q^3)\dots(1+q^{(2n-1)})/((1+q^2)(1+q^6)\dots(1+q^{(4n+2)}))$$

sum  $q^{((n+1)^2)}(1+q)(1+q^3)(1+q^{(2n-1)})/((1+q^2)(1+q^6)(1+q^{(4n+2)}))$ , n = 0 to 5

Result:

$$\sum_{n=0}^5 \frac{q^{(n+1)^2} (1+q)(1+q^3)(1+q^{2n-1})}{(1+q^2)(1+q^6)(1+q^{4n+2})} =$$

$$\frac{\left(\frac{1}{q} + 1\right)(q+1)(q^3+1)q}{(q^2+1)^2(q^6+1)} + \frac{(q+1)^2(q^3+1)q^4}{(q^2+1)(q^6+1)^2} + \frac{(q+1)(q^3+1)^2q^9}{(q^2+1)(q^6+1)(q^{10}+1)} +$$

$$\frac{(q+1)(q^3+1)(q^9+1)q^{36}}{(q^2+1)(q^6+1)(q^{22}+1)} + \frac{(q+1)(q^3+1)(q^7+1)q^{25}}{(q^2+1)(q^6+1)(q^{18}+1)} + \frac{(q+1)(q^3+1)(q^5+1)q^{16}}{(q^2+1)(q^6+1)(q^{14}+1)}$$

[Open code](#)

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Result:

$$\sum_{n=0}^5 \frac{q^{(n+1)^2} (1+q)(1+q^3)(1+q^{2n-1})}{(1+q^2)(1+q^6)(1+q^{4n+2})} = \frac{1}{(q^2+1)^2(q^6+1)^2}$$

$$q(q+1)(q^3+1) \left( \left(\frac{1}{q} + 1\right)(q^6+1) + (q+1)(q^2+1)q^3 + \right.$$

$$\left. \frac{(q^2+1)(q^6+1)(q^9+1)q^{35}}{q^{22}+1} + \frac{(q^2+1)(q^6+1)(q^7+1)q^{24}}{q^{18}+1} + \right.$$

$$\left. \frac{(q^2+1)(q^5+1)(q^6+1)q^{15}}{q^{14}+1} + \frac{(q^2+1)(q^3+1)(q^6+1)q^8}{q^{10}+1} \right)$$

[Open code](#)

sum  $0.5^{((n+1)^2)}(1+0.5)(1+0.5^3)(1+0.5^{(2n-1)})/((1+0.5^2)(1+0.5^6)(1+0.5^{(4n+2)}))$ , n = 0 to 5

Sum:

$$\sum_{n=0}^5 \frac{(1+0.5)(1+0.5^3)0.5^{(n+1)^2}(0.5^{2n-1}+1)}{(1+0.5^2)(1+0.5^6)(0.5^{4n+2}+1)} = \frac{43\,313\,555\,682\,788\,469\,893\,707\,383}{25\,171\,851\,645\,640\,221\,982\,720\,000}$$

[Open code](#)

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Decimal approximation:

More digits

1.720713926513642100085597524089282549536162124629177698628...

[Open code](#)

$$8+10^3\left[\left(\left(\sum_{n=0}^5 0.5^{(n+1)^2} (1+0.5)(1+0.5^3)(1+0.5^{2n-1})\right)/\left((1+0.5^2)(1+0.5^6)(1+0.5^{4n+2})\right)\right), n = 0 \text{ to } 5\right]$$

Input interpretation:

$$8 + 10^3 \sum_{n=0}^5 0.5^{(n+1)^2} (1 + 0.5)(1 + 0.5^3) \times \frac{1 + 0.5^{2n-1}}{(1 + 0.5^2)(1 + 0.5^6)(1 + 0.5^{4n+2})}$$

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Result:

1728.71

1728.71

This result is very near to the mass of candidate glueball  $f_0(1710)$  meson. Furthermore, 1728 occurs in the algebraic formula for the [j-invariant](#) of an [elliptic curve](#). As a consequence, it is sometimes called a Zagier as a pun on the Gross–Zagier theorem. The number 1728 is one less than the Hardy–Ramanujan number [1729](#)

$$\left(\left(\left(\left(\left(\left(8+10^3\left[\left(\left(\sum_{n=0}^5 0.5^{(n+1)^2} (1+0.5)(1+0.5^3)(1+0.5^{2n-1})\right)/\left((1+0.5^2)(1+0.5^6)(1+0.5^{4n+2})\right)\right), n = 0 \text{ to } 5\right)\right]\right)\right)\right)\right)\right)^{1/3}$$

Input interpretation:

$$\sqrt[3]{8 + 10^3 \sum_{n=0}^5 0.5^{(n+1)^2} (1 + 0.5)(1 + 0.5^3) \times \frac{1 + 0.5^{2n-1}}{(1 + 0.5^2)(1 + 0.5^6)(1 + 0.5^{4n+2})}}$$

Enlarge Data Customize A [Plaintext](#) [Interactive](#)

Result:

12.0017

This result is very near to the value of black hole entropy 12,1904

$$2\left(\left(\left(\left(\left(\left(8+10^3\left[\left(\left(\sum_{n=0}^5 0.5^{(n+1)^2} (1+0.5)(1+0.5^3)(1+0.5^{2n-1})\right)/\left((1+0.5^2)(1+0.5^6)(1+0.5^{4n+2})\right)\right), n = 0 \text{ to } 5\right)\right]\right)\right)\right)\right)\right)^{1/3}$$

Input interpretation:

$$2 \sqrt[3]{8 + 10^3 \sum_{n=0}^5 0.5^{(n+1)^2} (1 + 0.5)(1 + 0.5^3) \times \frac{1 + 0.5^{2n-1}}{(1 + 0.5^2)(1 + 0.5^6)(1 + 0.5^{4n+2})}}$$

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Result:

24.0033

This value is linked to the "Ramanujan function" (an elliptic modular function that satisfies the need for "conformal symmetry") that has 24 "modes" corresponding to the physical vibrations of a [bosonic string](#).



$$\left(\frac{8+10^3 \sum_{n=0}^5 0.5^{(n+1)^2} (1+0.5)(1+0.5^3)(1+0.5^{2n-1})}{(1+0.5^2)(1+0.5^6)(1+0.5^{4n+2})}, n = 0 \text{ to } 5\right)^{1/15}$$

Input interpretation:

$$\sqrt[15]{8 + 10^3 \sum_{n=0}^5 0.5^{(n+1)^2} (1 + 0.5)(1 + 0.5^3) \times \frac{1 + 0.5^{2n-1}}{(1 + 0.5^2)(1 + 0.5^6)(1 + 0.5^{4n+2})}}$$

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Result:

1.6438

1.6438 is very near to the value of  $\zeta(2) = \pi^2/6 = 1.6449\dots$

Then:

$$\left(\frac{q^{(n+1)^2} (1+q)(1+q^3)(1+q^{2n-1})}{(1+q^2)(1+q^6)(1+q^{4n+2})}\right)$$

Input:

$$\frac{q^{(n+1)^2} (1+q)(1+q^3)(1+q^{2n-1})}{(1+q^2)(1+q^6)(1+q^{4n+2})}$$

[Open code](#)

Enlarge Data Customize A [Plaintext](#) [Interactive](#)

Expanded form:

Step-by-step solution

$$\begin{aligned} & \frac{q^{(n+1)^2}}{(q^2+1)(q^6+1)(q^{4n+2}+1)} + \frac{q^{(n+1)^2+1}}{(q^2+1)(q^6+1)(q^{4n+2}+1)} + \frac{q^{(n+1)^2+3}}{(q^2+1)(q^6+1)(q^{4n+2}+1)} + \\ & \frac{q^{(n+1)^2+4}}{(q^2+1)(q^6+1)(q^{4n+2}+1)} + \frac{q^{(n+1)^2+2n-1}}{(q^2+1)(q^6+1)(q^{4n+2}+1)} + \\ & \frac{q^{(n+1)^2+2n}}{(q^2+1)(q^6+1)(q^{4n+2}+1)} + \frac{q^{(n+1)^2+2n+2}}{(q^2+1)(q^6+1)(q^{4n+2}+1)} + \frac{q^{(n+1)^2+2n+3}}{(q^2+1)(q^6+1)(q^{4n+2}+1)} \end{aligned}$$

Alternate form assuming n and q are positive:

$$\frac{(q+1)^2 (q^2 - q + 1) q^{n(n+2)} (q^{2n} + q)}{(q^2+1)^2 (q^4 - q^2 + 1) (q^{4n+2} + 1)}$$

Series expansion at  $q = 0$ :

$$\frac{q^{n(n+2)} (q^{2n} (1 + q - q^2 + 2q^4 + O(q^6)) + (q + q^2 - q^3 + 2q^5 + O(q^6)))}{q^{4n+2} + 1}$$

[Open code](#)

• [Big-O notation](#)

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Series expansion at  $q = \infty$ :

$$\frac{q^{n(n+2)} \left( q^{2n} \left( \left(\frac{1}{q}\right)^4 + \left(\frac{1}{q}\right)^5 - \left(\frac{1}{q}\right)^6 + \frac{2}{q^8} + O\left(\left(\frac{1}{q}\right)^9\right) \right) + \left( \left(\frac{1}{q}\right)^3 + \left(\frac{1}{q}\right)^4 - \left(\frac{1}{q}\right)^5 + \frac{2}{q^7} + O\left(\left(\frac{1}{q}\right)^8\right) \right) \right)}{q^{4n+2} + 1}$$

[Open code](#)

Derivative:

Step-by-step solution

$$\frac{\partial}{\partial q} \left( \frac{q^{(n+1)^2} (1+q)(1+q^3)(1+q^{2n-1})}{(1+q^2)(1+q^6)(1+q^{4n+2})} \right) =$$

$$\left( \frac{q^{(n+1)^2} \left( -2(1+q)(1+q^3)(1+q^6)(q^{2n}+q)(q^{4n+2}+1) + 3(1+q)(1+q^2)(1+q^6) \right. \right.}{6(1+q)(1+q^2)(1+q^3)q^4(q^{2n}+q)(q^{4n+2}+1) -$$

$$\left. \left. (q^{4n+2}+1)q^{2n-2} + (1+q^2)(1+q^3)(1+q^6)(q^{2n-1}+1)(q^{4n+2}+1) + \right. \right.$$

$$\left. \left. (n+1)^2(1+q)(1+q^2)(1+q^3)(1+q^6)(q^{2n-1}+1)(q^{4n+2}+1) \right) \right) /$$

$$\left( (1+q^2)^2(1+q^6)^2(q^{4n+2}+1)^2 \right)$$

$$\frac{(((0.5^{(2+1)^2})(1+0.5)(1+0.5^3)(1+0.5^{(2*2-1)})))}{(((1+0.5^2)(1+0.5^6)(1+0.5^{(4*2+2)})))}$$

Input:

$$\frac{0.5^{(2+1)^2} (1+0.5)(1+0.5^3)(1+0.5^{2*2-1})}{(1+0.5^2)(1+0.5^6)(1+0.5^{4*2+2})}$$

[Open code](#)

Enlarge Data Customize A [Plaintext](#) [Interactive](#)

Result:

More digits

0.002917823639774859287054409005628517823639774859287054409...

[Open code](#)

And:

$$\frac{(((1/ [(((0.5^{(2+1)^2})(1+0.5)(1+0.5^3)(1+0.5^{(2*2-1)}))) /$$

$$(((1+0.5^2)(1+0.5^6)(1+0.5^{(4*2+2)})))]))^{1/12}}$$

Input:

$$\frac{1}{\sqrt[12]{\frac{0.5^{(2+1)^2} (1+0.5)(1+0.5^3)(1+0.5^{2*2-1})}{(1+0.5^2)(1+0.5^6)(1+0.5^{4*2+2})}}}$$

[Open code](#)

Enlarge Data Customize A [Plaintext](#) [Interactive](#)

Result:

Fewer digits



3.141592653589793238374480837648859414940262834340694258080

Now:

$$U_0(q) = \sum_{n \geq 0} \frac{q^{n^2} (-q; q^2)_n}{(-q^4; q^4)_n}$$

Sum\_{n >= 0}

$$q^{n^2} (1+q)(1+q^3)\dots(1+q^{2n-1})/((1+q^4)(1+q^8)\dots(1+q^{4n})).$$

sum q^{n^2} (1+q)(1+q^3)(1+q^{2n-1})/((1+q^4)(1+q^8)(1+q^{4n})), n = 0 to 5

Result:

$$\sum_{n=0}^5 \frac{q^{n^2} (1+q)(1+q^3)(1+q^{2n-1})}{(1+q^4)(1+q^8)(1+q^{4n})} = \frac{1}{2(q^4+1)^2(q^8+1)^2} \\ (q+1)(q^3+1) \left( 2(q+1)(q^8+1)q + \left(\frac{1}{q}+1\right)(q^4+1)(q^8+1) + \right. \\ \left. 2(q^3+1)(q^4+1)q^4 + \frac{2(q^4+1)(q^7+1)(q^8+1)q^{16}}{q^{16}+1} + \right. \\ \left. \frac{2(q^5+1)(q^8+1)q^9}{q^8-q^4+1} + \frac{2(q^4+1)(q^8+1)(q^9+1)q^{25}}{q^{20}+1} \right)$$

[Open code](#)

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Sum:

$$\sum_{n=0}^5 \frac{(q+1)(q^3+1)q^{n^2}(q^{2n-1}+1)}{(q^4+1)(q^8+1)(q^{4n}+1)} = \\ \frac{(q+1)(q^3+1)^2q^4}{(q^4+1)(q^8+1)^2} + \frac{(q+1)^2(q^3+1)q}{(q^4+1)^2(q^8+1)} + \frac{\left(\frac{1}{q}+1\right)(q+1)(q^3+1)}{2(q^4+1)(q^8+1)} + \\ \frac{(q+1)(q^3+1)(q^7+1)q^{16}}{(q^4+1)(q^8+1)(q^{16}+1)} + \frac{(q+1)(q^3+1)(q^9+1)q^{25}}{(q^4+1)(q^8+1)(q^{20}+1)} + \frac{(q+1)(q^3+1)(q^5+1)q^9}{(q^4+1)(q^8+1)(q^{12}+1)}$$

[Open code](#)

sum 0.5^{n^2} (1+0.5)(1+0.5^3)(1+0.5^{2n-1})/((1+0.5^4)(1+0.5^8)(1+0.5^{4n})), n = 0 to 5

Sum:

$$\sum_{n=0}^5 \frac{(1+0.5)(1+0.5^3)0.5^{n^2}(0.5^{2n-1}+1)}{(1+0.5^4)(1+0.5^8)(0.5^{4n}+1)} = \frac{4289088487767912062955}{1190142788577664467008}$$

[Open code](#)

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[Decimal approximation:](#)

[More digits](#)

3.603843613499340604148347497397255268795565619063424356565...

[Open code](#)

$$\left(\frac{(27 \times 3 + 2)}{25} \sum_{n=0}^5 0.5^{n^2} (1 + 0.5)(1 + 0.5^3)(1 + 0.5^{2n-1}) / ((1 + 0.5^4)(1 + 0.5^8)(1 + 0.5^{4n}))\right)^3$$

Input interpretation:

$$\left(\left(\frac{1}{25} (27 \times 3 + 2)\right) \sum_{n=0}^5 0.5^{n^2} (1 + 0.5)(1 + 0.5^3) \times \frac{1 + 0.5^{2n-1}}{(1 + 0.5^4)(1 + 0.5^8)(1 + 0.5^{4n})}\right)^3$$

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Result:

1712.82

$$\left[\left(\frac{(27 \times 3 + 2)}{25} \sum_{n=0}^5 0.5^{n^2} (1 + 0.5)(1 + 0.5^3)(1 + 0.5^{2n-1}) / ((1 + 0.5^4)(1 + 0.5^8)(1 + 0.5^{4n}))\right)^3\right]^{1/3}$$

Input interpretation:

$$\sqrt[3]{\left(\left(\frac{1}{25} (27 \times 3 + 2)\right) \sum_{n=0}^5 0.5^{n^2} (1 + 0.5)(1 + 0.5^3) \times \frac{1 + 0.5^{2n-1}}{(1 + 0.5^4)(1 + 0.5^8)(1 + 0.5^{4n})}\right)^3}$$

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Result:

11.9648

This result is very near to the value of black hole entropy 12,1904

$$2 * \left[\left(\frac{(27 \times 3 + 2)}{25} \sum_{n=0}^5 0.5^{n^2} (1 + 0.5)(1 + 0.5^3)(1 + 0.5^{2n-1}) / ((1 + 0.5^4)(1 + 0.5^8)(1 + 0.5^{4n}))\right)^3\right]^{1/3}$$

Input interpretation:

$$2 \sqrt[3]{\left(\left(\frac{1}{25} (27 \times 3 + 2)\right) \sum_{n=0}^5 0.5^{n^2} (1 + 0.5)(1 + 0.5^3) \times \frac{1 + 0.5^{2n-1}}{(1 + 0.5^4)(1 + 0.5^8)(1 + 0.5^{4n})}\right)^3}$$

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Result:

23.9295

This result is very near to the value of black hole entropy 23.90

$$\left[\left(\frac{(27 \times 3 + 2)}{25} \sum_{n=0}^5 0.5^{n^2} (1 + 0.5)(1 + 0.5^3)(1 + 0.5^{2n-1}) / ((1 + 0.5^4)(1 + 0.5^8)(1 + 0.5^{4n}))\right)^3\right]^{1/15}$$

Input interpretation:

$$\sqrt[15]{\left(\left(\frac{1}{25} (27 \times 3 + 2)\right) \sum_{n=0}^5 0.5^{n^2} (1 + 0.5)(1 + 0.5^3) \times \frac{1 + 0.5^{2n-1}}{(1 + 0.5^4)(1 + 0.5^8)(1 + 0.5^{4n})}\right)^3}$$

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Result:

1.64279

1.64279 is very near to the value of  $\zeta(2) = \pi^2/6 = 1.6449\dots$

Then:

$$\frac{((q^{n^2}(1+q)(1+q^3)(1+q^{2n-1})))}{(((1+q^4)(1+q^8)(1+q^{4n})))}$$

Input:

$$\frac{q^{n^2} (1+q)(1+q^3)(1+q^{2n-1})}{(1+q^4)(1+q^8)(1+q^{4n})}$$

[Open code](#)

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Values:

$n$	
0	$\frac{\left(\frac{1}{q} + 1\right)(q + 1)(q^3 + 1)}{2(q^4 + 1)(q^8 + 1)}$
1	$\frac{q(q + 1)^2(q^3 + 1)}{(q^4 + 1)^2(q^8 + 1)}$
2	$\frac{q^4(q + 1)(q^3 + 1)^2}{(q^4 + 1)(q^8 + 1)^2}$
3	$\frac{q^9(q + 1)(q^3 + 1)(q^5 + 1)}{(q^4 + 1)(q^8 + 1)(q^{12} + 1)}$

Alternate form:

$$\frac{(q + 1)^2 (q^2 - q + 1) q^{n^2 - 1} (q^{2n} + q)}{(q^4 + 1)(q^8 + 1)(q^{4n} + 1)}$$

Expanded form:

Step-by-step solution

$$\begin{aligned} & \frac{q^{n^2}}{(q^4 + 1)(q^8 + 1)(q^{4n} + 1)} + \frac{q^{n^2+1}}{(q^4 + 1)(q^8 + 1)(q^{4n} + 1)} + \\ & \frac{q^{n^2+3}}{(q^4 + 1)(q^8 + 1)(q^{4n} + 1)} + \frac{q^{n^2+4}}{(q^4 + 1)(q^8 + 1)(q^{4n} + 1)} + \frac{q^{n^2+2n-1}}{(q^4 + 1)(q^8 + 1)(q^{4n} + 1)} + \\ & \frac{q^{n^2+2n}}{(q^4 + 1)(q^8 + 1)(q^{4n} + 1)} + \frac{q^{n^2+2n+2}}{(q^4 + 1)(q^8 + 1)(q^{4n} + 1)} + \frac{q^{n^2+2n+3}}{(q^4 + 1)(q^8 + 1)(q^{4n} + 1)} \end{aligned}$$

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Real roots:

Exact forms

More digits

$$n = m + 1, \quad q = -1, \quad m^2 + 4m \in \mathbb{Z}, \quad m^2 + 2m \in \mathbb{Z}, \quad 4m \in \mathbb{Z}, \quad m \in \mathbb{Z}$$

[Open code](#)

$$n > 0.5, \quad q = 0$$

Integer root:

$$q = -1$$

[Open code](#)

## Enlarge Data Customize A Plaintext Interactive

Roots for the variable  $q$ :

Step-by-step solution

$$q = -1$$

[Open code](#)

$$q = \sqrt[3]{-1}$$

$$q = -(-1)^{2/3}$$

$$q = (-1)^{-1/(2n-1)}$$

Series expansion at  $q = 0$ :

$$\frac{q^{n^2} (q^{2n} + q) \left( \frac{1}{q} + 1 + q^2 - q^4 + O(q^5) \right)}{q^{4n} + 1}$$

[Open code](#)

## Enlarge Data Customize A Plaintext Interactive

Series expansion at  $q = \infty$ :

$$\frac{q^{n^2} (q^{2n} + q) \left( \left( \frac{1}{q} \right)^9 + \left( \frac{1}{q} \right)^{10} + \left( \frac{1}{q} \right)^{12} - \left( \frac{1}{q} \right)^{14} - \left( \frac{1}{q} \right)^{16} - \left( \frac{1}{q} \right)^{17} + O\left( \left( \frac{1}{q} \right)^{25} \right) \right)}{q^{4n} + 1}$$

[Open code](#)

• [Big-O notation](#)

Derivative:

Step-by-step solution

• [Big-O notation](#)

$$\frac{\partial}{\partial q} \left( \frac{q^{n^2} (1+q)(1+q^3)(1+q^{2n-1})}{(1+q^4)(1+q^8)(1+q^{4n})} \right) =$$

$$\left( \frac{q^{n^2} \left( \frac{n^2 (1+q)(1+q^3)(1+q^4)(1+q^8)(q^{2n}+q)(1+q^{4n})}{q^2} + \right. \right.$$

$$3(1+q)(1+q^4)(1+q^8)q(q^{2n}+q)(1+q^{4n}) +$$

$$(2n-1)(1+q)(1+q^3)(1+q^4)(1+q^8)(1+q^{4n})q^{2n-2} +$$

$$(1+q^3)(1+q^4)(1+q^8)(q^{2n-1}+1)(1+q^{4n}) - 4(1+q)(1+q^3)(1+q^8)q^2$$

$$(q^{2n}+q)(1+q^{4n}) - 8(1+q)(1+q^3)(1+q^4)q^6(q^{2n}+q)(1+q^{4n}) -$$

$$\left. \left. 4n(1+q)(1+q^3)(1+q^4)(1+q^8)(q^{2n}+q)q^{4n-2} \right) \right) /$$

$$((1+q^4)^2 (1+q^8)^2 (1+q^{4n})^2)$$

For  $q = 0,5$  and  $n = 2$ , we obtain:

$$\frac{((0.5^{2^2})(1+0.5)(1+0.5^3)(1+0.5^{2*2-1}))}{((1+0.5^4)(1+0.5^8)(1+0.5^{4*2}))}$$

Input:

$$\frac{0.5^{2^2} (1+0.5)(1+0.5^3)(1+0.5^{2*2-1})}{(1+0.5^4)(1+0.5^8)(1+0.5^{4*2})}$$

[Open code](#)

Enlarge Data Customize A [Plaintext](#) [Interactive](#)

Result:

More digits

0.110805435892960039471586602816269204770433359190547481237...

$$1 / \left( \frac{(((((0.5^{2^2})(1+0.5)(1+0.5^3)(1+0.5^{2*2-1}))) / ((1+0.5^4)(1+0.5^8)(1+0.5^{4*2}))))^{1/4}} \right)$$

Input:

$$\frac{1}{\sqrt[4]{\frac{0.5^{2^2} (1+0.5)(1+0.5^3)(1+0.5^{2*2-1})}{(1+0.5^4)(1+0.5^8)(1+0.5^{4*2})}}}$$

[Open code](#)

Enlarge Data Customize A [Plaintext](#) [Interactive](#)

Result:

Fewer digits

More digits

1.733244111338517103315069811787548605396295692557710550305...

$$1/2 * \left[ \frac{(((((1 / \left( \frac{(((((0.5^{2^2})(1+0.5)(1+0.5^3)(1+0.5^{2*2-1}))) / ((1+0.5^4)(1+0.5^8)(1+0.5^{4*2}))))^{1/5}} \right) \right) \right) + 1.7332441113385171033)} \right]$$



Input interpretation:

$$\frac{1}{2} \left( \frac{1}{\sqrt[5]{\frac{0.5 \cdot 2^2 (1+0.5)(1+0.5^3)(1+0.5^{2 \times 2-1})}{(1+0.5^4)(1+0.5^8)(1+0.5^{4 \times 2})}}} + 1.7332441113385171033 \right)$$

[Open code](#)

### Enlarge Data Customize A [Plaintext](#) [Interactive](#)

Result:

Fewer digits  
More digits

1.642972473414248159524389175683432397300064819460733440825...

1.6429724734142481595243891756834323973000648194607334

is very near to the value of  $\zeta(2) = \pi^2/6 = 1.6449...$

Continued fraction:

Linear form

$$1 + \frac{1}{1 + \frac{1}{1 + \frac{1}{1 + \frac{1}{4 + \frac{1}{44 + \frac{1}{41 + \frac{1}{2 + \frac{1}{4 + \frac{1}{4 + \frac{1}{1 + \frac{1}{8 + \frac{1}{1 + \frac{1}{1 + \frac{1}{1 + \frac{1}{1 + \frac{1}{1 + \frac{1}{2 + \frac{1}{1 + \frac{1}{1 + \frac{1}{2 + \frac{1}{1 + \frac{1}{1 + \frac{1}{1 + \frac{1}{1 + \frac{1}{1 + \frac{1}{1 + \frac{1}{19 + \frac{1}{1 + \frac{1}{\dots}}$$

[Open code](#)

### Enlarge Data Customize A [Plaintext](#) [Interactive](#)

Possible closed forms:

More

$$\frac{4525961533 \pi}{8654270070} \approx 1.642972473414248159545293$$

$$\frac{1}{200} (48 e^\pi - 213 \pi + 2063 \log(\pi) - 1253 \log(2\pi) - 136 \tan^{-1}(\pi)) \approx$$

$$1.64297247341424815945204$$

$$\frac{-551 - 204 e + 413 e^2}{4(-101 - 17 e + 60 e^2)} \approx 1.64297247341424815916355$$

$$(1.6429724734142481595 \times 8654270070) / 4525961533$$

Input interpretation:  
 $1.6429724734142481595 \times 8654270070$

$$\underline{\hspace{2em}} \\ 4525961533$$

[Open code](#)

### Enlarge Data Customize A Plaintext Interactive

Result:  
 More digits  
 3.141592653589793238376035522748222173190971307356005316715...  
 3.141592653589793238376035522748222173190971307356005316715

From the four results, we obtain the following mean:

$\frac{1}{4} *$

$$(1.616953587550403225806451 + 1.621996449817778272583121 + 1.626466340780163593638612 + 1.642972473414248159524389)$$

Input interpretation:  
 $\frac{1}{4} (1.616953587550403225806451 + 1.621996449817778272583121 + 1.626466340780163593638612 + 1.642972473414248159524389)$

[Open code](#)

### Enlarge Data Customize A Plaintext Interactive

Result:  
 1.62709721289064831288814325  
 1.62709721289064831288814325 this result is a golden number

Continued fraction:  
 Linear form

$$1 + \frac{1}{1 + \frac{1}{1 + \frac{1}{1 + \frac{1}{2 + \frac{1}{7 + \frac{1}{13 + \frac{1}{3 + \frac{1}{1 + \frac{1}{2 + \frac{1}{1 + \frac{1}{1 + \frac{1}{3 + \frac{1}{1 + \frac{1}{2 + \frac{1}{1 + \frac{1}{1 + \frac{1}{3 + \frac{1}{1 + \frac{1}{2 + \frac{1}{1 + \frac{1}{15 + \frac{1}{3 + \frac{1}{1 + \frac{1}{45 + \frac{1}{\dots}}}}}}}}}}}}}}}}}}}}}}}}}}}}}}}}$$

[Open code](#)

### Enlarge Data Customize A Plaintext Interactive

Possible closed forms:

More

$$\frac{10 \sqrt{\frac{2660970}{10183879}}}{\pi} \approx 1.627097212890648363302$$

$$\frac{3418811179 \pi}{6601026662} \approx 1.62709721289064831290552$$

$$\frac{2311583 - 16788 \pi^2}{419802 \pi} \approx 1.62709721289064831277120$$

$$(1.627097212890648312 * 6601026662) / 3418811179$$

Input interpretation:

$$\frac{1.627097212890648312 \times 6601026662}{3418811179}$$

[Open code](#)

### Enlarge Data Customize A Plaintext Interactive

Result:

More digits

3.141592653589793236714256848988734396553814474350061787954...  
 3.141592653589793236714256848988734396553814474350061787954

Now, from the sum of the four results, we obtain:

$$(1.616953587550403225806451 + 1.621996449817778272583121 + 1.626466340780163593638612 + 1.642972473414248159524389)^{(e+1.2619)}$$

where “e” is the Euler number and 1.2619 is a Hausdorff dimension

Input interpretation:

$$(1.616953587550403225806451 + 1.621996449817778272583121 + 1.626466340780163593638612 + 1.642972473414248159524389)^{e+1.2619}$$

[Open code](#)

### Enlarge Data Customize A Plaintext Interactive

Result:

Fewer digits

More digits

1728.910105392591315967102785492505711099335938132036375855...  
 1728.9101053925913159671027854925057110993359381320363

This result is very near to the mass of candidate glueball  $f_0(1710)$  meson. Furthermore, 1728 occurs in the algebraic formula for the j-invariant of an elliptic curve. As a consequence, it is sometimes called a Zagier as a pun on the Gross–

Zagier theorem. The number 1728 is one less than the Hardy–Ramanujan number 1729

Continued fraction:

Linear form

$$1728 + \frac{1}{1 + \frac{1}{10 + \frac{1}{8 + \frac{1}{17 + \frac{1}{1 + \frac{1}{353 + \frac{1}{3 + \frac{1}{1 + \frac{1}{1 + \frac{1}{1 + \frac{1}{26 + \frac{1}{1 + \frac{1}{19 + \frac{1}{1 + \frac{1}{13 + \frac{1}{1 + \frac{1}{8 + \frac{1}{\dots}}}}}}}}}}}}}}}}}}}}}}}}}}$$

Open code

Enlarge Data Customize A [Plaintext](#) [Interactive](#)

Possible closed forms:

More

$\pi$ 
root of  $x^5 - 549x^4 - 732x^3 + 281x^2 + 1053x + 641$  near  $x = 550.329$ 
 $\approx$   
 $1728.910105392591315918484$   
 $\frac{2603651\pi}{3890} - \frac{6852523}{5835\pi} \approx 1728.910105392591315985733$   
 $\frac{-1635 + 267\sqrt{\pi} + 1138\pi + 3987\pi^{3/2} + 4110\pi^2}{12\pi} \approx 1728.9101053925913159641787$

Now:

$$(1728.9101053925913159671027854925057110993359381320363)^{1/3}$$

Input interpretation:

$$\sqrt[3]{1728.9101053925913159671027854925057110993359381320363}$$

Open code

Enlarge Data Customize A [Plaintext](#) [Interactive](#)

Result:

More digits

12.002106355696320546684685767692813791725171142081534...  
 12.002106355696320546684685767692813791725171142081534

This result 12,0021 is very near to the value of black hole entropy 12,1904  
 $2*(1728.9101053925913159671027854925057110993359381320363)^{1/3}$

Input interpretation:

$$2\sqrt[3]{1728.9101053925913159671027854925057110993359381320363}$$

[Open code](#)

- [Units »](#)

Enlarge Data Customize A [Plaintext](#) [Interactive](#)

Result:

More digits

24.004212711392641093369371535385627583450342284163069...  
 24.004212711392641093369371535385627583450342284163069

Continued fraction:

Linear form

$$24 + \cfrac{1}{237 + \cfrac{1}{2 + \cfrac{1}{1 + \cfrac{1}{1 + \cfrac{1}{7 + \cfrac{1}{1 + \cfrac{1}{610 + \cfrac{1}{13 + \cfrac{1}{1 + \cfrac{1}{6 + \cfrac{1}{3 + \cfrac{1}{1 + \cfrac{1}{1 + \cfrac{1}{3 + \cfrac{1}{6 + \cfrac{1}{3 + \cfrac{1}{2 + \cfrac{1}{5 + \cfrac{1}{\dots}}}}}}}}}}}}}}}}}}}}$$

[Open code](#)

Enlarge Data Customize A [Plaintext](#) [Interactive](#)

Possible closed forms:

More

$\pi$    $\approx$

24.00421271139264110259

$\approx$

24.00421271139264111474

$\frac{1875\,894\,986\,\pi}{245\,510\,985} \approx 24.0042127113926410942825$

This value is linked to the "Ramanujan function" (an elliptic modular function that satisfies the need for "conformal symmetry") that has 24 "modes" corresponding to the physical vibrations of a bosonic string.



1

root of  $65552x^3 - 7600x^2 - 21965x + 4220$  near  $x = 0.202781$   $\approx$

4.931428592699612369233

Where 4.93142859... is very near to the first value of upper bound dark photon energy range ( $1.8 * 10^{15} - 4.95 * 10^{16} - 5.4 * 10^{16}$ ) (Physics Letters B 731 (2014) 265–271 - **Searching a dark photon with HADES**)

From the product of the results, we obtain:

$(1.616953587550403225806451 * 1.621996449817778272583121 * 1.626466340780163593638612 * 1.642972473414248159524389)^{(2 * 1.2619 + 1.3057)}$

Where 1.2619 and 1.3057 are Hausdorff dimensions

Input interpretation:

$(1.616953587550403225806451 \times 1.621996449817778272583121 \times 1.626466340780163593638612 \times 1.642972473414248159524389)^{2 * 1.2619 + 1.3057}$

[Open code](#)

Enlarge Data Customize A [Plaintext](#) Interactive

Result:

- Fewer digits
- More digits

1731.061879997675135109524574096610559864760525844826370122...  
1731.0618799976751351095245740966105598647605258448263

This result is very near to the mass of candidate glueball  $f_0(1710)$  meson.

We have that:

$(1731.0618799976751351095245740966105598647605258448263)^{1/3}$

Input interpretation:

$\sqrt[3]{1731.0618799976751351095245740966105598647605258448263}$

[Open code](#)

Enlarge Data Customize A [Plaintext](#) Interactive

Result:

- More digits

12.007083503022478426187741052296711086819185061476107...  
12.007083503022478426187741052296711086819185061476107

This result 12,007 is very near to the value of black hole entropy 12,1904

$2 * (1731.0618799976751351095245740966105598647605258448263)^{1/3}$

Input interpretation:

$$2 \sqrt[3]{1731.0618799976751351095245740966105598647605258448263}$$

[Open code](#)

Enlarge Data Customize A [Plaintext](#) [Interactive](#)

Result:

- More digits

24.014167006044956852375482104593422173638370122952213...  
 24.014167006044956852375482104593422173638370122952213

Continued fraction:

- Linear form

$$24 + \frac{1}{70 + \frac{1}{1 + \frac{1}{1 + \frac{1}{2 + \frac{1}{2 + \frac{1}{1 + \frac{1}{1 + \frac{1}{2 + \frac{1}{1 + \frac{1}{15 + \frac{1}{13 + \frac{1}{2 + \frac{1}{2 + \frac{1}{1 + \frac{1}{3 + \frac{1}{1 + \frac{1}{4 + \frac{1}{1 + \frac{1}{55 + \frac{1}{3 + \frac{1}{\dots}}$$

[Open code](#)

Enlarge Data Customize A [Plaintext](#) [Interactive](#)

Possible closed forms:

- More

$\pi \sqrt{\text{root of } 397x^3 + 289x^2 - 16912x - 64926 \text{ near } x = 7.64395} \approx$   
 24.0141670060449568536240

$\sqrt{\text{root of } 51x^5 - 1219x^4 - 119x^3 - 440x^2 - 40x - 410 \text{ near } x = 24.0142} \approx$   
 24.0141670060449568573623

$\frac{1}{\sqrt{\text{root of } 410x^5 + 40x^4 + 440x^3 + 119x^2 + 1219x - 51 \text{ near } x = 0.0416421}} \approx$   
 24.0141670060449568573623

This value is linked to the "Ramanujan function" (an elliptic modular function that satisfies the need for "conformal symmetry") that has 24 "modes" corresponding to the physical vibrations of a bosonic string.

Now:



$$(1731.0618799976751351095245740966105598647605258448263)^{1/15}$$

Input interpretation:

$$\sqrt[15]{1731.0618799976751351095245740966105598647605258448263}$$

[Open code](#)

Enlarge Data Customize A Plaintext Interactive

Result:

More digits

1.64394584239714816931331532543343460906488619072871746...

1.64394584239714816931331532543343460906488619072871746

is very near to the value of  $\zeta(2) = \pi^2/6 = 1.6449\dots$

We have that:

$$1.64394584239714816931331532543343460906488619072871746 * 3$$

Result:

4.93183752719144450793994597630030382719465857218615238

4.93183752719144450793994597630030382719465857218615238

Continued fraction:

Linear form

$$\begin{aligned}
 & 4 + \cfrac{1}{1 + \cfrac{1}{13 + \cfrac{1}{1 + \cfrac{1}{2 + \cfrac{1}{26 + \cfrac{1}{2 + \cfrac{1}{1 + \cfrac{1}{3 + \cfrac{1}{1 + \cfrac{1}{1 + \cfrac{1}{2 + \cfrac{1}{2 + \cfrac{1}{68 + \cfrac{1}{1 + \cfrac{1}{1 + \cfrac{1}{1 + \cfrac{1}{2 + \cfrac{1}{8 + \cfrac{1}{2 + \cfrac{1}{1 + \cfrac{1}{\dots}}}}}}}}}}}}}}}}}}}}}}}}}}}}}}}}}}
 \end{aligned}$$

[Open code](#)

Possible closed forms:

More

$$-\frac{2(6691 C_{CR} - 3000)}{11 C_{CR} - 3010} \approx 4.931837527191444512541$$

Enlarge Data Customize A Plaintext Interactive

$$\text{root of } 3241x^3 - 34765x^2 + 77336x + 75400 \text{ near } x = 4.93184 \approx$$

$$4.93183752719144450782336$$

$$\frac{5089603373\pi}{3242089886} \approx 4.931837527191444507918267$$

- $c_{CR}$  is the ratio of sides to base of Calabi's triangle

This value 4,9318375 is very near to the first value of upper bound dark photon energy range ( $1.8 * 10^{15} - 4.95 * 10^{16} - 5.4 * 10^{16}$ ) (Physics Letters B 731 (2014) 265–271 - **Searching a dark photon with HADES**)

Now, we have also:

integrate  $[0.008748559907834101382488479262672811059907834101382488479]x$   
 $x, [0, -\tan(1727 * 1.08753^{11})]$

where 1.08753 is a result of Ramanujan mock theta function (see our previous papers)

Definite integral:

Step-by-step solution

$$\int_0^{-\tan(1727^\circ) 1.08753^{11}}$$

$$0.008748559907834101382488479262672811059907834101382488479$$

$$x x dx = 1.62689$$

Riemann sums:

Fewer cases

left sum	$1.62689 + \frac{0.813447}{n^2} - \frac{2.44034}{n} = 1.62689 - \frac{2.44034}{n} + O\left(\left(\frac{1}{n}\right)^2\right)$
midpoint sum	$1.62689 - \frac{0.406724}{n^2}$
right sum	$1.62689 + \frac{0.813447}{n^2} + \frac{2.44034}{n} = 1.62689 + \frac{2.44034}{n} + O\left(\left(\frac{1}{n}\right)^2\right)$

1.62689

Continued fraction:

Linear form



Continued fraction:

Linear form

$$1 + \frac{1}{1 + \frac{1}{1 + \frac{1}{1 + \frac{1}{11 + \frac{1}{114 + \frac{1}{\dots}}}}}}}$$

Open code

Enlarge Data Customize A [Plaintext](#) [Interactive](#)

Possible closed forms:

More

$$\frac{58}{35} \approx 1.657142857$$

$$\frac{2}{\log^2(3)} \approx 1.65707089$$

$$\sqrt{-\frac{11M}{7}} \approx 1.6571580897$$

- $\log(x)$  is the natural logarithm
- $M$  is the Madelung constant

We note that  $1.65715 * 3 = 4.97145$ .

This value 4,97145 is very near to the first value of upper bound dark photon energy range ( $1.8 * 10^{15} - 4.95 * 10^{16} - 5.4 * 10^{16}$ ) (Physics Letters B 731 (2014) 265–271 - **Searching a dark photon with HADES**)

Continued fraction:

Linear form

$$4 + \frac{1}{1 + \frac{1}{34 + \frac{1}{38 + \frac{1}{\dots}}}}}$$

Open code

Enlarge Data Customize A [Plaintext](#) [Interactive](#)

Possible closed forms:

More

$$-e! - 3 + \frac{9e}{2} \approx 4.971447751$$

$$\frac{174}{35} \approx 4.971428571$$

$$3\sqrt{-\frac{11M}{7}} \approx 4.971474269$$

•

integrate [-0.380102040816326530612244897]x x,[0, tan1733]

Definite integral:

Step-by-step solution

$$\int_0^{\tan(1733^\circ)} -0.380102040816326530612244897 x x dx = 1.65662$$

Riemann sums:

Fewer cases

left sum	$\frac{0.82831049511939708771515621}{n^2} - \frac{2.48493148535819126314546863}{n} +$ $1.65662099023879417543031242 =$ $\frac{1.65662099023879417543031242 -}{n} - \frac{2.48493148535819126314546863}{n} + O\left(\left(\frac{1}{n}\right)^2\right)$
midpoint sum	$- \frac{0.4141552475596985438575781}{n^2} +$ $\frac{0.x10^{-26}}{n} + 1.6566209902387941754303124 =$ $1.6566209902387941754303124 - \frac{0.4141552475596985438575781}{n^2} +$ $O\left(\left(\frac{1}{n}\right)^4\right)$
right sum	$\frac{0.82831049511939708771515621}{n^2} + \frac{2.48493148535819126314546863}{n} +$ $1.65662099023879417543031242 =$ $1.65662099023879417543031242 +$ $\frac{2.48493148535819126314546863}{n} + O\left(\left(\frac{1}{n}\right)^2\right)$

1.65662

where 1,65662 is very near to the 14th root of the following Ramanujan's class

invariant  $Q = (G_{505}/G_{101/5})^3 = 1164,2696$  i.e. 1,65578...

Continued fraction:

Linear form

$$1 + \frac{1}{1 + \frac{1}{1 + \frac{1}{1 + \frac{1}{10 + \frac{1}{2 + \frac{1}{1 + \frac{1}{1 + \frac{1}{4 + \frac{1}{1 + \frac{1}{\dots}}}}}}}}}}}}$$

[Open code](#)

Enlarge Data Customize A Plaintext Interactive

Possible closed forms:

- More
 
$$\sqrt{\frac{\pi}{\log(\pi)}} \approx 1.6566220046$$

$$\frac{2(1+\pi)}{5} \approx 1.656637061$$

$$-\frac{2e}{e-6} \approx 1.6566211273$$

We note that  $1.65662 * 3 = 4.96986$

This value 4,96986 is very near to the first value of upper bound dark photon energy range ( $1.8 * 10^{15} - 4.95 * 10^{16} - 5.4 * 10^{16}$ ) (Physics Letters B 731 (2014) 265–271 - **Searching a dark photon with HADES**)

Continued fraction:  
Linear form

- $$4 + \frac{1}{1 + \frac{1}{32 + \frac{1}{5 + \frac{1}{1 + \frac{1}{1 + \frac{1}{1 + \frac{1}{1 + \frac{1}{17 + \frac{1}{\dots}}}}}}}}}}$$

[Open code](#)

Enlarge Data Customize A [Plaintext](#) [Interactive](#)

Possible closed forms:

- More
 
$$3 \sqrt{\frac{\pi}{\log(\pi)}} \approx 4.9698660139$$

$$\frac{164}{33} \approx 4.96969696$$

$$e^{1+1/(2\pi)} \sqrt{\pi \sin(e\pi)} \approx 4.969857446$$

`integrate [0.002917823639774859287054409]x x,[0, -tan1729*1.08663428^17]`

where 1.08663428 is a mean of the some results of Ramanujan mock theta function (see our previous papers)

Definite integral:  
Step-by-step solution

$$\int_0^{-\tan(1729^\circ) 1.08663428^{17}} 0.002917823639774859287054409 x x dx = 1.64921$$

Riemann sums:  
Fewer cases

left sum	$1.64921 + \frac{0.824605}{n^2} - \frac{2.47381}{n} = 1.64921 - \frac{2.47381}{n} + O\left(\left(\frac{1}{n}\right)^2\right)$
midpoint sum	$1.64921 - \frac{0.412302}{n^2}$
right sum	$1.64921 + \frac{0.824605}{n^2} + \frac{2.47381}{n} = 1.64921 + \frac{2.47381}{n} + O\left(\left(\frac{1}{n}\right)^2\right)$

$$1.64921 \approx \zeta(2) = \frac{\pi^2}{6} = 1.6449$$

Continued fraction:  
Linear form

$$1 + \frac{1}{1 + \frac{1}{1 + \frac{1}{1 + \frac{1}{5 + \frac{1}{1 + \frac{1}{2 + \frac{1}{3 + \frac{1}{5 + \frac{1}{1 + \frac{1}{1 + \frac{1}{2 + \frac{1}{2 + \frac{1}{\dots}}}}}}}}}}}}}}}}}}$$

[Open code](#)

Enlarge Data Customize A Plaintext Interactive

Possible closed forms:

More

$$\frac{21\pi}{40} \approx 1.64933614$$

$$\frac{5}{4}\pi \operatorname{sech}^2(1) \approx 1.649235382$$

$$\frac{20\gamma}{7} \approx 1.64918761$$

$\gamma$  is the Euler-Mascheroni constant

$\operatorname{sech}(x)$  is the hyperbolic secant function

We note that  $1.64921 * 3 = 4.94763$

Continued fraction:  
Linear form

$$4 + \frac{1}{1 + \frac{1}{18 + \frac{1}{10 + \frac{1}{1 + \frac{1}{1 + \frac{1}{6 + \frac{1}{4 + \frac{1}{1 + \frac{1}{1 + \frac{1}{1 + \frac{1}{\dots}}}}}}}}}}}}}}}$$

Open code

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Possible closed forms:

- More
  - $\frac{7\pi}{2} - \frac{19}{\pi} \approx 4.947686450$
  - $7C_T + \frac{1}{5} \approx 4.947641443$
  - $13\lambda + 1 \approx 4.947619037$

This value 4,94763 is very near to the first value of upper bound dark photon energy range ( $1.8 * 10^{15} - 4.95 * 10^{16} - 5.4 * 10^{16}$ ) (Physics Letters B 731 (2014) 265–271 - **Searching a dark photon with HADES**)

integrate [0.11080543589296003947158660]x x,[0, -tan1727\*1.08753454]

Definite integral:

Step-by-step solution

- $\int_0^{-\tan(1727^\circ) 1.08753454} 0.11080543589296003947158660 x x dx = 1.66246$

Or

integrate [0.11080543589296003947158660]x x,[0, -tan1727\*1.08663428]

Definite integral:

Step-by-step solution

- $\int_0^{-\tan(1727^\circ) 1.08663428} 0.11080543589296003947158660 x x dx = 1.65834$

Riemann sums:

- Fewer cases



left sum	$1.65834 + \frac{0.829169}{n^2} - \frac{2.48751}{n} = 1.65834 - \frac{2.48751}{n} + O\left(\left(\frac{1}{n}\right)^2\right)$
midpoint sum	$1.65834 - \frac{0.414584}{n^2}$
right sum	$1.65834 + \frac{0.829169}{n^2} + \frac{2.48751}{n} = 1.65834 + \frac{2.48751}{n} + O\left(\left(\frac{1}{n}\right)^2\right)$

1.65834 is very near to the 14th root of the following Ramanujan's class invariant  $Q = (G_{505}/G_{101/5})^3 = 1164,2696$  i.e. 1,65578...

Continued fraction:

Linear form

$$1 + \frac{1}{1 + \frac{1}{1 + \frac{1}{1 + \frac{1}{12 + \frac{1}{1 + \frac{1}{2 + \frac{1}{10 + \frac{1}{13 + \frac{1}{\dots}}}}}}}}}}$$

[Open code](#)

Possible closed forms:

$$5\pi - 2\sqrt{5}\pi \approx 1.658333805$$

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$$\frac{\sqrt{11}}{2} \approx 1.658312395$$

$$\sqrt{\frac{2}{5}} L \approx 1.658334805$$

- $L$  is the lemniscate constant

We have that  $1.65834 * 3 = 4.97502$

Continued fraction:

Linear form

$$4 + \frac{1}{1 + \frac{1}{39 + \frac{1}{31 + \frac{1}{4 + \frac{1}{2 + \frac{1}{\dots}}}}}}}}$$

[Open code](#)

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Possible closed forms:

$$\frac{199}{40} \approx 4.975000000$$

$$\frac{e(4+e)}{-1-e+e^2} \approx 4.9750221798$$

$$\frac{4}{3} e^{3/2} \sqrt{\log(2)} \approx 4.975001201$$

This value 4,97502 is very near to the first value of upper bound dark photon energy range ( $1.8 * 10^{15} - 4.95 * 10^{16} - 5.4 * 10^{16}$ ) (Physics Letters B 731 (2014) 265–271 - **Searching a dark photon with HADES**)

The mean of the four results of the integrals is:

$$(1.65715 + 1.65662 + 1.64921 + 1.65834) / 4$$

Input interpretation:

$$\frac{1}{4} (1.65715 + 1.65662 + 1.64921 + 1.65834)$$

[Open code](#)

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Result:

1.65533

1.65533

Continued fraction:

Linear form

$$1 + \frac{1}{1 + \frac{1}{1 + \frac{1}{1 + \frac{1}{9 + \frac{1}{7 + \frac{1}{2 + \frac{1}{3 + \frac{1}{1 + \frac{1}{4 + \frac{1}{3 + \frac{1}{\dots}}}}}}}}}}}}}}$$

[Open code](#)

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Possible closed forms:

More

$$1 - \frac{3M}{8} \approx 1.6553367229$$

$$\frac{26}{5\pi} \approx 1.65521140$$

$$\frac{10G_g}{7} \approx 1.655326390$$

- $M$  is the Madelung constant
- $G_g$  is the tether length at which a goat tied to the boundary of a unit circular field can graze exactly half the field

We have that  $1.65533 * 3 = 4.96599$

Continued fraction:

Linear form

$$4 + \frac{1}{1 + \frac{1}{28 + \frac{1}{2 + \frac{1}{2 + \frac{1}{12 + \frac{1}{2 + \frac{1}{3 + \frac{1}{3 + \frac{1}{\dots}}}}}}}}}}$$

Open code

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Possible closed forms:

More

$$\frac{144}{29} \approx 4.96551724$$

$$\frac{30 G_g}{7} \approx 4.965979170$$

$$P_{\text{pen}} + 3 \approx 4.965948236$$

- $G_g$  is the tether length at which a goat tied to the boundary of a unit circular field can graze exactly half the field
- $P_{\text{pen}}$  is the pentanacci constant

This value 4,96599 is very near to the first value of upper bound dark photon energy range ( $1.8 * 10^{15} - 4.95 * 10^{16} - 5.4 * 10^{16}$ ) (Physics Letters B 731 (2014) 265–271 - **Searching a dark photon with HADES**)

The final result is 1,65533 practically equal to the 14th root of the following Ramanujan's class invariant  $Q = (G_{505}/G_{101/5})^3 = 1164,2696$  i.e. 1,65578...

$$\sqrt[14]{\left(\sqrt{\frac{113 + 5\sqrt{505}}{8}} + \sqrt{\frac{105 + 5\sqrt{505}}{8}}\right)^3} = 1,65578 \dots$$

From the mean value  $1.65533 * 11$  we obtain:

18.20863

Continued fraction:

Linear form

$$18 + \frac{1}{4 + \frac{1}{1 + \frac{1}{3 + \frac{1}{1 + \frac{1}{5 + \frac{1}{16 + \frac{1}{1 + \frac{1}{1 + \frac{1}{3 + \frac{1}{1 + \frac{1}{\dots}}}}}}}}}}}}$$

Open code

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Possible closed forms:

More

$$\frac{59 b_4(2)}{2} + 100 \approx 18.2086326939$$

$$-9 + \frac{46}{\pi} + 4\pi \approx 18.208625378$$

$$\frac{1733 \pi}{299} \approx 18.208628992$$

This result 18,2086 is very near to the value of black hole entropy 18,2773.

Now, we have (mock theta functions order 8):

$$S_0(q) = \sum_{n \geq 0} \frac{q^{n^2} (-q; q^2)_n}{(-q^2; q^2)_n} \text{ (sequence [A153148](#) in the [OEIS](#))}$$

$$S_1(q) = \sum_{n \geq 0} \frac{q^{n(n+2)} (-q; q^2)_n}{(-q^2; q^2)_n} \text{ (sequence [A153149](#) in the [OEIS](#))}$$

$$T_0(q) = \sum_{n \geq 0} \frac{q^{(n+1)(n+2)} (-q^2; q^2)_n}{(-q; q^2)_{n+1}} \text{ (sequence [A153155](#) in the [OEIS](#))}$$

$$T_1(q) = \sum_{n \geq 0} \frac{q^{n(n+1)} (-q^2; q^2)_n}{(-q; q^2)_{n+1}} \text{ (sequence [A153156](#) in the [OEIS](#))}$$

For  $q = 0.5$  and  $n = 2$ , we obtain for  $S_0(q)$ :

$$\text{Sum}_{\{n \geq 0\}} q^{(n^2)}(1+q)(1+q^3)\dots(1+q^{(2n-1)}) / ((1+q^2)(1+q^4)\dots(1+q^{(2n)})).$$

$$\text{sum } q^{(n^2)}(1+q)(1+q^3)(1+q^{(2n-1)}) * 1 / ((1+q^2)(1+q^4)(1+q^{(2n)})), n = 0 \text{ to } 5$$

Result:

$$\sum_{n=0}^5 \frac{q^{n^2} (1+q)(1+q^3)(1+q^{2n-1})}{(1+q^2)(1+q^4)(1+q^{2n})} = \frac{1}{2(q^2+1)^2(q^4+1)^2} \\ (q+1)(q^3+1) \left( 2(q+1)(q^4+1)q + \left(\frac{1}{q}+1\right)(q^2+1)(q^4+1) + \right. \\ \left. 2(q^2+1)(q^3+1)q^4 + \frac{2(q^2+1)(q^4+1)(q^9+1)q^{25}}{q^{10}+1} + \right. \\ \left. \frac{2(q^2+1)(q^4+1)(q^7+1)q^{16}}{q^8+1} + \frac{2(q^2+1)(q^4+1)(q^5+1)q^9}{q^6+1} \right)$$

[Open code](#)

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Sum:

$$\sum_{n=0}^5 \frac{(q+1)(q^3+1)q^{n^2}(q^{2n-1}+1)}{(q^2+1)(q^4+1)(q^{2n}+1)} = \\ \frac{(q+1)(q^3+1)^2q^4}{(q^2+1)(q^4+1)^2} + \frac{(q+1)^2(q^3+1)q}{(q^2+1)^2(q^4+1)} + \frac{\left(\frac{1}{q}+1\right)(q+1)(q^3+1)}{2(q^2+1)(q^4+1)} + \\ \frac{(q+1)(q^3+1)(q^9+1)q^{25}}{(q^2+1)(q^4+1)(q^{10}+1)} + \frac{(q+1)(q^3+1)(q^7+1)q^{16}}{(q^2+1)(q^4+1)(q^8+1)} + \frac{(q+1)(q^3+1)(q^5+1)q^9}{(q^2+1)(q^4+1)(q^6+1)}$$

[Open code](#)

sum  $0.5^{(n^2)}(1+0.5)(1+0.5^3)(1+0.5^{(2n-1)}) * 1/((1+0.5^2)(1+0.5^4)(1+0.5^{(2n)}))$ ,  
 $n = 0$  to  $5$

Sum:

$$\sum_{n=0}^5 \frac{(1+0.5)(1+0.5^3)0.5^{n^2}(0.5^{2n-1}+1)}{(1+0.5^2)(1+0.5^4)(0.5^{2n}+1)} = \frac{57177775264014927}{20755255918592000}$$

[Open code](#)

### Enlarge Data Customize A Plaintext Interactive

Decimal approximation:

More digits

2.754857636460006923978015191294304473666175863704887316855...

[Open code](#)

$\left[ \left( \left( \left( \sum_{n=0}^5 0.5^{(n^2)}(1+0.5)(1+0.5^3)(1+0.5^{(2n-1)}) * \right. \right. \right. \right. \\ \left. \left. \left. \left. 1/((1+0.5^2)(1+0.5^4)(1+0.5^{(2n)})) \right) \right) \right) \right]^{1/2}$ ,  $n = 0$  to  $5$

Input interpretation:

$$\sqrt{\sum_{n=0}^5 0.5^{n^2} (1+0.5)(1+0.5^3) \left( (1+0.5^{2n-1}) \times \frac{1}{(1+0.5^2)(1+0.5^4)(1+0.5^{2n})} \right)}$$

### Enlarge Data Customize A Plaintext Interactive

Result:

1.65978

1.65978 is very near to the 14th root of the following Ramanujan's class invariant

$$Q = (G_{505}/G_{101/5})^3 = 1164,2696 \text{ i.e. } 1,65578\dots$$

$$24 \times 3 + 10^3 * [(((\sum_{n=0}^5 0.5^{n^2} (1+0.5)(1+0.5^3)(1+0.5^{2n-1})) * 1/((1+0.5^2)(1+0.5^4)(1+0.5^{2n}))), n = 0 \text{ to } 5))]^{1/2}$$

input interpretation:

$$24 \times 3 + 10^3 \sqrt{\sum_{n=0}^5 0.5^{n^2} (1+0.5)(1+0.5^3) \left( (1+0.5^{2n-1}) \times \frac{1}{(1+0.5^2)(1+0.5^4)(1+0.5^{2n})} \right)}$$

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Result:

1731.78

This result is very near to the mass of candidate glueball  $f_0(1710)$  meson.

$$(((((((24 \times 3 + 10^3 * [(((\sum_{n=0}^5 0.5^{n^2} (1+0.5)(1+0.5^3)(1+0.5^{2n-1})) * 1/((1+0.5^2)(1+0.5^4)(1+0.5^{2n}))), n = 0 \text{ to } 5))]^{1/2}))))))^{1/3}$$

Input interpretation:

$$\left( 24 \times 3 + 10^3 \sqrt{\sum_{n=0}^5 0.5^{n^2} (1+0.5)(1+0.5^3) \left( (1+0.5^{2n-1}) \times \frac{1}{(1+0.5^2)(1+0.5^4)(1+0.5^{2n})} \right)} \right)^{1/3}$$

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Result:

12.0087

This result is very near to the value of black hole entropy 12,1904

$$2(((((((24 \times 3 + 10^3 * [(((\sum_{n=0}^5 0.5^{n^2} (1+0.5)(1+0.5^3)(1+0.5^{2n-1})) * 1/((1+0.5^2)(1+0.5^4)(1+0.5^{2n}))), n = 0 \text{ to } 5))]^{1/2}))))))^{1/3}$$

Input interpretation:

$$2 \left( 24 \times 3 + 10^3 \sqrt{\sum_{n=0}^5 0.5^{n^2} (1+0.5)(1+0.5^3) \left( (1+0.5^{2n-1}) \times \frac{1}{(1+0.5^2)(1+0.5^4)(1+0.5^{2n})} \right)} \right)^{1/3}$$

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Result:

24.0175

This value is linked to the "Ramanujan function" (an elliptic modular function that satisfies the need for "conformal symmetry") that has 24 "modes" corresponding to the physical vibrations of a bosonic string.

$$\left( \left( \left( \left( \left( 24 \times 3 + 10^3 \times \left[ \left( \sum_{n=0}^5 0.5^{n^2} (1+0.5)(1+0.5^3)(1+0.5^{2n-1}) \right) \times \frac{1}{(1+0.5^2)(1+0.5^4)(1+0.5^{2n})} \right) \right]^{1/2} \right) \right) \right) \right)^{1/15}$$

Input interpretation:

$$\left( 24 \times 3 + 10^3 \sqrt{\sum_{n=0}^5 0.5^{n^2} (1+0.5)(1+0.5^3) \left( (1+0.5^{2n-1}) \times \frac{1}{(1+0.5^2)(1+0.5^4)(1+0.5^{2n})} \right)} \right)^{1/15}$$

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Result:

1.64399

$$1.64399 \approx \zeta(2) = \frac{\pi^2}{6} = 1.6449$$

Then:

$$\left( \left( \left( q^{n^2} (1+q)(1+q^3)(1+q^{2n-1}) \right) \right) \right) / \left( \left( \left( (1+q^2)(1+q^4)(1+q^{2n}) \right) \right) \right)$$

Input:

$$\frac{q^{n^2} (1+q)(1+q^3)(1+q^{2n-1})}{(1+q^2)(1+q^4)(1+q^{2n})}$$

[Open code](#)

Series expansion at q = 0:

$$\frac{q^{n^2} (q^{2n} + q) \left( \frac{1}{q} + 1 - q + q^3 - q^4 + O(q^5) \right)}{q^{2n} + 1}$$

[Open code](#)

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Series expansion at  $q = \infty$ :

$$\frac{q^{n^2} (q^{2n} + q) \left( \left( \frac{1}{q} \right)^3 + \left( \frac{1}{q} \right)^4 - \left( \frac{1}{q} \right)^5 + \left( \frac{1}{q} \right)^7 - \left( \frac{1}{q} \right)^8 - \left( \frac{1}{q} \right)^9 + \left( \frac{1}{q} \right)^{11} + \left( \frac{1}{q} \right)^{12} + O\left( \left( \frac{1}{q} \right)^{13} \right) \right)}{q^{2n} + 1}$$

[Open code](#)

• [Big-O notation](#)

Derivative:

Step-by-step solution

$$\frac{\partial}{\partial q} \left( \frac{q^{n^2} (1+q)(1+q^3)(1+q^{2n-1})}{(1+q^2)(1+q^4)(1+q^{2n})} \right) =$$

$$\left( (1+q)q^{n^2-2} \left( n^2 (q^9 + q^7 + q^6 + q^5 + q^4 + (1+q^3) + q^2) (q^{2n} + q) (1+q^{2n}) + q^{2n+1} - \right. \right.$$

$$3q^{2n+2} + q^{2n+3} - q^{2n+4} - 3q^{2n+5} - 5q^{2n+6} - 7q^{2n+7} - 4q^{2n+9} + q^{4n+1} -$$

$$4q^{4n+2} + 4q^{4n+3} - 6q^{4n+4} + 2q^{4n+5} - 8q^{4n+6} - q^{4n+8} - 3q^{4n+9} -$$

$$q^{4n} - q^{2n} - 2q^{2(n+5)} - 2n(q^{10} - q^9 + q^8 - q^2 + q - 1)q^{2n} - 2q^{10} - q^9 +$$

$$\left. \left. q^8 - 7q^7 + 3q^6 - 5q^5 + 5q^4 - 3q^3 + q^2 \right) \right) / \left( (1+q^2)^2 (1+q^4)^2 (1+q^{2n})^2 \right)$$

• [Big-O notation](#)

$$\frac{(((0.5^{(2^2)}(1+0.5)(1+0.5^3)(1+0.5^{(2*2-1))})) / (((1+0.5^2)(1+0.5^4)(1+0.5^{(2*2))})))$$

Input:

$$\frac{0.5^{2^2} (1+0.5)(1+0.5^3)(1+0.5^{2*2-1})}{(1+0.5^2)(1+0.5^4)(1+0.5^{2*2})}$$

[Open code](#)

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Result:

More digits

0.084083044982698961937716262975778546712802768166089965397...

[Open code](#)

$$2\text{Pi}^2 \frac{(((0.5^{(2^2)}(1+0.5)(1+0.5^3)(1+0.5^{(2*2-1))})) / (((1+0.5^2)(1+0.5^4)(1+0.5^{(2*2))})))$$



Input:

$$2\pi^2 \times \frac{0.5^{2^2} (1+0.5)(1+0.5^3)(1+0.5^{2^2-1})}{(1+0.5^2)(1+0.5^4)(1+0.5^{2^2})}$$

[Open code](#)

Enlarge Data Customize A [Plaintext](#) [Interactive](#)

Result:

Fewer digits

More digits

1.659732781636480376731336548768100156319189588899316347556...

1.6597327816364803767313365487681001563191895888993163

is very near to the 14th root of the following Ramanujan's class invariant  $Q = (G_{505}/G_{101/5})^3 = 1164,2696$  i.e. 1,65578...

Series representations:

More

$$\frac{(2\pi^2)(0.5^{2^2}(1+0.5)(1+0.5^3)(1+0.5^{2^2-1}))}{(1+0.5^2)(1+0.5^4)(1+0.5^{2^2})} = 2.69066 \left( \sum_{k=0}^{\infty} \frac{(-1)^k}{1+2k} \right)^2$$

[Open code](#)

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$$\frac{(2\pi^2)(0.5^{2^2}(1+0.5)(1+0.5^3)(1+0.5^{2^2-1}))}{(1+0.5^2)(1+0.5^4)(1+0.5^{2^2})} = 0.672664 \left( -1 + \sum_{k=1}^{\infty} \frac{2^k}{\binom{2k}{k}} \right)^2$$

[Open code](#)

$$\frac{(2\pi^2)(0.5^{2^2}(1+0.5)(1+0.5^3)(1+0.5^{2^2-1}))}{(1+0.5^2)(1+0.5^4)(1+0.5^{2^2})} = 0.168166 \left( \sum_{k=0}^{\infty} \frac{2^{-k}(-6+50k)}{\binom{3k}{k}} \right)^2$$

Integral representations:

More

$$\frac{(2\pi^2)(0.5^{2^2}(1+0.5)(1+0.5^3)(1+0.5^{2^2-1}))}{(1+0.5^2)(1+0.5^4)(1+0.5^{2^2})} = 0.672664 \left( \int_0^{\infty} \frac{1}{1+t^2} dt \right)^2$$

[Open code](#)

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$$\frac{(2\pi^2)(0.5^{2^2}(1+0.5)(1+0.5^3)(1+0.5^{2^2-1}))}{(1+0.5^2)(1+0.5^4)(1+0.5^{2^2})} = 2.69066 \left( \int_0^1 \sqrt{1-t^2} dt \right)^2$$

[Open code](#)

$$\frac{(2\pi^2)(0.5^{2^2}(1+0.5)(1+0.5^3)(1+0.5^{2^2-1}))}{(1+0.5^2)(1+0.5^4)(1+0.5^{2^2})} = 0.672664 \left( \int_0^\infty \frac{\sin(t)}{t} dt \right)^2$$

$$96/5 \left( \frac{0.5^{2^2}(1+0.5)(1+0.5^3)(1+0.5^{2^2-1})}{(1+0.5^2)(1+0.5^4)(1+0.5^{2^2})} \right) /$$

Input:

$$\frac{96}{5} \times \frac{0.5^{2^2}(1+0.5)(1+0.5^3)(1+0.5^{2^2-1})}{(1+0.5^2)(1+0.5^4)(1+0.5^{2^2})}$$

[Open code](#)

Enlarge Data Customize A [Plaintext](#) [Interactive](#)

Result:

More digits

1.614394463667820069204152249134948096885813148788927335640...

1.614394463667820069204152249134948096885813148788927335640

This result is a golden number, near to the value of golden ratio

Continued fraction:

Linear form

$$1 + \frac{1}{1 + \frac{1}{1 + \frac{1}{1 + \frac{1}{1 + \frac{1}{2 + \frac{1}{5 + \frac{1}{1 + \frac{1}{1 + \frac{1}{2 + \frac{1}{4 + \frac{1}{4 + \frac{1}{\dots}}}}}}}}}}}}}}$$

[Open code](#)

Enlarge Data Customize A [Plaintext](#) [Interactive](#)

Possible closed forms:

More

$$\frac{7776 \sqrt{W_{\text{Wad}}}}{1445} \approx 1.61439446366782006920415224913494809688581314878892733564013840$$

$$\frac{11664}{7225} \approx 1.61439446366782006920415224913494809688581314878892733564013840$$

$$\frac{1755691273\pi}{3416554584} \approx 1.61439446366782006917974$$

$$\frac{\exp(((0.5^{(2^2)}(1+0.5)(1+0.5^3)(1+0.5^{(2*2-1))}))}{(((1+0.5^2)(1+0.5^4)(1+0.5^{(2*2))}))}$$

Input:

$$\exp\left(\frac{0.5^{2^2} (1 + 0.5) (1 + 0.5^3) (1 + 0.5^{2 \times 2 - 1})}{(1 + 0.5^2) (1 + 0.5^4) (1 + 0.5^{2 \times 2})}\right)$$

[Open code](#)

Enlarge Data Customize A [Plaintext](#) [Interactive](#)

Result:

- Fewer digits
- More digits

1.087719219681998777516154019807133726448011315189367700807...  
 1.0877192196819987775161540198071337264480113151893677

$$\left(\frac{\exp(((0.5^{(2^2)}(1+0.5)(1+0.5^3)(1+0.5^{(2*2-1))}))}{(((1+0.5^2)(1+0.5^4)(1+0.5^{(2*2))}))}\right)^6$$

Input:

$$\exp^6\left(\frac{0.5^{2^2} (1 + 0.5) (1 + 0.5^3) (1 + 0.5^{2 \times 2 - 1})}{(1 + 0.5^2) (1 + 0.5^4) (1 + 0.5^{2 \times 2})}\right)$$

[Open code](#)

Enlarge Data Customize A [Plaintext](#) [Interactive](#)

Result:

- Fewer digits
- More digits

1.656154369467837212509803842711975818427745850601446717495...  
 1.6561543694678372125098038427119758184277458506014467

is very near to the 14th root of the following Ramanujan's class invariant  $Q = (G_{505}/G_{101/5})^3 = 1164,2696$  i.e. 1,65578...

$$(15.8174+22.6589) * \int \left(\frac{\exp(((0.5^{(2^2)}(1+0.5)(1+0.5^3)(1+0.5^{(2*2-1))}))}{(((1+0.5^2)(1+0.5^4)(1+0.5^{(2*2))}))}\right)^6 x dx$$

Input interpretation:

$$(15.8174 + 22.6589) \int \frac{0.5^{2^2} (1 + 0.5) (1 + 0.5^3) (1 + 0.5^{2 \times 2 - 1})}{(1 + 0.5^2) (1 + 0.5^4) (1 + 0.5^{2 \times 2})} x dx$$

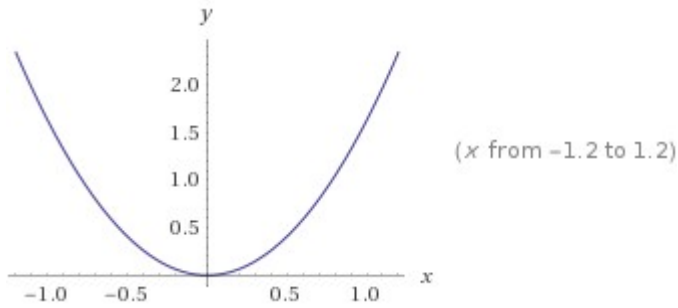
[Open code](#)

Enlarge Data Customize A [Plaintext](#) [Interactive](#)

Result:

1.6176  $x^2$

Plot:



[Open code](#)

$1.6176 x^2$  for  $x = 1$

Input interpretation:

$1.6176 x^2$  where  $x = 1$

[Open code](#)

Enlarge Data Customize A Plaintext Interactive

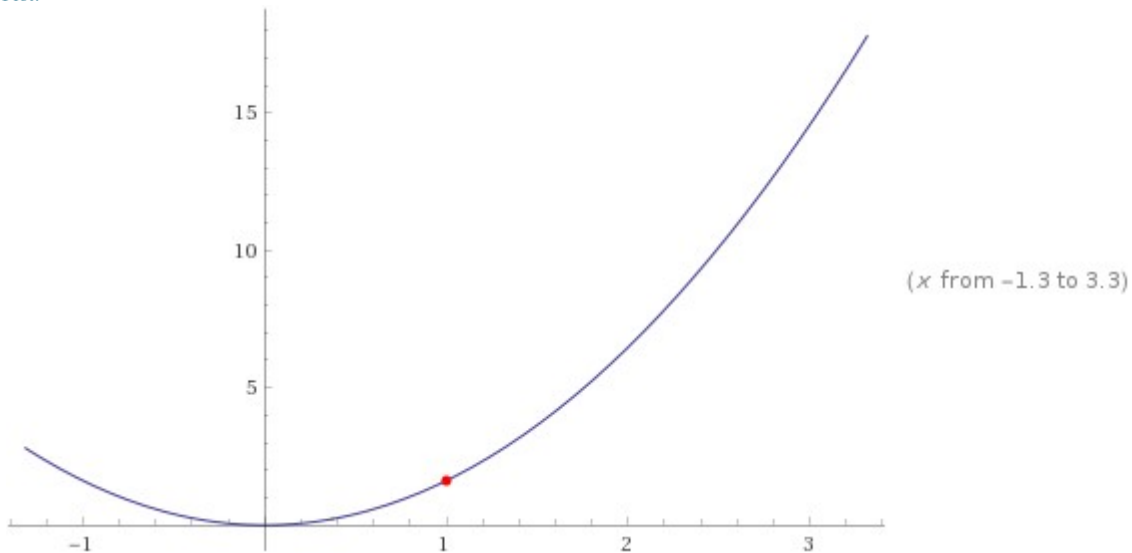
Result:

1.6176

1.6176.

This result is a very good approximation to the value of the golden ratio 1,618033988749...

Plot:



$$12 * \text{colog} \left( \frac{((0.5^{(2^2)}(1+0.5)(1+0.5^3)(1+0.5^{(2*2-1))}))}{((1+0.5^2)(1+0.5^4)(1+0.5^{(2*2)}))} \right)$$

Input:

$$12 \left( -\log \left( \frac{0.5^{2^2} (1 + 0.5) (1 + 0.5^3) (1 + 0.5^{2 \times 2 - 1})}{(1 + 0.5^2) (1 + 0.5^4) (1 + 0.5^{2 \times 2})} \right) \right)$$

Open code

- $\log(x)$  is the natural logarithm

### Enlarge Data Customize A [Plaintext](#) [Interactive](#)

Result:

More digits

29.7114...

29.7114...

This result is very near to the value of black hole entropy 29.7668

Series representations:

More

$$12 (-1) \log \left( \frac{0.5^{2^2} (1 + 0.5) (1 + 0.5^3) (1 + 0.5^{2 \times 2 - 1})}{(1 + 0.5^2) (1 + 0.5^4) (1 + 0.5^{2 \times 2})} \right) = 12 \sum_{k=1}^{\infty} \frac{(-1)^k (-0.915917)^k}{k}$$

Open code

### Enlarge Data Customize A [Plaintext](#) [Interactive](#)

$$12 (-1) \log \left( \frac{0.5^{2^2} (1 + 0.5) (1 + 0.5^3) (1 + 0.5^{2 \times 2 - 1})}{(1 + 0.5^2) (1 + 0.5^4) (1 + 0.5^{2 \times 2})} \right) = -24 i \pi \left\lfloor \frac{\arg(0.084083 - x)}{2 \pi} \right\rfloor - 12 \log(x) + 12 \sum_{k=1}^{\infty} \frac{(-1)^k (0.084083 - x)^k x^{-k}}{k} \text{ for } x < 0$$

Open code

$$12 (-1) \log \left( \frac{0.5^{2^2} (1 + 0.5) (1 + 0.5^3) (1 + 0.5^{2 \times 2 - 1})}{(1 + 0.5^2) (1 + 0.5^4) (1 + 0.5^{2 \times 2})} \right) = -12 \left\lfloor \frac{\arg(0.084083 - z_0)}{2 \pi} \right\rfloor \log \left( \frac{1}{z_0} \right) - 12 \log(z_0) - 12 \left\lfloor \frac{\arg(0.084083 - z_0)}{2 \pi} \right\rfloor \log(z_0) + 12 \sum_{k=1}^{\infty} \frac{(-1)^k (0.084083 - z_0)^k z_0^{-k}}{k}$$

Open code

- $\arg(z)$  is the complex argument
  - $\lfloor x \rfloor$  is the floor function

- $i$  is the imaginary unit
- [More information](#)

Integral representation:

$$12(-1) \log \left( \frac{0.5^{2^2} (1+0.5)(1+0.5^3)(1+0.5^{2^2-1})}{(1+0.5^2)(1+0.5^4)(1+0.5^{2^2})} \right) = -12 \int_1^{0.084083} \frac{1}{t} dt$$

$$64\pi * \operatorname{colog} \left( \frac{((0.5^{(2^2)}(1+0.5)(1+0.5^3)(1+0.5^{(2^2-1)})))}{(((1+0.5^2)(1+0.5^4)(1+0.5^{(2^2)})))} \right)$$

Input:

$$64\pi \left( -\log \left( \frac{0.5^{2^2} (1+0.5)(1+0.5^3)(1+0.5^{2^2-1})}{(1+0.5^2)(1+0.5^4)(1+0.5^{2^2})} \right) \right)$$

[Open code](#)

- $\log(x)$  is the natural logarithm

Enlarge Data Customize A [Plaintext](#) [Interactive](#)

Result:

- More digits  
497.819...

This result is practically equal to the rest mass of Kaon meson  $497.614 \pm 0.024$

Series representations:

More

$$(64\pi)(-1) \log \left( \frac{0.5^{2^2} (1+0.5)(1+0.5^3)(1+0.5^{2^2-1})}{(1+0.5^2)(1+0.5^4)(1+0.5^{2^2})} \right) = 64\pi \sum_{k=1}^{\infty} \frac{(-1)^k (-0.915917)^k}{k}$$

[Open code](#)

Enlarge Data Customize A [Plaintext](#) [Interactive](#)

$$(64\pi)(-1) \log \left( \frac{0.5^{2^2} (1+0.5)(1+0.5^3)(1+0.5^{2^2-1})}{(1+0.5^2)(1+0.5^4)(1+0.5^{2^2})} \right) =$$

$$-128 i \pi^2 \left[ \frac{\arg(0.084083 - x)}{2\pi} \right] - 64\pi \log(x) +$$

$$64\pi \sum_{k=1}^{\infty} \frac{(-1)^k (0.084083 - x)^k x^{-k}}{k} \text{ for } x < 0$$

[Open code](#)

$$(64 \pi) (-1) \log \left( \frac{0.5^{2^2} (1 + 0.5) (1 + 0.5^3) (1 + 0.5^{2 \times 2 - 1})}{(1 + 0.5^2) (1 + 0.5^4) (1 + 0.5^{2 \times 2})} \right) =$$

$$-128 i \pi^2 \left[ -\frac{-\pi + \arg\left(\frac{0.084083}{z_0}\right) + \arg(z_0)}{2 \pi} \right] -$$

$$64 \pi \log(z_0) + 64 \pi \sum_{k=1}^{\infty} \frac{(-1)^k (0.084083 - z_0)^k z_0^{-k}}{k}$$

Open code

- $\arg(z)$  is the complex argument
  - $\lfloor x \rfloor$  is the floor function
  - $i$  is the imaginary unit
    - [More information](#)

Integral representation:

$$(64 \pi) (-1) \log \left( \frac{0.5^{2^2} (1 + 0.5) (1 + 0.5^3) (1 + 0.5^{2 \times 2 - 1})}{(1 + 0.5^2) (1 + 0.5^4) (1 + 0.5^{2 \times 2})} \right) = -64 \pi \int_1^{0.084083} \frac{1}{t} dt$$

$$71 \pi^2 * \text{colog} \left( \frac{0.5^{2^2} (1 + 0.5) (1 + 0.5^3) (1 + 0.5^{2 \times 2 - 1})}{(1 + 0.5^2) (1 + 0.5^4) (1 + 0.5^{2 \times 2})} \right) /$$

Input:

$$71 \pi^2 \left( -\log \left( \frac{0.5^{2^2} (1 + 0.5) (1 + 0.5^3) (1 + 0.5^{2 \times 2 - 1})}{(1 + 0.5^2) (1 + 0.5^4) (1 + 0.5^{2 \times 2})} \right) \right)$$

Open code

- $\log(x)$  is the natural logarithm

Enlarge Data Customize A Plaintext Interactive

Result:

More digits

1735.00...

This result is very near to the mass of candidate glueball  $f_0(1710)$  meson.

Series representations:

More

$$(71 \pi^2) (-1) \log \left( \frac{0.5^{2^2} (1 + 0.5) (1 + 0.5^3) (1 + 0.5^{2 \times 2 - 1})}{(1 + 0.5^2) (1 + 0.5^4) (1 + 0.5^{2 \times 2})} \right) = 71 \pi^2 \sum_{k=1}^{\infty} \frac{(-1)^k (-0.915917)^k}{k}$$

Open code

Enlarge Data Customize A Plaintext Interactive

$$(71 \pi^2) (-1) \log \left( \frac{0.5^{2^2} (1+0.5)(1+0.5^3)(1+0.5^{2^2-1})}{(1+0.5^2)(1+0.5^4)(1+0.5^{2^2})} \right) =$$

$$-142 i \pi^3 \left\lfloor \frac{\arg(0.084083 - x)}{2 \pi} \right\rfloor - 71 \pi^2 \log(x) +$$

$$71 \pi^2 \sum_{k=1}^{\infty} \frac{(-1)^k (0.084083 - x)^k x^{-k}}{k} \text{ for } x < 0$$

Open code

$$(71 \pi^2) (-1) \log \left( \frac{0.5^{2^2} (1+0.5)(1+0.5^3)(1+0.5^{2^2-1})}{(1+0.5^2)(1+0.5^4)(1+0.5^{2^2})} \right) =$$

$$-142 i \pi^3 \left\lfloor \frac{-\pi + \arg\left(\frac{0.084083}{z_0}\right) + \arg(z_0)}{2 \pi} \right\rfloor -$$

$$71 \pi^2 \log(z_0) + 71 \pi^2 \sum_{k=1}^{\infty} \frac{(-1)^k (0.084083 - z_0)^k z_0^{-k}}{k}$$

Open code

- $\arg(z)$  is the complex argument
  - $\lfloor x \rfloor$  is the floor function
  - $i$  is the imaginary unit
    - [More information](#)

Integral representation:

$$(71 \pi^2) (-1) \log \left( \frac{0.5^{2^2} (1+0.5)(1+0.5^3)(1+0.5^{2^2-1})}{(1+0.5^2)(1+0.5^4)(1+0.5^{2^2})} \right) = -71 \pi^2 \int_1^{0.084083} \frac{1}{t} dt$$

Now, we have for  $S_1(q)$ :

$$\text{Sum}_{\{n \geq 0\}} q^{(n^2+2n)} (1+q)(1+q^3)\dots(1+q^{(2n-1)}) / ((1+q^2)(1+q^4)\dots(1+q^{(2n)}))$$

$$\text{sum}(((q^{(n^2+2n)} (1+q)(1+q^3)(1+q^{(2n-1)}))) / (((1+q^2)(1+q^4)(1+q^{(2n)}))), n$$

$$= 0 \text{ to } 5$$

Result:



$$\sum_{n=0}^5 \frac{q^{n^2+2n} (1+q)(1+q^3)(1+q^{2n-1})}{(1+q^2)(1+q^4)(1+q^{2n})} = \frac{1}{2(q^2+1)^2(q^4+1)^2} \\ (q+1)(q^3+1) \left( 2(q+1)(q^4+1)q^3 + \left(\frac{1}{q}+1\right)(q^2+1)(q^4+1) + \right. \\ \left. 2(q^2+1)(q^3+1)q^8 + \frac{2(q^2+1)(q^4+1)(q^9+1)q^{35}}{q^{10}+1} + \right. \\ \left. \frac{2(q^2+1)(q^4+1)(q^7+1)q^{24}}{q^8+1} + \frac{2(q^2+1)(q^4+1)(q^5+1)q^{15}}{q^6+1} \right)$$

Open code

### Enlarge Data Customize A Plaintext Interactive

Sum:

$$\sum_{n=0}^5 \frac{(q+1)(q^3+1)q^{n^2+2n}(q^{2n-1}+1)}{(q^2+1)(q^4+1)(q^{2n}+1)} = \\ \frac{(q+1)^2(q^3+1)q^3}{(q^2+1)^2(q^4+1)} + \frac{\left(\frac{1}{q}+1\right)(q+1)(q^3+1)}{2(q^2+1)(q^4+1)} + \frac{(q+1)(q^3+1)^2q^8}{(q^2+1)(q^4+1)^2} + \\ \frac{(q+1)(q^3+1)(q^9+1)q^{35}}{(q^2+1)(q^4+1)(q^{10}+1)} + \frac{(q+1)(q^3+1)(q^7+1)q^{24}}{(q^2+1)(q^4+1)(q^8+1)} + \frac{(q+1)(q^3+1)(q^5+1)q^{15}}{(q^2+1)(q^4+1)(q^6+1)}$$

Open code

$$\text{sum}(((0.5^{(n^2+2n)}(1+0.5)(1+0.5^3)(1+0.5^{(2n-1))}))) / \\ (((1+0.5^2)(1+0.5^4)(1+0.5^{(2n)}))), n=0 \text{ to } 5$$

Sum:

$$\sum_{n=0}^5 \frac{(1+0.5)(1+0.5^3)0.5^{n^2+2n}(0.5^{2n-1}+1)}{(1+0.5^2)(1+0.5^4)(0.5^{2n}+1)} = \frac{44669619359076482127}{21253382060638208000}$$

Open code

### Enlarge Data Customize A Plaintext Interactive

Decimal approximation:

More digits

2.101765226429807978776375749652386702804412371589230330203...

Open code

$$(13*3)/(24*2+2) * \text{sum}(((0.5^{(n^2+2n)}(1+0.5)(1+0.5^3)(1+0.5^{(2n-1))}))) / \\ (((1+0.5^2)(1+0.5^4)(1+0.5^{(2n)}))), n=0 \text{ to } 5$$

Input interpretation:

$$\frac{13 \times 3}{24 \times 2 + 2} \sum_{n=0}^5 \frac{0.5^{n^2+2n} (1+0.5)(1+0.5^3)(1+0.5^{2n-1})}{(1+0.5^2)(1+0.5^4)(1+0.5^{2n})}$$

### Enlarge Data Customize A Plaintext Interactive

Result:

1.63938

$$1.63938 \approx \zeta(2) = \frac{\pi^2}{6} = 1.6449$$

$24 \times 4 + 10^3 [(((13 \times 3) / (24 \times 2 + 2) \times \sum_{n=0}^5 ((0.5^{n^2+2n}) (1+0.5)(1+0.5^3)(1+0.5^{(2n-1))})) / (((1+0.5^2)(1+0.5^4)(1+0.5^{(2n))})), n = 0 \text{ to } 5)]$

Input interpretation:

$$24 \times 4 + 10^3 \left( \frac{13 \times 3}{24 \times 2 + 2} \sum_{n=0}^5 \frac{0.5^{n^2+2n} (1+0.5)(1+0.5^3)(1+0.5^{2n-1})}{(1+0.5^2)(1+0.5^4)(1+0.5^{2n})} \right)$$

Enlarge Data Customize A [Plaintext](#) [Interactive](#)

Result:

1735.38

This result is very near to the mass of candidate glueball  $f_0(1710)$  meson.

$(((((24 \times 4 + 10^3 [(((13 \times 3) / (24 \times 2 + 2) \times \sum_{n=0}^5 ((0.5^{n^2+2n}) (1+0.5)(1+0.5^3)(1+0.5^{(2n-1))})) / (((1+0.5^2)(1+0.5^4)(1+0.5^{(2n))})), n = 0 \text{ to } 5)]))))))^{1/3}$

Input interpretation:

$$\sqrt[3]{24 \times 4 + 10^3 \left( \frac{13 \times 3}{24 \times 2 + 2} \sum_{n=0}^5 \frac{0.5^{n^2+2n} (1+0.5)(1+0.5^3)(1+0.5^{2n-1})}{(1+0.5^2)(1+0.5^4)(1+0.5^{2n})} \right)}$$

Enlarge Data Customize A [Plaintext](#) [Interactive](#)

Result:

12.0171

This result is very near to the value of black hole entropy 12,1904

$2(((24 \times 4 + 10^3 [(((13 \times 3) / (24 \times 2 + 2) \times \sum_{n=0}^5 ((0.5^{n^2+2n}) (1+0.5)(1+0.5^3)(1+0.5^{(2n-1))})) / (((1+0.5^2)(1+0.5^4)(1+0.5^{(2n))})), n = 0 \text{ to } 5)]))))^{1/3}$

Input interpretation:

$$2 \sqrt[3]{24 \times 4 + 10^3 \left( \frac{13 \times 3}{24 \times 2 + 2} \sum_{n=0}^5 \frac{0.5^{n^2+2n} (1+0.5)(1+0.5^3)(1+0.5^{2n-1})}{(1+0.5^2)(1+0.5^4)(1+0.5^{2n})} \right)}$$

Enlarge Data Customize A [Plaintext](#) [Interactive](#)

Result:

24.0341

This value is linked to the "Ramanujan function" (an elliptic modular function that satisfies the need for "conformal symmetry") that has 24 "modes" corresponding to the physical vibrations of a bosonic string.

$$\left( \frac{24 \times 4 + 10^3 \left[ \frac{13 \times 3}{24 \times 2 + 2} \sum_{n=0}^5 \frac{0.5^{n^2+2n} (1+0.5)(1+0.5^3)(1+0.5^{2n-1})}{(1+0.5^2)(1+0.5^4)(1+0.5^{2n})} \right]}{1} \right)^{1/15}$$

Input interpretation:

$$\sqrt[15]{24 \times 4 + 10^3 \left( \frac{13 \times 3}{24 \times 2 + 2} \sum_{n=0}^5 \frac{0.5^{n^2+2n} (1+0.5)(1+0.5^3)(1+0.5^{2n-1})}{(1+0.5^2)(1+0.5^4)(1+0.5^{2n})} \right)}$$

Enlarge Data Customize A [Plaintext](#) [Interactive](#)

Result:

1.64422

$$1.64422 \approx \zeta(2) = \frac{\pi^2}{6} = 1.6449$$

Then:

$$\left( \frac{q^{n^2+2n} (1+q)(1+q^3)(1+q^{2n-1})}{(1+q^2)(1+q^4)(1+q^{2n})} \right)$$

Input:

$$\frac{q^{n^2+2n} (1+q)(1+q^3)(1+q^{2n-1})}{(1+q^2)(1+q^4)(1+q^{2n})}$$

Values:

n	
0	$\frac{\left(\frac{1}{q} + 1\right)(q+1)(q^3+1)}{2(q^2+1)(q^4+1)}$
1	$\frac{q^3(q+1)^2(q^3+1)}{(q^2+1)^2(q^4+1)}$
2	$\frac{q^8(q+1)(q^3+1)^2}{(q^2+1)(q^4+1)^2}$
3	$\frac{q^{15}(q+1)(q^3+1)(q^5+1)}{(q^2+1)(q^4+1)(q^6+1)}$

Expanded form:

Step-by-step solution

$$\begin{aligned} & \frac{q^{n^2+2n}}{(q^2+1)(q^4+1)(q^{2n}+1)} + \frac{q^{n^2+2n+1}}{(q^2+1)(q^4+1)(q^{2n}+1)} + \\ & \frac{q^{n^2+2n+3}}{(q^2+1)(q^4+1)(q^{2n}+1)} + \frac{q^{n^2+2n+4}}{(q^2+1)(q^4+1)(q^{2n}+1)} + \frac{q^{n^2+4n-1}}{(q^2+1)(q^4+1)(q^{2n}+1)} + \\ & \frac{q^{n^2+4n}}{(q^2+1)(q^4+1)(q^{2n}+1)} + \frac{q^{n^2+4n+2}}{(q^2+1)(q^4+1)(q^{2n}+1)} + \frac{q^{n^2+4n+3}}{(q^2+1)(q^4+1)(q^{2n}+1)} + \\ & \frac{q^{n^2+4n+3}}{(q^2+1)(q^4+1)(q^{2n}+1)} + \frac{q^{n^2+4n+2}}{(q^2+1)(q^4+1)(q^{2n}+1)} + \frac{q^{n^2+4n-1}}{(q^2+1)(q^4+1)(q^{2n}+1)} \end{aligned}$$

Real roots:  
 Exact forms  
 More digits

$$n = m + 1, \quad q = -1, \quad m^2 + 6m \in \mathbb{Z}, \quad m^2 + 4m \in \mathbb{Z}, \quad 2m \in \mathbb{Z}, \quad m \in \mathbb{Z}$$

[Open code](#)

### Enlarge Data Customize A Plaintext Interactive

$$n > 0.5, \quad q = 0$$

Integer root:

$$q = -1$$

[Open code](#)

### Enlarge Data Customize A Plaintext Interactive

Roots for the variable  $q$ :  
 Step-by-step solution

$$q = -1$$

[Open code](#)

$$q = 0$$

$$q = \sqrt[3]{-1}$$

$$q = -(-1)^{2/3}$$

$$q = \sqrt[2n-1]{-1}$$

Series expansion at  $q = 0$ :

$$\frac{q^{n(n+2)} (q^{2n} + q) \left( \frac{1}{q} + 1 - q + q^3 - q^4 + O(q^5) \right)}{q^{2n} + 1}$$

[Open code](#)

### Enlarge Data Customize A Plaintext Interactive

Series expansion at  $q = \infty$ :

$$\frac{q^{n(n+2)} (q^{2n} + q) \left( \left(\frac{1}{q}\right)^3 + \left(\frac{1}{q}\right)^4 - \left(\frac{1}{q}\right)^5 + \left(\frac{1}{q}\right)^7 - \left(\frac{1}{q}\right)^8 - \left(\frac{1}{q}\right)^9 + \left(\frac{1}{q}\right)^{11} + \left(\frac{1}{q}\right)^{12} + O\left(\left(\frac{1}{q}\right)^{13}\right) \right)}{q^{2n} + 1}$$

[Big-O notation](#)

Derivative:

Step-by-step solution

$$\frac{\partial}{\partial q} \left( \frac{q^{n^2+2n} (1+q)(1+q^3)(1+q^{2n-1})}{(1+q^2)(1+q^4)(1+q^{2n})} \right) =$$

$$\left( \frac{q^{n(n+2)} \left( -2(1+q)(1+q^3)(1+q^4)(q^{2n}+q)(1+q^{2n}) + \right. \right.}{n(n+2)(1+q)^2(1+q^2)((1+q^2)-q)(1+q^4)(q^{2n}+q)(1+q^{2n})} -$$

$$\frac{4(1+q)(1+q^2)(1+q^3)q^2(q^{2n}+q)(1+q^{2n}) + (2n-1)(1+q)(1+q^2)(1+q^3)(1+q^4)(1+q^{2n})q^{2n-2} + (1+q^2)(1+q^3)(1+q^4)(q^{2n-1}+1)(1+q^{2n}) - 2n(1+q)(1+q^2)(1+q^3)(1+q^4)(q^{2n}+q)q^{2n-2}}{(1+q^2)^2(1+q^4)^2(1+q^{2n})^2} \left. \right) / ((1+q^2)^2(1+q^4)^2(1+q^{2n})^2)$$

For  $q = 0.5$  and  $n = 2$ , we obtain for  $S_1(q)$ :

$$\frac{((((0.5^{2^2+2*2})(1+0.5)(1+0.5^3)(1+0.5^{2*2-1}))))}{((((1+0.5^2)(1+0.5^4)(1+0.5^{2*2}))))}$$

Input:

$$\frac{0.5^{2^2+2*2} (1+0.5)(1+0.5^3)(1+0.5^{2*2-1})}{(1+0.5^2)(1+0.5^4)(1+0.5^{2*2})}$$

[Open code](#)

Enlarge Data Customize A [Plaintext](#) [Interactive](#)

Result:

More digits

0.005255190311418685121107266435986159169550173010380622837...

[Open code](#)

0.005255190311418685121107266435986159169550173010380622837

$$(21*5)\pi * \operatorname{colog} \left( \frac{((((0.5^{2^2+2*2})(1+0.5)(1+0.5^3)(1+0.5^{2*2-1}))))}{((((1+0.5^2)(1+0.5^4)(1+0.5^{2*2}))))} \right)$$

Input:

$$(21 \times 5)\pi \left( -\log \left( \frac{0.5^{2^2+2*2} (1+0.5)(1+0.5^3)(1+0.5^{2*2-1})}{(1+0.5^2)(1+0.5^4)(1+0.5^{2*2})} \right) \right)$$

[Open code](#)

- $\log(x)$  is the natural logarithm

Enlarge Data Customize A [Plaintext](#) [Interactive](#)

Result:

More digits

1731.32...

This result is very near to the mass of candidate glueball  $f_0(1710)$  meson.

Series representations:

More

$$\left( \pi \left( -\log \left( \frac{0.5^{2^2+2 \times 2} (1+0.5)(1+0.5^3)(1+0.5^{2 \times 2-1})}{(1+0.5^2)(1+0.5^4)(1+0.5^{2 \times 2})} \right) \right) \right) 21 \times 5 =$$

$$105 \pi \sum_{k=1}^{\infty} \frac{(-1)^k (-0.994745)^k}{k}$$

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$$\left( \pi \left( -\log \left( \frac{0.5^{2^2+2 \times 2} (1+0.5)(1+0.5^3)(1+0.5^{2 \times 2-1})}{(1+0.5^2)(1+0.5^4)(1+0.5^{2 \times 2})} \right) \right) \right) 21 \times 5 =$$

$$-210 i \pi^2 \left\lfloor \frac{\arg(0.00525519 - x)}{2 \pi} \right\rfloor - 105 \pi \log(x) +$$

$$105 \pi \sum_{k=1}^{\infty} \frac{(-1)^k (0.00525519 - x)^k x^{-k}}{k} \quad \text{for } x < 0$$

[Open code](#)

$$\left( \pi \left( -\log \left( \frac{0.5^{2^2+2 \times 2} (1+0.5)(1+0.5^3)(1+0.5^{2 \times 2-1})}{(1+0.5^2)(1+0.5^4)(1+0.5^{2 \times 2})} \right) \right) \right) 21 \times 5 =$$

$$-210 i \pi^2 \left[ -\frac{-\pi + \arg\left(\frac{0.00525519}{z_0}\right) + \arg(z_0)}{2 \pi} \right] -$$

$$105 \pi \log(z_0) + 105 \pi \sum_{k=1}^{\infty} \frac{(-1)^k (0.00525519 - z_0)^k z_0^{-k}}{k}$$

[Open code](#)

- $\arg(z)$  is the complex argument
  - $\lfloor x \rfloor$  is the floor function
  - $i$  is the imaginary unit
  - [More information](#)

Integral representation:

$$\left( \pi \left( -\log \left( \frac{0.5^{2^2+2 \times 2} (1+0.5)(1+0.5^3)(1+0.5^{2 \times 2-1})}{(1+0.5^2)(1+0.5^4)(1+0.5^{2 \times 2})} \right) \right) \right) 21 \times 5 = -105 \pi \int_1^{0.00525519} \frac{1}{t} dt$$

$$\frac{\sqrt[3]{(21 \times 5) \pi \cdot \operatorname{colog} \left( \frac{0.5^{2^2+2 \times 2} (1+0.5)(1+0.5^3)(1+0.5^{2 \times 2-1})}{(1+0.5^2)(1+0.5^4)(1+0.5^{2 \times 2})} \right)}}{\sqrt[3]{(1+0.5^2)(1+0.5^4)(1+0.5^{2 \times 2})}}^{1/3}}$$

Input:

$$\sqrt[3]{(21 \times 5) \pi \left( -\log \left( \frac{0.5^{2^2+2 \times 2} (1+0.5)(1+0.5^3)(1+0.5^{2 \times 2-1})}{(1+0.5^2)(1+0.5^4)(1+0.5^{2 \times 2})} \right) \right)}$$

[Open code](#)

- $\log(x)$  is the natural logarithm

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Result:

More digits

12.0077...

This result is very near to the value of black hole entropy 12,1904

$$\frac{2 \sqrt[3]{(21 \times 5) \pi \cdot \operatorname{colog} \left( \frac{0.5^{2^2+2 \times 2} (1+0.5)(1+0.5^3)(1+0.5^{2 \times 2-1})}{(1+0.5^2)(1+0.5^4)(1+0.5^{2 \times 2})} \right)}}{\sqrt[3]{(1+0.5^2)(1+0.5^4)(1+0.5^{2 \times 2})}}^{1/3}}$$

Input:

$$2 \sqrt[3]{(21 \times 5) \pi \left( -\log \left( \frac{0.5^{2^2+2 \times 2} (1+0.5)(1+0.5^3)(1+0.5^{2 \times 2-1})}{(1+0.5^2)(1+0.5^4)(1+0.5^{2 \times 2})} \right) \right)}$$

[Open code](#)

- $\log(x)$  is the natural logarithm

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Result:

More digits

24.0154...

This value is linked to the "Ramanujan function" (an elliptic modular function that satisfies the need for "conformal symmetry") that has 24 "modes" corresponding to the physical vibrations of a bosonic string.

Series representations:

More

$$2 \sqrt[3]{\left[ \pi \left( -\log \left( \frac{0.5^{2^2+2 \times 2} (1+0.5)(1+0.5^3)(1+0.5^{2 \times 2-1})}{(1+0.5^2)(1+0.5^4)(1+0.5^{2 \times 2})} \right) \right) \right]} 21 \times 5 = 2 \sqrt[3]{105} \sqrt[3]{\pi \sum_{k=1}^{\infty} \frac{(-1)^k (-0.994745)^k}{k}}$$

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$$2 \sqrt[3]{\left(\pi \left(-\log \left(\frac{0.5^{2^2+2 \times 2} (1+0.5)(1+0.5^3)(1+0.5^{2 \times 2-1})}{(1+0.5^2)(1+0.5^4)(1+0.5^{2 \times 2})}\right)\right)\right)} 21 \times 5 = 2 \sqrt[3]{105}$$

$$\sqrt[3]{-\pi \left(2 i \pi \left[\frac{\arg(0.00525519 - x)}{2 \pi}\right] + \log(x) - \sum_{k=1}^{\infty} \frac{(-1)^k (0.00525519 - x)^k x^{-k}}{k}\right)}$$

for  $x < 0$

[Open code](#)

$$2 \sqrt[3]{\left(\pi \left(-\log \left(\frac{0.5^{2^2+2 \times 2} (1+0.5)(1+0.5^3)(1+0.5^{2 \times 2-1})}{(1+0.5^2)(1+0.5^4)(1+0.5^{2 \times 2})}\right)\right)\right)} 21 \times 5 =$$

$$2 \sqrt[3]{105} \left(-\pi \left(\log(z_0) + \left[\frac{\arg(0.00525519 - z_0)}{2 \pi}\right] \left(\log\left(\frac{1}{z_0}\right) + \log(z_0)\right) - \sum_{k=1}^{\infty} \frac{(-1)^k (0.00525519 - z_0)^k z_0^{-k}}{k}\right)\right)^{1/3}$$

[Open code](#)

- $\arg(z)$  is the complex argument
  - $\lfloor x \rfloor$  is the floor function
  - $i$  is the imaginary unit
  - [More information](#)

Integral representation:

$$2 \sqrt[3]{\left(\pi \left(-\log \left(\frac{0.5^{2^2+2 \times 2} (1+0.5)(1+0.5^3)(1+0.5^{2 \times 2-1})}{(1+0.5^2)(1+0.5^4)(1+0.5^{2 \times 2})}\right)\right)\right)} 21 \times 5 =$$

$$2 \sqrt[3]{105} \sqrt[3]{-\pi \int_1^{0.00525519} \frac{1}{t} dt}$$

$$\left(\frac{\pi \cdot \operatorname{colog}\left(\frac{0.5^{2^2+2 \times 2} (1+0.5)(1+0.5^3)(1+0.5^{2 \times 2-1})}{(1+0.5^2)(1+0.5^4)(1+0.5^{2 \times 2})}\right)}{15}\right)^{1/3}$$

Input:

$$\sqrt[15]{(21 \times 5) \pi \left(-\log \left(\frac{0.5^{2^2+2 \times 2} (1+0.5)(1+0.5^3)(1+0.5^{2 \times 2-1})}{(1+0.5^2)(1+0.5^4)(1+0.5^{2 \times 2})}\right)\right)}$$



[Open code](#)

- $\log(x)$  is the natural logarithm

### Enlarge Data Customize A Plaintext Interactive

Result:

- Fewer digits
- More digits

1.643962248723371608808542655060252532517999740271402583071...

$$1.6439622487233716088\dots \approx \zeta(2) = \frac{\pi^2}{6} = 1.6449$$

Continued fraction:

Linear form

$$\begin{aligned} & 1 + \frac{1}{1 + \frac{1}{1 + \frac{1}{1 + \frac{1}{4 + \frac{1}{4 + \frac{1}{2 + \frac{1}{2 + \frac{1}{15 + \frac{1}{1 + \frac{1}{1 + \frac{1}{2 + \frac{1}{1 + \frac{1}{2 + \frac{1}{2 + \frac{1}{39 + \frac{1}{7 + \frac{1}{2 + \frac{1}{5 + \frac{1}{1 + \frac{1}{5 + \frac{1}{1 + \frac{1}{\dots}}}}}}}}}}}}}}}}}}}}}}}}}}}}}}}}}}}} \end{aligned}$$

[Open code](#)

### Enlarge Data Customize A Plaintext Interactive

Possible closed forms:

More

$$\frac{2\,289\,028\,079\,\pi}{4\,374\,305\,920} \approx 1.64396224872337160877978$$

$$\frac{\log\left(\frac{560\,160\,117}{180\,626}\right)}{\log(133)} \approx 1.64396224872337160877616$$

$$\pi \sqrt{\text{root of } 766x^4 + 4829x^3 + 22x^2 - 1929x + 254 \text{ near } x = 0.523289} \approx 1.64396224872337160874570$$

Now, for  $T_0(q)$ , we have:

$$\sum_{n=0}^m (q^{(n+1)(n+2)} (1+q^2)(1+q^4)(1+q^{(2n)}) / (((1+q)(1+q^3)(1+q^{(2n+1))}))), n$$

Input interpretation:

$$\sum_{n=0}^m \frac{q^{(n+1)(n+2)} (1+q^2)(1+q^4)(1+q^{2n})}{(1+q)(1+q^3)(1+q^{2n+1})}$$

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Result:

$$\sum_{n=0}^m \frac{q^{(n+1)(n+2)} (1+q^2)(1+q^4)(1+q^{2n})}{(1+q)(1+q^3)(1+q^{2n+1})}$$

sum (q<sup>((n+1)(n+2))</sup> (1+q<sup>2</sup>)(1+q<sup>4</sup>)(1+q<sup>(2n)</sup>)/(((1+q)(1+q<sup>3</sup>)(1+q<sup>(2n+1)</sup>)))) , n =0 to 5

Result:

$$\sum_{n=0}^5 \frac{q^{(n+1)(n+2)} (1+q^2)(1+q^4)(1+q^{2n})}{(1+q)(1+q^3)(1+q^{2n+1})} = \frac{2(q^2+1)(q^4+1)q^2}{(q+1)^2(q^3+1)} + \frac{(q^2+1)^2(q^4+1)q^6}{(q+1)(q^3+1)^2} + \frac{(q^2+1)(q^4+1)^2q^{12}}{(q+1)(q^3+1)(q^5+1)} + \frac{(q^2+1)(q^4+1)(q^{10}+1)q^{42}}{(q+1)(q^3+1)(q^{11}+1)} + \frac{(q^2+1)(q^4+1)(q^8+1)q^{30}}{(q+1)(q^3+1)(q^9+1)} + \frac{(q^2+1)(q^4+1)(q^6+1)q^{20}}{(q+1)(q^3+1)(q^7+1)}$$

[Open code](#)

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Result:

$$\sum_{n=0}^5 \frac{q^{(n+1)(n+2)} (1+q^2)(1+q^4)(1+q^{2n})}{(1+q)(1+q^3)(1+q^{2n+1})} = \frac{1}{(q+1)^2(q^3+1)^2} \left( q^2(q^2+1)(q^4+1) \left( 2(q^3+1) + (q+1)(q^2+1)q^4 + \frac{(q+1)(q^3+1)(q^{10}+1)q^{40}}{q^{11}+1} + \frac{(q+1)(q^3+1)(q^8+1)q^{28}}{q^9+1} + \frac{(q+1)(q^3+1)(q^6+1)q^{18}}{q^7+1} + \frac{(q+1)(q^3+1)(q^4+1)q^{10}}{q^5+1} \right) \right)$$

[Open code](#)

sum (0.5<sup>((n+1)(n+2))</sup> (1+0.5<sup>2</sup>)(1+0.5<sup>4</sup>)(1+0.5<sup>(2n)</sup>)/(((1+0.5)(1+0.5<sup>3</sup>)(1+0.5<sup>(2n+1)</sup>)))) , n =0 to 5

Sum:

$$\sum_{n=0}^5 \frac{(1+0.5^2)(1+0.5^4)0.5^{(n+1)(n+2)}(0.5^{2n}+1)}{(1+0.5)(1+0.5^3)(0.5^{2n+1}+1)} = \frac{10871498844334353191855}{39359791323927357161472}$$

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Decimal approximation:

More digits

0.276208243963057291099201936449639811558345854974960010689...

[Open code](#)

$$6 * \sum_{n=0}^5 \frac{0.5^{(n+1)(n+2)}}{(1+0.5^2)(1+0.5^4)(1+0.5^{2n})} \times \frac{1+0.5^{2n}}{((1+0.5)(1+0.5^3)(1+0.5^{2n+1}))}, n=0 \text{ to } 5$$

Input interpretation:

$$6 \sum_{n=0}^5 0.5^{(n+1)(n+2)} (1+0.5^2)(1+0.5^4) \times \frac{1+0.5^{2n}}{(1+0.5)(1+0.5^3)(1+0.5^{2n+1})}$$

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Result:

1.65725

1.65725 is very near to the 14th root of the following Ramanujan's class invariant

$$Q = (G_{505}/G_{101/5})^3 = 1164,2696 \text{ i.e. } 1,65578\dots$$

$$24*3 + 10^3 [(((6 * \sum_{n=0}^5 \frac{0.5^{(n+1)(n+2)}}{(1+0.5^2)(1+0.5^4)(1+0.5^{2n})} \times \frac{1+0.5^{2n}}{((1+0.5)(1+0.5^3)(1+0.5^{2n+1}))}, n=0 \text{ to } 5)))]$$

Input interpretation:

$$24 \times 3 + 10^3 \left( 6 \sum_{n=0}^5 0.5^{(n+1)(n+2)} (1+0.5^2)(1+0.5^4) \times \frac{1+0.5^{2n}}{(1+0.5)(1+0.5^3)(1+0.5^{2n+1})} \right)$$

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Result:

1729.25

This result is very near to the mass of candidate glueball  $f_0(1710)$  meson. Furthermore, 1728 occurs in the algebraic formula for the j-invariant of an elliptic curve. As a consequence, it is sometimes called a Zagier as a pun on the Gross–Zagier theorem. The number 1728 is one less than the Hardy–Ramanujan number 1729

$$((((((((24*3 + 10^3 [(((6 * \sum_{n=0}^5 \frac{0.5^{(n+1)(n+2)}}{(1+0.5^2)(1+0.5^4)(1+0.5^{2n})} \times \frac{1+0.5^{2n}}{((1+0.5)(1+0.5^3)(1+0.5^{2n+1}))}, n=0 \text{ to } 5)))]))))))^{1/3}$$

Input interpretation:

$$\sqrt[3]{24 \times 3 + 10^3 \left( 6 \sum_{n=0}^5 0.5^{(n+1)(n+2)} (1+0.5^2)(1+0.5^4) \times \frac{1+0.5^{2n}}{(1+0.5)(1+0.5^3)(1+0.5^{2n+1})} \right)}$$

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Result:

12.0029

This result is very near to the value of black hole entropy 12,1904

$$2((((((((24*3 + 10^3 [(((6 * \sum_{n=0}^5 \frac{0.5^{(n+1)(n+2)}}{(1+0.5^2)(1+0.5^4)(1+0.5^{2n})} \times \frac{1+0.5^{2n}}{((1+0.5)(1+0.5^3)(1+0.5^{2n+1}))}, n=0 \text{ to } 5)))]))))))^{1/3}$$

Input interpretation:

$$2 \left( 24 \times 3 + 10^3 \left( 6 \sum_{n=0}^5 0.5^{(n+1)(n+2)} (1 + 0.5^2) (1 + 0.5^4) \times \frac{1 + 0.5^{2n}}{(1 + 0.5)(1 + 0.5^3)(1 + 0.5^{2n+1})} \right) \right)^{1/3}$$

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Result:

24.0058

This value is linked to the "Ramanujan function" (an elliptic modular function that satisfies the need for "conformal symmetry") that has 24 "modes" corresponding to the physical vibrations of a bosonic string.

$$\left( \left( \left( \left( \left( 24 \times 3 + 10^3 \left[ \left( 6 \sum_{n=0}^5 (0.5^{(n+1)(n+2)} (1 + 0.5^2) (1 + 0.5^4) \times \frac{1 + 0.5^{2n}}{(1 + 0.5)(1 + 0.5^3)(1 + 0.5^{2n+1})} \right) \right] \right) \right) \right) \right) \right) \right)^{1/15}$$

Input interpretation:

$$15 \sqrt{24 \times 3 + 10^3 \left( 6 \sum_{n=0}^5 0.5^{(n+1)(n+2)} (1 + 0.5^2) (1 + 0.5^4) \times \frac{1 + 0.5^{2n}}{(1 + 0.5)(1 + 0.5^3)(1 + 0.5^{2n+1})} \right)}$$

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Result:

1.64383

$$1.64383 \approx \zeta(2) = \frac{\pi^2}{6} = 1.6449$$

For which, for  $q = 0.5$ , and  $n = 2$ , we obtain:

$$\frac{(0.5^{(2+1)(2+2)} (1 + 0.5^2) (1 + 0.5^4) (1 + 0.5^{2 \times 2}))}{(1 + 0.5)(1 + 0.5^3)(1 + 0.5^{2 \times 2 + 1})}$$

Input:

$$\frac{0.5^{(2+1)(2+2)} (1 + 0.5^2) (1 + 0.5^4) (1 + 0.5^{2 \times 2})}{(1 + 0.5)(1 + 0.5^3)(1 + 0.5^{2 \times 2 + 1})}$$

[Open code](#)

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Result:

More digits

0.000197970372124017957351290684624017957351290684624017957...

0.000197970372124017957351290684624017957351290684624017957

$$(89+5)\pi * \operatorname{colog}\left(\frac{0.5^{(2+1)(2+2)} (1+0.5^2) (1+0.5^4) (1+0.5^{2 \times 2})}{(1+0.5)(1+0.5^3)(1+0.5^{2 \times 2+1})}\right)$$

Input:

$$(89 + 5) \pi \left( -\log \left( \frac{0.5^{(2+1)(2+2)} (1 + 0.5^2) (1 + 0.5^4) (1 + 0.5^{2 \times 2})}{(1 + 0.5) (1 + 0.5^3) (1 + 0.5^{2 \times 2+1})} \right) \right)$$

[Open code](#)

- $\log(x)$  is the natural logarithm

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Result:

More digits

2518.22...

This result 2518,22 is practically equal to the rest mass of charmed Sigma baryon  $2517.9 \pm 0.6$

Series representations:

More

$$\begin{aligned} & ((89 + 5) \pi) (-1) \log \left( \frac{0.5^{(2+1)(2+2)} (1 + 0.5^2) (1 + 0.5^4) (1 + 0.5^{2 \times 2})}{(1 + 0.5) (1 + 0.5^3) (1 + 0.5^{2 \times 2+1})} \right) = \\ & 94 \pi \sum_{k=1}^{\infty} \frac{(-1)^k (-0.999802)^k}{k} \end{aligned}$$

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$$\begin{aligned} & ((89 + 5) \pi) (-1) \log \left( \frac{0.5^{(2+1)(2+2)} (1 + 0.5^2) (1 + 0.5^4) (1 + 0.5^{2 \times 2})}{(1 + 0.5) (1 + 0.5^3) (1 + 0.5^{2 \times 2+1})} \right) = \\ & -188 i \pi^2 \left[ \frac{\arg(0.00019797 - x)}{2 \pi} \right] - 94 \pi \log(x) + \\ & 94 \pi \sum_{k=1}^{\infty} \frac{(-1)^k (0.00019797 - x)^k x^{-k}}{k} \quad \text{for } x < 0 \end{aligned}$$

[Open code](#)

$$\begin{aligned} & ((89 + 5) \pi) (-1) \log \left( \frac{0.5^{(2+1)(2+2)} (1 + 0.5^2) (1 + 0.5^4) (1 + 0.5^{2 \times 2})}{(1 + 0.5) (1 + 0.5^3) (1 + 0.5^{2 \times 2+1})} \right) = \\ & -188 i \pi^2 \left[ \frac{-\pi + \arg\left(\frac{0.00019797}{z_0}\right) + \arg(z_0)}{2 \pi} \right] - \\ & 94 \pi \log(z_0) + 94 \pi \sum_{k=1}^{\infty} \frac{(-1)^k (0.00019797 - z_0)^k z_0^{-k}}{k} \end{aligned}$$

Open code

- $\arg(z)$  is the complex argument
  - $\lfloor x \rfloor$  is the floor function
  - $i$  is the imaginary unit
  - [More information](#)

Integral representation:

$$((89 + 5) \pi) (-1) \log \left( \frac{0.5^{(2+1)(2+2)} (1 + 0.5^2) (1 + 0.5^4) (1 + 0.5^{2 \times 2})}{(1 + 0.5) (1 + 0.5^3) (1 + 0.5^{2 \times 2+1})} \right) = -94 \pi \int_1^{0.00019797} \frac{1}{t} dt$$

$$\left( \frac{94 \pi \cdot \text{colog} \left( \frac{0.5^{(2+1)(2+2)} (1 + 0.5^2) (1 + 0.5^4) (1 + 0.5^{2 \times 2})}{(1 + 0.5) (1 + 0.5^3) (1 + 0.5^{2 \times 2+1})} \right)}{1} \right)^{1/3}$$

Input:

$$\sqrt[3]{94 \pi \left( -\log \left( \frac{0.5^{(2+1)(2+2)} (1 + 0.5^2) (1 + 0.5^4) (1 + 0.5^{2 \times 2})}{(1 + 0.5) (1 + 0.5^3) (1 + 0.5^{2 \times 2+1})} \right) \right)}$$

Open code

- $\log(x)$  is the natural logarithm

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Result:

More digits

13.6050...

This result is practically equal to the Rydberg constant 13,605693 eV that is used to express the limiting value of the highest wavenumber (inverse wavelength) of any photon that can be emitted from an atom, or, alternatively, the wavenumber of the lowest-energy photon capable of ionizing an atom from its ground state.

$$\frac{5}{3} * \left( \frac{94 \pi \cdot \text{colog} \left( \frac{0.5^{(2+1)(2+2)} (1 + 0.5^2) (1 + 0.5^4) (1 + 0.5^{2 \times 2})}{(1 + 0.5) (1 + 0.5^3) (1 + 0.5^{2 \times 2+1})} \right)}{1} \right)^{1/3}$$

Input:

$$\frac{5}{3} \sqrt[3]{94 \pi \left( -\log \left( \frac{0.5^{(2+1)(2+2)} (1 + 0.5^2) (1 + 0.5^4) (1 + 0.5^{2 \times 2})}{(1 + 0.5) (1 + 0.5^3) (1 + 0.5^{2 \times 2+1})} \right) \right)}$$

Open code

- $\log(x)$  is the natural logarithm

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Result:

More digits

22.6750...

This result is very near to the value of black hole entropy 22,6589

Series representations:

More

$$\frac{1}{3} \sqrt[3]{94 \pi \left( -\log \left( \frac{0.5^{(2+1)(2+2)} (1+0.5^2) (1+0.5^4) (1+0.5^{2 \times 2})}{(1+0.5) (1+0.5^3) (1+0.5^{2 \times 2+1})} \right) \right)} 5 =$$

$$\frac{5}{3} \sqrt[3]{94} \sqrt[3]{\pi \sum_{k=1}^{\infty} \frac{(-1)^k (-0.999802)^k}{k}}$$

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$$\frac{1}{3} \sqrt[3]{94 \pi \left( -\log \left( \frac{0.5^{(2+1)(2+2)} (1+0.5^2) (1+0.5^4) (1+0.5^{2 \times 2})}{(1+0.5) (1+0.5^3) (1+0.5^{2 \times 2+1})} \right) \right)} 5 = \frac{5}{3} \sqrt[3]{94}$$

$$\sqrt[3]{-\pi \left( 2 i \pi \left[ \frac{\arg(0.00019797 - x)}{2 \pi} \right] + \log(x) - \sum_{k=1}^{\infty} \frac{(-1)^k (0.00019797 - x)^k x^{-k}}{k} \right)}$$

for  $x < 0$

[Open code](#)

$$\frac{1}{3} \sqrt[3]{94 \pi \left( -\log \left( \frac{0.5^{(2+1)(2+2)} (1+0.5^2) (1+0.5^4) (1+0.5^{2 \times 2})}{(1+0.5) (1+0.5^3) (1+0.5^{2 \times 2+1})} \right) \right)} 5 =$$

$$\frac{5}{3} \sqrt[3]{94} \left( -\pi \left( \log(z_0) + \left[ \frac{\arg(0.00019797 - z_0)}{2 \pi} \right] \left( \log\left(\frac{1}{z_0}\right) + \log(z_0) \right) - \sum_{k=1}^{\infty} \frac{(-1)^k (0.00019797 - z_0)^k z_0^{-k}}{k} \right) \right)^{1/3}$$

Integral representation:

$$\frac{1}{3} \sqrt[3]{94 \pi \left( -\log \left( \frac{0.5^{(2+1)(2+2)} (1+0.5^2) (1+0.5^4) (1+0.5^{2 \times 2})}{(1+0.5) (1+0.5^3) (1+0.5^{2 \times 2+1})} \right) \right)} 5 =$$

$$\frac{5}{3} \sqrt[3]{94} \sqrt[3]{-\pi \int_1^{0.00019797} \frac{1}{t} dt}$$

$$\left( \frac{94 \pi \cdot \text{colog} \left( \frac{0.5^{(2+1)(2+2)} (1+0.5^2) (1+0.5^4) (1+0.5^{2 \times 2})}{(1+0.5) (1+0.5^3) (1+0.5^{2 \times 2+1})} \right)}{1} \right)^{1/16}$$

Input:

$$16 \sqrt[94]{\pi \left( -\log \left( \frac{0.5^{(2+1)(2+2)} (1 + 0.5^2) (1 + 0.5^4) (1 + 0.5^{2 \times 2})}{(1 + 0.5) (1 + 0.5^3) (1 + 0.5^{2 \times 2 + 1})} \right) \right)}$$

Open code

- $\log(x)$  is the natural logarithm

### Enlarge Data Customize A [Plaintext](#) [Interactive](#)

Result:

- Fewer digits
- More digits

1.631429743343197487113605286196873395757977605587834541840...

1.6314297433431974871136052861968733957579776055878345

This result is a golden number

Continued fraction:

Linear form

$$1 + \frac{1}{1 + \frac{1}{1 + \frac{1}{1 + \frac{1}{2 + \frac{1}{2 + \frac{1}{18 + \frac{1}{6 + \frac{1}{1 + \frac{1}{10 + \frac{1}{3 + \frac{1}{2 + \frac{1}{1 + \frac{1}{1 + \frac{1}{3 + \frac{1}{2 + \frac{1}{1 + \frac{1}{1 + \frac{1}{15 + \frac{1}{1 + \frac{1}{1 + \frac{1}{\dots}}$$

Open code

### Enlarge Data Customize A [Plaintext](#) [Interactive](#)

Possible closed forms:

- More

$$\frac{1754809196 \pi}{3379180563} \approx 1.63142974334319748707458$$

$$\text{root of } 8228x^3 + 32559x^2 - 60588x - 23540 \text{ near } x = 1.63143 \approx$$

1.631429743343197487104480

1

$$\text{root of } 23540x^3 + 60588x^2 - 32559x - 8228 \text{ near } x = 0.612959 \approx$$

1.631429743343197487104480

Now, for  $T_1(q)$ , we have:



sum (q^(n^2+n) (1+q^2)(1+q^4)(1+q^(2n))) / ((1+q)(1+q^3)(1+q^(2n+1))), n= 0 to k

Input interpretation:

$$\sum_{n=0}^k \frac{q^{n^2+n} (1+q^2)(1+q^4)(1+q^{2n})}{(1+q)(1+q^3)(1+q^{2n+1})}$$

Enlarge Data Customize A [Plaintext](#) [Interactive](#)

Result:

$$\sum_{n=0}^k \frac{q^{n^2+n} (1+q^2)(1+q^4)(1+q^{2n})}{(1+q)(1+q^3)(1+q^{2n+1})}$$

For q = 0.5, we obtain:

sum (0.5^(n^2+n) (1+0.5^2)(1+0.5^4)(1+0.5^(2n))) / ((1+0.5)(1+0.5^3)(1+0.5^(2n+1))), n= 0 to 5

Sum:

$$\sum_{n=0}^5 \frac{(1+0.5^2)(1+0.5^4) 0.5^{n^2+n} (0.5^{2n} + 1)}{(1+0.5)(1+0.5^3)(0.5^{2n+1} + 1)} = \frac{12\,308\,281\,534\,536\,114\,095}{9\,609\,324\,053\,693\,202\,432}$$

[Open code](#)

Enlarge Data Customize A [Plaintext](#) [Interactive](#)

Decimal approximation:

More digits

1.280868609047023139259735645997459065034349956716746645287...

[Open code](#)

((((sum (0.5^(n^2+n) (1+0.5^2)(1+0.5^4)(1+0.5^(2n))) / ((1+0.5)(1+0.5^3)(1+0.5^(2n+1))), n= 0 to 5))))^2

Input interpretation:

$$\left( \sum_{n=0}^5 \frac{0.5^{n^2+n} (1+0.5^2)(1+0.5^4)(1+0.5^{2n})}{(1+0.5)(1+0.5^3)(1+0.5^{2n+1})} \right)^2$$

Enlarge Data Customize A [Plaintext](#) [Interactive](#)

Result:

1.64062

$$1.64062 \approx \zeta(2) = \frac{\pi^2}{6} = 1.6449$$

24\*4 + 10^3(((sum (0.5^(n^2+n) (1+0.5^2)(1+0.5^4)(1+0.5^(2n))) / ((1+0.5)(1+0.5^3)(1+0.5^(2n+1))), n= 0 to 5))))^2

Input interpretation:

$$24 \times 4 + 10^3 \left( \sum_{n=0}^5 \frac{0.5^{n^2+n} (1+0.5^2)(1+0.5^4)(1+0.5^{2n})}{(1+0.5)(1+0.5^3)(1+0.5^{2n+1})} \right)^2$$

Enlarge Data Customize A [Plaintext](#) [Interactive](#)

Result:

1736.62

This result is very near to the mass of candidate glueball  $f_0(1710)$  meson.

$$\left[ \left( \left( \left( 24 \times 4 + 10^3 \left( \sum_{n=0}^5 \frac{0.5^{n^2+n} (1+0.5^2)(1+0.5^4)(1+0.5^{2n})}{(1+0.5)(1+0.5^3)(1+0.5^{2n+1})} \right) \right) \right) \right)^2 \right]^{1/3}$$

Input interpretation:

$$\sqrt[3]{24 \times 4 + 10^3 \left( \sum_{n=0}^5 \frac{0.5^{n^2+n} (1+0.5^2)(1+0.5^4)(1+0.5^{2n})}{(1+0.5)(1+0.5^3)(1+0.5^{2n+1})} \right)^2}$$

Enlarge Data Customize A [Plaintext](#) [Interactive](#)

Result:

12.0199

This result is very near to the value of black hole entropy 12,1904

$$2 \left[ \left( \left( \left( 24 \times 4 + 10^3 \left( \sum_{n=0}^5 \frac{0.5^{n^2+n} (1+0.5^2)(1+0.5^4)(1+0.5^{2n})}{(1+0.5)(1+0.5^3)(1+0.5^{2n+1})} \right) \right) \right) \right)^2 \right]^{1/3}$$

Input interpretation:

$$2 \sqrt[3]{24 \times 4 + 10^3 \left( \sum_{n=0}^5 \frac{0.5^{n^2+n} (1+0.5^2)(1+0.5^4)(1+0.5^{2n})}{(1+0.5)(1+0.5^3)(1+0.5^{2n+1})} \right)^2}$$

Enlarge Data Customize A [Plaintext](#) [Interactive](#)

Result:

24.0399

This value is linked to the "Ramanujan function" (an elliptic modular function that satisfies the need for "conformal symmetry") that has 24 "modes" corresponding to the physical vibrations of a bosonic string.

$$\left[ \left( \left( \left( 24 \times 4 + 10^3 \left( \sum_{n=0}^5 \frac{0.5^{n^2+n} (1+0.5^2)(1+0.5^4)(1+0.5^{2n})}{(1+0.5)(1+0.5^3)(1+0.5^{2n+1})} \right) \right) \right) \right)^2 \right]^{1/15}$$

Input interpretation:

$$\sqrt[15]{24 \times 4 + 10^3 \left( \sum_{n=0}^5 \frac{0.5^{n^2+n} (1+0.5^2)(1+0.5^4)(1+0.5^{2n})}{(1+0.5)(1+0.5^3)(1+0.5^{2n+1})} \right)^2}$$

Enlarge Data Customize A [Plaintext](#) [Interactive](#)

Result:

1.6443

$$1.6443 \approx \zeta(2) = \frac{\pi^2}{6} = 1.6449$$

From which, for  $q = 0.5$ , and  $n = 2$ , we obtain:

$$(0.5^{2^2+2} (1+0.5^2)(1+0.5^4)(1+0.5^{(2*2)})) / ((1+0.5)(1+0.5^3)(1+0.5^{(2*2+1)}))$$

Input:

$$\frac{0.5^{2^2+2} (1 + 0.5^2) (1 + 0.5^4) (1 + 0.5^{2 \times 2})}{(1 + 0.5) (1 + 0.5^3) (1 + 0.5^{2 \times 2 + 1})}$$

[Open code](#)

Enlarge Data Customize A [Plaintext](#) [Interactive](#)

Result:

More digits

0.012670103815937149270482603815937149270482603815937149270...  
0.012670103815937149270482603815937149270482603815937149270

$$(64+48)\pi * \operatorname{colog}(\frac{((0.5^{(2^2+2)} (1+0.5^2)(1+0.5^4)(1+0.5^{(2*2)}))}{((1+0.5)(1+0.5^3)(1+0.5^{(2*2+1))}))})$$

Input:

$$(64 + 48) \pi \left( -\log \left( \frac{0.5^{2^2+2} (1 + 0.5^2) (1 + 0.5^4) (1 + 0.5^{2 \times 2})}{(1 + 0.5) (1 + 0.5^3) (1 + 0.5^{2 \times 2 + 1})} \right) \right)$$

[Open code](#)

- $\log(x)$  is the natural logarithm

Enlarge Data Customize A [Plaintext](#) [Interactive](#)

Result:

More digits

1537.10...  
1537.10...

This result is very near to the rest mass of Xi baryon,  $1535.0 \pm 0.6$

Series representations:

More

$$\left( (64 + 48) \pi (-1) \log \left( \frac{0.5^{2^2+2} (1 + 0.5^2) (1 + 0.5^4) (1 + 0.5^{2 \times 2})}{(1 + 0.5) (1 + 0.5^3) (1 + 0.5^{2 \times 2 + 1})} \right) \right) = 112 \pi \sum_{k=1}^{\infty} \frac{(-1)^k (-0.98733)^k}{k}$$

[Open code](#)

Enlarge Data Customize A [Plaintext](#) [Interactive](#)

$$\begin{aligned}
& ((64 + 48) \pi) (-1) \log \left( \frac{0.5^{2^2+2} (1 + 0.5^2) (1 + 0.5^4) (1 + 0.5^{2 \times 2})}{(1 + 0.5) (1 + 0.5^3) (1 + 0.5^{2 \times 2+1})} \right) = \\
& -224 i \pi^2 \left\lfloor \frac{\arg(0.0126701 - x)}{2 \pi} \right\rfloor - 112 \pi \log(x) + \\
& 112 \pi \sum_{k=1}^{\infty} \frac{(-1)^k (0.0126701 - x)^k x^{-k}}{k} \quad \text{for } x < 0
\end{aligned}$$

Open code

$$\begin{aligned}
& ((64 + 48) \pi) (-1) \log \left( \frac{0.5^{2^2+2} (1 + 0.5^2) (1 + 0.5^4) (1 + 0.5^{2 \times 2})}{(1 + 0.5) (1 + 0.5^3) (1 + 0.5^{2 \times 2+1})} \right) = \\
& -224 i \pi^2 \left\lfloor \frac{-\pi + \arg\left(\frac{0.0126701}{z_0}\right) + \arg(z_0)}{2 \pi} \right\rfloor - \\
& 112 \pi \log(z_0) + 112 \pi \sum_{k=1}^{\infty} \frac{(-1)^k (0.0126701 - z_0)^k z_0^{-k}}{k}
\end{aligned}$$

Open code

- $\arg(z)$  is the complex argument
  - $\lfloor x \rfloor$  is the floor function
  - $i$  is the imaginary unit
  - [More information](#)

Integral representation:

$$((64 + 48) \pi) (-1) \log \left( \frac{0.5^{2^2+2} (1 + 0.5^2) (1 + 0.5^4) (1 + 0.5^{2 \times 2})}{(1 + 0.5) (1 + 0.5^3) (1 + 0.5^{2 \times 2+1})} \right) = -112 \pi \int_1^{0.0126701} \frac{1}{t} dt$$

$$21/10 * (((((64+48)Pi * colog((((0.5^(2^2+2) (1+0.5^2)(1+0.5^4)(1+0.5^(2*2))) / ((1+0.5)(1+0.5^3)(1+0.5^(2*2+1)))))))))) ^{1/3}$$

Input:

$$\frac{21}{10} \sqrt[3]{(64 + 48) \pi \left( -\log \left( \frac{0.5^{2^2+2} (1 + 0.5^2) (1 + 0.5^4) (1 + 0.5^{2 \times 2})}{(1 + 0.5) (1 + 0.5^3) (1 + 0.5^{2 \times 2+1})} \right) \right)}$$

Open code

- $\log(x)$  is the natural logarithm

Enlarge Data Customize A Plaintext Interactive

Result:

More digits

24.2356...

This result is very near to the value of black hole entropy 24,2477

Series representations:

More

$$\frac{1}{10} \sqrt[3]{(64 + 48)\pi \left( -\log \left( \frac{0.5^{2^2+2} (1 + 0.5^2) (1 + 0.5^4) (1 + 0.5^{2 \times 2})}{(1 + 0.5) (1 + 0.5^3) (1 + 0.5^{2 \times 2+1})} \right) \right)} 21 =$$

$$\frac{21}{5} \sqrt[3]{14} \sqrt[3]{\pi \sum_{k=1}^{\infty} \frac{(-1)^k (-0.98733)^k}{k}}$$

[Open code](#)

Enlarge Data Customize A [Plaintext](#) Interactive

$$\frac{1}{10} \sqrt[3]{(64 + 48)\pi \left( -\log \left( \frac{0.5^{2^2+2} (1 + 0.5^2) (1 + 0.5^4) (1 + 0.5^{2 \times 2})}{(1 + 0.5) (1 + 0.5^3) (1 + 0.5^{2 \times 2+1})} \right) \right)} 21 = \frac{21}{5} \sqrt[3]{14}$$

$$\sqrt[3]{-\pi \left( 2i\pi \left\lfloor \frac{\arg(0.0126701 - x)}{2\pi} \right\rfloor + \log(x) - \sum_{k=1}^{\infty} \frac{(-1)^k (0.0126701 - x)^k x^{-k}}{k} \right)} \text{ for } x < 0$$

[Open code](#)

$$\frac{1}{10} \sqrt[3]{(64 + 48)\pi \left( -\log \left( \frac{0.5^{2^2+2} (1 + 0.5^2) (1 + 0.5^4) (1 + 0.5^{2 \times 2})}{(1 + 0.5) (1 + 0.5^3) (1 + 0.5^{2 \times 2+1})} \right) \right)} 21 =$$

$$\frac{21}{5} \sqrt[3]{14} \left( -\pi \left( \log(z_0) + \left\lfloor \frac{\arg(0.0126701 - z_0)}{2\pi} \right\rfloor \left( \log\left(\frac{1}{z_0}\right) + \log(z_0) \right) - \sum_{k=1}^{\infty} \frac{(-1)^k (0.0126701 - z_0)^k z_0^{-k}}{k} \right) \right)^{1/3}$$

Integral representation:

$$\frac{1}{10} \sqrt[3]{(64 + 48)\pi \left( -\log \left( \frac{0.5^{2^2+2} (1 + 0.5^2) (1 + 0.5^4) (1 + 0.5^{2 \times 2})}{(1 + 0.5) (1 + 0.5^3) (1 + 0.5^{2 \times 2+1})} \right) \right)} 21 =$$

$$\frac{21}{5} \sqrt[3]{14} \sqrt[3]{-\pi \int_1^{0.0126701} \frac{1}{t} dt}$$

`(((((64+48)Pi * colog((((0.5^(2^2+2) (1+0.5^2)(1+0.5^4)(1+0.5^(2*2)))) / ((1+0.5)(1+0.5^3)(1+0.5^(2*2+1))))))))))^(1/15)`

Input:

$$\sqrt[15]{(64 + 48)\pi \left( -\log \left( \frac{0.5^{2^2+2} (1 + 0.5^2)(1 + 0.5^4)(1 + 0.5^{2 \times 2})}{(1 + 0.5)(1 + 0.5^3)(1 + 0.5^{2 \times 2+1})} \right) \right)}$$

[Open code](#)

- $\log(x)$  is the natural logarithm

### Enlarge Data Customize A Plaintext Interactive

Result:

- Fewer digits
- More digits

1.630972916189556785819348558554576940564118336187924665693...  
 1.6309729161895567858193485585545769405641183361879246

This result is a golden number

Continued fraction:

Linear form

$$1 + \frac{1}{1 + \frac{1}{1 + \frac{1}{1 + \frac{1}{2 + \frac{1}{2 + \frac{1}{4 + \frac{1}{6 + \frac{1}{1 + \frac{1}{2 + \frac{1}{12 + \frac{1}{1 + \frac{1}{1 + \frac{1}{3 + \frac{1}{2 + \frac{1}{1 + \frac{1}{5 + \frac{1}{8 + \frac{1}{1 + \frac{1}{1 + \frac{1}{21 + \frac{1}{7 + \frac{1}{\dots}}}}}}}}}}}}}}}}}}}}}}}}}}}}}}$$

[Open code](#)

### Enlarge Data Customize A Plaintext Interactive

Possible closed forms:

More

$$\frac{6}{7} \pi \tanh^2 \left( \frac{843868}{810687} \right) \approx 1.630972916189556745194$$

$$-\frac{2(-154 - 50e + 105e^2)}{3(75 - 136e + 13e^2)} \approx 1.63097291618955678550412$$

$$\frac{1533309737\pi}{2953473082} \approx 1.63097291618955678579349$$

## Some development concerning some mock theta function of order 6

Ramanujan (1988) wrote down seven mock theta functions of order 6 in his lost notebook, and stated 11 identities between them, which were proved in (Andrews & Hickerson 1991). Two of Ramanujan's identities relate  $\varphi$  and  $\psi$  at various arguments, four of them express  $\varphi$  and  $\psi$  in terms of Appell–Lerch series, and the last five identities express the remaining five sixth-order mock theta functions in terms of  $\varphi$  and  $\psi$ . Berndt & Chan (2007) discovered two more sixth order functions.

We have the following function:

$$\sigma(q) = \sum_{n \geq 0} \frac{q^{(n+1)(n+2)/2} (-q; q)_n}{(q; q^2)_{n+1}}$$

That is:

$$\text{Sum}_{\{n \geq 0\}} q^{((n+1)(n+2)/2)} (1+q)(1+q^2)\dots(1+q^n)/((1-q)(1-q^3)\dots(1-q^{(2n+1)}))$$

We have that:

$$\text{sum } q^{((n+1)(n+2)/2)} (1+q)(1+q^2)(1+q^n)/((1-q)(1-q^3)(1-q^{(2n+1)})), n = 0 \text{ to } k$$

Input interpretation:

$$\sum_{n=0}^k \frac{q^{1/2(n+1)(n+2)} (1+q)(1+q^2)(1+q^n)}{(1-q)(1-q^3)(1-q^{2n+1})}$$

Enlarge Data Customize A Plaintext Interactive

Result:

$$\sum_{n=0}^k \frac{q^{1/2(n+1)(n+2)} (1+q)(1+q^2)(1+q^n)}{(1-q)(1-q^3)(1-q^{2n+1})}$$

[Open code](#)

For  $q = 0.5$  and  $n = 2$ , we develop the above formula in the following way:

$$(((0.5^{(2+1)(2+2)/2} (1+0.5)(1+0.5^2)(1+0.5^2)))/(((1-0.5)(1-0.5^3)(1-0.5^{(2*2+1)})))$$

Input:

$$\frac{0.5^{(2+1)(2+2)/2} (1+0.5)(1+0.5^2)(1+0.5^2)}{(1-0.5)(1-0.5^3)(1-0.5^{2*2+1})}$$

[Open code](#)

Enlarge Data Customize A Plaintext Interactive

Result:

- More digits  
0.086405529953917050691244239631336405529953917050691244239...  
[Open code](#)  
0.0864055...

$$1 + \left( \frac{0.5^{(2+1)(2+2)/2} (1+0.5)(1+0.5^2)(1+0.5^2)}{(1-0.5)(1-0.5^3)(1-0.5^{2 \times 2+1})} \right)$$

Input:

$$1 + \frac{0.5^{(2+1)(2+2)/2} (1+0.5)(1+0.5^2)(1+0.5^2)}{(1-0.5)(1-0.5^3)(1-0.5^{2 \times 2+1})}$$

[Open code](#)

Enlarge Data Customize A [Plaintext](#) [Interactive](#)

Result:

- More digits  
1.086405529953917050691244239631336405529953917050691244239...  
[Open code](#)  
1.0864055...

(Multiplying this value by 9, we obtain: 9.7776497695, value very near to the next result considering the various summations)

$$\left( \frac{1 + \left( \frac{0.5^{(2+1)(2+2)/2} (1+0.5)(1+0.5^2)(1+0.5^2)}{(1-0.5)(1-0.5^3)(1-0.5^{2 \times 2+1})} \right)}{1} \right)^6$$

Input:

$$\left( 1 + \frac{0.5^{(2+1)(2+2)/2} (1+0.5)(1+0.5^2)(1+0.5^2)}{(1-0.5)(1-0.5^3)(1-0.5^{2 \times 2+1})} \right)^6$$

[Open code](#)

Enlarge Data Customize A [Plaintext](#) [Interactive](#)

Result:

- More digits  
1.644189253712856553963579894421995177787385161124603000667...  
[Open code](#)  
1.6441892537128565539...  $\approx \zeta(2) = \frac{\pi^2}{6} = 1.6449$

$$24 \times 3 + 10^3 * \left( \frac{1 + \left( \frac{0.5^{(2+1)(2+2)/2} (1+0.5)(1+0.5^2)(1+0.5^2)}{(1-0.5)(1-0.5^3)(1-0.5^{2 \times 2+1})} \right)}{1} \right)^6$$

Input:

$$24 \times 3 + 10^3 \left( 1 + \frac{0.5^{(2+1)(2+2)/2} (1+0.5)(1+0.5^2)(1+0.5^2)}{(1-0.5)(1-0.5^3)(1-0.5^{2 \times 2+1})} \right)^6$$

[Open code](#)

Enlarge Data Customize A [Plaintext](#) [Interactive](#)

Result:

- More digits



1716.189253712856553963579894421995177787385161124603000667...

This result is very near to the mass of candidate glueball  $f_0(1710)$  meson.

$$\left(\left(\left(\left(\left(24 \times 3 + 10^3 * \left(\left(\left(\left(1 + \left(\frac{0.5^{(2+1)(2+2)/2}}{(1+0.5)(1+0.5^2)(1+0.5^2)}\right)\right)\right)\right)\right)\right)\right)\right)\right)\right)^6\right)^{1/3}$$

Input:

$$\sqrt[3]{24 \times 3 + 10^3 \left(1 + \frac{0.5^{(2+1) \times (2+2)/2} (1 + 0.5) (1 + 0.5^2) (1 + 0.5^2)}{(1 - 0.5) (1 - 0.5^3) (1 - 0.5^{2 \times 2 + 1})}\right)^6}$$

[Open code](#)

Enlarge Data Customize A [Plaintext](#) [Interactive](#)

Result:

More digits

11.9726...

This result is very near to the value of black hole entropy 12,1904

$$2 * \left(\left(\left(\left(\left(24 \times 3 + 10^3 * \left(\left(\left(\left(1 + \left(\frac{0.5^{(2+1)(2+2)/2}}{(1+0.5)(1+0.5^2)(1+0.5^2)}\right)\right)\right)\right)\right)\right)\right)\right)\right)^6\right)^{1/3}$$

Input:

$$2 \sqrt[3]{24 \times 3 + 10^3 \left(1 + \frac{0.5^{(2+1) \times (2+2)/2} (1 + 0.5) (1 + 0.5^2) (1 + 0.5^2)}{(1 - 0.5) (1 - 0.5^3) (1 - 0.5^{2 \times 2 + 1})}\right)^6}$$

[Open code](#)

Enlarge Data Customize A [Plaintext](#) [Interactive](#)

Result:

More digits

23.9452...

This result is very near to the value of black hole entropy 23,9078

$$\left(\left(\left(\left(\left(24 \times 3 + 10^3 * \left(\left(\left(\left(1 + \left(\frac{0.5^{(2+1)(2+2)/2}}{(1+0.5)(1+0.5^2)(1+0.5^2)}\right)\right)\right)\right)\right)\right)\right)\right)\right)^6\right)^{1/15}$$

Input:

$$\sqrt[15]{24 \times 3 + 10^3 \left(1 + \frac{0.5^{(2+1) \times (2+2)/2} (1 + 0.5) (1 + 0.5^2) (1 + 0.5^2)}{(1 - 0.5) (1 - 0.5^3) (1 - 0.5^{2 \times 2 + 1})}\right)^6}$$

[Open code](#)

Enlarge Data Customize A [Plaintext](#) [Interactive](#)

Result:

More digits

1.643000...

$$1.643 \approx \zeta(2) = \frac{\pi^2}{6} = 1.6449$$

For the same function, we have for n = 0 to infinity:

$$\sum_{n=0}^{\infty} \frac{0.5^{1/2(n+1)(n+2)} (1+0.5)(1+0.5^2)(1+0.5^n)}{(1-0.5)(1-0.5^3)(1-0.5^{2n+1})}, n = 0 \text{ to infinity}$$

Input interpretation:

$$\sum_{n=0}^{\infty} \frac{0.5^{1/2(n+1)(n+2)} (1+0.5)(1+0.5^2)(1+0.5^n)}{(1-0.5)(1-0.5^3)(1-0.5^{2n+1})}$$

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Approximated sum:

More digits

- $$\sum_{n=0}^{\infty} \frac{0.5^{1/2(n+1)(n+2)} (1+0.5)(1+0.5^2)(1+0.5^n)}{(1-0.5)(1-0.5^3)(1-0.5^{2n+1})} \approx 9.58109$$

For n = 0 to 2

$$\sum_{n=0}^2 \frac{0.5^{1/2(n+1)(n+2)} (1+0.5)(1+0.5^2)(1+0.5^n)}{(1-0.5)(1-0.5^3)(1-0.5^{2n+1})}, n = 0 \text{ to } 2$$

Sum:

$$\sum_{n=0}^2 \frac{(1+0.5)(1+0.5^2) 0.5^{1/2(n+1)(n+2)} (0.5^n + 1)}{(1-0.5)(1-0.5^3)(1-0.5^{2n+1})} = \frac{58185}{6076}$$

[Open code](#)

Enlarge Data Customize A [Plaintext](#) [Interactive](#)

Decimal approximation:

More digits

- 9.576201448321263989466754443712969058591178406846609611586...

Or;

$$\sum_{n=0}^{256} \frac{0.5^{1/2(n+1)(n+2)} (1+0.5)(1+0.5^2)(1+0.5^n)}{(1-0.5)(1-0.5^3)(1-0.5^{2n+1})}, n = 0 \text{ to } 256$$

Sum:

Exact form

Fewer digits

More digits

- $$\sum_{n=0}^{256} \frac{0.5^{1/2(n+1)(n+2)} (1+0.5)(1+0.5^2)(1+0.5^n)}{(1-0.5)(1-0.5^3)(1-0.5^{2n+1})} \approx 9.581088309560082244789784629413838843313$$

[Open code](#)

Enlarge Data Customize A [Plaintext](#) [Interactive](#)

Decimal approximation:

More digits

9.581088309560082244789784629413838843312764316015269774097...

The value is always about 9.58

$\frac{1}{12} * \exp \sum_{n=0}^5 0.5^{(n+1)(n+2)/2} (1+0.5)(1+0.5^2)(1+0.5^n) / ((1-0.5)(1-0.5^3)(1-0.5^{2n+1}))$ , n = 0 to 5

Input interpretation:

$$\frac{1}{12} \exp \left( \sum_{n=0}^5 0.5^{(n+1)(n+2)/2} (1+0.5)(1+0.5^2) \times \frac{1+0.5^n}{(1-0.5)(1-0.5^3)(1-0.5^{2n+1})} \right)$$

Enlarge Data Customize A [Plaintext](#) [Interactive](#)

Result:

1207.35

1207.35

$[\frac{1}{12} * \exp \sum_{n=0}^5 0.5^{(n+1)(n+2)/2} (1+0.5)(1+0.5^2)(1+0.5^n) / ((1-0.5)(1-0.5^3)(1-0.5^{2n+1}))]$ , n = 0 to 5<sup>1/14</sup>

Input interpretation:

$$\sqrt[14]{\frac{1}{12} \exp \left( \sum_{n=0}^5 0.5^{(n+1)(n+2)/2} (1+0.5)(1+0.5^2) \times \frac{1+0.5^n}{(1-0.5)(1-0.5^3)(1-0.5^{2n+1})} \right)}$$

Enlarge Data Customize A [Plaintext](#) [Interactive](#)

Result:

1.66009

1.66009 is very near to the 14th root of the following Ramanujan's class invariant

$$Q = (G_{505}/G_{101/5})^3 = 1164,2696 \text{ i.e. } 1,65578...$$

$[\frac{1}{12} * \exp \sum_{n=0}^5 0.5^{(n+1)(n+2)/2} (1+0.5)(1+0.5^2)(1+0.5^n) / ((1-0.5)(1-0.5^3)(1-0.5^{2n+1}))]$ , n = 0 to 5<sup>1/15</sup>

Input interpretation:

$$\sqrt[15]{\frac{1}{12} \exp \left( \sum_{n=0}^5 0.5^{(n+1)(n+2)/2} (1+0.5)(1+0.5^2) \times \frac{1+0.5^n}{(1-0.5)(1-0.5^3)(1-0.5^{2n+1})} \right)}$$

Enlarge Data Customize A [Plaintext](#) [Interactive](#)

Result:

1.60493

1.60493

This result is a golden number, very near to the electric charge of positron

The mean between the two results is:  $(1.66009 + 1.60493) / 2 = 1,63251$

$$2 * [\text{sum } 0.5^{((n+1)(n+2)/2)} (1+0.5)(1+0.5^2)(1+0.5^n)] / ((1-0.5)(1-0.5^3)(1-0.5^{(2n+1)})), n = 0 \text{ to } 5]^3$$

Input interpretation:

$$2 \left( \sum_{n=0}^5 0.5^{(n+1) \times (n+2) / 2} (1 + 0.5) (1 + 0.5^2) \times \frac{1 + 0.5^n}{(1 - 0.5)(1 - 0.5^3)(1 - 0.5^{2n+1})} \right)^3$$

Enlarge Data Customize A [Plaintext](#) [Interactive](#)

Result:

1759.04

result in the range of the mass of candidate “glueball”  $f_0(1710)$  and the hypothetical mass of Gluino

$$((((((2 * [\text{sum } 0.5^{((n+1)(n+2)/2)} (1+0.5)(1+0.5^2)(1+0.5^n)] / ((1-0.5)(1-0.5^3)(1-0.5^{(2n+1)})), n = 0 \text{ to } 5]^3))))))^{1/3}$$

Input interpretation:

$$\sqrt[3]{2 \left( \sum_{n=0}^5 0.5^{(n+1) \times (n+2) / 2} (1 + 0.5) (1 + 0.5^2) \times \frac{1 + 0.5^n}{(1 - 0.5)(1 - 0.5^3)(1 - 0.5^{2n+1})} \right)^3}$$

Enlarge Data Customize A [Plaintext](#) [Interactive](#)

Result:

12.0714

This result is very near to the value of black hole entropy 12,1904

$$2 * ((((((2 * [\text{sum } 0.5^{((n+1)(n+2)/2)} (1+0.5)(1+0.5^2)(1+0.5^n)] / ((1-0.5)(1-0.5^3)(1-0.5^{(2n+1)})), n = 0 \text{ to } 5]^3))))))^{1/3}$$

Input interpretation:

$$2 \sqrt[3]{2 \left( \sum_{n=0}^5 0.5^{(n+1) \times (n+2) / 2} (1 + 0.5) (1 + 0.5^2) \times \frac{1 + 0.5^n}{(1 - 0.5)(1 - 0.5^3)(1 - 0.5^{2n+1})} \right)^3}$$

Enlarge Data Customize A [Plaintext](#) [Interactive](#)

Result:

24.1428

This result is very near to the values of black hole entropies 24.2477

$$(((((((2 * [\text{sum } 0.5^{((n+1)(n+2)/2)} (1+0.5)(1+0.5^2)(1+0.5^n)] / ((1-0.5)(1-0.5^3)(1-0.5^{(2n+1)})), n = 0 \text{ to } 5]^3))))))^{1/15}$$

Input interpretation:

$$\sqrt[15]{2 \left( \sum_{n=0}^5 0.5^{(n+1) \times (n+2) / 2} (1 + 0.5) (1 + 0.5^2) \times \frac{1 + 0.5^n}{(1 - 0.5)(1 - 0.5^3)(1 - 0.5^{2n+1})} \right)^3}$$

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Result:

1.6457

$$1.6457 \approx \zeta(2) = \frac{\pi^2}{6} = 1.6449$$

Continued fraction:  
Linear form

$$\begin{array}{l}
 1 + \frac{1}{1 + \frac{1}{1 + \frac{1}{1 + \frac{1}{4 + \frac{1}{1 + \frac{1}{1 + \frac{1}{1 + \frac{1}{2 + \frac{1}{1 + \frac{1}{1 + \frac{1}{1 + \frac{1}{1 + \frac{1}{3 + \frac{1}{1 + \frac{1}{\dots}}}}}}}}}}}}}}}}}}}} \\
 \end{array}$$

Open code

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Possible closed forms:

More

$$\begin{array}{l}
 \sqrt{7} - 1 \approx 1.645751311 \\
 \frac{\pi^2}{6} \approx 1.6449340 \\
 \frac{130}{79} \approx 1.64556962
 \end{array}$$

Now:

$$\rho(q) = \sum_{n \geq 0} \frac{q^{n(n+1)/2} (-q; q)_n}{(q; q^2)_{n+1}}$$

That is:

$$\text{Sum}_{\{n \geq 0\}} ( q^{(n(n+1)/2)} * (1+q) * (1+q^2) \dots (1+q^n) / ((1-q) * (1-q^3) \dots (1-q^{(2n+1)})) )$$

$$\text{sum} ( 0.5^{(n(n+1)/2)} * (1+0.5) * (1+0.5^2) (1+0.5^n) / ((1-0.5) * (1-0.5^3) (1-0.5^{(2n+1)})) ), n = 0 \text{ to } 5$$

Sum:

$$\sum_{n=0}^5 \frac{(1+0.5)(1+0.5^2) 0.5^{1/2 n(n+1)} (0.5^n + 1)}{(1-0.5)(1-0.5^3)(1-0.5^{2n+1})} = \frac{159\,314\,890\,064\,385}{7\,379\,758\,720\,768}$$

Open code

Enlarge Data Customize A [Plaintext](#) [Interactive](#)

Decimal approximation:

More digits

21.58808927127164225765681655593742296293395065963169130459...

This result is very near to the black hole entropy 21,7656

$$\frac{1}{13} * (\text{sum} ( 0.5^{(n(n+1)/2)} * (1+0.5) * (1+0.5^2) * (1+0.5^n) / ((1-0.5) * (1-0.5^3) * (1-0.5^{2n+1}))), n = 0 \text{ to } 5))$$

Input interpretation:

$$\frac{1}{13} \sum_{n=0}^5 0.5^{n \times (n+1)/2} (1 + 0.5) (1 + 0.5^2) \times \frac{1 + 0.5^n}{(1 - 0.5) (1 - 0.5^3) (1 - 0.5^{2n+1})}$$

Enlarge Data Customize A [Plaintext](#) [Interactive](#)

Result:

1.66062

1.66062 is very near to the 14th root of the following Ramanujan's class invariant

$$Q = (G_{505}/G_{101/5})^3 = 1164,2696 \text{ i.e. } 1,65578...$$

$$(((((((\text{sum} ( 0.5^{(n(n+1)/2)} * (1+0.5) * (1+0.5^2) * (1+0.5^n) / ((1-0.5) * (1-0.5^3) * (1-0.5^{2n+1}))), n = 0 \text{ to } 5)))))))))^{1/6}$$

Input interpretation:

$$\sqrt[6]{\sum_{n=0}^5 0.5^{n \times (n+1)/2} (1 + 0.5) (1 + 0.5^2) \times \frac{1 + 0.5^n}{(1 - 0.5) (1 - 0.5^3) (1 - 0.5^{2n+1})}}$$

Enlarge Data Customize A [Plaintext](#) [Interactive](#)

Result:

1.66866

1.66866

This result is a golden number, very near to the proton mass

$$8^2 + 10^3 (((((((\text{sum} ( 0.5^{(n(n+1)/2)} * (1+0.5) * (1+0.5^2) * (1+0.5^n) / ((1-0.5) * (1-0.5^3) * (1-0.5^{2n+1}))), n = 0 \text{ to } 5)))))))))^{1/6}$$

Input interpretation:

$$8^2 + 10^3 \sqrt[6]{\sum_{n=0}^5 0.5^{n \times (n+1)/2} (1 + 0.5) (1 + 0.5^2) \times \frac{1 + 0.5^n}{(1 - 0.5) (1 - 0.5^3) (1 - 0.5^{2n+1})}}$$

Enlarge Data Customize A [Plaintext](#) [Interactive](#)

Result:

1732.66

This result is very near to the mass of candidate glueball  $f_0(1710)$  meson.

$$\left( \left( 8^2 + 10^3 \left( \left( \sum_{n=0}^5 \left( 0.5^{n(n+1)/2} \right) \cdot (1+0.5) \cdot (1+0.5^2) \cdot (1+0.5^n) \right) / \left( (1-0.5) \cdot (1-0.5^3) \cdot (1-0.5^{2n+1}) \right) \right) \right)^{1/6} \right)^{1/3}$$

Input interpretation:

$$\sqrt[3]{8^2 + 10^3 \sqrt[6]{\sum_{n=0}^5 0.5^{n \times (n+1)/2} (1+0.5)(1+0.5^2) \times \frac{1+0.5^n}{(1-0.5)(1-0.5^3)(1-0.5^{2n+1})}}}$$

Enlarge Data Customize A [Plaintext](#) [Interactive](#)

Result:

12.0108

This result is very near to the value of black hole entropy 12,1904

$$2 * \left( \left( 8^2 + 10^3 \left( \left( \sum_{n=0}^5 \left( 0.5^{n(n+1)/2} \right) \cdot (1+0.5) \cdot (1+0.5^2) \cdot (1+0.5^n) \right) / \left( (1-0.5) \cdot (1-0.5^3) \cdot (1-0.5^{2n+1}) \right) \right) \right)^{1/6} \right)^{1/3}$$

Input interpretation:

$$2 \sqrt[3]{8^2 + 10^3 \sqrt[6]{\sum_{n=0}^5 0.5^{n \times (n+1)/2} (1+0.5)(1+0.5^2) \times \frac{1+0.5^n}{(1-0.5)(1-0.5^3)(1-0.5^{2n+1})}}}$$

Enlarge Data Customize A [Plaintext](#) [Interactive](#)

Result:

24.0216

This value is linked to the "Ramanujan function" (an elliptic modular function that satisfies the need for "conformal symmetry") that has 24 "modes" corresponding to the physical vibrations of a bosonic string.

$$\left( \left( 8^2 + 10^3 \left( \left( \sum_{n=0}^5 \left( 0.5^{n(n+1)/2} \right) \cdot (1+0.5) \cdot (1+0.5^2) \cdot (1+0.5^n) \right) / \left( (1-0.5) \cdot (1-0.5^3) \cdot (1-0.5^{2n+1}) \right) \right) \right)^{1/6} \right)^{1/15}$$

Input interpretation:

$$\sqrt[15]{8^2 + 10^3 \sqrt[6]{\sum_{n=0}^5 0.5^{n \times (n+1)/2} (1+0.5)(1+0.5^2) \times \frac{1+0.5^n}{(1-0.5)(1-0.5^3)(1-0.5^{2n+1})}}}$$

Enlarge Data Customize A [Plaintext](#) [Interactive](#)

Result:

1.64405

$$1.64405 \approx \zeta(2) = \frac{\pi^2}{6} = 1.6449$$

For q = 0.5 and n = 2, we develop the above formula in the following way:

$$31 * \left( \left( 0.5^{(2(2+1)/2)} \cdot (1+0.5) \cdot (1+0.5^2) \cdot (1+0.5^2) \right) / \left( (1-0.5) \cdot (1-0.5^3) \cdot (1-0.5^{(2*2+1)}) \right) \right)$$

Input:

$$31 \times \frac{0.5^{2 \times (2+1)/2} (1 + 0.5) (1 + 0.5^2) (1 + 0.5^2)}{(1 - 0.5) (1 - 0.5^3) (1 - 0.5^{2 \times 2+1})}$$

[Open code](#)

### Enlarge Data Customize A [Plaintext](#) [Interactive](#)

Result:

More digits

21.42857142857142857142857142857142857142857142857142857142...

[Open code](#)

This result is very near to the previous result 21.588, and very near to the black hole entropy 21,7656

$$1 + ((-0.0864055 + (((0.5^{(2(2+1)/2}) * (1+0.5) * (1+0.5^2) * (1+0.5^2)))) / (((1-0.5) * (1-0.5^3) * (1-0.5^{(2*2+1))}))))$$

Where 0.0864055 is the result of previous mock theta function

Input interpretation:

$$1 + \left( -0.0864055 + \frac{0.5^{2 \times (2+1)/2} (1 + 0.5) (1 + 0.5^2) (1 + 0.5^2)}{(1 - 0.5) (1 - 0.5^3) (1 - 0.5^{2 \times 2+1})} \right)$$

[Open code](#)

### Enlarge Data Customize A [Plaintext](#) [Interactive](#)

Result:

More digits

1.604838739631336405529953917050691244239631336405529953917...

[Open code](#)

This result is a golden number, very near to the electric charge of positron

$$2 \times 8^2 + 10^3 \left( \frac{1 + ((-0.0864055 + (((0.5^{(2(2+1)/2}) * (1+0.5) * (1+0.5^2) * (1+0.5^2)))) / (((1-0.5) * (1-0.5^3) * (1-0.5^{(2*2+1))}))))}{1} \right)$$

Input interpretation:

$$2 \times 8^2 + 10^3 \left( 1 + \left( -0.0864055 + \frac{0.5^{2 \times (2+1)/2} (1 + 0.5) (1 + 0.5^2) (1 + 0.5^2)}{(1 - 0.5) (1 - 0.5^3) (1 - 0.5^{2 \times 2+1})} \right) \right)$$

[Open code](#)

### Enlarge Data Customize A [Plaintext](#) [Interactive](#)

Result:

More digits

1732.838739631336405529953917050691244239631336405529953917...

This result is very near to the mass of candidate glueball  $f_0(1710)$  meson.



$$\left(\left(\left(\left(\left(\left(\left(\left(\left(\left(\left(\left(2 \times 8^2 + 10^3 \left(1 + \left(-0.0864055 + \left(\frac{0.5^{2 \times (2+1)/2} (1 + 0.5) (1 + 0.5^2) (1 + 0.5^2)}{(1 - 0.5) (1 - 0.5^3) (1 - 0.5^{2 \times 2+1})}\right)\right)\right)\right)\right)\right)\right)\right)\right)\right)\right)\right)\right)\right)^{1/3}$$

Input interpretation:

$$\sqrt[3]{2 \times 8^2 + 10^3 \left(1 + \left(-0.0864055 + \frac{0.5^{2 \times (2+1)/2} (1 + 0.5) (1 + 0.5^2) (1 + 0.5^2)}{(1 - 0.5) (1 - 0.5^3) (1 - 0.5^{2 \times 2+1})}\right)\right)}$$

[Open code](#)

Enlarge Data Customize A [Plaintext](#) [Interactive](#)

Result:

More digits

12.0112...

This result is very near to the value of black hole entropy 12,1904

$$2 \left(\left(\left(\left(\left(\left(\left(\left(\left(\left(\left(\left(2 \times 8^2 + 10^3 \left(1 + \left(-0.0864055 + \left(\frac{0.5^{2 \times (2+1)/2} (1 + 0.5) (1 + 0.5^2) (1 + 0.5^2)}{(1 - 0.5) (1 - 0.5^3) (1 - 0.5^{2 \times 2+1})}\right)\right)\right)\right)\right)\right)\right)\right)\right)\right)\right)\right)\right)\right)^{1/3}$$

Input interpretation:

$$2 \sqrt[3]{2 \times 8^2 + 10^3 \left(1 + \left(-0.0864055 + \frac{0.5^{2 \times (2+1)/2} (1 + 0.5) (1 + 0.5^2) (1 + 0.5^2)}{(1 - 0.5) (1 - 0.5^3) (1 - 0.5^{2 \times 2+1})}\right)\right)}$$

[Open code](#)

Enlarge Data Customize A [Plaintext](#) [Interactive](#)

Result:

More digits

24.0224...

This value is linked to the "Ramanujan function" (an elliptic modular function that satisfies the need for "conformal symmetry") that has 24 "modes" corresponding to the physical vibrations of a bosonic string.

$$\left(\left(\left(\left(\left(\left(\left(\left(\left(\left(\left(\left(2 \times 8^2 + 10^3 \left(1 + \left(-0.0864055 + \left(\frac{0.5^{2 \times (2+1)/2} (1 + 0.5) (1 + 0.5^2) (1 + 0.5^2)}{(1 - 0.5) (1 - 0.5^3) (1 - 0.5^{2 \times 2+1})}\right)\right)\right)\right)\right)\right)\right)\right)\right)\right)\right)\right)\right)\right)^{1/15}$$

Input interpretation:

$$\sqrt[15]{2 \times 8^2 + 10^3 \left(1 + \left(-0.0864055 + \frac{0.5^{2 \times (2+1)/2} (1 + 0.5) (1 + 0.5^2) (1 + 0.5^2)}{(1 - 0.5) (1 - 0.5^3) (1 - 0.5^{2 \times 2+1})}\right)\right)}$$

[Open code](#)

Result:

Fewer digits

More digits

1.644058284466750047206491612275236799861913416508532872238...



Result:

$$\sum_{n=0}^5 \frac{(-1)^n q^{n^2} ((1-q)(1-q^3)(1-q^{2n-1}))}{(1+q)(1+q^2)(1+q^{2n})} = \frac{1}{2(q+1)(q^2+1)^2}$$

$$(1-q)(1-q^3) \left( \frac{(q-1)(q^2+1)}{q} - \frac{2(q^2+1)(q^3-1)q^4}{q^4+1} - \frac{2(q^2+1)(1-q^9)q^{25}}{q^{10}+1} - \frac{2(q^2+1)(q^7-1)q^{16}}{q^8+1} + \frac{2(q^5-1)q^9}{q^4-q^2+1} + 2(q-1)q \right)$$

[Open code](#)

### Enlarge Data Customize A [Plaintext](#) [Interactive](#)

Sum:

$$\sum_{n=0}^5 \frac{(-1)^n q^{n^2} ((1-q)(1-q^3)(1-q^{2n-1}))}{(q+1)(q^2+1)(q^{2n}+1)} =$$

$$- \frac{(1-q)^2(1-q^3)q}{(q+1)(q^2+1)^2} + \frac{(1-\frac{1}{q})(1-q)(1-q^3)}{2(q+1)(q^2+1)} + \frac{(1-q)(1-q^3)^2 q^4}{(q+1)(q^2+1)(q^4+1)} -$$

$$\frac{(1-q)(1-q^3)(1-q^9)q^{25}}{(q+1)(q^2+1)(q^{10}+1)} + \frac{(1-q)(1-q^3)(1-q^7)q^{16}}{(q+1)(q^2+1)(q^8+1)} - \frac{(1-q)(1-q^3)(1-q^5)q^9}{(q+1)(q^2+1)(q^6+1)}$$

For  $q = 0.5$  and  $n = 2$ , we develop the above formula in the following way:

$$\frac{(((((-1)^2 \cdot 0.5^{2^2} (1-0.5)(1-0.5^3)(1-0.5^{(2 \cdot 2-1))})))}{(((1+0.5)(1+0.5^2)(1+0.5^{(2 \cdot 2)})))}$$

Input:

$$\frac{(-1)^2 \times 0.5^{2^2} ((1-0.5)(1-0.5^3)(1-0.5^{2 \cdot 2-1}))}{(1+0.5)(1+0.5^2)(1+0.5^{2 \cdot 2})}$$

[Open code](#)

### Enlarge Data Customize A [Plaintext](#) [Interactive](#)

Result:

More digits

0.0120098039215686274509803921568627450980392156862745098039215686274509803...

[Open code](#)

$$\frac{10^3 \cdot 12^2 \cdot ((((-1)^2 \cdot 0.5^{2^2} (1-0.5)(1-0.5^3)(1-0.5^{(2 \cdot 2-1))})))}{(((1+0.5)(1+0.5^2)(1+0.5^{(2 \cdot 2)})))}$$

Input:

$$10^3 \times 12^2 \times \frac{(-1)^2 \times 0.5^{2^2} ((1-0.5)(1-0.5^3)(1-0.5^{2 \cdot 2-1}))}{(1+0.5)(1+0.5^2)(1+0.5^{2 \cdot 2})}$$

[Open code](#)

### Enlarge Data Customize A [Plaintext](#) [Interactive](#)

Result:

More digits

1729.411764705882352941176470588235294117647058823529411764...

[Open code](#)

This result is very near to the mass of candidate glueball  $f_0(1710)$  meson. Furthermore, 1728 occurs in the algebraic formula for the j-invariant of an elliptic curve. As a consequence, it is sometimes called a Zagier as a pun on the Gross–Zagier theorem. The number 1728 is one less than the Hardy–Ramanujan number 1729

$$\frac{((((10^3 \cdot 12^2 \cdot (((-1)^2 \cdot 0.5^{2^2} (1-0.5)(1-0.5^3)(1-0.5^{(2 \cdot 2-1))})))) / (((1+0.5)(1+0.5^2)(1+0.5^{(2 \cdot 2)}))))))^{1/3}}$$

Input:

$$\sqrt[3]{10^3 \times 12^2 \times \frac{(-1)^2 \times 0.5^{2^2} (1-0.5)(1-0.5^3)(1-0.5^{2 \cdot 2-1})}{(1+0.5)(1+0.5^2)(1+0.5^{2 \cdot 2})}}$$

[Open code](#)

Enlarge Data Customize A [Plaintext](#) [Interactive](#)

Result:

More digits

12.0033...

This result is very near to the value of black hole entropy 12,1904

$$\frac{2 \cdot (((((10^3 \cdot 12^2 \cdot (((-1)^2 \cdot 0.5^{2^2} (1-0.5)(1-0.5^3)(1-0.5^{(2 \cdot 2-1))})))) / (((1+0.5)(1+0.5^2)(1+0.5^{(2 \cdot 2)}))))))^{1/3}}$$

Input:

$$2 \sqrt[3]{10^3 \times 12^2 \times \frac{(-1)^2 \times 0.5^{2^2} (1-0.5)(1-0.5^3)(1-0.5^{2 \cdot 2-1})}{(1+0.5)(1+0.5^2)(1+0.5^{2 \cdot 2})}}$$

[Open code](#)

Enlarge Data Customize A [Plaintext](#) [Interactive](#)

Result:

More digits

24.0065...

This value is linked to the "Ramanujan function" (an elliptic modular function that satisfies the need for "conformal symmetry") that has 24 "modes" corresponding to the physical vibrations of a bosonic string.

$$\frac{\left(\frac{10^3 \times 12^2 \times \left(\frac{(-1)^2 \times 0.5^{2^2} (1-0.5)(1-0.5^3)(1-0.5^{(2 \times 2 - 1)})}{(1+0.5)(1+0.5^2)(1+0.5^{(2 \times 2)})}\right)}{15}\right)^{1/15}}$$

Input:

$$15 \sqrt[15]{10^3 \times 12^2 \times \frac{(-1)^2 \times 0.5^{2^2} (1-0.5)(1-0.5^3)(1-0.5^{2 \times 2 - 1})}{(1+0.5)(1+0.5^2)(1+0.5^{2 \times 2})}}$$

[Open code](#)

Enlarge Data Customize A [Plaintext](#) [Interactive](#)

Result:

Fewer digits  
More digits

1.643841324367798515454832097542381144404664502737971722143...

$$1.6438413243677985154... \approx \zeta(2) = \frac{\pi^2}{6} = 1.6449$$

We have also that:

$$240 * 0.0864055 / \left(\frac{(-1)^2 \times 0.5^{2^2} (1-0.5)(1-0.5^3)(1-0.5^{(2 \times 2 - 1)})}{(1+0.5)(1+0.5^2)(1+0.5^{(2 \times 2)})}\right)$$

Where 0.0864055 is the result of a previous mock theta function

Input interpretation:

$$240 \times \frac{0.0864055}{\frac{(-1)^2 \times 0.5^{2^2} (1-0.5)(1-0.5^3)(1-0.5^{2 \times 2 - 1})}{(1+0.5)(1+0.5^2)(1+0.5^{2 \times 2})}}$$

[Open code](#)

Enlarge Data Customize A [Plaintext](#) [Interactive](#)

Result:

More digits

1726.699297959183673469387755102040816326530612244897959183...

[Open code](#)

This result is very near to the mass of candidate glueball  $f_0(1710)$  meson.

$$\frac{24}{21 \times 5} * 0.0864055 / \left(\frac{(-1)^2 \times 0.5^{2^2} (1-0.5)(1-0.5^3)(1-0.5^{(2 \times 2 - 1)})}{(1+0.5)(1+0.5^2)(1+0.5^{(2 \times 2)})}\right)$$

Input interpretation:

$$\frac{24}{21 \times 5} \times \frac{0.0864055}{\frac{(-1)^2 \times 0.5^{2^2} (1-0.5)(1-0.5^3)(1-0.5^{2 \times 2 - 1})}{(1+0.5)(1+0.5^2)(1+0.5^{2 \times 2})}}$$

[Open code](#)

Enlarge Data Customize A [Plaintext](#) [Interactive](#)

Result:

More digits

1.644475521865889212827988338192419825072886297376093294460...

[Open code](#)

$$1.6444755... \approx \zeta(2) = \frac{\pi^2}{6} = 1.6449$$

$$\left(\frac{(156871447\pi)}{(2191631483)}\right) * 0.0864055 / \left(\frac{((-1)^2 0.5^{2^2} (1-0.5)(1-0.5^3)(1-0.5^{2^2-1}))}{((1+0.5)(1+0.5^2)(1+0.5^{2^2}))}\right)$$

Input interpretation:

$$\frac{156871447\pi}{2191631483} \times \frac{0.0864055}{\frac{(-1)^2 \times 0.5^{2^2} ((1-0.5)(1-0.5^3)(1-0.5^{2^2-1}))}{(1+0.5)(1+0.5^2)(1+0.5^{2^2})}}$$

[Open code](#)

Enlarge Data Customize A [Plaintext](#) [Interactive](#)

Result:

Fewer digits

More digits

1.617825645412775413129541372159827994986362626745634200865...

1.6178256454127754131295413721598279949863626267456342

This result is a very good approximation to the value of the golden ratio 1,618033988749...

Series representations:

More

$$\frac{0.0864055 (156871447\pi)}{\frac{((-1)^2 0.5^{2^2} ((1-0.5)(1-0.5^3)(1-0.5^{2^2-1}))}{(1+0.5)(1+0.5^2)(1+0.5^{2^2})} 2191631483} = 2.05988 \sum_{k=0}^{\infty} \frac{(-1)^k}{1+2k}$$

[Open code](#)

Enlarge Data Customize A [Plaintext](#) [Interactive](#)

$$\frac{0.0864055 (156871447\pi)}{\frac{((-1)^2 0.5^{2^2} ((1-0.5)(1-0.5^3)(1-0.5^{2^2-1}))}{(1+0.5)(1+0.5^2)(1+0.5^{2^2})} 2191631483} = -1.02994 + 1.02994 \sum_{k=1}^{\infty} \frac{2^k}{\binom{2k}{k}}$$

[Open code](#)

$$\frac{0.0864055 (156871447\pi)}{\frac{((-1)^2 0.5^{2^2} ((1-0.5)(1-0.5^3)(1-0.5^{2^2-1}))}{(1+0.5)(1+0.5^2)(1+0.5^{2^2})} 2191631483} = 0.51497 \sum_{k=0}^{\infty} \frac{2^{-k} (-6 + 50k)}{\binom{3k}{k}}$$

[Open code](#)



$$\frac{573\,669\,041\pi}{1\,113\,985\,583} \approx 1.61782564541277541335752$$

$$\frac{-900 - 62\pi + 821\pi^2}{2(-196 - 43\pi + 253\pi^2)} \approx 1.61782564541277541324379$$

Or:

Input interpretation:

$$\frac{1}{4} \left( \frac{24}{105} + \frac{24}{106} + \frac{24}{107} + \frac{24}{109} \right) \times \frac{0.0864055}{\frac{(-1)^2 \times 0.5^{2^2} ((1-0.5)(1-0.5^3)(1-0.5^{2 \times 2 - 1}))}{(1+0.5)(1+0.5^2)(1+0.5^{2 \times 2})}}$$

[Open code](#)

Enlarge Data Customize A [Plaintext](#) [Interactive](#)

Result:

More digits

1.617825645412775411638688246013637558389843317766041441130...

[Open code](#)

1.6178256454127754.....

This result is a very good approximation to the value of the golden ratio 1,618033988749...

Now, from the above formula, performing the summation, we obtain:

$$\sum_{n=0}^5 (-1)^n 0.5^{n^2} (1-0.5)(1-0.5^3)(1-0.5^{(2n-1)}) / ((1+0.5)(1+0.5^2)(1+0.5^{(2n)})), n = 0 \text{ to } 5$$

Sum:

$$\sum_{n=0}^5 \frac{(-1)^n 0.5^{n^2} ((1-0.5)(1-0.5^3)(1-0.5^{2n-1}))}{(1+0.5)(1+0.5^2)(0.5^{2n}+1)} = -\frac{1\,482\,215\,485\,125\,543}{9\,767\,179\,255\,808\,000}$$

[Open code](#)

Enlarge Data Customize A [Plaintext](#) [Interactive](#)

Decimal approximation:

More digits

-0.15175471303489711491609182421499417909826051444661496635...

$$-11 * \sum_{n=0}^5 \frac{((-1)^n 0.5^{n^2} (1-0.5)(1-0.5^3)(1-0.5^{(2n-1)}))}{((1+0.5)(1+0.5^2)(1+0.5^{(2n)})), n = 0 \text{ to } 5}$$

Input interpretation:



$$-11 \sum_{n=0}^5 \frac{(-1)^n \times 0.5^{n^2} ((1 - 0.5)(1 - 0.5^3)(1 - 0.5^{2n-1}))}{(1 + 0.5)(1 + 0.5^2)(1 + 0.5^{2n})}$$

Enlarge Data Customize A [Plaintext](#) [Interactive](#)

Result:

1.6693

1.6693

This result is a golden number, very near to the proton mass

$$8^2 + 10^3 * -11 * \text{sum} (((-1)^n 0.5^{n^2} (1-0.5)(1-0.5^3)(1-0.5^{(2n-1)})) / ((1+0.5)(1+0.5^2)(1+0.5^{(2n)})), n = 0 \text{ to } 5$$

Input interpretation:

$$8^2 + 10^3 \times (-11) \sum_{n=0}^5 \frac{(-1)^n \times 0.5^{n^2} ((1 - 0.5)(1 - 0.5^3)(1 - 0.5^{2n-1}))}{(1 + 0.5)(1 + 0.5^2)(1 + 0.5^{2n})}$$

Enlarge Data Customize A [Plaintext](#) [Interactive](#)

Result:

1733.3

This result is very near to the mass of candidate glueball  $f_0(1710)$  meson.

$$(((8^2 + 10^3 * -11 * \text{sum} (((-1)^n 0.5^{n^2} (1-0.5)(1-0.5^3)(1-0.5^{(2n-1)})) / ((1+0.5)(1+0.5^2)(1+0.5^{(2n)})), n = 0 \text{ to } 5))))^{1/3}$$

Input interpretation:

$$\sqrt[3]{8^2 + 10^3 \times (-11) \sum_{n=0}^5 \frac{(-1)^n \times 0.5^{n^2} ((1 - 0.5)(1 - 0.5^3)(1 - 0.5^{2n-1}))}{(1 + 0.5)(1 + 0.5^2)(1 + 0.5^{2n})}}$$

Enlarge Data Customize A [Plaintext](#) [Interactive](#)

Result:

12.0123

This result is very near to the value of black hole entropy 12,1904

$$2 * (((8^2 + 10^3 * -11 * \text{sum} (((-1)^n 0.5^{n^2} (1-0.5)(1-0.5^3)(1-0.5^{(2n-1)})) / ((1+0.5)(1+0.5^2)(1+0.5^{(2n)})), n = 0 \text{ to } 5))))^{1/3}$$

Input interpretation:

$$2 \sqrt[3]{8^2 + 10^3 \times (-11) \sum_{n=0}^5 \frac{(-1)^n \times 0.5^{n^2} ((1 - 0.5)(1 - 0.5^3)(1 - 0.5^{2n-1}))}{(1 + 0.5)(1 + 0.5^2)(1 + 0.5^{2n})}}$$

Enlarge Data Customize A [Plaintext](#) [Interactive](#)

Result:

24.0245

This value is linked to the "Ramanujan function" (an elliptic modular function that satisfies the need for "conformal symmetry") that has 24 "modes" corresponding to the physical vibrations of a bosonic string.

$$\left( \frac{((8^2 + 10^3 * -11 * \sum_{n=0}^5 ((-1)^n * 0.5^{n^2} * (1-0.5)(1-0.5^3)(1-0.5^{(2n-1)))} / ((1+0.5)(1+0.5^2)(1+0.5^{(2n))))))^{1/15}}{15} \right)$$

Input interpretation:

$$\sqrt[15]{8^2 + 10^3 \times (-11) \sum_{n=0}^5 \frac{(-1)^n \times 0.5^{n^2} (1 - 0.5)(1 - 0.5^3)(1 - 0.5^{2n-1})}{(1 + 0.5)(1 + 0.5^2)(1 + 0.5^{2n})}}$$

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Result:

1.64409

$$1.64409 \approx \zeta(2) = \frac{\pi^2}{6} = 1.6449$$

Continued fraction:

Linear form

$$1 + \frac{1}{1 + \frac{1}{1 + \frac{1}{1 + \frac{1}{4 + \frac{1}{3 + \frac{1}{1 + \frac{1}{12 + \frac{1}{5 + \frac{1}{1 + \frac{1}{2 + \frac{1}{3 + \frac{1}{\dots}}}}}}}}}}}}}}}}$$

[Open code](#)

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Possible closed forms:

More

$$\frac{\pi^2}{6} \approx 1.64493406$$

$$\frac{97}{59} \approx 1.644067796$$

$$\pi \sqrt{\text{root of } 3x^3 + 3x - 2 \text{ near } x = 0.523336} \approx 1.64410698$$

Moreover, we obtain:

$$-(25\pi)/8 * 1.0864055 * \sum_{n=0}^5 \left( \frac{((-1)^n * 0.5^{n^2} * (1-0.5)(1-0.5^3)(1-0.5^{(2n-1)))}}{((1+0.5)(1+0.5^2)(1+0.5^{(2n))))} \right)$$

Input interpretation:

$$\left(-\frac{1}{8}(25\pi)\right) \times 1.0864055 \sum_{n=0}^5 \frac{(-1)^n \times 0.5^{n^2} ((1-0.5)(1-0.5^3)(1-0.5^{2n-1}))}{(1+0.5)(1+0.5^2)(1+0.5^{2n})}$$

Enlarge Data Customize A [Plaintext](#) [Interactive](#)

Result:

1.61858

1.61858

This result is a very good approximation to the value of the golden ratio 1,618033988749...

Now, we have:

$$\psi(q) = \sum_{n \geq 0} \frac{(-1)^n q^{(n+1)^2} (q; q^2)_n}{(-q; q)_{2n+1}}$$

That is:

$$\sum_{\{n \geq 0\}} \frac{(-1)^n q^{(n+1)^2} (1-q)(1-q^3)\dots(1-q^{2n-1})}{((1+q)(1+q^2)\dots(1+q^{2n+1}))}$$

We obtain:

$$\text{sum } (-1)^n q^{(n+1)^2} (1-q)(1-q^3)(1-q^{2n-1}) / ((1+q)(1+q^2)(1+q^{2n+1})), n = 0 \text{ to } 5$$

Result:

$$\sum_{n=0}^5 \frac{(-1)^n q^{(n+1)^2} ((1-q)(1-q^3)(1-q^{2n-1}))}{(1+q)(1+q^2)(1+q^{2n+1})} = \frac{(1-\frac{1}{q})(1-q)(1-q^3)q}{(q+1)^2(q^2+1)} - \frac{(1-q)^2(1-q^3)q^4}{(q+1)(q^2+1)(q^3+1)} + \frac{(1-q)(1-q^3)^2q^9}{(q+1)(q^2+1)(q^5+1)} - \frac{(1-q)(1-q^3)(1-q^9)q^{36}}{(q+1)(q^2+1)(q^{11}+1)} + \frac{(1-q)(1-q^3)(1-q^7)q^{25}}{(q+1)(q^2+1)(q^9+1)} - \frac{(1-q)(1-q^3)(1-q^5)q^{16}}{(q+1)(q^2+1)(q^7+1)}$$

[Open code](#)

Enlarge Data Customize A [Plaintext](#) [Interactive](#)

Result:

$$\sum_{n=0}^5 \frac{(-1)^n q^{(n+1)^2} ((1-q)(1-q^3)(1-q^{2n-1}))}{(1+q)(1+q^2)(1+q^{2n+1})} = \frac{1}{(q+1)^2(q^2+1)} (1-q)q(1-q^3) \left( \frac{(q-1)q^3}{q^2-q+1} - \frac{(q+1)(1-q^9)q^{35}}{q^{11}+1} + \frac{(q+1)(1-q^7)q^{24}}{q^9+1} - \frac{(q+1)(1-q^5)q^{15}}{q^7+1} + \frac{(q+1)(1-q^3)q^8}{q^5+1} - \frac{1}{q} + 1 \right)$$

$$\text{sum } (-1)^n 0.5^{(n+1)^2} (1-0.5)^*(1-0.5^3)(1-0.5^{(2n-1)})$$

$$/((1+0.5)*(1+0.5^2)(1+0.5^{(2n+1)})), n = 0 \text{ to } 5$$

Sum:

$$\sum_{n=0}^5 \frac{(-1)^n 0.5^{(n+1)^2} ((1-0.5)(1-0.5^3)(1-0.5^{2n-1}))}{(1+0.5)(1+0.5^2)(0.5^{2n+1}+1)} = -\frac{7164369962501918299}{85416213810606243840}$$

[Open code](#)

Enlarge Data Customize A [Plaintext](#) [Interactive](#)

Decimal approximation:

More digits

-0.08387599546834876259453164876479371979144506403526984363...

[Open code](#)

$$-1 + \text{sum } (-1)^n 0.5^{(n+1)^2} (1-0.5)^*(1-0.5^3)(1-0.5^{(2n-1)})$$

$$/((1+0.5)*(1+0.5^2)(1+0.5^{(2n+1)})), n = 0 \text{ to } 5$$

Input interpretation:

$$-1 + \sum_{n=0}^5 (-1)^n \times 0.5^{(n+1)^2} \left( (1-0.5)(1-0.5^3) \times \frac{1-0.5^{2n-1}}{(1+0.5)(1+0.5^2)(1+0.5^{2n+1})} \right)$$

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Result:

-1.08388

$$((( -1 + \text{sum } (-1)^n 0.5^{(n+1)^2} (1-0.5)^*(1-0.5^3)(1-0.5^{(2n-1)})$$

$$/((1+0.5)*(1+0.5^2)(1+0.5^{(2n+1)})), n = 0 \text{ to } 5)))^6$$

Input interpretation:

$$\left( -1 + \sum_{n=0}^5 (-1)^n \times 0.5^{(n+1)^2} \left( (1-0.5)(1-0.5^3) \times \frac{1-0.5^{2n-1}}{(1+0.5)(1+0.5^2)(1+0.5^{2n+1})} \right) \right)^6$$

Enlarge Data Customize A [Plaintext](#) [Interactive](#)

Result:

1.62135

1.62135

This result is a golden number

$$27*4+10^3((( -1 + \text{sum } (-1)^n 0.5^{(n+1)^2} (1-0.5)^*(1-0.5^3)(1-0.5^{(2n-1)})$$

$$/((1+0.5)*(1+0.5^2)(1+0.5^{(2n+1)})), n = 0 \text{ to } 5)))^6$$

Input interpretation:

$$27 \times 4 + 10^3 \left( -1 + \sum_{n=0}^5 (-1)^n \times 0.5^{(n+1)^2} \left( (1-0.5)(1-0.5^3) \times \frac{1-0.5^{2n-1}}{(1+0.5)(1+0.5^2)(1+0.5^{2n+1})} \right) \right)^6$$

Enlarge Data Customize A [Plaintext](#) [Interactive](#)

Result:

1729.35

This result is very near to the mass of candidate glueball  $f_0(1710)$  meson. Furthermore, 1728 occurs in the algebraic formula for the j-invariant of an elliptic curve. As a consequence, it is sometimes called a Zagier as a pun on the Gross-

Zagier theorem. The number 1728 is one less than the Hardy–Ramanujan number 1729

$$\left[ \left( \left( \left( \left( 27 \times 4 + 10^3 \left( \left( -1 + \sum_{n=0}^5 (-1)^n \times 0.5^{(n+1)^2} \right) \times \frac{1 - 0.5^{2n-1}}{(1+0.5)(1+0.5^2)(1+0.5^{2n+1})} \right) \right) \right) \right)^6 \right) \right]^{1/3}$$

Input interpretation:

$$\left( 27 \times 4 + 10^3 \left( -1 + \sum_{n=0}^5 (-1)^n \times 0.5^{(n+1)^2} \right) \left( (1 - 0.5)(1 - 0.5^3) \times \frac{1 - 0.5^{2n-1}}{(1 + 0.5)(1 + 0.5^2)(1 + 0.5^{2n+1})} \right) \right)^6 \right)^{1/3}$$

Enlarge Data Customize A [Plaintext](#) [Interactive](#)

Result:

12.0031

This result is very near to the value of black hole entropy 12,1904

$$2 \left[ \left( \left( \left( \left( 27 \times 4 + 10^3 \left( \left( -1 + \sum_{n=0}^5 (-1)^n \times 0.5^{(n+1)^2} \right) \times \frac{1 - 0.5^{2n-1}}{(1+0.5)(1+0.5^2)(1+0.5^{2n+1})} \right) \right) \right) \right)^6 \right) \right]^{1/3}$$

Input interpretation:

$$2 \left( 27 \times 4 + 10^3 \left( -1 + \sum_{n=0}^5 (-1)^n \times 0.5^{(n+1)^2} \right) \left( (1 - 0.5)(1 - 0.5^3) \times \frac{1 - 0.5^{2n-1}}{(1 + 0.5)(1 + 0.5^2)(1 + 0.5^{2n+1})} \right) \right)^6 \right)^{1/3}$$

Enlarge Data Customize A [Plaintext](#) [Interactive](#)

Result:

24.0063

This value is linked to the "Ramanujan function" (an elliptic modular function that satisfies the need for "conformal symmetry") that has 24 "modes" corresponding to the physical vibrations of a bosonic string.

$$\left[ \left( \left( \left( \left( 27 \times 4 + 10^3 \left( \left( -1 + \sum_{n=0}^5 (-1)^n \times 0.5^{(n+1)^2} \right) \times \frac{1 - 0.5^{2n-1}}{(1+0.5)(1+0.5^2)(1+0.5^{2n+1})} \right) \right) \right) \right)^6 \right) \right]^{1/15}$$

Input interpretation:

$$\left( 27 \times 4 + 10^3 \left( -1 + \sum_{n=0}^5 (-1)^n \times 0.5^{(n+1)^2} \right) \left( (1 - 0.5)(1 - 0.5^3) \times \frac{1 - 0.5^{2n-1}}{(1 + 0.5)(1 + 0.5^2)(1 + 0.5^{2n+1})} \right) \right)^6 \right)^{1/15}$$

Enlarge Data Customize A [Plaintext](#) [Interactive](#)

Result:

1.64384

$$1.64384 \approx \zeta(2) = \frac{\pi^2}{6} = 1.6449$$

For  $q = 0.5$  and  $n = 2$ , we develop the above formula in the following way:

$$\frac{(((((-1)^2 \cdot 0.5^{(2+1)^2} (1-0.5)^{(1-0.5^3)} (1-0.5^{(2*2-1)}))))}{(((((1+0.5)^{(1+0.5^2)} (1+0.5^{(2*2+1)})))))$$

Input:

$$\frac{(-1)^2 \times 0.5^{(2+1)^2} ((1-0.5)(1-0.5^3)(1-0.5^{2 \times 2-1}))}{(1+0.5)(1+0.5^2)(1+0.5^{2 \times 2+1})}$$

[Open code](#)

Enlarge Data Customize A [Plaintext](#) [Interactive](#)

Result:

More digits

0.00038667929...

[Open code](#)

$$5 * (0.0864055) / \frac{(((((-1)^2 \cdot 0.5^{(2+1)^2} (1-0.5)^{(1-0.5^3)} (1-0.5^{(2*2-1)}))))}{(((((1+0.5)^{(1+0.5^2)} (1+0.5^{(2*2+1)})))))$$

Input interpretation:

$$5 \times \frac{0.0864055}{\frac{(-1)^2 \times 0.5^{(2+1)^2} ((1-0.5)(1-0.5^3)(1-0.5^{2 \times 2-1}))}{(1+0.5)(1+0.5^2)(1+0.5^{2 \times 2+1})}}$$

[Open code](#)

Enlarge Data Customize A [Plaintext](#) [Interactive](#)

Result:

More digits

1117.276016326530612244897959183673469387755102040816326530...

[Open code](#)

This result 1117.276 is very near to the rest mass of the Lambda baryon  $1115.683 \pm 0.006$

$$2 * \frac{((((5 * (0.0864055) / \frac{(((((-1)^2 \cdot 0.5^{(2+1)^2} (1-0.5)^{(1-0.5^3)} (1-0.5^{(2*2-1)}))))}{(((((1+0.5)^{(1+0.5^2)} (1+0.5^{(2*2+1)})))))$$

Input interpretation:

$$2 \sqrt[3]{5 \times \frac{0.0864055}{\frac{(-1)^2 \times 0.5^{(2+1)^2} ((1-0.5)(1-0.5^3)(1-0.5^{2 \times 2-1}))}{(1+0.5)(1+0.5^2)(1+0.5^{2 \times 2+1})}}$$

[Open code](#)

Enlarge Data Customize A [Plaintext](#) [Interactive](#)

Result:

More digits

20.7531...

This result is very near to the value of black hole entropy 20,5520

$$\left( \frac{5 \times 0.0864055}{((-1)^2 \times 0.5^{(2+1)^2} ((1-0.5)(1-0.5^3)(1-0.5^{2 \times 2-1}))) / ((1+0.5)(1+0.5^2)(1+0.5^{(2 \times 2+1)}))} \right)^{1/14}$$

Input interpretation:

$$\sqrt[14]{5 \times \frac{0.0864055}{\frac{(-1)^2 \times 0.5^{(2+1)^2} ((1-0.5)(1-0.5^3)(1-0.5^{2 \times 2-1})))}{(1+0.5)(1+0.5^2)(1+0.5^{2 \times 2+1})}}$$

[Open code](#)

Enlarge Data Customize A [Plaintext](#) [Interactive](#)

Result:

More digits

1.65092...

1.65092 is very near to the 14th root of the following Ramanujan's class invariant

$$Q = (G_{505}/G_{101/5})^3 = 1164,2696 \text{ i.e. } 1,65578...$$

$$\left( \frac{(24 \times 8)/(5^2) \times 0.0864055}{((-1)^2 \times 0.5^{(2+1)^2} ((1-0.5)(1-0.5^3)(1-0.5^{(2 \times 2-1)}))) / ((1+0.5)(1+0.5^2)(1+0.5^{(2 \times 2+1)}))} \right)$$

Input interpretation:

$$\frac{24 \times 8}{5^2} \times \frac{0.0864055}{\frac{(-1)^2 \times 0.5^{(2+1)^2} ((1-0.5)(1-0.5^3)(1-0.5^{2 \times 2-1})))}{(1+0.5)(1+0.5^2)(1+0.5^{2 \times 2+1})}}$$

[Open code](#)

Enlarge Data Customize A [Plaintext](#) [Interactive](#)

Result:

More digits

1716.135961077551020408163265306122448979591836734693877551...

[Open code](#)

This result is very near to the mass of candidate glueball  $f_0(1710)$  meson.

$$\left[ \left( \frac{(24 \times 8)/(5^2) \times 0.0864055}{((-1)^2 \times 0.5^{(2+1)^2} ((1-0.5)(1-0.5^3)(1-0.5^{(2 \times 2-1)}))) / ((1+0.5)(1+0.5^2)(1+0.5^{(2 \times 2+1)}))} \right) \right]^{1/3}$$

Input interpretation:

$$\sqrt[3]{\frac{24 \times 8}{5^2} \times \frac{0.0864055}{\frac{(-1)^2 \times 0.5^{(2+1)^2} ((1-0.5)(1-0.5^3)(1-0.5^{2 \times 2-1})))}{(1+0.5)(1+0.5^2)(1+0.5^{2 \times 2+1})}}$$

[Open code](#)

Enlarge Data Customize A Plaintext Interactive

Result:

More digits

11.9725...

This result is very near to the value of black hole entropy 12,1904

$$2[\frac{(((\frac{24 \times 8}{5^2}) * 0.0864055 / ((((-1)^2 * 0.5^{(2+1)^2} (1-0.5)*(1-0.5^3)(1-0.5^{(2*2-1))}))) / (((1+0.5)*(1+0.5^2)(1+0.5^{(2*2+1))})))))]^{1/3}$$

Input interpretation:

$$2 \sqrt[3]{\frac{24 \times 8}{5^2} \times \frac{0.0864055}{\frac{(-1)^2 \times 0.5^{(2+1)^2} (1-0.5)(1-0.5^3)(1-0.5^{2 \times 2-1}}{(1+0.5)(1+0.5^2)(1+0.5^{2 \times 2+1}})}}$$

[Open code](#)

Enlarge Data Customize A Plaintext Interactive

Result:

More digits

23.9449...

This result is very near to the value of black hole entropy 23,9078

$$[\frac{(((\frac{24 \times 8}{5^2}) * 0.0864055 / ((((-1)^2 * 0.5^{(2+1)^2} (1-0.5)*(1-0.5^3)(1-0.5^{(2*2-1))}))) / (((1+0.5)*(1+0.5^2)(1+0.5^{(2*2+1))})))))]^{1/15}$$

Input interpretation:

$$15 \sqrt{\frac{24 \times 8}{5^2} \times \frac{0.0864055}{\frac{(-1)^2 \times 0.5^{(2+1)^2} (1-0.5)(1-0.5^3)(1-0.5^{2 \times 2-1}}{(1+0.5)(1+0.5^2)(1+0.5^{2 \times 2+1}})}}$$

[Open code](#)

Enlarge Data Customize A Plaintext Interactive

Result:

More digits

1.64300...

$$1.64300... \approx \zeta(2) = \frac{\pi^2}{6} = 1.6449$$

Now, we have:



$$\lambda(q) = \sum_{n \geq 0} \frac{(-1)^n q^n (q; q^2)_n}{(-q; q)_n}$$

That is:

$$\text{Sum}_{\{n \geq 0\}} (-q)^n (1-q)(1-q^3)\dots(1-q^{2n-1})/((1+q)(1+q^2)\dots(1+q^n))$$

$$\text{sum } (-q)^n (1-q)(1-q^3)(1-q^{2n-1})/((1+q)(1+q^2)(1+q^n)), n = 0 \text{ to } 5$$

Result:

$$\sum_{n=0}^5 \frac{(-q)^n (1-q)(1-q^3)(1-q^{2n-1})}{(1+q)(1+q^2)(1+q^n)} = \frac{(q-1)^3 (q^2+q+1)^2 (2q^{20} - 2q^{19} + 2q^{18} + 2q^{17} + 6q^{14} - 6q^{13} + 10q^{12} - 7q^{11} + 10q^{10} - 13q^9 + 22q^8 - 25q^7 + 25q^6 - 21q^5 + 17q^4 - 10q^3 + 5q^2 - 2q + 1)}{(2q(q+1)^2(q^2+1)^2(q^2-q+1)(q^4+1)(q^4-q^3+q^2-q+1))}$$

[Open code](#)

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Sum:

$$\sum_{n=0}^5 \frac{(1-q)(1-q^3)(-q)^n(1-q^{2n-1})}{(q+1)(q^2+1)(q^n+1)} = \frac{(1-q)(1-q^3)^2 q^2}{(q+1)(q^2+1)^2} - \frac{(1-q)^2(1-q^3)q}{(q+1)^2(q^2+1)} + \frac{(1-\frac{1}{q})(1-q)(1-q^3)}{2(q+1)(q^2+1)} - \frac{(1-q)(1-q^3)(1-q^5)q^3}{(q+1)(q^2+1)(q^3+1)} - \frac{(1-q)(1-q^3)(1-q^9)q^5}{(q+1)(q^2+1)(q^5+1)} + \frac{(1-q)(1-q^3)(1-q^7)q^4}{(q+1)(q^2+1)(q^4+1)}$$

[Open code](#)

We obtain:

$$\text{sum } (-0.5)^n (1-0.5)(1-0.5^3)(1-0.5^{2n-1})/((1+0.5)(1+0.5^2)(1+0.5^n)), n = 0 \text{ to } 5$$

Sum:

$$\sum_{n=0}^5 \frac{(1-0.5)(1-0.5^3)(-0.5)^n(1-0.5^{2n-1})}{(1+0.5)(1+0.5^2)(0.5^n+1)} = -\frac{17226587}{129254400}$$

[Open code](#)

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Decimal approximation:

More digits

• -0.13327660025500099029510794216676569617746088334323628441...

[Open code](#)

Multiplying by  $10^3$  and changing the sign, we obtain 133,276, result very near to the rest mass of Pion  $\pi^0$ :  $134.9766(6) \text{ MeV}/c^2$

$$-(-1 + 0.0864055 / \sum_{n=0}^5 (-0.5)^n (1-0.5)(1-0.5^3)(1-0.5^{2n-1})) / ((1+0.5)(1+0.5^2)(1+0.5^n)), n = 0 \text{ to } 5$$

Input interpretation:

$$-\left( -1 + \frac{0.0864055}{\sum_{n=0}^5 (-0.5)^n (1-0.5)(1-0.5^3) \times \frac{1-0.5^{2n-1}}{(1+0.5)(1+0.5^2)(1+0.5^n)}} \right)$$

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Result:

1.64832

$$1.64832 \approx \zeta(2) = \frac{\pi^2}{6} = 1.6449$$

$$24 \times 4 + 10^3 * -(-1 + 0.0864055 / \sum_{n=0}^5 (-0.5)^n (1-0.5)(1-0.5^3)(1-0.5^{2n-1})) / ((1+0.5)(1+0.5^2)(1+0.5^n)), n = 0 \text{ to } 5$$

Input interpretation:

$$24 \times 4 + 10^3 \times (-1) \left( -1 + \frac{0.0864055}{\sum_{n=0}^5 (-0.5)^n (1-0.5)(1-0.5^3) \times \frac{1-0.5^{2n-1}}{(1+0.5)(1+0.5^2)(1+0.5^n)}} \right)$$

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Result:

1744.32

This result is very near to the mass of candidate glueball  $f_0(1710)$  meson.

$$[\left( (24 \times 4 + 10^3 * -(-1 + 0.0864055 / \sum_{n=0}^5 ((-0.5)^n (1-0.5)(1-0.5^3)(1-0.5^{2n-1})) / ((1+0.5)(1+0.5^2)(1+0.5^n))), n = 0 \text{ to } 5) \right)]^{1/3}$$

Input interpretation:

$$\sqrt[3]{24 \times 4 + 10^3 \times (-1) \left( -1 + \frac{0.0864055}{\sum_{n=0}^5 (-0.5)^n (1-0.5)(1-0.5^3) \times \frac{1-0.5^{2n-1}}{(1+0.5)(1+0.5^2)(1+0.5^n)}} \right)}$$

Enlarge Data Customize A [Plaintext](#) [Interactive](#)

Result:

12.0377

This result is very near to the value of black hole entropy 12,1904

$$2\left[\left(\frac{24 \times 4 + 10^3 \times (-1) + 0.0864055}{\sum_{n=0}^5 ((-0.5)^n (1-0.5)(1-0.5^3)(1-0.5^{2n-1})) / ((1+0.5)(1+0.5^2)(1+0.5^n))}\right)\right]^{1/3}$$

Input interpretation:

$$2 \left( 24 \times 4 + 10^3 \times (-1) \left( -1 + \frac{0.0864055}{\sum_{n=0}^5 (-0.5)^n (1-0.5)(1-0.5^3) \times \frac{1-0.5^{2n-1}}{(1+0.5)(1+0.5^2)(1+0.5^n)}} \right) \right)^{1/3}$$

Enlarge Data Customize A [Plaintext](#) [Interactive](#)

Result:

24.0753

This value is linked to the "Ramanujan function" (an elliptic modular function that satisfies the need for "conformal symmetry") that has 24 "modes" corresponding to the physical vibrations of a bosonic string.

$$\left[\left(\frac{24 \times 4 + 10^3 \times (-1) + 0.0864055}{\sum_{n=0}^5 ((-0.5)^n (1-0.5)(1-0.5^3)(1-0.5^{2n-1})) / ((1+0.5)(1+0.5^2)(1+0.5^n))}\right)\right]^{1/15}$$

Input interpretation:

$$\sqrt[15]{24 \times 4 + 10^3 \times (-1) \left( -1 + \frac{0.0864055}{\sum_{n=0}^5 (-0.5)^n (1-0.5)(1-0.5^3) \times \frac{1-0.5^{2n-1}}{(1+0.5)(1+0.5^2)(1+0.5^n)}} \right)}$$

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Result:

1.64478

We note that this result is  $\cong \frac{\pi^2}{6} = 1.6449 \dots$  Indeed, we have that:

$$\left[\left[6 \left(\frac{24 \times 4 + 10^3 \times (-1) + 0.0864055}{\sum_{n=0}^5 ((-0.5)^n (1-0.5)(1-0.5^3)(1-0.5^{2n-1})) / ((1+0.5)(1+0.5^2)(1+0.5^n))}\right)\right]^{1/15}\right]^{1/2}$$

Input interpretation:

$$\sqrt[6]{24 \times 4 + 10^3 \times (-1)}$$
$$\sqrt[15]{-1 + \frac{0.0864055}{\sum_{n=0}^5 (-0.5)^n (1 - 0.5)(1 - 0.5^3) \times \frac{1 - 0.5^{2n-1}}{(1 + 0.5)(1 + 0.5^2)(1 + 0.5^n)}}}$$

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Result:

3.14145  
3.14145

Possible closed forms:

More

$\pi \approx 3.14159265$

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$\sqrt[3]{31} \approx 3.14138065$   
 $\frac{311}{99} \approx 3.141414141$

This result is a very good approximation to  $\pi$

For  $q = 0.5$  and  $n = 2$ , we develop the above formula in the following way:

$$\frac{((-0.5)^2 (1-0.5)(1-0.5^3)(1-0.5^{(2 \times 2 - 1))})}{((1+0.5)(1+0.5^2)(1+0.5^2))}$$

Input:

$$\frac{(-0.5)^2 (1 - 0.5)(1 - 0.5^3)(1 - 0.5^{2 \times 2 - 1})}{(1 + 0.5)(1 + 0.5^2)(1 + 0.5^2)}$$

[Open code](#)

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Result:

More digits

0.040833...

[Open code](#)

$$10^3 \left( \frac{1.0864055}{((-0.5)^2 (1-0.5)(1-0.5^3)(1-0.5^{(2 \times 2 - 1))})} \right) / ((1+0.5)(1+0.5^2)(1+0.5^2)))^{1/6}$$

Input interpretation:

$$10^3 \sqrt[6]{\frac{1.0864055}{\frac{(-0.5)^2 (1-0.5)(1-0.5^3)(1-0.5^{2 \times 2-1})}{(1+0.5)(1+0.5^2)(1+0.5^2)}}}$$

[Open code](#)

Enlarge Data Customize A [Plaintext](#) [Interactive](#)

Result:

More digits

1727.81...

This result is very near to the mass of candidate glueball  $f_0(1710)$  meson. Furthermore, 1728 occurs in the algebraic formula for the j-invariant of an elliptic curve. As a consequence, it is sometimes called a Zagier as a pun on the Gross–Zagier theorem. The number 1728 is one less than the Hardy–Ramanujan number 1729

$$\left(\left(\left(\left(\left(10^3 \left(\frac{1.0864055}{((-0.5)^2 (1-0.5)(1-0.5^3)(1-0.5^{2 \times 2-1}))}\right)\right)\right)\right)\right)\right)^{1/6}\right)^{1/3}$$

Input interpretation:

$$\sqrt[3]{10^3 \sqrt[6]{\frac{1.0864055}{\frac{(-0.5)^2 (1-0.5)(1-0.5^3)(1-0.5^{2 \times 2-1})}{(1+0.5)(1+0.5^2)(1+0.5^2)}}}}$$

[Open code](#)

Enlarge Data Customize A [Plaintext](#) [Interactive](#)

Result:

More digits

11.99956...

This result is very near to the value of black hole entropy 12,1904

$$2\left(\left(\left(\left(\left(10^3 \left(\frac{1.0864055}{((-0.5)^2 (1-0.5)(1-0.5^3)(1-0.5^{2 \times 2-1}))}\right)\right)\right)\right)\right)\right)^{1/6}\right)^{1/3}$$

Input interpretation:

$$2 \sqrt[3]{10^3 \sqrt[6]{\frac{1.0864055}{\frac{(-0.5)^2 (1-0.5)(1-0.5^3)(1-0.5^{2 \times 2-1})}{(1+0.5)(1+0.5^2)(1+0.5^2)}}}}$$

[Open code](#)

Enlarge Data Customize A [Plaintext](#) [Interactive](#)

Result:

More digits

23.99912...

This value is linked to the "Ramanujan function" (an elliptic modular function that satisfies the need for "conformal symmetry") that has 24 "modes" corresponding to the physical vibrations of a bosonic string.

$$\left( \frac{10^3 \left( \frac{1.0864055}{((-0.5)^2 (1-0.5)(1-0.5^3)(1-0.5^{2*2-1}))} \right)}{((1+0.5)(1+0.5^2)(1+0.5^2))} \right)^{1/6} \Bigg)^{1/15}$$

Input interpretation:

$$\sqrt[15]{10^3 \sqrt[6]{\frac{1.0864055}{\frac{(-0.5)^2 (1-0.5)(1-0.5^3)(1-0.5^{2*2-1})}{(1+0.5)(1+0.5^2)(1+0.5^2)}}}}$$

[Open code](#)

Enlarge Data Customize A [Plaintext](#) [Interactive](#)

Result:

More digits

1.643740...

$$1.643740... \approx \zeta(2) = \frac{\pi^2}{6} = 1.6449$$

Now, we have:

$$2\mu(q) = \sum_{n \geq 0} \frac{(-1)^n q^{n+1} (1+q^n)(q; q^2)_n}{(-q; q)_{n+1}}$$

That is:

$$1 + \sum_{n \geq 0} \frac{(-1)^n q^{n+1} (1+q^n) (1-q)(1-q^3) \dots (1-q^{2n-1})}{(1+q)(1+q^2) \dots (1+q^{n+1})}$$

$$1 + \sum_{n=0}^5 \frac{(-1)^n q^{n+1} (1+q^n) (1-q)(1-q^3)(1-q^{2n-1})}{(1+q)(1+q^2)(1+q^{n+1})},$$

Input interpretation:

$$1 + \sum_{n=0}^5 \frac{(-1)^n q^{n+1} (1+q^n) (1-q)(1-q^3) \dots (1-q^{2n-1})}{(1+q)(1+q^2)(1+q^{n+1})}$$

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Result:

$$\begin{aligned} & ((q^2 + q + 1)^2 (q^{28} - q^{27} + q^{26} + q^{24} + q^{23} + 4q^{20} - 2q^{19} + 2q^{18} + 6q^{16} - \\ & 8q^{15} + 10q^{14} - 7q^{13} + 9q^{12} - 10q^{11} + 13q^{10} - 14q^9 + 18q^8 - \\ & 18q^7 + 13q^6 - 7q^5 + 9q^4 - 12q^3 + 11q^2 - 6q + 2)(q-1)^3) / \\ & ((q+1)^2 (q^2+1)^2 (q^2-q+1)(q^4+1)(q^4-q^2+1)(q^4-q^3+q^2-q+1)) + 1 \end{aligned}$$

For  $q = 0.5$  we obtain:

$$1 + \sum_{n=0}^5 (-1)^n 0.5^{n+1} (1+0.5^n)(1-0.5)(1-0.5^3)(1-0.5^{2n-1}) / ((1+0.5)(1+0.5^2)(1+0.5^{n+1})), n = 0 \text{ to } 5$$

Input interpretation:

$$1 + \sum_{n=0}^5 (-1)^n \times 0.5^{n+1} (1 + 0.5^n) (1 - 0.5) (1 - 0.5^3) \times \frac{1 - 0.5^{2n-1}}{(1 + 0.5)(1 + 0.5^2)(1 + 0.5^{n+1})}$$

Enlarge Data Customize A [Plaintext](#) [Interactive](#)

Result:

0.826602

$$2 * (((1 + \sum_{n=0}^5 (-1)^n 0.5^{n+1} (1+0.5^n)(1-0.5)(1-0.5^3)(1-0.5^{2n-1}) / ((1+0.5)(1+0.5^2)(1+0.5^{n+1}))), n = 0 \text{ to } 5)))$$

Input interpretation:

$$2 \left( 1 + \sum_{n=0}^5 (-1)^n \times 0.5^{n+1} (1 + 0.5^n) (1 - 0.5) (1 - 0.5^3) \times \frac{1 - 0.5^{2n-1}}{(1 + 0.5)(1 + 0.5^2)(1 + 0.5^{n+1})} \right)$$

Enlarge Data Customize A [Plaintext](#) [Interactive](#)

Result:

1.6532

1.6532 is very near to the 14th root of the following Ramanujan's class invariant

$$Q = (G_{505}/G_{101/5})^3 = 1164,2696 \text{ i.e. } 1,65578...$$

$$24*3 + 10^3 * 2 * (((1 + \sum_{n=0}^5 (-1)^n 0.5^{n+1} (1+0.5^n)(1-0.5)(1-0.5^3)(1-0.5^{2n-1}) / ((1+0.5)(1+0.5^2)(1+0.5^{n+1}))), n = 0 \text{ to } 5)))$$

Input interpretation:

$$24 \times 3 + 10^3 \times 2 \left( 1 + \sum_{n=0}^5 (-1)^n \times 0.5^{n+1} (1 + 0.5^n) (1 - 0.5) (1 - 0.5^3) \times \frac{1 - 0.5^{2n-1}}{(1 + 0.5)(1 + 0.5^2)(1 + 0.5^{n+1})} \right)$$

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Result:

1725.2

This result is very near to the mass of candidate glueball  $f_0(1710)$  meson.

$$[24*3 + 10^3 * 2 * (((1 + \sum_{n=0}^5 (-1)^n 0.5^{n+1} (1+0.5^n)(1-0.5)(1-0.5^3)(1-0.5^{2n-1}) / ((1+0.5)(1+0.5^2)(1+0.5^{n+1}))), n = 0 \text{ to } 5)))]^{1/3}$$

Input interpretation:

$$\left( 24 \times 3 + 10^3 \times 2 \left( 1 + \sum_{n=0}^5 (-1)^n \times 0.5^{n+1} (1 + 0.5^n) (1 - 0.5) \right) \right. \\ \left. (1 - 0.5^3) \times \frac{1 - 0.5^{2n-1}}{(1 + 0.5)(1 + 0.5^2)(1 + 0.5^{n+1})} \right) \right)^{(1/3)}$$

Enlarge Data Customize A [Plaintext](#) [Interactive](#)

Result:

11.9935

This result is very near to the value of black hole entropy 12,1904

$$2 * [24 * 3 + 10^3 * 2 * (((1 + \sum_{n=0}^5 (-1)^n 0.5^{n+1} (1 + 0.5^n) (1 - 0.5) (1 - 0.5^3) (1 - 0.5^{2n-1})) / ((1 + 0.5)(1 + 0.5^2)(1 + 0.5^{n+1}))), n = 0 \text{ to } 5))]^{1/3}$$

Input interpretation:

$$2 \left( 24 \times 3 + 10^3 \times 2 \left( 1 + \sum_{n=0}^5 (-1)^n \times 0.5^{n+1} (1 + 0.5^n) (1 - 0.5) \right) \right. \\ \left. (1 - 0.5^3) \times \frac{1 - 0.5^{2n-1}}{(1 + 0.5)(1 + 0.5^2)(1 + 0.5^{n+1})} \right) \right)^{(1/3)}$$

Enlarge Data Customize A [Plaintext](#) [Interactive](#)

Result:

23.987

This value is linked to the "Ramanujan function" (an elliptic modular function that satisfies the need for "conformal symmetry") that has 24 "modes" corresponding to the physical vibrations of a bosonic string.

$$[24 * 3 + 10^3 * 2 * (((1 + \sum_{n=0}^5 (-1)^n 0.5^{n+1} (1 + 0.5^n) (1 - 0.5) (1 - 0.5^3) (1 - 0.5^{2n-1})) / ((1 + 0.5)(1 + 0.5^2)(1 + 0.5^{n+1}))), n = 0 \text{ to } 5))]^{1/15}$$

Input interpretation:

$$\left( 24 \times 3 + 10^3 \times 2 \left( 1 + \sum_{n=0}^5 (-1)^n \times 0.5^{n+1} (1 + 0.5^n) (1 - 0.5) \right) \right. \\ \left. (1 - 0.5^3) \times \frac{1 - 0.5^{2n-1}}{(1 + 0.5)(1 + 0.5^2)(1 + 0.5^{n+1})} \right) \right)^{(1/15)}$$

Enlarge Data Customize A [Plaintext](#) [Interactive](#)

Result:

1.64357

$$1.64357 \approx \zeta(2) = \frac{\pi^2}{6} = 1.6449$$

For  $q = 0.5$  and  $n = 2$ , we develop the above formula in the following way:



$$1 + \frac{((-1)^2 \cdot 0.5^{(2+1)})(1+0.5^2)(1-0.5)(1-0.5^3)(1-0.5^{(2*2-1)})}{((1+0.5)(1+0.5^2)(1+0.5^{(2+1))})}$$

Input:

$$1 + (-1)^2 \times 0.5^{2+1} (1 + 0.5^2) (1 - 0.5) (1 - 0.5^3) \times \frac{1 - 0.5^{2 \times 2 - 1}}{(1 + 0.5)(1 + 0.5^2)(1 + 0.5^{2+1})}$$

[Open code](#)

Enlarge Data Customize A [Plaintext](#) [Interactive](#)

Result:

More digits

1.028356481...

1.028356481 (period 3)

Input:

$$1 + \frac{(-1)^2 \times 0.5^{2+1} (1 + 0.5^2) (1 - 0.5) (1 - 0.5^3) (1 - 0.5^{2 \times 2 - 1})}{(1 + 0.5)(1 + 0.5^2)(1 + 0.5^{2+1})}$$

[Open code](#)

Enlarge Data Customize A [Plaintext](#) [Interactive](#)

Result:

More digits

1.028356481...

$$10^3 * \left( \frac{1 + \frac{((-1)^2 \cdot 0.5^{(2+1)})(1+0.5^2)(1-0.5)(1-0.5^3)(1-0.5^{(2*2-1)})}{((1+0.5)(1+0.5^2)(1+0.5^{(2+1))})}}{1} \right)$$

Input:

$$10^3 \left( 1 + \frac{(-1)^2 \times 0.5^{2+1} (1 + 0.5^2) (1 - 0.5) (1 - 0.5^3) (1 - 0.5^{2 \times 2 - 1})}{(1 + 0.5)(1 + 0.5^2)(1 + 0.5^{2+1})} \right)$$

[Open code](#)

Enlarge Data Customize A [Plaintext](#) [Interactive](#)

Result:

More digits

1028.356481...

[Open code](#)

Repeating decimal:

1028.356481 (period 3)

This result is very near to the rest mass of Phi meson  $1019.445 \pm 0.020$

$$\left( \frac{1 + \frac{((-1)^2 \cdot 0.5^{(2+1)})(1+0.5^2)(1-0.5)(1-0.5^3)(1-0.5^{(2*2-1)})}{((1+0.5)(1+0.5^2)(1+0.5^{(2+1))})}}{1} \right)^{18}$$

Input:

$$\left(1 + (-1)^2 \times 0.5^{2+1} (1 + 0.5^2) (1 - 0.5) (1 - 0.5^3) \times \frac{1 - 0.5^{2 \times 2 - 1}}{(1 + 0.5)(1 + 0.5^2)(1 + 0.5^{2+1})}\right)^{18}$$

[Open code](#)

Enlarge Data Customize A [Plaintext](#) [Interactive](#)

Result:

More digits

1.654193895940540585938941330423539168330463876866415721388...

1.6541938959405405859... is very near to the 14th root of the following

Ramanujan's class invariant  $Q = (G_{505}/G_{101/5})^3 = 1164,2696$  i.e. 1,65578...

$$24 \times 3 + 10^3 * (((1 + (((-1)^2 0.5^{(2+1)}(1+0.5^2)(1-0.5)(1-0.5^3)(1-0.5^{(2*2-1)})))/((1+0.5)(1+0.5^2)(1+0.5^{(2+1))))))^18$$

Input:

$$24 \times 3 + 10^3 \left(1 + (-1)^2 \times 0.5^{2+1} (1 + 0.5^2) (1 - 0.5) (1 - 0.5^3) \times \frac{1 - 0.5^{2 \times 2 - 1}}{(1 + 0.5)(1 + 0.5^2)(1 + 0.5^{2+1})}\right)^{18}$$

[Open code](#)

Enlarge Data Customize A [Plaintext](#) [Interactive](#)

Result:

More digits

1726.193895940540585938941330423539168330463876866415721388...

[Open code](#)

This result is very near to the mass of candidate glueball  $f_0(1710)$  meson.

$$((((24 \times 3 + 10^3 * (((1 + (((-1)^2 0.5^{(2+1)}(1+0.5^2)(1-0.5)(1-0.5^3)(1-0.5^{(2*2-1)})))/((1+0.5)(1+0.5^2)(1+0.5^{(2+1))))))^18))))))^{1/3}$$

Input:

$$\left(24 \times 3 + 10^3 \left(1 + (-1)^2 \times 0.5^{2+1} (1 + 0.5^2) (1 - 0.5) (1 - 0.5^3) \times \frac{1 - 0.5^{2 \times 2 - 1}}{(1 + 0.5)(1 + 0.5^2)(1 + 0.5^{2+1})}\right)^{18}\right)^{(1/3)}$$

[Open code](#)

Enlarge Data Customize A [Plaintext](#) [Interactive](#)

Result:

More digits

11.9958...

This result is very near to the value of black hole entropy 12,1904



sum  $q^{n^2}/((1+q+q^2)(1+q^2+q^4)(1+q^n+q^{2n}))$ ,  $n = 0$  to  $5$

Result:

$$\sum_{n=0}^5 \frac{q^{n^2}}{(1+q+q^2)(1+q^2+q^4)(1+q^n+q^{2n})} = \frac{q^4}{(q^2+q+1)(q^4+q^2+1)^2} + \frac{q}{(q^2+q+1)^2(q^4+q^2+1)} + \frac{1}{3(q^2+q+1)(q^4+q^2+1)} + \frac{q^{16}}{(q^2+q+1)(q^4+q^2+1)(q^8+q^4+1)} + \frac{q^{25}}{(q^2+q+1)(q^4+q^2+1)(q^{10}+q^5+1)} + \frac{q^9}{(q^2+q+1)(q^4+q^2+1)(q^6+q^3+1)}$$

[Open code](#)

Enlarge Data Customize A [Plaintext](#) [Interactive](#)

Result:

$$\sum_{n=0}^5 \frac{q^{n^2}}{(1+q+q^2)(1+q^2+q^4)(1+q^n+q^{2n})} = \left( 3(q^2+q+1)q^4 + 3(q^4+q^2+1)q + (q^2+q+1)(q^4+q^2+1) + \frac{3(q^2+q+1)q^{16}}{q^4-q^2+1} + \frac{3(q^2+q+1)(q^4+q^2+1)q^{25}}{q^{10}+q^5+1} + \frac{3(q^2+q+1)(q^4+q^2+1)q^9}{q^6+q^3+1} \right) / (3(q^2+q+1)^2(q^4+q^2+1)^2)$$

[Open code](#)

For  $q = 0.5$ , we obtain:

sum  $0.5^{n^2}/((1+0.5+0.5^2)(1+0.5^2+0.5^4)(1+0.5^n+0.5^{2n}))$ ,  $n = 0$  to  $5$

Sum:

$$\sum_{n=0}^5 \frac{0.5^{n^2}}{(1+0.5^2+0.5^4)(0.5+1+0.5^2)(0.5^n+0.5^{2n}+1)} = \frac{21\,969\,655\,349}{75\,496\,791\,552}$$

[Open code](#)

Enlarge Data Customize A [Plaintext](#) [Interactive](#)

Decimal approximation:

More digits

0.291001178955637322604721012873169680735308819074303858437...

[Open code](#)

$2 * ((((((\text{sum } 0.5^{n^2}/((1+0.5+0.5^2)(1+0.5^2+0.5^4)(1+0.5^n+0.5^{2n})), n = 0 \text{ to } 5))))))^{1/6}$

Input interpretation:

$$2 \sqrt[6]{\sum_{n=0}^5 \frac{0.5^{n^2}}{(1+0.5+0.5^2)(1+0.5^2+0.5^4)(1+0.5^n+0.5^{2n})}}$$

Enlarge Data Customize A [Plaintext](#) [Interactive](#)

Result:

1.62809

This result is a golden number

$$27 \cdot 4 + 10^3 \cdot 2 \cdot \left( \left( \left( \left( \left( \sum_{n=0}^5 \frac{0.5^{n^2}}{(1+0.5+0.5^2)(1+0.5^2+0.5^4)(1+0.5^n+0.5^{2n})} \right) \right) \right) \right) \right)^{1/6}$$

Input interpretation:

$$27 \times 4 + 10^3 \times 2 \sqrt[6]{\sum_{n=0}^5 \frac{0.5^{n^2}}{(1+0.5+0.5^2)(1+0.5^2+0.5^4)(1+0.5^n+0.5^{2n})}}$$

Enlarge Data Customize A [Plaintext](#) [Interactive](#)

Result:

1736.09

This result is very near to the mass of candidate glueball  $f_0(1710)$  meson.

$$\left( \left( \left( \left( \left( \left( \left( \left( \left( \left( \sum_{n=0}^5 \frac{0.5^{n^2}}{(1+0.5+0.5^2)(1+0.5^2+0.5^4)(1+0.5^n+0.5^{2n})} \right) \right) \right) \right) \right) \right) \right) \right)^{1/6} \right)^{1/3}$$

Input interpretation:

$$\sqrt[3]{27 \times 4 + 10^3 \times 2 \sqrt[6]{\sum_{n=0}^5 \frac{0.5^{n^2}}{(1+0.5+0.5^2)(1+0.5^2+0.5^4)(1+0.5^n+0.5^{2n})}}}$$

Enlarge Data Customize A [Plaintext](#) [Interactive](#)

Result:

12.0187

This result is very near to the value of black hole entropy 12,1904

$$2 \left( \left( \left( \left( \left( \left( \left( \left( \left( \left( \sum_{n=0}^5 \frac{0.5^{n^2}}{(1+0.5+0.5^2)(1+0.5^2+0.5^4)(1+0.5^n+0.5^{2n})} \right) \right) \right) \right) \right) \right) \right) \right)^{1/6} \right)^{1/3}$$

Input interpretation:

$$2 \sqrt[3]{27 \times 4 + 10^3 \times 2} \sqrt[6]{\sum_{n=0}^5 \frac{0.5^{n^2}}{(1 + 0.5 + 0.5^2)(1 + 0.5^2 + 0.5^4)(1 + 0.5^n + 0.5^{2n})}}$$

Enlarge Data Customize A [Plaintext](#) [Interactive](#)

Result:

24.0374

This value is linked to the "Ramanujan function" (an elliptic modular function that satisfies the need for "conformal symmetry") that has 24 "modes" corresponding to the physical vibrations of a bosonic string.

$$\left( \left( \left( \left( \left( \left( 27 \times 4 + 10^3 \times 2 \right) \times \left( \sum_{n=0}^5 \frac{0.5^{n^2}}{(1 + 0.5 + 0.5^2)(1 + 0.5^2 + 0.5^4)(1 + 0.5^n + 0.5^{2n})} \right) \right) \right) \right) \right) \right)^{1/6} \right)^{1/15}$$

Input interpretation:

$$15 \sqrt{27 \times 4 + 10^3 \times 2} \sqrt[6]{\sum_{n=0}^5 \frac{0.5^{n^2}}{(1 + 0.5 + 0.5^2)(1 + 0.5^2 + 0.5^4)(1 + 0.5^n + 0.5^{2n})}}$$

Enlarge Data Customize A [Plaintext](#) [Interactive](#)

Result:

1.64426

$$1.64426 \approx \zeta(2) = \frac{\pi^2}{6} = 1.6449$$

$$\text{sqrt}[6 \left( \left( \left( \left( \left( \left( 27 \times 4 + 10^3 \times 2 \right) \times \left( \sum_{n=0}^5 \frac{0.5^{n^2}}{(1 + 0.5 + 0.5^2)(1 + 0.5^2 + 0.5^4)(1 + 0.5^n + 0.5^{2n})} \right) \right) \right) \right) \right) \right)^{1/6} \right)^{1/15}]$$

Input interpretation:

$$\sqrt[6]{15 \sqrt{27 \times 4 + 10^3 \times 2} \sqrt[6]{\sum_{n=0}^5 \frac{0.5^{n^2}}{(1 + 0.5 + 0.5^2)(1 + 0.5^2 + 0.5^4)(1 + 0.5^n + 0.5^{2n})}}}$$

Enlarge Data Customize A [Plaintext](#) [Interactive](#)

Result:

3.14095

3.14095

From the above expression, for  $q = 0.5$  and  $n = 2$ , we obtain:

Input:

$$\frac{0.5^{2^2}}{(1 + 0.5 + 0.5^2)(1 + 0.5^2 + 0.5^4)(1 + 0.5^2 + 0.5^{2 \times 2})}$$

[Open code](#)

Enlarge Data Customize A [Plaintext](#) [Interactive](#)

Result:

More digits

0.020732102364755425979915775834143181081956592160673793326...

[Open code](#)

$$(54+24) * 0.5^{2^2} / ((1+0.5+0.5^2)(1+0.5^2+0.5^4)(1+0.5^2+0.5^{(2*2)}))$$

Input:

$$(54 + 24) \times \frac{0.5^{2^2}}{(1 + 0.5 + 0.5^2)(1 + 0.5^2 + 0.5^4)(1 + 0.5^2 + 0.5^{2 \times 2})}$$

[Open code](#)

Enlarge Data Customize A Plaintext Interactive

Result:

More digits

1.617103984450923226433430515063168124392614188532555879494...

[Open code](#)

Or:

Input:

$$(55 + 21 + 2) \times \frac{0.5^{2^2}}{(1 + 0.5 + 0.5^2)(1 + 0.5^2 + 0.5^4)(1 + 0.5^2 + 0.5^{2 \times 2})}$$

[Open code](#)

Enlarge Data Customize A Plaintext Interactive

Result:

More digits

1.617103984450923226433430515063168124392614188532555879494...

This result is a very good approximation to the value of the golden ratio 1,618033988749...

$$27*4 + 10^3*(55+21+2) * 0.5^{2^2} / ((1+0.5+0.5^2)(1+0.5^2+0.5^4)(1+0.5^2+0.5^{(2*2)}))$$

Input:

$$27 \times 4 + 10^3 (55 + 21 + 2) \times \frac{0.5^{2^2}}{(1 + 0.5 + 0.5^2)(1 + 0.5^2 + 0.5^4)(1 + 0.5^2 + 0.5^{2 \times 2})}$$

[Open code](#)

Enlarge Data Customize A Plaintext Interactive

Result:

More digits

1725.103984450923226433430515063168124392614188532555879494...

[Open code](#)

This result is very near to the mass of candidate glueball  $f_0(1710)$  meson.

$$\left(\left(\left(\left(27 \times 4 + 10^3 \times (55 + 21 + 2) \times 0.5^{2^2} / \left( (1 + 0.5 + 0.5^2)(1 + 0.5^2 + 0.5^4)(1 + 0.5^2 + 0.5^{(2 \times 2)}) \right) \right)\right)\right)\right)^{1/3}$$

Input:

$$\sqrt[3]{27 \times 4 + 10^3 (55 + 21 + 2) \times \frac{0.5^{2^2}}{(1 + 0.5 + 0.5^2)(1 + 0.5^2 + 0.5^4)(1 + 0.5^2 + 0.5^{2 \times 2})}}$$

[Open code](#)

Enlarge Data Customize A [Plaintext](#) [Interactive](#)

Result:

More digits

11.9933...

This result is very near to the value of black hole entropy 12,1904

$$2 \left(\left(\left(\left(27 \times 4 + 10^3 \times (55 + 21 + 2) \times 0.5^{2^2} / \left( (1 + 0.5 + 0.5^2)(1 + 0.5^2 + 0.5^4)(1 + 0.5^2 + 0.5^{(2 \times 2)}) \right) \right)\right)\right)\right)^{1/3}$$

Input:

$$2 \sqrt[3]{27 \times 4 + 10^3 (55 + 21 + 2) \times \frac{0.5^{2^2}}{(1 + 0.5 + 0.5^2)(1 + 0.5^2 + 0.5^4)(1 + 0.5^2 + 0.5^{2 \times 2})}}$$

[Open code](#)

Enlarge Data Customize A [Plaintext](#) [Interactive](#)

Result:

More digits

23.9866...

This value is linked to the "Ramanujan function" (an elliptic modular function that satisfies the need for "conformal symmetry") that has 24 "modes" corresponding to the physical vibrations of a bosonic string.

$$\left(\left(\left(\left(27 \times 4 + 10^3 \times (55 + 21 + 2) \times 0.5^{2^2} / \left( (1 + 0.5 + 0.5^2)(1 + 0.5^2 + 0.5^4)(1 + 0.5^2 + 0.5^{(2 \times 2)}) \right) \right)\right)\right)\right)^{1/15}$$

Input:

$$\sqrt[15]{27 \times 4 + 10^3 (55 + 21 + 2) \times \frac{0.5^{2^2}}{(1 + 0.5 + 0.5^2)(1 + 0.5^2 + 0.5^4)(1 + 0.5^2 + 0.5^{2 \times 2})}}$$

[Open code](#)

Enlarge Data Customize A [Plaintext](#) [Interactive](#)

Result:

Fewer digits

More digits

1.643568030985126263487230283183810464038877569960072506370...



$$1.64356803098512626\dots \approx \zeta(2) = \frac{\pi^2}{6} = 1.6449$$

Continued fraction:

Linear form

$$1 + \frac{1}{1 + \frac{1}{1 + \frac{1}{1 + \frac{1}{4 + \frac{1}{6 + \frac{1}{1 + \frac{1}{25 + \frac{1}{1 + \frac{1}{4 + \frac{1}{2 + \frac{1}{4 + \frac{1}{1 + \frac{1}{3 + \frac{1}{1 + \frac{1}{4 + \frac{1}{1 + \frac{1}{12 + \frac{1}{2 + \frac{1}{1 + \frac{1}{1 + \frac{1}{\dots}}}}}}}}}}}}}}}}}}}}}}}}}}}}}}}}}}}}$$

[Open code](#)

Enlarge Data Customize A [Plaintext](#) [Interactive](#)

Possible closed forms:

More

$$-\operatorname{csc}\left(\cot\left(\frac{96\,921\,172}{32\,323\,975}\right)\right) \approx 1.6435680309851262650630$$

$$\frac{4\,294\,734\,226\,\pi}{8\,209\,155\,471} \approx 1.6435680309851262634819812$$

$$\frac{44\,472 + 2376\,\pi + 21\,733\,\pi^2}{51\,600\,\pi} \approx 1.643568030985126263491012$$

$$\operatorname{sqrt}\left(\left(\left(\left(\left(\left(6\left(\left(\left(\left(27*4 + 10^3*(55+21+2) * 0.5^{2^2}/((1+0.5+0.5^2)(1+0.5^2+0.5^4)(1+0.5^2+0.5^{(2*2)}))\right)\right)\right)\right)\right)\right)\right)^{1/15}\right)\right)\right)$$

Input:

$$\sqrt[6]{\sqrt[15]{27 \times 4 + 10^3 (55 + 21 + 2) \times \frac{0.5^{2^2}}{(1 + 0.5 + 0.5^2)(1 + 0.5^2 + 0.5^4)(1 + 0.5^2 + 0.5^{2 \cdot 2})}}}}$$

[Open code](#)

Enlarge Data Customize A [Plaintext](#) [Interactive](#)

Result:

More digits

3.140288...

3.140288

We now, analyze the following **mock theta function  $\mu$  of order 2** that was found by Srinivasa Ramanujan in his lost notebook.

$$\mu(q) = \sum_{n \geq 0} \frac{(-1)^n q^{n^2} (q; q^2)_n}{(-q^2; q^2)_n^2}$$

That is:

$$\sum_{n=0}^5 \frac{(-1)^n q^{n^2} (1-q)(1-q^3)\dots(1-q^{2n-1})}{((1+q^2)^2 (1+q^4)^2 \dots (1+q^{2n})^2)}, n = 0 \text{ to } 5$$

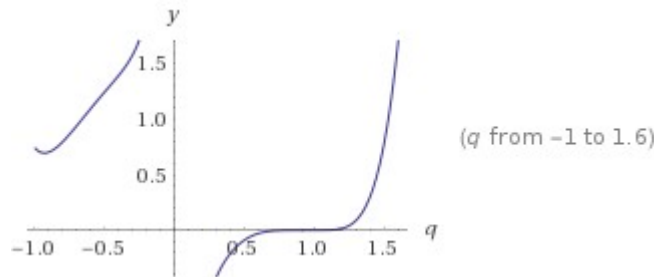
Sum:

$$\sum_{n=0}^5 \frac{(-1)^n q^{n^2} (1-q)(1-q^3)(1-q^{2n-1})}{(q^2+1)^2 (q^4+1)^2 (q^{2n}+1)^2} = \frac{(1-q)(1-q^3)^2 q^4}{(q^2+1)^2 (q^4+1)^4} - \frac{(1-q)^2 (1-q^3) q}{(q^2+1)^4 (q^4+1)^2} + \frac{(1-\frac{1}{q})(1-q)(1-q^3)}{4(q^2+1)^2 (q^4+1)^2} - \frac{(1-q)(1-q^3)(1-q^9) q^{25}}{(q^2+1)^2 (q^4+1)^2 (q^{10}+1)^2} + \frac{(1-q)(1-q^3)(1-q^7) q^{16}}{(q^2+1)^2 (q^4+1)^2 (q^8+1)^2} - \frac{(1-q)(1-q^3)(1-q^5) q^9}{(q^2+1)^2 (q^4+1)^2 (q^6+1)^2}$$

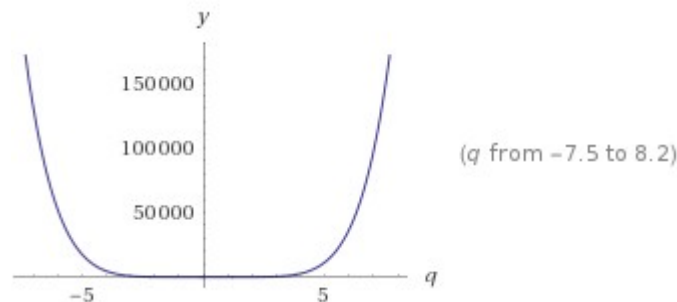
[Open code](#)

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Plots:



[Open code](#)



Alternate forms:

$$\frac{1}{4(q^2+1)^4(q^4+1)^4} (1-q)(1-q^3) \left( 4(q-1)(q^4+1)^2 q + \frac{(q-1)(q^2+1)^2(q^4+1)^2}{q} - 4(q^2+1)^2(q^3-1)q^4 + \frac{4(q^4+1)^2(q^5-1)q^9}{(q^4-q^2+1)^2} - \frac{4(q^2+1)^2(q^4+1)^2(1-q^9)q^{25}}{(q^{10}+1)^2} - \frac{4(q^2+1)^2(q^4+1)^2(q^7-1)q^{16}}{(q^8+1)^2} \right)$$

[Open code](#)

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$$(q-1)^3(q^2+q+1)^2 \left( 4q^{64} - 8q^{62} + 4q^{61} + 20q^{60} - 8q^{59} - 20q^{58} + 16q^{57} + 32q^{56} - 16q^{55} - 24q^{54} + 20q^{53} + 48q^{52} - 20q^{51} - 39q^{50} + 35q^{49} + 66q^{48} - 61q^{47} - 40q^{46} + 109q^{45} + 65q^{44} - 186q^{43} - 53q^{42} + 279q^{41} + 126q^{40} - 429q^{39} - 166q^{38} + 607q^{37} + 259q^{36} - 866q^{35} - 271q^{34} + 1101q^{33} + 284q^{32} - 1365q^{31} - 191q^{30} + 1536q^{29} + 109q^{28} - 1665q^{27} + 31q^{26} + 1638q^{25} - 113q^{24} - 1561q^{23} + 209q^{22} + 1380q^{21} - 235q^{20} - 1181q^{19} + 272q^{18} + 921q^{17} - 255q^{16} - 682q^{15} + 243q^{14} + 447q^{13} - 190q^{12} - 265q^{11} + 142q^{10} + 123q^9 - 81q^8 - 46q^7 + 41q^6 + 9q^5 - 12q^4 - q^3 + 2q^2 - q + 1 \right) / \left( 4q(q^2+1)^4(q^4+1)^4(q^4-q^2+1)^2(q^8+1)^2(q^8-q^6+q^4-q^2+1)^2 \right)$$

[Open code](#)

$$q^6 - q^5 - 2q^4 + q^3 + 2q^2 + \frac{2911q + 43811}{10800(q^2+1)} + \frac{-667q - 5755}{5400(q^2+1)^2} + \frac{157 - 15q}{900(q^2+1)^3} - \frac{259(q+1)}{450(q^2+1)^4} + \frac{q^2+q-1}{9(q^4-q^2+1)^2} - \frac{3(11q^3-5q^2-9q-1)}{16(q^4+1)} + \frac{-q^3-2q^2-11q-3}{27(q^4-q^2+1)} + \frac{11q^3-5q^2-13q+3}{8(q^4+1)^2} + \frac{q^3+5q^2+q-3}{4(q^4+1)^3} + \frac{-q^3-q^2+q+1}{2(q^4+1)^4} + \frac{4q^6+3q^5-4q^4-5q^3+5q+3}{4(q^8+1)} + \frac{2q^7-48q^6-67q^5+51q^4+110q^3+4q^2-58q-42}{25(q^8-q^6+q^4-q^2+1)} + \frac{-q^7-3q^6-q^5+3q^4+3q^3-q^2-3q-1}{4(q^8+1)^2} + \frac{-5q^7+8q^5+5q^4-5q^3-8q^2+5}{5(q^8-q^6+q^4-q^2+1)^2} + q - \frac{1}{4q} - 2$$

For  $q = 0.5$ , we obtain:

$$\text{sum } (-1)^n 0.5^{n^2} (1-0.5)(1-0.5^3)(1-0.5^{(2n-1)}) / ((1+0.5^2)^2 (1+0.5^4)^2 (1+0.5^{(2n)})^2), n = 0 \text{ to } 5$$

Sum:

$$\sum_{n=0}^5 \frac{(-1)^n 0.5^{n^2} ((1-0.5)(1-0.5^3)(1-0.5^{2n-1}))}{(1+0.5^2)^2 (1+0.5^4)^2 (0.5^{2n}+1)^2} = -\frac{141\,245\,053\,289\,140\,078\,593}{1\,567\,170\,868\,825\,081\,000\,000}$$

[Open code](#)

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Decimal approximation:

More digits

-0.09012741118333349726766616666134756886310924364046879548...

[Open code](#)

$$\left( \left( \left( -1 + \sum_{n=0}^5 (-1)^n 0.5^{n^2} (1-0.5)(1-0.5^3)(1-0.5^{2n-1}) \right) / \left( (1+0.5^2)^2 (1+0.5^4)^2 (1+0.5^{2n})^2 \right) \right) \right)^6$$

Input interpretation:

$$\left( -1 + \sum_{n=0}^5 (-1)^n \times 0.5^{n^2} \left( (1-0.5)(1-0.5^3) \times \frac{1-0.5^{2n-1}}{(1+0.5^2)^2 (1+0.5^4)^2 (1+0.5^{2n})^2} \right) \right)^6$$

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Result:

1.67828

$$-64/10^3 + \left( \left( \left( -1 + \sum_{n=0}^5 (-1)^n 0.5^{n^2} (1-0.5)(1-0.5^3)(1-0.5^{2n-1}) \right) / \left( (1+0.5^2)^2 (1+0.5^4)^2 (1+0.5^{2n})^2 \right) \right) \right)^6$$

Input interpretation:

$$-\frac{64}{10^3} + \left( -1 + \sum_{n=0}^5 (-1)^n \times 0.5^{n^2} \left( (1-0.5)(1-0.5^3) \times \frac{1-0.5^{2n-1}}{(1+0.5^2)^2 (1+0.5^4)^2 (1+0.5^{2n})^2} \right) \right)^6$$

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Result:

1.61428

This result is a golden number

$$24 \times 2 + 10^3 * \left( \left( \left( -1 + \sum_{n=0}^5 (-1)^n 0.5^{n^2} (1-0.5)(1-0.5^3)(1-0.5^{2n-1}) \right) / \left( (1+0.5^2)^2 (1+0.5^4)^2 (1+0.5^{2n})^2 \right) \right) \right)^6$$

Input interpretation:

$$24 \times 2 + 10^3 \left( -1 + \sum_{n=0}^5 (-1)^n \times 0.5^{n^2} \left( (1-0.5)(1-0.5^3) \times \frac{1-0.5^{2n-1}}{(1+0.5^2)^2 (1+0.5^4)^2 (1+0.5^{2n})^2} \right) \right)^6$$

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Result:

1726.28

This result is very near to the mass of candidate glueball  $f_0(1710)$  meson.

$$\left(\left(\left(\left(\left(24 \times 2 + 10^3 \times \left(-1 + \sum_{n=0}^5 (-1)^n \times 0.5^{n^2} \frac{(1-0.5)(1-0.5^3)(1-0.5^{2n-1})}{(1+0.5^2)^2 (1+0.5^4)^2 (1+0.5^{2n})^2}\right)\right)\right)\right)\right)\right)^{1/3}$$

Input interpretation:

$$\left(24 \times 2 + 10^3 \left(-1 + \sum_{n=0}^5 (-1)^n \times 0.5^{n^2} \frac{(1-0.5)(1-0.5^3) \times (1-0.5^{2n-1})}{(1+0.5^2)^2 (1+0.5^4)^2 (1+0.5^{2n})^2}\right)\right)^{1/3}$$

Enlarge Data Customize A [Plaintext](#) [Interactive](#)

Result:

11.996

This result is very near to the value of black hole entropy 12,1904

$$2 \left(\left(\left(\left(\left(24 \times 2 + 10^3 \times \left(-1 + \sum_{n=0}^5 (-1)^n \times 0.5^{n^2} \frac{(1-0.5)(1-0.5^3)(1-0.5^{2n-1})}{(1+0.5^2)^2 (1+0.5^4)^2 (1+0.5^{2n})^2}\right)\right)\right)\right)\right)\right)^{1/3}$$

Input interpretation:

$$2 \left(24 \times 2 + 10^3 \left(-1 + \sum_{n=0}^5 (-1)^n \times 0.5^{n^2} \frac{(1-0.5)(1-0.5^3) \times (1-0.5^{2n-1})}{(1+0.5^2)^2 (1+0.5^4)^2 (1+0.5^{2n})^2}\right)\right)^{1/3}$$

Enlarge Data Customize A [Plaintext](#) [Interactive](#)

Result:

23.992

This value is linked to the "Ramanujan function" (an elliptic modular function that satisfies the need for "conformal symmetry") that has 24 "modes" corresponding to the physical vibrations of a bosonic string.

$$\left(\left(\left(\left(\left(24 \times 2 + 10^3 \times \left(-1 + \sum_{n=0}^5 (-1)^n \times 0.5^{n^2} \frac{(1-0.5)(1-0.5^3)(1-0.5^{2n-1})}{(1+0.5^2)^2 (1+0.5^4)^2 (1+0.5^{2n})^2}\right)\right)\right)\right)\right)\right)^{1/15}$$

Input interpretation:

$$\left(24 \times 2 + 10^3 \left(-1 + \sum_{n=0}^5 (-1)^n \times 0.5^{n^2} \frac{(1-0.5)(1-0.5^3) \times (1-0.5^{2n-1})}{(1+0.5^2)^2 (1+0.5^4)^2 (1+0.5^{2n})^2}\right)\right)^{1/15}$$

Enlarge Data Customize A [Plaintext](#) [Interactive](#)

Result:

1.64364

$$1.64364 \approx \zeta(2) = \frac{\pi^2}{6} = 1.6449$$

For q =0.5 and n =2, we have the following develop of the above function:

$$\frac{((-1)^2 \cdot 0.5^{2^2} (1-0.5)(1-0.5^3)(1-0.5^{(2 \cdot 2-1)}))}{((1+0.5^2)^2 (1+0.5^4)^2 (1+0.5^{(2 \cdot 2)})^2)}$$

Input:

$$\frac{(-1)^2 \times 0.5^{2^2} ((1 - 0.5)(1 - 0.5^3)(1 - 0.5^{2 \times 2 - 1}))}{(1 + 0.5^2)^2 (1 + 0.5^4)^2 (1 + 0.5^{2 \times 2})^2}$$

[Open code](#)

Enlarge Data Customize A [Plaintext](#) [Interactive](#)

Result:

More digits

0.012015181810562612995534057302953748159145604099567773374...

[Open code](#)

0.012015181810562612995534057302953748159145604099567773374

$$\frac{((((10^5 \frac{((-1)^2 \cdot 0.5^{2^2} (1-0.5)(1-0.5^3)(1-0.5^{(2 \cdot 2-1)}))}{((1+0.5^2)^2 (1+0.5^4)^2 (1+0.5^{(2 \cdot 2)})^2)}))))^{1/14}}$$

Input:

$$\sqrt[14]{10^5 \times \frac{(-1)^2 \times 0.5^{2^2} ((1 - 0.5)(1 - 0.5^3)(1 - 0.5^{2 \times 2 - 1}))}{(1 + 0.5^2)^2 (1 + 0.5^4)^2 (1 + 0.5^{2 \times 2})^2}}$$

[Open code](#)

Enlarge Data Customize A [Plaintext](#) [Interactive](#)

Result:

Fewer digits

More digits

1.659513306462121259314750377488713863612510498425599719157...

1.659513... is very near to the 14th root of the following Ramanujan's class invariant

$$Q = (G_{505}/G_{101/5})^3 = 1164,2696 \text{ i.e. } 1,65578...$$

$$10^3 \frac{((-1)^2 \cdot 0.5^{2^2} (1-0.5)(1-0.5^3)(1-0.5^{(2 \cdot 2-1)}))}{((1+0.5^2)^2 (1+0.5^4)^2 (1+0.5^{(2 \cdot 2)})^2)}$$

Input:

$$10^3 \times \frac{(-1)^2 \times 0.5^{2^2} ((1 - 0.5)(1 - 0.5^3)(1 - 0.5^{2 \times 2 - 1}))}{(1 + 0.5^2)^2 (1 + 0.5^4)^2 (1 + 0.5^{2 \times 2})^2}$$

[Open code](#)

Enlarge Data Customize A [Plaintext](#) [Interactive](#)

Result:

More digits

12.01518181056261299553405730295374815914560409956777337436...

[Open code](#)

This result is very near to the value of black hole entropy 12,1904

$$\left( \frac{10^3 \times (-1)^2 \times 0.5^{2^2} (1-0.5)(1-0.5^3)(1-0.5^{2 \times 2-1})}{(1+0.5^2)^2 (1+0.5^4)^2 (1+0.5^{2 \times 2})^2} \right)^3$$

Input:

$$\left( 10^3 \times \frac{(-1)^2 \times 0.5^{2^2} (1-0.5)(1-0.5^3)(1-0.5^{2 \times 2-1})}{(1+0.5^2)^2 (1+0.5^4)^2 (1+0.5^{2 \times 2})^2} \right)^3$$

[Open code](#)

Enlarge Data Customize A [Plaintext](#) [Interactive](#)

Result:

More digits

1734.566843207654958658514074263974836550968921529874304506...

This result is very near to the mass of candidate glueball  $f_0(1710)$  meson.

$$2 * \left( \frac{10^3 \times (-1)^2 \times 0.5^{2^2} (1-0.5)(1-0.5^3)(1-0.5^{2 \times 2-1})}{(1+0.5^2)^2 (1+0.5^4)^2 (1+0.5^{2 \times 2})^2} \right)^{3/3}$$

Input:

$$2 \sqrt[3]{ \left( 10^3 \times \frac{(-1)^2 \times 0.5^{2^2} (1-0.5)(1-0.5^3)(1-0.5^{2 \times 2-1})}{(1+0.5^2)^2 (1+0.5^4)^2 (1+0.5^{2 \times 2})^2} \right)^3 }$$

[Open code](#)

Enlarge Data Customize A [Plaintext](#) [Interactive](#)

Result:

More digits

24.03036362112522599106811460590749631829120819913554674872...

[Open code](#)

This value is linked to the "Ramanujan function" (an elliptic modular function that satisfies the need for "conformal symmetry") that has 24 "modes" corresponding to the physical vibrations of a bosonic string.

$$\left( \frac{10^3 \times (-1)^2 \times 0.5^{2^2} (1-0.5)(1-0.5^3)(1-0.5^{2 \times 2-1})}{(1+0.5^2)^2 (1+0.5^4)^2 (1+0.5^{2 \times 2})^2} \right)^{3/15}$$

Input:

$$\sqrt[15]{ \left( 10^3 \times \frac{(-1)^2 \times 0.5^{2^2} (1-0.5)(1-0.5^3)(1-0.5^{2 \times 2-1})}{(1+0.5^2)^2 (1+0.5^4)^2 (1+0.5^{2 \times 2})^2} \right)^3 }$$

[Open code](#)

Enlarge Data Customize A [Plaintext](#) Interactive

Result:

More digits

1.64417...

$$1.64417... \approx \zeta(2) = \frac{\pi^2}{6} = 1.6449$$

$$\sqrt[15]{\left(6 \times \left( \frac{10^3 \times ((-1)^2 \times 0.5^2 \times (1-0.5)(1-0.5^3)(1-0.5^{2 \times 2-1}))}{(1+0.5^2)^2 (1+0.5^4)^2 (1+0.5^{2 \times 2})^2} \right)^3 \right)^{1/15}}$$

Input:

$$\sqrt[6]{\sqrt[15]{\left(10^3 \times \frac{(-1)^2 \times 0.5^2 \times (1-0.5)(1-0.5^3)(1-0.5^{2 \times 2-1})}{(1+0.5^2)^2 (1+0.5^4)^2 (1+0.5^{2 \times 2})^2}\right)^3}}$$

[Open code](#)

Enlarge Data Customize A [Plaintext](#) Interactive

Result:

Fewer digits

More digits

3.140860587175079239571433610911208148312482781128066729236...

3.1408605871750792395714336109112081483124827811280667

Multiplying by  $0.0864055^2$  and adding 1 to the result, we obtain:

$$1 + 0.0864055^2 \times \left( \frac{10^3 \times ((-1)^2 \times 0.5^2 \times (1-0.5)(1-0.5^3)(1-0.5^{2 \times 2-1}))}{(1+0.5^2)^2 (1+0.5^4)^2 (1+0.5^{2 \times 2})^2} \right)^3$$

Input interpretation:

$$1 + \frac{0.0864055^2}{\frac{(-1)^2 \times 0.5^2 \times (1-0.5)(1-0.5^3)(1-0.5^{2 \times 2-1})}{(1+0.5^2)^2 (1+0.5^4)^2 (1+0.5^{2 \times 2})^2}}$$

[Open code](#)

Enlarge Data Customize A [Plaintext](#) Interactive

Result:

More digits

1.621373071832061393893494897959183673469387755102040816326...

[Open code](#)

This result is a golden number

$$10^3 \times \left( \frac{3 + 0.0864055^2 \times \left( \frac{10^3 \times ((-1)^2 \times 0.5^2 \times (1-0.5)(1-0.5^3)(1-0.5^{2 \times 2-1}))}{(1+0.5^2)^2 (1+0.5^4)^2 (1+0.5^{2 \times 2})^2} \right)^3}{\left( \frac{10^3 \times ((-1)^2 \times 0.5^2 \times (1-0.5)(1-0.5^3)(1-0.5^{2 \times 2-1}))}{(1+0.5^2)^2 (1+0.5^4)^2 (1+0.5^{2 \times 2})^2} \right)^3} \right)^{1/15}$$

Input interpretation:



$$10^3 \left( 3 + \frac{0.0864055^2}{\frac{(-1)^2 \times 0.5^{2^2} ((1-0.5)(1-0.5^3)(1-0.5^{2 \times 2-1}))}{(1+0.5^2)^2 (1+0.5^4)^2 (1+0.5^{2 \times 2})^2}} \right)$$

[Open code](#)

Enlarge Data Customize A [Plaintext](#) [Interactive](#)

Result:

More digits

3621.373071832061393893494897959183673469387755102040816326...

This result is equal to the rest mass of double charmed Xi baryon  $3621.40 \pm 0.78$

$$\frac{1}{3} * 10^3 * \left( \frac{(((((3+0.0864055^2/((((((-1)^2 * 0.5^{2^2} * (1-0.5)(1-0.5^3)(1-0.5^{(2*2-1)})))))) / (((((1+0.5^2)^2 * (1+0.5^4)^2 * (1+0.5^{(2*2)})^2)))))))))) \right)$$

Input interpretation:

$$\frac{1}{3} \times 10^3 \left( 3 + \frac{0.0864055^2}{\frac{(-1)^2 \times 0.5^{2^2} ((1-0.5)(1-0.5^3)(1-0.5^{2 \times 2-1}))}{(1+0.5^2)^2 (1+0.5^4)^2 (1+0.5^{2 \times 2})^2}} \right)$$

[Open code](#)

Enlarge Data Customize A [Plaintext](#) [Interactive](#)

Result:

More digits

1207.124357277353797964498299319727891156462585034013605442...

[Open code](#)

This result is very near to the rest mass of Sigma baryon  $1197.449 \pm 0.030$

$$\frac{8}{21} * 10^3 * \left( \frac{(((((3+0.0864055^2/((((((-1)^2 * 0.5^{2^2} * (1-0.5)(1-0.5^3)(1-0.5^{(2*2-1)})))))) / (((((1+0.5^2)^2 * (1+0.5^4)^2 * (1+0.5^{(2*2)})^2)))))))))) \right)$$

Input interpretation:

$$\frac{8}{21} \times 10^3 \left( 3 + \frac{0.0864055^2}{\frac{(-1)^2 \times 0.5^{2^2} ((1-0.5)(1-0.5^3)(1-0.5^{2 \times 2-1}))}{(1+0.5^2)^2 (1+0.5^4)^2 (1+0.5^{2 \times 2})^2}} \right)$$

[Open code](#)

Enlarge Data Customize A [Plaintext](#) [Interactive](#)

Result:

More digits

1379.570694031261483387998056365403304178814382896015549076...

[Open code](#)

This result is very near to the rest mass of Sigma baryon  $1382.8 \pm 0.4$

$$1.616255^2 * 10^3 * \left( \frac{(((((3+0.0864055^2/((((( (-1)^2 * 0.5^2^2 (1-0.5)(1-0.5^3)(1-0.5^{(2*2-1)})))))) / (((1+0.5^2)^2 (1+0.5^4)^2 (1+0.5^{(2*2)})^2)))))))))) \right)$$

Where 1.616255 is the Planck length without exponent

Input interpretation:

$$1.616255^2 \times 10^3 \left( 3 + \frac{0.0864055^2}{\frac{(-1)^2 \times 0.5^2^2 ((1-0.5)(1-0.5^3)(1-0.5^{2 \times 2 - 1}))}{(1+0.5^2)^2 (1+0.5^4)^2 (1+0.5^{2 \times 2})^2}} \right)$$

[Open code](#)

Enlarge Data Customize A [Plaintext](#) [Interactive](#)

Result:

More digits

9460.041262984932827049714012924505739795918367346938775510...

[Open code](#)

This result is practically equal to the rest mass of Upsilon meson  $9460.30 \pm 0.26$

$$1/8 * 1.616255^2 * 10^3 * \left( \frac{(((((3+0.0864055^2/((((( (-1)^2 * 0.5^2^2 (1-0.5)(1-0.5^3)(1-0.5^{(2*2-1)})))))) / (((1+0.5^2)^2 (1+0.5^4)^2 (1+0.5^{(2*2)})^2)))))))))) \right)$$

Input interpretation:

$$\frac{1}{8} \times 1.616255^2 \times 10^3 \left( 3 + \frac{0.0864055^2}{\frac{(-1)^2 \times 0.5^2^2 ((1-0.5)(1-0.5^3)(1-0.5^{2 \times 2 - 1}))}{(1+0.5^2)^2 (1+0.5^4)^2 (1+0.5^{2 \times 2})^2}} \right)$$

[Open code](#)

Enlarge Data Customize A [Plaintext](#) [Interactive](#)

Result:

More digits

1182.505157873116603381214251615563217474489795918367346938...

[Open code](#)

$$1/8 * 1.618034^2 * 10^3 * \left( \frac{(((((3+0.0864055^2/((((( (-1)^2 * 0.5^2^2 (1-0.5)(1-0.5^3)(1-0.5^{(2*2-1)})))))) / (((1+0.5^2)^2 (1+0.5^4)^2 (1+0.5^{(2*2)})^2)))))))))) \right)$$

Input interpretation:

$$\frac{1}{8} \times 1.618034^2 \times 10^3 \left( 3 + \frac{0.0864055^2}{\frac{(-1)^2 \times 0.5^2^2 ((1-0.5)(1-0.5^3)(1-0.5^{2 \times 2 - 1}))}{(1+0.5^2)^2 (1+0.5^4)^2 (1+0.5^{2 \times 2})^2}} \right)$$

[Open code](#)

Enlarge Data Customize A [Plaintext](#) [Interactive](#)

Result:

More digits

1185.109739980005001788487305808553890306122448979591836734...

[Open code](#)

These results 1182.505 and 1185.109 are very near to the rest mass of Sigma baryon  
1189.37±0.07

$$10^3 * \left( \frac{3 + 0.0864055^2 / \left( \frac{(-1)^2 \cdot 0.5^{2^2} (1-0.5)(1-0.5^3)(1-0.5^{(2*2-1)})}{(1+0.5^2)^2 (1+0.5^4)^2 (1+0.5^{(2*2)})^2} \right)}{0.0864055^2} \right)$$

Input interpretation:

$$10^3 \left( 3 + \frac{0.0864055^2}{\frac{(-1)^2 \times 0.5^{2^2} ((1-0.5)(1-0.5^3)(1-0.5^{2 \times 2 - 1}))}{(1+0.5^2)^2 (1+0.5^4)^2 (1+0.5^{2 \times 2})^2}} \right)$$

[Open code](#)

Enlarge Data Customize A [Plaintext](#) [Interactive](#)

Result:

More digits

3621.373071832061393893494897959183673469387755102040816326...

[Open code](#)

3621.373071832061393893494897959183673469387755102040816326

$$1/(24\pi) * 1.618034^2 *$$

3621.373071832061393893494897959183673469387755102040816326

Input interpretation:

$$\frac{1}{24 \pi} \times 1.618034^2 \times 3621.373071832061393893494897959183673469387755102040816326$$

[Open code](#)

Enlarge Data Customize A [Plaintext](#) [Interactive](#)

Result:

More digits

125.7440...

This result is in the range of the Higgs boson mass 125,18

Series representations:

More

$$\frac{1}{24 \pi} 1.61803^2 \times \frac{3621.3730718320613938934948979591836734693877551020408163260000}{98.7591} = \frac{\sum_{k=0}^{\infty} \frac{(-1)^k}{1+2^k}}$$

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$$\frac{1}{24\pi} 1.61803^2 \times \frac{3621.3730718320613938934948979591836734693877551020408163260000}{197.518} = -1 + \sum_{k=1}^{\infty} \frac{2^k}{\binom{2k}{k}}$$

[Open code](#)

$$\frac{1}{24\pi} 1.61803^2 \times \frac{3621.3730718320613938934948979591836734693877551020408163260000}{395.037} = \sum_{k=0}^{\infty} \frac{2^{-k}(-6+50k)}{\binom{3k}{k}}$$

[Open code](#)

Integral representations:  
 More

$$\frac{1}{24\pi} 1.61803^2 \times \frac{3621.3730718320613938934948979591836734693877551020408163260000}{197.518} = \int_0^{\infty} \frac{1}{1+t^2} dt$$

[Open code](#)

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$$\frac{1}{24\pi} 1.61803^2 \times \frac{3621.3730718320613938934948979591836734693877551020408163260000}{98.7591} = \int_0^1 \sqrt{1-t^2} dt$$

[Open code](#)

$$\frac{1}{24\pi} 1.61803^2 \times \frac{3621.3730718320613938934948979591836734693877551020408163260000}{197.518} = \int_0^{\infty} \frac{\sin(t)}{t} dt$$

[Open code](#)

$$10^3 * \left( \frac{(((((3+0.0864055^2/(((((-1)^2 0.5^2^2 (1-0.5)(1-0.5^3)(1-0.5^{(2*2-1))})))))) / (((1+0.5^2)^2 (1+0.5^4)^2 (1+0.5^{(2*2)})^2)))))) \right)$$

Input interpretation:

$$10^3 \left( 3 + \frac{0.0864055^2}{\frac{(-1)^2 \times 0.5^{2^2} ((1-0.5)(1-0.5^3)(1-0.5^{2 \times 2 - 1}))}{(1+0.5^2)^2 (1+0.5^4)^2 (1+0.5^{2 \times 2})^2}} \right)$$

[Open code](#)

Enlarge Data Customize A [Plaintext](#) [Interactive](#)

Result:

More digits

3621.373071832061393893494897959183673469387755102040816326...

3621.373071832061393893494897959183673469387755102040816326

$$1/(22\pi) * 1.618034^2 *$$

3621.373071832061393893494897959183673469387755102040816326

Input interpretation:

$$\frac{1}{22\pi} \times 1.618034^2 \times$$

3621.373071832061393893494897959183673469387755102040816326

[Open code](#)

Enlarge Data Customize A [Plaintext](#) [Interactive](#)

Result:

More digits

137.1753...

137.1753...

This result is practically equal to the inverse of fine-structure constant 137,035... and

very near to the masses of two Pions  $\pi^\pm$  : 139.57018(35) MeV/c<sup>2</sup>  $\pi^0$  :

134.9766(6) MeV/c<sup>2</sup> (note that the mean of masses is 137,27339, very near to the above result)

Series representations:

More

$$\frac{1}{22\pi} 1.61803^2 \times$$

3621.3730718320613938934948979591836734693877551020408163260000 =  
107.737

$$\sum_{k=0}^{\infty} \frac{(-1)^k}{1+2^k}$$

[Open code](#)

Enlarge Data Customize A [Plaintext](#) [Interactive](#)

$$\frac{\frac{1}{22\pi} 1.61803^2 \times 3621.3730718320613938934948979591836734693877551020408163260000}{215.474} = \frac{-1 + \sum_{k=1}^{\infty} \frac{2^k}{\binom{2k}{k}}}{-1 + \sum_{k=1}^{\infty} \frac{2^k}{\binom{2k}{k}}}$$

[Open code](#)

$$\frac{\frac{1}{22\pi} 1.61803^2 \times 3621.3730718320613938934948979591836734693877551020408163260000}{430.949} = \frac{\sum_{k=0}^{\infty} \frac{2^{-k} (-6+50k)}{\binom{3k}{k}}}{\sum_{k=0}^{\infty} \frac{2^{-k} (-6+50k)}{\binom{3k}{k}}}$$

[Open code](#)

Integral representations:

More

$$\frac{\frac{1}{22\pi} 1.61803^2 \times 3621.3730718320613938934948979591836734693877551020408163260000}{215.474} = \frac{\int_0^{\infty} \frac{1}{1+t^2} dt}{\int_0^{\infty} \frac{1}{1+t^2} dt}$$

[Open code](#)

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$$\frac{\frac{1}{22\pi} 1.61803^2 \times 3621.3730718320613938934948979591836734693877551020408163260000}{107.737} = \frac{\int_0^1 \sqrt{1-t^2} dt}{\int_0^1 \sqrt{1-t^2} dt}$$

[Open code](#)

$$\frac{\frac{1}{22\pi} 1.61803^2 \times 3621.3730718320613938934948979591836734693877551020408163260000}{215.474} = \frac{\int_0^{\infty} \frac{\sin(t)}{t} dt}{\int_0^{\infty} \frac{\sin(t)}{t} dt}$$

[Open code](#)

$$10^3 * ((((((3+0.0864055^2/(((((-1)^2 0.5^2^2 (1-0.5)(1-0.5^3)(1-0.5^(2*2-1)))) / (((1+0.5^2)^2 (1+0.5^4)^2 (1+0.5^(2*2))^2))))))))))$$

Input interpretation:

$$10^3 \left( 3 + \frac{0.0864055^2}{\frac{(-1)^2 \times 0.5^2 \times ((1-0.5)(1-0.5^3)(1-0.5^{2 \times 2-1}))}{(1+0.5^2)^2 (1+0.5^4)^2 (1+0.5^{2 \times 2})^2}} \right)$$

[Open code](#)

Enlarge Data Customize A [Plaintext](#) [Interactive](#)

Result:

More digits

3621.373071832061393893494897959183673469387755102040816326...

[Open code](#)

3621.373071832061393893494897959183673469387755102040816326

$$1/(33\pi) * 1.618034^2 *$$

3621.373071832061393893494897959183673469387755102040816326

Input interpretation:

$$\frac{1}{33\pi} \times 1.618034^2 \times$$

3621.373071832061393893494897959183673469387755102040816326

[Open code](#)

Enlarge Data Customize A [Plaintext](#) [Interactive](#)

Result:

More digits

91.45022...

This result is very near to the value of Z boson mass  $91.1876 \pm 0.0021 \text{ GeV}/c^2$

Series representations:

More

$$\frac{1}{33\pi} 1.61803^2 \times$$

3621.3730718320613938934948979591836734693877551020408163260000 =  
71.8248

$$\sum_{k=0}^{\infty} \frac{(-1)^k}{1+2k}$$

[Open code](#)

Enlarge Data Customize A [Plaintext](#) [Interactive](#)

$$\frac{1}{33\pi} 1.61803^2 \times$$

3621.3730718320613938934948979591836734693877551020408163260000 =  
143.65

$$-1 + \sum_{k=1}^{\infty} \frac{2^k}{\binom{2k}{k}}$$

[Open code](#)

$$\frac{\frac{1}{33\pi} 1.61803^2 \times 3621.3730718320613938934948979591836734693877551020408163260000}{287.299} = \frac{\sum_{k=0}^{\infty} \frac{2^{-k(-6+50k)}}{\binom{3k}{k}}}{}$$

[Open code](#)

Integral representations:

More

$$\frac{\frac{1}{33\pi} 1.61803^2 \times 3621.3730718320613938934948979591836734693877551020408163260000}{143.65} = \frac{\int_0^{\infty} \frac{1}{1+t^2} dt}{}$$

[Open code](#)

Enlarge Data Customize A [Plaintext](#) [Interactive](#)

$$\frac{\frac{1}{33\pi} 1.61803^2 \times 3621.3730718320613938934948979591836734693877551020408163260000}{71.8248} = \frac{\int_0^1 \sqrt{1-t^2} dt}{}$$

[Open code](#)

$$\frac{\frac{1}{33\pi} 1.61803^2 \times 3621.3730718320613938934948979591836734693877551020408163260000}{143.65} = \frac{\int_0^{\infty} \frac{\sin(t)}{t} dt}{}$$

[Open code](#)

$$10^3 * \left( \frac{(((((3+0.0864055^2)/(((((-1)^2 0.5^2^2 (1-0.5)(1-0.5^3)(1-0.5^{(2*2-1))}))))))}{(((1+0.5^2)^2 (1+0.5^4)^2 (1+0.5^{(2*2)})^2)} \right)$$

Input interpretation:

$$10^3 \left( 3 + \frac{0.0864055^2}{\frac{(-1)^2 \times 0.5^{2^2} ((1-0.5)(1-0.5^3)(1-0.5^{2 \times 2 - 1}))}{(1+0.5^2)^2 (1+0.5^4)^2 (1+0.5^{2 \times 2})^2}} \right)$$

[Open code](#)

Enlarge Data Customize A [Plaintext](#) [Interactive](#)

Result:

More digits



3621.373071832061393893494897959183673469387755102040816326...

[Open code](#)

3621.373071832061393893494897959183673469387755102040816326

$$1/(39\pi) * 1.6449^2 *$$

3621.373071832061393893494897959183673469387755102040816326

Input interpretation:

$$\frac{1}{39\pi} \times 1.6449^2 \times$$

3621.373071832061393893494897959183673469387755102040816326

[Open code](#)

Enlarge Data Customize A [Plaintext](#) [Interactive](#)

Result:

More digits

79.9720...

This result is very near to the value of W boson mass  $80.379 \pm 0.012 \text{ GeV}/c^2$

Series representations:

More

$$\frac{1}{39\pi} 1.6449^2 \times$$

3621.3730718320613938934948979591836734693877551020408163260000 =  
62.8098

$$\sum_{k=0}^{\infty} \frac{(-1)^k}{1+2k}$$

[Open code](#)

Enlarge Data Customize A [Plaintext](#) [Interactive](#)

$$\frac{1}{39\pi} 1.6449^2 \times$$

3621.3730718320613938934948979591836734693877551020408163260000 =  
125.62

$$-1 + \sum_{k=1}^{\infty} \frac{2^k}{\binom{2k}{k}}$$

[Open code](#)

$$\frac{1}{39\pi} 1.6449^2 \times$$

3621.3730718320613938934948979591836734693877551020408163260000 =  
251.239

$$\sum_{k=0}^{\infty} \frac{2^{-k}(-6+50k)}{\binom{3k}{k}}$$

[Open code](#)

Integral representations:

More

$$\frac{1}{39\pi} 1.6449^2 \times \frac{3621.3730718320613938934948979591836734693877551020408163260000}{125.62} = \frac{\int_0^\infty \frac{1}{1+t^2} dt}{125.62}$$

[Open code](#)

Enlarge Data Customize A [Plaintext](#) [Interactive](#)

$$\frac{1}{39\pi} 1.6449^2 \times \frac{3621.3730718320613938934948979591836734693877551020408163260000}{62.8098} = \frac{\int_0^1 \sqrt{1-t^2} dt}{62.8098}$$

[Open code](#)

$$\frac{1}{39\pi} 1.6449^2 \times \frac{3621.3730718320613938934948979591836734693877551020408163260000}{125.62} = \frac{\int_0^\infty \frac{\sin(t)}{t} dt}{125.62}$$

We note that:

$$1 / [16\pi * ((((-1)^2 0.5^2^2 (1-0.5)(1-0.5^3)(1-0.5^{2*2-1}))) / (((1+0.5^2)^2 (1+0.5^4)^2 (1+0.5^{2*2})^2)))]$$

Input:

$$\frac{1}{16\pi \times \frac{(-1)^2 \times 0.5^{2^2} ((1-0.5)(1-0.5^3)(1-0.5^{2 \times 2 - 1}))}{(1+0.5^2)^2 (1+0.5^4)^2 (1+0.5^{2 \times 2})^2}}$$

[Open code](#)

Enlarge Data Customize A [Plaintext](#) [Interactive](#)

Result:

More digits

1.65577...

1.65577... is practically equal to the 14th root of the following Ramanujan's class

invariant  $Q = (G_{505}/G_{101/5})^3 = 1164,2696$  i.e. 1,65578...

Series representations:

More

$$\frac{1}{(16\pi)\left(\frac{(-1)^2 0.5^{2^2} ((1-0.5)(1-0.5^3)(1-0.5^{2 \times 2-1}))}{(1+0.5^2)^2 (1+0.5^4)^2 (1+0.5^{2 \times 2})^2}\right)} = \frac{1.30044}{\sum_{k=0}^{\infty} \frac{(-1)^k}{1+2k}}$$

Open code

Enlarge Data Customize A [Plaintext](#) [Interactive](#)

$$\frac{1}{(16\pi)\left(\frac{(-1)^2 0.5^{2^2} ((1-0.5)(1-0.5^3)(1-0.5^{2 \times 2-1}))}{(1+0.5^2)^2 (1+0.5^4)^2 (1+0.5^{2 \times 2})^2}\right)} = \frac{2.60088}{-1 + \sum_{k=1}^{\infty} \frac{2^k}{\binom{2k}{k}}}$$

Open code

$$\frac{1}{(16\pi)\left(\frac{(-1)^2 0.5^{2^2} ((1-0.5)(1-0.5^3)(1-0.5^{2 \times 2-1}))}{(1+0.5^2)^2 (1+0.5^4)^2 (1+0.5^{2 \times 2})^2}\right)} = \frac{5.20175}{\sum_{k=0}^{\infty} \frac{2^{-k} (-6+50k)}{\binom{3k}{k}}}$$

Open code

Integral representations:

More

$$\frac{1}{(16\pi)\left(\frac{(-1)^2 0.5^{2^2} ((1-0.5)(1-0.5^3)(1-0.5^{2 \times 2-1}))}{(1+0.5^2)^2 (1+0.5^4)^2 (1+0.5^{2 \times 2})^2}\right)} = \int_0^{\infty} \frac{1}{1+t^2} dt$$

Open code

Enlarge Data Customize A [Plaintext](#) [Interactive](#)

$$\frac{1}{(16\pi)\left(\frac{(-1)^2 0.5^{2^2} ((1-0.5)(1-0.5^3)(1-0.5^{2 \times 2-1}))}{(1+0.5^2)^2 (1+0.5^4)^2 (1+0.5^{2 \times 2})^2}\right)} = \int_0^1 \sqrt{1-t^2} dt$$

Open code

$$\frac{1}{(16\pi)\left(\frac{(-1)^2 0.5^{2^2} ((1-0.5)(1-0.5^3)(1-0.5^{2 \times 2-1}))}{(1+0.5^2)^2 (1+0.5^4)^2 (1+0.5^{2 \times 2})^2}\right)} = \int_0^{\infty} \frac{\sin(t)}{t} dt$$

$$16 + 10^3 * 1 / [16\pi * (((-1)^2 0.5^{2^2} (1-0.5)(1-0.5^3)(1-0.5^{(2*2-1)}))) / (((1+0.5^2)^2 (1+0.5^4)^2 (1+0.5^{(2*2)})^2)))]$$

Input:

$$16 + 10^3 \times \frac{1}{16\pi \times \frac{(-1)^2 \times 0.5^{2^2} ((1-0.5)(1-0.5^3)(1-0.5^{2 \times 2-1}))}{(1+0.5^2)^2 (1+0.5^4)^2 (1+0.5^{2 \times 2})^2}}$$

Open code

### Enlarge Data Customize A Plaintext Interactive

Result:

More digits

1671.77...

This result is very near to the rest mass of Omega baryon  $1672.45 \pm 0.29$

Series representations:

More

$$16 + \frac{10^3}{16\pi \left( (-1)^2 0.5^{2^2} \left( (1-0.5)(1-0.5^3)(1-0.5^2 \times 2^{-1}) \right) \right)} \frac{1}{(1+0.5^2)^2 (1+0.5^4)^2 (1+0.5^2 \times 2)^2} = 16 + \frac{1300.44}{\sum_{k=0}^{\infty} \frac{(-1)^k}{1+2k}}$$

[Open code](#)

### Enlarge Data Customize A Plaintext Interactive

$$16 + \frac{10^3}{16\pi \left( (-1)^2 0.5^{2^2} \left( (1-0.5)(1-0.5^3)(1-0.5^2 \times 2^{-1}) \right) \right)} \frac{1}{(1+0.5^2)^2 (1+0.5^4)^2 (1+0.5^2 \times 2)^2} = 16 + \frac{2600.88}{-1 + \sum_{k=1}^{\infty} \frac{2^k}{\binom{2k}{k}}}$$

[Open code](#)

$$16 + \frac{10^3}{16\pi \left( (-1)^2 0.5^{2^2} \left( (1-0.5)(1-0.5^3)(1-0.5^2 \times 2^{-1}) \right) \right)} \frac{1}{(1+0.5^2)^2 (1+0.5^4)^2 (1+0.5^2 \times 2)^2} = 16 + \frac{5201.75}{\sum_{k=0}^{\infty} \frac{2^{-k}(-6+50k)}{\binom{3k}{k}}}$$

Integral representations:

More

$$16 + \frac{10^3}{16\pi \left( (-1)^2 0.5^{2^2} \left( (1-0.5)(1-0.5^3)(1-0.5^2 \times 2^{-1}) \right) \right)} \frac{1}{(1+0.5^2)^2 (1+0.5^4)^2 (1+0.5^2 \times 2)^2} = 16 + \frac{2600.88}{\int_0^{\infty} \frac{1}{1+t^2} dt}$$

[Open code](#)

### Enlarge Data Customize A Plaintext Interactive

$$16 + \frac{10^3}{16\pi \left( (-1)^2 0.5^{2^2} \left( (1-0.5)(1-0.5^3)(1-0.5^2 \times 2^{-1}) \right) \right)} \frac{1}{(1+0.5^2)^2 (1+0.5^4)^2 (1+0.5^2 \times 2)^2} = 16 + \frac{1300.44}{\int_0^1 \sqrt{1-t^2} dt}$$

[Open code](#)

$$16 + \frac{10^3}{16\pi \left( (-1)^2 0.5^{2^2} \left( (1-0.5)(1-0.5^3)(1-0.5^2 \times 2^{-1}) \right) \right)} \frac{1}{(1+0.5^2)^2 (1+0.5^4)^2 (1+0.5^2 \times 2)^2} = 16 + \frac{2600.88}{\int_0^{\infty} \frac{\sin(t)}{t} dt}$$

[Open code](#)

$$(1/1.65577) * 1/[16*(((1-0.5)^2 * 0.5^2 * (1-0.5)(1-0.5^3)(1-0.5^{(2*2-1))})) / (((1+0.5^2)^2 (1+0.5^4)^2 (1+0.5^{(2*2)})^2)))]$$

Input interpretation:

$$\frac{1}{1.65577} \times \frac{1}{16 \times \frac{(-1)^2 \times 0.5^2 \times ((1-0.5)(1-0.5^3)(1-0.5^{2 \times 2-1}))}{(1+0.5^2)^2 (1+0.5^4)^2 (1+0.5^{2 \times 2})^2}}$$

[Open code](#)

Enlarge Data Customize A [Plaintext](#) [Interactive](#)

Result:

More digits

3.141591121829778191859191722009107791639699539261158844278...

[Open code](#)

This result is a very good approximation to  $\pi$

$$(2/1.65577) * 1/[16*(((1-0.5)^2 * 0.5^2 * (1-0.5)(1-0.5^3)(1-0.5^{(2*2-1))})) / (((1+0.5^2)^2 (1+0.5^4)^2 (1+0.5^{(2*2)})^2)))]$$

Input interpretation:

$$\frac{2}{1.65577} \times \frac{1}{16 \times \frac{(-1)^2 \times 0.5^2 \times ((1-0.5)(1-0.5^3)(1-0.5^{2 \times 2-1}))}{(1+0.5^2)^2 (1+0.5^4)^2 (1+0.5^{2 \times 2})^2}}$$

[Open code](#)

Enlarge Data Customize A [Plaintext](#) [Interactive](#)

Result:

More digits

6.283182243659556383718383444018215583279399078522317688557...

[Open code](#)

6.283182243659556383718383444018215583279399078522317688557

This result is a very good approximation to the circle length with radius equal to 1:  
 $2\pi$

## Mock theta functions of order 7

From the following mock theta function of order 7:

$$F_1(q) = \sum_{n \geq 0} \frac{q^{n^2}}{(q^n; q)_n}$$

That is:

$$\text{Sum}_{\{n \geq 1\}} q^{n^2} / ((1-q^n)(1-q^{(n+1)}) \dots (1-q^{(2n-1)}))$$

sum  $q^{n^2} / ((1-q^n)(1-q^{(n+1)})(1-q^{(2n-1)}))$ ,  $n = 1$  to  $k$

Input interpretation:

$$\sum_{n=1}^k \frac{q^{n^2}}{(1-q^n)(1-q^{n+1})(1-q^{2n-1})}$$

Enlarge Data Customize A [Plaintext](#) [Interactive](#)

Result:

$$\sum_{n=1}^k \frac{q^{n^2}}{(1-q^n)(1-q^{n+1})(1-q^{2n-1})}$$

[Open code](#)

Sum:

$$\sum_{n=1}^k \frac{q^{n^2}}{(1-q^n)(1-q^{n+1})(1-q^{2n-1})}$$

From which, for  $q = 0.93201$ , we obtain:

sum  $0.93201^{n^2} / ((1-0.93201^n)(1-0.93201^{(n+1)})(1-0.93201^{(2n-1)}))$ ,  $n = 1$  to infinity

Input interpretation:

$$\sum_{n=1}^{\infty} \frac{0.93201^{n^2}}{(1-0.93201^n)(1-0.93201^{n+1})(1-0.93201^{2n-1})}$$

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Infinite sum:

$$\sum_{n=1}^{\infty} \frac{0.93201^{n^2}}{(1-0.93201^n)(1-0.93201^{n+1})(1-0.93201^{2n-1})} = 1748.11$$

1748

This result is very near to the mass of candidate glueball  $f_0(1710)$  meson.

For  $n = 123$ ,  $a(n) = 1748$  (OEIS), value that correspond the above result.

$(((((sum 0.93201^{n^2} / ((1-0.93201^n)(1-0.93201^{(n+1)})(1-0.93201^{(2n-1)}))$ ,  $n = 1$  to infinity))))))<sup>1/3</sup>

Input interpretation:

$$\sqrt[3]{\sum_{n=1}^{\infty} \frac{0.93201^{n^2}}{(1-0.93201^n)(1-0.93201^{n+1})(1-0.93201^{2n-1})}}$$

Enlarge Data Customize A [Plaintext](#) [Interactive](#)

Result:

12.0464

This result is very near to the value of black hole entropy 12,1904

$$2 * \left( \left( \left( \left( \sum_{n=1}^{\infty} \frac{0.93201^{n^2}}{(1-0.93201^n)(1-0.93201^{n+1})(1-0.93201^{2n-1})} \right) \right) \right) \right)^{1/3}$$

Input interpretation:

$$2 \sqrt[3]{\sum_{n=1}^{\infty} \frac{0.93201^{n^2}}{(1-0.93201^n)(1-0.93201^{n+1})(1-0.93201^{2n-1})}}$$

Enlarge Data Customize A [Plaintext](#) [Interactive](#)

Result:

24.0927

This value is linked to the "Ramanujan function" (an elliptic modular function that satisfies the need for "conformal symmetry") that has 24 "modes" corresponding to the physical vibrations of a bosonic string.

$$\left( \left( \left( \left( \sum_{n=1}^{\infty} \frac{0.93201^{n^2}}{(1-0.93201^n)(1-0.93201^{n+1})(1-0.93201^{2n-1})} \right) \right) \right) \right)^{1/15}$$

Input interpretation:

$$\sqrt[15]{\sum_{n=1}^{\infty} \frac{0.93201^{n^2}}{(1-0.93201^n)(1-0.93201^{n+1})(1-0.93201^{2n-1})}}$$

Enlarge Data Customize A [Plaintext](#) [Interactive](#)

Result:

1.64502

$$1.64502 \approx \zeta(2) = \frac{\pi^2}{6} = 1.6449$$

Continued fraction:

Linear form

$$1 + \frac{1}{1 + \frac{1}{1 + \frac{1}{1 + \frac{1}{4 + \frac{1}{2 + \frac{1}{6 + \frac{1}{1 + \frac{1}{10 + \frac{1}{1 + \frac{1}{1 + \frac{1}{\dots}}}}}}}}}}}}$$

[Open code](#)

Enlarge Data Customize A [Plaintext](#) [Interactive](#)

Possible closed forms:

[More](#)

$$\sigma_S - \frac{1}{60} \approx 1.6450212829$$

$$\sqrt{\frac{2}{\mathcal{D}_{\mathcal{D}_0}}} \approx 1.645007053$$

- $\sigma_S$  is Somos's quadratic recurrence constant
- $\mathcal{D}_{\mathcal{D}_0}$  is the Dottie number

We have also that:

$$\sum_{n=1}^k \frac{q^{n^2}}{(1-q^n)(1-q^{n+1})(1-q^{2n-1})}, n = 1 \text{ to } k$$

$$\sum_{n=1}^5 \frac{q^{n^2}}{(1-q^n)(1-q^{n+1})(1-q^{2n-1})}, n = 1 \text{ to } 5$$

Result:

$$\sum_{n=1}^5 \frac{q^{n^2}}{(1-q^n)(1-q^{n+1})(1-q^{2n-1})} = \frac{q}{(1-q)^2(1-q^2)} + \frac{q^4}{(1-q^2)(1-q^3)^2} + \frac{q^{25}}{q^{16}(1-q^5)(1-q^6)(1-q^9)} + \frac{q^{16}}{(1-q^4)(1-q^5)(1-q^7)} + \frac{q^9}{(1-q^3)(1-q^4)(1-q^5)}$$

[Open code](#)

Enlarge Data Customize A [Plaintext](#) [Interactive](#)

Result:

$$\sum_{n=1}^5 \frac{q^{n^2}}{(1-q^n)(1-q^{n+1})(1-q^{2n-1})} = \frac{q^4}{(1-q^2)(q^3-1)^2} - \frac{q^{25}}{(q^5-1)(q^6-1)(q^9-1)} - \frac{q^{16}}{(q^4-1)(q^5-1)(q^7-1)} - \frac{q^9}{(q^3-1)(q^4-1)(q^5-1)} - \frac{q}{(q-1)^3(q+1)}$$

[Open code](#)

For  $q = 0.5$ , we obtain:

$$\sum_{n=1}^5 \frac{0.5^{n^2}}{(1-0.5^n)(1-0.5^{n+1})(1-0.5^{2n-1})}, n = 1 \text{ to } 5$$

Sum:

$$\sum_{n=1}^5 \frac{0.5^{n^2}}{(1-0.5^n)(1-0.5^{n+1})(1-0.5^{2n-1})} = \frac{8047828109}{2897002080}$$

[Open code](#)

Enlarge Data Customize A [Plaintext](#) [Interactive](#)

Decimal approximation:

More digits

2.777984926058458335659876364327636243878706500618045810999...

[Open code](#)



$$\left(\left(\left(\left(\sum_{n=1}^5 \frac{0.5^{n^2}}{(1-0.5^n)(1-0.5^{n+1})(1-0.5^{2n-1})}\right), n = 1 \text{ to } 5\right)\right)\right)^{1/2}$$

Input interpretation:

$$\sqrt{\sum_{n=1}^5 \frac{0.5^{n^2}}{(1-0.5^n)(1-0.5^{n+1})(1-0.5^{2n-1})}}$$

Enlarge Data Customize A [Plaintext](#) [Interactive](#)

Result:

1.66673

1.66673

This result is a golden number, very near to the proton mass

$$8^2 + 10^3 \left(\left(\left(\left(\sum_{n=1}^5 \frac{0.5^{n^2}}{(1-0.5^n)(1-0.5^{n+1})(1-0.5^{2n-1})}\right), n = 1 \text{ to } 5\right)\right)\right)^{1/2}$$

Input interpretation:

$$8^2 + 10^3 \sqrt{\sum_{n=1}^5 \frac{0.5^{n^2}}{(1-0.5^n)(1-0.5^{n+1})(1-0.5^{2n-1})}}$$

Enlarge Data Customize A [Plaintext](#) [Interactive](#)

Result:

1730.73

This result is very near to the mass of candidate glueball  $f_0(1710)$  meson.

$$\left[\left(\left(8^2 + 10^3 \left(\left(\left(\left(\sum_{n=1}^5 \frac{0.5^{n^2}}{(1-0.5^n)(1-0.5^{n+1})(1-0.5^{2n-1})}\right), n = 1 \text{ to } 5\right)\right)\right)\right)^{1/2}\right)\right]^{1/3}$$

Input interpretation:

$$\sqrt[3]{8^2 + 10^3 \sqrt{\sum_{n=1}^5 \frac{0.5^{n^2}}{(1-0.5^n)(1-0.5^{n+1})(1-0.5^{2n-1})}}$$

Enlarge Data Customize A [Plaintext](#) [Interactive](#)

Result:

12.0063

This result is very near to the value of black hole entropy 12,1904

$$2 \cdot \left[\left(\left(8^2 + 10^3 \left(\left(\left(\left(\sum_{n=1}^5 \frac{0.5^{n^2}}{(1-0.5^n)(1-0.5^{n+1})(1-0.5^{2n-1})}\right), n = 1 \text{ to } 5\right)\right)\right)\right)^{1/2}\right)\right]^{1/3}$$

Input interpretation:

$$2 \sqrt[3]{8^2 + 10^3 \sqrt{\sum_{n=1}^5 \frac{0.5^{n^2}}{(1-0.5^n)(1-0.5^{n+1})(1-0.5^{2n-1})}}$$

Enlarge Data Customize A [Plaintext](#) [Interactive](#)

Result:

24.0126

This value is linked to the "Ramanujan function" (an elliptic modular function that satisfies the need for "conformal symmetry") that has 24 "modes" corresponding to the physical vibrations of a bosonic string.

$$[\left(\left(\left(8^2 + 10^3 \left(\sum_{n=1}^5 \frac{0.5^{n^2}}{(1-0.5^n)(1-0.5^{n+1})(1-0.5^{2n-1})}\right)\right)\right)^{1/2}\right)^{1/15}]$$

Input interpretation:

$$15 \sqrt{8^2 + 10^3 \sqrt{\sum_{n=1}^5 \frac{0.5^{n^2}}{(1-0.5^n)(1-0.5^{n+1})(1-0.5^{2n-1})}}}$$

Enlarge Data Customize A [Plaintext](#) [Interactive](#)

Result:

1.64392

$$1.64392 \approx \zeta(2) = \frac{\pi^2}{6} = 1.6449$$

For  $q = 0.5$  and  $n = 2$ , we obtain:

$$\left(\frac{0.5^{2^2}}{(1-0.5^2)(1-0.5^{2+1})(1-0.5^{2*2-1})}\right)$$

Input:

$$\frac{0.5^{2^2}}{(1-0.5^2)(1-0.5^{2+1})(1-0.5^{2*2-1})}$$

[Open code](#)

Enlarge Data Customize A [Plaintext](#) [Interactive](#)

Result:

More digits

0.108843537414965986394557823129251700680272108843537414965...

[Open code](#)

$$\left(\frac{27 * 0.0864055}{13 * \left(\frac{0.5^{2^2}}{(1-0.5^2)(1-0.5^{2+1})(1-0.5^{2*2-1})}\right)}\right)$$

Input interpretation:

$$13 \times \frac{27 \times 0.0864055}{\frac{0.5^{2^2}}{(1-0.5^2)(1-0.5^{2+1})(1-0.5^{2*2-1})}}$$

[Open code](#)

Enlarge Data Customize A [Plaintext](#) [Interactive](#)

Result:

More digits

1.648766487980769230769230769230769230769230769230769230769230769...

[Open code](#)

$$1.648766... \approx \zeta(2) = \frac{\pi^2}{6} = 1.6449$$

$$24 \times 4 + ((((((10^3((27 * 0.0864055)) / ((13 * ((0.5^2^2 / ((1 - 0.5^2)(1 - 0.5^{(2+1))}(1 - 0.5^{(2*2-1)))))$$

Input interpretation:

$$24 \times 4 + 10^3 \times \frac{27 \times 0.0864055}{13 \times \frac{0.5^{2^2}}{(1-0.5^2)(1-0.5^{2+1})(1-0.5^{2 \times 2-1})}}$$

[Open code](#)

Enlarge Data Customize A [Plaintext](#) [Interactive](#)

Result:

More digits

1744.766487980769230769230769230769230769230769230769230769230769...

[Open code](#)

This result is very near to the mass of candidate glueball  $f_0(1710)$  meson.

$$((((((((24 \times 4 + 10^3((27 * 0.0864055)) / ((13 * ((0.5^2^2 / ((1 - 0.5^2)(1 - 0.5^{(2+1))}(1 - 0.5^{(2*2-1)))))$$

Input interpretation:

$$\sqrt[3]{24 \times 4 + 10^3 \times \frac{27 \times 0.0864055}{13 \times \frac{0.5^{2^2}}{(1-0.5^2)(1-0.5^{2+1})(1-0.5^{2 \times 2-1})}}$$

[Open code](#)

Enlarge Data Customize A [Plaintext](#) [Interactive](#)

Result:

More digits

12.0387...

This result is very near to the value of black hole entropy 12,1904

$$2 * (((((((24 \times 4 + 10^3((27 * 0.0864055)) / ((13 * ((0.5^2^2 / ((1 - 0.5^2)(1 - 0.5^{(2+1))}(1 - 0.5^{(2*2-1)))))$$

Input interpretation:

$$2 \sqrt[3]{24 \times 4 + 10^3 \times \frac{27 \times 0.0864055}{13 \times \frac{0.5^{2^2}}{(1-0.5^2)(1-0.5^{2+1})(1-0.5^{2 \times 2-1})}}$$

[Open code](#)

Enlarge Data Customize A [Plaintext](#) [Interactive](#)

Result:

More digits

24.0774...

This value is linked to the "Ramanujan function" (an elliptic modular function that satisfies the need for "conformal symmetry") that has 24 "modes" corresponding to the physical vibrations of a bosonic string.

$$\left(\left(\left(\left(\left(24 \times 4 + 10^3 \left( \frac{27 \times 0.0864055}{13 \times \frac{0.5^{2^2}}{(1-0.5^2)(1-0.5^{2+1})(1-0.5^{2 \times 2-1})} \right) \right) \right) \right) \right) \right)^{1/15}$$

Input interpretation:

$$\sqrt[15]{24 \times 4 + 10^3 \times \frac{27 \times 0.0864055}{13 \times \frac{0.5^{2^2}}{(1-0.5^2)(1-0.5^{2+1})(1-0.5^{2 \times 2-1})}}}$$

[Open code](#)

Enlarge Data Customize A [Plaintext](#) [Interactive](#)

Result:

More digits

1.64481...

$$1.64481... \approx \zeta(2) = \frac{\pi^2}{6} = 1.6449$$

Now, we have that:

$$F_0(q) = \sum_{n \geq 0} \frac{q^{n^2}}{(q^{n+1}; q)_n}$$

That is:

$$\text{Sum}_{\{n \geq 0\}} \frac{q^{n^2}}{(1-q^{n+1})(1-q^{n+2}) \dots (1-q^{2n})}$$

$$\text{sum } \frac{q^{n^2}}{(1-q^{n+1})(1-q^{n+2})(1-q^{2n})}, n = 1 \text{ to } 5$$

Result:

$$\sum_{n=1}^5 \frac{q^{n^2}}{(1-q^{n+1})(1-q^{n+2})(1-q^{2n})} = \frac{q^4}{(1-q^3)(1-q^4)^2} + \frac{q}{(1-q^2)^2(1-q^3)} + \frac{q^9}{(1-q^6)(1-q^7)(1-q^{10})} + \frac{q^{16}}{(1-q^5)(1-q^6)(1-q^8)} + \frac{q^{25}}{(1-q^4)(1-q^5)(1-q^6)}$$

[Open code](#)

Enlarge Data Customize A [Plaintext](#) [Interactive](#)

Result:

$$\sum_{n=1}^5 \frac{q^{n^2}}{(1-q^{n+1})(1-q^{n+2})(1-q^{2n})} = \frac{q^4}{(1-q^3)(q^4-1)^2} + \frac{q}{(q^2-1)^2(1-q^3)} - \frac{q^{25}}{(q^6-1)(q^7-1)(q^{10}-1)} - \frac{q^{16}}{(q^5-1)(q^6-1)(q^8-1)} - \frac{q^9}{(q^4-1)(q^5-1)(q^6-1)}$$

[Open code](#)

For  $q = 0.5$ , we obtain:

$\text{sum } 0.5^{n^2}/((1-0.5^{(n+1)})(1-0.5^{(n+2)})(1-0.5^{(2n)})), n = 1 \text{ to } 5$

Sum:

$$\sum_{n=1}^5 \frac{0.5^{n^2}}{(1-0.5^{n+1})(1-0.5^{n+2})(1-0.5^{2n})} = \frac{5\,098\,953\,283}{4\,638\,179\,700}$$

[Open code](#)

Enlarge Data Customize A [Plaintext](#) [Interactive](#)

Decimal approximation:

More digits

1.099343624612043384175046085428729723430077536668102790411...

[Open code](#)

$(((((\text{sum } 0.5^{n^2}/((1-0.5^{(n+1)})(1-0.5^{(n+2)})(1-0.5^{(2n)})), n = 1 \text{ to } 5))))))^5$

Input interpretation:

$$\left( \sum_{n=1}^5 \frac{0.5^{n^2}}{(1-0.5^{n+1})(1-0.5^{n+2})(1-0.5^{2n})} \right)^5$$

Enlarge Data Customize A [Plaintext](#) [Interactive](#)

Result:

1.60571

This result is a golden number very near to the electric charge of positron

$16 \times 2^3 + 10^3 (((((\text{sum } 0.5^{n^2}/((1-0.5^{(n+1)})(1-0.5^{(n+2)})(1-0.5^{(2n)})), n = 1 \text{ to } 5))))))^5$

Input interpretation:

$$16 \times 2^3 + 10^3 \left( \sum_{n=1}^5 \frac{0.5^{n^2}}{(1-0.5^{n+1})(1-0.5^{n+2})(1-0.5^{2n})} \right)^5$$

Enlarge Data Customize A [Plaintext](#) [Interactive](#)

Result:

1733.71

This result is very near to the mass of candidate glueball  $f_0(1710)$  meson.

$(((((((((16 \times 2^3 + 10^3 (((((\text{sum } 0.5^{n^2}/((1-0.5^{(n+1)})(1-0.5^{(n+2)})(1-0.5^{(2n)})), n = 1 \text{ to } 5))))))^5))))))^1/3$

Input interpretation:

$$\sqrt[3]{16 \times 2^3 + 10^3 \left( \sum_{n=1}^5 \frac{0.5^{n^2}}{(1 - 0.5^{n+1})(1 - 0.5^{n+2})(1 - 0.5^{2n})} \right)^5}$$

Enlarge Data Customize A [Plaintext](#) [Interactive](#)

Result:

12.0132

This result is very near to the value of black hole entropy 12,1904

$$2 * (((((((((16 * 2^3 + 10^3 * (((((\sum_{n=1}^5 0.5^{n^2} / (((1 - 0.5^{n+1})(1 - 0.5^{n+2})(1 - 0.5^{2n}))), n = 1 \text{ to } 5)))))))))^5))))))^{1/3}$$

Input interpretation:

$$2 \sqrt[3]{16 \times 2^3 + 10^3 \left( \sum_{n=1}^5 \frac{0.5^{n^2}}{(1 - 0.5^{n+1})(1 - 0.5^{n+2})(1 - 0.5^{2n})} \right)^5}$$

Enlarge Data Customize A [Plaintext](#) [Interactive](#)

Result:

24.0264

This value is linked to the "Ramanujan function" (an elliptic modular function that satisfies the need for "conformal symmetry") that has 24 "modes" corresponding to the physical vibrations of a bosonic string.

$$((((((((((16 * 2^3 + 10^3 * (((((\sum_{n=1}^5 0.5^{n^2} / (((1 - 0.5^{n+1})(1 - 0.5^{n+2})(1 - 0.5^{2n}))), n = 1 \text{ to } 5)))))))))^5))))))^{1/15}$$

Input interpretation:

$$\sqrt[15]{16 \times 2^3 + 10^3 \left( \sum_{n=1}^5 \frac{0.5^{n^2}}{(1 - 0.5^{n+1})(1 - 0.5^{n+2})(1 - 0.5^{2n})} \right)^5}$$

Enlarge Data Customize A [Plaintext](#) [Interactive](#)

Result:

1.64411

$$1.64411 \approx \zeta(2) = \frac{\pi^2}{6} = 1.6449$$

For q = 0.5 and n = 2, we obtain:

$$0.5^{2^2} / (((1 - 0.5^{2+1})(1 - 0.5^{2+2})(1 - 0.5^{2*2})))$$

Input:

$$\frac{0.5^{2^2}}{(1 - 0.5^{2+1})(1 - 0.5^{2+2})(1 - 0.5^{2 \times 2})}$$

Open code

### Enlarge Data Customize A Plaintext Interactive

Result:

More digits

0.081269841269841269841269841269841269841269841269841269841269841...

Open code

$$\left( \left( \left( \left( 1 + 0.5^{2^2} / \left( (1 - 0.5^{2+1})(1 - 0.5^{2+2})(1 - 0.5^{2 \times 2}) \right) \right) \right) \right) \right)^{2\pi}$$

Input:

$$\left( 1 + \frac{0.5^{2^2}}{(1 - 0.5^{2+1})(1 - 0.5^{2+2})(1 - 0.5^{2 \times 2})} \right)^{2\pi}$$

Open code

### Enlarge Data Customize A Plaintext Interactive

Result:

More digits

1.63386...

This result is a golden number

Series representations:

More

$$\left( 1 + \frac{0.5^{2^2}}{(1 - 0.5^{2+1})(1 - 0.5^{2+2})(1 - 0.5^{2 \times 2})} \right)^{2\pi} = 1.08127^8 \sum_{k=0}^{\infty} (-1)^k / (1+2k)$$

Open code

### Enlarge Data Customize A Plaintext Interactive

$$\left( 1 + \frac{0.5^{2^2}}{(1 - 0.5^{2+1})(1 - 0.5^{2+2})(1 - 0.5^{2 \times 2})} \right)^{2\pi} = 0.731583 e^{0.312545 \times \sum_{k=1}^{\infty} 2^k / \binom{2k}{k}}$$

Open code

$$\left( 1 + \frac{0.5^{2^2}}{(1 - 0.5^{2+1})(1 - 0.5^{2+2})(1 - 0.5^{2 \times 2})} \right)^{2\pi} = 1.08127^2 \sum_{k=0}^{\infty} (2^{-k} (-6+50k)) / \binom{3k}{k}$$

Open code

•  $\binom{n}{m}$  is the binomial coefficient

Integral representations:

More

$$\left( 1 + \frac{0.5^{2^2}}{(1 - 0.5^{2+1})(1 - 0.5^{2+2})(1 - 0.5^{2 \times 2})} \right)^{2\pi} = e^{0.312545 \int_0^{\infty} 1/(1+t^2) dt}$$

Open code

Enlarge Data Customize A Plaintext Interactive

$$\left(1 + \frac{0.5^{2^2}}{(1 - 0.5^{2+1})(1 - 0.5^{2+2})(1 - 0.5^{2 \times 2})}\right)^{2\pi} = e^{0.625089} \int_0^1 \sqrt{1-t^2} dt$$

[Open code](#)

$$\left(1 + \frac{0.5^{2^2}}{(1 - 0.5^{2+1})(1 - 0.5^{2+2})(1 - 0.5^{2 \times 2})}\right)^{2\pi} = e^{0.312545} \int_0^\infty \sin(t)/t dt$$

[Open code](#)

$$24 \times 4 + 10^3 \left( \left( \left( \left( \left( 1 + 0.5^{2^2} / ((1 - 0.5^{2+1})(1 - 0.5^{2+2})(1 - 0.5^{2 \times 2})) \right) \right) \right) \right) \right)^{2\pi}$$

Input:

$$24 \times 4 + 10^3 \left(1 + \frac{0.5^{2^2}}{(1 - 0.5^{2+1})(1 - 0.5^{2+2})(1 - 0.5^{2 \times 2})}\right)^{2\pi}$$

[Open code](#)

Enlarge Data Customize A Plaintext Interactive

Result:

More digits

1729.86...

This result is very near to the mass of candidate glueball  $f_0(1710)$  meson. Furthermore, 1728 occurs in the algebraic formula for the j-invariant of an elliptic curve. As a consequence, it is sometimes called a Zagier as a pun on the Gross–Zagier theorem. The number 1728 is one less than the Hardy–Ramanujan number 1729

Series representations:

More

$$24 \times 4 + 10^3 \left(1 + \frac{0.5^{2^2}}{(1 - 0.5^{2+1})(1 - 0.5^{2+2})(1 - 0.5^{2 \times 2})}\right)^{2\pi} = 96 + 1000 \times 1.08127^8 \sum_{k=0}^\infty (-1)^k / (1+2k)$$

[Open code](#)

Enlarge Data Customize A Plaintext Interactive

$$24 \times 4 + 10^3 \left(1 + \frac{0.5^{2^2}}{(1 - 0.5^{2+1})(1 - 0.5^{2+2})(1 - 0.5^{2 \times 2})}\right)^{2\pi} = 96 + 731.583 e^{0.312545 \times \sum_{k=1}^\infty 2^k / \binom{2k}{k}}$$

[Open code](#)



$$24 \times 4 + 10^3 \left( 1 + \frac{0.5^{2^2}}{(1 - 0.5^{2+1})(1 - 0.5^{2+2})(1 - 0.5^{2 \times 2})} \right)^{2\pi} = 96 + 1000 \times 1.08127^{2 \sum_{k=0}^{\infty} (2^{-k} (-6+50k)) / \binom{3k}{k}}$$

[Open code](#)

•  $\binom{n}{m}$  is the binomial coefficient •

Integral representations:

More

$$24 \times 4 + 10^3 \left( 1 + \frac{0.5^{2^2}}{(1 - 0.5^{2+1})(1 - 0.5^{2+2})(1 - 0.5^{2 \times 2})} \right)^{2\pi} = 96 + 1000 e^{0.312545 \int_0^{\infty} 1/(1+t^2) dt}$$

[Open code](#)

Enlarge Data Customize A [Plaintext](#) [Interactive](#)

$$24 \times 4 + 10^3 \left( 1 + \frac{0.5^{2^2}}{(1 - 0.5^{2+1})(1 - 0.5^{2+2})(1 - 0.5^{2 \times 2})} \right)^{2\pi} = 96 + 1000 e^{0.625089 \int_0^1 \sqrt{1-t^2} dt}$$

[Open code](#)

$$24 \times 4 + 10^3 \left( 1 + \frac{0.5^{2^2}}{(1 - 0.5^{2+1})(1 - 0.5^{2+2})(1 - 0.5^{2 \times 2})} \right)^{2\pi} = 96 + 1000 e^{0.312545 \int_0^{\infty} \sin(t)/t dt}$$

[Open code](#)

$$((((((((24*4 + 10^3((((((1 + 0.5^2^2/((1-0.5^(2+1))(1-0.5^(2+2))(1-0.5^(2*2))))))^(2Pi))))))))))^(1/3$$

Input:

$$\sqrt[3]{24 \times 4 + 10^3 \left( 1 + \frac{0.5^{2^2}}{(1 - 0.5^{2+1})(1 - 0.5^{2+2})(1 - 0.5^{2 \times 2})} \right)^{2\pi}}$$

[Open code](#)

Enlarge Data Customize A [Plaintext](#) [Interactive](#)

Result:

More digits

12.0043...

This result is very near to the value of black hole entropy 12,1904

$$2*((((((((24*4 + 10^3((((((1 + 0.5^2^2/((1-0.5^(2+1))(1-0.5^(2+2))(1-0.5^(2*2))))))^(2Pi))))))))))^(1/3$$

Input:

$$2 \sqrt[3]{24 \times 4 + 10^3 \left( 1 + \frac{0.5^{2^2}}{(1 - 0.5^{2+1})(1 - 0.5^{2+2})(1 - 0.5^{2 \times 2})} \right)^{2\pi}}$$

[Open code](#)

Enlarge Data Customize A [Plaintext](#) [Interactive](#)

Result:

More digits

24.0086...

This value is linked to the "Ramanujan function" (an elliptic modular function that satisfies the need for "conformal symmetry") that has 24 "modes" corresponding to the physical vibrations of a bosonic string.

Series representations:

More

$$2 \sqrt[3]{24 \times 4 + 10^3 \left( 1 + \frac{0.5^{2^2}}{(1 - 0.5^{2+1})(1 - 0.5^{2+2})(1 - 0.5^{2 \times 2})} \right)^{2\pi}} = 4 \sqrt[3]{12 + 125 \times 1.08127^{8 \sum_{k=0}^{\infty} (-1)^k / (1+2k)}}$$

[Open code](#)

Enlarge Data Customize A [Plaintext](#) [Interactive](#)

$$2 \sqrt[3]{24 \times 4 + 10^3 \left( 1 + \frac{0.5^{2^2}}{(1 - 0.5^{2+1})(1 - 0.5^{2+2})(1 - 0.5^{2 \times 2})} \right)^{2\pi}} = 2 \sqrt[3]{96 + 731.583 e^{0.312545 \times \sum_{k=1}^{\infty} 2^k / \binom{2k}{k}}}$$

[Open code](#)

$$2 \sqrt[3]{24 \times 4 + 10^3 \left( 1 + \frac{0.5^{2^2}}{(1 - 0.5^{2+1})(1 - 0.5^{2+2})(1 - 0.5^{2 \times 2})} \right)^{2\pi}} = 4 \sqrt[3]{12 + 125 \times 1.08127^{2 \sum_{k=0}^{\infty} (2^{-k} (-6+50k)) / \binom{3k}{k}}}$$

[Open code](#)

•  $\binom{n}{m}$  is the binomial coefficient

Integral representations:

More

$$\frac{2^3 \sqrt[3]{24 \times 4 + 10^3 \left( 1 + \frac{0.5^{2^2}}{(1-0.5^{2+1})(1-0.5^{2+2})(1-0.5^{2 \times 2})} \right)^{2\pi}}}{4 \sqrt[3]{12 + 125 e^{0.312545 \int_0^\infty \frac{1}{(1+t^2)} dt}}} =$$

[Open code](#)

Enlarge Data Customize A [Plaintext](#) [Interactive](#)

$$\frac{2^3 \sqrt[3]{24 \times 4 + 10^3 \left( 1 + \frac{0.5^{2^2}}{(1-0.5^{2+1})(1-0.5^{2+2})(1-0.5^{2 \times 2})} \right)^{2\pi}}}{4 \sqrt[3]{12 + 125 e^{0.625089 \int_0^1 \sqrt{1-t^2} dt}}} =$$

[Open code](#)

$$\frac{2^3 \sqrt[3]{24 \times 4 + 10^3 \left( 1 + \frac{0.5^{2^2}}{(1-0.5^{2+1})(1-0.5^{2+2})(1-0.5^{2 \times 2})} \right)^{2\pi}}}{4 \sqrt[3]{12 + 125 e^{0.312545 \int_0^\infty \frac{\sin(t)}{t} dt}}} =$$

[Open code](#)

((((((((24\*4 + 10^3((((((1 + 0.5^2^2/((1-0.5^(2+1))(1-0.5^(2+2)))(1-0.5^(2\*2))))))^(2Pi))))))))^(1/15

Input:

$$15 \sqrt[3]{24 \times 4 + 10^3 \left( 1 + \frac{0.5^{2^2}}{(1-0.5^{2+1})(1-0.5^{2+2})(1-0.5^{2 \times 2})} \right)^{2\pi}}$$

[Open code](#)

Enlarge Data Customize A [Plaintext](#) [Interactive](#)

Result:

- More digits

1.643870...

$$1.643870... \approx \zeta(2) = \frac{\pi^2}{6} = 1.6449$$

Now, we have that:

$$F_2(q) = \sum_{n \geq 0} \frac{q^{n(n+1)}}{(q^{n+1}; q)_{n+1}}$$

That is:

Sum\_{n >= 0} q^(n(n+1))/((1-q^(n+1))(1-q^(n+2))...(1-q^(2n+1)))

sum q^(n(n+1))/((1-q^(n+1))(1-q^(n+2))(1-q^(2n+1))), n = 0 to 5

Result:

$$\sum_{n=0}^5 \frac{q^{n(n+1)}}{(1-q^{n+1})(1-q^{n+2})(1-q^{2n+1})} = \frac{1}{(1-q)^2(1-q^2)} + \frac{q^2}{(1-q^2)(1-q^3)^2} + \frac{q^{30}}{(1-q^6)(1-q^7)(1-q^{11})} + \frac{q^{20}}{(1-q^5)(1-q^6)(1-q^9)} + \frac{q^{12}}{(1-q^4)(1-q^5)(1-q^7)} + \frac{q^6}{(1-q^3)(1-q^4)(1-q^5)}$$

[Open code](#)

Enlarge Data Customize A [Plaintext](#) [Interactive](#)

Result:

$$\sum_{n=0}^5 \frac{q^{n(n+1)}}{(1-q^{n+1})(1-q^{n+2})(1-q^{2n+1})} = \frac{q^2}{(1-q^2)(q^3-1)^2} - \frac{q^{30}}{(q^6-1)(q^7-1)(q^{11}-1)} - \frac{q^{20}}{(q^5-1)(q^6-1)(q^9-1)} - \frac{q^{12}}{(q^4-1)(q^5-1)(q^7-1)} - \frac{q^6}{(q^3-1)(q^4-1)(q^5-1)} - \frac{1}{(q-1)^3(q+1)}$$

[Open code](#)

For q = 0.5, we obtain:

sum 0.5^(n(n+1))/((1-0.5^(n+1))(1-0.5^(n+2))(1-0.5^(2n+1))), n = 0 to 5

Sum:

$$\sum_{n=0}^5 \frac{0.5^{n(n+1)}}{(1-0.5^{n+1})(1-0.5^{n+2})(1-0.5^{2n+1})} = \frac{13731035602583}{2372065303104}$$

[Open code](#)

Enlarge Data Customize A [Plaintext](#) [Interactive](#)

Decimal approximation:

More digits

5.788641478215231617620274222175362884979462245421215507942...

[Open code](#)

1.0864055 (((((sum 0.5^(n(n+1))/((1-0.5^(n+1))(1-0.5^(n+2))(1-0.5^(2n+1))), n = 0 to 5))))))

Input interpretation:

$$1.0864055 \sum_{n=0}^5 \frac{0.5^{n(n+1)}}{(1-0.5^{n+1})(1-0.5^{n+2})(1-0.5^{2n+1})}$$

Enlarge Data Customize A [Plaintext](#) [Interactive](#)

Result:

6.28881

$6.28881 \approx 2\pi$

$$1/2 * 1.0864055 \left( \left( \left( \left( \sum_{n=0}^5 0.5^{n(n+1)} \right) / \left( (1-0.5^{n+1})(1-0.5^{n+2})(1-0.5^{2n+1}) \right) \right) \right) \right), n = 0 \text{ to } 5 \right)$$

Input interpretation:

$$\frac{1}{2} \times 1.0864055 \sum_{n=0}^5 \frac{0.5^{n(n+1)}}{(1-0.5^{n+1})(1-0.5^{n+2})(1-0.5^{2n+1})}$$

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Result:

3.14441

$$1.0864055^3 \left( \left( \left( \left( \sum_{n=0}^5 0.5^{n(n+1)} \right) / \left( (1-0.5^{n+1})(1-0.5^{n+2})(1-0.5^{2n+1}) \right) \right) \right) \right), n = 0 \text{ to } 5 \right)^{1/7}$$

Input interpretation:

$$1.0864055^3 \sqrt[7]{\sum_{n=0}^5 \frac{0.5^{n(n+1)}}{(1-0.5^{n+1})(1-0.5^{n+2})(1-0.5^{2n+1})}}$$

Enlarge Data Customize A [Plaintext](#) [Interactive](#)

Result:

1.64784

$1.64784 \approx \zeta(2) = \frac{\pi^2}{6} = 1.6449$

$$24 \times 4 + 10^3 * 1.0864055^3 \left( \left( \left( \left( \sum_{n=0}^5 0.5^{n(n+1)} \right) / \left( (1-0.5^{n+1})(1-0.5^{n+2})(1-0.5^{2n+1}) \right) \right) \right) \right), n = 0 \text{ to } 5 \right)^{1/7}$$

Input interpretation:

$$24 \times 4 + 10^3 \times 1.0864055^3 \sqrt[7]{\sum_{n=0}^5 \frac{0.5^{n(n+1)}}{(1-0.5^{n+1})(1-0.5^{n+2})(1-0.5^{2n+1})}}$$

Enlarge Data Customize A [Plaintext](#) [Interactive](#)

Result:

1743.84

This result is very near to the mass of candidate glueball  $f_0(1710)$  meson.

$$\left(\left(\left(\left(\left(24 \times 4 + 10^3 \times 1.0864055^3 \left(\left(\left(\left(\sum_{n=0}^5 \frac{0.5^{n(n+1)}}{(1-0.5^{n+1})(1-0.5^{n+2})(1-0.5^{2n+1})}\right)\right)\right)\right)\right)\right)^{1/7}\right)\right)\right)^{1/3}$$

Input interpretation:

$$\sqrt[3]{24 \times 4 + 10^3 \times 1.0864055^3 \sqrt[7]{\sum_{n=0}^5 \frac{0.5^{n(n+1)}}{(1-0.5^{n+1})(1-0.5^{n+2})(1-0.5^{2n+1})}}}$$

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Result:

12.0366

This result is very near to the value of black hole entropy 12,1904

$$2 \left(\left(\left(\left(\left(24 \times 4 + 10^3 \times 1.0864055^3 \left(\left(\left(\left(\sum_{n=0}^5 \frac{0.5^{n(n+1)}}{(1-0.5^{n+1})(1-0.5^{n+2})(1-0.5^{2n+1})}\right)\right)\right)\right)\right)\right)^{1/7}\right)\right)\right)^{1/3}$$

Input interpretation:

$$2 \sqrt[3]{24 \times 4 + 10^3 \times 1.0864055^3 \sqrt[7]{\sum_{n=0}^5 \frac{0.5^{n(n+1)}}{(1-0.5^{n+1})(1-0.5^{n+2})(1-0.5^{2n+1})}}}$$

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Result:

24.0731

This value is linked to the "Ramanujan function" (an elliptic modular function that satisfies the need for "conformal symmetry") that has 24 "modes" corresponding to the physical vibrations of a bosonic string.

$$\left(\left(\left(\left(\left(24 \times 4 + 10^3 \times 1.0864055^3 \left(\left(\left(\left(\sum_{n=0}^5 \frac{0.5^{n(n+1)}}{(1-0.5^{n+1})(1-0.5^{n+2})(1-0.5^{2n+1})}\right)\right)\right)\right)\right)\right)^{1/7}\right)\right)\right)^{1/15}$$

Input interpretation:

$$\sqrt[15]{24 \times 4 + 10^3 \times 1.0864055^3 \sqrt[7]{\sum_{n=0}^5 \frac{0.5^{n(n+1)}}{(1-0.5^{n+1})(1-0.5^{n+2})(1-0.5^{2n+1})}}}$$

Enlarge Data Customize A [Plaintext](#) [Interactive](#)

Result:

1.64475

$$1.64475 \approx \zeta(2) = \frac{\pi^2}{6} = 1.6449$$

For  $q = 0.5$  and  $n = 2$ , we obtain:

$$0.5^{(2(2+1))} / ((1-0.5^{(2+1)})(1-0.5^{(2+2)})(1-0.5^{(2*2+1)}))$$

Input:

$$\frac{0.5^{2(2+1)}}{(1-0.5^{2+1})(1-0.5^{2+2})(1-0.5^{2 \times 2+1})}$$

[Open code](#)

### Enlarge Data Customize A Plaintext Interactive

Result:

More digits

0.019662058371735791090629800307219662058371735791090629800...

[Open code](#)

$$(8/250) 1 / ((((((( 0.5^{2(2+1)} / ((1-0.5^{(2+1)})(1-0.5^{(2+2)})(1-0.5^{(2*2+1)})))))$$

Input:

$$\frac{8}{250} \times \frac{1}{\frac{0.5^{2(2+1)}}{(1-0.5^{2+1})(1-0.5^{2+2})(1-0.5^{2 \times 2+1})}}$$

[Open code](#)

### Enlarge Data Customize A Plaintext Interactive

Result:

1.6275

This result is a golden number

$$27 \times 4 + 10^3 (8/250) 1 / ((((((( 0.5^{2(2+1)} / ((1-0.5^{(2+1)})(1-0.5^{(2+2)})(1-0.5^{(2*2+1)}))$$

Input:

$$27 \times 4 + 10^3 \times \frac{8}{250} \times \frac{1}{\frac{0.5^{2(2+1)}}{(1-0.5^{2+1})(1-0.5^{2+2})(1-0.5^{2 \times 2+1})}}$$

[Open code](#)

### Enlarge Data Customize A Plaintext Interactive

Result:

1735.5

[Open code](#)

This result is very near to the mass of candidate glueball  $f_0(1710)$  meson.

$$((((((((((27 \times 4 + 10^3 (8/250) 1 / ((((((( 0.5^{2(2+1)} / ((1-0.5^{(2+1)})(1-0.5^{(2+2)})(1-0.5^{(2*2+1)}))$$

Input:

$$\sqrt[3]{27 \times 4 + 10^3 \times \frac{8}{250} \times \frac{1}{\frac{0.5^{2(2+1)}}{(1-0.5^{2+1})(1-0.5^{2+2})(1-0.5^{2 \times 2+1})}}$$

[Open code](#)

Enlarge Data Customize A [Plaintext](#) [Interactive](#)

Result:

More digits

12.0173...

This result is very near to the value of black hole entropy 12,1904

$$2 * ((((((((((27 * 4 + 10^3 (8/250) 1 / ((((((( 0.5^{(2(2+1)) / ((1 - 0.5^{(2+1)) (1 - 0.5^{(2+2)) (1 - 0.5^{(2 * 2 + 1))}))))))))))))))))))))))^{1/3}$$

Input:

$$2 \sqrt[3]{27 \times 4 + 10^3 \times \frac{8}{250} \times \frac{1}{\frac{0.5^{2(2+1)}}{(1-0.5^{2+1})(1-0.5^{2+2})(1-0.5^{2 \times 2 + 1})}}}$$

[Open code](#)

Enlarge Data Customize A [Plaintext](#) [Interactive](#)

Result:

More digits

24.0347...

This value is linked to the "Ramanujan function" (an elliptic modular function that satisfies the need for "conformal symmetry") that has 24 "modes" corresponding to the physical vibrations of a bosonic string.

$$(((((((((((27 * 4 + 10^3 (8/250) 1 / ((((((( 0.5^{(2(2+1)) / ((1 - 0.5^{(2+1)) (1 - 0.5^{(2+2)) (1 - 0.5^{(2 * 2 + 1))}))))))))))))))))))))))^{1/15}$$

Input:

$$15 \sqrt{27 \times 4 + 10^3 \times \frac{8}{250} \times \frac{1}{\frac{0.5^{2(2+1)}}{(1-0.5^{2+1})(1-0.5^{2+2})(1-0.5^{2 \times 2 + 1})}}}$$

[Open code](#)

Enlarge Data Customize A [Plaintext](#) [Interactive](#)

Result:

More digits

1.64423...

$$1.64423... \approx \zeta(2) = \frac{\pi^2}{6} = 1.6449$$



## Conclusion

Also in this work we obtain several interesting results, such as the particle type solutions and the values close to the mass of the "glueball" candlestick  $f_0(1710)$ . Furthermore, the mathematical connection between  $\zeta(2) = \pi^2/6 = 1.6449\dots$ ,  $\pi$  and various Ramanujan's Mock Theta functions, appears even more evident here.

## References

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