Remainder Theorem and the Division by Zero Calculus

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Abstract: In this short note, for the elementary theorem of remainder in polynomials we recall the division by zero calculus that appears naturally in order to show the importance of the division by zero calculus.

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1 Introduction

In this short note, for the elementary theorem of remainder in polynomials we recall the division by zero calculus that appears naturally in order to show the importance of the division by zero calculus.

We found a very interesting question on the relation of the remainder theorem and division by zero on 2019.10.22 at Atsu Gake

Accademia Nuts: https://twitter.com/search?q=

On the good question, we expressed our opinions in the site. We feel that the question is very natural and the problem may be contributed to a good understanding on the division by zero calculus.

The remaider theorem on polynomials may be stated as follows:

For a polynomial $f(x)$ when we look for the value $f(a)$, we divide it by the factor $(x-a)$ as follows:

$$\frac{f(x)}{x-a} = Q(x) \cdots R \tag{1.1}$$

with the remainder R. Then we obtain

$$f(a) = R. \tag{1.2}$$

Here, it seems that we divided there the function $f(x)$ by the zero $(x-a)|_{x=a}$ that was proposed as a question there. The method and idea deriving the theorem look like against for the principle: **"Thou shalt not divide by zero".**

However, for the theorem there is no problem from the identity

$$f(x) = (x-a)Q(x) + R. \tag{1.3}$$

However, we would like to state some general and good view-point on this problem.

2 Division by zero calculus

In order to state our answer for the question in a self-contained manner, we will recall the division by zero calculus.

For any Laurent expansion around $z = a$,

$$f(z) = \sum_{n=-\infty}^{-1} C_n(z-a)^n + C_0 + \sum_{n=1}^{\infty} C_n(z-a)^n,$$

we **define** the division by zero calculus by the identity

$$f(a) = C_0.$$

For many basic properties and applications of the division by zero calculus, see [8] and the references.

In particular, note that for the base of our division by zero, we need the assumption of the definition of the division by zero calculus only, as in an axiom.

3 Our answer and conclusion

Among any polynomials (or generally, analytic functions) we can consider the division; there we do not consider any singular points and zero points. Except singular points and zero points, of course, there is no problem for any division.

Now, by the division by zero calculus, we can consider the values at singular points and there is no problem in the logic for deriving the remainder theorem.

Indeed, in the identity

$$\frac{f(x)}{x-a}|_{x=a} = Q(x)|_{x=a} + \frac{R}{x-a}|_{x=a}, \tag{3.1}$$

the identity

$$\frac{f(x)}{x-a}|_{x=a} = f'(a) = Q(x)|_{x=a} \tag{3.2}$$

is valid, by the division by zero calculus.

For any analytic function $f(z)$ around the origin $z = 0$ that is permitted to have any singularity at $z = 0$ (of course, any constant function is permitted), we can consider the value, by the division by zero calculus

$$\frac{f(z)}{z^n} \tag{3.3}$$

at the point $z = 0$, for any positive integer n. This will mean that from **the form** we can consider it as follows:

$$\frac{f(z)}{z^n} \ |_{z=0} \ . \tag{3.4}$$

For example,

$$\frac{e^x}{x^n} \ |_{x=0} = \frac{1}{n!}.$$

In this sense, we can divide the numbers and analytic functions by zero. For $z \neq 0$, $\frac{f(z)}{z^n}$ means the usual division of the function $f(z)$ by z^n.

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