A Model of Charmonium (Revised)

R.WAYTE

29 Audley Way, Ascot, Berkshire SL5 8EE, England UK.

email: rwayte@googlemail.com

Submitted to vixra.org   19 October 2019

Abstract

A composite model of charmonium has been developed, based on the logarithmic confinement potential. The quark and antiquark pair orbit around the centre of mass, with their colour fields confined within a toroidal flux-tube of characteristic radius.

PACS: 14.40.Lb

1. Introduction

A model of charmonium structure has been developed, based on Einstein's equations of general relativity, as was done for previous models of the electron and proton, (Wayte, Papers 1 and 2). Essentially, the model is compatible with measured spectra of charmonium and bottomium. Although different types of quark/antiquark potentials have been invoked in the literature, this application of Einstein's equations is quite selective. The Coulomb + linear potential presented by Eichten *et al* (1978), (1980) appears to cover the data in many ways, but the energy density of such a field sums to infinity and is physically impossible, in contrast to the logarithmic potential described by Quigg and Rosner (1977), (1979). Whereas the former is effectively two superimposed fields, involving a variable strength factor, the latter describes a single field extending smoothly from a source radius to infinity. Furthermore, the corresponding theoretical leptonic decay widths of vector mesons appear to fit the observations better.

2 The Potential

Previous work on the Coulomb and Yukawa potentials (Papers 1 and 2) indicated that those potential functions were inherently relativistic and very simple in form. Likewise, a logarithmic confinement potential, based on the work of Quigg and Rosner (1979, pp217-223), can be very successful when introduced into Einstein's field equations, namely:

$$V(r) = C \ln(r / r_q) \quad , \tag{2.1}$$

where C = 0.733GeV, r_q = 0.89GeV^{-1} (0.18fm) is the *diametrical* separation of quark and antiquark. Given that the electron classical radius is defined as ($r_{oe} = e^2/m_e c^2$= 2.818fm), and the Quigg-Rosner fundamental charmed quark mass is (m_c = 1.08GeV/c^2), then the field strength ratio could be of the order:

$$\frac{q^2}{e^2} = \frac{(r_q / 2)(2m_c)c^2}{r_{oe} m_e c^2} \approx 135 \quad , \tag{2.2}$$

which is comparable with the strong force, so the non-relativistic Schrodinger equation appears adequate. They determined the empirical charmonium masses from their expression:

$$M_{nl} = CE_{nl} + E_o \quad , \tag{2.3}$$

where E_{nl} is the eigenvalue, C = 0.733GeV, and E_o is a reference mass constant given by:

$$E_o = 2.329\text{GeV} = 2m_c - \tfrac{1}{2} C \ln\left(C m_c r_q{}^2\right) \quad . \tag{2.4}$$

Compatibility of proton and charmonium as electromagnetic structures is confirmed since the proton has spin radius ($r_p = \hbar/m_p c$ = 1.066GeV^{-1} = 0.2103fm), and mass m_p = 0.938GeV, so

$$r_p m_p = \hbar / c \approx (r_q / 2)(2m_c) \quad . \tag{2.5}$$

Quigg and Rosner (1979) also give the fundamental mass of the upsilon family as (m_Q = 4.52GeV), while the field strength q^2 and the radius r_q remain the same as for charmonium. Consequently, extra mass is incorporated *passively* in the quark and the antiquark mechanisms, as was found for baryons (Paper 3). In this case we have:

$$m_Q = m_c(37.7 / 9) \quad , \tag{2.6}$$

which will be interpreted by postulating that a charmonium quark is made of 3 pearls, each of 3 grains, whereas an upsilon quark consists of 37 such grains, but the overall charge is saturated. The three pearls allow for the concept of 3 colours to be introduced and the charmonium grains might be identified as gluons carrying the colour field.

3 Application of Einstein' Equations:

In order to interpret the inter-quark force in a way compatible with the electromagnetic and hadronic forces, the diametrical distances used by Quigg & Rosner will be discontinued in favour of actual *radii* ($r_1 = r/2$ and $r_{q1} = r_q/2$) respectively. Then the metric tensor component can be given its familiar form:

$$\gamma = \left[1 + V(r_1)/M_C c^2\right] \ , \tag{3.1}$$

For this, ($M_C = 2m_c$) is the fundamental charmonium mass, and ($C = 0.733$GeV) in (2.1) will be proposed equal to ($M_C c^2/2\sqrt{2}$). Then the potential energy, for an antiquark in the field of a quark, may be written as:

$$V(r_1) = \frac{q^2}{2\sqrt{2}(r_{q1})}\ln\left(\frac{r_1}{r_{q1}}\right) = \frac{M_C c^2}{2\sqrt{2}}\ln\left(\frac{r_1}{r_{q1}}\right) \ , \tag{3.2}$$

which means that:

$$\gamma = \left\{1 + \frac{1}{2\sqrt{2}}\ln\left(\frac{r_1}{r_{q1}}\right)\right\} \quad . \tag{3.3}$$

We shall now demonstrate that potential (3.2) cannot be applied to a spherically symmetric field, but can apply for a flux tube of colour charge.

3.1 Spherically symmetric field test

As derived for the electromagnetic and gravitational forces, the energy-momentum tensor components for a conserved spherically symmetric radial field of a quark are:

$$8\pi\left(\frac{S}{c^4}\right)T_1^1 = 8\pi\left(\frac{S}{c^4}\right)T_4^4 = \left(\frac{1}{r_1^{\,2}}\right)\frac{d}{dr_1}\left[r_1(1-\gamma^2)\right] \ , \tag{3.4}$$

$$8\pi\left(\frac{S}{c^4}\right)T_2^2 = 8\pi\left(\frac{S}{c^4}\right)T_3^3 = -\gamma\left(\frac{1}{r_1^{\,2}}\right)\frac{d}{dr}\left(r_1^{\,2}\frac{d\gamma}{dr_1}\right) - \left(\frac{d\gamma}{dr_1}\right)^2 \ , \tag{3.5}$$

where S is a quark constant. After introducing (3.3), we have:

$$8\pi\left(\frac{S}{c^4}\right)T_2^2 = -\left[\frac{\gamma}{2\sqrt{2}\,r_1{}^2}\right] - \left[\frac{1}{2\sqrt{2}\,r_1}\right]^2 \quad , \tag{3.6}$$

$$8\pi\left(\frac{S}{c^4}\right)T_4^4 = -\left[\frac{\gamma}{\sqrt{2}r_1{}^2}\right] - \left[\frac{1}{r_1{}^2}\left(\gamma^2 - 1\right)\right] \quad . \tag{3.7}$$

If a spherically symmetric integration is carried out on this energy density it approaches infinity; therefore empirical potential (2.1) cannot apply to a spherical field.

3.2 Linear field

Let the charmonium gluonic field of colour quanta from a quark and antiquark be *confined* to a toroidal flux-tube, analogous to the spin-loop of a proton, see Figure 1.

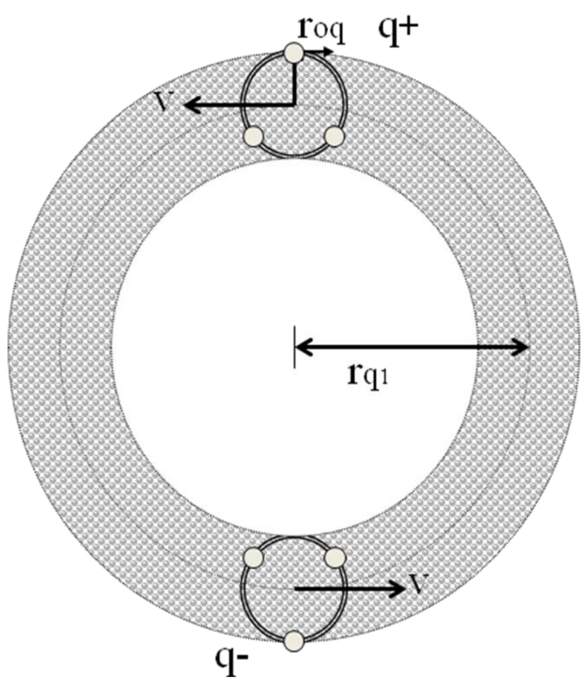

Figure 1 Schematic diagram of charmonium, in which a quark and antiquark orbit with their constituent gluons emitting the colour field into a toroidal flux-tube.

There is a solution of Einstein's Equations for a static linear field. First, consider a quark placed at the origin and an antiquark placed on the x-axis with their gluonic colour field confined by a flux-tube. It will have components of the energy-momentum tensor as derived from Dingle's formulae (Tolman, 1934, p. 253), for the line element:

$$ds^2 = -\gamma^{-2}dx^2 - dy^2 - dz^2 + \gamma^2 dt^2 \quad . \tag{3.8}$$

These components are mathematically:

$$8\pi\left(\frac{S'}{c^4}\right)T_1^1 = 8\pi\left(\frac{S'}{c^4}\right)T_4^4 = 0 \quad , \tag{3.9}$$

$$8\pi\left(\frac{S'}{c^4}\right)T_2^2 = 8\pi\left(\frac{S'}{c^4}\right)T_3^3 = -\gamma\frac{d^2\gamma}{dx^2} - \left(\frac{d\gamma}{dx}\right)^2 \quad . \tag{3.10}$$

This expression for tangential momentum density T_2^2 can be interpreted like (3.5) as the colour field charge component shown separate from its quantum component. Now, let *x* represent distance around a circumferential flux-tube with internal linear colour field. Then, upon introducing γ from (3.3) and working with radius r_1 for convenience, we get:

$$8\pi\left(\frac{S}{c^4}\right)T_2^2 = \left(\frac{\gamma}{2\sqrt{2}\ r_1{}^2}\right) - \left(\frac{1}{2\sqrt{2}\ r_1}\right)^2 \quad . \tag{3.11}$$

Given the form of (3.4) for a radial field of a quark, we will interpret the longitudinal momentum density T_1^1 in (3.9) as describing the zero *sum* of linear opposing fields propagating both ways around the circumference, and between matter and anti-matter. It can be re-stated in material terms more like (3.11) by proposing the form:

$$8\pi\left(\frac{S}{c^4}\right)T_1^1 = 8\pi\left(\frac{S}{c^4}\right)T_4^4 \Rightarrow \left|\frac{-1}{8\ r_1{}^2}\right| + \left|\frac{1}{8r_1{}^2}\right| \quad . \tag{3.12}$$

Integration of this energy density T_4^4 from ($r_1 = r_z = r_{q1}\exp(-2\sqrt{2})$, where $\gamma = 0$) to ($r_1 = \infty$) will then lead to the total colour field energy. This integration represents the colour field wrapped in many turns around the circumference:

$$\left(\frac{S}{c^4}\right)\int_{r_1=r_z}^{\infty} T_4^4(4\pi\ r_z{}^2)dr \Rightarrow \left|\frac{-r_z}{16}\right| + \left|\frac{r_z}{16}\right| \quad . \tag{3.13}$$

On the left, $(4\pi r_z^2)$ is a workable representative area for the flux-tube. Upon setting [$r_z = SM_C/c^2$], analogous to the proton, then the total *colour* field energy is 12.5% of the charmonium total mass energy:

$$W = -\int_{r_z}^{\infty} T_4^4 (4\pi\, r_z{}^2)dr = \frac{1}{8} M_C c^2 \quad . \tag{3.14}$$

The gluons carrying the colour field account for the same amount of mass energy. Tangential momentum density may also be integrated to get a similar result:

$$\left(\frac{S}{c^4}\right)\int_{r_z}^{\infty} T_2^2 (4\pi\, r_z{}^2)dr = \left|\frac{-r_z}{16}\right| + \left|\frac{r_z}{16}\right| \quad . \tag{3.15}$$

This means that on average the colour field charge and quanta have unitary helicity. Apparently, proposing (3.12) was reasonable in order to interpret (3.9).

Given these qualities of the logarithmic potential, it appears that the compound Coulomb + linear potential [$V(r) = -\kappa/r + r/a^2$] cannot be viable because integration of the *corresponding* tangential momentum density T_2^2 in (3.10) would go to infinity. In fact, the basic *classical* energy density term $(dV(r)/dr)^2$ would integrate to infinity. Quigg (1998) showed how the logarithmic potential fits the data well but he saw no reason to attach fundamental significance to it. Upon inspection here, only the logarithmic potential has viable status.

4 Conclusion

A physical model for charmonium has been developed by applying the logarithmic confinement potential to Einstein's equations of general relativity. The quark and antiquark orbit around a common centre of mass, with their gluonic colour field confined to a toroidal flux-tube. The *fundamental* radius is $r_q/2 \approx 0.09$fm, but the empirical charmonium excited states are larger and more massive, as derived by Quigg and Rosner. Half the charmonium total energy exists in the external radial hadronic field, an eighth in the colour field, and an eighth in the gluon bodies located within the quark and anti-quark. This leaves a quarter of the energy for binding the internal mechanism of the quark and anti-quark. Self-consistency of this theory is conclusive in contrast to the Coulomb + linear potential which is invalid.

Acknowledgements

I would like to thank Imperial College Libraries and R. Simpson for typing.

References

Eichten E. et al. 1978 *Phys. Rev* D **17** 3090-3117

Eichten E. et al. 1980 *Phys Rev* D **21** 203-233

Quigg C. and Rosner J. L. 1977 *Phys Lett* **71B** 153-157

Quigg C. and Rosner J. L. 1979 *Phys Reports* **56** 167-235

Quigg C. 1998 Realizing the Potential of Quarkonium, arXiv:hep-ph/9707493v2

Wayte R. (Paper 1) 2012 A Model of the Electron, http://vixra.org/

Wayte R. (Paper 2) 2019 A Model of the Proton (Revised), http://vixra.org/

Wayte R. (Paper 3) 2018 A Model of Baryons (Revised), http://vixra.org/