ON THE NATURE OF MAXWELL AND DIRAC FIELD

Vu B Ho

Advanced Study, 9 Adela Court, Mulgrave, Victoria 3170, Australia

Email: vubho@bigpond.net.au

Abstract: In this work we discuss the nature of Maxwell and Dirac field by examining the mathematical structures of the subfields that are coupled to form these two physical fields. We show that while Maxwell and Dirac field are hyperbolic fields which are described by wave equations the subfields are elliptic fields which are described by elliptic equations. Therefore, it is reasonable to suggest that the subfields are more fundamental and should be used to represent quantum particles in stable states with invariant physical properties. Furthermore, since the subfields are described by elliptic equations therefore they comply with the Euclidean relativity rather than the pseudo-Euclidean relativity as Maxwell and Dirac fields do, and this results in profound implications such as if quantum particles possess physical properties that are represented by subfields which are described by elliptic equations, hence acting in accordance with the Euclidean relativity, then they can be used to explain physical phenomena that require physical transmissions with speeds greater than the speed of light in vacuum, such as the Einstein-Podosky-Rosen paradox in quantum entanglement. As a further discussion, we also show that it is possible to formulate a Dirac-like elliptic field that complies with the Euclidean relativity.

1. Introduction

We have shown in our previous works that both Maxwell field equations of the electromagnetic field and Dirac equation of massive quantum particles can be formulated from a general system of linear first order partial differential equations, and, as a consequence, the field equations of the two physical fields have many common features that specify characteristics that are not typical in classical physics [1] [2] [3] [4]. In this work we discuss further the similarity between the Maxwell and Dirac field by examining the subfields that are coupled to form these two physical fields. We show that the subfields have the mathematical structures and physical properties that are essentially different from the coupled field of Maxwell and Dirac. In particular, we show that the subfields of both Maxwell and Dirac field satisfy elliptic equations rather than hyperbolic equations therefore while Maxwell and Dirac fields are described by wave equations therefore they comply with the laws of the pseudo-Euclidean relativity [5] [6], the Maxwell and Dirac subfields are described by elliptic equations therefore they comply with those of the Euclidean relativity instead [7] [8]. The fact that the subfields of Maxwell and Dirac fields are Euclidean relativistic has profound implications, such as they can be used to explain the stability of elementary particles because if elementary particles are represented by subfields which are described by elliptic equations then since elliptic equations are used to describe equilibrium states of physical systems therefore elementary particles associated with those subfields are also stable. Furthermore, if quantum particles possess physical properties that are represented by subfields which are described by elliptic equations, hence acting in accordance with the Euclidean relativity, then they can be used to explain physical phenomena that require physical transmissions with speeds greater than the speed of light in vacuum, such as the Einstein-Podosky-Rosen paradox in quantum entanglement [9] [10] [11].

The system of linear first order partial differential equations that we need to use in this work is given as follows [12] [13]

$$\sum_{i=1}^{n}\sum_{j=1}^{n} a_{ij}^{r}\frac{\partial \psi_i}{\partial x_j} = k_1\sum_{l=1}^{n} b_l^r\,\psi_l + k_2 c^r, \qquad r = 1, 2, \ldots, n \tag{1}$$

Equation (1) can be rewritten in a matrix form as

$$\left(\sum_{i=1}^{n} A_i\frac{\partial}{\partial x_i}\right)\psi = k_1\sigma\psi + k_2 J \tag{2}$$

where $\psi = (\psi_1, \psi_2, \ldots, \psi_n)^T$, $\partial\psi/\partial x_i = (\partial\psi_1/\partial x_i, \partial\psi_2/\partial x_i, \ldots, \partial\psi_n/\partial x_i)^T$, A_i, σ and J are matrices representing the quantities a_{ij}^k, b_l^r and c^r, and k_1 and k_2 are undetermined constants. Now, if we apply the operator $\sum_{i=1}^{n} A_i\,\partial/\partial x_i$ on the left on both sides of Equation (2) then we obtain

$$\left(\sum_{i=1}^{n} A_i\frac{\partial}{\partial x_i}\right)\left(\sum_{j=1}^{n} A_j\frac{\partial}{\partial x_j}\right)\psi = \left(\sum_{i=1}^{n} A_i\frac{\partial}{\partial x_i}\right)(k_1\sigma\psi + k_2 J) \tag{3}$$

If we assume further that the coefficients a_{ij}^k and b_l^r are constants and $A_i\sigma = \sigma A_i$, then Equation (3) can be rewritten in the following form

$$\left(\sum_{i=1}^{n} {A_i}^2\frac{\partial^2}{\partial x_i^2} + \sum_{i=1}^{n}\sum_{j>i}^{n}\left(A_iA_j + A_jA_i\right)\frac{\partial^2}{\partial x_i\partial x_j}\right)\psi = k_1^2\sigma^2\psi + k_1k_2\sigma J + k_2\sum_{i=1}^{n} A_i\frac{\partial J}{\partial x_i} \tag{4}$$

In order for the above systems of partial differential equations to be applied to physical phenomena, the matrices A_i must be determined. For the case of Maxwell and Dirac field, the matrices A_i must take a form so that Equation (4) reduces to a wave equation

$$\left(\sum_{i=1}^{n} {A_i}^2\frac{\partial^2}{\partial x_i^2}\right)\psi = k_1^2\sigma^2\psi + k_1k_2\sigma J + k_2\sum_{i=1}^{n} A_i\frac{\partial J}{\partial x_i} \tag{5}$$

It is seen from Equation (4) that for Dirac field we simply require the matrices A_i to satisfy the conditions $A_iA_j + A_jA_i = 0$ and $A_i^2 = \pm 1$ for all values of i and j. For the case of Maxwell field the conditions required for the matrices A_i can be determined from the classical form of Maxwell field equations [14] [15]. Furthermore, in order to reduce Equation

(4) to Equation (5) for the case of Maxwell field, we will also need an extra condition on the components of the wavefuction ψ in the form of a divergence or Gauss's law

$$\sum_{i=1}^{n}\frac{\partial\psi_i}{\partial x_i}=\rho \tag{6}$$

In this work we only discuss Maxwell and Dirac fields therefore we will set $\sigma=1$.

2. Maxwell field as coupling of two elliptic fields

In this section we show that Maxwell field of electromagnetism is a coupled field that is formed from the coupling of two subfields that satisfy elliptic equations [16]. In order to distinguish a field that satisfies an elliptic equation from a field that satisfies a hyperbolic equation, or wave equation, we refer to the former as an elliptic field and the latter as a hyperbolic field. From the general equation given in Equation (2), the two subfields that are coupled to form the Maxwell field can be rewritten in the following simple form

$$\left(A_0\frac{\partial}{\partial t}+A_1\frac{\partial}{\partial x_1}+A_2\frac{\partial}{\partial x_2}+A_3\frac{\partial}{\partial x_3}\right)\psi=k_1\psi+k_2J \tag{7}$$

where $\psi=(\psi_1,\psi_2,\psi_3)^T$ and $J=(j_1,j_2,j_3)^T$, and the matrices A_i are given as follows

$$A_0=\mp\begin{pmatrix}1&0&0\\0&1&0\\0&0&1\end{pmatrix},\ A_1=\begin{pmatrix}0&0&0\\0&0&-1\\0&1&0\end{pmatrix},\ A_2=\begin{pmatrix}0&0&1\\0&0&0\\-1&0&0\end{pmatrix},\ A_3=\begin{pmatrix}0&-1&0\\1&0&0\\0&0&0\end{pmatrix} \tag{8}$$

In Equation (8), the negative sign in front of the matrix A_0 together with other matrices form one subfield and the positive sign in front of the matrix A_0 together with other matrices form another subfield. Then we obtain the following results

$$A_0^2=\begin{pmatrix}1&0&0\\0&1&0\\0&0&1\end{pmatrix},\ A_1^2=\begin{pmatrix}0&0&0\\0&-1&0\\0&0&-1\end{pmatrix},\ A_2^2=\begin{pmatrix}-1&0&0\\0&0&0\\0&0&-1\end{pmatrix},\ A_3^2=\begin{pmatrix}-1&0&0\\0&-1&0\\0&0&0\end{pmatrix} \tag{9}$$

$$A_0A_i+A_iA_0=\mp 2A_i\qquad\text{for}\qquad i=1,2,3 \tag{10}$$

$$A_1A_2+A_2A_1=\begin{pmatrix}0&1&0\\1&0&0\\0&0&0\end{pmatrix},\ A_1A_3+A_3A_1=\begin{pmatrix}0&0&1\\0&0&0\\1&0&0\end{pmatrix},\ A_2A_3+A_3A_2=\begin{pmatrix}0&0&0\\0&0&1\\0&1&0\end{pmatrix} \tag{11}$$

Using the matrices A_i given in Equation (8) with the negative time for the matrix A_0 we obtain the following system of differential equations from Equation (7)

$$-\frac{\partial\psi_1}{\partial t}+\frac{\partial\psi_3}{\partial x_2}-\frac{\partial\psi_2}{\partial x_3}=k_1\psi_1+k_2j_1 \tag{12}$$

$$-\frac{\partial\psi_2}{\partial t}-\frac{\partial\psi_3}{\partial x_1}+\frac{\partial\psi_1}{\partial x_3}=k_1\psi_2+k_2j_2 \tag{13}$$

$$-\frac{\partial\psi_3}{\partial t}+\frac{\partial\psi_2}{\partial x_1}-\frac{\partial\psi_1}{\partial x_2}=k_1\psi_3+k_2j_3 \tag{14}$$

Using the matrices A_i given in Equation (8) with the positive time for the matrix A_0 we obtain the following system of differential equations from Equation (7)

$$\frac{\partial\psi_4}{\partial t}+\frac{\partial\psi_6}{\partial x_2}-\frac{\partial\psi_5}{\partial x_3}=k_1\psi_4+k_2j_4 \tag{15}$$

$$\frac{\partial\psi_5}{\partial t}-\frac{\partial\psi_6}{\partial x_1}+\frac{\partial\psi_4}{\partial x_3}=k_1\psi_5+k_2j_5 \tag{16}$$

$$\frac{\partial\psi_6}{\partial t}+\frac{\partial\psi_5}{\partial x_1}-\frac{\partial\psi_4}{\partial x_2}=k_1\psi_6+k_2j_6 \tag{17}$$

In Equations (15-17) we have used different subscripts for the field components ψ_i with positive time because it is a different field from the field with negative time given in Equations (12-14). However, for simplicity, we have used the same k_1 and k_2 for the system of equations given in Equations (15-17) even though they may have different dimensional values from those given in Equations (12-14).

On the other hand, using the matrices A_i given in Equation (8) with negative time for the matrix A_0 we obtain the following system of differential equations from Equation (4)

$$\begin{aligned}\frac{\partial^2\psi_1}{\partial t^2}-\frac{\partial^2\psi_1}{\partial x_2^2}-\frac{\partial^2\psi_1}{\partial x_3^2}-2\frac{\partial}{\partial t}\left(\frac{\partial\psi_3}{\partial x_2}-\frac{\partial\psi_2}{\partial x_3}\right)+\frac{\partial}{\partial x_1}\left(\frac{\partial\psi_2}{\partial x_2}+\frac{\partial\psi_3}{\partial x_3}\right)\\ =k_1^2\psi_1+k_1k_2j_1+k_2\left(-\frac{\partial j_1}{\partial t}+\frac{\partial j_3}{\partial x_2}-\frac{\partial j_2}{\partial x_3}\right)\end{aligned} \tag{18}$$

$$\begin{aligned}\frac{\partial^2\psi_2}{\partial t^2}-\frac{\partial^2\psi_2}{\partial x_1^2}-\frac{\partial^2\psi_2}{\partial x_3^2}+2\frac{\partial}{\partial t}\left(\frac{\partial\psi_3}{\partial x_1}-\frac{\partial\psi_1}{\partial x_3}\right)+\frac{\partial}{\partial x_2}\left(\frac{\partial\psi_1}{\partial x_1}+\frac{\partial\psi_3}{\partial x_3}\right)\\ =k_1^2\psi_2+k_1k_2j_2+k_2\left(-\frac{\partial j_2}{\partial t}+\frac{\partial j_1}{\partial x_3}-\frac{\partial j_3}{\partial x_1}\right)\end{aligned} \tag{19}$$

$$\begin{aligned}\frac{\partial^2\psi_3}{\partial t^2}-\frac{\partial^2\psi_3}{\partial x_1^2}-\frac{\partial^2\psi_3}{\partial x_2^2}-2\frac{\partial}{\partial t}\left(\frac{\partial\psi_2}{\partial x_1}-\frac{\partial\psi_1}{\partial x_2}\right)+\frac{\partial}{\partial x_3}\left(\frac{\partial\psi_1}{\partial x_1}+\frac{\partial\psi_2}{\partial x_2}\right)\\ =k_1^2\psi_3+k_1k_2j_3+k_2\left(-\frac{\partial j_3}{\partial t}+\frac{\partial j_2}{\partial x_1}-\frac{\partial j_1}{\partial x_2}\right)\end{aligned} \tag{20}$$

Similarly, using the matrices A_i given in Equation (8) with positive time for the matrix A_0 we obtain the following system of differential equations from Equation (4)

$$\begin{aligned}\frac{\partial^2\psi_4}{\partial t^2}-\frac{\partial^2\psi_4}{\partial x_2^2}-\frac{\partial^2\psi_4}{\partial x_3^2}+2\frac{\partial}{\partial t}\left(\frac{\partial\psi_6}{\partial x_2}-\frac{\partial\psi_5}{\partial x_3}\right)+\frac{\partial}{\partial x_1}\left(\frac{\partial\psi_5}{\partial x_2}+\frac{\partial\psi_6}{\partial x_3}\right)\\ =k_1^2\psi_4+k_1k_2j_4+k_2\left(\frac{\partial j_4}{\partial t}+\frac{\partial j_6}{\partial x_2}-\frac{\partial j_5}{\partial x_3}\right)\end{aligned} \tag{21}$$

$$\frac{\partial^2\psi_5}{\partial t^2}-\frac{\partial^2\psi_5}{\partial x_1^2}-\frac{\partial^2\psi_5}{\partial x_3^2}-2\frac{\partial}{\partial t}\left(\frac{\partial\psi_6}{\partial x_1}-\frac{\partial\psi_4}{\partial x_3}\right)+\frac{\partial}{\partial x_2}\left(\frac{\partial\psi_4}{\partial x_1}+\frac{\partial\psi_6}{\partial x_3}\right)$$
$$=k_1^2\psi_5+k_1k_2j_5+k_2\left(\frac{\partial j_5}{\partial t}+\frac{\partial j_6}{\partial x_2}-\frac{\partial j_4}{\partial x_3}\right) \tag{22}$$

$$\frac{\partial^2\psi_6}{\partial t^2}-\frac{\partial^2\psi_6}{\partial x_1^2}-\frac{\partial^2\psi_6}{\partial x_2^2}+2\frac{\partial}{\partial t}\left(\frac{\partial\psi_5}{\partial x_1}-\frac{\partial\psi_4}{\partial x_2}\right)+\frac{\partial}{\partial x_3}\left(\frac{\partial\psi_4}{\partial x_1}+\frac{\partial\psi_5}{\partial x_2}\right)$$
$$=k_1^2\psi_6+k_1k_2j_6+k_2\left(\frac{\partial j_6}{\partial t}+\frac{\partial j_5}{\partial x_1}-\frac{\partial j_4}{\partial x_2}\right) \tag{23}$$

The equations given in Equations (18-20) and Equations (21-23) contain cross derivatives that involve both space and time. Even though the cross derivatives that involve the time coordinate can be removed by using the system of equations given in Equations (12-14) and Equations (15-17), the cross derivatives that involve the spatial coordinates can only be removed by imposing on the wave function ψ an additional condition that is commonly known as the divergence of a vector field as given in Equation (6). The divergence of a field in fact endows the field with a physical character and gives a direct relationship between a mathematical object and a physical entity. Using Equation (6), Gauss's laws for the field $\psi=(\psi_1,\psi_2,\psi_3)^T$ and the field $\psi=(\psi_4,\psi_5,\psi_6)^T$ are written as follows

$$\frac{\partial\psi_1}{\partial x_1}+\frac{\partial\psi_2}{\partial x_2}+\frac{\partial\psi_3}{\partial x_3}=\rho_1 \tag{24}$$

$$\frac{\partial\psi_4}{\partial x_1}+\frac{\partial\psi_5}{\partial x_2}+\frac{\partial\psi_6}{\partial x_3}=\rho_2 \tag{25}$$

where ρ_1 and ρ_2 are physical quantities that can be identified with the electric and magnetic charge densities. Using Equation (24) and Equations (12-14) then from Equations (18-20) we obtain the following system of equations

$$\frac{\partial^2\psi_1}{\partial t^2}+\frac{\partial^2\psi_1}{\partial x_1^2}+\frac{\partial^2\psi_1}{\partial x_2^2}+\frac{\partial^2\psi_1}{\partial x_3^2}+2k_1\frac{\partial\psi_1}{\partial t}$$
$$=-k_1^2\psi_1-k_1k_2j_1-k_2\left(\frac{\partial j_1}{\partial t}+\frac{\partial j_3}{\partial x_2}-\frac{\partial j_2}{\partial x_3}\right)+\frac{\partial\rho_1}{\partial x_1} \tag{26}$$

$$\frac{\partial^2\psi_2}{\partial t^2}+\frac{\partial^2\psi_2}{\partial x_1^2}+\frac{\partial^2\psi_2}{\partial x_2^2}+\frac{\partial^2\psi_2}{\partial x_3^2}+2k_1\frac{\partial\psi_2}{\partial t}$$
$$=-k_1^2\psi_2-k_1k_2j_2-k_2\left(\frac{\partial j_2}{\partial t}+\frac{\partial j_3}{\partial x_2}-\frac{\partial j_1}{\partial x_3}\right)+\frac{\partial\rho_1}{\partial x_2} \tag{27}$$

$$\frac{\partial^2\psi_3}{\partial t^2}+\frac{\partial^2\psi_3}{\partial x_1^2}+\frac{\partial^2\psi_3}{\partial x_2^2}+\frac{\partial^2\psi_3}{\partial x_1^2}+2k_1\frac{\partial\psi_3}{\partial t}$$
$$=-k_1^2\psi_3-k_1k_2j_3-k_2\left(\frac{\partial j_3}{\partial t}+\frac{\partial j_2}{\partial x_1}-\frac{\partial j_1}{\partial x_2}\right)+\frac{\partial\rho_1}{\partial x_3} \tag{28}$$

In order to obtain a system of differential equations that can be applied to the electromagnetic field we set $k_1 = 0$. Then Equations (26-28) reduce to the following system of equations

$$\frac{\partial^2\psi_1}{\partial t^2}+\frac{\partial^2\psi_1}{\partial x_1^2}+\frac{\partial^2\psi_1}{\partial x_2^2}+\frac{\partial^2\psi_1}{\partial x_3^2}=-k_2\left(\frac{\partial j_1}{\partial t}+\frac{\partial j_3}{\partial x_2}-\frac{\partial j_2}{\partial x_3}\right)+\frac{\partial\rho_1}{\partial x_1} \tag{29}$$

$$\frac{\partial^2\psi_2}{\partial t^2}+\frac{\partial^2\psi_2}{\partial x_1^2}+\frac{\partial^2\psi_2}{\partial x_2^2}+\frac{\partial^2\psi_2}{\partial x_3^2}=-k_2\left(\frac{\partial j_2}{\partial t}+\frac{\partial j_3}{\partial x_2}-\frac{\partial j_1}{\partial x_3}\right)+\frac{\partial\rho_1}{\partial x_2} \tag{30}$$

$$\frac{\partial^2\psi_3}{\partial t^2}+\frac{\partial^2\psi_3}{\partial x_1^2}+\frac{\partial^2\psi_3}{\partial x_2^2}+\frac{\partial^2\psi_3}{\partial x_1^2}=-k_2\left(\frac{\partial j_3}{\partial t}+\frac{\partial j_2}{\partial x_1}-\frac{\partial j_1}{\partial x_2}\right)+\frac{\partial\rho_1}{\partial x_3} \tag{31}$$

The equations given in Equations (29-31) are elliptic equations rather than hyperbolic or wave equations therefore subfields are more suitable to represent stable quantum particles with invariant physical properties. Moreover, since elliptic equations comply with the Euclidean relativity instead of the pseudo-Euclidean relativity therefore there may exist some physical properties associated with quantum particles that can travel with speeds greater than the speed of light in vacuum, which is a speed limit of transmission for physical events that comply with the pseudo-Euclidean relativity.

Now, similar to the case of negative time, by using the matrices A_i given in Equation (8), Equations (15-17), Gauss's laws given in Equation (25), and $k_1 = 0$, a system of equations with positive time can be obtained and given as follows

$$\frac{\partial^2\psi_4}{\partial t^2}+\frac{\partial^2\psi_4}{\partial x_1^2}+\frac{\partial^2\psi_4}{\partial x_2^2}+\frac{\partial^2\psi_4}{\partial x_3^2}=-k_2\left(-\frac{\partial j_4}{\partial t}+\frac{\partial j_6}{\partial x_2}-\frac{\partial j_5}{\partial x_3}\right)+\frac{\partial\rho_2}{\partial x_1} \tag{32}$$

$$\frac{\partial^2\psi_5}{\partial t^2}+\frac{\partial^2\psi_5}{\partial x_1^2}+\frac{\partial^2\psi_5}{\partial x_2^2}+\frac{\partial^2\psi_5}{\partial x_3^2}=-k_2\left(-\frac{\partial j_5}{\partial t}+\frac{\partial j_6}{\partial x_2}-\frac{\partial j_4}{\partial x_3}\right)+\frac{\partial\rho_2}{\partial x_2} \tag{33}$$

$$\frac{\partial^2\psi_6}{\partial t^2}+\frac{\partial^2\psi_6}{\partial x_1^2}+\frac{\partial^2\psi_6}{\partial x_2^2}+\frac{\partial^2\psi_6}{\partial x_1^2}=-k_2\left(-\frac{\partial j_6}{\partial t}+\frac{\partial j_5}{\partial x_1}-\frac{\partial j_4}{\partial x_2}\right)+\frac{\partial\rho_2}{\partial x_3} \tag{34}$$

The equations of the subfield with positive time also satisfy elliptic equations rather than wave equations therefore they are also suitable to represent quantum particles with stable properties that accompany the stable properties associated with the stable properties associated with the negative time whose field equations are given in Equations (29-31).

Having shown the basic equations for the two subfields by using the matrices A_i with negative and positive time, each of which can be used to represent stable properties of quantum particles due to the fact that they satisfy elliptic equations rather than wave equations, we now show that a coupling of the two subfields can give rise to a coupled field that satisfies wave equations such as Maxwell field equations of the electromagnetic field. A coupled field from the two subfields with the matrices given in Equation (8) can be formulated by using the following coupled matrices

$$A_0 = \begin{pmatrix} -1 & 0 & 0 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 & 0 & 0 \\ 0 & 0 & -1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix} \qquad A_1 = \begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & -1 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & -1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \end{pmatrix}$$

$$A_2 = \begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & -1 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ -1 & 0 & 0 & 0 & 0 & 0 \end{pmatrix} \qquad A_3 = \begin{pmatrix} 0 & 0 & 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix} \tag{35}$$

Then we obtain the following results

$$A_0^2 = \begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix} \qquad A_1^2 = \begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 & 0 & 0 \\ 0 & 0 & -1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & 0 & 0 & -1 \end{pmatrix}$$

$$A_2^2 = \begin{pmatrix} -1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & -1 & 0 & 0 & 0 \\ 0 & 0 & 0 & -1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & -1 \end{pmatrix} \qquad A_3^2 = \begin{pmatrix} -1 & 0 & 0 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & -1 & 0 & 0 \\ 0 & 0 & 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}$$

$$A_1A_2 + A_2A_1 = \begin{pmatrix} 0 & 1 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix} \qquad A_1A_3 + A_3A_1 = \begin{pmatrix} 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \end{pmatrix}$$

$$A_2A_3 + A_3A_2 = \begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 1 & 0 \end{pmatrix} \qquad A_0A_i + A_iA_0 = 0 \quad \text{for} \quad i = 1,2,3 \tag{36}$$

It is noticed that by coupling the two subfields with negative and positive time the cross derivatives that involve time are automatically removed. This shows that the electromagnetic field may be considered as a resonant field which is formed from the superposition of two physical fields that flow in opposite directions. Similar to the case of subfields, we write Equation (2) for the coupled field in the following simple form

$$\left(A_0\frac{\partial}{\partial t} + A_1\frac{\partial}{\partial x_1} + A_2\frac{\partial}{\partial x_2} + A_3\frac{\partial}{\partial x_3}\right)\psi = k_1\psi + k_2J \tag{37}$$

where $\psi = (\psi_1, \psi_2, \psi_3, \psi_4, \psi_5, \psi_6)^T$ and $J = (j_1, j_2, j_3, j_4, j_5, j_6)^T$. Using the matrices given in Equation (35) we obtain the following system of equations for the coupled field from Equation (37)

$$-\frac{\partial \psi_1}{\partial t}+\frac{\partial \psi_6}{\partial x_2}-\frac{\partial \psi_5}{\partial x_3}=k_1\psi_1+k_2j_1 \tag{38}$$

$$-\frac{\partial \psi_2}{\partial t}+\frac{\partial \psi_4}{\partial x_3}-\frac{\partial \psi_6}{\partial x_1}=k_1\psi_2+k_2j_2 \tag{39}$$

$$-\frac{\partial \psi_3}{\partial t}+\frac{\partial \psi_5}{\partial x_1}-\frac{\partial \psi_4}{\partial x_2}=k_1\psi_3+k_2j_3 \tag{40}$$

$$\frac{\partial \psi_4}{\partial t}+\frac{\partial \psi_3}{\partial x_2}-\frac{\partial \psi_2}{\partial x_3}=k_1\psi_4+k_2j_4 \tag{41}$$

$$\frac{\partial \psi_5}{\partial t}+\frac{\partial \psi_1}{\partial x_3}-\frac{\partial \psi_3}{\partial x_1}=k_1\psi_5+k_2j_5 \tag{42}$$

$$\frac{\partial \psi_6}{\partial t}+\frac{\partial \psi_2}{\partial x_1}-\frac{\partial \psi_1}{\partial x_2}=k_1\psi_6+k_2j_6 \tag{43}$$

Using the results obtained for the matrices A_i given in Equation (36) we obtain the following system of equations for the coupled field from Equation (4)

$$\begin{aligned}\frac{\partial^2 \psi_1}{\partial t^2}-\frac{\partial^2 \psi_1}{\partial x_2^2}-\frac{\partial^2 \psi_1}{\partial x_3^2}+\frac{\partial}{\partial x_1}\left(\frac{\partial \psi_2}{\partial x_2}+\frac{\partial \psi_3}{\partial x_3}\right)\\ =k_1^2\psi_1+k_1k_2j_1+k_2\left(-\frac{\partial j_1}{\partial t}+\frac{\partial j_6}{\partial x_2}-\frac{\partial j_5}{\partial x_3}\right)\end{aligned} \tag{44}$$

$$\begin{aligned}\frac{\partial^2 \psi_2}{\partial t^2}-\frac{\partial^2 \psi_2}{\partial x_1^2}-\frac{\partial^2 \psi_2}{\partial x_3^2}+\frac{\partial}{\partial x_2}\left(\frac{\partial \psi_1}{\partial x_1}+\frac{\partial \psi_3}{\partial x_3}\right)\\ =k_1^2\psi_2+k_1k_2j_2+k_2\left(-\frac{\partial j_2}{\partial t}+\frac{\partial j_4}{\partial x_3}-\frac{\partial j_6}{\partial x_1}\right)\end{aligned} \tag{45}$$

$$\begin{aligned}\frac{\partial^2 \psi_3}{\partial t^2}-\frac{\partial^2 \psi_3}{\partial x_1^2}-\frac{\partial^2 \psi_3}{\partial x_2^2}+\frac{\partial}{\partial x_3}\left(\frac{\partial \psi_1}{\partial x_1}+\frac{\partial \psi_2}{\partial x_2}\right)\\ =k_1^2\psi_3+k_1k_2j_3+k_2\left(-\frac{\partial j_3}{\partial t}+\frac{\partial j_5}{\partial x_1}-\frac{\partial j_4}{\partial x_2}\right)\end{aligned} \tag{46}$$

$$\frac{\partial^2 \psi_4}{\partial t^2}-\frac{\partial^2 \psi_4}{\partial x_2^2}-\frac{\partial^2 \psi_4}{\partial x_3^2}+\frac{\partial}{\partial x_1}\left(\frac{\partial \psi_5}{\partial x_2}+\frac{\partial \psi_6}{\partial x_3}\right)=k_1^2\psi_4+k_1k_2j_4+k_2\left(\frac{\partial j_4}{\partial t}+\frac{\partial j_3}{\partial x_2}-\frac{\partial j_2}{\partial x_3}\right) \tag{47}$$

$$\begin{aligned}\frac{\partial^2 \psi_5}{\partial t^2}-\frac{\partial^2 \psi_5}{\partial x_1^2}-\frac{\partial^2 \psi_5}{\partial x_3^2}+\frac{\partial}{\partial x_2}\left(\frac{\partial \psi_4}{\partial x_1}+\frac{\partial \psi_6}{\partial x_3}\right)\\ =k_1^2\psi_5+k_1k_2j_5+k_2\left(\frac{\partial j_5}{\partial t}+\frac{\partial j_1}{\partial x_3}-\frac{\partial j_3}{\partial x_1}\right)\end{aligned} \tag{48}$$

$$\frac{\partial^2\psi_6}{\partial t^2}-\frac{\partial^2\psi_6}{\partial x_1^2}-\frac{\partial^2\psi_6}{\partial x_2^2}+\frac{\partial}{\partial x_3}\left(\frac{\partial\psi_4}{\partial x_1}+\frac{\partial\psi_5}{\partial x_2}\right)$$
$$=k_1^2\psi_6+k_1k_2j_6+k_2\left(\frac{\partial j_6}{\partial t}+\frac{\partial j_2}{\partial x_1}-\frac{\partial j_1}{\partial x_2}\right) \quad (49)$$

Using the divergence conditions or Gauss's laws given in Equations (24-25) the system of equations given in Equations (44-49) reduces to the following system of equations

$$\frac{\partial^2\psi_1}{\partial t^2}-\frac{\partial^2\psi_1}{\partial x_1^2}-\frac{\partial^2\psi_1}{\partial x_2^2}-\frac{\partial^2\psi_1}{\partial x_3^2}=k_1^2\psi_1+k_1k_2j_1+k_2\left(-\frac{\partial j_1}{\partial t}+\frac{\partial j_6}{\partial x_2}-\frac{\partial j_5}{\partial x_3}\right)-\frac{\partial\rho_1}{\partial x_1} \quad (50)$$

$$\frac{\partial^2\psi_2}{\partial t^2}-\frac{\partial^2\psi_2}{\partial x_1^2}-\frac{\partial^2\psi_2}{\partial x_2^2}-\frac{\partial^2\psi_2}{\partial x_3^2}=k_1^2\psi_2+k_1k_2j_2+k_2\left(-\frac{\partial j_2}{\partial t}+\frac{\partial j_4}{\partial x_3}-\frac{\partial j_6}{\partial x_1}\right)-\frac{\partial\rho_1}{\partial x_2} \quad (51)$$

$$\frac{\partial^2\psi_3}{\partial t^2}-\frac{\partial^2\psi_3}{\partial x_1^2}-\frac{\partial^2\psi_3}{\partial x_2^2}-\frac{\partial^2\psi_3}{\partial x_3^2}=k_1^2\psi_3+k_1k_2j_3+k_2\left(-\frac{\partial j_3}{\partial t}+\frac{\partial j_5}{\partial x_1}-\frac{\partial j_4}{\partial x_2}\right)-\frac{\partial\rho_1}{\partial x_3} \quad (52)$$

$$\frac{\partial^2\psi_4}{\partial t^2}-\frac{\partial^2\psi_4}{\partial x_1^2}-\frac{\partial^2\psi_4}{\partial x_2^2}-\frac{\partial^2\psi_4}{\partial x_3^2}=k_1^2\psi_4+k_1k_2j_4+k_2\left(\frac{\partial j_4}{\partial t}+\frac{\partial j_3}{\partial x_2}-\frac{\partial j_2}{\partial x_3}\right)-\frac{\partial\rho_2}{\partial x_1} \quad (53)$$

$$\frac{\partial^2\psi_5}{\partial t^2}-\frac{\partial^2\psi_5}{\partial x_1^2}-\frac{\partial^2\psi_5}{\partial x_2^2}-\frac{\partial^2\psi_5}{\partial x_3^2}=k_1^2\psi_5+k_1k_2j_5+k_2\left(\frac{\partial j_5}{\partial t}+\frac{\partial j_1}{\partial x_3}-\frac{\partial j_3}{\partial x_1}\right)-\frac{\partial\rho_2}{\partial x_2} \quad (54)$$

$$\frac{\partial^2\psi_6}{\partial t^2}-\frac{\partial^2\psi_6}{\partial x_1^2}-\frac{\partial^2\psi_6}{\partial x_2^2}-\frac{\partial^2\psi_6}{\partial x_3^2}=k_1^2\psi_6+k_1k_2j_6+k_2\left(\frac{\partial j_6}{\partial t}+\frac{\partial j_2}{\partial x_1}-\frac{\partial j_1}{\partial x_2}\right)-\frac{\partial\rho_2}{\partial x_3} \quad (55)$$

Now, to obtain Maxwell field equations of the electromagnetic field we set $k_1=0$ and obtain the following system of equations from Equations (38-43)

$$-\frac{\partial\psi_1}{\partial t}+\frac{\partial\psi_6}{\partial x_2}-\frac{\partial\psi_5}{\partial x_3}=k_2j_1 \quad (56)$$

$$-\frac{\partial\psi_2}{\partial t}+\frac{\partial\psi_4}{\partial x_3}-\frac{\partial\psi_6}{\partial x_1}=k_2j_2 \quad (57)$$

$$-\frac{\partial\psi_3}{\partial t}+\frac{\partial\psi_5}{\partial x_1}-\frac{\partial\psi_4}{\partial x_2}=k_2j_3 \quad (58)$$

$$\frac{\partial\psi_4}{\partial t}+\frac{\partial\psi_3}{\partial x_2}-\frac{\partial\psi_2}{\partial x_3}=k_2j_4 \quad (59)$$

$$\frac{\partial\psi_5}{\partial t}+\frac{\partial\psi_1}{\partial x_3}-\frac{\partial\psi_3}{\partial x_1}=k_2j_5 \quad (60)$$

$$\frac{\partial\psi_6}{\partial t}+\frac{\partial\psi_2}{\partial x_1}-\frac{\partial\psi_1}{\partial x_2}=k_2j_6 \quad (61)$$

By identifying $\mathbf{E}=(\psi_1,\psi_2,\psi_3)$, $\mathbf{B}=(\psi_4,\psi_5,\psi_6)$, $\mathbf{j}_1=(j_1,j_2,j_3)$ and $\mathbf{j}_2=(j_4,j_5,j_6)$ the above system of equations, together with Gauss's laws given in Equations (24-25), can be rewritten in the familiar form in classical electrodynamics as

$$\nabla.\mathbf{E}=\rho_1 \tag{62}$$

$$\nabla.\mathbf{B}=\rho_2 \tag{63}$$

$$\nabla\times\mathbf{E}+\frac{\partial\mathbf{B}}{\partial t}=k_2\mathbf{j}_2 \tag{64}$$

$$\nabla\times\mathbf{B}-\frac{\partial\mathbf{E}}{\partial t}=k_2\mathbf{j}_1 \tag{65}$$

With $k_1=0$ we also obtain the following system of equations from Equations (50-55)

$$\frac{\partial^2\psi_1}{\partial t^2}-\frac{\partial^2\psi_1}{\partial x_1^2}-\frac{\partial^2\psi_1}{\partial x_2^2}-\frac{\partial^2\psi_1}{\partial x_3^2}=k_2\left(-\frac{\partial j_1}{\partial t}+\frac{\partial j_6}{\partial x_2}-\frac{\partial j_5}{\partial x_3}\right)-\frac{\partial\rho_1}{\partial x_1} \tag{66}$$

$$\frac{\partial^2\psi_2}{\partial t^2}-\frac{\partial^2\psi_2}{\partial x_1^2}-\frac{\partial^2\psi_2}{\partial x_2^2}-\frac{\partial^2\psi_2}{\partial x_3^2}=k_2\left(-\frac{\partial j_2}{\partial t}+\frac{\partial j_4}{\partial x_3}-\frac{\partial j_6}{\partial x_1}\right)-\frac{\partial\rho_1}{\partial x_2} \tag{67}$$

$$\frac{\partial^2\psi_3}{\partial t^2}-\frac{\partial^2\psi_3}{\partial x_1^2}-\frac{\partial^2\psi_3}{\partial x_2^2}-\frac{\partial^2\psi_3}{\partial x_3^2}=k_2\left(-\frac{\partial j_3}{\partial t}+\frac{\partial j_5}{\partial x_1}-\frac{\partial j_4}{\partial x_2}\right)-\frac{\partial\rho_1}{\partial x_3} \tag{68}$$

$$\frac{\partial^2\psi_4}{\partial t^2}-\frac{\partial^2\psi_4}{\partial x_1^2}-\frac{\partial^2\psi_4}{\partial x_2^2}-\frac{\partial^2\psi_4}{\partial x_3^2}=k_2\left(\frac{\partial j_4}{\partial t}+\frac{\partial j_3}{\partial x_2}-\frac{\partial j_2}{\partial x_3}\right)-\frac{\partial\rho_2}{\partial x_1} \tag{69}$$

$$\frac{\partial^2\psi_5}{\partial t^2}-\frac{\partial^2\psi_5}{\partial x_1^2}-\frac{\partial^2\psi_5}{\partial x_2^2}-\frac{\partial^2\psi_5}{\partial x_3^2}=k_2\left(\frac{\partial j_5}{\partial t}+\frac{\partial j_1}{\partial x_3}-\frac{\partial j_3}{\partial x_1}\right)-\frac{\partial\rho_2}{\partial x_2} \tag{70}$$

$$\frac{\partial^2\psi_6}{\partial t^2}-\frac{\partial^2\psi_6}{\partial x_1^2}-\frac{\partial^2\psi_6}{\partial x_2^2}-\frac{\partial^2\psi_6}{\partial x_3^2}=k_2\left(\frac{\partial j_6}{\partial t}+\frac{\partial j_2}{\partial x_1}-\frac{\partial j_1}{\partial x_2}\right)-\frac{\partial\rho_2}{\partial x_3} \tag{71}$$

Equations (66-71) can be rewritten in a vector form as a system of two equations as in classical electrodynamics

$$\frac{\partial^2\mathbf{E}}{\partial t^2}-\nabla^2\mathbf{E}=\nabla(\rho_1)-k_2\frac{\partial\mathbf{J}_e}{\partial t}+k_2\nabla\times\mathbf{J}_2 \tag{72}$$

$$\frac{\partial^2\mathbf{B}}{\partial t^2}-\nabla^2\mathbf{B}=\nabla(\rho_2)-k_2\frac{\partial\mathbf{J}_b}{\partial t}+k_2\nabla\times\mathbf{J}_1 \tag{73}$$

where the charge density ρ_i and the current density $\mathbf{j}_i$ satisfy the conservation law

$$\nabla.\mathbf{j}_i+\frac{\partial\rho_i}{\partial t}=0 \tag{74}$$

3. Dirac field as coupling of two elliptic fields

In this section we formulate Dirac equation using the same procedure that we have applied to the Maxwell field of electromagnetism in the previous section. We have shown that Maxwell field is represented by matrices of rank six but the two subfields that are coupled to form Maxwell field are represented by matrices of rank three. Now, as it has been known that Dirac equation is formulated with matrices of rank four which are built upon Pauli matrices therefore we will simply use Pauli matrices as the required matrices for the two subfields. The Dirac equation then can be seen as a coupling of two systems of field equations similar to the case of Maxwell field equations of the electromagnetic field. Although the formulation of Dirac equation we consider in this work is straightforward from the known results there are new features that emerge with regard to the nature of the subfields that are coupled to form the Dirac field, such as the subfields also satisfy elliptic equations and therefore comply with the Euclidean relativity instead of wave equations and the pseudo-Euclidean relativity. Except for the dimensions, these characteristics show that the quantum behaviours of both Maxwell and Dirac are similar when they are represented by the subfields. The Pauli matrices $A_i = \sigma_i$ that we use for Dirac subfields are given as follows

$$A_0 = \mp\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} \quad A_1 = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \quad A_2 = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix} \quad A_3 = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} \tag{75}$$

We then obtain the following results

$$A_i^2 = 1 \qquad \text{and} \qquad A_iA_j + A_jA_i = 0 \qquad \text{for} \qquad i, j = 0, 1, 2, 3 \tag{76}$$

Using the Pauli matrices A_i given in Equation (75) with negative time we obtain the following system of differential equations from Equation (2)

$$-\frac{\partial \psi_1}{\partial t} + \frac{\partial \psi_2}{\partial x_1} - i\frac{\partial \psi_2}{\partial x_2} + \frac{\partial \psi_1}{\partial x_3} = k_1\psi_1 + k_2 j_1 \tag{77}$$

$$-\frac{\partial \psi_2}{\partial t} + \frac{\partial \psi_1}{\partial x_1} + i\frac{\partial \psi_1}{\partial x_2} - \frac{\partial \psi_2}{\partial x_3} = k_1\psi_2 + k_2 j_2 \tag{78}$$

Using the Pauli matrices A_i given in Equation (75) with positive time we obtain the following system of differential equations from Equation (2)

$$\frac{\partial \psi_3}{\partial t} + \frac{\partial \psi_4}{\partial x_1} - i\frac{\partial \psi_4}{\partial x_2} + \frac{\partial \psi_3}{\partial x_3} = k_1\psi_3 + k_2 j_1 \tag{79}$$

$$\frac{\partial \psi_4}{\partial t} + \frac{\partial \psi_3}{\partial x_1} + i\frac{\partial \psi_3}{\partial x_2} - \frac{\partial \psi_4}{\partial x_3} = k_1\psi_4 + k_2 j_2 \tag{80}$$

On the other hand, using the results obtained in Equation (76) with negative time we obtain the following equation for the components of the function $\psi = (\psi_1, \psi_2)^T$ from Equation (4)

$$\frac{\partial^2\psi_1}{\partial t^2}+\frac{\partial^2\psi_1}{\partial x_1^2}+\frac{\partial^2\psi_1}{\partial x_2^2}+\frac{\partial^2\psi_1}{\partial x_3^2}=k_1^2\psi_1+k_1k_2j_1+k_2\left(-\frac{\partial j_1}{\partial t}+\frac{\partial j_2}{\partial x_1}-i\frac{\partial j_2}{\partial x_2}+\frac{\partial j_1}{\partial x_3}\right) \quad (81)$$

$$\frac{\partial^2\psi_2}{\partial t^2}+\frac{\partial^2\psi_2}{\partial x_1^2}+\frac{\partial^2\psi_2}{\partial x_2^2}+\frac{\partial^2\psi_2}{\partial x_3^2}=k_1^2\psi_2+k_1k_2j_2+k_2\left(-\frac{\partial j_2}{\partial t}+\frac{\partial j_1}{\partial x_1}+i\frac{\partial j_1}{\partial x_2}-\frac{\partial j_2}{\partial x_3}\right) \quad (82)$$

Similarly, using the results obtained in Equation (76) with positive time we obtain the following equation for the components of the function $\psi=(\psi_3,\psi_4)^T$ from Equation (4)

$$\frac{\partial^2\psi_3}{\partial t^2}+\frac{\partial^2\psi_3}{\partial x_1^2}+\frac{\partial^2\psi_3}{\partial x_2^2}+\frac{\partial^2\psi_3}{\partial x_3^2}=k_1^2\psi_3+k_1k_2j_3+k_2\left(\frac{\partial j_3}{\partial t}+\frac{\partial j_4}{\partial x_1}-i\frac{\partial j_4}{\partial x_2}+\frac{\partial j_3}{\partial x_3}\right) \quad (83)$$

$$\frac{\partial^2\psi_4}{\partial t^2}+\frac{\partial^2\psi_4}{\partial x_1^2}+\frac{\partial^2\psi_4}{\partial x_2^2}+\frac{\partial^2\psi_4}{\partial x_3^2}=k_1^2\psi_4+k_1k_2j_4+k_2\left(\frac{\partial j_4}{\partial t}+\frac{\partial j_3}{\partial x_1}+i\frac{\partial j_3}{\partial x_2}-\frac{\partial j_4}{\partial x_3}\right) \quad (84)$$

As in the case of the subfields of Maxwell field of the electromagnetic field, the equations given in Equations (81-84) are elliptic equations therefore they can be used to describe the steady states of physical systems, in particular they can be used to explain the stability of elementary particles. Furthermore, if quantum particles possess physical properties that are represented by subfields which are described by elliptic equations, hence complying with the Euclidean relativity, then they can be used to explain physical phenomena that require physical transmissions with speeds greater than the speed of light in vacuum, such as the Einstein-Podosky-Rosen paradox in quantum entanglement. As being well-known the coupled field which can be used to represent Dirac field is formulated by using the familiar gamma matrices γ^μ written in terms of the Pauli and unit matrices as

$$\gamma_0=\begin{pmatrix}I & 0\\ 0 & -I\end{pmatrix} \qquad \gamma_i=\begin{pmatrix}0 & \sigma_i\\ -\sigma_i & 0\end{pmatrix} \quad (85)$$

$$\gamma_0=\begin{pmatrix}1&0&0&0\\0&1&0&0\\0&0&-1&0\\0&0&0&-1\end{pmatrix} \quad \gamma_1=\begin{pmatrix}0&0&0&1\\0&0&1&0\\0&-1&0&0\\-1&0&0&0\end{pmatrix} \quad \gamma_2=\begin{pmatrix}0&0&0&-i\\0&0&i&0\\0&i&0&0\\-i&0&0&0\end{pmatrix} \quad \gamma_3=\begin{pmatrix}0&0&1&0\\0&0&0&-1\\-1&0&0&0\\0&1&0&0\end{pmatrix} \quad (86)$$

With $k_2=0$, Equation (2) reduces to Dirac equation for a free particle which is written in the form

$$\gamma^\mu\partial_\mu\psi=-im\psi \quad (87)$$

Using the gamma matrices given in Equation (86), Dirac equation given in Equation (85) can be written out for the wavefunction $\psi=(\psi_1,\psi_2,\psi_3,\psi_4)^T$ as

$$\frac{\partial\psi_1}{\partial t}+im\psi_1=-\frac{\partial\psi_3}{\partial z}-\left(\frac{\partial}{\partial x}+i\frac{\partial}{\partial y}\right)\psi_4 \quad (88)$$

$$\frac{\partial\psi_2}{\partial t}+im\psi_2=-\left(\frac{\partial}{\partial x}+i\frac{\partial}{\partial y}\right)\psi_3+\frac{\partial\psi_4}{\partial z} \quad (89)$$

$$\frac{\partial \psi_3}{\partial t} - im\psi_3 = -\frac{\partial \psi_1}{\partial z} - \left(\frac{\partial}{\partial x} - i\frac{\partial}{\partial y}\right)\psi_2 \tag{90}$$

$$\frac{\partial \psi_4}{\partial t} - im\psi_4 = -\left(\frac{\partial}{\partial x} + i\frac{\partial}{\partial y}\right)\psi_1 - \frac{\partial \psi_2}{\partial z} \tag{91}$$

Dirac equation written as a system of linear first order partial differential equations given in Equations (88-91) suggests that matter wave can be interpreted as a coupling of two different physical subfields represented by the field (ψ_1, ψ_2) and the field (ψ_3, ψ_4) whose temporal rates of change will convert one field to the other. From the gamma matrices given in Equation (86) we obtain the following relations

$$\gamma_0^2 = 1, \qquad \gamma_i^2 = -1 \quad \text{for} \quad i = 1, 2, 3 \qquad \text{and} \qquad \gamma_i\gamma_j + \gamma_j\gamma_i = 0 \quad \text{for} \ \ i \neq j \tag{92}$$

With the relations obtained in Equation (92), it can be shown that all components of Dirac wavefunction $\psi = (\psi_1, \psi_2, \psi_3, \psi_4)^T$ satisfy the Klein-Gordon equation

$$\frac{\partial^2 \psi_\mu}{\partial t^2} - \frac{\partial^2 \psi_\mu}{\partial x^2} - \frac{\partial^2 \psi_\mu}{\partial y^2} - \frac{\partial^2 \psi_\mu}{\partial z^2} = -m^2\psi_\mu \tag{93}$$

The Klein-Gordon equation is a wave equation that is Lorentz invariant in the pseudo-Euclidean space which was proposed and developed by Minkowski based on Einstein's theory of special relativity.

In fact it is possible to formulate a coupled field that is similar to Dirac field from the subfields represented by the Pauli matrices but instead satisfies an elliptic equation rather than a wave equation. Such field therefore will be Euclidean invariant. Consider a coupled field that is formed from the subfields represented by Pauli matrices with the coupled matrices given as follows

$$A_0 = \begin{pmatrix} -I & 0 \\ 0 & I \end{pmatrix} \qquad A_i = \begin{pmatrix} 0 & \sigma_i \\ \sigma_i & 0 \end{pmatrix} \tag{94}$$

Then we obtain the following results

$$\gamma_0^2 = 1, \qquad \gamma_i^2 = 1 \quad \text{for} \quad i = 1, 2, 3 \qquad \text{and} \qquad \gamma_i\gamma_j + \gamma_j\gamma_i = 0 \quad \text{for} \ \ i \neq j \tag{95}$$

From the relations obtained in Equation (95), then it can be shown that all components of the wavefunction $\psi = (\psi_1, \psi_2, \psi_3, \psi_4)^T$ satisfy the following elliptic equation

$$\frac{\partial^2 \psi_\mu}{\partial t^2} + \frac{\partial^2 \psi_\mu}{\partial x^2} + \frac{\partial^2 \psi_\mu}{\partial y^2} + \frac{\partial^2 \psi_\mu}{\partial z^2} = -m^2\psi_\mu \tag{96}$$

As a further remark, we would like to mention here that we have formulated Maxwell and Dirac field essentially from a general system of linear first order partial differential equations which is a purely mathematical framework that can be used to formulate any physical theory that requires such mathematical structure, similar to the case of Laplace or Poisson's equation. Nonetheless, with such perspective, it has been suggested that they should be referred to as Maxwell-like and Dirac-like field equations instead of Maxwell and Dirac. The approach that we have used to formulate Maxwell and Dirac field is quite different from other mathematical methods such as gauge theories whose formulation is based on the variational principle [17] [18]. However, as we have shown in our work on the principle of least action that the variational principle with quantum objects may not lead to a least action as the principle is supposed to provide but only complies with Feynman's integral method of random paths or random surfaces, which itself is not related to the principle of least action [19]. Therefore, physical theories such as gauge theories that rely on the variational principle with a Lagrangian function to establish a deterministic least action should not be regarded as statistical theories therefore they are not in accordance with the current interpretation of the quantum theory which relies on the probability view for their interpretation of experimental results.

Appendix: Euclidean Relativity

In this appendix we show the possibility to formulate a Euclidean relativity so that spacetime has a positive definite Euclidean metric instead of a pseudo-Euclidean metric. In physics, the concept of a pseudo-Euclidean spacetime was introduced by Minkowski in order to accommodate Einstein's theory of special relativity in which the coordinate transformation between the inertial frame S with spacetime coordinates (ct, x, y, z) and the inertial frame S' with coordinates (ct', x', y', z') are derived from the principle of relativity and the postulate of a universal speed c. The coordinate transformation is the Lorentz transformation given by

$$x' = \gamma(x - \beta ct) \qquad y' = y \qquad z' = z \qquad ct' = \gamma(-\beta x + ct) \tag{1}$$

where $\beta = v/c$ and $\gamma = 1/\sqrt{1-\beta^2}$. It is shown that the Lorentz transformation given in Equation (1) leaves the Minkowski spacetime interval $-c^2t^2 + x^2 + y^2 + z^2$ invariant. Spacetime with this metric is a pseudo-Euclidean space. We now show that it is possible to construct a special relativistic transformation that will make spacetime a Euclidean space rather than a pseudo-Euclidean space as in the case of the Lorentz transformation. Consider the following modified Lorentz transformation

$$x' = \gamma_E(x - \beta ct) \qquad y' = y \qquad z' = z \qquad ct' = \gamma_E(\beta x + ct) \tag{2}$$

where $\beta = v/c$ and γ_E will be determined from the principle of relativity and the postulate of a universal speed of a physical field. Instead of the invariance of the Minkowski spacetime interval, if we now assume the invariance of the Euclidean interval $c^2t^2 + x^2 + y^2 + z^2 = c^2t'^2 + x'^2 + y'^2 + z'^2$ then from the modified Lorentz transformation given in Equation (2), we obtain the following expression for γ_E

$$\gamma_E = \frac{1}{\sqrt{1+\beta^2}} \tag{3}$$

It is seen from the expression of γ_E given in Equation (3) that there is no upper limit in the relative speed v between inertial frames. The value of γ_E at the universal speed $v = c$ is $\gamma_E = 1/\sqrt{2}$. For the values of $v \ll c$, the modified Lorentz transformation given in Equation (2) also reduces to the Galilean transformation. However, it is interesting to observe that when $v \to \infty$ we have $\gamma_E \to 0$ and $\beta\gamma_E \to 1$, and in this case we have $x' \to -ct$ and $ct' \to x$. This result shows that there is a conversion between space and time when $v \to \infty$. As in the case of the Lorentz transformation given in Equation (1), we can also derive the relativistic kinematics from the modified Lorentz transformation given in Equations (2), such as the transformation of a length, the transformation of a time interval and the transformation of velocities. Let L_0 be the proper length and Δt_0 is the proper time interval then the length and the time interval transformations can be found as follows

$$L = \sqrt{1+\beta^2}L_0 \qquad \Delta t = \frac{1}{\sqrt{1+\beta^2}}\Delta t_0 \tag{4}$$

It is observed from the length transformation given in Equation (4) that the length of a moving object is expanding rather than contracting as in Einstein theory of special relativity. It is also observed from the time interval transformation given in Equation (4) that the proper time interval is longer than the same time interval measured by a moving observer. With the modified Lorentz transformation given in Equation (2), the transformation of velocities can be found as

$$v'_x = \frac{dx'}{dt'} = \frac{v_x - \beta c}{1 + \frac{\beta v_x}{c}} \qquad v'_y = \frac{dy'}{dt'} = \frac{v_y}{\gamma_E\left(1 + \frac{\beta v_x}{c}\right)} \qquad v'_z = \frac{dz'}{dt'} = \frac{v_z}{\gamma_E\left(1 + \frac{\beta v_x}{c}\right)} \tag{5}$$

From Equation (5), if we let $v_x = c$ then we obtain $v'_x = ((c - v)/(c + v))c$. Therefore in this case $v'_x = c$ only when the relative speed v between two inertial frames vanishes, $v = 0$. In other words, the universal speed c is not the common speed of any moving physical object or physical field in inertial reference frames. In order to specify the nature of the assumed universal speed we observe that in Einstein theory of special relativity it is assumed that spatial space of an inertial frame remains steady and this assumption is contradicted to Einstein theory of general relativity that shows that spatial space is actually expanding. Therefore it seems reasonable to suggest that the universal speed c in the modified Lorentz transformation given in Equation (2) is the universal speed of expansion of the spatial space of all inertial frames.

References

[1] Vu B Ho, *Spacetime Structures of Quantum Particles*, ResearchGate (2017), viXra 1708.0192v2, Int. J. Phys. vol 6, no 4 (2018): 105-115.

[2] Vu B Ho, *A Classification of Quantum Particles*, ResearchGate (2018), viXra 1809.0249v2, GJSFR-A. vol 18, no 9 (2018): 37-58.

[3] Vu B Ho, Fluid State of Dirac Quantum Particles. *Journal of Modern Physics* (2018), **9**, 2402-2419.

[4] Vu B Ho, *Fluid State of the Electromagnetic Field*, ResearchGate (2018), viXra 1812.0298v1.

[5] Einstein, A. (1952) The Principle of Relativity. Dover Publications, New York.

[6] P. A. M. Dirac, *The Quantum Theory of the Electron*, Proceedings of the Royal Society A: Mathematical, Physical and Engineering Sciences, **117** (1928).

[7] Vu B Ho, *On the motion of quantum particles and Euclidean relativity*. Preprint, ResearchGate, 2017, viXra 1710.0253v1.

[8] Vu B Ho, *Euclidean Relativity* (Preprint, ResearchGate, 2017), viXra 1710.0302v1.

[9] A. Einstein, B. Podolsky and N. Rosen, *Can Quantum-Mechanical Description of Physical Reality Be Considered Complete?* Phys. Rev. **47** (1935).

[10] J. S. Bell, *On the Einstein Podolsky Rosen paradox*, Physics Vol 1, No 33 (1964).

[11] W. Rosenfeld, D. Burchardt, R. Garthoff, K. Redeker, N. Ortegel M. Rau and H. Weinfurter , *Event-Ready Bell Test Using Entangled Atoms Simultaneously Closing Detection and Locality Loopholes*, Phys. Rev. Lett. **119** (2017).

[12] S. V. Melshko, *Methods for Constructing Exact Solutions of Partial Differential Equations*, Springer Science & Business Media, Inc, 2005.

[13] S. L. Sobolev, *Partial Differential Equations of Mathematical Physics*, Dover Publications, Inc, New York, 1964.

[14] Jackson, J.D. (1975) Classical Electrodynamics. John Wiley & Sons, New York.

[15] Landau, L.D. and Lifshitz, E.M. (1987) The Classical Theory of Fields. Pergamon Press, Sydney.

[16] Walter A. Strauss, *Partial Differential Equation* (John Wiley & Sons, Inc., New York, 1992).

[17] Jose Leite Lopes, *Gauge Field Theories: An Introduction* (Pergamon Press, Sydney, 2011)

[18] Lim Zheng Liang (2019) *Aspects of Newton Cartan Gauge Theory*. National University of Singapore.

[19] Vu B Ho, *On the Principle of Least Action*, ResearchGate (2016), viXra 1708.0226v2, Int. J. Phys, vol. 6, no. 2 (2018): 47-52.