

On the Ramanujan formulas: mathematical connections with some sectors of Particle physics, in particular on the masses of the dilaton, of the candidate glueball and of the two Pion mesons.

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Abstract

In this research thesis, we have analyzed various Ramanujan equations and described the new possible mathematical connections with some sectors of Particle physics, in particular on the masses of the dilaton, of the candidate glueball and of the two Pion mesons.

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<https://twitter.com/royalsociety/status/1076386910845710337>



<https://biografieonline.it/biografia-enrico-fermi>

From:

Dynamical Gauge Boson of Hidden Local Symmetry within the Standard Model

Koichi Yamawaki - <https://arxiv.org/abs/1803.07271v2>

We now study the nonperturbative dynamics in the large N limit of Eq.(40). The F_π (and hence G) in the classical Lagrangian Eq.(40) should be regarded as the bare quantity and receives quantum corrections in the large N limit. The effective action at leading order of $1/N$ expansion reads:

$$\Gamma_{\text{eff}}[\phi, \eta, \sigma, \rho_\mu] = \int d^D x \frac{1}{2} \text{tr}_{p \times p} [D_\mu \phi (D^\mu \phi)^\dagger - \eta(x) (\phi \phi^\dagger - N \sigma^2 \mathbf{1})] - V(\sigma) + \frac{i}{2} N \text{TrLn}(-D_\mu D^\mu - \eta), \quad (2 \leq D \leq 4), \quad (41)$$

where in D dimensions $\phi(x)$ and $\sigma(x)$ and $\eta(x)$ have a canonical dimension $d_{\phi/\sigma} = D/2 - 1$, and $d_\eta = 2$, respectively, while ρ_μ scales in the same way as the derivative in the covariant derivative, $d_{\rho_\mu} = 1$.

The effective potential for $\langle \phi_{i,\beta}(x) \rangle = \sqrt{N} v(\delta_{i,j}, 0)$ and $\langle \eta_{i,j}(x) \rangle = \eta \delta_{i,j}$, $\langle \sigma(x) \rangle = \sigma$ takes the form:

$$\frac{1}{Np} V_{\text{eff}}(v, \eta, \sigma) = \eta (v^2 - \sigma^2) + \frac{1}{Np} V(\sigma) + \int \frac{d^D k}{i(2\pi)^D} \ln(k^2 - \eta). \quad (42)$$

This yields the gap equation:

$$\frac{1}{Np} \frac{\partial V_{\text{eff}}}{\partial v} = 2\eta v = 0, \quad (43)$$

$$\frac{1}{Np} \frac{\partial V_{\text{eff}}}{\partial \sigma} = -2\eta \sigma + \frac{\hat{\lambda}}{p} \sigma \left(\sigma^2 - \frac{1}{G} \right) = 0, \quad (44)$$

$$\frac{1}{Np} \frac{\partial V_{\text{eff}}}{\partial \eta} = v^2 - \sigma^2 + \int \frac{d^D k}{i(2\pi)^D} \frac{1}{\eta - k^2} = 0. \quad (45)$$

Eq.(45) together with (43) is the same form as that of CP^{N-1} in D dimensions (see e.g., [7, 10]), and implies either of the two cases:

$$\begin{cases} \eta = 0, & v \neq 0; & \text{case (i)} \\ v = 0, & \eta \neq 0; & \text{case (ii)}. \end{cases} \quad (46)$$

Eq.(44) yields two cases:

$$\begin{cases} \sigma = 0, \\ \sigma \neq 0, & -2\eta + \frac{\hat{\lambda}}{p} \left(\sigma^2 - \frac{1}{G} \right) = 0. \end{cases} \quad (47)$$

where the first solution $\sigma = 0$ in Eq.(47) contradicts Eqs.(45) and (43), and hence we are left with the second one, which implies $\eta = 0$ for $\hat{\lambda} \rightarrow 0$, the BPS limit in the broken phase, case (i), while for $\hat{\lambda} \neq 0$ we have:

$$\sigma^2 = \frac{1}{G} + \frac{2p\eta}{\hat{\lambda}}. \quad (48)$$

The stationary condition in Eq.(45) gives a relation between η and v . By putting $\eta = v = 0$ in Eq. (45), the critical point $G(\equiv G(\Lambda)) = G_{\text{crit}}(\equiv G_{\text{crit}}(\Lambda))$ separating the two phases in Eq. (46) is determined as

$$\frac{1}{G_{\text{crit}}} = \int \frac{d^D k}{i(2\pi)^D} \frac{1}{-k^2} = \frac{1}{\left(\frac{D}{2} - 1\right) \Gamma\left(\frac{D}{2}\right)} \frac{\Lambda^{D-2}}{(4\pi)^{\frac{D}{2}}}, \quad (49)$$

by which the integral in Eq.(45) reads:

$$\int \frac{d^D k}{i(2\pi)^D} \frac{1}{\eta - k^2} = \frac{1}{G_{\text{crit}}} - \frac{\Gamma(2 - D/2)}{(D/2 - 1)} \cdot \frac{\eta^{D/2-1}}{(4\pi)^{D/2}}. \quad (50)$$

$$v^2 - \int \frac{d^D k}{i(2\pi)^D} \left(\frac{1}{-k^2} - \frac{1}{\eta - k^2} \right) = \frac{1}{G} - \frac{1}{G_{\text{crit}}} = \frac{1}{G^{(R)}} - \frac{1}{G_{\text{crit}}^{(R)}}, \quad (\text{B13})$$

The stationary condition in Eq. (B13), combined with Eq. (B9), leads to the cases (i) (broken phase of $SU(N)_{\text{global}} \times U(1)_{\text{local}}$) and (ii) (unbroken phase of $SU(N)_{\text{global}} \times U(1)_{\text{local}}$) in Eq. (B11), respectively;

$$\begin{aligned} \text{(i)} \quad G < G_{\text{cr}} &\Rightarrow \langle \phi_N \rangle = \sqrt{N}v \neq 0, \quad \langle \eta(x) \rangle = \eta = 0 \\ \frac{1}{G(\Lambda)} - \frac{1}{G_{\text{crit}}(\Lambda)} &= \frac{1}{G^{(R)}(\mu)} - \frac{1}{G_{\text{crit}}^{(R)}(\mu)} = v^2 > 0, \end{aligned} \quad (\text{B15})$$

$$\begin{aligned} \text{(ii)} \quad G > G_{\text{cr}} &\Rightarrow \langle \phi_N \rangle = \sqrt{N}v = 0, \quad \langle \eta(x) \rangle = \eta \neq 0 \\ \frac{1}{G(\Lambda)} - \frac{1}{G_{\text{crit}}(\Lambda)} &= \frac{1}{G^{(R)}(\mu)} - \frac{1}{G_{\text{crit}}^{(R)}(\mu)} \\ &= -\frac{\Gamma(2 - D/2)}{(D/2 - 1)} \cdot \frac{\eta^{D/2-1}}{(4\pi)^{D/2}} \equiv -v_\eta^2 < 0. \end{aligned} \quad (\text{B16})$$

The gap equations Eq.(B15) and Eq.(B16) take the same form as that of the D -dimensional NJL model which is also renormalizable for $2 \leq D < 4$ [48, 49], with opposite sign and the same sign, respectively. (See also Eq. (C3) for

We have that:

$$\begin{aligned} \int \frac{d^D k}{i(2\pi)^4} \frac{1}{\eta - k^2} &= \frac{1}{G_{\text{crit}}} - \frac{\Gamma(2 - D/2)}{(D/2 - 1)} \cdot \frac{\eta^{D/2-1}}{(4\pi)^{D/2}} \\ \frac{1}{G^{(R)}(\mu)} - \frac{1}{G_{\text{crit}}^{(R)}(\mu)} &= -\frac{\Gamma(2 - D/2)}{(D/2 - 1)} \cdot \frac{\eta^{D/2-1}}{(4\pi)^{D/2}} \equiv -v_\eta^2 < 0 \end{aligned}$$

For $D = 3$ and $\eta = 5$, we obtain:

$$-\left(\frac{\Gamma(2-3/2) \cdot 5^{0.5}}{(3/2-1) (4\pi)^{3/2}}\right)$$

Input:

$$\frac{\Gamma\left(2 - \frac{3}{2}\right) \sqrt{5}}{\left(\frac{3}{2} - 1\right) (4\pi)^{3/2}}$$

$\Gamma(x)$ is the gamma function

Exact result:

$$-\frac{\sqrt{5}}{4\pi}$$

Decimal approximation:

$$-0.17794063585429426461919066910095076625888875596247909884\dots$$

$$-0.17794063585\dots$$

Property:

$$-\frac{\sqrt{5}}{4\pi} \text{ is a transcendental number}$$

Alternative representations:

$$-\frac{\Gamma\left(2 - \frac{3}{2}\right) \sqrt{5}}{\left(\frac{3}{2} - 1\right) (4\pi)^{3/2}} = -\frac{\sqrt{5} e^{-\log G(1/2) + \log G(3/2)}}{\frac{1}{2} (4\pi)^{3/2}}$$

$$-\frac{\Gamma\left(2 - \frac{3}{2}\right) \sqrt{5}}{\left(\frac{3}{2} - 1\right) (4\pi)^{3/2}} = -\frac{\left(-\frac{1}{2}\right)! \sqrt{5}}{\frac{1}{2} (4\pi)^{3/2}}$$

$$-\frac{\Gamma\left(2 - \frac{3}{2}\right) \sqrt{5}}{\left(\frac{3}{2} - 1\right) (4\pi)^{3/2}} = -\frac{\Gamma\left(\frac{1}{2}, 0\right) \sqrt{5}}{\frac{1}{2} (4\pi)^{3/2}}$$

Series representations:

$$-\frac{\Gamma\left(2 - \frac{3}{2}\right) \sqrt{5}}{\left(\frac{3}{2} - 1\right) (4\pi)^{3/2}} = -\frac{\sqrt{5} \sum_{k=0}^{\infty} \frac{2^{-k} \Gamma^{(k)}(1)}{k!}}{2\pi^{3/2}}$$

$$-\frac{\Gamma\left(2 - \frac{3}{2}\right) \sqrt{5}}{\left(\frac{3}{2} - 1\right) (4\pi)^{3/2}} = -\frac{\sqrt{5} \sum_{k=0}^{\infty} \frac{\left(\frac{1}{2} - z_0\right)^k \Gamma^{(k)}(z_0)}{k!}}{4\pi^{3/2}} \quad \text{for } (z_0 \notin \mathbb{Z} \text{ or } z_0 > 0)$$

$$-\frac{\Gamma\left(2 - \frac{3}{2}\right) \sqrt{5}}{\left(\frac{3}{2} - 1\right) (4\pi)^{3/2}} = -\frac{\sqrt{5}}{4\sqrt{\pi} \sum_{k=0}^{\infty} \left(\frac{1}{2} - z_0\right)^k \sum_{j=0}^k \frac{(-1)^j \pi^{-j+k} \sin\left(\frac{1}{2} \pi (-j+k+2z_0)\right) \Gamma^{(j)}(1-z_0)}{j!(-j+k)!}}$$

Integral representations:

$$-\frac{\Gamma\left(2 - \frac{3}{2}\right) \sqrt{5}}{\left(\frac{3}{2} - 1\right) (4\pi)^{3/2}} = -\frac{\sqrt{5}}{4\pi^{3/2}} \int_0^{\infty} \frac{e^{-t}}{\sqrt{t}} dt$$

$$-\frac{\Gamma\left(2 - \frac{3}{2}\right) \sqrt{5}}{\left(\frac{3}{2} - 1\right) (4\pi)^{3/2}} = -\frac{\sqrt{5}}{4\pi^{3/2}} \int_0^1 \frac{1}{\sqrt{\log\left(\frac{1}{t}\right)}} dt$$

$$-\frac{\Gamma\left(2 - \frac{3}{2}\right) \sqrt{5}}{\left(\frac{3}{2} - 1\right) (4\pi)^{3/2}} = -\frac{\sqrt{5} \csc\left(\frac{\pi}{4}\right)}{4\pi^{3/2}} \int_0^{\infty} \frac{\sin(t)}{\sqrt{t}} dt$$

From:

Collected Papers of
SRINIVASA RAMANUJAN

Edited by
G. H. HARDY
P. V. SESHU AIYAR
and
B. M. WILSON

Cambridge
AT THE UNIVERSITY PRESS
1927

From the following Ramanujan equation:

$$\int_0^{\infty} |\Gamma(a+ix)\Gamma(b+ix)|^2 dx = \frac{1}{2} \sqrt{\pi} \frac{\Gamma(a)\Gamma(a+\frac{1}{2})\Gamma(b)\Gamma(b+\frac{1}{2})\Gamma(a+b)}{\Gamma(a+b+\frac{1}{2})}$$

For a = 3 and b = 5, we obtain:

$$\frac{\sqrt{\pi}/2 * (((\Gamma(3)\Gamma(3+1/2)\Gamma(5)\Gamma(5+1/2)\Gamma(8))))}{((\Gamma(3+5+1/2)))}$$

Input:

$$\frac{\sqrt{\pi}}{2} \times \frac{\Gamma(3)\Gamma(3+\frac{1}{2})\Gamma(5)\Gamma(5+\frac{1}{2})\Gamma(8)}{\Gamma(3+5+\frac{1}{2})}$$

Γ(x) is the gamma function

Exact result:

$$\frac{120960\pi}{143}$$

Decimal approximation:

2657.391939707841888982107298192228453653773500338551049686...

2657.3919397....

Property:

$\frac{120960\pi}{143}$ is a transcendental number

Alternative representations:

$$\frac{(\Gamma(3)\Gamma(3+\frac{1}{2})\Gamma(5)\Gamma(5+\frac{1}{2})\Gamma(8))\sqrt{\pi}}{\Gamma(3+5+\frac{1}{2})2} = \frac{2! \times \frac{5}{2}! \times 4! \times \frac{9}{2}! \times 7! \sqrt{\pi}}{2 \times \frac{15}{2}!}$$

$$\frac{(\Gamma(3)\Gamma(3+\frac{1}{2})\Gamma(5)\Gamma(5+\frac{1}{2})\Gamma(8))\sqrt{\pi}}{\Gamma(3+5+\frac{1}{2})2} = \frac{(e^{\log(2)} e^{-\log(12)+\log(288)} e^{-\log(24883200)+\log(125411328000)} e^{-\log G(7/2)+\log G(9/2)} e^{-\log G(11/2)+\log G(13/2)} \sqrt{\pi}) / (2 e^{-\log G(17/2)+\log G(19/2)})}{\Gamma(3+5+\frac{1}{2})2}$$

$$\frac{(\Gamma(3)\Gamma(3+\frac{1}{2})\Gamma(5)\Gamma(5+\frac{1}{2})\Gamma(8))\sqrt{\pi}}{\Gamma(3+5+\frac{1}{2})2} = \frac{(1)_2 (1)_{\frac{5}{2}} (1)_4 (1)_{\frac{9}{2}} (1)_7 \sqrt{\pi}}{2 (1)_{\frac{15}{2}}}$$

Series representations:

$$\frac{(\Gamma(3)\Gamma(3+\frac{1}{2})\Gamma(5)\Gamma(5+\frac{1}{2})\Gamma(8))\sqrt{\pi}}{\Gamma(3+5+\frac{1}{2})2} = \frac{483840}{143} \sum_{k=0}^{\infty} \frac{(-1)^k}{1+2k}$$

$$\frac{(\Gamma(3)\Gamma(3+\frac{1}{2})\Gamma(5)\Gamma(5+\frac{1}{2})\Gamma(8))\sqrt{\pi}}{\Gamma(3+5+\frac{1}{2})2} = \sum_{k=0}^{\infty} \frac{96768 \left(-\frac{1}{25}\right)^k 239^{-1-2k} (5^{1+2k} - 4 \times 239^{1+2k})}{143 (1+2k)}$$

$$\frac{(\Gamma(3)\Gamma(3+\frac{1}{2})\Gamma(5)\Gamma(5+\frac{1}{2})\Gamma(8))\sqrt{\pi}}{\Gamma(3+5+\frac{1}{2})2} = \frac{120960}{143} \sum_{k=0}^{\infty} \left(-\frac{1}{4}\right)^k \left(\frac{1}{1+2k} + \frac{2}{1+4k} + \frac{1}{3+4k}\right)$$

Integral representations:

$$\frac{(\Gamma(3)\Gamma(3+\frac{1}{2})\Gamma(5)\Gamma(5+\frac{1}{2})\Gamma(8))\sqrt{\pi}}{\Gamma(3+5+\frac{1}{2})2} = \frac{483840}{143} \int_0^1 \sqrt{1-t^2} dt$$

$$\frac{(\Gamma(3)\Gamma(3+\frac{1}{2})\Gamma(5)\Gamma(5+\frac{1}{2})\Gamma(8))\sqrt{\pi}}{\Gamma(3+5+\frac{1}{2})2} = \frac{241920}{143} \int_0^1 \frac{1}{\sqrt{1-t^2}} dt$$

$$\frac{(\Gamma(3)\Gamma(3+\frac{1}{2})\Gamma(5)\Gamma(5+\frac{1}{2})\Gamma(8))\sqrt{\pi}}{\Gamma(3+5+\frac{1}{2})^2} = \frac{241920}{143} \int_0^\infty \frac{1}{1+t^2} dt$$

We note that 2657.3919397.... value very near to the rest mass of charmed Xi baryon 2645.49 or to the average mass of:

$D_s^*(2600)$ MASS

VALUE (MeV)	EVTS	DOCUMENT ID	TECN	CHG	COMMENT
2623 ±12 OUR AVERAGE		Error includes scale factor of 4.8. See the ideogram below.			
2681.1 ± 5.6 ±14.0	28k	¹ AAIJ	16AH LHCb		$B^- \rightarrow D^+ \pi^- \pi^-$
2649.2 ± 3.5 ± 3.5	51k	AAIJ	13CC LHCb		$p p \rightarrow D^{*+} \pi^- X$
2608.7 ± 2.4 ± 2.5	26k	DEL-AMO-SA..10P	BABR	0	$e^+ e^- \rightarrow D^+ \pi^- X$
2621.3 ± 3.7 ± 4.2	13k	² DEL-AMO-SA..10P	BABR	+	$e^+ e^- \rightarrow D^0 \pi^+ X$

¹ From the amplitude analysis in the model describing the $D^+ \pi^-$ wave together with virtual contributions from the $D^*(2007)^0$ and B^{*0} states, and components corresponding to the $D_2^*(2460)^0$, $D_1^*(2680)^0$, $D_3^*(2760)^0$, and $D_2^*(3000)^0$ resonances.

² At a fixed width of 93 MeV.

Indeed: $(2623+2681.1+2649.2) / 3 = 2651.1$

From this expression, we obtain also:

$$-1/\pi * 1/((((sqrt(\pi)/2 * (((gamma(3) gamma(3+1/2) gamma(5) gamma(5+1/2) gamma(8)))) / (((gamma(3+5+1/2))))))))^1/13$$

Input:

$$\frac{1}{\pi \sqrt[13]{\frac{\sqrt{\pi}}{2} \times \frac{\Gamma(3)\Gamma(3+\frac{1}{2})\Gamma(5)\Gamma(5+\frac{1}{2})\Gamma(8)}{\Gamma(3+5+\frac{1}{2})}}}$$

$\Gamma(x)$ is the gamma function

Exact result:

$$\frac{\sqrt[13]{\frac{143}{35}}}{2^{7/13} \times 3^{3/13} \pi^{14/13}}$$

Decimal approximation:

-0.17355233644890782676397396090563169425991430099077432357...

-0.1735523364489...

Property:

$$-\frac{\sqrt[13]{\frac{143}{35}}}{2^{7/13} \times 3^{3/13} \pi^{14/13}}$$

is a transcendental number

Alternate form:

$\text{root of } 120960x^{13} + 143 \text{ near } x = -0.595419$
$\pi^{14/13}$

Alternative representations:

$$-\frac{1}{\sqrt[13]{\frac{\sqrt{\pi} \Gamma(3)\Gamma(3+\frac{1}{2})\Gamma(5)\Gamma(5+\frac{1}{2})\Gamma(8)}{2\Gamma(3+5+\frac{1}{2})}} \pi} = -\frac{1}{\pi \sqrt[13]{\frac{2! \times \frac{5}{2}! \times 4! \times \frac{9}{2}! \times 7! \sqrt{\pi}}{2 \times \frac{15}{2}!}}}$$

$$-\frac{1}{\sqrt[13]{\frac{\sqrt{\pi} \Gamma(3)\Gamma(3+\frac{1}{2})\Gamma(5)\Gamma(5+\frac{1}{2})\Gamma(8)}{2\Gamma(3+5+\frac{1}{2})}} \pi} = -\frac{1}{\pi \sqrt[13]{\frac{(1)_2 (1)_5 (1)_4 (1)_9 (1)_7 \sqrt{\pi}}{2 (1)_{15/2}}}}$$

$$-\frac{1}{\sqrt[13]{\frac{\sqrt{\pi} \Gamma(3)\Gamma(3+\frac{1}{2})\Gamma(5)\Gamma(5+\frac{1}{2})\Gamma(8)}{2\Gamma(3+5+\frac{1}{2})}} \pi} = -\frac{1}{\pi \sqrt[13]{\frac{\Gamma(3,0)\Gamma(\frac{7}{2},0)\Gamma(5,0)\Gamma(\frac{11}{2},0)\Gamma(8,0)\sqrt{\pi}}{2\Gamma(\frac{17}{2},0)}}$$

Series representations:

$$-\frac{1}{\sqrt[13]{\frac{\sqrt{\pi} \Gamma(3)\Gamma(3+\frac{1}{2})\Gamma(5)\Gamma(5+\frac{1}{2})\Gamma(8)}{2\Gamma(3+5+\frac{1}{2})}} \pi} = -\frac{\sqrt[13]{\frac{143}{35}}}{4 \times 2^{9/13} \times 3^{3/13} \left(\sum_{k=0}^{\infty} \frac{(-1)^k}{1+2k} \right)^{14/13}}$$

$$\frac{1}{\sqrt[13]{\frac{\sqrt{\pi} \Gamma(3)\Gamma(3+\frac{1}{2})\Gamma(5)\Gamma(5+\frac{1}{2})\Gamma(8)}{2\Gamma(3+5+\frac{1}{2})}} \pi} = \frac{\sqrt[13]{\frac{143}{35}}}{2^{7/13} \times 3^{3/13} \left(\sum_{k=0}^{\infty} \frac{4(-1)^k 1195^{-1-2k} (5^{1+2k} - 4 \times 239^{1+2k})}{1+2k} \right)^{14/13}}$$

$$\frac{1}{\sqrt[13]{\frac{\sqrt{\pi} \Gamma(3)\Gamma(3+\frac{1}{2})\Gamma(5)\Gamma(5+\frac{1}{2})\Gamma(8)}{2\Gamma(3+5+\frac{1}{2})}} \pi} = \frac{\sqrt[13]{\frac{143}{35}}}{2^{7/13} \times 3^{3/13} \left(\sum_{k=0}^{\infty} \left(-\frac{1}{4}\right)^k \left(\frac{1}{1+2k} + \frac{2}{1+4k} + \frac{1}{3+4k}\right) \right)^{14/13}}$$

Integral representations:

$$\frac{1}{\sqrt[13]{\frac{\sqrt{\pi} \Gamma(3)\Gamma(3+\frac{1}{2})\Gamma(5)\Gamma(5+\frac{1}{2})\Gamma(8)}{2\Gamma(3+5+\frac{1}{2})}} \pi} = \frac{\sqrt[13]{\frac{143}{35}}}{2 \times 2^{8/13} \times 3^{3/13} \left(\int_0^1 \frac{1}{\sqrt{1-t^2}} dt \right)^{14/13}}$$

$$\frac{1}{\sqrt[13]{\frac{\sqrt{\pi} \Gamma(3)\Gamma(3+\frac{1}{2})\Gamma(5)\Gamma(5+\frac{1}{2})\Gamma(8)}{2\Gamma(3+5+\frac{1}{2})}} \pi} = \frac{\sqrt[13]{\frac{143}{35}}}{4 \times 2^{9/13} \times 3^{3/13} \left(\int_0^1 \sqrt{1-t^2} dt \right)^{14/13}}$$

$$\frac{1}{\sqrt[13]{\frac{\sqrt{\pi} \Gamma(3)\Gamma(3+\frac{1}{2})\Gamma(5)\Gamma(5+\frac{1}{2})\Gamma(8)}{2\Gamma(3+5+\frac{1}{2})}} \pi} = \frac{\sqrt[13]{\frac{143}{35}}}{2 \times 2^{8/13} \times 3^{3/13} \left(\int_0^{\infty} \frac{1}{1+t^2} dt \right)^{14/13}}$$

and:

$$-1/(1.1424432422+0.9243408674589+1) \ 1/((((((\sqrt{\text{Pi}})/2 * (((\text{gamma}(3) \text{gamma}(3+1/2) \text{gamma}(5) \text{gamma}(5+1/2) \text{gamma}(8)))))) / (((\text{gamma}(3+5+1/2))))))))))^{1/13}$$

where 1.1424432422 and 0.9243408674589 are two results of Ramanujan mock theta functions

Input interpretation:

$$\frac{1}{\sqrt[13]{\frac{\sqrt{\pi}}{2} \times \frac{\Gamma(3)\Gamma(3+\frac{1}{2})\Gamma(5)\Gamma(5+\frac{1}{2})\Gamma(8)}{\Gamma(3+5+\frac{1}{2})}}} - \frac{1}{1.1424432422 + 0.9243408674589 + 1}$$

Γ(x) is the gamma function

Result:

-0.17778582571...
-0.17778582571...

Alternative representations:

$$\frac{1}{\sqrt[13]{\frac{\sqrt{\pi} (\Gamma(3)\Gamma(3+\frac{1}{2})\Gamma(5)\Gamma(5+\frac{1}{2})\Gamma(8))}{2\Gamma(3+5+\frac{1}{2})}} (1.14244324220000 + 0.92434086745890000 + 1)}} =$$

$$\frac{1}{3.06678410965890 \sqrt[13]{\frac{2! \times \frac{5!}{2} \times 4! \times \frac{9!}{2} \times 7! \sqrt{\pi}}{2 \times \frac{15!}{2}}}}$$

$$\frac{1}{\sqrt[13]{\frac{\sqrt{\pi} (\Gamma(3)\Gamma(3+\frac{1}{2})\Gamma(5)\Gamma(5+\frac{1}{2})\Gamma(8))}{2\Gamma(3+5+\frac{1}{2})}} (1.14244324220000 + 0.92434086745890000 + 1)}} =$$

$$\frac{1}{3.06678410965890 \sqrt[13]{\frac{(1)_2 (1)_5 (1)_4 (1)_9 (1)_7 \sqrt{\pi}}{2 \frac{(1)_{15}}{2}}}}$$

$$\frac{1}{\sqrt[13]{\frac{\sqrt{\pi} (\Gamma(3)\Gamma(3+\frac{1}{2})\Gamma(5)\Gamma(5+\frac{1}{2})\Gamma(8))}{2\Gamma(3+5+\frac{1}{2})}} (1.14244324220000 + 0.92434086745890000 + 1)}} =$$

$$\frac{1}{3.06678410965890 \sqrt[13]{\frac{\Gamma(3,0)\Gamma(\frac{7}{2},0)\Gamma(5,0)\Gamma(\frac{11}{2},0)\Gamma(8,0)\sqrt{\pi}}{2\Gamma(\frac{17}{2},0)}}$$

Integral representations:

$$\begin{aligned}
& \frac{1}{\sqrt[13]{\frac{\sqrt{\pi} \Gamma(3)\Gamma(3+\frac{1}{2})\Gamma(5)\Gamma(5+\frac{1}{2})\Gamma(8)}{2\Gamma(3+5+\frac{1}{2})} (1.14244324220000 + 0.92434086745890000 + 1)}} \\
& \frac{1}{\sqrt[13]{\exp\left(\int_0^1 \frac{-25-25\sqrt{x}+8x+8x^{3/2}+8x^2+8x^{5/2}+6x^3+4x^{7/2}+4x^4+4x^{9/2}+2x^5-2x^8}{2(1+\sqrt{x})\log(x)} dx\right)\sqrt{\pi}}} \\
& \frac{1}{\sqrt[13]{\frac{\sqrt{\pi} \Gamma(3)\Gamma(3+\frac{1}{2})\Gamma(5)\Gamma(5+\frac{1}{2})\Gamma(8)}{2\Gamma(3+5+\frac{1}{2})} (1.14244324220000 + 0.92434086745890000 + 1)}} \\
& -\left(0.343932288275408 / \left(\exp\left(-\frac{33\gamma}{2} + \int_0^1 \left((4-x^3-x^{7/2}-x^5-x^{11/2}-x^8+x^{17/2} + \log(x^3) + \log(x^{7/2}) + \log(x^5) + \log(x^{11/2}) + \log(x^8) - \log(x^{17/2})) / (\log(x) - x \log(x)) \right) dx \right) \sqrt{\pi} \right)^{1/13}\right) \\
& \frac{1}{\sqrt[13]{\frac{\sqrt{\pi} \Gamma(3)\Gamma(3+\frac{1}{2})\Gamma(5)\Gamma(5+\frac{1}{2})\Gamma(8)}{2\Gamma(3+5+\frac{1}{2})} (1.14244324220000 + 0.92434086745890000 + 1)}} \\
& -\left(0.343932288275408 \left(\int_0^1 \log^{15/2}\left(\frac{1}{t}\right) dt\right) \left(\int_0^1 \int_0^1 \int_0^1 \int_0^1 \int_0^1 \log^2\left(\frac{1}{t_1}\right) \log^{5/2}\left(\frac{1}{t_2}\right) \log^4\left(\frac{1}{t_3}\right) \log^{9/2}\left(\frac{1}{t_4}\right) \log^7\left(\frac{1}{t_5}\right) dt_5 dt_4 dt_3 dt_2 dt_1\right)^{12/13} / \left(\int_0^1 \log^2\left(\frac{1}{t}\right) dt\right) \left(\int_0^1 \log^{5/2}\left(\frac{1}{t}\right) dt\right) \left(\int_0^1 \log^4\left(\frac{1}{t}\right) dt\right) \left(\int_0^1 \log^{9/2}\left(\frac{1}{t}\right) dt\right) \left(\int_0^1 \log^7\left(\frac{1}{t}\right) dt\right) \sqrt{\pi}\right)
\end{aligned}$$

Thence, the following mathematical connection:

$$\left(-\frac{\Gamma\left(2-\frac{3}{2}\right)\sqrt{5}}{\left(\frac{3}{2}-1\right)(4\pi)^{3/2}} \right) = -0.17794063585... \Rightarrow$$

$$\Rightarrow \left(\frac{\sqrt[13]{\frac{\sqrt{\pi}}{2} \times \frac{\Gamma(3)\Gamma(3+\frac{1}{2})\Gamma(5)\Gamma(5+\frac{1}{2})\Gamma(8)}{\Gamma(3+5+\frac{1}{2})}}}{1.1424432422 + 0.9243408674589 + 1} \right) = -0.17778582571\dots$$

$$-0.17794063585 \approx -0.17778582571\dots$$

Now, we have that:

$$\int_0^{\infty} \frac{dx}{\{1 + x^2/a^2\} \{1 + x^2/(a+1)^2\} \dots \{1 + x^2/b^2\} \{1 + x^2/(b+1)^2\} \dots} = \frac{1}{2} \sqrt{\pi} \frac{\Gamma(a + \frac{1}{2}) \Gamma(b + \frac{1}{2}) \Gamma(a+b)}{\Gamma(a) \Gamma(b) \Gamma(a+b + \frac{1}{2})},$$

From

$$\frac{1}{2} \sqrt{\pi} \frac{\Gamma(a + \frac{1}{2}) \Gamma(b + \frac{1}{2}) \Gamma(a+b)}{\Gamma(a) \Gamma(b) \Gamma(a+b + \frac{1}{2})}$$

We obtain:

$$\sqrt{\pi}/2 * ((((((\text{gamma}(3+1/2) \text{gamma}(5+1/2) \text{gamma}(8)))))/(((\text{gamma}(3) \text{gamma}(5) \text{gamma}(3+5+1/2))))))))$$

Input:

$$\frac{\sqrt{\pi}}{2} \times \frac{\Gamma(3 + \frac{1}{2}) \Gamma(5 + \frac{1}{2}) \Gamma(8)}{\Gamma(3) \Gamma(5) \Gamma(3 + 5 + \frac{1}{2})}$$

$\Gamma(x)$ is the gamma function

Exact result:

$$\frac{105 \pi}{286}$$

Decimal approximation:

$$1.153381918275973042092928514840376933009450303966385004204\dots$$

1.1533819182...

Property:

$\frac{105\pi}{286}$ is a transcendental number

Alternative representations:

$$\frac{\left(\Gamma\left(3 + \frac{1}{2}\right)\Gamma\left(5 + \frac{1}{2}\right)\Gamma(8)\right)\sqrt{\pi}}{\left(\Gamma(3)\Gamma(5)\Gamma\left(3 + 5 + \frac{1}{2}\right)\right)2} = \frac{\frac{5}{2}! \times \frac{9}{2}! \times 7! \sqrt{\pi}}{2\left(2! \times 4! \times \frac{15}{2}!\right)}$$

$$\frac{\left(\Gamma\left(3 + \frac{1}{2}\right)\Gamma\left(5 + \frac{1}{2}\right)\Gamma(8)\right)\sqrt{\pi}}{\left(\Gamma(3)\Gamma(5)\Gamma\left(3 + 5 + \frac{1}{2}\right)\right)2} = \frac{e^{-\log(24\,883\,200)+\log(125\,411\,328\,000)} e^{-\log G(7/2)+\log G(9/2)} e^{-\log G(11/2)+\log G(13/2)} \sqrt{\pi}}{2\left(e^{\log(2)} e^{-\log(12)+\log(288)} e^{-\log G(17/2)+\log G(19/2)}\right)}$$

$$\frac{\left(\Gamma\left(3 + \frac{1}{2}\right)\Gamma\left(5 + \frac{1}{2}\right)\Gamma(8)\right)\sqrt{\pi}}{\left(\Gamma(3)\Gamma(5)\Gamma\left(3 + 5 + \frac{1}{2}\right)\right)2} = \frac{(1)_{\frac{5}{2}} (1)_{\frac{9}{2}} (1)_7 \sqrt{\pi}}{2\left((1)_2 (1)_4 (1)_{\frac{15}{2}}\right)}$$

Series representations:

$$\frac{\left(\Gamma\left(3 + \frac{1}{2}\right)\Gamma\left(5 + \frac{1}{2}\right)\Gamma(8)\right)\sqrt{\pi}}{\left(\Gamma(3)\Gamma(5)\Gamma\left(3 + 5 + \frac{1}{2}\right)\right)2} = \frac{210}{143} \sum_{k=0}^{\infty} \frac{(-1)^k}{1+2k}$$

$$\frac{\left(\Gamma\left(3 + \frac{1}{2}\right)\Gamma\left(5 + \frac{1}{2}\right)\Gamma(8)\right)\sqrt{\pi}}{\left(\Gamma(3)\Gamma(5)\Gamma\left(3 + 5 + \frac{1}{2}\right)\right)2} = \sum_{k=0}^{\infty} \frac{42 \times 239^{-1-2k} \left(-5(-1)^k + 4\left(-\frac{1}{25}\right)^k 239^{1+2k}\right)}{143(1+2k)}$$

$$\frac{\left(\Gamma\left(3 + \frac{1}{2}\right)\Gamma\left(5 + \frac{1}{2}\right)\Gamma(8)\right)\sqrt{\pi}}{\left(\Gamma(3)\Gamma(5)\Gamma\left(3 + 5 + \frac{1}{2}\right)\right)2} = \frac{105}{286} \sum_{k=0}^{\infty} \left(-\frac{1}{4}\right)^k \left(\frac{1}{1+2k} + \frac{2}{1+4k} + \frac{1}{3+4k}\right)$$

Integral representations:

$$\frac{\left(\Gamma\left(3 + \frac{1}{2}\right)\Gamma\left(5 + \frac{1}{2}\right)\Gamma(8)\right)\sqrt{\pi}}{\left(\Gamma(3)\Gamma(5)\Gamma\left(3 + 5 + \frac{1}{2}\right)\right)2} = \frac{210}{143} \int_0^1 \sqrt{1-t^2} dt$$

$$\frac{(\Gamma(3 + \frac{1}{2})\Gamma(5 + \frac{1}{2})\Gamma(8))\sqrt{\pi}}{(\Gamma(3)\Gamma(5)\Gamma(3 + 5 + \frac{1}{2}))2} = \frac{105}{143} \int_0^1 \frac{1}{\sqrt{1-t^2}} dt$$

$$\frac{(\Gamma(3 + \frac{1}{2})\Gamma(5 + \frac{1}{2})\Gamma(8))\sqrt{\pi}}{(\Gamma(3)\Gamma(5)\Gamma(3 + 5 + \frac{1}{2}))2} = \frac{105}{143} \int_0^\infty \frac{1}{1+t^2} dt$$

1/((((sqrt(Pi)/2 * ((((((gamma (3+1/2) gamma (5+1/2) gamma (8))))/(((gamma (3) gamma (5) gamma (3+5+1/2))))))))))))))^(1/8

Input:

$$\sqrt[8]{\frac{1}{\frac{\sqrt{\pi}}{2} \times \frac{\Gamma(3+\frac{1}{2})\Gamma(5+\frac{1}{2})\Gamma(8)}{\Gamma(3)\Gamma(5)\Gamma(3+5+\frac{1}{2})}}$$

Γ(x) is the gamma function

Exact result:

$$\sqrt[8]{\frac{286}{105\pi}}$$

Decimal approximation:

0.982320839865782115693825278108315242791464593090816733233...

0.9823208398.... result very near to the value of the following Rogers-Ramanujan continued fraction:

$$\frac{e^{-\frac{\pi}{\sqrt{5}}}}{\sqrt{5}} = 1 - \frac{e^{-\pi\sqrt{5}}}{1 + \frac{e^{-2\pi\sqrt{5}}}{1 + \frac{e^{-3\pi\sqrt{5}}}{1 + \frac{e^{-4\pi\sqrt{5}}}{1 + \dots}}}} - \varphi + 1 \approx 0.9991104684$$

and to the dilaton value $0.989117352243 = \phi$

Property:

$$\sqrt[8]{\frac{286}{105\pi}}$$
 is a transcendental number

Alternative representations:

$$\frac{1}{\sqrt[8]{\frac{(\Gamma(3+\frac{1}{2})\Gamma(5+\frac{1}{2})\Gamma(8))\sqrt{\pi}}{(\Gamma(3)\Gamma(5)\Gamma(3+5+\frac{1}{2}))^2}}} = \frac{1}{\sqrt[8]{\frac{\frac{5!}{2} \times \frac{9!}{2} \times 7! \sqrt{\pi}}{2(2! \times 4! \times \frac{15!}{2})}}}$$

$$\frac{1}{\sqrt[8]{\frac{(\Gamma(3+\frac{1}{2})\Gamma(5+\frac{1}{2})\Gamma(8))\sqrt{\pi}}{(\Gamma(3)\Gamma(5)\Gamma(3+5+\frac{1}{2}))^2}}} = \frac{1}{\sqrt[8]{\frac{(1)_5 (1)_9 (1)_7 \sqrt{\pi}}{2 \left((1)_2 (1)_4 (1)_{\frac{15}{2}} \right)}}$$

$$\frac{1}{\sqrt[8]{\frac{(\Gamma(3+\frac{1}{2})\Gamma(5+\frac{1}{2})\Gamma(8))\sqrt{\pi}}{(\Gamma(3)\Gamma(5)\Gamma(3+5+\frac{1}{2}))^2}}} = \frac{1}{\sqrt[8]{\frac{\Gamma(\frac{7}{2},0)\Gamma(\frac{11}{2},0)\Gamma(8,0)\sqrt{\pi}}{2(\Gamma(3,0)\Gamma(5,0)\Gamma(\frac{17}{2},0))}}}$$

Series representations:

$$\frac{1}{\sqrt[8]{\frac{(\Gamma(3+\frac{1}{2})\Gamma(5+\frac{1}{2})\Gamma(8))\sqrt{\pi}}{(\Gamma(3)\Gamma(5)\Gamma(3+5+\frac{1}{2}))^2}}} = \sqrt[8]{\frac{143}{210}} \sqrt[8]{\frac{1}{\sum_{k=0}^{\infty} \frac{(-1)^k}{1+2k}}}$$

$$\frac{1}{\sqrt[8]{\frac{(\Gamma(3+\frac{1}{2})\Gamma(5+\frac{1}{2})\Gamma(8))\sqrt{\pi}}{(\Gamma(3)\Gamma(5)\Gamma(3+5+\frac{1}{2}))^2}}} = \sqrt[8]{\frac{286}{105}} \sqrt[8]{\frac{1}{\sum_{k=0}^{\infty} \left(-\frac{1}{4}\right)^k \left(\frac{1}{1+2k} + \frac{2}{1+4k} + \frac{1}{3+4k}\right)}}$$

$$\frac{1}{\sqrt[8]{\frac{(\Gamma(3+\frac{1}{2})\Gamma(5+\frac{1}{2})\Gamma(8))\sqrt{\pi}}{(\Gamma(3)\Gamma(5)\Gamma(3+5+\frac{1}{2}))^2}}} = \sqrt[8]{\frac{143}{210}} \sqrt[8]{\frac{1}{\sum_{k=0}^{\infty} \frac{(-1)^{1+k} 1195^{-1-2k} (5^{1+2k} - 4 \times 239^{1+2k})}{1+2k}}}$$

Integral representations:

$$\frac{1}{\sqrt[8]{\frac{(\Gamma(3+\frac{1}{2})\Gamma(5+\frac{1}{2})\Gamma(8))\sqrt{\pi}}{(\Gamma(3)\Gamma(5)\Gamma(3+5+\frac{1}{2}))^2}}} = \sqrt[8]{\frac{143}{105}} \sqrt[8]{\int_0^\infty \frac{1}{1+t^2} dt}$$

$$\frac{1}{\sqrt[8]{\frac{(\Gamma(3+\frac{1}{2})\Gamma(5+\frac{1}{2})\Gamma(8))\sqrt{\pi}}{(\Gamma(3)\Gamma(5)\Gamma(3+5+\frac{1}{2}))^2}}} = \sqrt[8]{\frac{143}{210}} \sqrt[8]{\int_0^1 \sqrt{1-t^2} dt}$$

$$\frac{1}{\sqrt[8]{\frac{(\Gamma(3+\frac{1}{2})\Gamma(5+\frac{1}{2})\Gamma(8))\sqrt{\pi}}{(\Gamma(3)\Gamma(5)\Gamma(3+5+\frac{1}{2}))^2}}} = \sqrt[8]{\frac{143}{105}} \sqrt[8]{\int_0^1 \frac{1}{\sqrt{1-t^2}} dt}$$

From which, we obtain:

$$-1/(4*\text{golden ratio}) * \text{sqrt}(\text{Pi})/2 * (((((\text{gamma}(3+1/2) \text{gamma}(5+1/2) \text{gamma}(8))))/((\text{gamma}(3) \text{gamma}(5) \text{gamma}(3+5+1/2))))))))$$

Input:

$$\frac{\frac{\sqrt{\pi}}{2} \times \frac{\Gamma(3+\frac{1}{2})\Gamma(5+\frac{1}{2})\Gamma(8)}{\Gamma(3)\Gamma(5)\Gamma(3+5+\frac{1}{2})}}{4\phi}$$

$\Gamma(x)$ is the gamma function

ϕ is the golden ratio

Exact result:

$$\frac{105\pi}{1144\phi}$$

Decimal approximation:

-0.17820730687602621557816511463818450660108421228482086665...

-0.178207306876....

Property:

$-\frac{105\pi}{1144\phi}$ is a transcendental number

Alternate forms:

$$\frac{(105 - 105\sqrt{5})\pi}{2288}$$

$$-\frac{105\pi}{572(1 + \sqrt{5})}$$

Alternative representations:

$$\frac{(\sqrt{\pi} \left(\Gamma\left(3 + \frac{1}{2}\right) \Gamma\left(5 + \frac{1}{2}\right) \Gamma(8) \right)) (-1)}{\left(2 \left(\Gamma(3) \Gamma(5) \Gamma\left(3 + 5 + \frac{1}{2}\right) \right) \right) (4\phi)} = -\frac{\frac{5}{2}! \times \frac{9}{2}! \times 7! \sqrt{\pi}}{2(4\phi) \left(2! \times 4! \times \frac{15}{2}! \right)}$$

$$\frac{(\sqrt{\pi} \left(\Gamma\left(3 + \frac{1}{2}\right) \Gamma\left(5 + \frac{1}{2}\right) \Gamma(8) \right)) (-1)}{\left(2 \left(\Gamma(3) \Gamma(5) \Gamma\left(3 + 5 + \frac{1}{2}\right) \right) \right) (4\phi)} = \frac{e^{-\log(24883200) + \log(125411328000)} e^{-\log G(7/2) + \log G(9/2)} e^{-\log G(11/2) + \log G(13/2)} \sqrt{\pi}}{2(4\phi) \left(e^{\log(2)} e^{-\log(12) + \log(288)} e^{-\log G(17/2) + \log G(19/2)} \right)}$$

$$\frac{(\sqrt{\pi} \left(\Gamma\left(3 + \frac{1}{2}\right) \Gamma\left(5 + \frac{1}{2}\right) \Gamma(8) \right)) (-1)}{\left(2 \left(\Gamma(3) \Gamma(5) \Gamma\left(3 + 5 + \frac{1}{2}\right) \right) \right) (4\phi)} = -\frac{\binom{1}{2} \binom{1}{2} \binom{1}{7} \sqrt{\pi}}{2(4\phi) \left(\binom{1}{2} \binom{1}{4} \binom{1}{\frac{15}{2}} \right)}$$

Series representations:

$$\frac{(\sqrt{\pi} \left(\Gamma\left(3 + \frac{1}{2}\right) \Gamma\left(5 + \frac{1}{2}\right) \Gamma(8) \right)) (-1)}{\left(2 \left(\Gamma(3) \Gamma(5) \Gamma\left(3 + 5 + \frac{1}{2}\right) \right) \right) (4\phi)} = -\frac{105 \sum_{k=0}^{\infty} \frac{(-1)^k}{1+2k}}{286\phi}$$

$$\frac{(\sqrt{\pi} \left(\Gamma\left(3 + \frac{1}{2}\right) \Gamma\left(5 + \frac{1}{2}\right) \Gamma(8) \right)) (-1)}{\left(2 \left(\Gamma(3) \Gamma(5) \Gamma\left(3 + 5 + \frac{1}{2}\right) \right) \right) (4\phi)} = -\frac{105 \sum_{k=0}^{\infty} \left(-\frac{1}{4}\right)^k \left(\frac{1}{1+2k} + \frac{2}{1+4k} + \frac{1}{3+4k} \right)}{1144\phi}$$

$$\frac{(\sqrt{\pi} (\Gamma(3 + \frac{1}{2}) \Gamma(5 + \frac{1}{2}) \Gamma(8)))^{(-1)}}{(2 (\Gamma(3) \Gamma(5) \Gamma(3 + 5 + \frac{1}{2})))^{(4 \phi)}} = \sum_{k=0}^{\infty} \frac{21 \times 239^{-1-2k} (5 (-1)^k - 4 (-\frac{1}{25})^k 239^{1+2k})}{143 (1 + \sqrt{5}) (1 + 2k)}$$

Integral representations:

$$\frac{(\sqrt{\pi} (\Gamma(3 + \frac{1}{2}) \Gamma(5 + \frac{1}{2}) \Gamma(8)))^{(-1)}}{(2 (\Gamma(3) \Gamma(5) \Gamma(3 + 5 + \frac{1}{2})))^{(4 \phi)}} = -\frac{105}{286 \phi} \int_0^1 \sqrt{1-t^2} dt$$

$$\frac{(\sqrt{\pi} (\Gamma(3 + \frac{1}{2}) \Gamma(5 + \frac{1}{2}) \Gamma(8)))^{(-1)}}{(2 (\Gamma(3) \Gamma(5) \Gamma(3 + 5 + \frac{1}{2})))^{(4 \phi)}} = -\frac{105}{572 \phi} \int_0^1 \frac{1}{\sqrt{1-t^2}} dt$$

$$\frac{(\sqrt{\pi} (\Gamma(3 + \frac{1}{2}) \Gamma(5 + \frac{1}{2}) \Gamma(8)))^{(-1)}}{(2 (\Gamma(3) \Gamma(5) \Gamma(3 + 5 + \frac{1}{2})))^{(4 \phi)}} = -\frac{105}{572 \phi} \int_0^{\infty} \frac{1}{1+t^2} dt$$

Thence, we have another mathematical connection:

$$\left(-\frac{\Gamma(2 - \frac{3}{2}) \sqrt{5}}{(\frac{3}{2} - 1) (4\pi)^{3/2}} \right) = -0.17794063585 \Rightarrow$$

$$\Rightarrow \left(-\frac{\frac{\sqrt{\pi}}{2} \times \frac{\Gamma(3+\frac{1}{2})\Gamma(5+\frac{1}{2})\Gamma(8)}{\Gamma(3)\Gamma(5)\Gamma(3+5+\frac{1}{2})}}{4\phi} \right) = -0.178207306876.....$$

$$-0.17794063585 \approx -0.178207306876$$

We have also:

$$\int_0^{\infty} \left(\frac{1+x^2/b^2}{1+x^2/a^2} \right) \left(\frac{1+x^2/(b+1)^2}{1+x^2/(a+1)^2} \right) \left(\frac{1+x^2/(b+2)^2}{1+x^2/(a+2)^2} \right) \dots dx$$

$$= \frac{1}{2}\sqrt{\pi} \frac{\Gamma(a+\frac{1}{2})\Gamma(b)\Gamma(b-a-\frac{1}{2})}{\Gamma(a)\Gamma(b-\frac{1}{2})\Gamma(b-a)}, \dots\dots(3)$$

$$\int_0^{\infty} \left| \frac{\Gamma(a+ix)}{\Gamma(b+ix)} \right|^2 dx = \frac{1}{2}\sqrt{\pi} \frac{\Gamma(a)\Gamma(a+\frac{1}{2})\Gamma(b-a-\frac{1}{2})}{\Gamma(b-\frac{1}{2})\Gamma(b)\Gamma(b-a)}. \dots\dots(4)$$

For a = 3 and b = 5, we obtain:

$$\sqrt{\pi}/2 * (((\text{gamma}(3+1/2) \text{gamma}(5) \text{gamma}(5-3-1/2)))) / (((\text{gamma}(3) \text{gamma}(5-1/2) \text{gamma}(5-3))))))$$

Input:

$$\frac{\sqrt{\pi}}{2} \times \frac{\Gamma(3+\frac{1}{2})\Gamma(5)\Gamma(5-3-\frac{1}{2})}{\Gamma(3)\Gamma(5-\frac{1}{2})\Gamma(5-3)}$$

Γ(x) is the gamma function

Exact result:

$$\frac{6\pi}{7}$$

Decimal approximation:

2.692793703076965632967980042811002472169002342321519275121...

2.6927937030769....

Property:

$\frac{6\pi}{7}$ is a transcendental number

Alternative representations:

$$\frac{\left(\Gamma\left(3 + \frac{1}{2}\right)\Gamma(5)\Gamma\left(5 - 3 - \frac{1}{2}\right)\right)\sqrt{\pi}}{\left(\Gamma(3)\Gamma\left(5 - \frac{1}{2}\right)\Gamma(5 - 3)\right)2} = \frac{\frac{1}{2}! \times \frac{5}{2}! \times 4! \sqrt{\pi}}{2\left(1! \times 2! \times \frac{7}{2}!\right)}$$

$$\frac{\left(\Gamma\left(3 + \frac{1}{2}\right)\Gamma(5)\Gamma\left(5 - 3 - \frac{1}{2}\right)\right)\sqrt{\pi}}{\left(\Gamma(3)\Gamma\left(5 - \frac{1}{2}\right)\Gamma(5 - 3)\right)2} = \frac{e^{-\log(12)+\log(288)} e^{-\log\Gamma(3/2)+\log\Gamma(5/2)} e^{-\log\Gamma(7/2)+\log\Gamma(9/2)} \sqrt{\pi}}{2\left(e^0 e^{\log(2)} e^{-\log\Gamma(9/2)+\log\Gamma(11/2)}\right)}$$

$$\frac{\left(\Gamma\left(3 + \frac{1}{2}\right)\Gamma(5)\Gamma\left(5 - 3 - \frac{1}{2}\right)\right)\sqrt{\pi}}{\left(\Gamma(3)\Gamma\left(5 - \frac{1}{2}\right)\Gamma(5 - 3)\right)2} = \frac{(1)_{\frac{1}{2}} (1)_{\frac{5}{2}} (1)_4 \sqrt{\pi}}{2\left((1)_1 (1)_2 (1)_{\frac{7}{2}}\right)}$$

Series representations:

$$\frac{\left(\Gamma\left(3 + \frac{1}{2}\right)\Gamma(5)\Gamma\left(5 - 3 - \frac{1}{2}\right)\right)\sqrt{\pi}}{\left(\Gamma(3)\Gamma\left(5 - \frac{1}{2}\right)\Gamma(5 - 3)\right)2} = \frac{24}{7} \sum_{k=0}^{\infty} \frac{(-1)^k}{1+2k}$$

$$\frac{\left(\Gamma\left(3 + \frac{1}{2}\right)\Gamma(5)\Gamma\left(5 - 3 - \frac{1}{2}\right)\right)\sqrt{\pi}}{\left(\Gamma(3)\Gamma\left(5 - \frac{1}{2}\right)\Gamma(5 - 3)\right)2} = \sum_{k=0}^{\infty} -\frac{24(-1)^k 1195^{-1-2k} \left(5^{1+2k} - 4 \times 239^{1+2k}\right)}{7(1+2k)}$$

$$\frac{\left(\Gamma\left(3 + \frac{1}{2}\right)\Gamma(5)\Gamma\left(5 - 3 - \frac{1}{2}\right)\right)\sqrt{\pi}}{\left(\Gamma(3)\Gamma\left(5 - \frac{1}{2}\right)\Gamma(5 - 3)\right)2} = \frac{6}{7} \sum_{k=0}^{\infty} \left(-\frac{1}{4}\right)^k \left(\frac{1}{1+2k} + \frac{2}{1+4k} + \frac{1}{3+4k}\right)$$

Integral representations:

$$\frac{\left(\Gamma\left(3 + \frac{1}{2}\right)\Gamma(5)\Gamma\left(5 - 3 - \frac{1}{2}\right)\right)\sqrt{\pi}}{\left(\Gamma(3)\Gamma\left(5 - \frac{1}{2}\right)\Gamma(5 - 3)\right)2} = \frac{24}{7} \int_0^1 \sqrt{1-t^2} dt$$

$$\frac{\left(\Gamma\left(3+\frac{1}{2}\right)\Gamma(5)\Gamma\left(5-3-\frac{1}{2}\right)\right)\sqrt{\pi}}{\left(\Gamma(3)\Gamma\left(5-\frac{1}{2}\right)\Gamma(5-3)\right)2} = \frac{12}{7} \int_0^1 \frac{1}{\sqrt{1-t^2}} dt$$

$$\frac{\left(\Gamma\left(3+\frac{1}{2}\right)\Gamma(5)\Gamma\left(5-3-\frac{1}{2}\right)\right)\sqrt{\pi}}{\left(\Gamma(3)\Gamma\left(5-\frac{1}{2}\right)\Gamma(5-3)\right)2} = \frac{12}{7} \int_0^\infty \frac{1}{1+t^2} dt$$

1/(((sqrt(Pi)/2 * (((gamma (3+1/2) gamma (5) gamma (5-3-1/2)))))) / (((gamma (3) gamma (5-1/2) gamma (5-3))))))^1/64

Input:

$$\frac{1}{\sqrt[64]{\frac{\sqrt{\pi}}{2} \times \frac{\Gamma\left(3+\frac{1}{2}\right)\Gamma(5)\Gamma\left(5-3-\frac{1}{2}\right)}{\Gamma(3)\Gamma\left(5-\frac{1}{2}\right)\Gamma(5-3)}}}$$

$\Gamma(x)$ is the gamma function

Exact result:

$$\sqrt[64]{\frac{7}{6\pi}}$$

Decimal approximation:

0.984641365454763821899784453794638236359240744503499621802...

0.9846413654.... result very near to the value of the following Rogers-Ramanujan continued fraction:

$$\frac{\frac{e^{-\frac{\pi}{\sqrt{5}}}}{\sqrt{5}}}{1 + \sqrt[5]{\sqrt{\varphi^5 4\sqrt{5^3} - 1}} - \varphi + 1} = 1 - \frac{e^{-\pi\sqrt{5}}}{1 + \frac{e^{-2\pi\sqrt{5}}}{1 + \frac{e^{-3\pi\sqrt{5}}}{1 + \frac{e^{-4\pi\sqrt{5}}}{1 + \dots}}}} \approx 0.9991104684$$

and to the dilaton value **0.989117352243 = ϕ**

Property:

$\sqrt[64]{\frac{7}{6\pi}}$ is a transcendental number

Alternative representations:

$$\frac{1}{\sqrt[64]{\frac{(\Gamma(3+\frac{1}{2})\Gamma(5)\Gamma(5-3-\frac{1}{2}))\sqrt{\pi}}{(\Gamma(3)\Gamma(5-\frac{1}{2})\Gamma(5-3))2}}} = \frac{1}{\sqrt[64]{\frac{\frac{1! \times 5! \times 4! \sqrt{\pi}}{2(1! \times 2! \times \frac{7!}{2})}}{2}}$$

$$\frac{1}{\sqrt[64]{\frac{(\Gamma(3+\frac{1}{2})\Gamma(5)\Gamma(5-3-\frac{1}{2}))\sqrt{\pi}}{(\Gamma(3)\Gamma(5-\frac{1}{2})\Gamma(5-3))2}}} = \frac{1}{\sqrt[64]{\frac{e^{-\log(12)+\log(288)} e^{-\log G(3/2)+\log G(5/2)} e^{-\log G(7/2)+\log G(9/2)} \sqrt{\pi}}{2(e^0 e^{\log(2)} e^{-\log G(9/2)+\log G(11/2)})}}$$

$$\frac{1}{\sqrt[64]{\frac{(\Gamma(3+\frac{1}{2})\Gamma(5)\Gamma(5-3-\frac{1}{2}))\sqrt{\pi}}{(\Gamma(3)\Gamma(5-\frac{1}{2})\Gamma(5-3))2}}} = \frac{1}{\sqrt[64]{\frac{\frac{(1)_1 (1)_5 (1)_4 \sqrt{\pi}}{2 \cdot 2}}{2 \left((1)_1 (1)_2 (1)_7 \right)_2}}$$

Series representations:

$$\frac{1}{\sqrt[64]{\frac{(\Gamma(3+\frac{1}{2})\Gamma(5)\Gamma(5-3-\frac{1}{2}))\sqrt{\pi}}{(\Gamma(3)\Gamma(5-\frac{1}{2})\Gamma(5-3))2}}} = \frac{\sqrt[64]{\frac{7}{3}} \sqrt[64]{\frac{1}{\sum_{k=0}^{\infty} \frac{(-1)^k}{1+2k}}}}{2^{3/64}}$$

$$\frac{1}{\sqrt[64]{\frac{(\Gamma(3+\frac{1}{2})\Gamma(5)\Gamma(5-3-\frac{1}{2}))\sqrt{\pi}}{(\Gamma(3)\Gamma(5-\frac{1}{2})\Gamma(5-3))2}}} = \sqrt[64]{\frac{7}{6}} \sqrt[64]{\frac{1}{\sum_{k=0}^{\infty} \frac{4(-1)^k 1195^{-1-2k} (5^{1+2k} - 4 \times 239^{1+2k})}{1+2k}}}}$$

$$\frac{1}{\sqrt[64]{\frac{(\Gamma(3+\frac{1}{2})\Gamma(5)\Gamma(5-3-\frac{1}{2}))\sqrt{\pi}}{(\Gamma(3)\Gamma(5-\frac{1}{2})\Gamma(5-3))2}}} = \sqrt[64]{\frac{7}{6}} \sqrt[64]{\frac{1}{\sum_{k=0}^{\infty} \left(-\frac{1}{4}\right)^k \left(\frac{1}{1+2k} + \frac{2}{1+4k} + \frac{1}{3+4k}\right)}}$$

Integral representations:

$$\frac{1}{\sqrt[64]{\frac{(\Gamma(3+\frac{1}{2})\Gamma(5)\Gamma(5-3-\frac{1}{2}))\sqrt{\pi}}{(\Gamma(3)\Gamma(5-\frac{1}{2})\Gamma(5-3))2}}}} = \frac{\sqrt[64]{\frac{7}{3}} \sqrt[64]{\int_0^{\infty} \frac{1}{1+t^2} dt}}{\sqrt[32]{2}}$$

$$\frac{1}{\sqrt[64]{\frac{(\Gamma(3+\frac{1}{2})\Gamma(5)\Gamma(5-3-\frac{1}{2}))\sqrt{\pi}}{(\Gamma(3)\Gamma(5-\frac{1}{2})\Gamma(5-3))2}}}} = \frac{\sqrt[64]{\frac{7}{3}} \sqrt[64]{\int_0^1 \frac{1}{\sqrt{1-t^2}} dt}}{2^{3/64}}$$

$$\frac{1}{\sqrt[64]{\frac{(\Gamma(3+\frac{1}{2})\Gamma(5)\Gamma(5-3-\frac{1}{2}))\sqrt{\pi}}{(\Gamma(3)\Gamma(5-\frac{1}{2})\Gamma(5-3))2}}}} = \frac{\sqrt[64]{\frac{7}{3}} \sqrt[64]{\int_0^1 \frac{1}{\sqrt{1-t^2}} dt}}{\sqrt[32]{2}}$$

From which, we obtain:

$$\frac{1}{5 \times 3} \sqrt{\frac{\pi}{2}} * (((\text{gamma}(3+\frac{1}{2}) \text{gamma}(5) \text{gamma}(5-3-\frac{1}{2})))) / (((\text{gamma}(3) \text{gamma}(5-\frac{1}{2}) \text{gamma}(5-3))))$$

Input:

$$\frac{1}{5 \times 3} \times \frac{\sqrt{\pi}}{2} \times \frac{\Gamma(3 + \frac{1}{2}) \Gamma(5) \Gamma(5 - 3 - \frac{1}{2})}{\Gamma(3) \Gamma(5 - \frac{1}{2}) \Gamma(5 - 3)}$$

$\Gamma(x)$ is the gamma function

Exact result:

$$\frac{2\pi}{35}$$

Decimal approximation:

0.179519580205131042197865336187400164811266822821434618341...

0.179519580205131.....

Property:

$\frac{2\pi}{35}$ is a transcendental number

Alternative representations:

$$\frac{\sqrt{\pi} \left(\Gamma\left(3 + \frac{1}{2}\right) \Gamma(5) \Gamma\left(5 - 3 - \frac{1}{2}\right) \right)}{\left(2 \left(\Gamma(3) \Gamma\left(5 - \frac{1}{2}\right) \Gamma(5 - 3) \right) \right) (5 \times 3)} = \frac{\frac{1}{2}! \times \frac{5}{2}! \times 4! \sqrt{\pi}}{2 \times 15 \left(1! \times 2! \times \frac{7}{2}! \right)}$$

$$\frac{\sqrt{\pi} \left(\Gamma\left(3 + \frac{1}{2}\right) \Gamma(5) \Gamma\left(5 - 3 - \frac{1}{2}\right) \right)}{\left(2 \left(\Gamma(3) \Gamma\left(5 - \frac{1}{2}\right) \Gamma(5 - 3) \right) \right) (5 \times 3)} = \frac{e^{-\log(12)+\log(288)} e^{-\log\Gamma(3/2)+\log\Gamma(5/2)} e^{-\log\Gamma(7/2)+\log\Gamma(9/2)} \sqrt{\pi}}{2 \times 15 \left(e^0 e^{\log(2)} e^{-\log\Gamma(9/2)+\log\Gamma(11/2)} \right)}$$

$$\frac{\sqrt{\pi} \left(\Gamma\left(3 + \frac{1}{2}\right) \Gamma(5) \Gamma\left(5 - 3 - \frac{1}{2}\right) \right)}{\left(2 \left(\Gamma(3) \Gamma\left(5 - \frac{1}{2}\right) \Gamma(5 - 3) \right) \right) (5 \times 3)} = \frac{(1)_{\frac{1}{2}} (1)_{\frac{5}{2}} (1)_4 \sqrt{\pi}}{2 \times 15 \left((1)_1 (1)_2 (1)_{\frac{7}{2}} \right)}$$

Series representations:

$$\frac{\sqrt{\pi} \left(\Gamma\left(3 + \frac{1}{2}\right) \Gamma(5) \Gamma\left(5 - 3 - \frac{1}{2}\right) \right)}{\left(2 \left(\Gamma(3) \Gamma\left(5 - \frac{1}{2}\right) \Gamma(5 - 3) \right) \right) (5 \times 3)} = \frac{8}{35} \sum_{k=0}^{\infty} \frac{(-1)^k}{1 + 2k}$$

$$\frac{\sqrt{\pi} \left(\Gamma\left(3 + \frac{1}{2}\right) \Gamma(5) \Gamma\left(5 - 3 - \frac{1}{2}\right) \right)}{\left(2 \left(\Gamma(3) \Gamma\left(5 - \frac{1}{2}\right) \Gamma(5 - 3) \right) \right) (5 \times 3)} = \sum_{k=0}^{\infty} \frac{8 (-1)^k (956 \times 5^{-2k} - 5 \times 239^{-2k})}{41825 (1 + 2k)}$$

$$\frac{\sqrt{\pi} \left(\Gamma\left(3 + \frac{1}{2}\right) \Gamma(5) \Gamma\left(5 - 3 - \frac{1}{2}\right) \right)}{\left(2 \left(\Gamma(3) \Gamma\left(5 - \frac{1}{2}\right) \Gamma(5 - 3) \right) \right) (5 \times 3)} = \frac{2}{35} \sum_{k=0}^{\infty} \left(-\frac{1}{4} \right)^k \left(\frac{1}{1+2k} + \frac{2}{1+4k} + \frac{1}{3+4k} \right)$$

Integral representations:

$$\frac{\sqrt{\pi} \left(\Gamma\left(3 + \frac{1}{2}\right) \Gamma(5) \Gamma\left(5 - 3 - \frac{1}{2}\right) \right)}{\left(2 \left(\Gamma(3) \Gamma\left(5 - \frac{1}{2}\right) \Gamma(5 - 3) \right) \right) (5 \times 3)} = \frac{8}{35} \int_0^1 \sqrt{1-t^2} dt$$

$$\frac{\sqrt{\pi} \left(\Gamma\left(3 + \frac{1}{2}\right) \Gamma(5) \Gamma\left(5 - 3 - \frac{1}{2}\right) \right)}{\left(2 \left(\Gamma(3) \Gamma\left(5 - \frac{1}{2}\right) \Gamma(5 - 3) \right) \right) (5 \times 3)} = \frac{4}{35} \int_0^1 \frac{1}{\sqrt{1-t^2}} dt$$

$$\frac{\sqrt{\pi} \left(\Gamma\left(3 + \frac{1}{2}\right) \Gamma(5) \Gamma\left(5 - 3 - \frac{1}{2}\right) \right)}{\left(2 \left(\Gamma(3) \Gamma\left(5 - \frac{1}{2}\right) \Gamma(5 - 3) \right) \right) (5 \times 3)} = \frac{4}{35} \int_0^\infty \frac{1}{1+t^2} dt$$

And:

$$\frac{\sqrt{\pi} \left(\Gamma(3) \Gamma\left(3 + \frac{1}{2}\right) \Gamma\left(5 - 3 - \frac{1}{2}\right) \right)}{\left(\Gamma(5) \Gamma(5 - 3) \right)}$$

Input:

$$\frac{\sqrt{\pi}}{2} \times \frac{\Gamma(3) \Gamma\left(3 + \frac{1}{2}\right) \Gamma\left(5 - 3 - \frac{1}{2}\right)}{\Gamma\left(5 - \frac{1}{2}\right) \Gamma(5) \Gamma(5 - 3)}$$

$\Gamma(x)$ is the gamma function

Exact result:

$$\frac{\pi}{168}$$

Decimal approximation:

0.018699956271367816895610972519520850501173627377232772743...

0.0018699956271367.....

Property:

$\frac{\pi}{168}$ is a transcendental number

Alternative representations:

$$\frac{(\Gamma(3)\Gamma(3+\frac{1}{2})\Gamma(5-3-\frac{1}{2}))\sqrt{\pi}}{(\Gamma(5-\frac{1}{2})\Gamma(5)\Gamma(5-3))2} = \frac{\frac{1}{2}! \times 2! \times \frac{5}{2}! \sqrt{\pi}}{2(1! \times \frac{7}{2}! \times 4!)}$$

$$\frac{(\Gamma(3)\Gamma(3+\frac{1}{2})\Gamma(5-3-\frac{1}{2}))\sqrt{\pi}}{(\Gamma(5-\frac{1}{2})\Gamma(5)\Gamma(5-3))2} = \frac{e^{\log(2)} e^{-\log G(3/2)+\log G(5/2)} e^{-\log G(7/2)+\log G(9/2)} \sqrt{\pi}}{2(e^0 e^{-\log(12)+\log(288)} e^{-\log G(9/2)+\log G(11/2)})}$$

$$\frac{(\Gamma(3)\Gamma(3+\frac{1}{2})\Gamma(5-3-\frac{1}{2}))\sqrt{\pi}}{(\Gamma(5-\frac{1}{2})\Gamma(5)\Gamma(5-3))2} = \frac{(1)_{\frac{1}{2}} (1)_2 (1)_{\frac{5}{2}} \sqrt{\pi}}{2((1)_1 (1)_{\frac{7}{2}} (1)_4)}$$

Series representations:

$$\frac{(\Gamma(3)\Gamma(3+\frac{1}{2})\Gamma(5-3-\frac{1}{2}))\sqrt{\pi}}{(\Gamma(5-\frac{1}{2})\Gamma(5)\Gamma(5-3))2} = \frac{1}{42} \sum_{k=0}^{\infty} \frac{(-1)^k}{1+2k}$$

$$\frac{(\Gamma(3)\Gamma(3+\frac{1}{2})\Gamma(5-3-\frac{1}{2}))\sqrt{\pi}}{(\Gamma(5-\frac{1}{2})\Gamma(5)\Gamma(5-3))2} = \sum_{k=0}^{\infty} -\frac{(-1)^k 1195^{-1-2k} (5^{1+2k} - 4 \times 239^{1+2k})}{42(1+2k)}$$

$$\frac{(\Gamma(3)\Gamma(3+\frac{1}{2})\Gamma(5-3-\frac{1}{2}))\sqrt{\pi}}{(\Gamma(5-\frac{1}{2})\Gamma(5)\Gamma(5-3))2} = \frac{1}{168} \sum_{k=0}^{\infty} \left(-\frac{1}{4}\right)^k \left(\frac{1}{1+2k} + \frac{2}{1+4k} + \frac{1}{3+4k}\right)$$

Integral representations:

$$\frac{(\Gamma(3)\Gamma(3+\frac{1}{2})\Gamma(5-3-\frac{1}{2}))\sqrt{\pi}}{(\Gamma(5-\frac{1}{2})\Gamma(5)\Gamma(5-3))2} = \frac{1}{42} \int_0^1 \sqrt{1-t^2} dt$$

$$\frac{(\Gamma(3)\Gamma(3+\frac{1}{2})\Gamma(5-3-\frac{1}{2}))\sqrt{\pi}}{(\Gamma(5-\frac{1}{2})\Gamma(5)\Gamma(5-3))2} = \frac{1}{84} \int_0^1 \frac{1}{\sqrt{1-t^2}} dt$$

$$\frac{(\Gamma(3)\Gamma(3+\frac{1}{2})\Gamma(5-3-\frac{1}{2}))\sqrt{\pi}}{(\Gamma(5-\frac{1}{2})\Gamma(5)\Gamma(5-3))2} = \frac{1}{84} \int_0^{\infty} \frac{1}{1+t^2} dt$$

$((\sqrt{\pi}/2 * ((\Gamma(3) \Gamma(3+1/2) \Gamma(5-3-1/2)))) / (((\Gamma(5-1/2) \Gamma(5) \Gamma(5-3))))))^{1/256}$

Input:

$$\sqrt[256]{\frac{\sqrt{\pi}}{2} \times \frac{\Gamma(3)\Gamma(3+\frac{1}{2})\Gamma(5-3-\frac{1}{2})}{\Gamma(5-\frac{1}{2})\Gamma(5)\Gamma(5-3)}}$$

$\Gamma(x)$ is the gamma function

Exact result:

$$\frac{\sqrt[256]{\frac{\pi}{21}}}{2^{3/256}}$$

Decimal approximation:

0.984576299466753732842533575445391018920805996977764823977...

0.98457629946.... result very near to the value of the following Rogers-Ramanujan continued fraction:

$$\frac{e^{-\frac{\pi}{\sqrt{5}}}}{\sqrt{5}} = 1 - \frac{e^{-\pi\sqrt{5}}}{1 + \frac{e^{-2\pi\sqrt{5}}}{1 + \frac{e^{-3\pi\sqrt{5}}}{1 + \frac{e^{-4\pi\sqrt{5}}}{1 + \dots}}}} - \phi + 1 \approx 0.9991104684$$

and to the dilaton value **0.989117352243 = ϕ**

Property:

$\frac{\sqrt[256]{\frac{\pi}{21}}}{2^{3/256}}$ is a transcendental number

All 256th roots of $\pi/168$:

$$\frac{\sqrt[256]{\frac{\pi}{21} e^0}}{2^{3/256}} \approx 0.984576 \text{ (real, principal root)}$$

$$\frac{256 \sqrt{\frac{\pi}{21}} e^{(i\pi)/128}}{2^{3/256}} \approx 0.984280 + 0.024163 i$$

$$\frac{256 \sqrt{\frac{\pi}{21}} e^{(i\pi)/64}}{2^{3/256}} \approx 0.983390 + 0.048311 i$$

$$\frac{256 \sqrt{\frac{\pi}{21}} e^{(3i\pi)/128}}{2^{3/256}} \approx 0.981909 + 0.07243 i$$

$$\frac{256 \sqrt{\frac{\pi}{21}} e^{(i\pi)/32}}{2^{3/256}} \approx 0.979835 + 0.09651 i$$

Alternative representations:

$$256 \sqrt{\frac{\left(\Gamma(3) \Gamma\left(3 + \frac{1}{2}\right) \Gamma\left(5 - 3 - \frac{1}{2}\right)\right) \sqrt{\pi}}{\left(\Gamma\left(5 - \frac{1}{2}\right) \Gamma(5) \Gamma(5 - 3)\right) 2}} = 256 \sqrt{\frac{\frac{1}{2}! \times 2! \times \frac{5}{2}! \sqrt{\pi}}{2 \left(1! \times \frac{7}{2}! \times 4!\right)}}$$

$$256 \sqrt{\frac{\left(\Gamma(3) \Gamma\left(3 + \frac{1}{2}\right) \Gamma\left(5 - 3 - \frac{1}{2}\right)\right) \sqrt{\pi}}{\left(\Gamma\left(5 - \frac{1}{2}\right) \Gamma(5) \Gamma(5 - 3)\right) 2}} =$$

$$256 \sqrt{\frac{e^{\log(2)} e^{-\log(3/2) + \log(5/2)} e^{-\log(7/2) + \log(9/2)} \sqrt{\pi}}{2 \left(e^0 e^{-\log(12) + \log(288)} e^{-\log(9/2) + \log(11/2)}\right)}}$$

$$256 \sqrt{\frac{\left(\Gamma(3) \Gamma\left(3 + \frac{1}{2}\right) \Gamma\left(5 - 3 - \frac{1}{2}\right)\right) \sqrt{\pi}}{\left(\Gamma\left(5 - \frac{1}{2}\right) \Gamma(5) \Gamma(5 - 3)\right) 2}} = 256 \sqrt{\frac{\frac{(1)_1}{2} \frac{(1)_2}{2} \frac{(1)_5}{2} \sqrt{\pi}}{2 \left(\frac{(1)_1}{1} \frac{(1)_7}{2} \frac{(1)_4}{1}\right)}}$$

Series representations:

$$256 \sqrt{\frac{\left(\Gamma(3) \Gamma\left(3 + \frac{1}{2}\right) \Gamma\left(5 - 3 - \frac{1}{2}\right)\right) \sqrt{\pi}}{\left(\Gamma\left(5 - \frac{1}{2}\right) \Gamma(5) \Gamma(5 - 3)\right) 2}} = \frac{256 \sqrt{\sum_{k=0}^{\infty} \frac{(-1)^k}{1+2k}}}{256 \sqrt{42}}$$

$$\sqrt[256]{\frac{(\Gamma(3)\Gamma(3+\frac{1}{2})\Gamma(5-3-\frac{1}{2}))\sqrt{\pi}}{(\Gamma(5-\frac{1}{2})\Gamma(5)\Gamma(5-3))2}} = \frac{\sqrt[256]{\sum_{k=0}^{\infty} \frac{(-1)^{1+k} 1195^{-1-2k} (5^{1+2k} - 4 \times 230^{1+2k})}{1+2k}}}{\sqrt[256]{42}}$$

$$\sqrt[256]{\frac{(\Gamma(3)\Gamma(3+\frac{1}{2})\Gamma(5-3-\frac{1}{2}))\sqrt{\pi}}{(\Gamma(5-\frac{1}{2})\Gamma(5)\Gamma(5-3))2}} = \frac{\sqrt[256]{\sum_{k=0}^{\infty} (-\frac{1}{4})^k (\frac{1}{1+2k} + \frac{2}{1+4k} + \frac{1}{3+4k})}}{2^{3/256} \sqrt[256]{21}}$$

Integral representations:

$$\sqrt[256]{\frac{(\Gamma(3)\Gamma(3+\frac{1}{2})\Gamma(5-3-\frac{1}{2}))\sqrt{\pi}}{(\Gamma(5-\frac{1}{2})\Gamma(5)\Gamma(5-3))2}} = \frac{\sqrt[256]{\int_0^1 \sqrt{1-t^2} dt}}{\sqrt[256]{42}}$$

$$\sqrt[256]{\frac{(\Gamma(3)\Gamma(3+\frac{1}{2})\Gamma(5-3-\frac{1}{2}))\sqrt{\pi}}{(\Gamma(5-\frac{1}{2})\Gamma(5)\Gamma(5-3))2}} = \frac{\sqrt[256]{\int_0^{\infty} \frac{1}{1+t^2} dt}}{128\sqrt{2} \sqrt[256]{21}}$$

$$\sqrt[256]{\frac{(\Gamma(3)\Gamma(3+\frac{1}{2})\Gamma(5-3-\frac{1}{2}))\sqrt{\pi}}{(\Gamma(5-\frac{1}{2})\Gamma(5)\Gamma(5-3))2}} = \frac{\sqrt[256]{\int_0^1 \frac{1}{\sqrt{1-t^2}} dt}}{128\sqrt{2} \sqrt[256]{21}}$$

From which, we obtain:

$$\frac{30}{\pi} \sqrt{\frac{\pi}{2}} * (((\text{gamma}(3) \text{gamma}(3+1/2) \text{gamma}(5-3-1/2)))) / (((\text{gamma}(5-1/2) \text{gamma}(5) \text{gamma}(5-3))))$$

Input:

$$\frac{30}{\pi} \times \frac{\sqrt{\pi}}{2} \times \frac{\Gamma(3)\Gamma(3+\frac{1}{2})\Gamma(5-3-\frac{1}{2})}{\Gamma(5-\frac{1}{2})\Gamma(5)\Gamma(5-3)}$$

$\Gamma(x)$ is the gamma function

Exact result:

$$\frac{5}{28}$$

Decimal approximation:

0.178571428571428571428571428571428571428571428571428571428571428...

0.17857142857...

Repeating decimal:

0.17857142 (period 6)

Egyptian fraction expansion:

$$\frac{1}{6} + \frac{1}{84}$$

Alternative representations:

$$\frac{(\sqrt{\pi} \left(\Gamma(3) \Gamma\left(3 + \frac{1}{2}\right) \Gamma\left(5 - 3 - \frac{1}{2}\right) \right)) 30}{\left(2 \left(\Gamma\left(5 - \frac{1}{2}\right) \Gamma(5) \Gamma(5 - 3) \right) \right) \pi} = \frac{15 \times \frac{1}{2}! \times 2! \times \frac{5}{2}! \sqrt{\pi}}{\pi \left(1! \times \frac{7}{2}! \times 4! \right)}$$

$$\frac{(\sqrt{\pi} \left(\Gamma(3) \Gamma\left(3 + \frac{1}{2}\right) \Gamma\left(5 - 3 - \frac{1}{2}\right) \right)) 30}{\left(2 \left(\Gamma\left(5 - \frac{1}{2}\right) \Gamma(5) \Gamma(5 - 3) \right) \right) \pi} = \frac{15 e^{\log(2)} e^{-\log G(3/2) + \log G(5/2)} e^{-\log G(7/2) + \log G(9/2)} \sqrt{\pi}}{\pi \left(e^0 e^{-\log(12) + \log(288)} e^{-\log G(9/2) + \log G(11/2)} \right)}$$

$$\frac{(\sqrt{\pi} \left(\Gamma(3) \Gamma\left(3 + \frac{1}{2}\right) \Gamma\left(5 - 3 - \frac{1}{2}\right) \right)) 30}{\left(2 \left(\Gamma\left(5 - \frac{1}{2}\right) \Gamma(5) \Gamma(5 - 3) \right) \right) \pi} = \frac{15 \Gamma\left(\frac{3}{2}, 0\right) \Gamma(3, 0) \Gamma\left(\frac{7}{2}, 0\right) \sqrt{\pi}}{\pi \left(\Gamma(2, 0) \Gamma\left(\frac{9}{2}, 0\right) \Gamma(5, 0) \right)}$$

Series representations:

$$\frac{(\sqrt{\pi} \left(\Gamma(3) \Gamma\left(3 + \frac{1}{2}\right) \Gamma\left(5 - 3 - \frac{1}{2}\right) \right)) 30}{\left(2 \left(\Gamma\left(5 - \frac{1}{2}\right) \Gamma(5) \Gamma(5 - 3) \right) \right) \pi} = \frac{15 \exp\left(i \pi \left[\frac{\arg(\pi - x)}{2 \pi} \right]\right) \Gamma\left(\frac{3}{2}\right) \Gamma(3) \Gamma\left(\frac{7}{2}\right) \sqrt{x} \sum_{k=0}^{\infty} \frac{(-1)^k (\pi - x)^k x^{-k} \left(-\frac{1}{2}\right)_k}{k!}}{\pi \Gamma(2) \Gamma\left(\frac{9}{2}\right) \Gamma(5)}$$

for $(x \in \mathbb{R} \text{ and } x < 0)$

$$\frac{(\sqrt{\pi} \left(\Gamma(3) \Gamma\left(3 + \frac{1}{2}\right) \Gamma\left(5 - 3 - \frac{1}{2}\right) \right)) 30}{\left(2 \left(\Gamma\left(5 - \frac{1}{2}\right) \Gamma(5) \Gamma(5 - 3) \right) \right) \pi} =$$

$$\frac{15 \Gamma\left(\frac{3}{2}\right) \Gamma(3) \Gamma\left(\frac{7}{2}\right) \left(\frac{1}{z_0}\right)^{1/2 \lceil \operatorname{arg}(\pi - z_0) / (2\pi) \rceil} z_0^{1/2 (1 + \lceil \operatorname{arg}(\pi - z_0) / (2\pi) \rceil)} \sum_{k=0}^{\infty} \frac{(-1)^k \left(-\frac{1}{2}\right)_k (\pi - z_0)^k z_0^{-k}}{k!}}{\pi \Gamma(2) \Gamma\left(\frac{9}{2}\right) \Gamma(5)}$$

$$\frac{(\sqrt{\pi} \left(\Gamma(3) \Gamma\left(3 + \frac{1}{2}\right) \Gamma\left(5 - 3 - \frac{1}{2}\right) \right)) 30}{\left(2 \left(\Gamma\left(5 - \frac{1}{2}\right) \Gamma(5) \Gamma(5 - 3) \right) \right) \pi} =$$

$$\left(15 \sqrt{-1 + \pi} \sum_{k_1=0}^{\infty} \sum_{k_2=0}^{\infty} \sum_{k_3=0}^{\infty} \sum_{k_4=0}^{\infty} \frac{1}{k_2! k_3! k_4!} (-1 + \pi)^{-k_1} \left(\frac{1}{k_1}\right) \left(\frac{3}{2} - z_0\right)^{k_2} \right. \\ \left. (3 - z_0)^{k_3} \left(\frac{7}{2} - z_0\right)^{k_4} \Gamma^{(k_2)}(z_0) \Gamma^{(k_3)}(z_0) \Gamma^{(k_4)}(z_0) \right) /$$

$$\left(\pi \left(\sum_{k=0}^{\infty} \frac{(2 - z_0)^k \Gamma^{(k)}(z_0)}{k!} \right) \left(\sum_{k=0}^{\infty} \frac{\left(\frac{9}{2} - z_0\right)^k \Gamma^{(k)}(z_0)}{k!} \right) \sum_{k=0}^{\infty} \frac{(5 - z_0)^k \Gamma^{(k)}(z_0)}{k!} \right)$$

for $(z_0 \notin \mathbb{Z} \text{ or } z_0 > 0)$

Integral representations:

$$\frac{(\sqrt{\pi} \left(\Gamma(3) \Gamma\left(3 + \frac{1}{2}\right) \Gamma\left(5 - 3 - \frac{1}{2}\right) \right)) 30}{\left(2 \left(\Gamma\left(5 - \frac{1}{2}\right) \Gamma(5) \Gamma(5 - 3) \right) \right) \pi} =$$

$$\int_0^1 \int_0^1 \int_0^1 \sqrt{\log\left(\frac{1}{t_1}\right) \log^2\left(\frac{1}{t_2}\right) \log^{5/2}\left(\frac{1}{t_3}\right)} dt_3 dt_2 dt_1$$

$$\frac{(\sqrt{\pi} \left(\Gamma(3) \Gamma\left(3 + \frac{1}{2}\right) \Gamma\left(5 - 3 - \frac{1}{2}\right) \right)) 30}{\left(2 \left(\Gamma\left(5 - \frac{1}{2}\right) \Gamma(5) \Gamma(5 - 3) \right) \right) \pi} =$$

$$\frac{15 \exp\left(\int_0^1 \frac{-7-7\sqrt{x}+2x^{3/2}+2x^3+4x^{7/2}+4x^4+2x^{9/2}}{2(1+\sqrt{x})\log(x)} dx\right) \sqrt{\pi}}{\pi}$$

$$\frac{(\sqrt{\pi} \left(\Gamma(3) \Gamma\left(3 + \frac{1}{2}\right) \Gamma\left(5 - 3 - \frac{1}{2}\right) \right)) 30}{\left(2 \left(\Gamma\left(5 - \frac{1}{2}\right) \Gamma(5) \Gamma(5 - 3) \right) \right) \pi} = \frac{1}{\pi}$$

$$15 \exp\left(\frac{7\gamma}{2} + \int_0^1 \frac{1}{(-1+x)\log(x)} \left(x^{3/2} - x^2 + x^3 + x^{7/2} - x^{9/2} - x^5 - \log(x^{3/2}) + \right. \right. \\ \left. \left. \log(x^2) - \log(x^3) - \log(x^{7/2}) + \log(x^{9/2}) + \log(x^5) \right) dx\right) \sqrt{\pi}$$

γ is the Euler-Mascheroni constant

From the average between the two results, we obtain:

$$\frac{\left(\frac{1}{5 \times 3} \times \frac{\sqrt{\pi}}{2} \times \frac{\Gamma(3 + \frac{1}{2}) \Gamma(5) \Gamma(5 - 3 - \frac{1}{2})}{\Gamma(3) \Gamma(5 - \frac{1}{2}) \Gamma(5 - 3)} + \frac{30}{\pi} \times \frac{\sqrt{\pi}}{2} \times \frac{\Gamma(3) \Gamma(3 + \frac{1}{2}) \Gamma(5 - 3 - \frac{1}{2})}{\Gamma(5 - \frac{1}{2}) \Gamma(5) \Gamma(5 - 3)} \right)}{2} =$$

$$= \frac{(0.179519580205131 + 0.17857142857)}{2} =$$

$$= -0.17904550438756550 \approx$$

$$\approx \left(-\frac{\Gamma(2 - \frac{3}{2}) \sqrt{5}}{(\frac{3}{2} - 1) (4\pi)^{3/2}} \right) = -0.17794063585$$

Now, we have that:

$$\langle \sigma(x) \rangle = \sqrt{\frac{-m^2}{\lambda}} \equiv v = 246 \text{ GeV}.$$

$$\lambda = \frac{M_\phi^2}{2v^2} \simeq \frac{(125 \text{ GeV})^2}{2 \times (246 \text{ GeV})^2} \simeq \frac{1}{8} \ll 1$$

$$125^2 / (2 \times 246^2)$$

Input:

$$\frac{125^2}{2 \times 246^2}$$

Exact result:

$$\frac{15625}{121032}$$

Decimal approximation:

0.129098089761385418732236102848833366382444312248000528785...

0.12909808976.....

And:

$$(((125^2/(2*246^2))))^{1/256}$$

Input:

$$\sqrt[256]{\frac{125^2}{2 \times 246^2}}$$

Result:

$$\frac{5^{3/128}}{2^{3/256} \sqrt[128]{123}}$$

Decimal approximation:

0.992035081679943485912847869470544089706055278576032326957...

0.992035081679.... result very near to the value of the following Rogers-Ramanujan continued fraction:

$$\frac{e^{-\frac{\pi}{\sqrt{5}}}}{\sqrt{5}} = 1 - \frac{e^{-\pi\sqrt{5}}}{1 + \frac{e^{-2\pi\sqrt{5}}}{1 + \frac{e^{-3\pi\sqrt{5}}}{1 + \frac{e^{-4\pi\sqrt{5}}}{1 + \dots}}}} - \phi + 1 \approx 0.9991104684$$

and to the dilaton value **0.989117352243 = ϕ**

Alternate form:

$$\frac{1}{246} \times 5^{3/128} \times 2^{253/256} \times 123^{127/128}$$

$$f(q^2, \eta) = -\frac{1}{2} \frac{\Gamma(2 - \frac{D}{2})}{(4\pi)^{\frac{D}{2}} \Gamma(2)} \int_0^1 dx \frac{(1 - 2x)^2}{[x(1 - x)q^2 + \eta]^{2 - \frac{D}{2}}} \quad (68)$$

$$f(q^2, 0) = -\frac{1}{D-1} \frac{\Gamma(2 - \frac{D}{2}) [\Gamma(D/2 - 1)]^2}{2(4\pi)^{\frac{D}{2}} \Gamma(D-2)} (q^2)^{D/2-2}$$

for D = 3 and

$$q^2 = M_\rho^2$$

we obtain:

$$-\left(\left(\frac{1}{2} * (\text{gamma}(2-3/2) * (\text{gamma}(3/2-1))^2 * 1/(2300^2)^{(0.5)}\right)\right) / \left(\left(2(4\pi)^{1.5} \text{gamma}(1)\right)\right)$$

Input:

$$-\frac{\frac{1}{2} \left(\Gamma\left(2 - \frac{3}{2}\right) \Gamma\left(\frac{3}{2} - 1\right)^2 \times \frac{1}{\sqrt{2300^2}} \right)}{2 (4\pi)^{1.5} \Gamma(1)}$$

Γ(x) is the gamma function

Result:

$$-0.00001358695652173913043478260869565217391304347826086956...$$

$$-0.0000135869565.....$$

Repeating decimal:

$$-0.00001358695652173913043478260 \text{ (period 22)}$$

Rational approximation:

$$-\frac{1}{73600}$$

Alternative representations:

$$-\frac{\Gamma\left(2 - \frac{3}{2}\right) \Gamma\left(\frac{3}{2} - 1\right)^2}{\left(2\sqrt{2300^2}\right) (2(4\pi)^{1.5} \Gamma(1))} = -\frac{\left(-\frac{1}{2}\right)! \left(\left(-\frac{1}{2}\right)!\right)^2}{2(2 \times 0! (4\pi)^{1.5}) \sqrt{2300^2}}$$

$$-\frac{\Gamma\left(2 - \frac{3}{2}\right) \Gamma\left(\frac{3}{2} - 1\right)^2}{\left(2\sqrt{2300^2}\right) (2(4\pi)^{1.5} \Gamma(1))} = -\frac{e^{-\log G(1/2) + \log G(3/2)} \left(e^{-\log G(1/2) + \log G(3/2)}\right)^2}{2(2e^0 (4\pi)^{1.5}) \sqrt{2300^2}}$$

$$\frac{\Gamma\left(2 - \frac{3}{2}\right)\Gamma\left(\frac{3}{2} - 1\right)^2}{\left(2\sqrt{2300^2}\right)(2(4\pi)^{1.5}\Gamma(1))} = \frac{\Gamma\left(\frac{1}{2}, 0\right)\Gamma\left(\frac{1}{2}, 0\right)^2}{2(2\Gamma(1, 0)(4\pi)^{1.5})\sqrt{2300^2}}$$

Series representations:

$$\frac{\Gamma\left(2 - \frac{3}{2}\right)\Gamma\left(\frac{3}{2} - 1\right)^2}{\left(2\sqrt{2300^2}\right)(2(4\pi)^{1.5}\Gamma(1))} = \frac{0.000013587 \left(\sum_{k=0}^{\infty} \frac{\left(\frac{1}{2} - z_0\right)^k \Gamma^{(k)}(z_0)}{k!} \right)^3}{\pi^{1.5} \sum_{k=0}^{\infty} \frac{(1-z_0)^k \Gamma^{(k)}(z_0)}{k!}}$$

for $(z_0 \notin \mathbb{Z} \text{ or } z_0 > 0)$

$$\frac{\Gamma\left(2 - \frac{3}{2}\right)\Gamma\left(\frac{3}{2} - 1\right)^2}{\left(2\sqrt{2300^2}\right)(2(4\pi)^{1.5}\Gamma(1))} = \frac{0.000013587 \pi^{0.5} \sum_{k=0}^{\infty} (1-z_0)^k \sum_{j=0}^k \frac{(-1)^j \pi^{-j+k} \sin\left(\frac{1}{2}\pi(-j+k+2z_0)\right) \Gamma^{(j)}(1-z_0)}{j!(-j+k)!}}{\left(\sum_{k=0}^{\infty} \left(\frac{1}{2} - z_0\right)^k \sum_{j=0}^k \frac{(-1)^j \pi^{-j+k} \sin\left(\frac{1}{2}\pi(-j+k+2z_0)\right) \Gamma^{(j)}(1-z_0)}{j!(-j+k)!} \right)^3}$$

Integral representations:

$$\frac{\Gamma\left(2 - \frac{3}{2}\right)\Gamma\left(\frac{3}{2} - 1\right)^2}{\left(2\sqrt{2300^2}\right)(2(4\pi)^{1.5}\Gamma(1))} = \frac{0.000013587 e^3 \int_0^1 \frac{1-\sqrt{x}}{2\log(x)+2\sqrt{x}\log(x)} dx}{\pi^{1.5}}$$

$$\frac{\Gamma\left(2 - \frac{3}{2}\right)\Gamma\left(\frac{3}{2} - 1\right)^2}{\left(2\sqrt{2300^2}\right)(2(4\pi)^{1.5}\Gamma(1))} = \frac{0.000013587 \exp\left(-\frac{\gamma}{2} + \int_0^1 \frac{2-3\sqrt{x}+x+3\log(\sqrt{x})-\log(x)}{\log(x)-x\log(x)} dx\right)}{\pi^{1.5}}$$

$$\frac{\Gamma\left(2 - \frac{3}{2}\right)\Gamma\left(\frac{3}{2} - 1\right)^2}{\left(2\sqrt{2300^2}\right)(2(4\pi)^{1.5}\Gamma(1))} = \frac{0.000013587 \left(\int_0^1 \frac{1}{\sqrt{\log\left(\frac{1}{t}\right)}} dt \right)^3}{\pi^{1.5} \int_0^1 1 dt}$$

$$1 / ((((-(((1/2 * (\text{gamma}(2-3/2) * (\text{gamma}(3/2-1))^2 * 1/(2300^2)^{(0.5)})))) / ((2(4\pi)^{1.5} (\text{gamma}(1))))))))))$$

Input:

$$\frac{1}{\frac{\frac{1}{2} \left(\Gamma\left(2-\frac{3}{2}\right) \Gamma\left(\frac{3}{2}-1\right)^2 \times \frac{1}{\sqrt{2300^2}} \right)}{2(4\pi)^{1.5} \Gamma(1)}}$$

$\Gamma(x)$ is the gamma function

Result:

-73600

-73600

Alternative representations:

$$-\frac{1}{\frac{\Gamma\left(2-\frac{3}{2}\right) \Gamma\left(\frac{3}{2}-1\right)^2}{\left(2\sqrt{2300^2}\right) (2(4\pi)^{1.5} \Gamma(1))}} = -\frac{1}{\frac{\left(-\frac{1}{2}\right)! \left(\left(-\frac{1}{2}\right)!\right)^2}{2(2 \times 0! (4\pi)^{1.5}) \sqrt{2300^2}}}$$

$$-\frac{1}{\frac{\Gamma\left(2-\frac{3}{2}\right) \Gamma\left(\frac{3}{2}-1\right)^2}{\left(2\sqrt{2300^2}\right) (2(4\pi)^{1.5} \Gamma(1))}} = -\frac{1}{\frac{e^{-\log G(1/2) + \log G(3/2)} \left(e^{-\log G(1/2) + \log G(3/2)}\right)^2}{2(2 e^0 (4\pi)^{1.5}) \sqrt{2300^2}}}$$

$$-\frac{1}{\frac{\Gamma\left(2-\frac{3}{2}\right) \Gamma\left(\frac{3}{2}-1\right)^2}{\left(2\sqrt{2300^2}\right) (2(4\pi)^{1.5} \Gamma(1))}} = -\frac{1}{\frac{\Gamma\left(\frac{1}{2}, 0\right) \Gamma\left(\frac{1}{2}, 0\right)^2}{2(2 \Gamma(1, 0) (4\pi)^{1.5}) \sqrt{2300^2}}}$$

Series representations:

$$-\frac{1}{\frac{\Gamma\left(2-\frac{3}{2}\right) \Gamma\left(\frac{3}{2}-1\right)^2}{\left(2\sqrt{2300^2}\right) (2(4\pi)^{1.5} \Gamma(1))}} = -\frac{73600 \pi^{1.5} \sum_{k=0}^{\infty} \frac{(1-z_0)^k \Gamma^{(k)}(z_0)}{k!}}{\left(\sum_{k=0}^{\infty} \frac{\left(\frac{1}{2}-z_0\right)^k \Gamma^{(k)}(z_0)}{k!}\right)^3} \quad \text{for } (z_0 \notin \mathbb{Z} \text{ or } z_0 > 0)$$

$$-\frac{1}{\frac{\Gamma\left(2-\frac{3}{2}\right)\Gamma\left(\frac{3}{2}-1\right)^2}{\left(2\sqrt{2300^2}\right)\left(2(4\pi)^{1.5}\Gamma(1)\right)}} = \frac{73600 \left(\sum_{k=0}^{\infty} \left(\frac{1}{2} - z_0\right)^k \sum_{j=0}^k \frac{(-1)^j \pi^{-j+k} \sin\left(\frac{1}{2}\pi(-j+k+2z_0)\right) \Gamma^{(j)}(1-z_0)}{j!(-j+k)!} \right)^3}{\pi^{0.5} \sum_{k=0}^{\infty} (1-z_0)^k \sum_{j=0}^k \frac{(-1)^j \pi^{-j+k} \sin\left(\frac{1}{2}\pi(-j+k+2z_0)\right) \Gamma^{(j)}(1-z_0)}{j!(-j+k)!}}$$

Integral representations:

$$-\frac{1}{\frac{\Gamma\left(2-\frac{3}{2}\right)\Gamma\left(\frac{3}{2}-1\right)^2}{\left(2\sqrt{2300^2}\right)\left(2(4\pi)^{1.5}\Gamma(1)\right)}} = -73600 \exp\left(-3 \int_0^1 \frac{1-\sqrt{x}}{2 \log(x) + 2\sqrt{x} \log(x)} dx\right) \pi^{1.5}$$

$$-\frac{1}{\frac{\Gamma\left(2-\frac{3}{2}\right)\Gamma\left(\frac{3}{2}-1\right)^2}{\left(2\sqrt{2300^2}\right)\left(2(4\pi)^{1.5}\Gamma(1)\right)}} = -73600 \exp\left(\frac{\gamma}{2} + \int_0^1 \frac{2-3\sqrt{x}+x+3\log(\sqrt{x})-\log(x)}{(-1+x)\log(x)} dx\right) \pi^{1.5}$$

$$-\frac{1}{\frac{\Gamma\left(2-\frac{3}{2}\right)\Gamma\left(\frac{3}{2}-1\right)^2}{\left(2\sqrt{2300^2}\right)\left(2(4\pi)^{1.5}\Gamma(1)\right)}} = -\frac{73600 \pi^{1.5} \int_0^1 1 dt}{\left(\int_0^1 \frac{1}{\sqrt{\log\left(\frac{1}{t}\right)}} dt\right)^3}$$

$$-\left(\frac{1}{\left(\left(\left(\left(\left(\frac{1}{2} * \left(\Gamma\left(2-\frac{3}{2}\right) * \left(\Gamma\left(\frac{3}{2}-1\right)\right)^2 * \frac{1}{\left(2300^2\right)^{0.5}}\right)\right)\right)\right)\right)\right)\right) / \left(\left(\left(2(4\pi)^{1.5} \Gamma(1)\right)\right)\right)\right) + 27 * 4$$

Input:

$$\left[-\frac{1}{\frac{\frac{1}{2} \left(\Gamma\left(2-\frac{3}{2}\right) \Gamma\left(\frac{3}{2}-1\right)^2 \times \frac{1}{\sqrt{2300^2}} \right)}{2(4\pi)^{1.5} \Gamma(1)}} + 27 \times 4 \right]$$

$\Gamma(x)$ is the gamma function

Result:

73492

73492

Alternative representations:

$$-\left[\frac{1}{\frac{\Gamma(2-\frac{3}{2})\Gamma(\frac{3}{2}-1)^2}{(2\sqrt{2300^2})(2(4\pi)^{1.5}\Gamma(1))}} + 27 \times 4 \right] = -108 - \frac{1}{\frac{(-\frac{1}{2})!((-\frac{1}{2})!)^2}{2(2 \times 0!(4\pi)^{1.5})\sqrt{2300^2}}}$$

$$-\left[\frac{1}{\frac{\Gamma(2-\frac{3}{2})\Gamma(\frac{3}{2}-1)^2}{(2\sqrt{2300^2})(2(4\pi)^{1.5}\Gamma(1))}} + 27 \times 4 \right] = -108 - \frac{1}{\frac{e^{-\log G(1/2)+\log G(3/2)}(e^{-\log G(1/2)+\log G(3/2)})^2}{2(2e^0(4\pi)^{1.5})\sqrt{2300^2}}}$$

$$-\left[\frac{1}{\frac{\Gamma(2-\frac{3}{2})\Gamma(\frac{3}{2}-1)^2}{(2\sqrt{2300^2})(2(4\pi)^{1.5}\Gamma(1))}} + 27 \times 4 \right] = -108 - \frac{1}{\frac{\Gamma(\frac{1}{2},0)\Gamma(\frac{1}{2},0)^2}{2(2\Gamma(1,0)(4\pi)^{1.5})\sqrt{2300^2}}}$$

Series representations:

$$-\left[\frac{1}{\frac{\Gamma(2-\frac{3}{2})\Gamma(\frac{3}{2}-1)^2}{(2\sqrt{2300^2})(2(4\pi)^{1.5}\Gamma(1))}} + 27 \times 4 \right] = \frac{73600 \left(-0.00146739 \left(\sum_{k=0}^{\infty} \frac{(\frac{1}{2}-z_0)^k \Gamma^{(k)}(z_0)}{k!} \right)^3 + \pi^{1.5} \sum_{k=0}^{\infty} \frac{(1-z_0)^k \Gamma^{(k)}(z_0)}{k!} \right)}{\left(\sum_{k=0}^{\infty} \frac{(\frac{1}{2}-z_0)^k \Gamma^{(k)}(z_0)}{k!} \right)^3}$$

for $(z_0 \notin \mathbb{Z} \text{ or } z_0 > 0)$

$$\begin{aligned}
& - \left[- \frac{1}{\frac{\Gamma(2-\frac{3}{2})\Gamma(\frac{3}{2}-1)^2}{(2\sqrt{2300^2})(2(4\pi)^{1.5}\Gamma(1))}} + 27 \times 4 \right] = \\
& - \left(\left(108 \left(-681.481 \left(\sum_{k=0}^{\infty} \left(\frac{1}{2} - z_0 \right)^k \sum_{j=0}^k \frac{(-1)^j \pi^{-j+k} \sin\left(\frac{1}{2} \pi (-j+k+2z_0)\right) \Gamma^{(j)}(1-z_0)}{j!(-j+k)!} \right)^3 \right. \right. \right. \\
& \quad \left. \left. \left. + \pi^{0.5} \sum_{k=0}^{\infty} (1-z_0)^k \sum_{j=0}^k \frac{(-1)^j \pi^{-j+k} \sin\left(\frac{1}{2} \pi (-j+k+2z_0)\right) \Gamma^{(j)}(1-z_0)}{j!(-j+k)!} \right) \right) \right) / \\
& \quad \left(\pi^{0.5} \sum_{k=0}^{\infty} (1-z_0)^k \sum_{j=0}^k \frac{(-1)^j \pi^{-j+k} \sin\left(\frac{1}{2} \pi (-j+k+2z_0)\right) \Gamma^{(j)}(1-z_0)}{j!(-j+k)!} \right)
\end{aligned}$$

Integral representations:

$$\begin{aligned}
& - \left[- \frac{1}{\frac{\Gamma(2-\frac{3}{2})\Gamma(\frac{3}{2}-1)^2}{(2\sqrt{2300^2})(2(4\pi)^{1.5}\Gamma(1))}} + 27 \times 4 \right] = \\
& -108 + 73\,600 \exp\left(-3 \int_0^1 \frac{1-\sqrt{x}}{2 \log(x) + 2\sqrt{x} \log(x)} dx\right) \pi^{1.5}
\end{aligned}$$

$$\begin{aligned}
& - \left[- \frac{1}{\frac{\Gamma(2-\frac{3}{2})\Gamma(\frac{3}{2}-1)^2}{(2\sqrt{2300^2})(2(4\pi)^{1.5}\Gamma(1))}} + 27 \times 4 \right] = \\
& -108 + 73\,600 \exp\left(\frac{\gamma}{2} + \int_0^1 \frac{2-3\sqrt{x}+x+3\log(\sqrt{x})-\log(x)}{(-1+x)\log(x)} dx\right) \pi^{1.5}
\end{aligned}$$

$$-\left[\frac{1}{\frac{\Gamma(2-\frac{3}{2})\Gamma(\frac{3}{2}-1)^2}{(2\sqrt{2300^2})(2(4\pi)^{1.5}\Gamma(1))}} + 27 \times 4 \right] =$$

$$\frac{73600 \left(\pi^{1.5} \int_0^1 1 dt - 0.00146739 \left(\int_0^1 \frac{1}{\sqrt{\log(\frac{1}{t})}} dt \right)^3 \right)}{\left(\int_0^1 \frac{1}{\sqrt{\log(\frac{1}{t})}} dt \right)^3}$$

$\log(x)$ is the natural logarithm

γ is the Euler-Mascheroni constant

Thence, we have the following mathematical connection:

$$\left(- \left[\frac{1}{\frac{\frac{1}{2} \left(\Gamma(2-\frac{3}{2})\Gamma(\frac{3}{2}-1)^2 \times \frac{1}{\sqrt{2300^2}} \right)}{2(4\pi)^{1.5}\Gamma(1)}} + 27 \times 4 \right] \right) = 73492 \Rightarrow$$

$$\Rightarrow -3927 + 2 \left(\sqrt[13]{ N \exp \left[\int d\hat{\sigma} \left(-\frac{1}{4u^2} \mathbf{P}_i D \mathbf{P}_i \right) \right] |Bp\rangle_{NS} + \int [d\mathbf{X}^\mu] \exp \left\{ \int d\hat{\sigma} \left(-\frac{1}{4v^2} D\mathbf{X}^\mu D^2 \mathbf{X}^\mu \right) \right\} |X^\mu, X^i = 0\rangle_{NS} } \right) =$$

$$-3927 + 2 \sqrt[13]{ 2.2983717437 \times 10^{59} + 2.0823329825883 \times 10^{59} }$$

$$= 73490.8437525.... \Rightarrow$$

$$\Rightarrow \left(A(r) \times \frac{1}{B(r)} \left(-\frac{1}{\phi(r)} \right) \times \frac{1}{e^{\Lambda(r)}} \right) \Rightarrow$$

$$\Rightarrow \left(-0.000029211892 \times \frac{1}{0.0003644621} \left(-\frac{1}{0.0005946833} \right) \times \frac{1}{0.00183393} \right) =$$

$$= 73491.78832548118710549159572042220548025195726563413398700\dots$$

$$= 73491.7883254\dots \Rightarrow$$

$$\left(I_{21} \ll \int_{-\infty}^{+\infty} \exp \left(-\left(\frac{t}{H} \right)^2 \right) \left| \sum_{\lambda \leq P^{1-\epsilon_1}} \frac{a(\lambda)}{\sqrt{\lambda}} B(\lambda) \lambda^{-i(T+t)} \right|^2 dt \ll \right.$$

$$\left. \ll H \left\{ \left(\frac{4}{\epsilon_2 \log T} \right)^{2r} (\log T) (\log X)^{-2\beta} + (\epsilon_2^{-2r} (\log T)^{-2r} + \epsilon_2^{-r} h_1^r (\log T)^{-r} T^{-\epsilon_1} \right\} \right)$$

$$/(26 \times 4)^2 - 24 = \left(\frac{7.9313976505275 \times 10^8}{(26 \times 4)^2 - 24} \right) = 73493.30662\dots$$

Mathematical connections with the boundary state corresponding to the NSNS-sector of N Dp-branes in the limit of $u \rightarrow \infty$, with the ratio concerning the general asymptotically flat solution of the equations of motion of the p-brane and with the Karatsuba's equation concerning the zeros of a special type of function connected with Dirichlet series.

$$f(0, \eta) = -\frac{1}{3} \frac{\Gamma(2 - \frac{D}{2})}{2(4\pi)^{\frac{D}{2}} \Gamma(2)} \eta^{D-4}$$

$$-1/3 * (((\text{gamma}(2-3/2) * 1/(0.5)))) * 1/(((2*(4\text{Pi})^1.5) (\text{gamma}(2))))$$

Input:

$$-\frac{1}{3} \left(\Gamma\left(2 - \frac{3}{2}\right) \times \frac{1}{0.5} \right) \times \frac{1}{(2 (4 \pi)^{1.5}) \Gamma(2)}$$

$\Gamma(x)$ is the gamma function

Result:

-0.0132629...

-0.0132629...

Alternative representations:

$$\frac{\Gamma\left(2 - \frac{3}{2}\right)(-1)}{(0.5 ((2 (4 \pi)^{1.5}) \Gamma(2))) 3} = - \frac{e^{-\log G(1/2) + \log G(3/2)}}{3 \times 0.5 (2 e^0 (4 \pi)^{1.5})}$$

$$\frac{\Gamma\left(2 - \frac{3}{2}\right)(-1)}{(0.5 ((2 (4 \pi)^{1.5}) \Gamma(2))) 3} = - \frac{\left(-\frac{1}{2}\right)!}{3 \times 0.5 (2 \times 1! (4 \pi)^{1.5})}$$

$$\frac{\Gamma\left(2 - \frac{3}{2}\right)(-1)}{(0.5 ((2 (4 \pi)^{1.5}) \Gamma(2))) 3} = - \frac{\Gamma\left(\frac{1}{2}, 0\right)}{3 \times 0.5 (2 \Gamma(2, 0) (4 \pi)^{1.5})}$$

Series representations:

$$\frac{\Gamma\left(2 - \frac{3}{2}\right)(-1)}{(0.5 ((2 (4 \pi)^{1.5}) \Gamma(2))) 3} = - \frac{0.0416667 \sum_{k=0}^{\infty} \frac{\left(\frac{1}{2} - z_0\right)^k \Gamma^{(k)}(z_0)}{k!}}{\pi^{1.5} \sum_{k=0}^{\infty} \frac{(2 - z_0)^k \Gamma^{(k)}(z_0)}{k!}} \quad \text{for } (z_0 \notin \mathbb{Z} \text{ or } z_0 > 0)$$

$$\frac{\Gamma\left(2 - \frac{3}{2}\right)(-1)}{(0.5 ((2 (4 \pi)^{1.5}) \Gamma(2))) 3} = \frac{0.0416667 \sum_{k=0}^{\infty} (2 - z_0)^k \sum_{j=0}^k \frac{(-1)^j \pi^{-j+k} \sin\left(\frac{1}{2} \pi (-j+k+2 z_0)\right) \Gamma^{(j)}(1-z_0)}{j! (-j+k)!}}{\pi^{1.5} \sum_{k=0}^{\infty} \left(\frac{1}{2} - z_0\right)^k \sum_{j=0}^k \frac{(-1)^j \pi^{-j+k} \sin\left(\frac{1}{2} \pi (-j+k+2 z_0)\right) \Gamma^{(j)}(1-z_0)}{j! (-j+k)!}}$$

Integral representations:

$$\frac{\Gamma\left(2 - \frac{3}{2}\right)(-1)}{(0.5 ((2 (4 \pi)^{1.5}) \Gamma(2))) 3} = - \frac{0.0416667 \exp\left(\int_0^1 \frac{3+\sqrt{x}-2x-2x^{3/2}}{2 \log(x)+2 \sqrt{x} \log(x)} dx\right)}{\pi^{1.5}}$$

$$\frac{\Gamma\left(2 - \frac{3}{2}\right)(-1)}{(0.5 ((2 (4 \pi)^{1.5}) \Gamma(2))) 3} = - \frac{0.0416667 \exp\left(\frac{3\gamma}{2} + \int_0^1 \frac{\sqrt{x}-x^2-\log(\sqrt{x})+\log(x^2)}{(-1+x) \log(x)} dx\right)}{\pi^{1.5}}$$

$$\frac{\Gamma\left(2 - \frac{3}{2}\right)(-1)}{(0.5 ((2 (4 \pi)^{1.5}) \Gamma(2))) 3} = - \frac{0.0416667 \int_0^1 \frac{1}{\sqrt{\log\left(\frac{1}{t}\right)}} dt}{\pi^{1.5} \int_0^1 \log\left(\frac{1}{t}\right) dt}$$

$$(-0.9243408674589*2)/(((-1/3 * (((\Gamma(2-3/2)*1/(0.5)))) * 1/(((2*(4\pi)^1.5) (\Gamma(2)))))))$$

Where 0.9243408674589 is a Ramanujan mock theta function

Input interpretation:

$$-0.9243408674589 \times 2$$

$$- \frac{1}{3} \left(\Gamma\left(2 - \frac{3}{2}\right) \times \frac{1}{0.5} \right) \times \frac{1}{(2(4\pi)^{1.5})\Gamma(2)}$$

Γ(x) is the gamma function

Result:

139.387...

139.387.... result very near to the rest mass of Pion meson 139.57

Alternative representations:

$$\frac{-0.92434086745890000 \times 2}{- \frac{\Gamma\left(2 - \frac{3}{2}\right)}{3 \times 0.5 ((2 (4 \pi)^{1.5}) \Gamma(2))}} = \frac{-1.8486817349178000}{- \frac{\left(-\frac{1}{2}\right)!}{3 \times 0.5 (2 \times 1! (4 \pi)^{1.5})}}$$

$$\frac{-0.92434086745890000 \times 2}{- \frac{\Gamma\left(2 - \frac{3}{2}\right)}{3 \times 0.5 ((2 (4 \pi)^{1.5}) \Gamma(2))}} = \frac{-1.8486817349178000}{- \frac{e^{-\log G(1/2)+\log G(3/2)}}{3 \times 0.5 (2 e^0 (4 \pi)^{1.5})}}$$

$$\frac{-0.92434086745890000 \times 2}{-\frac{\Gamma(2-\frac{3}{2})}{3 \times 0.5 ((2(4\pi)^{1.5}) \Gamma(2))}} = \frac{-1.8486817349178000}{-\frac{\Gamma(\frac{1}{2}, 0)}{3 \times 0.5 (2 \Gamma(2, 0) (4\pi)^{1.5})}}$$

Series representations:

$$\frac{-0.92434086745890000 \times 2}{-\frac{\Gamma(2-\frac{3}{2})}{3 \times 0.5 ((2(4\pi)^{1.5}) \Gamma(2))}} = \frac{44.3684 \pi^{1.5} \sum_{k=0}^{\infty} \frac{(2-z_0)^k \Gamma^{(k)}(z_0)}{k!}}{\sum_{k=0}^{\infty} \frac{(\frac{1}{2}-z_0)^k \Gamma^{(k)}(z_0)}{k!}} \quad \text{for } (z_0 \notin \mathbb{Z} \text{ or } z_0 > 0)$$

$$\frac{-0.92434086745890000 \times 2}{-\frac{\Gamma(2-\frac{3}{2})}{3 \times 0.5 ((2(4\pi)^{1.5}) \Gamma(2))}} = \frac{44.3684 \pi^{1.5} \sum_{k=0}^{\infty} (\frac{1}{2} - z_0)^k \sum_{j=0}^k \frac{(-1)^j \pi^{-j+k} \sin(\frac{1}{2} \pi (-j+k+2z_0)) \Gamma^{(j)}(1-z_0)}{j!(-j+k)!}}{\sum_{k=0}^{\infty} (2-z_0)^k \sum_{j=0}^k \frac{(-1)^j \pi^{-j+k} \sin(\frac{1}{2} \pi (-j+k+2z_0)) \Gamma^{(j)}(1-z_0)}{j!(-j+k)!}}$$

Integral representations:

$$\frac{-0.92434086745890000 \times 2}{-\frac{\Gamma(2-\frac{3}{2})}{3 \times 0.5 ((2(4\pi)^{1.5}) \Gamma(2))}} = 44.3684 \exp\left(\int_0^1 -\frac{3 + \sqrt{x} - 2x - 2x^{3/2}}{2 \log(x) + 2\sqrt{x} \log(x)} dx\right) \pi^{1.5}$$

$$\frac{-0.92434086745890000 \times 2}{-\frac{\Gamma(2-\frac{3}{2})}{3 \times 0.5 ((2(4\pi)^{1.5}) \Gamma(2))}} = 44.3684 \exp\left(-\frac{3\gamma}{2} + \int_0^1 \frac{\sqrt{x} - x^2 - \log(\sqrt{x}) + \log(x^2)}{\log(x) - x \log(x)} dx\right) \pi^{1.5}$$

$$\frac{-0.92434086745890000 \times 2}{-\frac{\Gamma(2-\frac{3}{2})}{3 \times 0.5 ((2(4\pi)^{1.5}) \Gamma(2))}} = \frac{44.3684 \pi^{1.5} \int_0^1 \log\left(\frac{1}{t}\right) dt}{\int_0^1 \frac{1}{\sqrt{\log\left(\frac{1}{t}\right)}} dt}$$

$$\left(\left(-\frac{1}{3} \left(\left(\left(\Gamma\left(2-\frac{3}{2}\right) \frac{1}{(0.5)}\right)\right)\right) \right) \frac{1}{\left(\left(2 \cdot (4\pi)^{1.5} \Gamma(2)\right)\right)}\right)^{1/512}$$

Input:

$$^{512}\sqrt{-\frac{1}{3} \left(\Gamma\left(2 - \frac{3}{2}\right) \times \frac{1}{0.5}\right) \times \frac{1}{(2(4\pi)^{1.5}) \Gamma(2)}}$$

$\Gamma(x)$ is the gamma function

Result:

$$0.99157394\dots + \\ 0.0060842978\dots i$$

Polar coordinates:

$$r = 0.991593 \text{ (radius)}, \quad \theta = 0.351563^\circ \text{ (angle)}$$

0.991593 result very near to the value of the following Rogers-Ramanujan continued fraction:

$$\frac{\frac{e^{-\frac{\pi}{\sqrt{5}}}}{\sqrt{5}}}{1 + \sqrt[5]{\sqrt{\phi^5 4\sqrt{5^3}} - 1}} - \phi + 1 = 1 - \frac{e^{-\pi\sqrt{5}}}{1 + \frac{e^{-2\pi\sqrt{5}}}{1 + \frac{e^{-3\pi\sqrt{5}}}{1 + \frac{e^{-4\pi\sqrt{5}}}{1 + \dots}}}} \approx 0.9991104684$$

and to the dilaton value **0.989117352243 = ϕ**

Phenomenological implications of our result for the SM rho would be divided into two different scenarios depending on the possible value of a single extra free parameter existing in the nonperturbative theory, $M_\rho = g_{\rho\pi\pi} \cdot F_\rho$ (or $g_{\rho\pi\pi} = g_{\text{HLS}} \equiv g_{\text{HLS}}(M_\rho^2) = M_\rho/F_\rho = M_\rho/(350\text{GeV})$ or the cutoff Λ (or the Landau pole $\tilde{\Lambda}$) in view of Eqs.(86) -(88):

$$\Lambda = e^{-4/3} \cdot \tilde{\Lambda} = e^{-4/3} \cdot M_\rho \cdot \exp\left[\frac{\frac{3}{8}(4\pi F_\rho)^2}{M_\rho^2}\right], \quad (136)$$

which implies that $\Lambda < M_\rho$ ($g_{\text{HLS}} > 6.7$, $M_\rho > 2.3 \text{ TeV}$) and $\Lambda > M_\rho$ ($g_{\text{HLS}} < 6.7$, $M_\rho < 2.3 \text{ TeV}$).

1) "Low M_ρ scenario" ($M_\rho < 2.3 \text{ TeV}$, $\Lambda > M_\rho$):

2) "High M_ρ scenario" ($M_\rho \gg 2.3 \text{ TeV}$, $\Lambda < M_\rho$, as a stabilizer of the skyrmion dark matter X_s)

$$M_{X_s} \lesssim 11 \text{ GeV}, \quad \text{or equivalently, } \lambda_{\varphi X_s X_s} \equiv \frac{g_{\varphi X_s X_s}}{2F_\varphi} = \frac{M_{X_s}^2}{F_\pi^2} \lesssim 0.002, \quad (F_\varphi = F_\pi = \sqrt{N}v = 246 \text{ GeV})$$

$$M_{X_s} \simeq 35 \frac{F_\pi}{g_{\text{HLS}}} \simeq 11 \text{ GeV} \times \left(\frac{780}{g_{\text{HLS}}} \right), \quad \lambda_{\varphi X_s X_s} = \left(\frac{35}{g_{\text{HLS}}} \right)^2 = 0.002 \times \left(\frac{780}{g_{\text{HLS}}} \right)^2, \quad (138)$$

which would imply

$$g_{\text{HLS}} \simeq 780, \quad (139)$$

and

$$\langle r_{X_s}^2 \rangle_{X_s} \simeq \left(\frac{2.2}{g_{\text{HLS}} F_\pi} \right)^2 \simeq 1.3 \times 10^{-10} (\text{GeV})^{-2} \times \left(\frac{780}{g_{\text{HLS}}} \right)^2. \quad (140)$$

This leads to the annihilation cross section of the skyrmion dark matter and the relic abundance $\Omega_{X_s} h^2$ [1, 42]:

$$\begin{aligned} \langle \sigma_{\text{ann}} v_{\text{rel}} \rangle_{\text{radius}} &\simeq 4\pi \cdot \langle r_{X_s}^2 \rangle_{X_s} \simeq 1.7 \times 10^{-9} \text{ GeV}^{-2}, \\ \Omega_{X_s} h^2 &\simeq \mathcal{O}(0.1), \end{aligned} \quad (141)$$

We have:

$$F_\pi = 246; \quad g_{\text{HLS}} = 780$$

$$M_{X_s} \simeq 35 \frac{F_\pi}{g_{\text{HLS}}}$$

$$(35 \cdot 246) / 780$$

$$\frac{287}{26}$$

Decimal approximation:

11.03846153846153846153846153846153846153846153846153846153846153...

11.038461538...

And:

$$1 / (((35 \cdot 246) / 780))^{1/256}$$

Input:

$$\frac{1}{\sqrt[256]{\frac{35 \cdot 246}{780}}}$$

Result:

$$\sqrt[256]{\frac{26}{287}}$$

Decimal approximation:

0.990663446023417462789151326532461780371692878976507360691...

0.99066344602341.... result very near to the value of the following Rogers-Ramanujan continued fraction:

$$\frac{e^{-\frac{\pi}{\sqrt{5}}}}{\sqrt{5}} = 1 - \frac{e^{-\pi\sqrt{5}}}{1 + \frac{e^{-2\pi\sqrt{5}}}{1 + \frac{e^{-3\pi\sqrt{5}}}{1 + \frac{e^{-4\pi\sqrt{5}}}{1 + \dots}}}} \approx 0.9991104684$$

and to the dilaton value **0.989117352243** = ϕ

Alternate form:

$$\frac{1}{287} \sqrt[256]{26} 287^{255/256}$$

$$\lambda_{\varphi X_s X_s} = \left(\frac{35}{g_{\text{HLS}}} \right)^2$$

$$(35/780)^2$$

Input:

$$\left(\frac{35}{780} \right)^2$$

Exact result:

$$\frac{49}{24336}$$

Decimal approximation:

0.002013477975016436554898093359631821170282708744247205785...

0.002013477975....

And:

$$1/(35/780)^2$$

Input:

$$\frac{1}{\left(\frac{35}{780}\right)^2}$$

Exact result:

$$\frac{24\,336}{49}$$

Decimal approximation:

496.6530612244897959183673469387755102040816326530612244897...

496.653.....

$$1/\left(\left(\left(\sqrt{5}+5\right)/2\right)\right) * 1/\left(35/780\right)^2 - \pi$$

Input:

$$\frac{1}{\frac{1}{2}(\sqrt{5}+5)} \times \frac{1}{\left(\frac{35}{780}\right)^2} - \pi$$

Result:

$$\frac{48\,672}{49(5+\sqrt{5})} - \pi$$

Decimal approximation:

134.1299373455226923700809272243851124919602012138777716772...

134.129937.... result very near to the rest mass of Pion meson 134.9766

Property:

$$\frac{48\,672}{49(5+\sqrt{5})} - \pi \text{ is a transcendental number}$$

Alternate forms:

$$\frac{1}{245} \left(60\,840 - 12\,168\sqrt{5} - 245\pi \right)$$

$$-\frac{12\,168}{245} \left(\sqrt{5} - 5 \right) - \pi$$

$$\frac{1}{245} (60840 - 12168\sqrt{5})^{-\pi}$$

Series representations:

$$\frac{1}{\frac{1}{2} \left(\frac{35}{780}\right)^2 (\sqrt{5} + 5)}^{-\pi} = -\pi + \frac{48672}{49 \left(5 + \sqrt{4} \sum_{k=0}^{\infty} 4^{-k} \binom{\frac{1}{2}}{k}\right)}$$

$$\frac{1}{\frac{1}{2} \left(\frac{35}{780}\right)^2 (\sqrt{5} + 5)}^{-\pi} = -\pi + \frac{48672}{49 \left(5 + \sqrt{4} \sum_{k=0}^{\infty} \frac{(-\frac{1}{4})^k (-\frac{1}{2})_k}{k!}\right)}$$

$$\frac{1}{\frac{1}{2} \left(\frac{35}{780}\right)^2 (\sqrt{5} + 5)}^{-\pi} = -\pi + \frac{97344 \sqrt{\pi}}{49 \left(10 \sqrt{\pi} + \sum_{j=0}^{\infty} \text{Res}_{s=-\frac{1}{2}+j} 4^{-s} \Gamma(-\frac{1}{2}-s) \Gamma(s)\right)}$$

$$\langle r_{X_s}^2 \rangle_{X_s} \simeq \left(\frac{2.2}{g_{\text{HLS}} F_{\pi}} \right)^2$$

$$(2.2/(780*246))^2$$

Input:

$$\left(\frac{2.2}{780 \times 246} \right)^2$$

Result:

$$1.3145767351902283795692786067868055085675854754089626... \times 10^{-10}$$

$$1.31457673519... * 10^{-10}$$

$$\langle \sigma_{\text{ann}} v_{\text{rel}} \rangle_{\text{radius}} \simeq 4\pi \cdot \langle r_{X_s}^2 \rangle_{X_s} \simeq 1.7 \times 10^{-9} \text{ GeV}^{-2}$$

$$4\pi * (2.2/(780*246))^2$$

Input:

$$4\pi \left(\frac{2.2}{780 \times 246} \right)^2$$

Result:

$$1.65195... \times 10^{-9}$$

$$1.65195... * 10^{-9}$$

Alternative representations:

$$4\pi \left(\frac{2.2}{780 \times 246} \right)^2 = 720^\circ \left(\frac{2.2}{191880} \right)^2$$

$$4\pi \left(\frac{2.2}{780 \times 246} \right)^2 = -4i \log(-1) \left(\frac{2.2}{191880} \right)^2$$

$$4\pi \left(\frac{2.2}{780 \times 246} \right)^2 = 4 \cos^{-1}(-1) \left(\frac{2.2}{191880} \right)^2$$

Series representations:

$$4\pi \left(\frac{2.2}{780 \times 246} \right)^2 = 2.10332 \times 10^{-9} \sum_{k=0}^{\infty} \frac{(-1)^k}{1+2k}$$

$$4\pi \left(\frac{2.2}{780 \times 246} \right)^2 = -1.05166 \times 10^{-9} + 1.05166 \times 10^{-9} \sum_{k=1}^{\infty} \frac{2^k}{\binom{2k}{k}}$$

$$4\pi \left(\frac{2.2}{780 \times 246} \right)^2 = 5.25831 \times 10^{-10} \sum_{k=0}^{\infty} \frac{2^{-k} (-6 + 50k)}{\binom{3k}{k}}$$

$\binom{n}{m}$ is the binomial coefficient

Integral representations:

$$4\pi \left(\frac{2.2}{780 \times 246} \right)^2 = 1.05166 \times 10^{-9} \int_0^{\infty} \frac{1}{1+t^2} dt$$

$$4\pi \left(\frac{2.2}{780 \times 246} \right)^2 = 2.10332 \times 10^{-9} \int_0^1 \sqrt{1-t^2} dt$$

$$4\pi \left(\frac{2.2}{780 \times 246} \right)^2 = 1.05166 \times 10^{-9} \int_0^\infty \frac{\sin(t)}{t} dt$$

$$(((4\pi \cdot (2.2/(780 \cdot 246))^2)))^{1/2048}$$

Input:

$$\sqrt[2048]{4\pi \left(\frac{2.2}{780 \times 246} \right)^2}$$

Result:

0.990174897...

0.990174897.... result very near to the value of the following Rogers-Ramanujan continued fraction:

$$\frac{\frac{e^{-\frac{\pi}{\sqrt{5}}}}{\sqrt{5}}}{1 + \sqrt[5]{\sqrt{\varphi^5 4\sqrt{5^3} - 1}}} - \varphi + 1 = 1 - \frac{e^{-\pi\sqrt{5}}}{1 + \frac{e^{-2\pi\sqrt{5}}}{1 + \frac{e^{-3\pi\sqrt{5}}}{1 + \frac{e^{-4\pi\sqrt{5}}}{1 + \dots}}}} \approx 0.9991104684$$

and to the dilaton value **0.989117352243 = ϕ**

Now, we have that:

Then our main results for the SM Higgs case were obtained as the $D = 4$ and $N \rightarrow 4$ case of the above generic results. The dynamically generated kinetic term and the mass of the SM rho ρ_μ read as Eq.(86) and Eq.(87):

$$\begin{aligned} \text{SM: } \frac{1}{\lambda_{\text{HLS}}(\mu^2)} &= \frac{1}{N g_{\text{HLS}}^2(\mu^2)} = \frac{1}{3} \frac{1}{(4\pi)^2} \ln \left(\frac{\Lambda^2}{\mu^2} \right), \\ M_\rho^2(\mu^2) &= g_{\text{HLS}}^2(\mu^2) \cdot F_\rho^2, \\ F_\rho^2 &= 2 \cdot N v^2 = 2 \cdot F_\pi^2 \simeq 2 \cdot (246 \text{ GeV})^2 \simeq (350 \text{ GeV})^2, \end{aligned} \quad (145)$$

1) "Low M_ρ scenario" ($M_\rho < 2.3$ TeV, $\Lambda > M_\rho$, collider detection):

A typical example is $M_\rho = 2$ TeV ($g_{\rho\pi\pi} \simeq 5.7$), which is a simple scale-up of the QCD ρ meson, thus is perfectly natural with $\Lambda \simeq 3.3$ TeV $\simeq 4\pi F_\pi$. This yields the "broad width" $\Gamma_\rho \simeq \Gamma_{\rho \rightarrow WW} \simeq g_{\rho\pi\pi}^2 M_\rho / (48\pi) \simeq 433$ GeV, which, although a scale-up of the ρ meson width, may be barely detectable at LHC. For larger (smaller) M_ρ the width gets larger (smaller) as $\sim M_\rho^3$, and the production cross section gets smaller (larger) as $\sim 1/M_\rho^2$, thus more difficult for $M_\rho > 2$ TeV to be seen at LHC. The SM rho with narrow resonance $\Gamma_\rho \lesssim 100$ GeV if any could be detected at LHC for $M_\rho \lesssim 1.2$ TeV, which corresponds to $g_{\text{HLS}} \lesssim 3.5$ and $\Lambda \gtrsim 50$ TeV.

We have:

$$2 \cdot (246)^2 = 121032;$$

Input:

$$\sqrt{121032}$$

Result:

$$246\sqrt{2}$$

Decimal approximation:

$$347.8965363437813820052154261555857273281392813427292260014\dots$$

$$347.89653634\dots = F_\pi$$

$$(5.7^2 \cdot 2000) / (48\pi)$$

Input:

$$\frac{5.7^2 \times 2000}{48\pi}$$

Result:

$$430.912\dots$$

$$430.912$$

Alternative representations:

$$\frac{5.7^2 \times 2000}{48\pi} = \frac{2000 \times 5.7^2}{8640^\circ}$$

$$\frac{5.7^2 \times 2000}{48\pi} = -\frac{2000 \times 5.7^2}{48 i \log(-1)}$$

$$\frac{5.7^2 \times 2000}{48\pi} = \frac{2000 \times 5.7^2}{48 \cos^{-1}(-1)}$$

Series representations:

$$\frac{5.7^2 \times 2000}{48 \pi} = \frac{338.438}{\sum_{k=0}^{\infty} \frac{(-1)^k}{1+2k}}$$

$$\frac{5.7^2 \times 2000}{48 \pi} = \frac{676.875}{-1 + \sum_{k=1}^{\infty} \frac{2^k}{\binom{2k}{k}}}$$

$$\frac{5.7^2 \times 2000}{48 \pi} = \frac{1353.75}{\sum_{k=0}^{\infty} \frac{2^{-k}(-6+50k)}{\binom{3k}{k}}}$$

Integral representations:

$$\frac{5.7^2 \times 2000}{48 \pi} = \frac{676.875}{\int_0^{\infty} \frac{1}{1+t^2} dt}$$

$$\frac{5.7^2 \times 2000}{48 \pi} = \frac{338.438}{\int_0^1 \sqrt{1-t^2} dt}$$

$$\frac{5.7^2 \times 2000}{48 \pi} = \frac{676.875}{\int_0^{\infty} \frac{\sin(t)}{t} dt}$$

And:

$$4\pi \sqrt{2 \times 246^2}$$

Input:

$$4 \pi \sqrt{2 \times 246^2}$$

Result:

$$984 \sqrt{2} \pi$$

Decimal approximation:

4371.796811147832387063626894219722599436787742345679179516...

4371.7968111...

Property:

$984\sqrt{2}\pi$ is a transcendental number

Series representations:

$$4\pi\sqrt{2 \times 246^2} = 4\pi\sqrt{121031} \sum_{k=0}^{\infty} 121031^{-k} \binom{\frac{1}{2}}{k}$$

$$4\pi\sqrt{2 \times 246^2} = 4\pi\sqrt{121031} \sum_{k=0}^{\infty} \frac{\left(-\frac{1}{121031}\right)^k \left(-\frac{1}{2}\right)_k}{k!}$$

$$4\pi\sqrt{2 \times 246^2} = \frac{2\pi \sum_{j=0}^{\infty} \operatorname{Res}_{s=-\frac{1}{2}+j} 121031^{-s} \Gamma\left(-\frac{1}{2}-s\right) \Gamma(s)}{\sqrt{\pi}}$$

And:

$$55 + \pi + \frac{1}{\left(\frac{1}{2}(\sqrt{5} + 1)\right)^2} \times 4\pi\sqrt{2 \times (246)^2}$$

Input:

$$55 + \pi + \frac{1}{\left(\frac{1}{2}(\sqrt{5} + 1)\right)^2} \times 4\left(\pi\sqrt{2 \times 246^2}\right)$$

Result:

$$55 + \pi + \frac{3936\sqrt{2}\pi}{(1 + \sqrt{5})^2}$$

Decimal approximation:

1728.019382603656566492596054915250648651976071328252528446...

1728.0193826.....

This result is very near to the mass of candidate glueball $f_0(1710)$ meson. Furthermore, 1728 occurs in the algebraic formula for the j-invariant of an elliptic curve. As a consequence, it is sometimes called a Zagier as a pun on the Gross–Zagier theorem. The number 1728 is one less than the Hardy–Ramanujan number 1729

Property:

$55 + \pi + \frac{3936 \sqrt{2} \pi}{(1 + \sqrt{5})^2}$ is a transcendental number

Alternate forms:

$$55 + \pi + 1476 \sqrt{2} \pi - 492 \sqrt{10} \pi$$

$$55 + \pi + 984 \sqrt{7 - 3\sqrt{5}} \pi$$

$$55 + \pi + \frac{1968 \sqrt{2} \pi}{3 + \sqrt{5}}$$

Series representations:

$$55 + \pi + \frac{4 \left(\pi \sqrt{2 \times 246^2} \right)}{\left(\frac{1}{2} (\sqrt{5} + 1) \right)^2} =$$

$$\left(55 + \pi + 110 \sqrt{4} \sum_{k=0}^{\infty} 4^{-k} \binom{\frac{1}{2}}{k} + 2 \pi \sqrt{4} \sum_{k=0}^{\infty} 4^{-k} \binom{\frac{1}{2}}{k} + 55 \sqrt{4}^2 \left(\sum_{k=0}^{\infty} 4^{-k} \binom{\frac{1}{2}}{k} \right)^2 + \right.$$

$$\left. \pi \sqrt{4}^2 \left(\sum_{k=0}^{\infty} 4^{-k} \binom{\frac{1}{2}}{k} \right)^2 + 16 \pi \sqrt{121031} \sum_{k=0}^{\infty} 121031^{-k} \binom{\frac{1}{2}}{k} \right) /$$

$$\left(1 + \sqrt{4} \sum_{k=0}^{\infty} 4^{-k} \binom{\frac{1}{2}}{k} \right)^2$$

$$55 + \pi + \frac{4 \left(\pi \sqrt{2 \times 246^2} \right)}{\left(\frac{1}{2} (\sqrt{5} + 1) \right)^2} =$$

$$\left(55 + \pi + 110 \sqrt{4} \sum_{k=0}^{\infty} \frac{\left(-\frac{1}{4}\right)^k \left(-\frac{1}{2}\right)_k}{k!} + 2 \pi \sqrt{4} \sum_{k=0}^{\infty} \frac{\left(-\frac{1}{4}\right)^k \left(-\frac{1}{2}\right)_k}{k!} + \right.$$

$$55 \sqrt{4}^2 \left(\sum_{k=0}^{\infty} \frac{\left(-\frac{1}{4}\right)^k \left(-\frac{1}{2}\right)_k}{k!} \right)^2 + \pi \sqrt{4}^2 \left(\sum_{k=0}^{\infty} \frac{\left(-\frac{1}{4}\right)^k \left(-\frac{1}{2}\right)_k}{k!} \right)^2 +$$

$$\left. 16 \pi \sqrt{121031} \sum_{k=0}^{\infty} \frac{\left(-\frac{1}{121031}\right)^k \left(-\frac{1}{2}\right)_k}{k!} \right) / \left(1 + \sqrt{4} \sum_{k=0}^{\infty} \frac{\left(-\frac{1}{4}\right)^k \left(-\frac{1}{2}\right)_k}{k!} \right)^2$$

$$\begin{aligned}
55 + \pi + \frac{4 \left(\pi \sqrt{2 \times 246^2} \right)}{\left(\frac{1}{2} (\sqrt{5} + 1) \right)^2} &= \left(55 + \pi + 110 \sqrt{z_0} \sum_{k=0}^{\infty} \frac{(-1)^k \left(-\frac{1}{2} \right)_k (5 - z_0)^k z_0^{-k}}{k!} + \right. \\
& 2 \pi \sqrt{z_0} \sum_{k=0}^{\infty} \frac{(-1)^k \left(-\frac{1}{2} \right)_k (5 - z_0)^k z_0^{-k}}{k!} + 55 \sqrt{z_0}^2 \\
& \left. \left(\sum_{k=0}^{\infty} \frac{(-1)^k \left(-\frac{1}{2} \right)_k (5 - z_0)^k z_0^{-k}}{k!} \right)^2 + \pi \sqrt{z_0}^2 \left(\sum_{k=0}^{\infty} \frac{(-1)^k \left(-\frac{1}{2} \right)_k (5 - z_0)^k z_0^{-k}}{k!} \right)^2 + \right. \\
& \left. 16 \pi \sqrt{z_0} \sum_{k=0}^{\infty} \frac{(-1)^k \left(-\frac{1}{2} \right)_k (121\,032 - z_0)^k z_0^{-k}}{k!} \right) / \\
& \left(1 + \sqrt{z_0} \sum_{k=0}^{\infty} \frac{(-1)^k \left(-\frac{1}{2} \right)_k (5 - z_0)^k z_0^{-k}}{k!} \right)^2
\end{aligned}$$

for not $((z_0 \in \mathbb{R} \text{ and } -\infty < z_0 \leq 0))$

Finally, our results are not restricted to the SM Higgs Lagrangian but to the generic nonlinear sigma model of the same $G/H = O(4)/O(3) \simeq [SU(2)_L \times SU(2)_R]/SU(2)_V$, with/without nonlinearly realized (approximate) scale symmetry, since we showed that the dynamical results obtained in the large N limit are not sensitive to the presence of the pseudo-dilaton φ . Then it is readily applied to the *two-flavored QCD* in the chiral limit. #29

In particular, the so-called *successful $a = 2$ results of the ρ meson*, i.e., *ρ -universality*, *KSRF I and II*, and *vector meson dominance (VMD)*, are now proved to be realized for any a for the dynamical gauge boson of the HLS, and thus are simply *nonperturbative dynamical consequences in the large N limit but not a mysterious parameter choice $a = 2$* . The dynamically generated kinetic term has a new free parameter, the ρ coupling (related to the cutoff or Landau pole, Eq.(1)), which is adjusted to the reality as $g_{\rho\pi\pi} = g_{\text{HLS}} \simeq 5.9$ corresponding to $m_\rho = g_{\rho\pi\pi} f_\rho = \sqrt{2} g_{\text{HLS}} f_\pi \simeq 770$ MeV ($f_\pi \simeq 92$ MeV), Eq.(2). This implies the cutoff (related to the Landau pole) $\Lambda = \tilde{\Lambda} \cdot e^{-4/3} = m_\rho \cdot e^{3(4\pi)^2/(8g_{\text{HLS}}^2)} \cdot e^{-4/3} \simeq 1.1$ GeV which coincides with the breakdown scale of the chiral perturbation theory $\Lambda_\chi \simeq 4\pi f_\pi$.

The fact is a most remarkable triumph of the nonlinear sigma model as an effective field theory including full nonperturbative dynamics. It in fact becomes a direct evidence of the dynamical generation of the HLS gauge boson in QCD !! Phrased differently, QCD knows the Grassmannian manifold! Or, Nature chooses Grassmannian manifold as the effective theory of QCD-like theories.

We have:

$$\text{sqrt}(2) * 5.9 * 92$$

Input:

$$\sqrt{2} \times 5.9 \times 92$$

Result:

767.635...

767.635...

$$8 + (((\sqrt{2}) * 5.9 * 92)))$$

Input:

$$8 + \sqrt{2} \times 5.9 \times 92$$

Result:

775.635...

775.635.... result practically equal to the rest mass of Charged rho meson 775.11

$$((((1/(((\sqrt{2}) * 5.9 * 92))))))^{1/1024}$$

Input:

$$\sqrt[1024]{\frac{1}{\sqrt{2} \times 5.9 \times 92}}$$

Result:

0.993533387...

0.993533387.... result very near to the value of the following Rogers-Ramanujan continued fraction:

$$\frac{e^{-\frac{\pi}{\sqrt{5}}}}{\sqrt{5}} = 1 - \frac{e^{-\pi\sqrt{5}}}{1 + \frac{e^{-2\pi\sqrt{5}}}{1 + \frac{e^{-3\pi\sqrt{5}}}{1 + \frac{e^{-4\pi\sqrt{5}}}{1 + \dots}}}} \approx 0.9991104684$$

$$\frac{1 + \sqrt[5]{\sqrt{\varphi^5 4 \sqrt{5^3} - 1}}}{\sqrt{5}} - \varphi + 1$$

and to the dilaton value **0.989117352243 = ϕ**

We have also:

$$4\pi * 92$$

Input:

$$4\pi \times 92$$

Result:

$$368\pi$$

Decimal approximation:

1156.106096521043911754252765046857061384558338970038942118...

1156.106096...

Property:

368π is a transcendental number

We note that:

$$((4\pi \times 92))^{1/14}$$

Input:

$$\sqrt[14]{4\pi \times 92}$$

Exact result:

$$2^{2/7} \sqrt[14]{23\pi}$$

Decimal approximation:

1.654952561335743147543223624316835307075065918559826571025...

1.65495256.... is very near to the 14th root of the following Ramanujan's class invariant $Q = (G_{505}/G_{101/5})^3 = 1164,2696$ i.e. 1,65578...

Property:

$2^{2/7} \sqrt[14]{23\pi}$ is a transcendental number

All 14th roots of 368π :

- Polar form

$$2^{2/7} \sqrt[14]{23\pi} e^0 \approx 1.65495 \quad (\text{real, principal root})$$

$$2^{2/7} \sqrt[14]{23\pi} e^{(i\pi)/7} \approx 1.4911 + 0.7181i$$

$$2^{2/7} \sqrt[14]{23\pi} e^{(2i\pi)/7} \approx 1.0318 + 1.2939i$$

$$2^{2/7} \sqrt[14]{23\pi} e^{(3i\pi)/7} \approx 0.36826 + 1.61346i$$

$$2^{2/7} \sqrt[14]{23} \pi e^{(4i\pi)/7} \approx -0.3683 + 1.61346i$$

Alternative representations:

$$\sqrt[14]{4\pi 92} = \sqrt[14]{66240^\circ}$$

$$\sqrt[14]{4\pi 92} = \sqrt[14]{-368i \log(-1)}$$

$$\sqrt[14]{4\pi 92} = \sqrt[14]{368 \cos^{-1}(-1)}$$

Series representations:

$$\sqrt[14]{4\pi 92} = 2^{3/7} \sqrt[14]{23} \sqrt[14]{\sum_{k=0}^{\infty} \frac{(-1)^k}{1+2k}}$$

$$\sqrt[14]{4\pi 92} = 2^{3/7} \sqrt[14]{\sum_{k=0}^{\infty} -\frac{23(-1)^k 1195^{-1-2k} (5^{1+2k} - 4 \times 239^{1+2k})}{1+2k}}$$

$$\sqrt[14]{4\pi 92} = 2^{2/7} \sqrt[14]{23} \sqrt[14]{\sum_{k=0}^{\infty} \left(-\frac{1}{4}\right)^k \left(\frac{1}{1+2k} + \frac{2}{1+4k} + \frac{1}{3+4k}\right)}$$

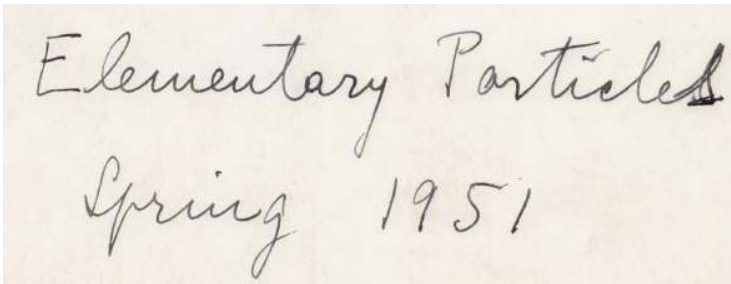
Integral representations:

$$\sqrt[14]{4\pi 92} = 2^{3/7} \sqrt[14]{23} \sqrt[14]{\int_0^1 \sqrt{1-t^2} dt}$$

$$\sqrt[14]{4\pi 92} = 2^{5/14} \sqrt[14]{23} \sqrt[14]{\int_0^{\infty} \frac{1}{1+t^2} dt}$$

$$\sqrt[14]{4\pi 92} = 2^{5/14} \sqrt[14]{23} \sqrt[14]{\int_0^1 \frac{1}{\sqrt{1-t^2}} dt}$$

From:



Enrico Fermi

C-1 - Pion decay

$$\pi \rightarrow \mu + \nu$$

$$\frac{e_3 \hbar c}{\sqrt{2\Omega \mu c^2}}$$

$$\frac{1}{\tau_\pi} = \frac{2\pi}{\hbar} \left(\frac{e_3 \hbar c}{\sqrt{2\Omega \mu c^2}} \right)^2 \frac{p^2 dp \Omega}{2\pi^2 \hbar^3 (v_\mu + v_\nu)} = \frac{e_3^2 p^2}{2\pi \hbar^2 \mu (v_\mu + v_\nu)}$$

$$cp + \sqrt{\mu^2 c^4 + c^2 p^2} = \mu c^2 \quad \begin{array}{l} \mu = 276 m \\ \mu_1 = 210 m \end{array}$$

$$p = 58.1 m c \quad v_\nu = c \quad v_\mu = .27c \quad (4.1 \text{ MeV})$$

$$\frac{1}{\tau_\pi} = 3.8 \times 10^{37} e_3^2 \quad \tau_\pi = 2.6 \times 10^{-8} \text{ sec}$$

$$e_3 = 10^{-15} \text{ esu} = 2 \times 10^{-6} e$$

(58.1)^{1/8}

Input:

$$\sqrt[8]{58.1}$$

Result:

1.661582909539033274740482638936982078096186008800838037791...

1.661582909539..... is very near to the 14th root of the following Ramanujan's class invariant $Q = (G_{505}/G_{101/5})^3 = 1164,2696$ i.e. 1,65578...

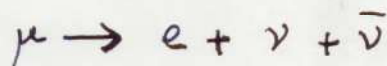
Possible closed forms:

$$\sqrt{\frac{14445233}{5232154}} \approx 1.66158290953903337167$$

$$\pi \sqrt{\text{root of } 961x^4 + 98x^3 - 4004x^2 + 341x + 850 \text{ near } x = 0.528898} \approx 1.661582909539033274714111$$

$$-\frac{3(-100 - 159e + 79e^2)}{105 - 853e + 287e^2} \approx 1.6615829095390332722040$$

C-2. Spontaneous muon decay

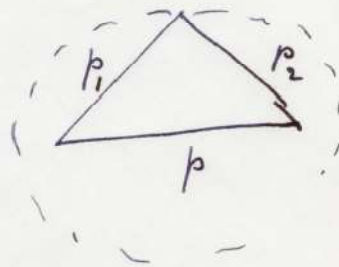


$$g_2 / \Omega$$

e relativistic

$$\text{Rate}(dp) = \frac{2\pi}{h} \frac{g_2}{\Omega^2} \frac{\Omega p^2 dp}{2\pi^2 h^3} \frac{dN}{dW}$$

$$|p_1| + |p_2| = \frac{W}{c} - p$$



Neutrino mom. space

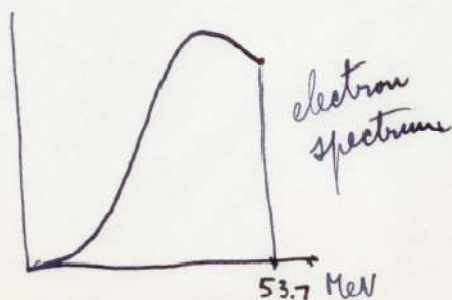
$$\frac{\pi}{6} \left(\frac{W^3}{c^3} - 3p \frac{W^2}{c^2} + 2p^2 \frac{W}{c} \right)$$

$$\frac{dN}{dW} = \frac{d}{dW} \left\{ \frac{\Omega}{8\pi^3 h^3} \frac{\pi}{6} \left(\dots \right) \right\} \quad W = \mu_i c^2$$

$$\text{Rate}(dp) = \frac{g_2^2 \mu_i^2 c}{48 \pi^3 h^7} \left(3 - \frac{6p}{\mu_i c} + \frac{2p^2}{\mu_i^2 c^2} \right) p^2 dp$$

Integrating

$$\frac{1}{\tau_\mu} = \frac{7}{7680 \pi^3} \frac{g_2^2 \mu_i^5 c^4}{h^7} = \frac{1}{2.15 \times 10^{-6}} \text{ sec}^{-1}$$



$$g_2 = 3.3 \times 10^{-49}$$

(53.7)^{1/8}

Input:

$$\sqrt[8]{53.7}$$

Result:

1.645306394929727369700052867839179083692696389708228192009...

$$1.645306394\dots \approx \zeta(2) = \frac{\pi^2}{6} = 1.644934 \dots$$

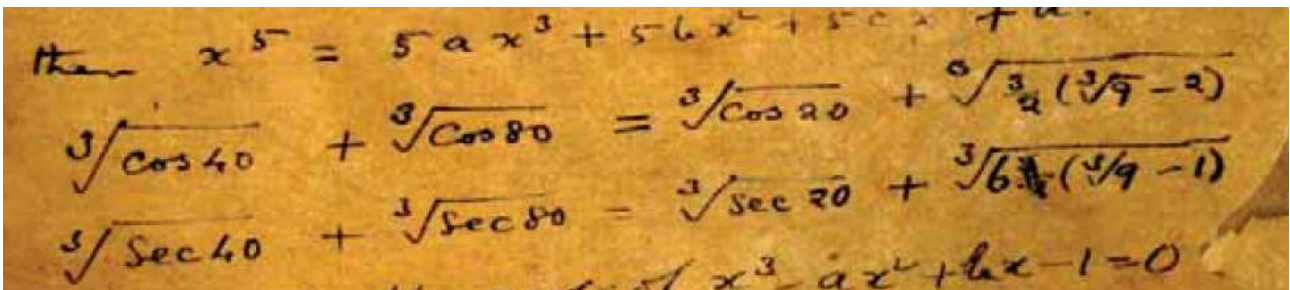
Possible closed forms:

$$\pi \sqrt{\text{root of } 61154x^3 - 8674x^2 + 891x - 6872 \text{ near } x = 0.523717} \approx 1.645306394929727369713919$$

$$\frac{2022798601\pi}{3862386510} \approx 1.64530639492972736968302$$

$$\sqrt{\text{root of } 13262x^3 + 1440x^2 - 39525x + 2065 \text{ near } x = 1.64531} \approx 1.6453063949297273697098827$$

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$$(\cos 40)^{1/3} + (\cos 80)^{1/3}$$

Input:

$$\sqrt[3]{\cos(40)} + \sqrt[3]{\cos(80)}$$

Decimal approximation:

0.67670128772606545120744355021838109939475430998924266628... +
 1.1720810118888308567868135834898319924398827643779341260... i

Polar coordinates:

$r \approx 1.3534$ (radius), $\theta \approx 60^\circ$ (angle)

1.3534

Alternate forms:

$$\sqrt[3]{\frac{1}{2}(e^{-40i} + e^{40i})} + \sqrt[3]{\frac{1}{2}(e^{-80i} + e^{80i})}$$

$$\frac{1}{2} \sqrt[3]{-\cos(40)} + i \left(\frac{1}{2} \sqrt{3} \sqrt[3]{-\cos(40)} + \frac{1}{2} \sqrt{3} \sqrt[3]{-\cos(80)} \right) + \frac{1}{2} \sqrt[3]{-\cos(80)}$$

Alternative representations:

$$\sqrt[3]{\cos(40)} + \sqrt[3]{\cos(80)} = \sqrt[3]{\frac{1}{\sec(40)}} + \sqrt[3]{\frac{1}{\sec(80)}}$$

$$\sqrt[3]{\cos(40)} + \sqrt[3]{\cos(80)} = \sqrt[3]{\cosh(-40i)} + \sqrt[3]{\cosh(-80i)}$$

$$\sqrt[3]{\cos(40)} + \sqrt[3]{\cos(80)} = \sqrt[3]{\cosh(40i)} + \sqrt[3]{\cosh(80i)}$$

Series representations:

$$\sqrt[3]{\cos(40)} + \sqrt[3]{\cos(80)} = \sqrt[3]{\sum_{k=0}^{\infty} \frac{(-6400)^k}{(2k)!}} + \sqrt[3]{\sum_{k=0}^{\infty} \frac{(-1600)^k}{(2k)!}}$$

$$\sqrt[3]{\cos(40)} + \sqrt[3]{\cos(80)} = \sqrt[3]{\sum_{k=0}^{\infty} \frac{\cos\left(\frac{k\pi}{2} + z_0\right) (40 - z_0)^k}{k!}} + \sqrt[3]{\sum_{k=0}^{\infty} \frac{\cos\left(\frac{k\pi}{2} + z_0\right) (80 - z_0)^k}{k!}}$$

$$\sqrt[3]{\cos(40)} + \sqrt[3]{\cos(80)} = \sqrt[3]{-\sum_{k=0}^{\infty} \frac{(-1)^k \left(40 - \frac{\pi}{2}\right)^{1+2k}}{(1+2k)!}} + \sqrt[3]{-\sum_{k=0}^{\infty} \frac{(-1)^k \left(80 - \frac{\pi}{2}\right)^{1+2k}}{(1+2k)!}}$$

Integral representations:

$$\sqrt[3]{\cos(40)} + \sqrt[3]{\cos(80)} = \sqrt[3]{-\int_{\frac{\pi}{2}}^{40} \sin(t) dt} + \sqrt[3]{-\int_{\frac{\pi}{2}}^{80} \sin(t) dt}$$

$$\sqrt[3]{\cos(40)} + \sqrt[3]{\cos(80)} = \sqrt[3]{1 - 40 \int_0^1 \sin(40t) dt} + \sqrt[3]{1 - 80 \int_0^1 \sin(80t) dt}$$

$$\sqrt[3]{\cos(40)} + \sqrt[3]{\cos(80)} = \frac{\sqrt[3]{-i \int_{-i\infty+\gamma}^{i\infty+\gamma} \frac{e^{-1600/s+s}}{\sqrt{s}} ds} + \sqrt[3]{-i \int_{-i\infty+\gamma}^{i\infty+\gamma} \frac{e^{-400/s+s}}{\sqrt{s}} ds}}{\sqrt[3]{2} \sqrt[6]{\pi}} \quad \text{for } \gamma > 0$$

$$\sqrt[3]{\cos(40)} + \sqrt[3]{\cos(80)} = \frac{\sqrt[3]{-i \int_{-i\infty+\gamma}^{i\infty+\gamma} \frac{20^{-2s} \Gamma(s)}{\Gamma(\frac{1}{2}-s)} ds} + \sqrt[3]{-i \int_{-i\infty+\gamma}^{i\infty+\gamma} \frac{40^{-2s} \Gamma(s)}{\Gamma(\frac{1}{2}-s)} ds}}{\sqrt[3]{2} \sqrt[6]{\pi}}$$

for $0 < \gamma < \frac{1}{2}$

$$(\cos 20)^{1/3} + (3/2(9^{1/3} - 2))^{1/3}$$

Input:

$$\sqrt[3]{\cos(20)} + \sqrt[3]{\frac{3}{2} (\sqrt[3]{9} - 2)}$$

Exact result:

$$\sqrt[3]{\frac{3}{2} (3^{2/3} - 2)} + \sqrt[3]{\cos(20)}$$

Decimal approximation:

1.235150302005868526995022813088258129210398609802262761649...

1.235150302....

Property:

$$\sqrt[3]{\frac{3}{2} (-2 + 3^{2/3})} + \sqrt[3]{\cos(20)} \text{ is a transcendental number}$$

Alternate forms:

$$\sqrt[3]{\frac{3 \times 3^{2/3}}{2} - 3} + \sqrt[3]{\cos(20)}$$

$$\sqrt[3]{\frac{3}{2} (3^{2/3} - 2)} + \sqrt[3]{\frac{1}{2} (e^{-20i} + e^{20i})}$$

$$\frac{1}{2} \left(2^{2/3} \sqrt[3]{3 (3^{2/3} - 2)} + 2 \sqrt[3]{\cos(20)} \right)$$

Alternative representations:

$$\sqrt[3]{\cos(20)} + \sqrt[3]{\frac{1}{2}(\sqrt[3]{9} - 2)3} = \sqrt[3]{\frac{3}{2}(-2 + \sqrt[3]{9})} + \sqrt[3]{\frac{1}{\sec(20)}}$$

$$\sqrt[3]{\cos(20)} + \sqrt[3]{\frac{1}{2}(\sqrt[3]{9} - 2)3} = \sqrt[3]{\cosh(-20i)} + \sqrt[3]{\frac{3}{2}(-2 + \sqrt[3]{9})}$$

$$\sqrt[3]{\cos(20)} + \sqrt[3]{\frac{1}{2}(\sqrt[3]{9} - 2)3} = \sqrt[3]{\cosh(20i)} + \sqrt[3]{\frac{3}{2}(-2 + \sqrt[3]{9})}$$

Series representations:

$$\sqrt[3]{\cos(20)} + \sqrt[3]{\frac{1}{2}(\sqrt[3]{9} - 2)3} = \sqrt[3]{\frac{3}{2}(-2 + 3^{2/3})} + \sqrt[3]{\sum_{k=0}^{\infty} \frac{(-400)^k}{(2k)!}}$$

$$\sqrt[3]{\cos(20)} + \sqrt[3]{\frac{1}{2}(\sqrt[3]{9} - 2)3} = \sqrt[3]{\frac{3}{2}(-2 + 3^{2/3})} + \sqrt[3]{-\sum_{k=0}^{\infty} \frac{(-1)^k (20 - \frac{\pi}{2})^{1+2k}}{(1+2k)!}}$$

$$\sqrt[3]{\cos(20)} + \sqrt[3]{\frac{1}{2}(\sqrt[3]{9} - 2)3} = \sqrt[3]{\frac{3}{2}(-2 + 3^{2/3})} + \sqrt[3]{\sum_{k=0}^{\infty} \frac{\cos(\frac{k\pi}{2} + z_0)(20 - z_0)^k}{k!}}$$

Integral representations:

$$\sqrt[3]{\cos(20)} + \sqrt[3]{\frac{1}{2}(\sqrt[3]{9} - 2)3} = \sqrt[3]{\frac{3}{2}(-2 + 3^{2/3})} + \sqrt[3]{1 - 20 \int_0^1 \sin(20t) dt}$$

$$\sqrt[3]{\cos(20)} + \sqrt[3]{\frac{1}{2}(\sqrt[3]{9} - 2)3} = \sqrt[3]{\frac{3}{2}(-2 + 3^{2/3})} + \sqrt[3]{-\int_{\frac{\pi}{2}}^{20} \sin(t) dt}$$

$$\sqrt[3]{\cos(20)} + \sqrt[3]{\frac{1}{2}(\sqrt[3]{9} - 2)3} = \sqrt[3]{\frac{3}{2}(-2 + 3^{2/3})} + \frac{\sqrt[3]{-i \int_{-i\infty+\gamma}^{i\infty+\gamma} \frac{e^{-100/s+s}}{\sqrt{s}} ds}}{\sqrt[3]{2} \sqrt[6]{\pi}} \quad \text{for } \gamma > 0$$

$$\sqrt[3]{\cos(20)} + \sqrt[3]{\frac{1}{2}(\sqrt[3]{9} - 2)}3 = \sqrt[3]{\frac{3}{2}(-2 + 3^{2/3})} + \frac{\sqrt[3]{-i \int_{-i\infty+\gamma}^{i\infty+\gamma} \frac{10^{-2s} \Gamma(s)}{\Gamma(\frac{1}{2}-s)} ds}}{\sqrt[3]{2} \sqrt[6]{\pi}}$$

for $0 < \gamma < \frac{1}{2}$

Multiplying the two results, we obtain:

$$(((\cos 40)^{1/3} + (\cos 80)^{1/3})) * (((\cos 20)^{1/3} + (3/2(9^{1/3} - 2))^{1/3}))$$

Input:

$$\left(\sqrt[3]{\cos(40)} + \sqrt[3]{\cos(80)}\right) \left(\sqrt[3]{\cos(20)} + \sqrt[3]{\frac{3}{2}(\sqrt[3]{9} - 2)}\right)$$

Exact result:

$$\left(\sqrt[3]{\frac{3}{2}(3^{2/3} - 2)} + \sqrt[3]{\cos(20)}\right) \left(\sqrt[3]{\cos(40)} + \sqrt[3]{\cos(80)}\right)$$

Decimal approximation:

0.83582779990260987510522728061193460310802661767979666827... +
1.4476962158098334122457748058747804911568007390944389625... i

Polar coordinates:

$r \approx 1.67166$ (radius), $\theta \approx 60^\circ$ (angle)

1.67166 result very near to the results of previous Fermi formulas

1.661582909539... to the result 1.645306394..... and to the value of the formula:

$$m_{p'} = 2 \times \frac{\eta}{R} m_P = 1.6714213 \times 10^{-24} \text{ gm}$$

that is the holographic proton mass (N. Haramein)

Alternate forms:

$$\left(\sqrt[3]{\frac{3 \times 3^{2/3}}{2} - 3} + \sqrt[3]{\cos(20)}\right) \left(\sqrt[3]{\cos(40)} + \sqrt[3]{\cos(80)}\right)$$

$$\frac{1}{2} \left(2^{2/3} \sqrt[3]{3(3^{2/3} - 2)} + 2 \sqrt[3]{\cos(20)}\right) \left(\sqrt[3]{\cos(40)} + \sqrt[3]{\cos(80)}\right)$$

$$\sqrt[3]{\frac{3}{2}(3^{2/3}-2)}\left(\sqrt[3]{\cos(40)}+\sqrt[3]{\cos(80)}\right)+\sqrt[3]{\cos(20)}\left(\sqrt[3]{\cos(40)}+\sqrt[3]{\cos(80)}\right)$$

Alternative representations:

$$\begin{aligned} & \left(\sqrt[3]{\cos(40)}+\sqrt[3]{\cos(80)}\right)\left(\sqrt[3]{\cos(20)}+\sqrt[3]{\frac{1}{2}\left(\sqrt[3]{9}-2\right)3}\right)= \\ & \left(\sqrt[3]{\frac{3}{2}\left(-2+\sqrt[3]{9}\right)}+\sqrt[3]{\frac{1}{\sec(20)}}\right)\left(\sqrt[3]{\frac{1}{\sec(40)}}+\sqrt[3]{\frac{1}{\sec(80)}}\right) \end{aligned}$$

$$\begin{aligned} & \left(\sqrt[3]{\cos(40)}+\sqrt[3]{\cos(80)}\right)\left(\sqrt[3]{\cos(20)}+\sqrt[3]{\frac{1}{2}\left(\sqrt[3]{9}-2\right)3}\right)= \\ & \left(\sqrt[3]{\cosh(-40i)}+\sqrt[3]{\cosh(-80i)}\right)\left(\sqrt[3]{\cosh(-20i)}+\sqrt[3]{\frac{3}{2}\left(-2+\sqrt[3]{9}\right)}\right) \end{aligned}$$

$$\begin{aligned} & \left(\sqrt[3]{\cos(40)}+\sqrt[3]{\cos(80)}\right)\left(\sqrt[3]{\cos(20)}+\sqrt[3]{\frac{1}{2}\left(\sqrt[3]{9}-2\right)3}\right)= \\ & \left(\sqrt[3]{\cosh(40i)}+\sqrt[3]{\cosh(80i)}\right)\left(\sqrt[3]{\cosh(20i)}+\sqrt[3]{\frac{3}{2}\left(-2+\sqrt[3]{9}\right)}\right) \end{aligned}$$

Series representations:

$$\begin{aligned} & \left(\sqrt[3]{\cos(40)}+\sqrt[3]{\cos(80)}\right)\left(\sqrt[3]{\cos(20)}+\sqrt[3]{\frac{1}{2}\left(\sqrt[3]{9}-2\right)3}\right)= \\ & \frac{1}{2}\left(\sqrt[3]{\sum_{k=0}^{\infty}\frac{(-6400)^k}{(2k)!}}+\sqrt[3]{\sum_{k=0}^{\infty}\frac{(-1600)^k}{(2k)!}}\right)\left(2^{2/3}\sqrt[3]{3\left(-2+3^{2/3}\right)}+2\sqrt[3]{\sum_{k=0}^{\infty}\frac{(-400)^k}{(2k)!}}\right) \end{aligned}$$

$$\begin{aligned} & \left(\sqrt[3]{\cos(40)}+\sqrt[3]{\cos(80)}\right)\left(\sqrt[3]{\cos(20)}+\sqrt[3]{\frac{1}{2}\left(\sqrt[3]{9}-2\right)3}\right)= \\ & \frac{1}{2}\left(2^{2/3}\sqrt[3]{3\left(-2+3^{2/3}\right)}+2\sqrt[3]{\sum_{k=0}^{\infty}\frac{\cos\left(\frac{k\pi}{2}+z_0\right)(20-z_0)^k}{k!}}\right) \\ & \left(\sqrt[3]{\sum_{k=0}^{\infty}\frac{\cos\left(\frac{k\pi}{2}+z_0\right)(40-z_0)^k}{k!}}+\sqrt[3]{\sum_{k=0}^{\infty}\frac{\cos\left(\frac{k\pi}{2}+z_0\right)(80-z_0)^k}{k!}}\right) \end{aligned}$$

$$\begin{aligned} & \left(\sqrt[3]{\cos(40)} + \sqrt[3]{\cos(80)} \right) \left(\sqrt[3]{\cos(20)} + \sqrt[3]{\frac{1}{2} (\sqrt[3]{9} - 2) 3} \right) = \\ & \frac{1}{2} \left(2^{2/3} \sqrt[3]{3(-2 + 3^{2/3})} + 2 \sqrt[3]{-\sum_{k=0}^{\infty} \frac{(-1)^k (20 - \frac{\pi}{2})^{1+2k}}{(1+2k)!}} \right) \\ & \left(\sqrt[3]{-\sum_{k=0}^{\infty} \frac{(-1)^k (40 - \frac{\pi}{2})^{1+2k}}{(1+2k)!}} + \sqrt[3]{-\sum_{k=0}^{\infty} \frac{(-1)^k (80 - \frac{\pi}{2})^{1+2k}}{(1+2k)!}} \right) \end{aligned}$$

Integral representations:

$$\begin{aligned} & \left(\sqrt[3]{\cos(40)} + \sqrt[3]{\cos(80)} \right) \left(\sqrt[3]{\cos(20)} + \sqrt[3]{\frac{1}{2} (\sqrt[3]{9} - 2) 3} \right) = \\ & \frac{1}{2} \left(2^{2/3} \sqrt[3]{3(-2 + 3^{2/3})} + 2 \sqrt[3]{-\int_{\frac{\pi}{2}}^{20} \sin(t) dt} \right) \left(\sqrt[3]{-\int_{\frac{\pi}{2}}^{40} \sin(t) dt} + \sqrt[3]{-\int_{\frac{\pi}{2}}^{80} \sin(t) dt} \right) \end{aligned}$$

$$\begin{aligned} & \left(\sqrt[3]{\cos(40)} + \sqrt[3]{\cos(80)} \right) \left(\sqrt[3]{\cos(20)} + \sqrt[3]{\frac{1}{2} (\sqrt[3]{9} - 2) 3} \right) = \\ & \frac{1}{2} \left(2^{2/3} \sqrt[3]{3(-2 + 3^{2/3})} + 2 \sqrt[3]{1 - 20 \int_0^1 \sin(20t) dt} \right) \\ & \left(\sqrt[3]{1 - 40 \int_0^1 \sin(40t) dt} + \sqrt[3]{1 - 80 \int_0^1 \sin(80t) dt} \right) \end{aligned}$$

$$\begin{aligned} & \left(\sqrt[3]{\cos(40)} + \sqrt[3]{\cos(80)} \right) \left(\sqrt[3]{\cos(20)} + \sqrt[3]{\frac{1}{2} (\sqrt[3]{9} - 2) 3} \right) = \\ & \frac{1}{2^{2/3} \sqrt[3]{\pi}} \left(\sqrt[3]{-i \int_{-i\infty+\gamma}^{i\infty+\gamma} \frac{e^{-1600/s+s}}{\sqrt{s}} ds} + \sqrt[3]{-i \int_{-i\infty+\gamma}^{i\infty+\gamma} \frac{e^{-400/s+s}}{\sqrt{s}} ds} \right) \\ & \left(\sqrt[3]{3(-2 + 3^{2/3})} \sqrt[6]{\pi} + \sqrt[3]{-i \int_{-i\infty+\gamma}^{i\infty+\gamma} \frac{e^{-100/s+s}}{\sqrt{s}} ds} \right) \text{ for } \gamma > 0 \end{aligned}$$

$$\begin{aligned} & \left(\sqrt[3]{\cos(40)} + \sqrt[3]{\cos(80)} \right) \left(\sqrt[3]{\cos(20)} + \sqrt[3]{\frac{1}{2} (\sqrt[3]{9} - 2) 3} \right) = \\ & \frac{1}{2^{2/3} \sqrt[3]{\pi}} \left(\sqrt[3]{3(-2 + 3^{2/3})} \sqrt[6]{\pi} + \sqrt[3]{-i \int_{-i\infty+\gamma}^{i\infty+\gamma} \frac{10^{-2s} \Gamma(s)}{\Gamma(\frac{1}{2}-s)} ds} \right) \\ & \left(\sqrt[3]{-i \int_{-i\infty+\gamma}^{i\infty+\gamma} \frac{20^{-2s} \Gamma(s)}{\Gamma(\frac{1}{2}-s)} ds} + \sqrt[3]{-i \int_{-i\infty+\gamma}^{i\infty+\gamma} \frac{40^{-2s} \Gamma(s)}{\Gamma(\frac{1}{2}-s)} ds} \right) \text{ for } 0 < \gamma < \frac{1}{2} \end{aligned}$$

Now, from the Fermi paper, we have the following equation:

$$\frac{1}{\tau_\mu} = \frac{7}{7680 \pi^3} \frac{g_2^2 \mu_1^5 c^4}{h^7} = \frac{1}{2.15 \times 10^{-6}} \text{ sec}^{-1}$$

We note that:

$$7/(7680\pi^3)$$

Input:

$$\frac{7}{7680 \pi^3}$$

Decimal approximation:

0.000029395929821926617746218016773430445332219186592753455...

Property:

$\frac{7}{7680 \pi^3}$ is a transcendental number

Alternative representations:

$$\frac{7}{7680 \pi^3} = \frac{7}{7680 (180^\circ)^3}$$

$$\frac{7}{7680 \pi^3} = \frac{7}{7680 (-i \log(-1))^3}$$

$$\frac{7}{7680 \pi^3} = \frac{7}{7680 \cos^{-1}(-1)^3}$$

Series representations:

$$\frac{7}{7680 \pi^3} = \frac{7}{491520 \left(\sum_{k=0}^{\infty} \frac{(-1)^k}{1+2k} \right)^3}$$

$$\frac{7}{7680 \pi^3} = \frac{7}{7680 \left(\sum_{k=0}^{\infty} \frac{4(-1)^k 1195^{-1-2k} (5^{1+2k} - 4 \times 239^{1+2k})}{1+2k} \right)^3}$$

$$\frac{7}{7680 \pi^3} = \frac{7}{7680 \left(\sum_{k=0}^{\infty} \left(-\frac{1}{4}\right)^k \left(\frac{1}{1+2k} + \frac{2}{1+4k} + \frac{1}{3+4k} \right) \right)^3}$$

Integral representations:

$$\frac{7}{7680 \pi^3} = \frac{7}{491520 \left(\int_0^1 \sqrt{1-t^2} dt \right)^3}$$

$$\frac{7}{7680 \pi^3} = \frac{7}{61440 \left(\int_0^{\infty} \frac{1}{1+t^2} dt \right)^3}$$

$$\frac{7}{7680 \pi^3} = \frac{7}{61440 \left(\int_0^1 \frac{1}{\sqrt{1-t^2}} dt \right)^3}$$

And that:

$$(2.67092537 - 0.50970737) * 1 / (((((7 * 1 / (7680 \pi^3)))))) - 29$$

Input interpretation:

$$(2.67092537 - 0.50970737) \times \frac{1}{7 \times \frac{1}{7680 \pi^3}} - 29$$

Result:

73491.995...

73491.995

Alternative representations:

$$\frac{2.67093 - 0.509707}{\frac{7}{7680 \pi^3}} - 29 = -29 + \frac{2.16122}{\frac{7}{7680 (180^\circ)^3}}$$

$$\frac{2.67093 - 0.509707}{\frac{7}{7680 \pi^3}} - 29 = -29 + \frac{2.16122}{\frac{7}{7680 \cos^{-1}(-1)^3}}$$

$$\frac{2.67093 - 0.509707}{\frac{7}{7680 \pi^3}} - 29 = -29 + \frac{2.16122}{\frac{7}{7680 (-i \log(-1))^3}}$$

Series representations:

$$\frac{2.67093 - 0.509707}{\frac{7}{7680 \pi^3}} - 29 = -29 + 151\,755 \cdot \left(\sum_{k=0}^{\infty} \frac{(-1)^k}{1+2k} \right)^3$$

$$\frac{2.67093 - 0.509707}{\frac{7}{7680 \pi^3}} - 29 = -29 + 18\,969.3 \left(-1 + \sum_{k=1}^{\infty} \frac{2^k}{\binom{2k}{k}} \right)^3$$

$$\frac{2.67093 - 0.509707}{\frac{7}{7680 \pi^3}} - 29 = -29 + 2371.16 \left(\sum_{k=0}^{\infty} \frac{2^{-k} (-6 + 50k)}{\binom{3k}{k}} \right)^3$$

Integral representations:

$$\frac{2.67093 - 0.509707}{\frac{7}{7680 \pi^3}} - 29 = -29 + 18\,969.3 \left(\int_0^{\infty} \frac{1}{1+t^2} dt \right)^3$$

$$\frac{2.67093 - 0.509707}{\frac{7}{7680 \pi^3}} - 29 = -29 + 151\,755 \cdot \left(\int_0^1 \sqrt{1-t^2} dt \right)^3$$

$$\frac{2.67093 - 0.509707}{\frac{7}{7680 \pi^3}} - 29 = -29 + 18\,969.3 \left(\int_0^{\infty} \frac{\sin(t)}{t} dt \right)^3$$

Where:

$$\frac{\left(\frac{1}{1-0.449329} + \frac{0.449329}{(1-0.449329^2)(1-0.449329^3)} \right) + \frac{0.449329^2}{(1-0.449329^3)(1-0.449329^4)(1-0.449329^5)}}{0.449329^2}$$

2.670925377482945723639317570028275016308835824074456769461...

$\chi(q) = 2.6709253774829...$

And

0.449329 + 0.449329⁴ (1 + 0.449329) + 0.449329⁹ (1 + 0.449329) (1 + 0.449329²)

0.509707374450926175465106350027401141383801983986000851664...

$\phi(q) = 0.50970737445...$

Are the values of two Ramanujan mock theta functions

Thence, we obtain the following mathematical connections:

$$\begin{aligned} & \left((2.67092537 - 0.50970737) \times \frac{1}{7 \times \frac{1}{7680 \pi^3}} - 29 \right) = 73491.995 \Rightarrow \\ & \Rightarrow -3927 + 2 \left(\sqrt[13]{N \exp \left[\int d\hat{\sigma} \left(-\frac{1}{4u^2} \mathbf{P}_i D \mathbf{P}_i \right) \right] |Bp\rangle_{NS} + \int [d\mathbf{X}^\mu] \exp \left\{ \int d\hat{\sigma} \left(-\frac{1}{4v^2} D\mathbf{X}^\mu D^2 \mathbf{X}^\mu \right) \right\} |X^\mu, X^i = 0\rangle_{NS}} \right) = \\ & -3927 + 2 \sqrt[13]{2.2983717437 \times 10^{59} + 2.0823329825883 \times 10^{59}} \\ & = 73490.8437525 \dots \Rightarrow \\ & \Rightarrow \left(A(r) \times \frac{1}{B(r)} \left(-\frac{1}{\phi(r)} \right) \times \frac{1}{e^{\Lambda(r)}} \right) \Rightarrow \\ & \Rightarrow \left(-0.000029211892 \times \frac{1}{0.0003644621} \left(-\frac{1}{0.0005946833} \right) \times \frac{1}{0.00183393} \right) = \\ & = 73491.78832548118710549159572042220548025195726563413398700 \dots \\ & = 73491.7883254 \dots \Rightarrow \end{aligned}$$

$$\left(I_{21} \ll \int_{-\infty}^{+\infty} \exp \left(-\left(\frac{t}{H} \right)^2 \right) \left| \sum_{\lambda \leq p^{1-\epsilon_1}} \frac{a(\lambda)}{\sqrt{\lambda}} B(\lambda) \lambda^{-i(T+t)} \right|^2 dt \ll \ll H \left\{ \left(\frac{4}{\epsilon_2 \log T} \right)^{2r} (\log T) (\log X)^{-2\beta} + (\epsilon_2^{-2r} (\log T)^{-2r} + \epsilon_2^{-r} h_1^r (\log T)^{-r} \right) T^{-\epsilon_1} \right\} \right)$$

$$/(26 \times 4)^2 - 24 = \left(\frac{7.9313976505275 \times 10^8}{(26 \times 4)^2 - 24} \right) = 73493.30662\dots$$

Mathematical connections with the boundary state corresponding to the NSNS-sector of N Dp-branes in the limit of $u \rightarrow \infty$, with the ratio concerning the general asymptotically flat solution of the equations of motion of the p-brane and with the Karatsuba's equation concerning the zeros of a special type of function connected with Dirichlet series.

Furthermore, we obtain also:

$$\left(\left(\left(\left(\frac{7}{7680 \pi^3} \right) \right) \right) \right)^{1/4096}$$

Input:

$$\sqrt[4096]{\frac{7}{7680 \pi^3}}$$

Exact result:

$$\frac{\sqrt[4096]{\frac{7}{15}}}{2^{9/4096} \pi^{3/4096}}$$

Decimal approximation:

$$0.997455719152116841448403004416878244502721880705041058496\dots$$

0.997455719152... result very near to the value of the following Rogers-Ramanujan continued fraction:

$$\frac{e^{-\frac{\pi}{\sqrt{5}}}}{\sqrt{5}} = 1 - \frac{e^{-\pi\sqrt{5}}}{1 + \frac{e^{-2\pi\sqrt{5}}}{1 + \frac{e^{-3\pi\sqrt{5}}}{1 + \frac{e^{-4\pi\sqrt{5}}}{1 + \dots}}}} \approx 0.9991104684$$

$$\frac{1 + \sqrt[5]{\sqrt{\phi^5 4\sqrt{5^3} - 1}}}{\sqrt{5}} - \phi + 1$$

and to the dilaton value **0.989117352243 = ϕ**

Property:

$$\frac{\sqrt[4096]{\frac{7}{15}}}{2^{9/4096} \pi^{3/4096}}$$
 is a transcendental number

All 4096th roots of $7/(7680 \pi^3)$:

$$\frac{\sqrt[4096]{\frac{7}{15}} e^0}{2^{9/4096} \pi^{3/4096}} \approx 0.9974557 \quad (\text{real, principal root})$$

$$\frac{\sqrt[4096]{\frac{7}{15}} e^{(i\pi)/2048}}{2^{9/4096} \pi^{3/4096}} \approx 0.9974545 + 0.0015301 i$$

$$\frac{\sqrt[4096]{\frac{7}{15}} e^{(i\pi)/1024}}{2^{9/4096} \pi^{3/4096}} \approx 0.9974510 + 0.0030602 i$$

$$\frac{\sqrt[4096]{\frac{7}{15}} e^{(3i\pi)/2048}}{2^{9/4096} \pi^{3/4096}} \approx 0.9974452 + 0.0045902 i$$

$$\frac{\sqrt[4096]{\frac{7}{15}} e^{(i\pi)/512}}{2^{9/4096} \pi^{3/4096}} \approx 0.9974369 + 0.006120 i$$

Alternative representations:

$$\sqrt[4096]{\frac{7}{7680 \pi^3}} = \sqrt[4096]{\frac{7}{7680 (180^\circ)^3}}$$

$$\sqrt[4096]{\frac{7}{7680 \pi^3}} = \sqrt[4096]{\frac{7}{7680 \cos^{-1}(-1)^3}}$$

$$\sqrt[4096]{\frac{7}{7680 \pi^3}} = \sqrt[4096]{\frac{7}{7680 (-i \log(-1))^3}}$$

Series representations:

$$\sqrt[4096]{\frac{7}{7680 \pi^3}} = \frac{4096 \sqrt{\frac{7}{15}}}{2^{15/4096} \left(\sum_{k=0}^{\infty} \frac{(-1)^k}{1+2k} \right)^{3/4096}}$$

$$\sqrt[4096]{\frac{7}{7680 \pi^3}} = \frac{4096 \sqrt{\frac{7}{15}}}{2^{15/4096} \left(\sum_{k=0}^{\infty} \frac{(-1)^{1+k} 1195^{-1-2k} (5^{1+2k} - 4 \times 239^{1+2k})}{1+2k} \right)^{3/4096}}$$

$$\sqrt[4096]{\frac{7}{7680 \pi^3}} = \frac{4096 \sqrt{\frac{7}{15}}}{2^{9/4096} \left(\sum_{k=0}^{\infty} \left(-\frac{1}{4}\right)^k \left(\frac{1}{1+2k} + \frac{2}{1+4k} + \frac{1}{3+4k} \right) \right)^{3/4096}}$$

$$\sqrt[4096]{\frac{7}{7680 \pi^3}} = \frac{4096 \sqrt{\frac{7}{15}}}{2^{9/4096} \left(\sum_{k=0}^{\infty} 16^{-k} \left(\frac{1}{-5-8k} - \frac{1}{2+4k} + \frac{4}{1+8k} - \frac{1}{6+8k} \right) \right)^{3/4096}}$$

$$\sqrt[4096]{\frac{7}{7680 \pi^3}} = \frac{4096 \sqrt{\frac{7}{5}}}{2^{3/1024} 1024 \sqrt[3]{3} \left(\sum_{k=1}^{\infty} \frac{-120+329k+568k^2}{k(1+k)(1+2k)(1+4k)(3+4k)(5+4k)} \right)^{3/4096}}$$

$$\sqrt[4096]{\frac{7}{7680 \pi^3}} = \frac{4096 \sqrt{\frac{7}{15}}}{2^{3/4096} \left(\sum_{k=0}^{\infty} 16^{-k} \left(\frac{8}{1+8k} + \frac{8}{2+8k} + \frac{4}{3+8k} - \frac{2}{5+8k} - \frac{2}{6+8k} - \frac{1}{7+8k} \right) \right)^{3/4096}}$$

$$\sqrt[4096]{\frac{7}{7680 \pi^3}} = \frac{4096 \sqrt{\frac{7}{15}} 2^{9/4096}}{\left(\sum_{k=0}^{\infty} (-1)^k 2^{-10k} \left(-\frac{32}{1+4k} - \frac{1}{3+4k} + \frac{256}{1+10k} - \frac{64}{3+10k} - \frac{4}{5+10k} - \frac{4}{7+10k} + \frac{1}{9+10k} \right) \right)^{3/4096}}$$

$$\sqrt[4096]{\frac{7}{7680 \pi^3}} = \frac{\sqrt[4096]{\frac{7}{15}}}{2^{9/4096} \left(\sum_{k=0}^{\infty} 16^{-k} \left(-\frac{8r}{2+8k} - \frac{4r}{3+8k} + \frac{r}{7+8k} - \frac{1+2r}{5+8k} - \frac{1+2r}{6+8k} - \frac{2+8r}{4+8k} + \frac{4+8r}{1+8k} \right) \right)^{3/4096}}$$

for $(r \in \mathbb{Z} \text{ and } r > 0)$

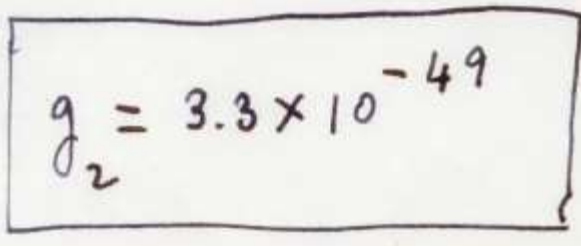
Integral representations:

$$\sqrt[4096]{\frac{7}{7680 \pi^3}} = \frac{\sqrt[4096]{\frac{7}{15}}}{2^{15/4096} \left(\int_0^1 \sqrt{1-t^2} dt \right)^{3/4096}}$$

$$\sqrt[4096]{\frac{7}{7680 \pi^3}} = \frac{\sqrt[4096]{\frac{7}{15}}}{2^{3/1024} \left(\int_0^{\infty} \frac{1}{1+t^2} dt \right)^{3/4096}}$$

$$\sqrt[4096]{\frac{7}{7680 \pi^3}} = \frac{\sqrt[4096]{\frac{7}{15}}}{2^{3/1024} \left(\int_0^1 \frac{1}{\sqrt{1-t^2}} dt \right)^{3/4096}}$$

We have that:



A handwritten equation in a rectangular box: $g_2 = 3.3 \times 10^{-49}$

From which:

$$(3.3 \times 10^{-49})^{1/4096}$$

Input interpretation:

$$\sqrt[4096]{3.3 \times 10^{-49}}$$

Result:

0.9731139529...

0.9731139529.... result very near to the spectral index n_s and to the mesonic Regge slope (see Appendix) and to the inflaton value at the end of the inflation 0.9402 and to the value of the following Rogers-Ramanujan continued fraction:

$$\frac{e^{-\frac{\pi}{5}}}{\sqrt{(\varphi-1)\sqrt{5} - \varphi + 1}} = 1 - \frac{e^{-\pi}}{1 + \frac{e^{-2\pi}}{1 + \frac{e^{-3\pi}}{1 + \frac{e^{-4\pi}}{1 + \dots}}}} \approx 0.9568666373$$

And:

$$\text{sqrt}(\text{((log base 0.9731139529 (3.3*10^-49))))$$

Input interpretation:

$$\sqrt{\log_{0.9731139529}(3.3 \times 10^{-49})}$$

$\log_b(x)$ is the base- b logarithm

Result:

64.000000...

64

Note that:

$$g_{22} = \sqrt{(1 + \sqrt{2})}$$

Hence

$$\begin{aligned} 64g_{22}^{24} &= e^{\pi\sqrt{22}} - 24 + 276e^{-\pi\sqrt{22}} - \dots, \\ 64g_{22}^{-24} &= 4096e^{-\pi\sqrt{22}} + \dots, \end{aligned}$$

so that

$$64(g_{22}^{24} + g_{22}^{-24}) = e^{\pi\sqrt{22}} - 24 + 4372e^{-\pi\sqrt{22}} + \dots = 64\{(1 + \sqrt{2})^{12} + (1 - \sqrt{2})^{12}\}.$$

Hence

$$e^{\pi\sqrt{22}} = 2508951.9982 \dots$$

Thence:

$$64g_{22}^{-24} = 4096e^{-\pi\sqrt{22}} + \dots$$

And

$$64(g_{22}^{24} + g_{22}^{-24}) = e^{\pi\sqrt{22}} - 24 + 4372e^{-\pi\sqrt{22}} + \dots = 64\{(1 + \sqrt{2})^{12} + (1 - \sqrt{2})^{12}\}$$

That are connected with 64 and $4096 = 64^2$

And:

$$27\sqrt{\log_{0.9731139529}(3.3 \times 10^{-49})}$$

Input interpretation:

$$27\sqrt{\log_{0.9731139529}(3.3 \times 10^{-49})}$$

$\log_b(x)$ is the base- b logarithm

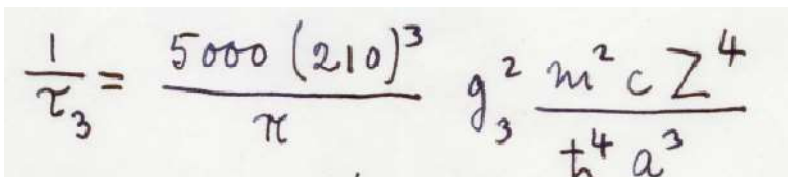
Result:

1728.0000...

1728

This result is very near to the mass of candidate glueball $f_0(1710)$ meson. Furthermore, 1728 occurs in the algebraic formula for the j -invariant of an elliptic curve. As a consequence, it is sometimes called a Zagier as a pun on the Gross–Zagier theorem. The number 1728 is one less than the Hardy–Ramanujan number 1729

We have that:



$$\frac{1}{\tau_3} = \frac{5000 (210)^3}{\pi} g_3^2 \frac{m^2 c Z^4}{h^4 a^3}$$

$$((5000(210)^3)/\pi)$$

Input:

$$\frac{5000 \times 210^3}{\pi}$$

Exact result:

$$\frac{46\,305\,000\,000}{\pi}$$

Decimal approximation:

$$1.4739339279740427045556325325928555068011307792023671... \times 10^{10}$$

$$1.473933927... \times 10^{10}$$

Property:

$$\frac{46\,305\,000\,000}{\pi} \text{ is a transcendental number}$$

Alternative representations:

$$\frac{5000 \times 210^3}{\pi} = \frac{5000 \times 210^3}{180^\circ}$$

$$\frac{5000 \times 210^3}{\pi} = -\frac{5000 \times 210^3}{i \log(-1)}$$

$$\frac{5000 \times 210^3}{\pi} = \frac{5000 \times 210^3}{\cos^{-1}(-1)}$$

Series representations:

$$\frac{5000 \times 210^3}{\pi} = \frac{11\,576\,250\,000}{\sum_{k=0}^{\infty} \frac{(-1)^k}{1+2k}}$$

$$\frac{5000 \times 210^3}{\pi} = \frac{11\,576\,250\,000}{\sum_{k=0}^{\infty} \frac{(-1)^{1+k} 1195^{-1-2k} (5^{1+2k} - 4 \times 239^{1+2k})}{1+2k}}$$

$$\frac{5000 \times 210^3}{\pi} = \frac{46\,305\,000\,000}{\sum_{k=0}^{\infty} \left(-\frac{1}{4}\right)^k \left(\frac{1}{1+2k} + \frac{2}{1+4k} + \frac{1}{3+4k}\right)}$$

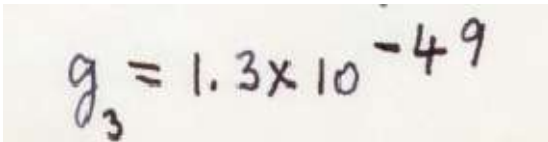
Integral representations:

$$\frac{5000 \times 210^3}{\pi} = \frac{11\,576\,250\,000}{\int_0^1 \sqrt{1-t^2} dt}$$

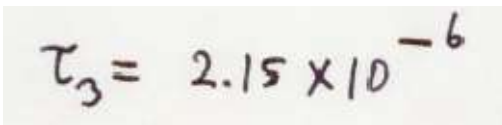
$$\frac{5000 \times 210^3}{\pi} = \frac{23\,152\,500\,000}{\int_0^\infty \frac{1}{1+t^2} dt}$$

$$\frac{5000 \times 210^3}{\pi} = \frac{23\,152\,500\,000}{\int_0^1 \frac{1}{\sqrt{1-t^2}} dt}$$

And for



$$g_3 = 1.3 \times 10^{-49}$$



$$\tau_3 = 2.15 \times 10^{-6}$$

We obtain:

$$(2.15 \times 10^{-6}) / (((5000(210)^3)) / \pi * (1.3 \times 10^{-49})^2)$$

Input interpretation:

$$\frac{2.15 \times 10^{-6}}{\frac{5000 \times 210^3}{\pi} (1.3 \times 10^{-49})^2}$$

Result:

$$8.63125... \times 10^{81}$$

$$8.63125... * 10^{81}$$

And:

$$(64 * 32 + 144 + 8) + 2 * (((((2.15 \times 10^{-6}) / (((5000(210)^3)) / \pi * (1.3 \times 10^{-49})^2))))))^{1/18}$$

Input interpretation:

$$(64 \times 32 + 144 + 8) + 2 \sqrt[18]{\frac{2.15 \times 10^{-6}}{\frac{5000 \times 210^3}{\pi} (1.3 \times 10^{-49})^2}}$$

Result:

73490.90...

73490.90...

Thence, the following mathematical connections:

$$\left((64 \times 32 + 144 + 8) + 2 \sqrt[18]{\frac{2.15 \times 10^{-6}}{\frac{5000 \times 210^3}{\pi} (1.3 \times 10^{-49})^2}} \right) = 73490.90 \Rightarrow$$

$$\Rightarrow -3927 + 2 \left(\sqrt[13]{N \exp \left[\int d\hat{\sigma} \left(-\frac{1}{4u^2} P_i D P_i \right) \right] |Bp\rangle_{NS} + \int [dX^\mu] \exp \left\{ \int d\hat{\sigma} \left(-\frac{1}{4v^2} D X^\mu D^2 X^\mu \right) \right\} |X^\mu, X^i = 0\rangle_{NS}} \right) =$$

$$-3927 + 2 \sqrt[13]{2.2983717437 \times 10^{59} + 2.0823329825883 \times 10^{59}}$$

$$= 73490.8437525.... \Rightarrow$$

$$\Rightarrow \left(A(r) \times \frac{1}{B(r)} \left(-\frac{1}{\phi(r)} \right) \times \frac{1}{e^{\Lambda(r)}} \right) \Rightarrow$$

$$\Rightarrow \left(-0.000029211892 \times \frac{1}{0.0003644621} \left(-\frac{1}{0.0005946833} \right) \times \frac{1}{0.00183393} \right) =$$

$$= 73491.78832548118710549159572042220548025195726563413398700...$$

$$= 73491.7883254... \Rightarrow$$

$$\left(I_{21} \ll \int_{-\infty}^{+\infty} \exp\left(-\left(\frac{t}{H}\right)^2\right) \left| \sum_{\lambda \leq P^{1-\epsilon_2}} \frac{a(\lambda)}{\sqrt{\lambda}} B(\lambda) \lambda^{-i(T+t)} \right|^2 dt \ll \right. \\
\left. \ll H \left\{ \left(\frac{4}{\epsilon_2 \log T} \right)^{2r} (\log T) (\log X)^{-2\beta} + (\epsilon_2^{-2r} (\log T)^{-2r} + \epsilon_2^{-r} h_1^r (\log T)^{-r}) T^{-\epsilon_1} \right\} \right) \\
/(26 \times 4)^2 - 24 = \left(\frac{7.9313976505275 \times 10^8}{(26 \times 4)^2 - 24} \right) = 73493.30662...$$

Mathematical connections with the boundary state corresponding to the NSNS-sector of N Dp-branes in the limit of $u \rightarrow \infty$, with the ratio concerning the general asymptotically flat solution of the equations of motion of the p-brane and with the Karatsuba's equation concerning the zeros of a special type of function connected with Dirichlet series.

We have also:

$$1/((((((2.15 \cdot 10^{-6}) / (((((5000(210)^3) / \pi * (1.3 \cdot 10^{-49})^2))))))))))^{1/4096}$$

Input interpretation:

$$\frac{1}{\sqrt[4096]{\frac{2.15 \times 10^{-6}}{\frac{5000 \cdot 210^3}{\pi} (1.3 \times 10^{-49})^2}}}$$

Result:

0.954983957...

0.954983957... result very near to the spectral index n_s , to the mesonic Regge slope (see Appendix), to the inflaton value at the end of the inflation 0.9402 and to the value of the following Rogers-Ramanujan continued fraction:

$$\frac{e^{\frac{\pi}{5}}}{\sqrt{(\varphi-1)\sqrt{5}} - \varphi + 1} = 1 - \frac{e^{-\pi}}{1 + \frac{e^{-2\pi}}{1 + \frac{e^{-3\pi}}{1 + \frac{e^{-4\pi}}{1 + \dots}}}}} \approx 0.9568666373$$

We have that:

$$U(r) = \frac{1}{(2\pi\hbar)^3} \int \frac{-e^2 \hbar^2 c^2}{\mu^2 c^4 + c^2 p^2} e^{\frac{i}{\hbar} \mathbf{p} \cdot \mathbf{r}} d^3r =$$

$$= -\frac{e^2}{2\pi\hbar} e^{-\frac{\mu c}{\hbar} r} \quad \frac{\hbar}{\mu c} = 1.4 \times 10^{-13}$$

$$(1.4 \times 10^{-13})^{1/4096}$$

Input interpretation:

$$\sqrt[4096]{1.4 \times 10^{-13}}$$

Result:

0.9928001810...

0.9928001810... result very near to the value of the following Rogers-Ramanujan continued fraction:

$$\frac{e^{-\frac{\pi}{\sqrt{5}}}}{\sqrt{5}} = 1 - \frac{e^{-\pi\sqrt{5}}}{1 + \frac{e^{-2\pi\sqrt{5}}}{1 + \frac{e^{-3\pi\sqrt{5}}}{1 + \frac{e^{-4\pi\sqrt{5}}}{1 + \dots}}}} \approx 0.9991104684$$

$$\frac{\sqrt{5}}{1 + \sqrt[5]{\sqrt{\phi^5 4\sqrt{5^3} - 1}}} - \phi + 1$$

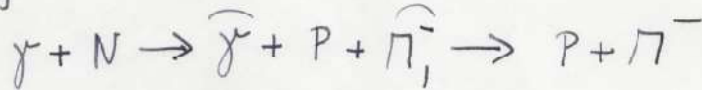
and to the dilaton value **0.989117352243 = ϕ**

D-4 - Production of pions by γ 's

$$\gamma + N \rightarrow P + \pi^-$$

D-4

Process



$$\frac{\frac{e_2 \hbar c}{\sqrt{2} \Omega \mu \mu c^2}}{\mu c^2} \times \frac{\sqrt{2\pi} e \hbar c}{\sqrt{4\pi \mu \mu \hbar \omega}} = \frac{\sqrt{\pi} e e_2 \hbar^{3/2}}{\Omega c \mu^{3/2} \omega^{1/2}}$$

$$\sigma = \frac{\sqrt{2} e^2 e_2^2}{c^3 \mu^{3/2}} \frac{\sqrt{\hbar \omega - \mu c^2}}{\hbar \omega} \quad \text{for } 335 \text{ MeV}$$

$$\sigma \approx 3 \times 10^{-28}$$

Discussion of other intermediate steps
 Difference between positive and neg. pion production
 Photo production of neutral pions

$$\frac{\pi^-}{\pi^+} \approx 1.3$$

$$(3 \times 10^{-28})^{1/4096}$$

Input interpretation:

$$\sqrt[4096]{3 \times 10^{-28}}$$

Result:

$$\frac{\sqrt[4096]{3}}{10^{7/1024}}$$

Decimal approximation:

0.984646966308441828238021915927473407248566395499039147463...

0.984646966308.... result very near to the value of the following Rogers-Ramanujan continued fraction:

$$\frac{e^{-\frac{\pi}{\sqrt{5}}}}{\sqrt{5}} = 1 - \frac{e^{-\pi\sqrt{5}}}{1 + \frac{e^{-2\pi\sqrt{5}}}{1 + \frac{e^{-3\pi\sqrt{5}}}{1 + \frac{e^{-4\pi\sqrt{5}}}{1 + \dots}}}} - \phi + 1 \approx 0.9991104684$$

and to the dilaton value **0.989117352243 = ϕ**

From:

Ramanujan's Notebooks

Working mostly in isolation, Ramanujan noted striking and sometimes still unproved results in series, special functions and number theory.

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University of Illinois,
Urbana, IL 61801

$$\begin{aligned} & \tan^{-1} \frac{1}{n+1} + \tan^{-1} \frac{1}{n+2} + \dots + \tan^{-1} \frac{1}{2n} + \tan^{-1} \frac{1}{2n+1} + \tan^{-1} \frac{1}{2n+3} + \dots + \tan^{-1} \frac{1}{4n+1} \\ &= \frac{\pi}{4} + \tan^{-1} \frac{9}{53} + \tan^{-1} \frac{18}{599} + \dots + \tan^{-1} \frac{9n}{32n^4 + 22n^2 - 1} \\ & \quad + \tan^{-1} \frac{4}{137} + \tan^{-1} \frac{8}{2081} + \dots + \tan^{-1} \frac{4n}{128n^4 + 8n^2 + 1}. \end{aligned}$$

$\frac{\pi}{4} + \tan^{-1} \left(\frac{9}{53} \right) + \tan^{-1} \left(\frac{18}{599} \right) + \tan^{-1} \left(\frac{18}{(32 \cdot 16 + 22 \cdot 4 - 1)} \right) + \tan^{-1} \left(\frac{4}{137} \right) + \tan^{-1} \left(\frac{8}{2081} \right) + \tan^{-1} \left(\frac{8}{(128 \cdot 16 + 8 \cdot 4 + 1)} \right)$

Input:

$$\begin{aligned} & \frac{\pi}{4} + \tan^{-1} \left(\frac{9}{53} \right) + \tan^{-1} \left(\frac{18}{599} \right) + \tan^{-1} \left(\frac{18}{32 \times 16 + 22 \times 4 - 1} \right) + \\ & \tan^{-1} \left(\frac{4}{137} \right) + \tan^{-1} \left(\frac{8}{2081} \right) + \tan^{-1} \left(\frac{8}{128 \times 16 + 8 \times 4 + 1} \right) \end{aligned}$$

$\tan^{-1}(x)$ is the inverse tangent function

Exact Result:

$$\frac{\pi}{4} + 2 \tan^{-1}\left(\frac{8}{2081}\right) + \tan^{-1}\left(\frac{4}{137}\right) + 2 \tan^{-1}\left(\frac{18}{599}\right) + \tan^{-1}\left(\frac{9}{53}\right)$$

(result in radians)

Decimal approximation:

$$1.050564383055564282645346089140019210234227249284812372396\dots$$

(result in radians)

$$1.050564383055\dots$$

Alternate forms:

$$\frac{1}{4} \left(\pi + 2 \tan^{-1}\left(\frac{79\,862\,893}{136\,200\,276}\right) \right)$$

$$\frac{\pi}{4} + \frac{1}{2} \tan^{-1}\left(\frac{79\,862\,893}{136\,200\,276}\right)$$

$$\frac{1}{4} \left(\pi + 8 \tan^{-1}\left(\frac{8}{2081}\right) + 4 \tan^{-1}\left(\frac{4}{137}\right) + 8 \tan^{-1}\left(\frac{18}{599}\right) \right) + \tan^{-1}\left(\frac{9}{53}\right)$$

Alternative representations:

$$\begin{aligned} & \frac{\pi}{4} + \tan^{-1}\left(\frac{9}{53}\right) + \tan^{-1}\left(\frac{18}{599}\right) + \tan^{-1}\left(\frac{18}{32 \times 16 + 22 \times 4 - 1}\right) + \\ & \quad \tan^{-1}\left(\frac{4}{137}\right) + \tan^{-1}\left(\frac{8}{2081}\right) + \tan^{-1}\left(\frac{8}{128 \times 16 + 8 \times 4 + 1}\right) = \\ & \quad \operatorname{sc}^{-1}\left(\frac{9}{53} \mid 0\right) + \operatorname{sc}^{-1}\left(\frac{4}{137} \mid 0\right) + 2 \operatorname{sc}^{-1}\left(\frac{18}{599} \mid 0\right) + 2 \operatorname{sc}^{-1}\left(\frac{8}{2081} \mid 0\right) + \frac{\pi}{4} \end{aligned}$$

$$\begin{aligned} & \frac{\pi}{4} + \tan^{-1}\left(\frac{9}{53}\right) + \tan^{-1}\left(\frac{18}{599}\right) + \tan^{-1}\left(\frac{18}{32 \times 16 + 22 \times 4 - 1}\right) + \\ & \quad \tan^{-1}\left(\frac{4}{137}\right) + \tan^{-1}\left(\frac{8}{2081}\right) + \tan^{-1}\left(\frac{8}{128 \times 16 + 8 \times 4 + 1}\right) = \\ & \quad \tan^{-1}\left(1, \frac{9}{53}\right) + \tan^{-1}\left(1, \frac{4}{137}\right) + 2 \tan^{-1}\left(1, \frac{18}{599}\right) + 2 \tan^{-1}\left(1, \frac{8}{2081}\right) + \frac{\pi}{4} \end{aligned}$$

$$\begin{aligned} & \frac{\pi}{4} + \tan^{-1}\left(\frac{9}{53}\right) + \tan^{-1}\left(\frac{18}{599}\right) + \tan^{-1}\left(\frac{18}{32 \times 16 + 22 \times 4 - 1}\right) + \\ & \quad \tan^{-1}\left(\frac{4}{137}\right) + \tan^{-1}\left(\frac{8}{2081}\right) + \tan^{-1}\left(\frac{8}{128 \times 16 + 8 \times 4 + 1}\right) = \\ & \quad \cot^{-1}\left(\frac{1}{53}\right) + \cot^{-1}\left(\frac{1}{137}\right) + 2 \cot^{-1}\left(\frac{1}{599}\right) + 2 \cot^{-1}\left(\frac{1}{2081}\right) + \frac{\pi}{4} \end{aligned}$$

Series representations:

$$\begin{aligned} & \frac{\pi}{4} + \tan^{-1}\left(\frac{9}{53}\right) + \tan^{-1}\left(\frac{18}{599}\right) + \tan^{-1}\left(\frac{18}{32 \times 16 + 22 \times 4 - 1}\right) + \\ & \quad \tan^{-1}\left(\frac{4}{137}\right) + \tan^{-1}\left(\frac{8}{2081}\right) + \tan^{-1}\left(\frac{8}{128 \times 16 + 8 \times 4 + 1}\right) = \\ & \frac{\pi}{4} + \sum_{k=0}^{\infty} \left(\frac{(-1)^k 9^{1+2k} \times 53^{-1-2k}}{1+2k} + \frac{(-1)^k 4^{1+2k} \times 137^{-1-2k}}{1+2k} + \right. \\ & \quad \left. 2 \left(\frac{(-1)^k 18^{1+2k} \times 599^{-1-2k}}{1+2k} + \frac{(-1)^k 8^{1+2k} \times 2081^{-1-2k}}{1+2k} \right) \right) \end{aligned}$$

$$\begin{aligned} & \frac{\pi}{4} + \tan^{-1}\left(\frac{9}{53}\right) + \tan^{-1}\left(\frac{18}{599}\right) + \tan^{-1}\left(\frac{18}{32 \times 16 + 22 \times 4 - 1}\right) + \\ & \quad \tan^{-1}\left(\frac{4}{137}\right) + \tan^{-1}\left(\frac{8}{2081}\right) + \tan^{-1}\left(\frac{8}{128 \times 16 + 8 \times 4 + 1}\right) = \\ & \frac{\pi}{4} - i \log\left(1 + \frac{8i}{2081}\right) - \frac{1}{2} i \log\left(1 + \frac{4i}{137}\right) - i \log\left(1 + \frac{18i}{599}\right) - \frac{1}{2} i \log\left(1 + \frac{9i}{53}\right) + \\ & \quad 3i \log(2) + \sum_{k=1}^{\infty} - \frac{i 2^{-1-k} \left(2 \left(1 + \frac{8i}{2081}\right)^k + \left(1 + \frac{4i}{137}\right)^k + 2 \left(1 + \frac{18i}{599}\right)^k + \left(1 + \frac{9i}{53}\right)^k \right)}{k} \end{aligned}$$

$$\begin{aligned} & \frac{\pi}{4} + \tan^{-1}\left(\frac{9}{53}\right) + \tan^{-1}\left(\frac{18}{599}\right) + \tan^{-1}\left(\frac{18}{32 \times 16 + 22 \times 4 - 1}\right) + \\ & \quad \tan^{-1}\left(\frac{4}{137}\right) + \tan^{-1}\left(\frac{8}{2081}\right) + \tan^{-1}\left(\frac{8}{128 \times 16 + 8 \times 4 + 1}\right) = \\ & \frac{\pi}{4} + i \log\left(1 - \frac{8i}{2081}\right) + \frac{1}{2} i \log\left(1 - \frac{4i}{137}\right) + i \log\left(1 - \frac{18i}{599}\right) + \frac{1}{2} i \log\left(1 - \frac{9i}{53}\right) - \\ & \quad 3i \log(2) + \sum_{k=1}^{\infty} \frac{i 2^{-1-k} \left(2 \left(1 - \frac{8i}{2081}\right)^k + \left(1 - \frac{4i}{137}\right)^k + 2 \left(1 - \frac{18i}{599}\right)^k + \left(1 - \frac{9i}{53}\right)^k \right)}{k} \end{aligned}$$

Integral representations:

$$\begin{aligned} & \frac{\pi}{4} + \tan^{-1}\left(\frac{9}{53}\right) + \tan^{-1}\left(\frac{18}{599}\right) + \tan^{-1}\left(\frac{18}{32 \times 16 + 22 \times 4 - 1}\right) + \\ & \quad \tan^{-1}\left(\frac{4}{137}\right) + \tan^{-1}\left(\frac{8}{2081}\right) + \tan^{-1}\left(\frac{8}{128 \times 16 + 8 \times 4 + 1}\right) = \\ & \frac{\pi}{4} + \int_0^1 \left(\frac{548}{18769 + 16t^2} + \frac{33296}{4330561 + 64t^2} + \frac{477}{2809 + 81t^2} + \frac{21564}{358801 + 324t^2} \right) dt \end{aligned}$$

$$\frac{\pi}{4} + \tan^{-1}\left(\frac{9}{53}\right) + \tan^{-1}\left(\frac{18}{599}\right) + \tan^{-1}\left(\frac{18}{32 \times 16 + 22 \times 4 - 1}\right) + \tan^{-1}\left(\frac{4}{137}\right) + \tan^{-1}\left(\frac{8}{2081}\right) + \tan^{-1}\left(\frac{8}{128 \times 16 + 8 \times 4 + 1}\right) = \frac{\pi}{4} + \int_{-i\infty+\gamma}^{i\infty+\gamma} \left(-\frac{9 i 2^{-2-s} \times 53^{-1+2s} \times 1445^{-s} \Gamma\left(\frac{1}{2}-s\right) \Gamma(1-s) \Gamma(s)^2}{\pi^{3/2}} - \frac{i 137^{-1+2s} \times 18 785^{-s} \Gamma\left(\frac{1}{2}-s\right) \Gamma(1-s) \Gamma(s)^2}{\pi^{3/2}} - \frac{9 i 599^{-1+2s} \times 359 125^{-s} \Gamma\left(\frac{1}{2}-s\right) \Gamma(1-s) \Gamma(s)^2}{\pi^{3/2}} - \frac{4 i 2081^{-1+2s} \times 4 330 625^{-s} \Gamma\left(\frac{1}{2}-s\right) \Gamma(1-s) \Gamma(s)^2}{\pi^{3/2}} \right) ds \text{ for } 0 < \gamma < \frac{1}{2}$$

$$\frac{\pi}{4} + \tan^{-1}\left(\frac{9}{53}\right) + \tan^{-1}\left(\frac{18}{599}\right) + \tan^{-1}\left(\frac{18}{32 \times 16 + 22 \times 4 - 1}\right) + \tan^{-1}\left(\frac{4}{137}\right) + \tan^{-1}\left(\frac{8}{2081}\right) + \tan^{-1}\left(\frac{8}{128 \times 16 + 8 \times 4 + 1}\right) = \frac{\pi}{4} + \int_{-i\infty+\gamma}^{i\infty+\gamma} \left(-\frac{i \left(\frac{9}{53}\right)^{1-2s} \Gamma\left(\frac{1}{2}-s\right) \Gamma(1-s) \Gamma(s)}{4 \pi \Gamma\left(\frac{3}{2}-s\right)} - \frac{i 16^{-s} \times 137^{-1+2s} \Gamma\left(\frac{1}{2}-s\right) \Gamma(1-s) \Gamma(s)}{\pi \Gamma\left(\frac{3}{2}-s\right)} - \frac{i 4^{-s} \times 9^{1-2s} \times 599^{-1+2s} \Gamma\left(\frac{1}{2}-s\right) \Gamma(1-s) \Gamma(s)}{\pi \Gamma\left(\frac{3}{2}-s\right)} - \frac{i 4^{1-3s} \times 2081^{-1+2s} \Gamma\left(\frac{1}{2}-s\right) \Gamma(1-s) \Gamma(s)}{\pi \Gamma\left(\frac{3}{2}-s\right)} \right) ds \text{ for } 0 < \gamma < \frac{1}{2}$$

(((((((Pi/4+tan^-1(9/53)+tan^-1(18/599)+tan^-1(18/(32*16+22*4-1))+tan^-1(4/137)+tan^-1(8/2081)+tan^-1(8/(128*16+8*4+1))))))))))^10

Input:

$$\left(\frac{\pi}{4} + \tan^{-1}\left(\frac{9}{53}\right) + \tan^{-1}\left(\frac{18}{599}\right) + \tan^{-1}\left(\frac{18}{32 \times 16 + 22 \times 4 - 1}\right) + \tan^{-1}\left(\frac{4}{137}\right) + \tan^{-1}\left(\frac{8}{2081}\right) + \tan^{-1}\left(\frac{8}{128 \times 16 + 8 \times 4 + 1}\right) \right)^{10}$$

$\tan^{-1}(x)$ is the inverse tangent function

Exact Result:

$$\left(\frac{\pi}{4} + 2 \tan^{-1}\left(\frac{8}{2081}\right) + \tan^{-1}\left(\frac{4}{137}\right) + 2 \tan^{-1}\left(\frac{18}{599}\right) + \tan^{-1}\left(\frac{9}{53}\right)\right)^{10}$$

(result in radians)

Decimal approximation:

1.637671268255303751988865082154298724800351052086303525617...

(result in radians)

$$1.637671268255\dots \approx \zeta(2) = \frac{\pi^2}{6} = 1.644934 \dots$$

Alternate forms:

$$\frac{\left(\pi + 2 \tan^{-1}\left(\frac{79862893}{136200276}\right)\right)^{10}}{1048576}$$

$$\left(\frac{\pi}{4} + \frac{1}{2} \tan^{-1}\left(\frac{79862893}{136200276}\right)\right)^{10}$$

$$\frac{\left(\pi + 4 \left(2 \tan^{-1}\left(\frac{8}{2081}\right) + \tan^{-1}\left(\frac{4}{137}\right) + 2 \tan^{-1}\left(\frac{18}{599}\right) + \tan^{-1}\left(\frac{9}{53}\right)\right)\right)^{10}}{1048576}$$

Alternative representations:

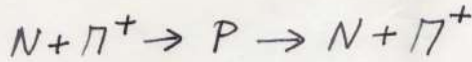
$$\left(\frac{\pi}{4} + \tan^{-1}\left(\frac{9}{53}\right) + \tan^{-1}\left(\frac{18}{599}\right) + \tan^{-1}\left(\frac{18}{32 \times 16 + 22 \times 4 - 1}\right) + \right. \\ \left. \tan^{-1}\left(\frac{4}{137}\right) + \tan^{-1}\left(\frac{8}{2081}\right) + \tan^{-1}\left(\frac{8}{128 \times 16 + 8 \times 4 + 1}\right)\right)^{10} = \\ \left(\operatorname{sc}^{-1}\left(\frac{9}{53} \mid 0\right) + \operatorname{sc}^{-1}\left(\frac{4}{137} \mid 0\right) + 2 \operatorname{sc}^{-1}\left(\frac{18}{599} \mid 0\right) + 2 \operatorname{sc}^{-1}\left(\frac{8}{2081} \mid 0\right) + \frac{\pi}{4}\right)^{10}$$

$$\left(\frac{\pi}{4} + \tan^{-1}\left(\frac{9}{53}\right) + \tan^{-1}\left(\frac{18}{599}\right) + \tan^{-1}\left(\frac{18}{32 \times 16 + 22 \times 4 - 1}\right) + \right. \\ \left. \tan^{-1}\left(\frac{4}{137}\right) + \tan^{-1}\left(\frac{8}{2081}\right) + \tan^{-1}\left(\frac{8}{128 \times 16 + 8 \times 4 + 1}\right)\right)^{10} = \\ \left(\tan^{-1}\left(1, \frac{9}{53}\right) + \tan^{-1}\left(1, \frac{4}{137}\right) + 2 \tan^{-1}\left(1, \frac{18}{599}\right) + 2 \tan^{-1}\left(1, \frac{8}{2081}\right) + \frac{\pi}{4}\right)^{10}$$

$$\left(\frac{\pi}{4} + \tan^{-1}\left(\frac{9}{53}\right) + \tan^{-1}\left(\frac{18}{599}\right) + \tan^{-1}\left(\frac{18}{32 \times 16 + 22 \times 4 - 1}\right) + \right. \\ \left. \tan^{-1}\left(\frac{4}{137}\right) + \tan^{-1}\left(\frac{8}{2081}\right) + \tan^{-1}\left(\frac{8}{128 \times 16 + 8 \times 4 + 1}\right)\right)^{10} = \\ \left(\cot^{-1}\left(\frac{1}{53}\right) + \cot^{-1}\left(\frac{1}{137}\right) + 2 \cot^{-1}\left(\frac{1}{599}\right) + 2 \cot^{-1}\left(\frac{1}{2081}\right) + \frac{\pi}{4}\right)^{10}$$

D-7 Scattering of pions by nucleons

D-7



$$\frac{\left(\frac{e_2 \hbar c}{\sqrt{2\Omega \mu c^2}}\right)^2}{\mu c^2} = \frac{e_2^2 \hbar^2}{2\Omega \mu^2 c^2} \quad \left(\frac{1}{\sqrt{w}}\right)^2 \approx \frac{1}{w^2}$$

$$\sigma = \frac{2\pi}{\hbar v} \left(\frac{e_2^2 \hbar^2}{2\mu^2 c^2}\right)^2 \frac{p^2}{2\pi^2 \hbar^3 v} \approx \frac{1}{4\pi} \left(\frac{e_2^2}{\mu c^2}\right)^2 = 1.6 \times 10^{-26}$$

Energy dependence

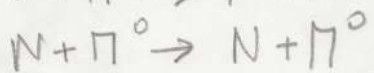
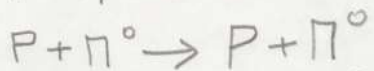
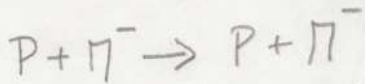
$$\frac{p^2}{v^2} \frac{1}{w^4} \rightarrow \frac{1}{w^2}$$

$$c^2 p = v w$$

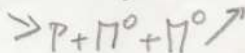
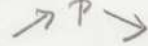
$$\frac{p}{v} = \frac{w}{c^2}$$

(Unreliable)

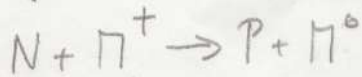
Discussion of similar processes



} here possible destructive interference like



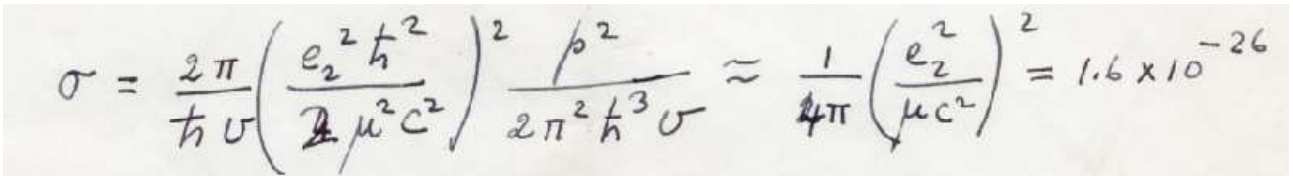
Scattering with exchange



V particles

$$2165 \pm 20 \quad Q \approx 30 \text{ m c}^2$$

We note that:



$$\sigma = \frac{2\pi}{h\nu} \left(\frac{e_2^2 h^2}{2\mu c^2} \right)^2 \frac{\rho^2}{2\pi^2 h^3 \nu} \approx \frac{1}{4\pi} \left(\frac{e_2^2}{\mu c^2} \right)^2 = 1.6 \times 10^{-26}$$

And that:

$$\left(\begin{array}{l} \left(\frac{\pi}{4} + 2 \tan^{-1}\left(\frac{8}{2081}\right) + \tan^{-1}\left(\frac{4}{137}\right) + 2 \tan^{-1}\left(\frac{18}{599}\right) + \tan^{-1}\left(\frac{9}{53}\right) \right)^{10} \\ \text{(result in radians)} \end{array} \right) =$$

$$= 1.637671268255\dots$$

$$1/10^{26} \left(\left(\left(\left(\left(\left(\left(\frac{\pi}{4} + \tan^{-1}\left(\frac{9}{53}\right) + \tan^{-1}\left(\frac{18}{599}\right) + \tan^{-1}\left(\frac{18}{32 \times 16 + 22 \times 4 - 1}\right) + \tan^{-1}\left(\frac{4}{137}\right) + \tan^{-1}\left(\frac{8}{2081}\right) + \tan^{-1}\left(\frac{8}{128 \times 16 + 8 \times 4 + 1}\right) \right) \right) \right) \right) \right) \right) \right)^{10}$$

Input:

$$\frac{1}{10^{26}} \left(\frac{\pi}{4} + \tan^{-1}\left(\frac{9}{53}\right) + \tan^{-1}\left(\frac{18}{599}\right) + \tan^{-1}\left(\frac{18}{32 \times 16 + 22 \times 4 - 1}\right) + \tan^{-1}\left(\frac{4}{137}\right) + \tan^{-1}\left(\frac{8}{2081}\right) + \tan^{-1}\left(\frac{8}{128 \times 16 + 8 \times 4 + 1}\right) \right)^{10}$$

$\tan^{-1}(x)$ is the inverse tangent function

Exact Result:

$$\frac{\left(\frac{\pi}{4} + 2 \tan^{-1}\left(\frac{8}{2081}\right) + \tan^{-1}\left(\frac{4}{137}\right) + 2 \tan^{-1}\left(\frac{18}{599}\right) + \tan^{-1}\left(\frac{9}{53}\right) \right)^{10}}{100\,000\,000\,000\,000\,000\,000\,000\,000}$$

(result in radians)

Decimal approximation:

$$1.6376712682553037519888650821542987248003510520863035\dots \times 10^{-26}$$

(result in radians)

$$1.637671268255\dots \times 10^{-26}$$

Alternate forms:

$$\frac{\left(\pi + 2 \tan^{-1}\left(\frac{79862893}{136200276}\right)\right)^{10}}{104857600000000000000000000000000}$$

$$\frac{\left(\frac{\pi}{4} + \frac{1}{2} \tan^{-1}\left(\frac{79862893}{136200276}\right)\right)^{10}}{1000000000000000000000000000000}$$

$$\frac{\left(\pi + 4\left(2 \tan^{-1}\left(\frac{8}{2081}\right) + \tan^{-1}\left(\frac{4}{137}\right) + 2 \tan^{-1}\left(\frac{18}{599}\right) + \tan^{-1}\left(\frac{9}{53}\right)\right)\right)^{10}}{104857600000000000000000000000000}$$

Alternative representations:

$$\frac{\frac{1}{10^{26}}\left(\frac{\pi}{4} + \tan^{-1}\left(\frac{9}{53}\right) + \tan^{-1}\left(\frac{18}{599}\right) + \tan^{-1}\left(\frac{18}{32 \times 16 + 22 \times 4 - 1}\right) + \tan^{-1}\left(\frac{4}{137}\right) + \tan^{-1}\left(\frac{8}{2081}\right) + \tan^{-1}\left(\frac{8}{128 \times 16 + 8 \times 4 + 1}\right)\right)^{10}}{\left(\operatorname{sc}^{-1}\left(\frac{9}{53} \mid 0\right) + \operatorname{sc}^{-1}\left(\frac{4}{137} \mid 0\right) + 2 \operatorname{sc}^{-1}\left(\frac{18}{599} \mid 0\right) + 2 \operatorname{sc}^{-1}\left(\frac{8}{2081} \mid 0\right) + \frac{\pi}{4}\right)^{10}} = \frac{\phantom{\frac{1}{10^{26}}\left(\frac{\pi}{4} + \tan^{-1}\left(\frac{9}{53}\right) + \tan^{-1}\left(\frac{18}{599}\right) + \tan^{-1}\left(\frac{18}{32 \times 16 + 22 \times 4 - 1}\right) + \tan^{-1}\left(\frac{4}{137}\right) + \tan^{-1}\left(\frac{8}{2081}\right) + \tan^{-1}\left(\frac{8}{128 \times 16 + 8 \times 4 + 1}\right)\right)^{10}}}{10^{26}}$$

$$\frac{\frac{1}{10^{26}}\left(\frac{\pi}{4} + \tan^{-1}\left(\frac{9}{53}\right) + \tan^{-1}\left(\frac{18}{599}\right) + \tan^{-1}\left(\frac{18}{32 \times 16 + 22 \times 4 - 1}\right) + \tan^{-1}\left(\frac{4}{137}\right) + \tan^{-1}\left(\frac{8}{2081}\right) + \tan^{-1}\left(\frac{8}{128 \times 16 + 8 \times 4 + 1}\right)\right)^{10}}{\left(\tan^{-1}\left(1, \frac{9}{53}\right) + \tan^{-1}\left(1, \frac{4}{137}\right) + 2 \tan^{-1}\left(1, \frac{18}{599}\right) + 2 \tan^{-1}\left(1, \frac{8}{2081}\right) + \frac{\pi}{4}\right)^{10}} = \frac{\phantom{\frac{1}{10^{26}}\left(\frac{\pi}{4} + \tan^{-1}\left(\frac{9}{53}\right) + \tan^{-1}\left(\frac{18}{599}\right) + \tan^{-1}\left(\frac{18}{32 \times 16 + 22 \times 4 - 1}\right) + \tan^{-1}\left(\frac{4}{137}\right) + \tan^{-1}\left(\frac{8}{2081}\right) + \tan^{-1}\left(\frac{8}{128 \times 16 + 8 \times 4 + 1}\right)\right)^{10}}}{10^{26}}$$

$$\frac{\frac{1}{10^{26}}\left(\frac{\pi}{4} + \tan^{-1}\left(\frac{9}{53}\right) + \tan^{-1}\left(\frac{18}{599}\right) + \tan^{-1}\left(\frac{18}{32 \times 16 + 22 \times 4 - 1}\right) + \tan^{-1}\left(\frac{4}{137}\right) + \tan^{-1}\left(\frac{8}{2081}\right) + \tan^{-1}\left(\frac{8}{128 \times 16 + 8 \times 4 + 1}\right)\right)^{10}}{\left(\cot^{-1}\left(\frac{1}{\frac{9}{53}}\right) + \cot^{-1}\left(\frac{1}{\frac{4}{137}}\right) + 2 \cot^{-1}\left(\frac{1}{\frac{18}{599}}\right) + 2 \cot^{-1}\left(\frac{1}{\frac{8}{2081}}\right) + \frac{\pi}{4}\right)^{10}} = \frac{\phantom{\frac{1}{10^{26}}\left(\frac{\pi}{4} + \tan^{-1}\left(\frac{9}{53}\right) + \tan^{-1}\left(\frac{18}{599}\right) + \tan^{-1}\left(\frac{18}{32 \times 16 + 22 \times 4 - 1}\right) + \tan^{-1}\left(\frac{4}{137}\right) + \tan^{-1}\left(\frac{8}{2081}\right) + \tan^{-1}\left(\frac{8}{128 \times 16 + 8 \times 4 + 1}\right)\right)^{10}}}{10^{26}}$$

Now, we have that, from:

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Gravitational waves from walking technicolor

Kohtaroh Miura, Hiroshi Ohki, Saeko Otani and Koichi Yamawaki

The phase transition dynamics is modified via the shift of $(2f_2/N_f)(s^0)^2 \rightarrow (\Delta m_s)^2 + (2f_2/N_f)(s^0)^2$ in $m_{s^i}^2$ with finite Δm_s . The details of the mass spectra at one loop with $(\Delta m_s)^2$ are summarized in appendix A. Using eq. (4.18), the total effective potential becomes,

$$V_{\text{eff}}(s^0, \Delta m_p, \Delta m_s, T) = \frac{N_f^2 - 1}{64\pi^2} \mathcal{M}_{s^i}^4(s^0, \Delta m_p, \Delta m_s, T) \left(\ln \frac{\mathcal{M}_{s^i}^2(s^0, \Delta m_p, \Delta m_s, T)}{\mu_{\text{GW}}^2} - \frac{3}{2} \right), \\ + \frac{T^4}{2\pi^2} (N_f^2 - 1) J_B(\mathcal{M}_{s^i}^2(s^0, \Delta m_p, \Delta m_s, T)/T^2) + C(T), \quad (4.19)$$

with,

$$\mathcal{M}_{s^i}^2(s^0, \Delta m_p, \Delta m_s, T) = m_{s^i}^2(s^0, \Delta m_p, \Delta m_s) + \Pi(T), \quad (4.20)$$

where the thermal mass $\Pi(T)$ is given in eq. (3.3). We require that the following properties remain intact for arbitrary Δm_s ; (1) the vev $\langle s^0 \rangle(T=0)$ determined by the minimum of the potential eq. (4.19) is identified with the dilaton decay constant favored by the walking technicolor model, $F_\phi = 1.25 \text{ TeV}$ or 1 TeV , (2) the dilaton mass given by the potential curvature at the vacuum is identified with the observed SM Higgs mass, $m_{s^0} = 125 \text{ GeV}$.

Thence $F_\phi = 1.25 \text{ TeV}$

$$2.0662356 + 1.00186743 + 0.655679 = 3.72378203 \div 3 = 1.2412606766666$$

$$\sqrt{\frac{e\pi}{2}} = \sum_{n=0}^{\infty} \frac{1}{(2n+1)!!} + \frac{1}{1 + \frac{1}{1 + \frac{2}{1 + \frac{3}{1 + \frac{4}{1 + \dots}}}}} \approx 2.0663656771$$

$$\sqrt{\frac{e\pi}{2}} \operatorname{erfc}\left(\frac{\sqrt{2}}{2}\right) \approx 0.6556795424$$

$$\frac{e^{-\frac{2\pi}{5}}}{\sqrt{\varphi\sqrt{5}-\varphi}} = 1 + \frac{e^{-2\pi}}{1 + \frac{e^{-4\pi}}{1 + \frac{e^{-6\pi}}{1 + \frac{e^{-8\pi}}{1 + \dots}}}} \approx 1.0018674362$$

From:

SUPERSTRING THEORY

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Loop amplitudes, anomalies and phenomenology

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The next term in the expansion of the integrand gives a divergence of the form $\int d\epsilon/\epsilon$ corresponding to the propagation of a massless dilaton, rather than a tachyon, down the long neck of fig. 8.22a. The coefficient of this divergence,

$$\int_F d^2\tau (\text{Im } \tau)^{-14} e^{4\pi \text{Im } \tau} |f(e^{2\pi i\tau})|^{-48}, \quad (8.2.47)$$

should be proportional to the coupling of a dilaton to a toroidal world sheet, *i.e.*, to the dilaton one-loop expectation value. This can be seen

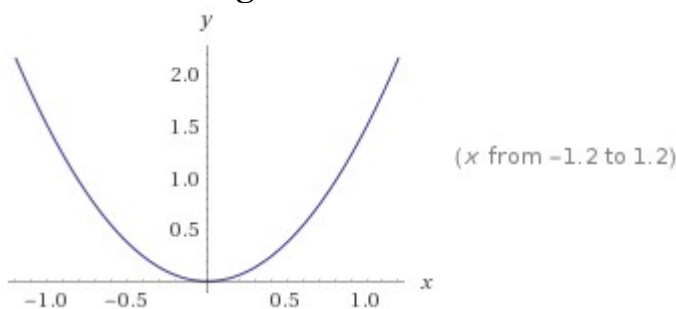
We calculate the following integral:

integrate[-exp(4Pi) (exp(2Pi))^-48]x

Indefinite integral:

$$\int -\frac{\exp(4\pi)x}{\exp^{48}(2\pi)} dx = -\frac{1}{2} e^{-92\pi} x^2 + \text{constant}$$

Plot of the integral:



For $x = 1$, we have:

$$-1/2 e^{(-92 \pi)}$$

Input:

$$-\frac{1}{2} e^{-92\pi}$$

Decimal approximation:

$$-1.50087820446173031810634934870291518491171201070408... \times 10^{-126}$$

$$-1.5008782... * 10^{-126}$$

Property:

$-\frac{1}{2} e^{-92\pi}$ is a transcendental number

Alternative representations:

$$\frac{1}{2} e^{-92\pi} (-1) = -\frac{1}{2} e^{-16560^\circ}$$

$$\frac{1}{2} e^{-92\pi} (-1) = -\frac{1}{2} e^{92i \log(-1)}$$

$$\frac{1}{2} e^{-92\pi} (-1) = \frac{1}{2} \exp^{-92\pi}(z)(-1) \text{ for } z = 1$$

Series representations:

$$\frac{1}{2} e^{-92\pi} (-1) = -\frac{1}{2} e^{-368 \sum_{k=0}^{\infty} (-1)^k / (1+2k)}$$

$$\frac{1}{2} e^{-92\pi} (-1) = -\frac{1}{2} \left(\sum_{k=0}^{\infty} \frac{1}{k!} \right)^{-92\pi}$$

$$\frac{1}{2} e^{-92\pi} (-1) = -\frac{1}{2} \left(\frac{1}{\sum_{k=0}^{\infty} \frac{(-1)^k}{k!}} \right)^{-92\pi}$$

Integral representations:

$$\frac{1}{2} e^{-92\pi} (-1) = -\frac{1}{2} e^{-184 \int_0^1 1/\sqrt{1-t^2} dt}$$

$$\frac{1}{2} e^{-92\pi} (-1) = -\frac{1}{2} e^{-368 \int_0^1 \sqrt{1-t^2} dt}$$

$$\frac{1}{2} e^{-92\pi} (-1) = -\frac{1}{2} e^{-184 \int_0^{\infty} 1/(1+t^2) dt}$$

$$[-((-1/2 e^{(-92 \pi)})^{1/4096}]$$

Input:

$$\sqrt[4096]{-\left(-\frac{1}{2} e^{-92 \pi}\right)}$$

Exact result:

$$\frac{e^{-(23 \pi)/1024}}{\sqrt[4096]{2}}$$

Decimal approximation:

0.931711239069052518334943626020824441131662057687785110881...

0.931711239.... result very near to the spectral index n_s , to the mesonic Regge slope (see Appendix), to the inflaton value at the end of the inflation 0.9402 and to the value of the following Rogers-Ramanujan continued fraction:

$$\frac{e^{-\frac{\pi}{5}}}{\sqrt{(\varphi-1)\sqrt{5}} - \varphi + 1} = 1 - \frac{e^{-\pi}}{1 + \frac{e^{-2\pi}}{1 + \frac{e^{-3\pi}}{1 + \frac{e^{-4\pi}}{1 + \dots}}}} \approx 0.9568666373$$

Property:

$\frac{e^{-(23 \pi)/1024}}{\sqrt[4096]{2}}$ is a transcendental number

All 4096th roots of $e^{(-92 \pi)/2}$:

$$\frac{e^{-(23 \pi)/1024} e^0}{\sqrt[4096]{2}} \approx 0.93171 \quad (\text{real, principal root})$$

$$\frac{e^{-(23 \pi)/1024} e^{(i \pi)/2048}}{\sqrt[4096]{2}} \approx 0.931710 + 0.001429 i$$

$$\frac{e^{-(23 \pi)/1024} e^{(i \pi)/1024}}{\sqrt[4096]{2}} \approx 0.931707 + 0.002858 i$$

$$\frac{e^{-(23\pi)/1024} e^{(3i\pi)/2048}}{4096\sqrt{2}} \approx 0.931701 + 0.004288i$$

$$\frac{e^{-(23\pi)/1024} e^{(i\pi)/512}}{4096\sqrt{2}} \approx 0.931694 + 0.005717i$$

Alternative representations:

$$4096\sqrt{-\frac{1}{2}(-1)e^{-92\pi}} = 4096\sqrt{\frac{e^{-16560^\circ}}{2}}$$

$$4096\sqrt{-\frac{1}{2}(-1)e^{-92\pi}} = 4096\sqrt{\frac{1}{2}e^{92i\log(-1)}}$$

$$4096\sqrt{-\frac{1}{2}(-1)e^{-92\pi}} = 4096\sqrt{-\frac{1}{2}(-1)\exp^{-92\pi}(z)} \text{ for } z = 1$$

Series representations:

$$4096\sqrt{-\frac{1}{2}(-1)e^{-92\pi}} = \frac{e^{-23/256 \sum_{k=0}^{\infty} (-1)^k / (1+2k)}}{4096\sqrt{2}}$$

$$4096\sqrt{-\frac{1}{2}(-1)e^{-92\pi}} = \frac{\left(\sum_{k=0}^{\infty} \frac{1}{k!}\right)^{-(23\pi)/1024}}{4096\sqrt{2}}$$

$$4096\sqrt{-\frac{1}{2}(-1)e^{-92\pi}} = \frac{\left(\frac{1}{\sum_{k=0}^{\infty} \frac{(-1)^k}{k!}}\right)^{-(23\pi)/1024}}{4096\sqrt{2}}$$

Integral representations:

$$4096\sqrt{-\frac{1}{2}(-1)e^{-92\pi}} = \frac{e^{-23/256 \int_0^1 \sqrt{1-t^2} dt}}{4096\sqrt{2}}$$

$$4096\sqrt{-\frac{1}{2}(-1)e^{-92\pi}} = \frac{e^{-23/512 \int_0^1 1/\sqrt{1-t^2} dt}}{4096\sqrt{2}}$$

$$4096 \sqrt{-\frac{1}{2} (-1) e^{-92\pi}} = \frac{e^{-23/512} \int_0^{\infty} 1/(1+t^2) dt}{4096 \sqrt{2}}$$

We have:

$$K_1 = \frac{4\pi}{3} \quad K_2 = \frac{4\pi^2}{45}$$

$$f = \frac{\pi^{1/2} \hbar}{\mu c} \frac{4^2 \pi^{1/4} \hbar^{7/2} c^{1/2}}{3 \times 2 \mu c^2 M \mu^{1/2}} = \frac{4\sqrt{2} \pi^{3/2} \hbar^{9/2}}{3 M \mu^{5/2} c^{5/2}} = 5.2 \times 10^{-62} \text{ ergs}^{-1}$$

$$f \approx 10^{-61}$$

Single pion production in nucleon collision E-2

$$\frac{S_3}{S_2} = \frac{V \mu^{3/2}}{8\sqrt{2} \pi \hbar^3} \frac{(T - \mu c^2)^2}{\sqrt{T}} \approx \frac{V \mu (T - \mu c^2)^2}{8\sqrt{2} \pi \hbar^3 c}$$

$$\frac{S_3}{S_2} = .004$$

From the values of above Fermi's formulas, we obtain:

$$1/\text{Pi}((1\text{e-}61*5.2\text{e-}62*1/(4\text{Pi}/3)*0.004))$$

Input interpretation:

$$\frac{1}{\pi} \left(1 \times 10^{-61} \times 5.2 \times 10^{-62} \times \frac{1}{4 \times \frac{\pi}{3}} \times 0.004 \right)$$

Result:

$$1.580610464820469234524519626071751166907996884887046... \times 10^{-126}$$

$$1.58061046... * 10^{-126}$$

$$[1/\text{Pi}((1\text{e-}61*5.2\text{e-}62*1/(4\text{Pi}/3)*0.004))]^{1/4096}$$

Input interpretation:

$$\sqrt[4096]{\frac{1}{\pi} \left(1 \times 10^{-61} \times 5.2 \times 10^{-62} \times \frac{1}{4 \times \frac{\pi}{3}} \times 0.004 \right)}$$

Result:

$$0.931723013...$$

$$0.931723013...$$

Thence, we have the following mathematical connection:

$$\left(\sqrt[4096]{-\left(-\frac{1}{2} e^{-92\pi}\right)} \right) = 0.931711239 \Rightarrow$$

$$\Rightarrow \left(\sqrt[4096]{\frac{1}{\pi} \left(1 \times 10^{-61} \times 5.2 \times 10^{-62} \times \frac{1}{4 \times \frac{\pi}{3}} \times 0.004 \right)} \right) = 0.93172301$$

$$0.931711239 \approx 0.931723013$$

Note that, the result 0.9317... is very near to the Regge slope of vector mesons ρ and ω as showed in the below description:

4.1.1 Light quark mesons

We begin by looking at mesons consisting only of light quarks - u and d . We assume for our analysis that the u and d quarks are equal in mass, as any difference between them would be too small to reveal itself in our fits.

This sector is where we have the most data, but it is also where our fits are the least conclusive. The trajectories we have analyzed are those of the π/b , ρ/a , η/h , and ω/f .

Of the four (J, M^2) trajectories, the two $I = 1$ trajectories, of the ρ and the π , show a weak dependence of χ^2 on m . Endpoint masses anywhere between 0 and 160 MeV are nearly equal in terms of χ^2 , and no clear optimum can be observed. For the two $I = 0$ trajectories, of the η and ω , the linear fit is optimal. If we allow an increase of up to 10% in χ^2 , we can add masses of only 60 MeV or less. Figure (2) presents the plots of χ^2 vs. α' and m for the trajectories of the ω and ρ and shows the difference in the allowed masses between them.

The slope for these trajectories is between $\alpha' = 0.81 - 0.86$ for the two trajectories starting with a pseudo-scalar (η and π), and $\alpha' = 0.88 - 0.93$ for the trajectories beginning with a vector meson (ρ and ω). The higher values for the slopes are obtained when we add masses, as increasing the mass generally requires an increase in α' to retain a good fit to a given trajectory. This can also be seen in figure (2), in the plot for the ρ trajectory fit.

Traj.	N	m	α'	a
π/b	4	$m_{u/d} = 90 - 185$	0.808 - 0.863	(0.23) 0.00
ρ/a	6	$m_{u/d} = 0 - 180$	0.883 - 0.933	0.47 - 0.66
η/h	5	$m_{u/d} = 0 - 70$	0.839 - 0.854	(-0.25) - (-0.21)
ω	6	$m_{u/d} = 0 - 60$	0.910 - 0.918	0.45 - 0.50
K^*	5	$m_{u/d} = 0 - 240$ $m_s = 0 - 390$	0.848 - 0.927	0.32 - 0.62
ϕ	3	$m_s = 400$	1.078	0.82
D	3	$m_{u/d} = 80$ $m_c = 1640$	1.073	-0.07
D_s^*	3	$m_s = 400$ $m_c = 1580$	1.093	0.89
Ψ	3	$m_c = 1500$	0.979	-0.09
Υ	3	$m_b = 4730$	0.635	1.00

Table 1. The results of the meson fits in the (J, M^2) plane. For the uneven K^* fit the higher values of m_s require $m_{u/d}$ to take a correspondingly low value. $m_{u/d} + m_s$ never exceeds 480 MeV, and the highest masses quoted for the s are obtained when $m_{u/d} = 0$. The ranges listed are those where χ^2 is within 10% of its optimal value. N is the number of data points in the trajectory.

Furthermore, the value is very near to the following Ramanujan mock theta function

Mock ϑ -functions (of 7th order)

$$(i) \quad 1 + \frac{q}{1-q^2} + \frac{q^4}{(1-q^3)(1-q^4)} + \frac{q^9}{(1-q^4)(1-q^5)(1-q^6)} + \dots$$

That is equal to **0.9243408674589**

From:

Ramanujan's Notebooks

Working mostly in isolation, Ramanujan noted striking and sometimes still unproved results in series, special functions and number theory.

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Now, we have that:

$$A_5 = \frac{1}{20} \log \frac{(1+x)^5}{1+x^5} + \frac{1}{4\sqrt{5}} \log \frac{1+x \frac{\sqrt{5}-1}{2} + x^2}{1-x \frac{\sqrt{5}-1}{2} + x^2} \\ + \frac{1}{10} \sqrt{10-2\sqrt{5}} \tan^{-1} \frac{x \sqrt{10-2\sqrt{5}}}{4-x(\sqrt{5}+1)} + \frac{\sqrt{10+2\sqrt{5}}}{10} \tan^{-1} \frac{x \sqrt{10+2\sqrt{5}}}{4+x(\sqrt{5}-1)}$$

$$\frac{1}{20} \ln\left(\frac{(1+2)^5}{1+2^5}\right) + \frac{1}{4\sqrt{5}} \ln\left(\frac{(((((1+2(1/\text{golden ratio})+2^2))))))}{(1-2(1/\text{golden ratio})+2^2))}\right)$$

Input:

$$\frac{1}{20} \log\left(\frac{(1+2)^5}{1+2^5}\right) + \frac{1}{4\sqrt{5}} \log\left(\frac{1+2 \times \frac{1}{\phi} + 2^2}{1-2 \times \frac{1}{\phi} + 2^2}\right)$$

$\log(x)$ is the natural logarithm

ϕ is the golden ratio

Exact result:

$$\frac{\log\left(\frac{\frac{2}{\phi}+5}{5-\frac{2}{\phi}}\right)}{4\sqrt{5}} + \frac{1}{20} \log\left(\frac{81}{11}\right)$$

Decimal approximation:

0.156275630312977622327464447184175443670743606014671252325...

0.15627563...

Alternate forms:

$$\frac{1}{20} \left(\sqrt{5} \log\left(\frac{5\phi+2}{5\phi-2}\right) + \log\left(\frac{81}{11}\right) \right)$$

$$\frac{1}{20} \log\left(\frac{81}{11}\right) + \frac{\log\left(\frac{1}{31} (29 + 10\sqrt{5})\right)}{4\sqrt{5}}$$

$$\frac{1}{20} \left(\sqrt{5} \log\left(\frac{\frac{2}{\phi}+5}{5-\frac{2}{\phi}}\right) + \log\left(\frac{81}{11}\right) \right)$$

Alternative representations:

$$\frac{1}{20} \log\left(\frac{(1+2)^5}{1+2^5}\right) + \frac{\log\left(\frac{1+\frac{2}{\phi}+2^2}{1-\frac{2}{\phi}+2^2}\right)}{4\sqrt{5}} = \frac{1}{20} \log(a) \log_a\left(\frac{3^5}{1+2^5}\right) + \frac{\log(a) \log_a\left(\frac{5+\frac{2}{\phi}}{5-\frac{2}{\phi}}\right)}{4\sqrt{5}}$$

$$\frac{1}{20} \log\left(\frac{(1+2)^5}{1+2^5}\right) + \frac{\log\left(\frac{1+\frac{2}{\phi}+2^2}{1-\frac{2}{\phi}+2^2}\right)}{4\sqrt{5}} = \frac{1}{20} \log_e\left(\frac{3^5}{1+2^5}\right) + \frac{\log_e\left(\frac{5+\frac{2}{\phi}}{5-\frac{2}{\phi}}\right)}{4\sqrt{5}}$$

$$\frac{1}{20} \log\left(\frac{(1+2)^5}{1+2^5}\right) + \frac{\log\left(\frac{1+\frac{2}{\phi}+2^2}{1-\frac{2}{\phi}+2^2}\right)}{4\sqrt{5}} = -\frac{1}{20} \operatorname{Li}_1\left(1 - \frac{3^5}{1+2^5}\right) - \frac{\operatorname{Li}_1\left(1 - \frac{5+\frac{2}{\phi}}{5-\frac{2}{\phi}}\right)}{4\sqrt{5}}$$

Series representations:

$$\begin{aligned} \frac{1}{20} \log\left(\frac{(1+2)^5}{1+2^5}\right) + \frac{\log\left(\frac{1+\frac{2}{\phi}+2^2}{1-\frac{2}{\phi}+2^2}\right)}{4\sqrt{5}} = \\ \frac{1}{10} i\pi \left[\frac{\arg\left(\frac{81}{11} - x\right)}{2\pi} \right] + \frac{i\pi \left[\frac{\arg(2+5\phi+2x-5\phi x)}{2\pi} \right]}{2\sqrt{5}} + \frac{\log(x)}{20} + \frac{\log(x)}{4\sqrt{5}} + \\ \sum_{k=1}^{\infty} \left(\frac{(-1)^{1+k} \left(\frac{81}{11} - x\right)^k x^{-k}}{20k} + \frac{(-1)^{1+k} (-2+5\phi)^{-k} x^{-k} (2+5\phi+2x-5\phi x)^k}{4\sqrt{5}k} \right) \text{ for } x < 0 \end{aligned}$$

$$\begin{aligned} \frac{1}{20} \log\left(\frac{(1+2)^5}{1+2^5}\right) + \frac{\log\left(\frac{1+\frac{2}{\phi}+2^2}{1-\frac{2}{\phi}+2^2}\right)}{4\sqrt{5}} = \frac{1}{10} i\pi \left[\frac{\arg\left(\frac{81}{11} - x\right)}{2\pi} \right] + \frac{i\pi \left[\frac{\arg\left(\frac{5+\frac{2}{\phi}-x}{5-\frac{2}{\phi}}\right)}{2\pi} \right]}{2\sqrt{5}} + \frac{\log(x)}{20} + \\ \frac{\log(x)}{4\sqrt{5}} + \sum_{k=1}^{\infty} \left(\frac{(-1)^{-1+k} \left(\frac{81}{11} - x\right)^k x^{-k}}{20k} + \frac{(-1)^{-1+k} \left(\frac{5+\frac{2}{\phi}-x}{5-\frac{2}{\phi}}\right)^k x^{-k}}{4\sqrt{5}k} \right) \text{ for } x < 0 \end{aligned}$$

$$\begin{aligned} \frac{1}{20} \log\left(\frac{(1+2)^5}{1+2^5}\right) + \frac{\log\left(\frac{1+\frac{2}{\phi}+2^2}{1-\frac{2}{\phi}+2^2}\right)}{4\sqrt{5}} = \\ \frac{1}{10} i\pi \left[\frac{\pi - \arg\left(\frac{1}{z_0}\right) - \arg(z_0)}{2\pi} \right] + \frac{i\pi \left[\frac{\pi - \arg\left(\frac{1}{z_0}\right) - \arg(z_0)}{2\pi} \right]}{2\sqrt{5}} + \frac{\log(z_0)}{20} + \frac{\log(z_0)}{4\sqrt{5}} + \\ \sum_{k=1}^{\infty} \frac{(-1)^{1+k} z_0^{-k} \left(\left(\frac{81}{11} - z_0\right)^k + \sqrt{5} (1+5\sqrt{5})^{-k} (9+5\sqrt{5} - (1+5\sqrt{5})z_0)^k \right)}{20k} \end{aligned}$$

Integral representations:

$$\frac{1}{20} \log\left(\frac{(1+2)^5}{1+2^5}\right) + \frac{\log\left(\frac{1+\frac{2}{\phi}+2^2}{1-\frac{2}{\phi}+2^2}\right)}{4\sqrt{5}} = \int_1^{\frac{81}{11}} \frac{-9+175\sqrt{5}+44(1+\sqrt{5})t}{20t(-9+175\sqrt{5}+44t)} dt$$

$$\frac{1}{20} \log\left(\frac{(1+2)^5}{1+2^5}\right) + \frac{\log\left(\frac{1+\frac{2}{\phi}+2^2}{1-\frac{2}{\phi}+2^2}\right)}{4\sqrt{5}} = \int_{-i\infty+\gamma}^{i\infty+\gamma} \left[-\frac{i\left(\frac{11}{7}\right)^s 2^{-3-s} \times 5^{-1-s} \Gamma(-s)^2 \Gamma(1+s)}{\pi \Gamma(1-s)} - \frac{i\left(-1+\frac{5+\frac{2}{\phi}}{5-\frac{2}{\phi}}\right)^{-s} \Gamma(-s)^2 \Gamma(1+s)}{8\sqrt{5} \pi \Gamma(1-s)} \right] ds \text{ for } -1 < \gamma < 0$$

$$\frac{1}{10} \sqrt{10-2\sqrt{5}} \tan^{-1} \left[\frac{2\sqrt{10-2\sqrt{5}}}{4-2(\sqrt{5}+1)} \right] + \frac{1}{10} \sqrt{10+2\sqrt{5}} \tan^{-1} \left[\frac{2\sqrt{10+2\sqrt{5}}}{4+2(\sqrt{5}-1)} \right]$$

Input:

$$\frac{1}{10} \sqrt{10-2\sqrt{5}} \tan^{-1} \left(\frac{2\sqrt{10-2\sqrt{5}}}{4-2(\sqrt{5}+1)} \right) + \left(\frac{1}{10} \sqrt{10+2\sqrt{5}} \right) \tan^{-1} \left(\frac{2\sqrt{10+2\sqrt{5}}}{4+2(\sqrt{5}-1)} \right)$$

$\tan^{-1}(x)$ is the inverse tangent function

Exact Result:

$$\frac{1}{10} \sqrt{10+2\sqrt{5}} \tan^{-1} \left(\frac{2\sqrt{10+2\sqrt{5}}}{4+2(\sqrt{5}-1)} \right) + \frac{1}{10} \sqrt{10-2\sqrt{5}} \tan^{-1} \left(\frac{2\sqrt{10-2\sqrt{5}}}{4-2(1+\sqrt{5})} \right)$$

(result in radians)

Decimal approximation:

0.073900830513950814579037443315566581309899511892543957758...

(result in radians)

0.07390083...

Alternate forms:

$$\frac{1}{5} \sqrt{\frac{1}{2}(5+\sqrt{5})} \tan^{-1} \left(\sqrt{\frac{1}{2}(5-\sqrt{5})} \right) - \frac{1}{5} \sqrt{\frac{1}{2}(5-\sqrt{5})} \tan^{-1} \left(\sqrt{\frac{1}{2}(5+\sqrt{5})} \right)$$

$$\frac{\frac{1}{10} \left(\sqrt{10-2\sqrt{5}} \tan^{-1} \left(\frac{\sqrt{2(5-\sqrt{5})}}{1-\sqrt{5}} \right) + \sqrt{2(5+\sqrt{5})} \tan^{-1} \left(\frac{\sqrt{2(5+\sqrt{5})}}{1+\sqrt{5}} \right) \right) + \sqrt{5+\sqrt{5}} \tan^{-1} \left(\frac{2\sqrt{10+2\sqrt{5}}}{4+2(\sqrt{5}-1)} \right) + \sqrt{5-\sqrt{5}} \tan^{-1} \left(\frac{2\sqrt{10-2\sqrt{5}}}{4-2(1+\sqrt{5})} \right)}{5\sqrt{2}}$$

Alternative representations:

$$\begin{aligned} & \frac{1}{10} \sqrt{10-2\sqrt{5}} \tan^{-1} \left(\frac{2\sqrt{10-2\sqrt{5}}}{4-2(\sqrt{5}+1)} \right) + \frac{1}{10} \tan^{-1} \left(\frac{2\sqrt{10+2\sqrt{5}}}{4+2(\sqrt{5}-1)} \right) \sqrt{10+2\sqrt{5}} = \\ & \frac{1}{10} \operatorname{sc}^{-1} \left(\frac{2\sqrt{10-2\sqrt{5}}}{4-2(1+\sqrt{5})} \mid 0 \right) \sqrt{10-2\sqrt{5}} + \\ & \frac{1}{10} \operatorname{sc}^{-1} \left(\frac{2\sqrt{10+2\sqrt{5}}}{4+2(-1+\sqrt{5})} \mid 0 \right) \sqrt{10+2\sqrt{5}} \end{aligned}$$

$$\begin{aligned} & \frac{1}{10} \sqrt{10-2\sqrt{5}} \tan^{-1} \left(\frac{2\sqrt{10-2\sqrt{5}}}{4-2(\sqrt{5}+1)} \right) + \frac{1}{10} \tan^{-1} \left(\frac{2\sqrt{10+2\sqrt{5}}}{4+2(\sqrt{5}-1)} \right) \sqrt{10+2\sqrt{5}} = \\ & \frac{1}{10} \tan^{-1} \left(1, \frac{2\sqrt{10-2\sqrt{5}}}{4-2(1+\sqrt{5})} \right) \sqrt{10-2\sqrt{5}} + \\ & \frac{1}{10} \tan^{-1} \left(1, \frac{2\sqrt{10+2\sqrt{5}}}{4+2(-1+\sqrt{5})} \right) \sqrt{10+2\sqrt{5}} \end{aligned}$$

$$\begin{aligned} & \frac{1}{10} \sqrt{10-2\sqrt{5}} \tan^{-1} \left(\frac{2\sqrt{10-2\sqrt{5}}}{4-2(\sqrt{5}+1)} \right) + \frac{1}{10} \tan^{-1} \left(\frac{2\sqrt{10+2\sqrt{5}}}{4+2(\sqrt{5}-1)} \right) \sqrt{10+2\sqrt{5}} = \\ & \frac{1}{10} i \tanh^{-1} \left(-\frac{2i\sqrt{10-2\sqrt{5}}}{4-2(1+\sqrt{5})} \right) \sqrt{10-2\sqrt{5}} + \\ & \frac{1}{10} i \tanh^{-1} \left(-\frac{2i\sqrt{10+2\sqrt{5}}}{4+2(-1+\sqrt{5})} \right) \sqrt{10+2\sqrt{5}} \end{aligned}$$

$$(0.1562756303129776) + 1/10 \operatorname{sqrt}(((10-2\operatorname{sqrt}(5)))) \tan^{-1} [(2\operatorname{sqrt}(10-2\operatorname{sqrt}(5)))/((4-2(\operatorname{sqrt}(5)+1)))] + \operatorname{sqrt}((10+2\operatorname{sqrt}(5)))/10 \tan^{-1} [(2\operatorname{sqrt}(10+2\operatorname{sqrt}(5)))/((4+2(\operatorname{sqrt}(5)-1)))]$$

Input interpretation:

$$0.1562756303129776 + \frac{1}{10} \sqrt{10 - 2\sqrt{5}} \tan^{-1} \left(\frac{2\sqrt{10 - 2\sqrt{5}}}{4 - 2(\sqrt{5} + 1)} \right) +$$

$$\left(\frac{1}{10} \sqrt{10 + 2\sqrt{5}} \right) \tan^{-1} \left(\frac{2\sqrt{10 + 2\sqrt{5}}}{4 + 2(\sqrt{5} - 1)} \right)$$

Result:

0.2301764608269284...

(result in radians)

0.23017646...

Alternative representations:

$$0.15627563031297760000 + \frac{1}{10} \sqrt{10 - 2\sqrt{5}} \tan^{-1} \left(\frac{2\sqrt{10 - 2\sqrt{5}}}{4 - 2(\sqrt{5} + 1)} \right) +$$

$$\frac{1}{10} \tan^{-1} \left(\frac{2\sqrt{10 + 2\sqrt{5}}}{4 + 2(\sqrt{5} - 1)} \right) \sqrt{10 + 2\sqrt{5}} =$$

$$0.15627563031297760000 + \frac{1}{10} \operatorname{sc}^{-1} \left(\frac{2\sqrt{10 - 2\sqrt{5}}}{4 - 2(1 + \sqrt{5})} \mid 0 \right) \sqrt{10 - 2\sqrt{5}} +$$

$$\frac{1}{10} \operatorname{sc}^{-1} \left(\frac{2\sqrt{10 + 2\sqrt{5}}}{4 + 2(-1 + \sqrt{5})} \mid 0 \right) \sqrt{10 + 2\sqrt{5}}$$

$$0.15627563031297760000 + \frac{1}{10} \sqrt{10 - 2\sqrt{5}} \tan^{-1} \left(\frac{2\sqrt{10 - 2\sqrt{5}}}{4 - 2(\sqrt{5} + 1)} \right) +$$

$$\frac{1}{10} \tan^{-1} \left(\frac{2\sqrt{10 + 2\sqrt{5}}}{4 + 2(\sqrt{5} - 1)} \right) \sqrt{10 + 2\sqrt{5}} =$$

$$0.15627563031297760000 + \frac{1}{10} \tan^{-1} \left(1, \frac{2\sqrt{10 - 2\sqrt{5}}}{4 - 2(1 + \sqrt{5})} \right) \sqrt{10 - 2\sqrt{5}} +$$

$$\frac{1}{10} \tan^{-1} \left(1, \frac{2\sqrt{10 + 2\sqrt{5}}}{4 + 2(-1 + \sqrt{5})} \right) \sqrt{10 + 2\sqrt{5}}$$

$$\begin{aligned}
& 0.15627563031297760000 + \frac{1}{10} \sqrt{10 - 2\sqrt{5}} \tan^{-1} \left(\frac{2\sqrt{10 - 2\sqrt{5}}}{4 - 2(\sqrt{5} + 1)} \right) + \\
& \frac{1}{10} \tan^{-1} \left(\frac{2\sqrt{10 + 2\sqrt{5}}}{4 + 2(\sqrt{5} - 1)} \right) \sqrt{10 + 2\sqrt{5}} = \\
& 0.15627563031297760000 + \frac{1}{10} i \tanh^{-1} \left(-\frac{2i\sqrt{10 - 2\sqrt{5}}}{4 - 2(1 + \sqrt{5})} \right) \sqrt{10 - 2\sqrt{5}} + \\
& \frac{1}{10} i \tanh^{-1} \left(-\frac{2i\sqrt{10 + 2\sqrt{5}}}{4 + 2(-1 + \sqrt{5})} \right) \sqrt{10 + 2\sqrt{5}}
\end{aligned}$$

Series representations:

$$\begin{aligned}
& 0.15627563031297760000 + \frac{1}{10} \sqrt{10 - 2\sqrt{5}} \tan^{-1} \left(\frac{2\sqrt{10 - 2\sqrt{5}}}{4 - 2(\sqrt{5} + 1)} \right) + \\
& \frac{1}{10} \tan^{-1} \left(\frac{2\sqrt{10 + 2\sqrt{5}}}{4 + 2(\sqrt{5} - 1)} \right) \sqrt{10 + 2\sqrt{5}} = \\
& 0.20000000000000000000 \left(0.7813781515648880000 + \right. \\
& 1.00000000000000000000 \exp \left(i\pi \left[\frac{\arg(10 - x - 2\sqrt{5})}{2\pi} \right] \right) \sqrt{x} \\
& \sum_{k_1=0}^{\infty} \sum_{k_2=0}^{\infty} \frac{1}{k_2!(1+2k_1)} (-1)^{k_1+k_2} x^{-k_2} T_{1+2k_1} \left(\frac{\sqrt{10 - 2\sqrt{5}}}{4 - 2(1 + \sqrt{5})} \right) \\
& {}_2F_1 \left(\frac{1}{2} + k_1, 1 + k_1; 2 + 2k_1; -4 \right) \left(-\frac{1}{2} \right)_{k_2} (10 - x - 2\sqrt{5})^{k_2} + \\
& 1.00000000000000000000 \exp \left(i\pi \left[\frac{\arg(10 - x + 2\sqrt{5})}{2\pi} \right] \right) \sqrt{x} \\
& \sum_{k_1=0}^{\infty} \sum_{k_2=0}^{\infty} \frac{1}{k_2!(1+2k_1)} (-1)^{k_1+k_2} x^{-k_2} T_{1+2k_1} \left(\frac{\sqrt{10 + 2\sqrt{5}}}{4 + 2(-1 + \sqrt{5})} \right) \\
& {}_2F_1 \left(\frac{1}{2} + k_1, 1 + k_1; 2 + 2k_1; -4 \right) \left(-\frac{1}{2} \right)_{k_2} \\
& \left. (10 - x + 2\sqrt{5})^{k_2} \right) \text{ for } (x \in \mathbb{R} \text{ and } x < 0)
\end{aligned}$$

$$\begin{aligned}
& 0.15627563031297760000 + \frac{1}{10} \sqrt{10 - 2\sqrt{5}} \tan^{-1} \left(\frac{2\sqrt{10 - 2\sqrt{5}}}{4 - 2(\sqrt{5} + 1)} \right) + \\
& \frac{1}{10} \tan^{-1} \left(\frac{2\sqrt{10 + 2\sqrt{5}}}{4 + 2(\sqrt{5} - 1)} \right) \sqrt{10 + 2\sqrt{5}} = \\
& \left. \begin{aligned}
& 0.10000000000000000000 \\
& 1.5627563031297760000 + 1.0000000000000000000
\end{aligned} \right\} \\
& \sqrt{9 - 2\sqrt{5}} \sum_{k_1=0}^{\infty} \sum_{k_2=0}^{\infty} \frac{1}{1+2k_2} \left(-\frac{1}{5}\right)^{k_2} 4^{1+2k_2} \binom{\frac{1}{2}}{k_1} F_{1+2k_2} \\
& \left((9 - 2\sqrt{5})^{-k_1} \left(\frac{\sqrt{10 - 2\sqrt{5}}}{(4 - 2(1 + \sqrt{5})) \left(1 + \sqrt{1 + \frac{16\sqrt{10 - 2\sqrt{5}}^2}{5(4 - 2(1 + \sqrt{5}))^2}} \right)} \right) \right)^{1+2k_2} + \\
& 1.00000000000000000000 \sqrt{9 + 2\sqrt{5}} \\
& \sum_{k_1=0}^{\infty} \sum_{k_2=0}^{\infty} \frac{1}{1+2k_2} \left(-\frac{1}{5}\right)^{k_2} 4^{1+2k_2} \binom{\frac{1}{2}}{k_1} F_{1+2k_2} (9 + 2\sqrt{5})^{-k_1} \\
& \left(\frac{\sqrt{10 + 2\sqrt{5}}}{(4 + 2(-1 + \sqrt{5})) \left(1 + \sqrt{1 + \frac{16\sqrt{10 + 2\sqrt{5}}^2}{5(4 + 2(-1 + \sqrt{5}))^2}} \right)} \right)^{1+2k_2}
\end{aligned}$$

$$0.15627563031297760000 + \frac{1}{10} \sqrt{10 - 2\sqrt{5}} \tan^{-1} \left(\frac{2\sqrt{10 - 2\sqrt{5}}}{4 - 2(\sqrt{5} + 1)} \right) +$$

$$\frac{1}{10} \tan^{-1} \left(\frac{2\sqrt{10 + 2\sqrt{5}}}{4 + 2(\sqrt{5} - 1)} \right) \sqrt{10 + 2\sqrt{5}} = 0.10000000000000000000$$

$$\left(1.5627563031297760000 + 1.00000000000000000000 \right)$$

$$\exp \left(i\pi \left[\frac{\arg(10 - x - 2\sqrt{5})}{2\pi} \right] \right) \sqrt{x} \sum_{k_1=0}^{\infty} \sum_{k_2=0}^{\infty} \frac{1}{k_1!(1+2k_2)} (-1)^{k_1+k_2}$$

$$4^{1+2k_2} \times 5^{-k_2} x^{-k_1} F_{1+2k_2} \left(-\frac{1}{2} \right)_{k_1} (10 - x - 2\sqrt{5})^{k_1}$$

$$\left(\frac{\sqrt{10 - 2\sqrt{5}}}{(4 - 2(1 + \sqrt{5})) \left(1 + \sqrt{1 + \frac{16\sqrt{10 - 2\sqrt{5}}^2}{5(4 - 2(1 + \sqrt{5}))^2}} \right)} \right)^{1+2k_2} +$$

$$1.00000000000000000000 \exp \left(i\pi \left[\frac{\arg(10 - x + 2\sqrt{5})}{2\pi} \right] \right) \sqrt{x}$$

$$\sum_{k_1=0}^{\infty} \sum_{k_2=0}^{\infty} \frac{1}{k_1!(1+2k_2)} (-1)^{k_1+k_2} 4^{1+2k_2} \times 5^{-k_2}$$

$$x^{-k_1} F_{1+2k_2} \left(-\frac{1}{2} \right)_{k_1} (10 - x + 2\sqrt{5})^{k_1}$$

$$\left(\frac{\sqrt{10 + 2\sqrt{5}}}{(4 + 2(-1 + \sqrt{5})) \left(1 + \sqrt{1 + \frac{16\sqrt{10 + 2\sqrt{5}}^2}{5(4 + 2(-1 + \sqrt{5}))^2}} \right)} \right)^{1+2k_2}$$

for $(x \in \mathbb{R} \text{ and } x < 0)$

Integral representations:

$$0.15627563031297760000 + \frac{1}{10} \sqrt{10 - 2\sqrt{5}} \tan^{-1} \left(\frac{2\sqrt{10 - 2\sqrt{5}}}{4 - 2(\sqrt{5} + 1)} \right) +$$

$$\frac{1}{10} \tan^{-1} \left(\frac{2\sqrt{10 + 2\sqrt{5}}}{4 + 2(\sqrt{5} - 1)} \right) \sqrt{10 + 2\sqrt{5}} =$$

$$0.15627563031297760000 + \int_0^1 \left(\left((-1 + \sqrt{5})^2 (1 + \sqrt{5}) \sqrt{2(5 + \sqrt{5})} \right)^2 + \sqrt{10 - 2\sqrt{5}}^2 \left(-(-1 + \sqrt{5})(1 + \sqrt{5})^2 + 2t^2 \sqrt{2(5 + \sqrt{5})} \right) \right) / \left(10 \left((-1 + \sqrt{5})^2 + t^2 \sqrt{10 - 2\sqrt{5}}^2 \right) \left((1 + \sqrt{5})^2 + t^2 \sqrt{2(5 + \sqrt{5})}^2 \right) \right) dt$$

$$0.15627563031297760000 + \frac{1}{10} \sqrt{10 - 2\sqrt{5}} \tan^{-1} \left(\frac{2\sqrt{10 - 2\sqrt{5}}}{4 - 2(\sqrt{5} + 1)} \right) +$$

$$\frac{1}{10} \tan^{-1} \left(\frac{2\sqrt{10 + 2\sqrt{5}}}{4 + 2(\sqrt{5} - 1)} \right) \sqrt{10 + 2\sqrt{5}} = 0.15627563031297760000 +$$

$$\int_{-i\infty+\gamma}^{i\infty+\gamma} \left(\frac{i\Gamma\left(\frac{1}{2} - s\right)\Gamma(1-s)\Gamma(s)^2 \sqrt{10 - 2\sqrt{5}}^2 \left(1 + \frac{4\sqrt{10 - 2\sqrt{5}}}{(4 - 2(1 + \sqrt{5}))^2} \right)^{-s}}{20\pi^{3/2} (4 - 2(1 + \sqrt{5}))} - \frac{i\Gamma\left(\frac{1}{2} - s\right)\Gamma(1-s)\Gamma(s)^2 \sqrt{10 + 2\sqrt{5}}^2 \left(1 + \frac{4\sqrt{10 + 2\sqrt{5}}}{(4 + 2(-1 + \sqrt{5}))^2} \right)^{-s}}{20\pi^{3/2} (4 + 2(-1 + \sqrt{5}))} \right)$$

$$ds \text{ for } 0 < \gamma < \frac{1}{2}$$

$$\begin{aligned}
& 0.15627563031297760000 + \frac{1}{10} \sqrt{10 - 2\sqrt{5}} \tan^{-1} \left(\frac{2\sqrt{10 - 2\sqrt{5}}}{4 - 2(\sqrt{5} + 1)} \right) + \\
& \frac{1}{10} \tan^{-1} \left(\frac{2\sqrt{10 + 2\sqrt{5}}}{4 + 2(\sqrt{5} - 1)} \right) \sqrt{10 + 2\sqrt{5}} = 0.15627563031297760000 + \\
& \int_{-i\infty+\gamma}^{i\infty+\gamma} \left(\frac{4^{-1-s} \Gamma\left(\frac{1}{2} - s\right) \Gamma(1-s) \Gamma(s) \sqrt{10 - 2\sqrt{5}}^2 \left(\frac{\sqrt{10 - 2\sqrt{5}}}{(4 - 2(1 + \sqrt{5}))} \right)^{-s}}{5i\pi \Gamma\left(\frac{3}{2} - s\right) (4 - 2(1 + \sqrt{5}))} + \right. \\
& \left. \frac{4^{-1-s} \Gamma\left(\frac{1}{2} - s\right) \Gamma(1-s) \Gamma(s) \sqrt{10 + 2\sqrt{5}}^2 \left(\frac{\sqrt{10 + 2\sqrt{5}}}{(4 + 2(-1 + \sqrt{5}))} \right)^{-s}}{5i\pi \Gamma\left(\frac{3}{2} - s\right) (4 + 2(-1 + \sqrt{5}))} \right) \\
& ds \text{ for } 0 < \gamma < \frac{1}{2}
\end{aligned}$$

(((0.1562756303129776) + 1/10 sqrt(((10-2sqrt(5)))) tan^-1 [(2sqrt(10-2sqrt(5)))/((4-2(sqrt(5)+1)))] + sqrt((10+2sqrt(5)))/10 tan^-1 [(2sqrt(10+2sqrt(5)))/((4+2(sqrt(5)-1)))])))*7

Input interpretation:

$$\left(0.1562756303129776 + \frac{1}{10} \sqrt{10 - 2\sqrt{5}} \tan^{-1} \left(\frac{2\sqrt{10 - 2\sqrt{5}}}{4 - 2(\sqrt{5} + 1)} \right) + \left(\frac{1}{10} \sqrt{10 + 2\sqrt{5}} \right) \tan^{-1} \left(\frac{2\sqrt{10 + 2\sqrt{5}}}{4 + 2(\sqrt{5} - 1)} \right) \right) \times 7$$

$\tan^{-1}(x)$ is the inverse tangent function

Result:

1.611235225788499...

(result in radians)

1.61123522....

This result is an approximation to the value of the golden ratio 1,618033988749...

Alternative representations:

$$\begin{aligned}
 & \left(0.15627563031297760000 + \frac{1}{10} \sqrt{10 - 2\sqrt{5}} \tan^{-1} \left(\frac{2\sqrt{10 - 2\sqrt{5}}}{4 - 2(\sqrt{5} + 1)} \right) + \right. \\
 & \quad \left. \frac{1}{10} \tan^{-1} \left(\frac{2\sqrt{10 + 2\sqrt{5}}}{4 + 2(\sqrt{5} - 1)} \right) \sqrt{10 + 2\sqrt{5}} \right) 7 = \\
 & 7 \left(0.15627563031297760000 + \frac{1}{10} \operatorname{sc}^{-1} \left(\frac{2\sqrt{10 - 2\sqrt{5}}}{4 - 2(1 + \sqrt{5})} \middle| 0 \right) \sqrt{10 - 2\sqrt{5}} + \right. \\
 & \quad \left. \frac{1}{10} \operatorname{sc}^{-1} \left(\frac{2\sqrt{10 + 2\sqrt{5}}}{4 + 2(-1 + \sqrt{5})} \middle| 0 \right) \sqrt{10 + 2\sqrt{5}} \right) \\
 & \left(0.15627563031297760000 + \frac{1}{10} \sqrt{10 - 2\sqrt{5}} \tan^{-1} \left(\frac{2\sqrt{10 - 2\sqrt{5}}}{4 - 2(\sqrt{5} + 1)} \right) + \right. \\
 & \quad \left. \frac{1}{10} \tan^{-1} \left(\frac{2\sqrt{10 + 2\sqrt{5}}}{4 + 2(\sqrt{5} - 1)} \right) \sqrt{10 + 2\sqrt{5}} \right) 7 = \\
 & 7 \left(0.15627563031297760000 + \frac{1}{10} \tan^{-1} \left(1, \frac{2\sqrt{10 - 2\sqrt{5}}}{4 - 2(1 + \sqrt{5})} \right) \sqrt{10 - 2\sqrt{5}} + \right. \\
 & \quad \left. \frac{1}{10} \tan^{-1} \left(1, \frac{2\sqrt{10 + 2\sqrt{5}}}{4 + 2(-1 + \sqrt{5})} \right) \sqrt{10 + 2\sqrt{5}} \right) \\
 & \left(0.15627563031297760000 + \frac{1}{10} \sqrt{10 - 2\sqrt{5}} \tan^{-1} \left(\frac{2\sqrt{10 - 2\sqrt{5}}}{4 - 2(\sqrt{5} + 1)} \right) + \right. \\
 & \quad \left. \frac{1}{10} \tan^{-1} \left(\frac{2\sqrt{10 + 2\sqrt{5}}}{4 + 2(\sqrt{5} - 1)} \right) \sqrt{10 + 2\sqrt{5}} \right) 7 = \\
 & 7 \left(0.15627563031297760000 + \frac{1}{10} i \tanh^{-1} \left(-\frac{2i\sqrt{10 - 2\sqrt{5}}}{4 - 2(1 + \sqrt{5})} \right) \sqrt{10 - 2\sqrt{5}} + \right. \\
 & \quad \left. \frac{1}{10} i \tanh^{-1} \left(-\frac{2i\sqrt{10 + 2\sqrt{5}}}{4 + 2(-1 + \sqrt{5})} \right) \sqrt{10 + 2\sqrt{5}} \right)
 \end{aligned}$$

Series representations:

$$\begin{aligned}
 & \left(0.15627563031297760000 + \frac{1}{10} \sqrt{10 - 2\sqrt{5}} \tan^{-1} \left(\frac{2\sqrt{10 - 2\sqrt{5}}}{4 - 2(\sqrt{5} + 1)} \right) + \right. \\
 & \quad \left. \frac{1}{10} \tan^{-1} \left(\frac{2\sqrt{10 + 2\sqrt{5}}}{4 + 2(\sqrt{5} - 1)} \right) \sqrt{10 + 2\sqrt{5}} \right) 7 = \\
 & 1.40000000000000000000 \left(0.7813781515648880000 + \right. \\
 & \quad 1.00000000000000000000 \exp \left(i\pi \left[\frac{\arg(10 - x - 2\sqrt{5})}{2\pi} \right] \right) \sqrt{x} \\
 & \quad \sum_{k_1=0}^{\infty} \sum_{k_2=0}^{\infty} \frac{1}{k_2!(1+2k_1)} (-1)^{k_1+k_2} x^{-k_2} T_{1+2k_1} \left(\frac{\sqrt{10 - 2\sqrt{5}}}{4 - 2(1 + \sqrt{5})} \right) \\
 & \quad \quad {}_2F_1 \left(\frac{1}{2} + k_1, 1 + k_1; 2 + 2k_1; -4 \right) \left(-\frac{1}{2} \right)_{k_2} (10 - x - 2\sqrt{5})^{k_2} + \\
 & 1.00000000000000000000 \exp \left(i\pi \left[\frac{\arg(10 - x + 2\sqrt{5})}{2\pi} \right] \right) \sqrt{x} \\
 & \quad \sum_{k_1=0}^{\infty} \sum_{k_2=0}^{\infty} \frac{1}{k_2!(1+2k_1)} (-1)^{k_1+k_2} x^{-k_2} T_{1+2k_1} \left(\frac{\sqrt{10 + 2\sqrt{5}}}{4 + 2(-1 + \sqrt{5})} \right) \\
 & \quad \quad {}_2F_1 \left(\frac{1}{2} + k_1, 1 + k_1; 2 + 2k_1; -4 \right) \left(-\frac{1}{2} \right)_{k_2} \\
 & \quad \quad \left. (10 - x + 2\sqrt{5})^{k_2} \right) \text{ for } (x \in \mathbb{R} \text{ and } x < 0)
 \end{aligned}$$

$$\begin{aligned}
& \left(0.15627563031297760000 + \frac{1}{10} \sqrt{10 - 2\sqrt{5}} \tan^{-1} \left(\frac{2\sqrt{10 - 2\sqrt{5}}}{4 - 2(\sqrt{5} + 1)} \right) + \right. \\
& \quad \left. \frac{1}{10} \tan^{-1} \left(\frac{2\sqrt{10 + 2\sqrt{5}}}{4 + 2(\sqrt{5} - 1)} \right) \sqrt{10 + 2\sqrt{5}} \right) 7 = \\
& \quad \left(\begin{array}{l} 0.70000000000000000000 \\ 1.5627563031297760000 + 1.00000000000000000000 \end{array} \right) \\
& \quad \sqrt{9 - 2\sqrt{5}} \sum_{k_1=0}^{\infty} \sum_{k_2=0}^{\infty} \frac{1}{1 + 2k_2} \left(-\frac{1}{5}\right)^{k_2} 4^{1+2k_2} \binom{\frac{1}{2}}{k_1} F_{1+2k_2} \\
& \quad \left((9 - 2\sqrt{5})^{-k_1} \frac{\sqrt{10 - 2\sqrt{5}}}{(4 - 2(1 + \sqrt{5})) \left(1 + \sqrt{1 + \frac{16\sqrt{10 - 2\sqrt{5}}^2}{5(4 - 2(1 + \sqrt{5}))^2}} \right)} \right)^{1+2k_2} + \\
& \quad 1.00000000000000000000 \sqrt{9 + 2\sqrt{5}} \sum_{k_1=0}^{\infty} \sum_{k_2=0}^{\infty} \frac{1}{1 + 2k_2} \left(-\frac{1}{5}\right)^{k_2} 4^{1+2k_2} \binom{\frac{1}{2}}{k_1} F_{1+2k_2} (9 + 2\sqrt{5})^{-k_1} \\
& \quad \left(\frac{\sqrt{10 + 2\sqrt{5}}}{(4 + 2(-1 + \sqrt{5})) \left(1 + \sqrt{1 + \frac{16\sqrt{10 + 2\sqrt{5}}^2}{5(4 + 2(-1 + \sqrt{5}))^2}} \right)} \right)^{1+2k_2}
\end{aligned}$$

$$\begin{aligned}
& \left(0.15627563031297760000 + \frac{1}{10} \sqrt{10 - 2\sqrt{5}} \tan^{-1} \left(\frac{2\sqrt{10 - 2\sqrt{5}}}{4 - 2(\sqrt{5} + 1)} \right) + \right. \\
& \quad \left. \frac{1}{10} \tan^{-1} \left(\frac{2\sqrt{10 + 2\sqrt{5}}}{4 + 2(\sqrt{5} - 1)} \right) \sqrt{10 + 2\sqrt{5}} \right) 7 = \\
& \quad \left(0.70000000000000000000 \left(1.5627563031297760000 + \right. \right. \\
& \quad \left. \left. 1.00000000000000000000 \exp \left(i \pi \left[\frac{\arg(10 - x - 2\sqrt{5})}{2\pi} \right] \right) \right. \right. \\
& \quad \left. \sqrt{x} \sum_{k_1=0}^{\infty} \sum_{k_2=0}^{\infty} \frac{1}{k_1! (1 + 2k_2)} (-1)^{k_1 + k_2} 4^{1+2k_2} \times \right. \\
& \quad \quad \left. 5^{-k_2} x^{-k_1} F_{1+2k_2} \left(-\frac{1}{2} \right)_{k_1} (10 - x - 2\sqrt{5})^{k_1} \right. \\
& \quad \quad \left. \left(\frac{\sqrt{10 - 2\sqrt{5}}}{(4 - 2(1 + \sqrt{5})) \left(1 + \sqrt{1 + \frac{16\sqrt{10 - 2\sqrt{5}}^2}{5(4 - 2(1 + \sqrt{5}))^2}} \right)} \right)^{1+2k_2} \right) + \\
& \quad \left. 1.00000000000000000000 \exp \left(i \pi \left[\frac{\arg(10 - x + 2\sqrt{5})}{2\pi} \right] \right) \sqrt{x} \right. \\
& \quad \left. \sum_{k_1=0}^{\infty} \sum_{k_2=0}^{\infty} \frac{1}{k_1! (1 + 2k_2)} (-1)^{k_1 + k_2} 4^{1+2k_2} \times 5^{-k_2} \right. \\
& \quad \quad \left. x^{-k_1} F_{1+2k_2} \left(-\frac{1}{2} \right)_{k_1} (10 - x + 2\sqrt{5})^{k_1} \right. \\
& \quad \quad \left. \left(\frac{\sqrt{10 + 2\sqrt{5}}}{(4 + 2(-1 + \sqrt{5})) \left(1 + \sqrt{1 + \frac{16\sqrt{10 + 2\sqrt{5}}^2}{5(4 + 2(-1 + \sqrt{5}))^2}} \right)} \right)^{1+2k_2} \right) \Bigg)
\end{aligned}$$

for $(x \in \mathbb{R} \text{ and } x < 0)$

Integral representations:

$$\begin{aligned}
 & \left(0.15627563031297760000 + \frac{1}{10} \sqrt{10 - 2\sqrt{5}} \tan^{-1} \left(\frac{2\sqrt{10 - 2\sqrt{5}}}{4 - 2(\sqrt{5} + 1)} \right) + \right. \\
 & \quad \left. \frac{1}{10} \tan^{-1} \left(\frac{2\sqrt{10 + 2\sqrt{5}}}{4 + 2(\sqrt{5} - 1)} \right) \sqrt{10 + 2\sqrt{5}} \right) 7 = \\
 & 1.0939294121908432000 + \int_0^1 \left(\left(7 \left((-1 + \sqrt{5})^2 (1 + \sqrt{5}) \sqrt{2(5 + \sqrt{5})^2} + \right. \right. \right. \\
 & \quad \left. \left. \left. \sqrt{10 - 2\sqrt{5}}^2 \left(-(-1 + \sqrt{5})(1 + \sqrt{5})^2 + 2t^2 \sqrt{2(5 + \sqrt{5})^2} \right) \right) \right) / \right. \\
 & \quad \left. \left(10 \left((-1 + \sqrt{5})^2 + t^2 \sqrt{10 - 2\sqrt{5}}^2 \right) \left((1 + \sqrt{5})^2 + t^2 \sqrt{2(5 + \sqrt{5})^2} \right) \right) \right) dt \\
 & \left(0.15627563031297760000 + \frac{1}{10} \sqrt{10 - 2\sqrt{5}} \tan^{-1} \left(\frac{2\sqrt{10 - 2\sqrt{5}}}{4 - 2(\sqrt{5} + 1)} \right) + \right. \\
 & \quad \left. \frac{1}{10} \tan^{-1} \left(\frac{2\sqrt{10 + 2\sqrt{5}}}{4 + 2(\sqrt{5} - 1)} \right) \sqrt{10 + 2\sqrt{5}} \right) 7 = 1.0939294121908432000 + \\
 & \int_{-i\infty+\gamma}^{i\infty+\gamma} \left(- \frac{7i\Gamma\left(\frac{1}{2} - s\right)\Gamma(1-s)\Gamma(s)^2 \sqrt{10 - 2\sqrt{5}}^2 \left(1 + \frac{4\sqrt{10 - 2\sqrt{5}}^2}{(4 - 2(1 + \sqrt{5}))^2} \right)^{-s}}{20\pi^{3/2}(4 - 2(1 + \sqrt{5}))} - \right. \\
 & \quad \left. \frac{7i\Gamma\left(\frac{1}{2} - s\right)\Gamma(1-s)\Gamma(s)^2 \sqrt{10 + 2\sqrt{5}}^2 \left(1 + \frac{4\sqrt{10 + 2\sqrt{5}}^2}{(4 + 2(-1 + \sqrt{5}))^2} \right)^{-s}}{20\pi^{3/2}(4 + 2(-1 + \sqrt{5}))} \right) ds \\
 & \text{for } 0 < \gamma < \frac{1}{2}
 \end{aligned}$$

$$\left(0.15627563031297760000 + \frac{1}{10} \sqrt{10 - 2\sqrt{5}} \tan^{-1} \left(\frac{2\sqrt{10 - 2\sqrt{5}}}{4 - 2(\sqrt{5} + 1)} \right) + \right. \\ \left. \frac{1}{10} \tan^{-1} \left(\frac{2\sqrt{10 + 2\sqrt{5}}}{4 + 2(\sqrt{5} - 1)} \right) \sqrt{10 + 2\sqrt{5}} \right) 7 = 1.0939294121908432000 + \\ \int_{-i\infty+\gamma}^{i\infty+\gamma} \left(\frac{7 \times 4^{-1-s} \Gamma\left(\frac{1}{2} - s\right) \Gamma(1-s) \Gamma(s) \sqrt{10 - 2\sqrt{5}}^2 \left(\frac{\sqrt{10 - 2\sqrt{5}}}{(4 - 2(1 + \sqrt{5}))} \right)^{-s}}{5 i \pi \Gamma\left(\frac{3}{2} - s\right) (4 - 2(1 + \sqrt{5}))} + \right. \\ \left. \frac{7 \times 4^{-1-s} \Gamma\left(\frac{1}{2} - s\right) \Gamma(1-s) \Gamma(s) \sqrt{10 + 2\sqrt{5}}^2 \left(\frac{\sqrt{10 + 2\sqrt{5}}}{(4 + 2(-1 + \sqrt{5}))} \right)^{-s}}{5 i \pi \Gamma\left(\frac{3}{2} - s\right) (4 + 2(-1 + \sqrt{5}))} \right) ds \text{ for } 0 < \gamma < \frac{1}{2}$$

$$(((0.2301764608269284 * 2\pi)^8))^{1/6}$$

Input interpretation:

$$\sqrt[6]{(0.2301764608269284 \times 2\pi)^8}$$

Result:

1.635514386305617...

$$1.63551438\dots \approx \zeta(2) = \frac{\pi^2}{6} = 1.644934 \dots$$

$$(5 * 1/\pi^3) (((\text{colog}(0.2301764608269284))))^6$$

Input interpretation:

$$\left(5 \times \frac{1}{\pi^3} \right) (-\log(0.2301764608269284))^6$$

$\log(x)$ is the natural logarithm

Result:

1.61990599812454...

1.619905998...

This result is a good approximation to the value of the golden ratio
1,618033988749...

Alternative representations:

$$\frac{(-\log(0.23017646082692840000))^6 5}{\pi^3} = \frac{5 (-\log_e(0.23017646082692840000))^6}{\pi^3}$$

$$\frac{(-\log(0.23017646082692840000))^6 5}{\pi^3} = \frac{5 \operatorname{Li}_1(0.76982353917307160000)^6}{\pi^3}$$

$$\frac{(-\log(0.23017646082692840000))^6 5}{\pi^3} = \frac{5 (-\log(a) \log_a(0.23017646082692840000))^6}{\pi^3}$$

Series representations:

$$\frac{(-\log(0.23017646082692840000))^6 5}{\pi^3} = \frac{5 \left(\sum_{k=1}^{\infty} \frac{(-1)^k (-0.76982353917307160000)^k}{k} \right)^6}{\pi^3}$$

$$\frac{(-\log(0.23017646082692840000))^6 5}{\pi^3} = \frac{1}{\pi^3} 5 \left(2 i \pi \left[\frac{\arg(0.23017646082692840000 - x)}{2 \pi} \right] + \log(x) - \sum_{k=1}^{\infty} \frac{(-1)^k (0.23017646082692840000 - x)^k x^{-k}}{k} \right)^6 \text{ for } x < 0$$

$$\frac{(-\log(0.23017646082692840000))^6 5}{\pi^3} = \frac{1}{\pi^3} 5 \left(\log(z_0) + \left[\frac{\arg(0.23017646082692840000 - z_0)}{2 \pi} \right] \left(\log\left(\frac{1}{z_0}\right) + \log(z_0) \right) - \sum_{k=1}^{\infty} \frac{(-1)^k (0.23017646082692840000 - z_0)^k z_0^{-k}}{k} \right)^6$$

Integral representation:

$$\frac{(-\log(0.23017646082692840000))^6 5}{\pi^3} = \frac{5 \left(\int_1^{0.23017646082692840000} \frac{1}{t} dt \right)^6}{\pi^3}$$

$$\left(\left(\left(\left(0.1562756303129776 + \frac{1}{10} \sqrt{10 - 2\sqrt{5}} \right) \tan^{-1} \left[\frac{2\sqrt{10 - 2\sqrt{5}}}{4 - 2(\sqrt{5} + 1)} \right] + \sqrt{\frac{10 + 2\sqrt{5}}{10}} \tan^{-1} \left[\frac{2\sqrt{10 + 2\sqrt{5}}}{4 + 2(\sqrt{5} - 1)} \right] \right) \right)^{1/128}$$

Input interpretation:

$$\left(0.1562756303129776 + \frac{1}{10} \sqrt{10 - 2\sqrt{5}} \tan^{-1} \left(\frac{2\sqrt{10 - 2\sqrt{5}}}{4 - 2(\sqrt{5} + 1)} \right) + \left(\frac{1}{10} \sqrt{10 + 2\sqrt{5}} \right) \tan^{-1} \left(\frac{2\sqrt{10 + 2\sqrt{5}}}{4 + 2(\sqrt{5} - 1)} \right) \right)^{(1/128)}$$

$\tan^{-1}(x)$ is the inverse tangent function

Result:

0.988589744523409512...

(result in radians)

0.98858974.... result very near to the value of the following Rogers-Ramanujan continued fraction:

$$\frac{e^{-\frac{\pi}{\sqrt{5}}}}{\sqrt{5}} = 1 - \frac{e^{-\pi\sqrt{5}}}{1 + \frac{e^{-2\pi\sqrt{5}}}{1 + \frac{e^{-3\pi\sqrt{5}}}{1 + \frac{e^{-4\pi\sqrt{5}}}{1 + \dots}}}} \approx 0.9991104684$$

and to the dilaton value **0.989117352243 = ϕ**

$$\frac{(64 \cdot 34)}{\left(\left(\left(\left(0.1562756303129776 + \frac{1}{10} \sqrt{10 - 2\sqrt{5}} \right) \tan^{-1} \left[\frac{2\sqrt{10 - 2\sqrt{5}}}{4 - 2(\sqrt{5} + 1)} \right] + \sqrt{\frac{10 + 2\sqrt{5}}{10}} \tan^{-1} \left[\frac{2\sqrt{10 + 2\sqrt{5}}}{4 + 2(\sqrt{5} - 1)} \right] \right) \right)^{1/128}} - 55$$

Where 34 and 55 are Fibonacci numbers

Input interpretation:

$$(64 \times 34) / \left(0.1562756303129776 + \frac{1}{10} \sqrt{10 - 2\sqrt{5}} \tan^{-1} \left(\frac{2\sqrt{10 - 2\sqrt{5}}}{4 - 2(\sqrt{5} + 1)} \right) + \left(\frac{1}{10} \sqrt{10 + 2\sqrt{5}} \right) \tan^{-1} \left(\frac{2\sqrt{10 + 2\sqrt{5}}}{4 + 2(\sqrt{5} - 1)} \right) \right) - 55$$

$\tan^{-1}(x)$ is the inverse tangent function

Result:

9398.616552198847...

(result in radians)

9398.616552.... result practically equal to the rest mass of Botton eta meson 9398

Alternative representations:

$$(64 \times 34) / \left(0.15627563031297760000 + \frac{1}{10} \sqrt{10 - 2\sqrt{5}} \tan^{-1} \left(\frac{2\sqrt{10 - 2\sqrt{5}}}{4 - 2(\sqrt{5} + 1)} \right) + \frac{1}{10} \sqrt{10 + 2\sqrt{5}} \tan^{-1} \left(\frac{2\sqrt{10 + 2\sqrt{5}}}{4 + 2(\sqrt{5} - 1)} \right) \right) - 55 = -55 + 2176 / \left(0.15627563031297760000 + \frac{1}{10} \operatorname{sc}^{-1} \left(\frac{2\sqrt{10 - 2\sqrt{5}}}{4 - 2(1 + \sqrt{5})} \middle| 0 \right) \sqrt{10 - 2\sqrt{5}} + \frac{1}{10} \operatorname{sc}^{-1} \left(\frac{2\sqrt{10 + 2\sqrt{5}}}{4 + 2(-1 + \sqrt{5})} \middle| 0 \right) \sqrt{10 + 2\sqrt{5}} \right) (64 \times 34) / \left(0.15627563031297760000 + \frac{1}{10} \sqrt{10 - 2\sqrt{5}} \tan^{-1} \left(\frac{2\sqrt{10 - 2\sqrt{5}}}{4 - 2(\sqrt{5} + 1)} \right) + \frac{1}{10} \sqrt{10 + 2\sqrt{5}} \tan^{-1} \left(\frac{2\sqrt{10 + 2\sqrt{5}}}{4 + 2(\sqrt{5} - 1)} \right) \right) - 55 = -55 + 2176 / \left(0.15627563031297760000 + \frac{1}{10} \tan^{-1} \left(1, \frac{2\sqrt{10 - 2\sqrt{5}}}{4 - 2(1 + \sqrt{5})} \right) \sqrt{10 - 2\sqrt{5}} + \frac{1}{10} \tan^{-1} \left(1, \frac{2\sqrt{10 + 2\sqrt{5}}}{4 + 2(-1 + \sqrt{5})} \right) \sqrt{10 + 2\sqrt{5}} \right)$$

$$\begin{aligned}
(64 \times 34) / & \left(0.15627563031297760000 + \frac{1}{10} \sqrt{10 - 2\sqrt{5}} \tan^{-1} \left(\frac{2\sqrt{10 - 2\sqrt{5}}}{4 - 2(\sqrt{5} + 1)} \right) + \right. \\
& \left. \frac{1}{10} \sqrt{10 + 2\sqrt{5}} \tan^{-1} \left(\frac{2\sqrt{10 + 2\sqrt{5}}}{4 + 2(\sqrt{5} - 1)} \right) \right) - 55 = \\
-55 + 2176 / & \left(0.15627563031297760000 + \frac{1}{10} i \tanh^{-1} \left(-\frac{2i\sqrt{10 - 2\sqrt{5}}}{4 - 2(1 + \sqrt{5})} \right) \right. \\
& \left. \sqrt{10 - 2\sqrt{5}} + \frac{1}{10} i \tanh^{-1} \left(-\frac{2i\sqrt{10 + 2\sqrt{5}}}{4 + 2(-1 + \sqrt{5})} \right) \sqrt{10 + 2\sqrt{5}} \right)
\end{aligned}$$

Series representations:

$$(64 \times 34) / \left(0.15627563031297760000 + \frac{1}{10} \sqrt{10 - 2\sqrt{5}} \tan^{-1} \left(\frac{2\sqrt{10 - 2\sqrt{5}}}{4 - 2(\sqrt{5} + 1)} \right) + \frac{1}{10} \sqrt{10 + 2\sqrt{5}} \tan^{-1} \left(\frac{2\sqrt{10 + 2\sqrt{5}}}{4 + 2(\sqrt{5} - 1)} \right) \right) - 55 =$$

$$- \left(\left(\begin{aligned} & 55.000000000000000000 \\ & - 394.0736073332338604 + \end{aligned} \right) \right)$$

$$\begin{aligned} & 1.00000000000000000000 \sqrt{9 - 2\sqrt{5}} \\ & \sum_{k_1=0}^{\infty} \sum_{k_2=0}^{\infty} \frac{1}{1+2k_2} \left(-\frac{1}{5}\right)^{k_2} 4^{1+2k_2} \binom{\frac{1}{2}}{k_1} F_{1+2k_2} (9 - 2\sqrt{5})^{-k_1} \\ & \left(\frac{\sqrt{10 - 2\sqrt{5}}}{(4 - 2(1 + \sqrt{5})) \left(1 + \sqrt{1 + \frac{16\sqrt{10 - 2\sqrt{5}}^2}{5(4 - 2(1 + \sqrt{5}))^2}} \right)} \right)^{1+2k_2} + \end{aligned}$$

$$\begin{aligned} & 1.00000000000000000000 \sqrt{9 + 2\sqrt{5}} \\ & \sum_{k_1=0}^{\infty} \sum_{k_2=0}^{\infty} \frac{1}{1+2k_2} \left(-\frac{1}{5}\right)^{k_2} 4^{1+2k_2} \binom{\frac{1}{2}}{k_1} F_{1+2k_2} (9 + 2\sqrt{5})^{-k_1} \\ & \left(\frac{\sqrt{10 + 2\sqrt{5}}}{(4 + 2(-1 + \sqrt{5})) \left(1 + \sqrt{1 + \frac{16\sqrt{10 + 2\sqrt{5}}^2}{5(4 + 2(-1 + \sqrt{5}))^2}} \right)} \right)^{1+2k_2} \Bigg) / \end{aligned}$$

$$\left(1.5627563031297760000 + 1.00000000000000000000 \sqrt{9 - 2\sqrt{5}} \right)$$

$$\begin{aligned} & \sum_{k_1=0}^{\infty} \sum_{k_2=0}^{\infty} \frac{1}{1+2k_2} \left(-\frac{1}{5}\right)^{k_2} 4^{1+2k_2} \binom{\frac{1}{2}}{k_1} F_{1+2k_2} (9 - 2\sqrt{5})^{-k_1} \\ & \left(\frac{\sqrt{10 - 2\sqrt{5}}}{(4 - 2(1 + \sqrt{5})) \left(1 + \sqrt{1 + \frac{16\sqrt{10 - 2\sqrt{5}}^2}{5(4 - 2(1 + \sqrt{5}))^2}} \right)} \right)^{1+2k_2} + \end{aligned}$$

$$1.00000000000000000000 \sqrt{9 + 2\sqrt{5}}$$

$$\sum_{k_1=0}^{\infty} \sum_{k_2=0}^{\infty} \frac{1}{1+2k_2} \left(-\frac{1}{5}\right)^{k_2} 4^{1+2k_2} \binom{\frac{1}{2}}{k_1} F_{1+2k_2} (9 + 2\sqrt{5})^{-k_1}$$

$$\begin{aligned}
(64 \times 34) / & \left(0.15627563031297760000 + \frac{1}{10} \sqrt{10 - 2\sqrt{5}} \tan^{-1} \left(\frac{2\sqrt{10 - 2\sqrt{5}}}{4 - 2(\sqrt{5} + 1)} \right) + \right. \\
& \left. \frac{1}{10} \sqrt{10 + 2\sqrt{5}} \tan^{-1} \left(\frac{2\sqrt{10 + 2\sqrt{5}}}{4 + 2(\sqrt{5} - 1)} \right) \right) - 55 = \\
- & \left(\left(55.000000000000000000 \right) \left(-197.03680366661693018 + \right. \right. \\
& 1.00000000000000000000 \exp \left(i\pi \left[\frac{\arg(10 - x - 2\sqrt{5})}{2\pi} \right] \right) \sqrt{x} \\
& \sum_{k_1=0}^{\infty} \sum_{k_2=0}^{\infty} \frac{1}{k_2!(1+2k_1)} (-1)^{k_1+k_2} x^{-k_2} T_{1+2k_1} \left(\frac{\sqrt{10 - 2\sqrt{5}}}{4 - 2(1 + \sqrt{5})} \right) {}_2F_1 \\
& \left(\frac{1}{2} + k_1, 1 + k_1; 2 + 2k_1; -4 \right) \left(-\frac{1}{2} \right)_{k_2} (10 - x - 2\sqrt{5})^{k_2} + \\
& 1.00000000000000000000 \exp \left(i\pi \left[\frac{\arg(10 - x + 2\sqrt{5})}{2\pi} \right] \right) \\
& \sqrt{x} \sum_{k_1=0}^{\infty} \sum_{k_2=0}^{\infty} \frac{1}{k_2!(1+2k_1)} (-1)^{k_1+k_2} x^{-k_2} \\
& T_{1+2k_1} \left(\frac{\sqrt{10 + 2\sqrt{5}}}{4 + 2(-1 + \sqrt{5})} \right) {}_2F_1 \left(\frac{1}{2} + k_1, 1 + k_1; 2 + 2k_1; -4 \right) \\
& \left. \left(-\frac{1}{2} \right)_{k_2} (10 - x + 2\sqrt{5})^{k_2} \right) / \\
& \left(0.7813781515648880000 + 1.00000000000000000000 \right. \\
& \exp \left(i\pi \left[\frac{\arg(10 - x - 2\sqrt{5})}{2\pi} \right] \right) \sqrt{x} \\
& \sum_{k_1=0}^{\infty} \sum_{k_2=0}^{\infty} \frac{1}{k_2!(1+2k_1)} (-1)^{k_1+k_2} x^{-k_2} T_{1+2k_1} \left(\frac{\sqrt{10 - 2\sqrt{5}}}{4 - 2(1 + \sqrt{5})} \right) \\
& {}_2F_1 \left(\frac{1}{2} + k_1, 1 + k_1; 2 + 2k_1; -4 \right) \left(-\frac{1}{2} \right)_{k_2} (10 - x - 2\sqrt{5})^{k_2} + \\
& 1.00000000000000000000 \exp \left(i\pi \left[\frac{\arg(10 - x + 2\sqrt{5})}{2\pi} \right] \right) \sqrt{x} \\
& \sum_{k_1=0}^{\infty} \sum_{k_2=0}^{\infty} \frac{1}{k_2!(1+2k_1)} (-1)^{k_1+k_2} x^{-k_2} T_{1+2k_1} \left(\frac{\sqrt{10 + 2\sqrt{5}}}{4 + 2(-1 + \sqrt{5})} \right) \\
& {}_2F_1 \left(\frac{1}{2} + k_1, 1 + k_1; 2 + 2k_1; -4 \right) \left(-\frac{1}{2} \right)_{k_2} \\
& \left. \left. (10 - x + 2\sqrt{5})^{k_2} \right) \right) \text{ for } (x \in \mathbb{R} \text{ and } x < 0)
\end{aligned}$$

$$\begin{aligned}
(64 \times 34) / & \left(0.15627563031297760000 + \frac{1}{10} \sqrt{10 - 2\sqrt{5}} \tan^{-1} \left(\frac{2\sqrt{10 - 2\sqrt{5}}}{4 - 2(\sqrt{5} + 1)} \right) + \right. \\
& \left. \frac{1}{10} \sqrt{10 + 2\sqrt{5}} \tan^{-1} \left(\frac{2\sqrt{10 + 2\sqrt{5}}}{4 + 2(\sqrt{5} - 1)} \right) \right) - 55 = \\
-55 + 2176 / & \left(0.15627563031297760000 + \frac{1}{10} \exp \left(i\pi \left[\frac{\arg(10 - x - 2\sqrt{5})}{2\pi} \right] \right) \right. \\
& \left. \sqrt{x} \left(\sum_{k=0}^{\infty} \frac{(-1)^k x^{-k} \left(-\frac{1}{2}\right)_k (10 - x - 2\sqrt{5})^k}{k!} \right) \right. \\
& \left(\tan^{-1}(x) + \pi \left[\frac{\arg \left(i \left(-x + \frac{2\sqrt{10 - 2\sqrt{5}}}{4 - 2(1 + \sqrt{5})} \right) \right)}{2\pi} \right] + \right. \\
& \left. \left. \frac{1}{2} i \sum_{k=1}^{\infty} \frac{(-(-i - x)^{-k} + (i - x)^{-k}) \left(-x + \frac{2\sqrt{10 - 2\sqrt{5}}}{4 - 2(1 + \sqrt{5})} \right)^k}{k} \right) + \frac{1}{10} \right. \\
& \left. \exp \left(i\pi \left[\frac{\arg(10 - x + 2\sqrt{5})}{2\pi} \right] \right) \sqrt{x} \left(\sum_{k=0}^{\infty} \frac{(-1)^k x^{-k} \left(-\frac{1}{2}\right)_k (10 - x + 2\sqrt{5})^k}{k!} \right) \right. \\
& \left(\tan^{-1}(x) + \pi \left[\frac{\arg \left(i \left(-x + \frac{2\sqrt{10 + 2\sqrt{5}}}{4 + 2(-1 + \sqrt{5})} \right) \right)}{2\pi} \right] + \right. \\
& \left. \left. \frac{1}{2} i \sum_{k=1}^{\infty} \frac{(-(-i - x)^{-k} + (i - x)^{-k}) \left(-x + \frac{2\sqrt{10 + 2\sqrt{5}}}{4 + 2(-1 + \sqrt{5})} \right)^k}{k} \right) \right) \right)
\end{aligned}$$

for ($i x \in \mathbb{R}$ and $i x < -1$ and $x \in \mathbb{R}$ and $x < 0$)

Integral representations:

$$(64 \times 34) / \left(0.15627563031297760000 + \frac{1}{10} \sqrt{10 - 2\sqrt{5}} \tan^{-1} \left(\frac{2\sqrt{10 - 2\sqrt{5}}}{4 - 2(\sqrt{5} + 1)} \right) + \frac{1}{10} \sqrt{10 + 2\sqrt{5}} \tan^{-1} \left(\frac{2\sqrt{10 + 2\sqrt{5}}}{4 + 2(\sqrt{5} - 1)} \right) \right) - 55 =$$

$$-55 + 2176 / \left(0.15627563031297760000 + \int_0^1 \left(\left((-1 + \sqrt{5})^2 (1 + \sqrt{5}) \sqrt{2(5 + \sqrt{5})^2} + \sqrt{10 - 2\sqrt{5}}^2 - (-1 + \sqrt{5})(1 + \sqrt{5})^2 + 2t^2 \sqrt{2(5 + \sqrt{5})^2} \right) / \left(10 \left((-1 + \sqrt{5})^2 + t^2 \sqrt{10 - 2\sqrt{5}}^2 \right) \left((1 + \sqrt{5})^2 + t^2 \sqrt{2(5 + \sqrt{5})^2} \right) \right) dt \right)$$

$$(64 \times 34) / \left(0.15627563031297760000 + \frac{1}{10} \sqrt{10 - 2\sqrt{5}} \tan^{-1} \left(\frac{2\sqrt{10 - 2\sqrt{5}}}{4 - 2(\sqrt{5} + 1)} \right) + \frac{1}{10} \sqrt{10 + 2\sqrt{5}} \tan^{-1} \left(\frac{2\sqrt{10 + 2\sqrt{5}}}{4 + 2(\sqrt{5} - 1)} \right) \right) - 55 =$$

$$-55 + 2176 / \left(0.15627563031297760000 + \int_{-i\infty+\gamma}^{i\infty+\gamma} \left(\frac{i\Gamma\left(\frac{1}{2} - s\right)\Gamma(1 - s)\Gamma(s)^2 \sqrt{10 - 2\sqrt{5}}^2 \left(1 + \frac{4\sqrt{10 - 2\sqrt{5}}^2}{(4 - 2(1 + \sqrt{5}))^2} \right)^{-s}}{20\pi^{3/2}(4 - 2(1 + \sqrt{5}))} - \frac{i\Gamma\left(\frac{1}{2} - s\right)\Gamma(1 - s)\Gamma(s)^2 \sqrt{10 + 2\sqrt{5}}^2 \left(1 + \frac{4\sqrt{10 + 2\sqrt{5}}^2}{(4 + 2(-1 + \sqrt{5}))^2} \right)^{-s}}{20\pi^{3/2}(4 + 2(-1 + \sqrt{5}))} \right) ds \right) \text{ for } 0 < \gamma < \frac{1}{2}$$

$$\begin{aligned}
(64 \times 34) / & \left(0.15627563031297760000 + \frac{1}{10} \sqrt{10 - 2\sqrt{5}} \tan^{-1} \left(\frac{2\sqrt{10 - 2\sqrt{5}}}{4 - 2(\sqrt{5} + 1)} \right) + \right. \\
& \left. \frac{1}{10} \sqrt{10 + 2\sqrt{5}} \tan^{-1} \left(\frac{2\sqrt{10 + 2\sqrt{5}}}{4 + 2(\sqrt{5} - 1)} \right) \right) - 55 = \\
-55 + 2176 / & \left(0.15627563031297760000 + \right. \\
& \int_{-i\infty+\gamma}^{i\infty+\gamma} \left(\frac{4^{-1-s} \Gamma\left(\frac{1}{2} - s\right) \Gamma(1-s) \Gamma(s) \sqrt{10 - 2\sqrt{5}}^2 \left(\frac{\sqrt{10 - 2\sqrt{5}}}{(4 - 2(1 + \sqrt{5}))} \right)^{-s}}{5i\pi \Gamma\left(\frac{3}{2} - s\right) (4 - 2(1 + \sqrt{5}))} + \right. \\
& \left. \frac{4^{-1-s} \Gamma\left(\frac{1}{2} - s\right) \Gamma(1-s) \Gamma(s) \sqrt{10 + 2\sqrt{5}}^2 \left(\frac{\sqrt{10 + 2\sqrt{5}}}{(4 + 2(-1 + \sqrt{5}))} \right)^{-s}}{5i\pi \Gamma\left(\frac{3}{2} - s\right) (4 + 2(-1 + \sqrt{5}))} \right) \\
& \left. ds \right) \text{ for } 0 < \gamma < \frac{1}{2}
\end{aligned}$$

Now, we have that:

$$\begin{aligned}
& \cos 2x - \left(1 + \frac{1}{3}\right) \cos 4x + \left(1 + \frac{1}{3} + \frac{1}{5}\right) \cos 6x - \&c \\
& = \frac{\pi}{4} (\cos x - \cos 3x + \cos 5x - \&c),
\end{aligned}$$

$$x = 6/13$$

$$\text{Pi}/4 * ((\cos(6/13) - \cos(3*6/13) + \cos(5*6/13)))$$

Input:

$$\frac{\pi}{4} \left(\cos\left(\frac{6}{13}\right) - \cos\left(3 \times \frac{6}{13}\right) + \cos\left(5 \times \frac{6}{13}\right) \right)$$

Exact result:

$$\frac{1}{4} \pi \left(\cos\left(\frac{6}{13}\right) - \cos\left(\frac{18}{13}\right) + \cos\left(\frac{30}{13}\right) \right)$$

Decimal approximation:

0.030056243651600759532848775534928356656353146768202475357...

0.0300562436516....

Alternate forms:

$$\frac{1}{4} \pi \cos\left(\frac{6}{13}\right) \left(1 - 2 \cos\left(\frac{12}{13}\right)\right)^2$$

$$\frac{1}{4} \left(\pi \cos\left(\frac{6}{13}\right) - \pi \cos\left(\frac{18}{13}\right) + \pi \cos\left(\frac{30}{13}\right) \right)$$

$$\frac{1}{4} \pi \left(\cos\left(\frac{6}{13}\right) - \cos\left(\frac{18}{13}\right) \right) + \frac{1}{4} \pi \cos\left(\frac{30}{13}\right)$$

Alternative representations:

$$\frac{1}{4} \left(\cos\left(\frac{6}{13}\right) - \cos\left(\frac{3 \times 6}{13}\right) + \cos\left(\frac{5 \times 6}{13}\right) \right) \pi = \frac{1}{4} \pi \left(\cosh\left(\frac{6i}{13}\right) - \cosh\left(\frac{18i}{13}\right) + \cosh\left(\frac{30i}{13}\right) \right)$$

$$\frac{1}{4} \left(\cos\left(\frac{6}{13}\right) - \cos\left(\frac{3 \times 6}{13}\right) + \cos\left(\frac{5 \times 6}{13}\right) \right) \pi = \frac{1}{4} \pi \left(\cosh\left(-\frac{6i}{13}\right) - \cosh\left(-\frac{18i}{13}\right) + \cosh\left(-\frac{30i}{13}\right) \right)$$

$$\frac{1}{4} \left(\cos\left(\frac{6}{13}\right) - \cos\left(\frac{3 \times 6}{13}\right) + \cos\left(\frac{5 \times 6}{13}\right) \right) \pi = \frac{1}{4} \pi \left(\frac{1}{\sec\left(\frac{6}{13}\right)} - \frac{1}{\sec\left(\frac{18}{13}\right)} + \frac{1}{\sec\left(\frac{30}{13}\right)} \right)$$

Series representations:

$$\frac{1}{4} \left(\cos\left(\frac{6}{13}\right) - \cos\left(\frac{3 \times 6}{13}\right) + \cos\left(\frac{5 \times 6}{13}\right) \right) \pi = \sum_{k=0}^{\infty} \frac{\left(\frac{9}{169}\right)^k 4^{-1+k} (1 - 9^k + 25^k) e^{i k \pi}}{(2k)!}$$

$$\frac{1}{4} \left(\cos\left(\frac{6}{13}\right) - \cos\left(\frac{3 \times 6}{13}\right) + \cos\left(\frac{5 \times 6}{13}\right) \right) \pi = \sum_{k=0}^{\infty} \frac{\pi \cos\left(\frac{k\pi}{2} + z_0\right) \left(\left(\frac{6}{13} - z_0\right)^k - \left(\frac{18}{13} - z_0\right)^k + \left(\frac{30}{13} - z_0\right)^k \right)}{4k!}$$

$$\frac{1}{4} \left(\cos\left(\frac{6}{13}\right) - \cos\left(\frac{3 \times 6}{13}\right) + \cos\left(\frac{5 \times 6}{13}\right) \right) \pi =$$

$$\sum_{k=0}^{\infty} \left(\frac{1}{4} \pi \left(\frac{(-1)^{-1+k} \left(\frac{6}{13} - \frac{\pi}{2}\right)^{1+2k}}{(1+2k)!} + \frac{(-1)^{-1+k} \left(\frac{30}{13} - \frac{\pi}{2}\right)^{1+2k}}{(1+2k)!} \right) - \frac{(-1)^{-1+k} \left(\frac{18}{13} - \frac{\pi}{2}\right)^{1+2k} \pi}{4(1+2k)!} \right)$$

Integral representations:

$$\frac{1}{4} \left(\cos\left(\frac{6}{13}\right) - \cos\left(\frac{3 \times 6}{13}\right) + \cos\left(\frac{5 \times 6}{13}\right) \right) \pi =$$

$$\frac{\pi}{4} + \int_0^1 -\frac{3}{26} \pi \left(\sin\left(\frac{6t}{13}\right) - 3 \sin\left(\frac{18t}{13}\right) + 5 \sin\left(\frac{30t}{13}\right) \right) dt$$

$$\frac{1}{4} \left(\cos\left(\frac{6}{13}\right) - \cos\left(\frac{3 \times 6}{13}\right) + \cos\left(\frac{5 \times 6}{13}\right) \right) \pi =$$

$$\int_{-i\infty+\gamma}^{i\infty+\gamma} -\frac{i e^{-225/(169s)+s} (1 - e^{144/(169s)} + e^{216/(169s)}) \sqrt{\pi}}{8 \sqrt{s}} ds \text{ for } \gamma > 0$$

$$\frac{1}{4} \left(\cos\left(\frac{6}{13}\right) - \cos\left(\frac{3 \times 6}{13}\right) + \cos\left(\frac{5 \times 6}{13}\right) \right) \pi =$$

$$\int_{-i\infty+\gamma}^{i\infty+\gamma} \frac{i \left(\frac{169}{225}\right)^s (-1 + \left(\frac{25}{9}\right)^s - 25^s) \sqrt{\pi} \Gamma(s)}{8 \Gamma\left(\frac{1}{2} - s\right)} ds \text{ for } 0 < \gamma < \frac{1}{2}$$

$$\frac{1}{4} \left(\cos\left(\frac{6}{13}\right) - \cos\left(\frac{3 \times 6}{13}\right) + \cos\left(\frac{5 \times 6}{13}\right) \right) \pi =$$

$$\int_{\frac{\pi}{2}}^{\frac{30}{13}} \left(-\frac{1}{4} \pi \sin(t) + \frac{1}{\frac{30}{13} - \frac{\pi}{2}} \left(\frac{6}{13} - \frac{\pi}{2} \right) \left(-\frac{1}{4} \pi \sin\left(\frac{-\frac{12\pi}{13} - \frac{6t}{13} + \frac{\pi t}{2}}{-\frac{30}{13} + \frac{\pi}{2}} \right) + \right. \right.$$

$$\left. \left. \frac{\left(\frac{18}{13} - \frac{\pi}{2}\right) \pi \sin\left(\frac{\frac{6\pi}{13} - \frac{18\left(-\frac{12\pi}{13} - \frac{6t}{13} + \frac{\pi t}{2}\right) + \pi\left(-\frac{12\pi}{13} - \frac{6t}{13} + \frac{\pi t}{2}\right)}{13\left(-\frac{30}{13} + \frac{\pi}{2}\right) + 2\left(-\frac{30}{13} + \frac{\pi}{2}\right)} \right)}{-\frac{6}{13} + \frac{\pi}{2}} \right) \right) dt$$

$$1 / \left(\frac{\pi}{4} \left(\cos\left(\frac{6}{13}\right) - \cos\left(3 \times \frac{6}{13}\right) + \cos\left(5 \times \frac{6}{13}\right) \right) \right)$$

Input:

$$\frac{1}{\frac{\pi}{4} \left(\cos\left(\frac{6}{13}\right) - \cos\left(3 \times \frac{6}{13}\right) + \cos\left(5 \times \frac{6}{13}\right) \right)}$$

Exact result:

$$\frac{4}{\pi \left(\cos\left(\frac{6}{13}\right) - \cos\left(\frac{18}{13}\right) + \cos\left(\frac{30}{13}\right) \right)}$$

Decimal approximation:

33.27095732891895151670851438881489360400123069525117845205...

33.2709573289...

Alternate forms:

$$\frac{4 \operatorname{sec}\left(\frac{6}{13}\right)}{\pi \left(1 - 2 \cos\left(\frac{12}{13}\right) \right)^2}$$

$$\frac{4}{\left(\frac{1}{2} \left(e^{-6i/13} + e^{6i/13} \right) + \frac{1}{2} \left(-e^{-18i/13} - e^{18i/13} \right) + \frac{1}{2} \left(e^{-30i/13} + e^{30i/13} \right) \right) \pi}$$

$$4 / \left(4 \pi \cos^5\left(\frac{6}{13}\right) - 4 \pi \cos^3\left(\frac{6}{13}\right) + \pi \cos\left(\frac{6}{13}\right) - 8 \pi \sin^2\left(\frac{6}{13}\right) \cos^3\left(\frac{6}{13}\right) + 4 \pi \sin^4\left(\frac{6}{13}\right) \cos\left(\frac{6}{13}\right) + 4 \pi \sin^2\left(\frac{6}{13}\right) \cos\left(\frac{6}{13}\right) \right)$$

Alternative representations:

$$\frac{1}{\frac{1}{4} \left(\cos\left(\frac{6}{13}\right) - \cos\left(\frac{3 \times 6}{13}\right) + \cos\left(\frac{5 \times 6}{13}\right) \right) \pi} = \frac{1}{\frac{1}{4} \pi \left(\cosh\left(\frac{6i}{13}\right) - \cosh\left(\frac{18i}{13}\right) + \cosh\left(\frac{30i}{13}\right) \right)}$$

$$\frac{1}{\frac{1}{4} \left(\cos\left(\frac{6}{13}\right) - \cos\left(\frac{3 \times 6}{13}\right) + \cos\left(\frac{5 \times 6}{13}\right) \right) \pi} = \frac{1}{\frac{1}{4} \pi \left(\cosh\left(-\frac{6i}{13}\right) - \cosh\left(-\frac{18i}{13}\right) + \cosh\left(-\frac{30i}{13}\right) \right)}$$

$$\frac{1}{\frac{1}{4} \left(\cos\left(\frac{6}{13}\right) - \cos\left(\frac{3 \times 6}{13}\right) + \cos\left(\frac{5 \times 6}{13}\right) \right) \pi} = \frac{1}{\frac{1}{4} \pi \left(\frac{1}{\sec\left(\frac{6}{13}\right)} - \frac{1}{\sec\left(\frac{18}{13}\right)} + \frac{1}{\sec\left(\frac{30}{13}\right)} \right)}$$

Series representations:

$$\frac{1}{4 \left(\cos\left(\frac{6}{13}\right) - \cos\left(\frac{3 \times 6}{13}\right) + \cos\left(\frac{5 \times 6}{13}\right) \right) \pi} = \frac{4}{\pi \sum_{k=0}^{\infty} \frac{\left(\frac{36}{169}\right)^k (1 - \varphi^k + 25^k) e^{i k \pi}}{(2k)!}}$$

$$\frac{1}{4 \left(\cos\left(\frac{6}{13}\right) - \cos\left(\frac{3 \times 6}{13}\right) + \cos\left(\frac{5 \times 6}{13}\right) \right) \pi} = \frac{4}{\pi \sum_{k=0}^{\infty} \frac{\cos\left(\frac{k \pi}{2} + z_0\right) \left(\left(\frac{6}{13} - z_0\right)^k - \left(\frac{18}{13} - z_0\right)^k + \left(\frac{30}{13} - z_0\right)^k \right)}{k!}}$$

$$\frac{1}{4 \left(\cos\left(\frac{6}{13}\right) - \cos\left(\frac{3 \times 6}{13}\right) + \cos\left(\frac{5 \times 6}{13}\right) \right) \pi} = \frac{\pi \sum_{k=0}^{\infty} \left(\frac{(-1)^{-1+k} \left(\frac{6}{13} - \frac{\pi}{2}\right)^{1+2k}}{(1+2k)!} + \frac{(-1)^k \left(\frac{18}{13} - \frac{\pi}{2}\right)^{1+2k}}{(1+2k)!} + \frac{(-1)^{-1+k} \left(\frac{30}{13} - \frac{\pi}{2}\right)^{1+2k}}{(1+2k)!} \right)}{4}$$

Integral representations:

$$\frac{1}{4 \left(\cos\left(\frac{6}{13}\right) - \cos\left(\frac{3 \times 6}{13}\right) + \cos\left(\frac{5 \times 6}{13}\right) \right) \pi} = \frac{\pi \left(-13 + \int_0^1 6 \left(\sin\left(\frac{6t}{13}\right) - 3 \sin\left(\frac{18t}{13}\right) + 5 \sin\left(\frac{30t}{13}\right) \right) dt \right)}{52}$$

$$\frac{1}{4 \left(\cos\left(\frac{6}{13}\right) - \cos\left(\frac{3 \times 6}{13}\right) + \cos\left(\frac{5 \times 6}{13}\right) \right) \pi} = \frac{8i}{\sqrt{\pi} \int_{-i\infty+\gamma}^{i\infty+\gamma} \frac{e^{-225/(169)s+s} \left(1 - e^{-144/(169)s} + e^{-216/(169)s} \right)}{\sqrt{s}} ds} \quad \text{for } \gamma > 0$$

$$\frac{1}{4 \left(\cos\left(\frac{6}{13}\right) - \cos\left(\frac{3 \times 6}{13}\right) + \cos\left(\frac{5 \times 6}{13}\right) \right) \pi} = \frac{8i}{\sqrt{\pi} \int_{-i\infty+\gamma}^{i\infty+\gamma} \frac{\left(\frac{169}{225}\right)^s \left(-1 + \left(\frac{25}{9}\right)^s - 25^s \right) \Gamma(s)}{\Gamma\left(\frac{1}{2}-s\right)} ds}$$

for $0 < \gamma < \frac{1}{2}$

$$\frac{1}{\frac{1}{4} \left(\cos\left(\frac{6}{13}\right) - \cos\left(\frac{3 \times 6}{13}\right) + \cos\left(\frac{5 \times 6}{13}\right) \right) \pi} =$$

$$- \left(4 / \left(\pi \int_{\frac{\pi}{2}}^{\frac{30}{13}} \sin(t) + \frac{1}{\frac{30}{13} - \frac{\pi}{2}} \left(\frac{6}{13} - \frac{\pi}{2} \right) \sin\left(\frac{-\frac{12\pi}{13} - \frac{6t}{13} + \frac{\pi t}{2}}{-\frac{30}{13} + \frac{\pi}{2}} \right) - \right. \right.$$

$$\left. \left. \frac{\left(\frac{18}{13} - \frac{\pi}{2} \right) \sin\left(\frac{\frac{6\pi}{13} - \frac{18\left(-\frac{12\pi}{13} - \frac{6t}{13} + \frac{\pi t}{2}\right) + \pi\left(-\frac{12\pi}{13} - \frac{6t}{13} + \frac{\pi t}{2}\right)}{13\left(-\frac{30}{13} + \frac{\pi}{2}\right) + 2\left(-\frac{30}{13} + \frac{\pi}{2}\right)} \right)}{\frac{-\frac{6}{13} + \frac{\pi}{2}}{\frac{6}{13} - \frac{\pi}{2}}} \right) dt \right)$$

We note that:

$$1/(((2+\text{sqrt}(10)))) * 1/ (((\text{Pi}/4 * (((\cos(6/13)-\cos(3*6/13)+\cos(5*6/13))))))))))$$

Input:

$$\frac{1}{2 + \sqrt{10}} \times \frac{1}{\frac{\pi}{4} \left(\cos\left(\frac{6}{13}\right) - \cos\left(3 \times \frac{6}{13}\right) + \cos\left(5 \times \frac{6}{13}\right) \right)}$$

Exact result:

$$\frac{4}{(2 + \sqrt{10}) \pi \left(\cos\left(\frac{6}{13}\right) - \cos\left(\frac{18}{13}\right) + \cos\left(\frac{30}{13}\right) \right)}$$

Decimal approximation:

6.445015072636318478083414584744417454350813563799129227586...

6.44501507263....

Alternate forms:

$$\frac{4 \sec\left(\frac{6}{13}\right)}{(2 + \sqrt{10}) \pi \left(1 - 2 \cos\left(\frac{12}{13}\right)\right)^2}$$

$$\frac{2\sqrt{10} - 4}{3\pi \left(\cos\left(\frac{6}{13}\right) - \cos\left(\frac{18}{13}\right) + \cos\left(\frac{30}{13}\right) \right)}$$

$$\frac{4}{(2 + \sqrt{10}) \left(\frac{1}{2} (e^{-(6i)/13} + e^{(6i)/13}) + \frac{1}{2} (-e^{-(18i)/13} - e^{(18i)/13}) + \frac{1}{2} (e^{-(30i)/13} + e^{(30i)/13}) \right) \pi}$$

Alternative representations:

$$\frac{1}{\frac{1}{4} \left(\pi \left(\cos\left(\frac{6}{13}\right) - \cos\left(\frac{3 \times 6}{13}\right) + \cos\left(\frac{5 \times 6}{13}\right) \right) \right) (2 + \sqrt{10})} =$$

$$\frac{1}{\frac{1}{4} \left(\pi \left(\cosh\left(\frac{6i}{13}\right) - \cosh\left(\frac{18i}{13}\right) + \cosh\left(\frac{30i}{13}\right) \right) \right) (2 + \sqrt{10})}$$

$$\frac{1}{\frac{1}{4} \left(\pi \left(\cos\left(\frac{6}{13}\right) - \cos\left(\frac{3 \times 6}{13}\right) + \cos\left(\frac{5 \times 6}{13}\right) \right) \right) (2 + \sqrt{10})} =$$

$$\frac{1}{\frac{1}{4} \left(\pi \left(\cosh\left(-\frac{6i}{13}\right) - \cosh\left(-\frac{18i}{13}\right) + \cosh\left(-\frac{30i}{13}\right) \right) \right) (2 + \sqrt{10})}$$

$$\frac{1}{\frac{1}{4} \left(\pi \left(\cos\left(\frac{6}{13}\right) - \cos\left(\frac{3 \times 6}{13}\right) + \cos\left(\frac{5 \times 6}{13}\right) \right) \right) (2 + \sqrt{10})} =$$

$$\frac{1}{\frac{1}{4} \left(\pi \left(\frac{1}{\sec\left(\frac{6}{13}\right)} - \frac{1}{\sec\left(\frac{18}{13}\right)} + \frac{1}{\sec\left(\frac{30}{13}\right)} \right) \right) (2 + \sqrt{10})}$$

Series representations:

$$\frac{1}{\frac{1}{4} \left(\pi \left(\cos\left(\frac{6}{13}\right) - \cos\left(\frac{3 \times 6}{13}\right) + \cos\left(\frac{5 \times 6}{13}\right) \right) \right) (2 + \sqrt{10})} =$$

$$(2 + \sqrt{10}) \pi \sum_{k=0}^{\infty} \frac{\left(\frac{36}{169}\right)^k (1-9^k + 25^k) e^{i k \pi}}{(2k)!}$$

$$\frac{1}{\frac{1}{4} \left(\pi \left(\cos\left(\frac{6}{13}\right) - \cos\left(\frac{3 \times 6}{13}\right) + \cos\left(\frac{5 \times 6}{13}\right) \right) \right) (2 + \sqrt{10})} =$$

$$(2 + \sqrt{10}) \pi \sum_{k=0}^{\infty} \frac{\cos\left(\frac{k\pi}{2} + z_0\right) \left(\left(\frac{6}{13} - z_0\right)^k - \left(\frac{18}{13} - z_0\right)^k + \left(\frac{30}{13} - z_0\right)^k \right)}{k!}$$

$$\frac{1}{4 \left(\pi \left(\cos\left(\frac{6}{13}\right) - \cos\left(\frac{3 \times 6}{13}\right) + \cos\left(\frac{5 \times 6}{13}\right) \right) \right) (2 + \sqrt{10})} =$$

$$(2 + \sqrt{10}) \pi \sum_{k=0}^{\infty} \frac{(-1)^k \left(-\left(\frac{6}{13} - \frac{\pi}{2}\right)^{1+2k} + \left(\frac{18}{13} - \frac{\pi}{2}\right)^{1+2k} - \left(\frac{30}{13} - \frac{\pi}{2}\right)^{1+2k} \right)}{(1+2k)!}$$

Integral representations:

$$\frac{1}{4 \left(\pi \left(\cos\left(\frac{6}{13}\right) - \cos\left(\frac{3 \times 6}{13}\right) + \cos\left(\frac{5 \times 6}{13}\right) \right) \right) (2 + \sqrt{10})} =$$

$$\frac{1}{52} \frac{1}{(2 + \sqrt{10}) \pi \left(-13 + \int_0^1 6 \left(\sin\left(\frac{6t}{13}\right) - 3 \sin\left(\frac{18t}{13}\right) + 5 \sin\left(\frac{30t}{13}\right) \right) dt \right)}$$

$$\frac{1}{4 \left(\pi \left(\cos\left(\frac{6}{13}\right) - \cos\left(\frac{3 \times 6}{13}\right) + \cos\left(\frac{5 \times 6}{13}\right) \right) \right) (2 + \sqrt{10})} =$$

$$\frac{1}{8i} \frac{1}{(2 + \sqrt{10}) \sqrt{\pi} \int_{-i\infty+\gamma}^{i\infty+\gamma} \frac{e^{-225/(169s)+s} \left(1 - e^{144/(169s)} + e^{216/(169s)} \right)}{\sqrt{s}} ds} \quad \text{for } \gamma > 0$$

$$\frac{1}{4 \left(\pi \left(\cos\left(\frac{6}{13}\right) - \cos\left(\frac{3 \times 6}{13}\right) + \cos\left(\frac{5 \times 6}{13}\right) \right) \right) (2 + \sqrt{10})} =$$

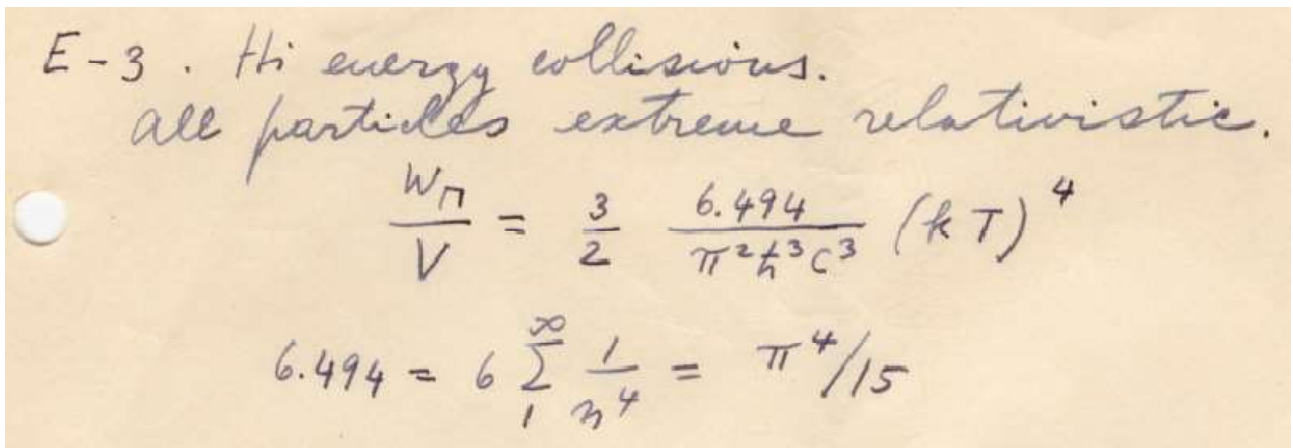
$$\frac{1}{8i} \frac{1}{(2 + \sqrt{10}) \sqrt{\pi} \int_{-i\infty+\gamma}^{i\infty+\gamma} \frac{\left(\frac{169}{225}\right)^s \left(-1 + \left(\frac{25}{9}\right)^s - 25^s \right) \Gamma(s)}{\Gamma\left(\frac{1}{2}-s\right)} ds} \quad \text{for } 0 < \gamma < \frac{1}{2}$$

$$\frac{1}{\frac{1}{4} \left(\pi \left(\cos\left(\frac{6}{13}\right) - \cos\left(\frac{3 \times 6}{13}\right) + \cos\left(\frac{5 \times 6}{13}\right) \right) \right) (2 + \sqrt{10})} =$$

$$-4 \left/ \left((2 + \sqrt{10}) \pi \int_{\frac{\pi}{2}}^{\frac{30}{13}} \sin(t) + \frac{1}{\frac{30}{13} - \frac{\pi}{2}} \left(\frac{6}{13} - \frac{\pi}{2} \right) \sin\left(\frac{-\frac{12\pi}{13} - \frac{6t}{13} + \frac{\pi t}{2}}{-\frac{30}{13} + \frac{\pi}{2}} \right) - \right. \right.$$

$$\left. \left. \frac{\left(\frac{18}{13} - \frac{\pi}{2} \right) \sin\left(\frac{\frac{6\pi}{13} - \frac{18\left(-\frac{12\pi}{13} - \frac{6t}{13} + \frac{\pi t}{2}\right) + \pi\left(-\frac{12\pi}{13} - \frac{6t}{13} + \frac{\pi t}{2}\right)}{13\left(-\frac{30}{13} + \frac{\pi}{2}\right) + 2\left(-\frac{30}{13} + \frac{\pi}{2}\right)} \right)}{-\frac{6}{13} + \frac{\pi}{2}} \right) \right| dt$$

The result 6.44501507263.... is very near to the following Fermi's formula:



Indeed:

$$6.494 \approx 6.445015\dots$$

And:

$$[55 * 1/ (((1/ (((Pi/4*(((cos(6/13)-cos(3*6/13)+cos(5*6/13))))))))))] * 1/10^26$$

Input:

$$\left(55 \times \frac{1}{\frac{1}{\frac{\pi}{4} \left(\cos\left(\frac{6}{13}\right) - \cos\left(3 \times \frac{6}{13}\right) + \cos\left(5 \times \frac{6}{13}\right) \right)}} \right) \times \frac{1}{10^{26}}$$

Exact result:

$$\frac{11 \pi \left(\cos\left(\frac{6}{13}\right) - \cos\left(\frac{18}{13}\right) + \cos\left(\frac{30}{13}\right) \right)}{80\,000\,000\,000\,000\,000\,000\,000\,000}$$

Decimal approximation:

$$1.6530934008380417743066826544210596160994230722511361... \times 10^{-26}$$

$$1.653093400838... * 10^{-26}$$

Alternate forms:

$$\frac{11 \pi \cos\left(\frac{6}{13}\right) \left(1 - 2 \cos\left(\frac{12}{13}\right)\right)^2}{80\,000\,000\,000\,000\,000\,000\,000\,000}$$

$$\frac{11 \left(\pi \cos\left(\frac{6}{13}\right) - \pi \cos\left(\frac{18}{13}\right) + \pi \cos\left(\frac{30}{13}\right) \right)}{80\,000\,000\,000\,000\,000\,000\,000\,000}$$

$$\frac{11 \pi \left(\cos\left(\frac{6}{13}\right) - \cos\left(\frac{18}{13}\right) \right)}{80\,000\,000\,000\,000\,000\,000\,000\,000} + \frac{11 \pi \cos\left(\frac{30}{13}\right)}{80\,000\,000\,000\,000\,000\,000\,000\,000}$$

Alternative representations:

$$\frac{55}{10^{26} \frac{1}{4} \pi \left(\cos\left(\frac{6}{13}\right) - \cos\left(\frac{3 \times 6}{13}\right) + \cos\left(\frac{5 \times 6}{13}\right) \right)} = \frac{55}{10^{26} \frac{1}{4} \pi \left(\cosh\left(\frac{6i}{13}\right) - \cosh\left(\frac{18i}{13}\right) + \cosh\left(\frac{30i}{13}\right) \right)}$$

$$\frac{55}{10^{26} \frac{1}{4} \pi \left(\cos\left(\frac{6}{13}\right) - \cos\left(\frac{3 \times 6}{13}\right) + \cos\left(\frac{5 \times 6}{13}\right) \right)} = \frac{55}{10^{26} \frac{1}{4} \pi \left(\cosh\left(-\frac{6i}{13}\right) - \cosh\left(-\frac{18i}{13}\right) + \cosh\left(-\frac{30i}{13}\right) \right)}$$

$$\frac{55}{10^{26} \frac{1}{4} \pi \left(\cos\left(\frac{6}{13}\right) - \cos\left(\frac{3 \times 6}{13}\right) + \cos\left(\frac{5 \times 6}{13}\right) \right)} = \frac{55}{10^{26} \frac{1}{4} \pi \left(\frac{1}{\sec\left(\frac{6}{13}\right)} - \frac{1}{\sec\left(\frac{18}{13}\right)} + \frac{1}{\sec\left(\frac{30}{13}\right)} \right)}$$

Series representations:

$$\frac{55}{\frac{10^{26}}{\frac{1}{4} \pi \left(\cos\left(\frac{6}{13}\right) - \cos\left(\frac{3 \times 6}{13}\right) + \cos\left(\frac{5 \times 6}{13}\right) \right)}} = \sum_{k=0}^{\infty} \frac{11 \left(\frac{9}{169}\right)^k 4^{-14+k} (1 - 9^k + 25^k) e^{i k \pi} \pi}{298\,023\,223\,876\,953\,125 (2k)!}$$

$$\frac{55}{\frac{10^{26}}{\frac{1}{4} \pi \left(\cos\left(\frac{6}{13}\right) - \cos\left(\frac{3 \times 6}{13}\right) + \cos\left(\frac{5 \times 6}{13}\right) \right)}} = \sum_{k=0}^{\infty} \frac{11 \pi \cos\left(\frac{k \pi}{2} + z_0\right) \left(\left(\frac{6}{13} - z_0\right)^k - \left(\frac{18}{13} - z_0\right)^k + \left(\frac{30}{13} - z_0\right)^k \right)}{80\,000\,000\,000\,000\,000\,000\,000\,000\,k!}$$

$$\frac{55}{\frac{10^{26}}{\frac{1}{4} \pi \left(\cos\left(\frac{6}{13}\right) - \cos\left(\frac{3 \times 6}{13}\right) + \cos\left(\frac{5 \times 6}{13}\right) \right)}} = \sum_{k=0}^{\infty} \left(\frac{11 \pi \left(\frac{(-1)^{-1+k} \left(\frac{6-\pi}{13-2}\right)^{1+2k}}{(1+2k)!} + \frac{(-1)^{-1+k} \left(\frac{30-\pi}{13-2}\right)^{1+2k}}{(1+2k)!} \right)}{80\,000\,000\,000\,000\,000\,000\,000\,000} - \frac{11 (-1)^{-1+k} \left(\frac{18}{13} - \frac{\pi}{2}\right)^{1+2k} \pi}{80\,000\,000\,000\,000\,000\,000\,000\,000 (1+2k)!} \right)$$

Integral representations:

$$\frac{55}{\frac{10^{26}}{\frac{1}{4} \pi \left(\cos\left(\frac{6}{13}\right) - \cos\left(\frac{3 \times 6}{13}\right) + \cos\left(\frac{5 \times 6}{13}\right) \right)}} = \frac{11 \pi}{80\,000\,000\,000\,000\,000\,000\,000\,000} + \int_0^1 - \frac{33 \pi \left(\sin\left(\frac{6t}{13}\right) - 3 \sin\left(\frac{18t}{13}\right) + 5 \sin\left(\frac{30t}{13}\right) \right)}{520\,000\,000\,000\,000\,000\,000\,000\,000} dt$$

$$\frac{55}{\frac{10^{26}}{\frac{1}{4} \pi \left(\cos\left(\frac{6}{13}\right) - \cos\left(\frac{3 \times 6}{13}\right) + \cos\left(\frac{5 \times 6}{13}\right) \right)}} = \int_{-i\infty+\gamma}^{i\infty+\gamma} \frac{11 i 5^{-25-2s} \left(\frac{169}{9}\right)^s \left(-1 + \left(\frac{25}{9}\right)^s - 25^s\right) \sqrt{\pi} \Gamma(s)}{536\,870\,912 \Gamma\left(\frac{1}{2}-s\right)} ds$$

for $0 < \gamma < \frac{1}{2}$

$$\frac{55}{\frac{10^{26}}{\frac{1}{4} \pi \left(\cos\left(\frac{6}{13}\right) - \cos\left(\frac{3 \times 6}{13}\right) + \cos\left(\frac{5 \times 6}{13}\right) \right)}} = \int_{-i\infty+\gamma}^{i\infty+\gamma} - \frac{11 i e^{-225/(169s)+s} (1 - e^{144/(169s)} + e^{216/(169s)}) \sqrt{\pi}}{160\,000\,000\,000\,000\,000\,000\,000\,000 \sqrt{s}} ds \text{ for } \gamma > 0$$

$$\frac{55}{10^{26} \left(\frac{1}{4} \pi \left(\cos\left(\frac{6}{13}\right) - \cos\left(\frac{3 \times 6}{13}\right) + \cos\left(\frac{5 \times 6}{13}\right) \right) \right)} =$$

$$\int_{\frac{\pi}{2}}^{\frac{30}{13}} \left(\frac{11 \pi \sin(t)}{80\,000\,000\,000\,000\,000\,000\,000\,000} + \frac{1}{\frac{30}{13} - \frac{\pi}{2}} \left(\frac{6}{13} - \frac{\pi}{2} \right) \right)$$

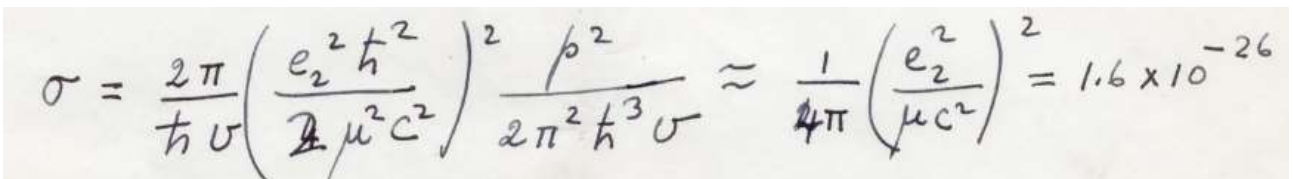
$$\left(\frac{11 \pi \sin\left(\frac{-\frac{12\pi - 6t + \pi t}{13} - \frac{30 + \pi}{13 \cdot 2}} \right)}{80\,000\,000\,000\,000\,000\,000\,000\,000} + \right)$$

$$\left. \frac{11 \left(\frac{18}{13} - \frac{\pi}{2} \right) \pi \sin\left(\frac{\frac{6\pi}{13} - \frac{18 \left(-\frac{12\pi - 6t + \pi t}{13} - \frac{30 + \pi}{13 \cdot 2} \right) + \pi \left(-\frac{12\pi - 6t + \pi t}{13} - \frac{30 + \pi}{13 \cdot 2} \right)}{-\frac{6}{13} + \frac{\pi}{2}} \right)}{80\,000\,000\,000\,000\,000\,000\,000\,000 \left(\frac{6}{13} - \frac{\pi}{2} \right)} \right) dt$$

The result

$$1.6530934008380417743066826544210596160994230722511361... \times 10^{-26}$$

1.6530934... * 10⁻²⁶ is very near to the following Fermi's formula:



$$\sigma = \frac{2\pi}{h\nu} \left(\frac{e_2^2 \hbar^2}{4\mu^2 c^2} \right)^2 \frac{\beta^2}{2\pi^2 \hbar^3 \nu} \approx \frac{1}{4\pi} \left(\frac{e_2^2}{\mu c^2} \right)^2 = 1.6 \times 10^{-26}$$

concerning:

and to the dilaton value $0.989117352243 = \phi$

Alternate forms:

$$\frac{256\sqrt{\pi \cos\left(\frac{6}{13}\right) - \pi \cos\left(\frac{18}{13}\right) + \pi \cos\left(\frac{30}{13}\right)}}{128\sqrt{2}}$$

$$\frac{256\sqrt{\left(\frac{1}{2} \left(e^{-(6i)/13} + e^{(6i)/13}\right) + \frac{1}{2} \left(-e^{-(18i)/13} - e^{(18i)/13}\right) + \frac{1}{2} \left(e^{-(30i)/13} + e^{(30i)/13}\right)\right)\pi}}{128\sqrt{2}}$$

All 256th roots of $1/4 \pi (\cos(6/13) - \cos(18/13) + \cos(30/13))$:

$$\frac{e^0 256\sqrt{\pi \left(\cos\left(\frac{6}{13}\right) - \cos\left(\frac{18}{13}\right) + \cos\left(\frac{30}{13}\right)\right)}}{128\sqrt{2}} \approx 0.98640 \text{ (real, principal root)}$$

$$\frac{e^{(i\pi)/128} 256\sqrt{\pi \left(\cos\left(\frac{6}{13}\right) - \cos\left(\frac{18}{13}\right) + \cos\left(\frac{30}{13}\right)\right)}}{128\sqrt{2}} \approx 0.98611 + 0.024208 i$$

$$\frac{e^{(i\pi)/64} 256\sqrt{\pi \left(\cos\left(\frac{6}{13}\right) - \cos\left(\frac{18}{13}\right) + \cos\left(\frac{30}{13}\right)\right)}}{128\sqrt{2}} \approx 0.98521 + 0.04840 i$$

$$\frac{e^{(3i\pi)/128} 256\sqrt{\pi \left(\cos\left(\frac{6}{13}\right) - \cos\left(\frac{18}{13}\right) + \cos\left(\frac{30}{13}\right)\right)}}{128\sqrt{2}} \approx 0.98373 + 0.07256 i$$

$$\frac{e^{(i\pi)/32} 256\sqrt{\pi \left(\cos\left(\frac{6}{13}\right) - \cos\left(\frac{18}{13}\right) + \cos\left(\frac{30}{13}\right)\right)}}{128\sqrt{2}} \approx 0.98165 + 0.09668 i$$

Alternative representations:

$$\sqrt[256]{\frac{1}{4} \left(\cos\left(\frac{6}{13}\right) - \cos\left(\frac{3 \times 6}{13}\right) + \cos\left(\frac{5 \times 6}{13}\right) \right)} \pi =$$

$$\sqrt[256]{\frac{1}{4} \pi \left(\cosh\left(\frac{6i}{13}\right) - \cosh\left(\frac{18i}{13}\right) + \cosh\left(\frac{30i}{13}\right) \right)}$$

$$\sqrt[256]{\frac{1}{4} \left(\cos\left(\frac{6}{13}\right) - \cos\left(\frac{3 \times 6}{13}\right) + \cos\left(\frac{5 \times 6}{13}\right) \right)} \pi =$$

$$\sqrt[256]{\frac{1}{4} \pi \left(\cosh\left(-\frac{6i}{13}\right) - \cosh\left(-\frac{18i}{13}\right) + \cosh\left(-\frac{30i}{13}\right) \right)}$$

$$\sqrt[256]{\frac{1}{4} \left(\cos\left(\frac{6}{13}\right) - \cos\left(\frac{3 \times 6}{13}\right) + \cos\left(\frac{5 \times 6}{13}\right) \right)} \pi = \sqrt[256]{\frac{1}{4} \pi \left(\frac{1}{\sec\left(\frac{6}{13}\right)} - \frac{1}{\sec\left(\frac{18}{13}\right)} + \frac{1}{\sec\left(\frac{30}{13}\right)} \right)}$$

Series representations:

$$\sqrt[256]{\frac{1}{4} \left(\cos\left(\frac{6}{13}\right) - \cos\left(\frac{3 \times 6}{13}\right) + \cos\left(\frac{5 \times 6}{13}\right) \right)} \pi = \frac{256\sqrt{\pi} \sqrt[256]{\sum_{k=0}^{\infty} \frac{\left(\frac{36}{169}\right)^k (1 - \varrho^{k+25k}) e^{i k \pi}}{(2k)!}}}{128\sqrt[2]{2}}$$

$$\sqrt[256]{\frac{1}{4} \left(\cos\left(\frac{6}{13}\right) - \cos\left(\frac{3 \times 6}{13}\right) + \cos\left(\frac{5 \times 6}{13}\right) \right)} \pi =$$

$$\frac{256\sqrt{\pi} \sqrt[256]{\sum_{k=0}^{\infty} \frac{\cos\left(\frac{k\pi}{2} + z_0\right) \left(\left(\frac{6}{13} - z_0\right)^k - \left(\frac{18}{13} - z_0\right)^k + \left(\frac{30}{13} - z_0\right)^k \right)}{k!}}}{128\sqrt[2]{2}}$$

$$\sqrt[256]{\frac{1}{4} \left(\cos\left(\frac{6}{13}\right) - \cos\left(\frac{3 \times 6}{13}\right) + \cos\left(\frac{5 \times 6}{13}\right) \right)} \pi =$$

$$\frac{256\sqrt{\pi} \sqrt[256]{\sum_{k=0}^{\infty} \left(\frac{(-1)^{-1+k} \left(\frac{6}{13} - \frac{\pi}{2}\right)^{1+2k}}{(1+2k)!} + \frac{(-1)^k \left(\frac{18}{13} - \frac{\pi}{2}\right)^{1+2k}}{(1+2k)!} + \frac{(-1)^{-1+k} \left(\frac{30}{13} - \frac{\pi}{2}\right)^{1+2k}}{(1+2k)!} \right)}}{128\sqrt[2]{2}}$$

Integral representations:

$$\frac{256 \sqrt{\frac{1}{4} \left(\cos\left(\frac{6}{13}\right) - \cos\left(\frac{3 \times 6}{13}\right) + \cos\left(\frac{5 \times 6}{13}\right) \right) \pi}}{256 \sqrt{\frac{\pi}{13}} \sqrt{256 \sqrt{13 + \int_0^1 -6 \left(\sin\left(\frac{6t}{13}\right) - 3 \sin\left(\frac{18t}{13}\right) + 5 \sin\left(\frac{30t}{13}\right) \right) dt}}}}{128 \sqrt{2}}$$

$$\frac{256 \sqrt{\frac{1}{4} \left(\cos\left(\frac{6}{13}\right) - \cos\left(\frac{3 \times 6}{13}\right) + \cos\left(\frac{5 \times 6}{13}\right) \right) \pi}}{512 \sqrt{\pi} \sqrt{256 \sqrt{-i \int_{-i \infty + \gamma}^{i \infty + \gamma} \frac{e^{-225/(169 s) + s} (1 - e^{-144/(169 s) + e^{216/(169 s)}})}{\sqrt{s}} ds}}}}{2^{3/256}} \quad \text{for } \gamma > 0$$

$$\frac{256 \sqrt{\frac{1}{4} \left(\cos\left(\frac{6}{13}\right) - \cos\left(\frac{3 \times 6}{13}\right) + \cos\left(\frac{5 \times 6}{13}\right) \right) \pi}}{512 \sqrt{\pi} \sqrt{256 \sqrt{-i \int_{-i \infty + \gamma}^{i \infty + \gamma} \frac{\left(\frac{169}{225}\right)^s \left(-1 + \left(\frac{25}{9}\right)^s - 25^s\right) \Gamma(s)}{\Gamma\left(\frac{1}{2} - s\right)} ds}}}}{2^{3/256}} \quad \text{for } 0 < \gamma < \frac{1}{2}$$

$$\begin{aligned}
& \sqrt[256]{\frac{1}{4} \left(\cos\left(\frac{6}{13}\right) - \cos\left(\frac{3 \times 6}{13}\right) + \cos\left(\frac{5 \times 6}{13}\right) \right) \pi} = \frac{1}{\sqrt[128]{2}} \\
& \left(\int_{\frac{\pi}{2}}^{\frac{30}{13}} -\sin(t) + \frac{1}{\frac{30}{13} - \frac{\pi}{2}} \left(\frac{6}{13} - \frac{\pi}{2} \right) -\sin\left(\frac{-\frac{12\pi}{13} - \frac{6t}{13} + \frac{\pi t}{2}}{-\frac{30}{13} + \frac{\pi}{2}} \right) + \right. \\
& \left. \left(\frac{18}{13} - \frac{\pi}{2} \right) \sin\left(\frac{\frac{6\pi}{13} - \frac{18 \left(\frac{-\frac{12\pi}{13} - \frac{6t}{13} + \frac{\pi t}{2}}{-\frac{30}{13} + \frac{\pi}{2}} \right) \pi \left(\frac{-\frac{12\pi}{13} - \frac{6t}{13} + \frac{\pi t}{2}}{-\frac{30}{13} + \frac{\pi}{2}} \right)}{\frac{-\frac{6}{13} + \frac{\pi}{2}}{2 \left(\frac{-\frac{30}{13} + \frac{\pi}{2}}{-\frac{30}{13} + \frac{\pi}{2}} \right)}} \right) \right) \\
& \left. \right)^{\wedge (1/256)}
\end{aligned}$$

Now, we have that:

On page 374, Ramanujan announces some expressions for certain values of the exponential function. For example, he states that

$$e^{\frac{\pi}{4}\sqrt{78}} = 4\sqrt{3} (75 + 52\sqrt{2}) \tag{4}$$

and

$$e^{\frac{\pi}{4}\sqrt{130}} = 12(323 + 40\sqrt{65}) \tag{5}$$

From the sum

we obtain:

$$\exp\left(\left(\frac{\pi}{4}\right)\sqrt{78}\right) + 4\sqrt{3}(75+52\sqrt{2})$$

Input:

$$\exp\left(\frac{\pi}{4} \sqrt{78}\right) + 4 \sqrt{3} \left(75 + 52 \sqrt{2}\right)$$

Exact result:

$$4 \sqrt{3} \left(75 + 52 \sqrt{2}\right) + e^{1/2 \sqrt{39/2} \pi}$$

Decimal approximation:

2058.218217515272934328477863254190789096533359619891371897...

2058.21821751...

Property:

$4 \sqrt{3} \left(75 + 52 \sqrt{2}\right) + e^{1/2 \sqrt{39/2} \pi}$ is a transcendental number

Alternate forms:

$$300 \sqrt{3} + 208 \sqrt{6} + e^{1/2 \sqrt{39/2} \pi}$$

$$4 \sqrt{3 \left(11033 + 7800 \sqrt{2}\right)} + e^{1/2 \sqrt{39/2} \pi}$$

Series representations:

$$\begin{aligned} \exp\left(\frac{\sqrt{78} \pi}{4}\right) + 4 \sqrt{3} \left(75 + 52 \sqrt{2}\right) &= \exp\left(\frac{1}{4} \pi \sqrt{z_0} \sum_{k=0}^{\infty} \frac{(-1)^k \left(-\frac{1}{2}\right)_k (78 - z_0)^k z_0^{-k}}{k!}\right) + \\ &300 \sqrt{z_0} \sum_{k=0}^{\infty} \frac{(-1)^k \left(-\frac{1}{2}\right)_k (3 - z_0)^k z_0^{-k}}{k!} + \\ &208 \sqrt{z_0}^2 \sum_{k_1=0}^{\infty} \sum_{k_2=0}^{\infty} \frac{(-1)^{k_1+k_2} \left(-\frac{1}{2}\right)_{k_1} \left(-\frac{1}{2}\right)_{k_2} (2 - z_0)^{k_1} (3 - z_0)^{k_2} z_0^{-k_1-k_2}}{k_1! k_2!} \end{aligned}$$

for not $((z_0 \in \mathbb{R} \text{ and } -\infty < z_0 \leq 0))$

$$\begin{aligned} & \exp\left(\frac{\sqrt{78} \pi}{4}\right) + 4 \sqrt{3} (75 + 52 \sqrt{2}) = \\ & \exp\left(\frac{1}{4} \pi \exp\left(i \pi \left[\frac{\arg(78-x)}{2 \pi}\right]\right)\right) \sqrt{x} \sum_{k=0}^{\infty} \frac{(-1)^k (78-x)^k x^{-k} \left(-\frac{1}{2}\right)_k}{k!} + \\ & 300 \exp\left(i \pi \left[\frac{\arg(3-x)}{2 \pi}\right]\right) \sqrt{x} \sum_{k=0}^{\infty} \frac{(-1)^k (3-x)^k x^{-k} \left(-\frac{1}{2}\right)_k}{k!} + \\ & 208 \exp\left(i \pi \left[\frac{\arg(2-x)}{2 \pi}\right]\right) \exp\left(i \pi \left[\frac{\arg(3-x)}{2 \pi}\right]\right) \sqrt{x}^2 \\ & \sum_{k_1=0}^{\infty} \sum_{k_2=0}^{\infty} \frac{(-1)^{k_1+k_2} (2-x)^{k_1} (3-x)^{k_2} x^{-k_1-k_2} \left(-\frac{1}{2}\right)_{k_1} \left(-\frac{1}{2}\right)_{k_2}}{k_1! k_2!} \end{aligned}$$

for ($x \in \mathbb{R}$ and $x < 0$)

$$\begin{aligned} & \exp\left(\frac{\sqrt{78} \pi}{4}\right) + 4 \sqrt{3} (75 + 52 \sqrt{2}) = \\ & \exp\left(\frac{1}{4} \pi \left(\frac{1}{z_0}\right)^{1/2 [\arg(78-z_0)/(2 \pi)]} z_0^{1/2 (1+[\arg(78-z_0)/(2 \pi)])} \sum_{k=0}^{\infty} \frac{(-1)^k \left(-\frac{1}{2}\right)_k (78-z_0)^k z_0^{-k}}{k!}\right) + \\ & 300 \left(\frac{1}{z_0}\right)^{1/2 [\arg(3-z_0)/(2 \pi)]} z_0^{1/2+1/2 [\arg(3-z_0)/(2 \pi)]} \sum_{k=0}^{\infty} \frac{(-1)^k \left(-\frac{1}{2}\right)_k (3-z_0)^k z_0^{-k}}{k!} + \\ & 208 \left(\frac{1}{z_0}\right)^{1/2 [\arg(2-z_0)/(2 \pi)]+1/2 [\arg(3-z_0)/(2 \pi)]} z_0^{1+1/2 [\arg(2-z_0)/(2 \pi)]+1/2 [\arg(3-z_0)/(2 \pi)]} \\ & \sum_{k_1=0}^{\infty} \sum_{k_2=0}^{\infty} \frac{(-1)^{k_1+k_2} \left(-\frac{1}{2}\right)_{k_1} \left(-\frac{1}{2}\right)_{k_2} (2-z_0)^{k_1} (3-z_0)^{k_2} z_0^{-k_1-k_2}}{k_1! k_2!} \end{aligned}$$

And:

$$55 + \exp\left(\left(\frac{\pi}{4}\right) \sqrt{78}\right) + 4 \sqrt{3} (75 + 52 \sqrt{2})$$

Input:

$$55 + \exp\left(\frac{\pi}{4} \sqrt{78}\right) + 4 \sqrt{3} (75 + 52 \sqrt{2})$$

Exact result:

$$55 + 4 \sqrt{3} (75 + 52 \sqrt{2}) + e^{1/2 \sqrt{39/2} \pi}$$

Decimal approximation:

2113.218217515272934328477863254190789096533359619891371897...

2113.2182175... result very near to the rest mass of strange D meson 2112.3

Property:

$55 + 4\sqrt{3} (75 + 52\sqrt{2}) + e^{1/2\sqrt{39/2}\pi}$ is a transcendental number

Alternate forms:

$$55 + 300\sqrt{3} + 208\sqrt{6} + e^{1/2\sqrt{39/2}\pi}$$

$$55 + 4\sqrt{3(11033 + 7800\sqrt{2})} + e^{1/2\sqrt{39/2}\pi}$$

Series representations:

$$55 + \exp\left(\frac{\sqrt{78}\pi}{4}\right) + 4\sqrt{3} (75 + 52\sqrt{2}) =$$

$$55 + \exp\left(\frac{1}{4}\pi\sqrt{z_0} \sum_{k=0}^{\infty} \frac{(-1)^k \left(-\frac{1}{2}\right)_k (78 - z_0)^k z_0^{-k}}{k!}\right) +$$

$$300\sqrt{z_0} \sum_{k=0}^{\infty} \frac{(-1)^k \left(-\frac{1}{2}\right)_k (3 - z_0)^k z_0^{-k}}{k!} +$$

$$208\sqrt{z_0}^2 \sum_{k_1=0}^{\infty} \sum_{k_2=0}^{\infty} \frac{(-1)^{k_1+k_2} \left(-\frac{1}{2}\right)_{k_1} \left(-\frac{1}{2}\right)_{k_2} (2 - z_0)^{k_1} (3 - z_0)^{k_2} z_0^{-k_1-k_2}}{k_1! k_2!}$$

for not $((z_0 \in \mathbb{R} \text{ and } -\infty < z_0 \leq 0))$

$$55 + \exp\left(\frac{\sqrt{78}\pi}{4}\right) + 4\sqrt{3} (75 + 52\sqrt{2}) =$$

$$55 + \exp\left(\frac{1}{4}\pi \exp\left(i\pi \left\lfloor \frac{\arg(78-x)}{2\pi} \right\rfloor\right) \sqrt{x} \sum_{k=0}^{\infty} \frac{(-1)^k (78-x)^k x^{-k} \left(-\frac{1}{2}\right)_k}{k!}\right) +$$

$$300 \exp\left(i\pi \left\lfloor \frac{\arg(3-x)}{2\pi} \right\rfloor\right) \sqrt{x} \sum_{k=0}^{\infty} \frac{(-1)^k (3-x)^k x^{-k} \left(-\frac{1}{2}\right)_k}{k!} +$$

$$208 \exp\left(i\pi \left\lfloor \frac{\arg(2-x)}{2\pi} \right\rfloor\right) \exp\left(i\pi \left\lfloor \frac{\arg(3-x)}{2\pi} \right\rfloor\right) \sqrt{x}^2$$

$$\sum_{k_1=0}^{\infty} \sum_{k_2=0}^{\infty} \frac{(-1)^{k_1+k_2} (2-x)^{k_1} (3-x)^{k_2} x^{-k_1-k_2} \left(-\frac{1}{2}\right)_{k_1} \left(-\frac{1}{2}\right)_{k_2}}{k_1! k_2!}$$

for $(x \in \mathbb{R} \text{ and } x < 0)$

$$\begin{aligned}
& 55 + \exp\left(\frac{\sqrt{78} \pi}{4}\right) + 4\sqrt{3} (75 + 52\sqrt{2}) = 55 + \\
& \exp\left(\frac{1}{4} \pi \left(\frac{1}{z_0}\right)^{1/2 [\arg(78-z_0)/(2\pi)]} z_0^{1/2 (1+[\arg(78-z_0)/(2\pi)])} \sum_{k=0}^{\infty} \frac{(-1)^k \left(-\frac{1}{2}\right)_k (78-z_0)^k z_0^{-k}}{k!}\right) + \\
& 300 \left(\frac{1}{z_0}\right)^{1/2 [\arg(3-z_0)/(2\pi)]} z_0^{1/2+1/2 [\arg(3-z_0)/(2\pi)]} \sum_{k=0}^{\infty} \frac{(-1)^k \left(-\frac{1}{2}\right)_k (3-z_0)^k z_0^{-k}}{k!} + \\
& 208 \left(\frac{1}{z_0}\right)^{1/2 [\arg(2-z_0)/(2\pi)]+1/2 [\arg(3-z_0)/(2\pi)]} z_0^{1+1/2 [\arg(2-z_0)/(2\pi)]+1/2 [\arg(3-z_0)/(2\pi)]} \\
& \sum_{k_1=0}^{\infty} \sum_{k_2=0}^{\infty} \frac{(-1)^{k_1+k_2} \left(-\frac{1}{2}\right)_{k_1} \left(-\frac{1}{2}\right)_{k_2} (2-z_0)^{k_1} (3-z_0)^{k_2} z_0^{-k_1-k_2}}{k_1! k_2!}
\end{aligned}$$

And:

$$[\exp(((\pi/4)*\sqrt{130})) + 12(323+40\sqrt{65})]$$

Input:

$$\exp\left(\frac{\pi}{4} \sqrt{130}\right) + 12(323 + 40\sqrt{65})$$

Exact result:

$$12(323 + 40\sqrt{65}) + e^{1/2 \sqrt{65/2} \pi}$$

Decimal approximation:

15491.76743836655172138137828341012317278305215849229906315...

15491.7674383...

Property:

$12(323 + 40\sqrt{65}) + e^{1/2 \sqrt{65/2} \pi}$ is a transcendental number

Alternate form:

$$3876 + 480\sqrt{65} + e^{1/2 \sqrt{65/2} \pi}$$

Series representations:

$$\exp\left(\frac{\sqrt{130} \pi}{4}\right) + 12(323 + 40\sqrt{65}) = 3876 + \exp\left(\frac{1}{4} \pi \sqrt{129} \sum_{k=0}^{\infty} 129^{-k} \binom{\frac{1}{2}}{k}\right) + 480 \sqrt{64} \sum_{k=0}^{\infty} 64^{-k} \binom{\frac{1}{2}}{k}$$

$$\exp\left(\frac{\sqrt{130} \pi}{4}\right) + 12(323 + 40\sqrt{65}) = 3876 + \exp\left(\frac{1}{4} \pi \sqrt{129} \sum_{k=0}^{\infty} \frac{\left(-\frac{1}{129}\right)^k \left(-\frac{1}{2}\right)_k}{k!}\right) + 480 \sqrt{64} \sum_{k=0}^{\infty} \frac{\left(-\frac{1}{64}\right)^k \left(-\frac{1}{2}\right)_k}{k!}$$

$$\exp\left(\frac{\sqrt{130} \pi}{4}\right) + 12(323 + 40\sqrt{65}) = 3876 + \exp\left(\frac{1}{4} \pi \sqrt{z_0} \sum_{k=0}^{\infty} \frac{(-1)^k \left(-\frac{1}{2}\right)_k (130 - z_0)^k z_0^{-k}}{k!}\right) + 480 \sqrt{z_0} \sum_{k=0}^{\infty} \frac{(-1)^k \left(-\frac{1}{2}\right)_k (65 - z_0)^k z_0^{-k}}{k!} \text{ for not } ((z_0 \in \mathbb{R} \text{ and } -\infty < z_0 \leq 0))$$

From which we obtain:

$$144 - 29 + \frac{1}{3} * [\exp(((\pi/4)*\sqrt{130})) + 12(323+40\sqrt{65})]$$

Where 144 is a Fibonacci number and 29 is a Lucas number

Input:

$$144 - 29 + \frac{1}{3} \left(\exp\left(\frac{\pi}{4} \sqrt{130}\right) + 12(323 + 40\sqrt{65}) \right)$$

Exact result:

$$115 + \frac{1}{3} \left(12(323 + 40\sqrt{65}) + e^{1/2 \sqrt{65/2} \pi} \right)$$

Decimal approximation:

5278.922479455517240460459427803374390927684052830766354383...

5278.9224....result very near to the rest mass of B meson 5279.15

Property:

$115 + \frac{1}{3} \left(12 \left(323 + 40 \sqrt{65} \right) + e^{1/2 \sqrt{65/2} \pi} \right)$ is a transcendental number

Alternate forms:

$$1407 + 160 \sqrt{65} + \frac{1}{3} e^{1/2 \sqrt{65/2} \pi}$$

$$\frac{1}{3} \left(4221 + 480 \sqrt{65} + e^{1/2 \sqrt{65/2} \pi} \right)$$

$$115 + \frac{1}{3} \left(3876 + 480 \sqrt{65} + e^{1/2 \sqrt{65/2} \pi} \right)$$

Series representations:

$$144 - 29 + \frac{1}{3} \left(\exp \left(\frac{\pi \sqrt{130}}{4} \right) + 12 \left(323 + 40 \sqrt{65} \right) \right) =$$

$$\frac{1}{3} \left(4221 + \exp \left(\frac{1}{4} \pi \sqrt{129} \sum_{k=0}^{\infty} 129^{-k} \binom{\frac{1}{2}}{k} \right) + 480 \sqrt{64} \sum_{k=0}^{\infty} 64^{-k} \binom{\frac{1}{2}}{k} \right)$$

$$144 - 29 + \frac{1}{3} \left(\exp \left(\frac{\pi \sqrt{130}}{4} \right) + 12 \left(323 + 40 \sqrt{65} \right) \right) =$$

$$\frac{1}{3} \left(4221 + \exp \left(\frac{1}{4} \pi \sqrt{129} \sum_{k=0}^{\infty} \frac{\left(-\frac{1}{129}\right)^k \left(-\frac{1}{2}\right)_k}{k!} \right) + 480 \sqrt{64} \sum_{k=0}^{\infty} \frac{\left(-\frac{1}{64}\right)^k \left(-\frac{1}{2}\right)_k}{k!} \right)$$

$$144 - 29 + \frac{1}{3} \left(\exp \left(\frac{\pi \sqrt{130}}{4} \right) + 12 \left(323 + 40 \sqrt{65} \right) \right) =$$

$$\frac{1}{3} \left(4221 + \exp \left(\frac{1}{4} \pi \sqrt{z_0} \sum_{k=0}^{\infty} \frac{(-1)^k \left(-\frac{1}{2}\right)_k (130 - z_0)^k z_0^{-k}}{k!} \right) + \right.$$

$$\left. 480 \sqrt{z_0} \sum_{k=0}^{\infty} \frac{(-1)^k \left(-\frac{1}{2}\right)_k (65 - z_0)^k z_0^{-k}}{k!} \right) \text{ for not } ((z_0 \in \mathbb{R} \text{ and } -\infty < z_0 \leq 0))$$

We have also:

$$(64*32+64*16+64*3)+29+4((((\exp(((\text{Pi}/4)*\text{sqrt}(78)))) + 4\text{sqrt}(3)(75+52\text{sqrt}(2)))] + [\exp(((\text{Pi}/4)*\text{sqrt}(130))) + 12(323+40\text{sqrt}(65))]))))$$

Where 29 and 4 are Lucas numbers

Input:

$$(64 \times 32 + 64 \times 16 + 64 \times 3) + 29 + 4 \left(\left(\exp\left(\frac{\pi}{4} \sqrt{78}\right) + 4 \sqrt{3} (75 + 52 \sqrt{2}) \right) + \left(\exp\left(\frac{\pi}{4} \sqrt{130}\right) + 12 (323 + 40 \sqrt{65}) \right) \right)$$

Exact result:

$$3293 + 4 \left(4 \sqrt{3} (75 + 52 \sqrt{2}) + 12 (323 + 40 \sqrt{65}) + e^{1/2 \sqrt{39/2} \pi} + e^{1/2 \sqrt{65/2} \pi} \right)$$

Decimal approximation:

73492.94262352729862283942458665725584751834207244876174019...

73492.94262...

Alternate forms:

$$18797 + 1200 \sqrt{3} + 832 \sqrt{6} + 1920 \sqrt{65} + 4 e^{1/2 \sqrt{39/2} \pi} + 4 e^{1/2 \sqrt{65/2} \pi}$$

$$1200 \sqrt{3} + 832 \sqrt{6} + 1920 \sqrt{65} + 4 e^{1/2 \sqrt{65/2} \pi} + 4 e^{1/2 \sqrt{39/2} \pi} + 18797$$

$$3293 + 4 \left(4 \left(969 + \sqrt{3 \left(323033 + 6000 \sqrt{195} + 520 \sqrt{30 (847 + 16 \sqrt{195})} \right)} \right) + e^{1/2 \sqrt{39/2} \pi} + e^{1/2 \sqrt{65/2} \pi} \right)$$

Series representations:

$$(64 \times 32 + 64 \times 16 + 64 \times 3) + 29 + 4 \left(\left(\exp\left(\frac{\sqrt{78} \pi}{4}\right) + 4 \sqrt{3} (75 + 52 \sqrt{2}) \right) + \left(\exp\left(\frac{\sqrt{130} \pi}{4}\right) + 12 (323 + 40 \sqrt{65}) \right) \right) =$$

$$18797 + 4 \exp\left(\frac{1}{4} \pi \sqrt{z_0} \sum_{k=0}^{\infty} \frac{(-1)^k \left(-\frac{1}{2}\right)_k (78 - z_0)^k z_0^{-k}}{k!}\right) +$$

$$4 \exp\left(\frac{1}{4} \pi \sqrt{z_0} \sum_{k=0}^{\infty} \frac{(-1)^k \left(-\frac{1}{2}\right)_k (130 - z_0)^k z_0^{-k}}{k!}\right) +$$

$$\sum_{k=0}^{\infty} \frac{240 (-1)^k \left(-\frac{1}{2}\right)_k \sqrt{z_0} (5 (3 - z_0)^k + 8 (65 - z_0)^k) z_0^{-k}}{k!} +$$

$$832 \sqrt{z_0}^2 \sum_{k_1=0}^{\infty} \sum_{k_2=0}^{\infty} \frac{(-1)^{k_1+k_2} \left(-\frac{1}{2}\right)_{k_1} \left(-\frac{1}{2}\right)_{k_2} (2 - z_0)^{k_1} (3 - z_0)^{k_2} z_0^{-k_1-k_2}}{k_1! k_2!}$$

for not $((z_0 \in \mathbb{R} \text{ and } -\infty < z_0 \leq 0))$

$$\begin{aligned}
& (64 \times 32 + 64 \times 16 + 64 \times 3) + 29 + 4 \left(\exp\left(\frac{\sqrt{78} \pi}{4}\right) + 4 \sqrt{3} (75 + 52 \sqrt{2}) \right) + \\
& \left(\exp\left(\frac{\sqrt{130} \pi}{4}\right) + 12 (323 + 40 \sqrt{65}) \right) = \\
& 18\,797 + 4 \exp\left(\frac{1}{4} \pi \exp\left(i \pi \left\lfloor \frac{\arg(78-x)}{2\pi} \right\rfloor\right)\right) \sqrt{x} \sum_{k=0}^{\infty} \frac{(-1)^k (78-x)^k x^{-k} \left(-\frac{1}{2}\right)_k}{k!} + \\
& 4 \exp\left(\frac{1}{4} \pi \exp\left(i \pi \left\lfloor \frac{\arg(130-x)}{2\pi} \right\rfloor\right)\right) \sqrt{x} \sum_{k=0}^{\infty} \frac{(-1)^k (130-x)^k x^{-k} \left(-\frac{1}{2}\right)_k}{k!} + \sum_{k=0}^{\infty} \frac{1}{k!} 240 \\
& (-1)^k x^{-k} \left(5 (3-x)^k \exp\left(i \pi \left\lfloor \frac{\arg(3-x)}{2\pi} \right\rfloor\right) + 8 (65-x)^k \exp\left(i \pi \left\lfloor \frac{\arg(65-x)}{2\pi} \right\rfloor\right) \right) \\
& \left(-\frac{1}{2}\right)_k \sqrt{x} + 832 \exp\left(i \pi \left\lfloor \frac{\arg(2-x)}{2\pi} \right\rfloor\right) \exp\left(i \pi \left\lfloor \frac{\arg(3-x)}{2\pi} \right\rfloor\right) \\
& \sqrt{x}^2 \sum_{k_1=0}^{\infty} \sum_{k_2=0}^{\infty} \frac{(-1)^{k_1+k_2} (2-x)^{k_1} (3-x)^{k_2} x^{-k_1-k_2} \left(-\frac{1}{2}\right)_{k_1} \left(-\frac{1}{2}\right)_{k_2}}{k_1! k_2!}
\end{aligned}$$

for $(x \in \mathbb{R} \text{ and } x < 0)$

$$\begin{aligned}
& (64 \times 32 + 64 \times 16 + 64 \times 3) + 29 + \\
& 4 \left(\exp\left(\frac{\sqrt{78} \pi}{4}\right) + 4 \sqrt{3} (75 + 52 \sqrt{2}) \right) + \left(\exp\left(\frac{\sqrt{130} \pi}{4}\right) + 12 (323 + 40 \sqrt{65}) \right) = \\
& 18\,797 + 4 \exp\left(\frac{1}{4} \pi \left(\frac{1}{z_0}\right)^{1/2 \lfloor \arg(78-z_0)/(2\pi) \rfloor} z_0^{1/2 (1 + \lfloor \arg(78-z_0)/(2\pi) \rfloor)} \right. \\
& \left. \sum_{k=0}^{\infty} \frac{(-1)^k \left(-\frac{1}{2}\right)_k (78-z_0)^k z_0^{-k}}{k!} \right) + 4 \exp\left(\frac{1}{4} \pi \left(\frac{1}{z_0}\right)^{1/2 \lfloor \arg(130-z_0)/(2\pi) \rfloor} \right. \\
& \left. z_0^{1/2 (1 + \lfloor \arg(130-z_0)/(2\pi) \rfloor)} \sum_{k=0}^{\infty} \frac{(-1)^k \left(-\frac{1}{2}\right)_k (130-z_0)^k z_0^{-k}}{k!} \right) + \\
& \sum_{k=0}^{\infty} \frac{1}{k!} 240 (-1)^k \left(-\frac{1}{2}\right)_k z_0^{1/2-k} \left(5 (3-z_0)^k \left(\frac{1}{z_0}\right)^{1/2 \lfloor \arg(3-z_0)/(2\pi) \rfloor} z_0^{1/2 \lfloor \arg(3-z_0)/(2\pi) \rfloor} + \right. \\
& \left. 8 (65-z_0)^k \left(\frac{1}{z_0}\right)^{1/2 \lfloor \arg(65-z_0)/(2\pi) \rfloor} z_0^{1/2 \lfloor \arg(65-z_0)/(2\pi) \rfloor} \right) + \\
& 832 \left(\frac{1}{z_0}\right)^{1/2 \lfloor \arg(2-z_0)/(2\pi) \rfloor + 1/2 \lfloor \arg(3-z_0)/(2\pi) \rfloor} z_0^{1 + 1/2 \lfloor \arg(2-z_0)/(2\pi) \rfloor + 1/2 \lfloor \arg(3-z_0)/(2\pi) \rfloor} \\
& \sum_{k_1=0}^{\infty} \sum_{k_2=0}^{\infty} \frac{(-1)^{k_1+k_2} \left(-\frac{1}{2}\right)_{k_1} \left(-\frac{1}{2}\right)_{k_2} (2-z_0)^{k_1} (3-z_0)^{k_2} z_0^{-k_1-k_2}}{k_1! k_2!}
\end{aligned}$$

Thence, we obtain the following mathematical connection:

$$\left(3293 + 4 \left(4 \sqrt{3} \left(75 + 52 \sqrt{2} \right) + 12 \left(323 + 40 \sqrt{65} \right) + e^{1/2 \sqrt{39/2} \pi} + e^{1/2 \sqrt{65/2} \pi} \right) \right) = 73492.94262 \Rightarrow$$

$$\Rightarrow -3927 + 2 \left(\sqrt[13]{ N \exp \left[\int d\hat{\sigma} \left(-\frac{1}{4u^2} \mathbf{P}_i D \mathbf{P}_i \right) \right] |Bp\rangle_{NS} + \int [d\mathbf{X}^\mu] \exp \left\{ \int d\hat{\sigma} \left(-\frac{1}{4v^2} D \mathbf{X}^\mu D^2 \mathbf{X}^\mu \right) \right\} | \mathbf{X}^\mu, \mathbf{X}^i = 0 \rangle_{NS} } \right) =$$

$$-3927 + 2 \sqrt[13]{ 2.2983717437 \times 10^{50} + 2.0823329825883 \times 10^{50} }$$

$$= 73490.8437525 \dots \Rightarrow$$

$$\Rightarrow \left(A(r) \times \frac{1}{B(r)} \left(-\frac{1}{\phi(r)} \right) \times \frac{1}{e^{\Lambda(r)}} \right) \Rightarrow$$

$$\Rightarrow \left(-0.000029211892 \times \frac{1}{0.0003644621} \left(-\frac{1}{0.0005946833} \right) \times \frac{1}{0.00183393} \right) =$$

$$= 73491.78832548118710549159572042220548025195726563413398700 \dots$$

$$= 73491.7883254 \dots \Rightarrow$$

$$\left(I_{21} \ll \int_{-\infty}^{+\infty} \exp \left(-\left(\frac{t}{H} \right)^2 \right) \left| \sum_{\lambda \leq p^{1-\epsilon_1}} \frac{a(\lambda)}{\sqrt{\lambda}} B(\lambda) \lambda^{-i(T+t)} \right|^2 dt \ll \right.$$

$$\left. \ll H \left\{ \left(\frac{4}{\epsilon_2 \log T} \right)^{2r} (\log T) (\log X)^{-2\beta} + (\epsilon_2^{-2r} (\log T)^{-2r} + \epsilon_2^{-r} h_1^r (\log T)^{-r} \right) T^{-\epsilon_1} \right\} \right)$$

$$/(26 \times 4)^2 - 24 = \left(\frac{7.9313976505275 \times 10^8}{(26 \times 4)^2 - 24} \right) = 73493.30662 \dots$$

Mathematical connections with the boundary state corresponding to the NSNS-sector of N Dp-branes in the limit of $u \rightarrow \infty$, with the ratio concerning the general asymptotically flat solution of the equations of motion of the p-brane and with the Karatsuba's equation concerning the zeros of a special type of function connected with Dirichlet series.

Now, we have that:

$$\begin{aligned}
 K_1 &= \frac{4\pi}{3} & K_2 &= \frac{4\pi^2}{45} \\
 Q_n &= K_n \omega^{3n} \\
 K_{n+1} \omega^{3n+3} &= 4\pi \int_0^\omega p^2 dp K_n (\omega - p)^{3n} \\
 \frac{K_{n+1}}{K_n} &= 4\pi \int_0^1 (1-x)^{3n} x^2 dx \\
 &= 4\pi \int_0^1 y^{3n} (1-2y+y^2) dy = \\
 &= 4\pi \left(\frac{1}{3n+1} - \frac{2}{3n+2} + \frac{1}{3n+3} \right) \\
 &= 4\pi \frac{9n^2 + 15n + 6 + 9n^2 + 9n + 2 - 18n}{(3n+1)(3n+2)(3n+3)} \\
 &= \frac{8\pi}{(3n+1)(3n+2)(3n+3)}
 \end{aligned}$$

$\frac{20 - 45 + 36 + 10}{60} = \frac{4}{15}$
 $\frac{2(9n^2 + 12n + 3)}{18n}$

We analyze:

$$\begin{aligned}
 \frac{K_{n+1}}{K_n} &= 4\pi \int_0^1 (1-x)^{3n} x^2 dx \\
 &= \frac{8\pi}{(3n+1)(3n+2)(3n+3)}
 \end{aligned}$$

For $n = 2$, we obtain:

$$(8\pi) / (((3 \times 2 + 1)(3 \times 2 + 2)(3 \times 3 + 3)))$$

Input:

$$\frac{8\pi}{(3 \times 2 + 1)(3 \times 2 + 2)(3 \times 3 + 3)}$$

Result:

$$\frac{\pi}{84}$$

Decimal approximation:

0.037399912542735633791221945039041701002347254754465545487...

0.0373999125427....

Property:

$\frac{\pi}{84}$ is a transcendental number

Alternative representations:

$$\frac{8\pi}{(3 \times 2 + 1)(3 \times 2 + 2)(3 \times 3 + 3)} = \frac{1440^\circ}{672}$$

$$\frac{8\pi}{(3 \times 2 + 1)(3 \times 2 + 2)(3 \times 3 + 3)} = -\frac{8}{672} i \log(-1)$$

$$\frac{8\pi}{(3 \times 2 + 1)(3 \times 2 + 2)(3 \times 3 + 3)} = \frac{8}{672} \cos^{-1}(-1)$$

Series representations:

$$\frac{8\pi}{(3 \times 2 + 1)(3 \times 2 + 2)(3 \times 3 + 3)} = \frac{1}{21} \sum_{k=0}^{\infty} \frac{(-1)^k}{1+2k}$$

$$\frac{8\pi}{(3 \times 2 + 1)(3 \times 2 + 2)(3 \times 3 + 3)} = \sum_{k=0}^{\infty} -\frac{(-1)^k 1195^{-1-2k} (5^{1+2k} - 4 \times 239^{1+2k})}{21(1+2k)}$$

$$\frac{8\pi}{(3 \times 2 + 1)(3 \times 2 + 2)(3 \times 3 + 3)} = \frac{1}{84} \sum_{k=0}^{\infty} \left(-\frac{1}{4}\right)^k \left(\frac{1}{1+2k} + \frac{2}{1+4k} + \frac{1}{3+4k}\right)$$

Integral representations:

$$\frac{8\pi}{(3 \times 2 + 1)(3 \times 2 + 2)(3 \times 3 + 3)} = \frac{1}{21} \int_0^1 \sqrt{1-t^2} dt$$

$$\frac{8\pi}{(3 \times 2 + 1)(3 \times 2 + 2)(3 \times 3 + 3)} = \frac{1}{42} \int_0^1 \frac{1}{\sqrt{1-t^2}} dt$$

$$\frac{8\pi}{(3 \times 2 + 1)(3 \times 2 + 2)(3 \times 3 + 3)} = \frac{1}{42} \int_0^\infty \frac{1}{1+t^2} dt$$

And for the previous Ramanujan formula

$$\begin{aligned} \cos 2x - \left(1 + \frac{1}{3}\right) \cos 4x + \left(1 + \frac{1}{3} + \frac{1}{5}\right) \cos 6x - \&c \\ = \frac{\pi}{4} (\cos x - \cos 3x + \cos 5x - \&c), \end{aligned}$$

For $n = 0.45418 \approx 5/11$ we obtain

$$\pi/4 * ((\cos(0.45418) - \cos(3*0.45418) + \cos(5*0.45418)))$$

Input:

$$\frac{\pi}{4} (\cos(0.45418) - \cos(3 \times 0.45418) + \cos(5 \times 0.45418))$$

Result:

0.037361304485236103253372193064995066345212530662528957648...

0.0373613044852....

Rational approximation:

$$\frac{4374}{117073}$$

Alternative representations:

$$\begin{aligned} \frac{1}{4} (\cos(0.45418) - \cos(3 \times 0.45418) + \cos(5 \times 0.45418)) \pi = \\ \frac{1}{4} \pi (\cosh(0.45418 i) - \cosh(1.36254 i) + \cosh(2.2709 i)) \end{aligned}$$

$$\frac{1}{4} (\cos(0.45418) - \cos(3 \times 0.45418) + \cos(5 \times 0.45418)) \pi =$$

$$\frac{1}{4} \pi \left(\frac{1}{\sec(0.45418)} - \frac{1}{\sec(1.36254)} + \frac{1}{\sec(2.2709)} \right)$$

$$\frac{1}{4} (\cos(0.45418) - \cos(3 \times 0.45418) + \cos(5 \times 0.45418)) \pi =$$

$$\frac{1}{4} \pi (\cosh(-0.45418 i) - \cosh(-1.36254 i) + \cosh(-2.2709 i))$$

Series representations:

$$\frac{1}{4} (\cos(0.45418) - \cos(3 \times 0.45418) + \cos(5 \times 0.45418)) \pi =$$

$$\sum_{k=0}^{\infty} \frac{(-1)^k e^{-1.57852k} (1 - e^{2.19722k} + e^{3.21888k}) \pi}{4(2k)!}$$

$$\frac{1}{4} (\cos(0.45418) - \cos(3 \times 0.45418) + \cos(5 \times 0.45418)) \pi =$$

$$\sum_{k=0}^{\infty} \frac{\pi \cos\left(\frac{k\pi}{2} + z_0\right) \left((0.45418 - z_0)^k - (1.36254 - z_0)^k + (2.2709 - z_0)^k \right)}{4k!}$$

$$\frac{1}{4} (\cos(0.45418) - \cos(3 \times 0.45418) + \cos(5 \times 0.45418)) \pi =$$

$$\sum_{k=0}^{\infty} \frac{(-1)^k \left(-\left(0.45418 - \frac{\pi}{2}\right)^{1+2k} + \left(1.36254 - \frac{\pi}{2}\right)^{1+2k} - \left(2.2709 - \frac{\pi}{2}\right)^{1+2k} \right) \pi}{4(1+2k)!}$$

Integral representations:

$$\frac{1}{4} (\cos(0.45418) - \cos(3 \times 0.45418) + \cos(5 \times 0.45418)) \pi =$$

$$0.25 \pi + \int_0^1 \pi (-0.113545 \sin(0.45418 t) +$$

$$0.340635 \sin(1.36254 t) - 0.567725 \sin(2.2709 t)) dt$$

$$\frac{1}{4} (\cos(0.45418) - \cos(3 \times 0.45418) + \cos(5 \times 0.45418)) \pi =$$

$$\int_{-i\infty+\gamma}^{i\infty+\gamma} \frac{e^{-1.80495/s+s} (e^{0.515699/s} - e^{1.34082/s} + e^{1.75338/s}) \sqrt{\pi}}{8i\sqrt{s}} ds \text{ for } \gamma > 0$$

$$\frac{1}{4} (\cos(0.45418) - \cos(3 \times 0.45418) + \cos(5 \times 0.45418)) \pi =$$

$$\int_{-i\infty+\gamma}^{i\infty+\gamma} \frac{e^{-0.254058s} (-1 + e^{1.02165s} - e^{3.21888s}) \Gamma(s) \sqrt{\pi}}{8i\Gamma\left(\frac{1}{2} - s\right)} ds \text{ for } 0 < \gamma < \frac{1}{2}$$

$$\frac{1}{4} (\cos(0.45418) - \cos(3 \times 0.45418) + \cos(5 \times 0.45418)) \pi =$$

$$\int_{\frac{\pi}{2}}^{2.2709} \frac{1}{-4.5418 + \pi} \pi \left((0.22709 - 0.25 \pi) \sin\left(\frac{\pi(-1.81672 + t) - 0.90836 t}{-4.5418 + \pi}\right) + \right.$$

$$\left. (1.13545 - 0.25 \pi) \sin(t) + (-0.68127 + 0.25 \pi) \sin\left(\frac{\pi(0.825118 - 3.63344 t) + \pi^2(-0.90836 + t) + 2.47535 t}{(-4.5418 + \pi)(-0.90836 + \pi)}\right) \right) dt$$

Or:

$$\pi/4 * ((\cos(5/11) - \cos(3*5/11) + \cos(5*5/11)))$$

Input:

$$\frac{\pi}{4} \left(\cos\left(\frac{5}{11}\right) - \cos\left(3 \times \frac{5}{11}\right) + \cos\left(5 \times \frac{5}{11}\right) \right)$$

Exact result:

$$\frac{1}{4} \pi \left(\cos\left(\frac{5}{11}\right) - \cos\left(\frac{15}{11}\right) + \cos\left(\frac{25}{11}\right) \right)$$

Decimal approximation:

0.036981193275882485818809620579675010044251744554803733743...

0.03698119327...

Alternate forms:

$$\frac{1}{4} \pi \cos\left(\frac{5}{11}\right) \left(1 - 2 \cos\left(\frac{10}{11}\right)\right)^2$$

$$\frac{1}{4} \left(\pi \cos\left(\frac{5}{11}\right) - \pi \cos\left(\frac{15}{11}\right) + \pi \cos\left(\frac{25}{11}\right) \right)$$

$$\frac{1}{4} \pi \left(\cos\left(\frac{5}{11}\right) - \cos\left(\frac{15}{11}\right) \right) + \frac{1}{4} \pi \cos\left(\frac{25}{11}\right)$$

Alternative representations:

$$\frac{1}{4} \left(\cos\left(\frac{5}{11}\right) - \cos\left(\frac{3 \times 5}{11}\right) + \cos\left(\frac{5 \times 5}{11}\right) \right) \pi = \frac{1}{4} \pi \left(\cosh\left(\frac{5i}{11}\right) - \cosh\left(\frac{15i}{11}\right) + \cosh\left(\frac{25i}{11}\right) \right)$$

$$\frac{1}{4} \left(\cos\left(\frac{5}{11}\right) - \cos\left(\frac{3 \times 5}{11}\right) + \cos\left(\frac{5 \times 5}{11}\right) \right) \pi =$$

$$\frac{1}{4} \pi \left(\cosh\left(-\frac{5i}{11}\right) - \cosh\left(-\frac{15i}{11}\right) + \cosh\left(-\frac{25i}{11}\right) \right)$$

$$\frac{1}{4} \left(\cos\left(\frac{5}{11}\right) - \cos\left(\frac{3 \times 5}{11}\right) + \cos\left(\frac{5 \times 5}{11}\right) \right) \pi = \frac{1}{4} \pi \left(\frac{1}{\sec\left(\frac{5}{11}\right)} - \frac{1}{\sec\left(\frac{15}{11}\right)} + \frac{1}{\sec\left(\frac{25}{11}\right)} \right)$$

Series representations:

$$\frac{1}{4} \left(\cos\left(\frac{5}{11}\right) - \cos\left(\frac{3 \times 5}{11}\right) + \cos\left(\frac{5 \times 5}{11}\right) \right) \pi = \sum_{k=0}^{\infty} \frac{\left(\frac{25}{121}\right)^k (1 - 9^k + 25^k) e^{i k \pi} \pi}{4 (2k)!}$$

$$\frac{1}{4} \left(\cos\left(\frac{5}{11}\right) - \cos\left(\frac{3 \times 5}{11}\right) + \cos\left(\frac{5 \times 5}{11}\right) \right) \pi = \sum_{k=0}^{\infty} \frac{\pi \cos\left(\frac{k\pi}{2} + z_0\right) \left(\left(\frac{5}{11} - z_0\right)^k - \left(\frac{15}{11} - z_0\right)^k + \left(\frac{25}{11} - z_0\right)^k \right)}{4 k!}$$

$$\frac{1}{4} \left(\cos\left(\frac{5}{11}\right) - \cos\left(\frac{3 \times 5}{11}\right) + \cos\left(\frac{5 \times 5}{11}\right) \right) \pi = \sum_{k=0}^{\infty} \left(\frac{1}{4} \pi \left(\frac{(-1)^{-1+k} \left(\frac{5}{11} - \frac{\pi}{2}\right)^{1+2k}}{(1+2k)!} + \frac{(-1)^{-1+k} \left(\frac{25}{11} - \frac{\pi}{2}\right)^{1+2k}}{(1+2k)!} \right) - \frac{(-1)^{-1+k} \left(\frac{15}{11} - \frac{\pi}{2}\right)^{1+2k} \pi}{4 (1+2k)!} \right)$$

Integral representations:

$$\frac{1}{4} \left(\cos\left(\frac{5}{11}\right) - \cos\left(\frac{3 \times 5}{11}\right) + \cos\left(\frac{5 \times 5}{11}\right) \right) \pi = \frac{\pi}{4} + \int_0^1 -\frac{5}{44} \pi \left(\sin\left(\frac{5t}{11}\right) - 3 \sin\left(\frac{15t}{11}\right) + 5 \sin\left(\frac{25t}{11}\right) \right) dt$$

$$\frac{1}{4} \left(\cos\left(\frac{5}{11}\right) - \cos\left(\frac{3 \times 5}{11}\right) + \cos\left(\frac{5 \times 5}{11}\right) \right) \pi = \int_{-i\infty+\gamma}^{i\infty+\gamma} \frac{i e^{-625/(484s)+s} (1 - e^{100/(121s)} + e^{150/(121s)}) \sqrt{\pi}}{8 \sqrt{s}} ds \text{ for } \gamma > 0$$

$$\frac{1}{4} \left(\cos\left(\frac{5}{11}\right) - \cos\left(\frac{3 \times 5}{11}\right) + \cos\left(\frac{5 \times 5}{11}\right) \right) \pi = \int_{-i\infty+\gamma}^{i\infty+\gamma} \frac{i \left(\frac{121}{625}\right)^s 2^{-3+2s} \left(-1 + \left(\frac{25}{9}\right)^s - 25^s\right) \sqrt{\pi} \Gamma(s)}{\Gamma\left(\frac{1}{2} - s\right)} ds \text{ for } 0 < \gamma < \frac{1}{2}$$

$$\frac{1}{4} \left(\cos\left(\frac{5}{11}\right) - \cos\left(\frac{3 \times 5}{11}\right) + \cos\left(\frac{5 \times 5}{11}\right) \right) \pi =$$

$$\int_{\frac{\pi}{2}}^{\frac{25}{11}} \left(-\frac{1}{4} \pi \sin(t) + \frac{1}{\frac{25}{11} - \frac{\pi}{2}} \left(\frac{5}{11} - \frac{\pi}{2} \right) \left(-\frac{1}{4} \pi \sin\left(\frac{-\frac{10\pi}{11} - \frac{5t}{11} + \frac{\pi t}{2} \right) + \right. \right.$$

$$\left. \left. \frac{\left(\frac{15}{11} - \frac{\pi}{2} \right) \pi \sin\left(\frac{\frac{5\pi}{11} - \frac{15\left(-\frac{10\pi}{11} - \frac{5t}{11} + \frac{\pi t}{2}\right) + \pi\left(-\frac{10\pi}{11} - \frac{5t}{11} + \frac{\pi t}{2}\right)}{11\left(-\frac{25}{11} + \frac{\pi}{2}\right) + 2\left(-\frac{25}{11} + \frac{\pi}{2}\right)}{-\frac{5}{11} + \frac{\pi}{2}} \right)}{4\left(\frac{5}{11} - \frac{\pi}{2}\right)} \right) dt$$

We note that the two results 0.0373999125427.... and 0.0373613044852.... are very near. Furthermore:

$$\left(\frac{8\pi}{((3 \times 2 + 1)(3 \times 2 + 2)(3 \times 3 + 3))} \right)^{1/256}$$

Input:

$$\sqrt[256]{\frac{8\pi}{(3 \times 2 + 1)(3 \times 2 + 2)(3 \times 3 + 3)}}$$

Exact result:

$$\frac{\sqrt[256]{\frac{\pi}{21}}}{\sqrt[128]{2}}$$

Decimal approximation:

0.987245756622518632898983325424964734972583221117935454100...

0.9872457566225...

Property:

$$\frac{256\sqrt{\frac{\pi}{21}}}{128\sqrt[2]{2}} \text{ is a transcendental number}$$

All 256th roots of $\pi/84$:

$$\frac{256\sqrt{\frac{\pi}{21}} e^0}{128\sqrt[2]{2}} \approx 0.987246 \text{ (real, principal root)}$$

$$\frac{256\sqrt{\frac{\pi}{21}} e^{(i\pi)/128}}{128\sqrt[2]{2}} \approx 0.986948 + 0.024228 i$$

$$\frac{256\sqrt{\frac{\pi}{21}} e^{(i\pi)/64}}{128\sqrt[2]{2}} \approx 0.986057 + 0.048442 i$$

$$\frac{256\sqrt{\frac{\pi}{21}} e^{(3i\pi)/128}}{128\sqrt[2]{2}} \approx 0.984571 + 0.07263 i$$

$$\frac{256\sqrt{\frac{\pi}{21}} e^{(i\pi)/32}}{128\sqrt[2]{2}} \approx 0.982492 + 0.09677 i$$

Alternative representations:

$$256\sqrt{\frac{8\pi}{(3 \times 2 + 1)(3 \times 2 + 2)(3 \times 3 + 3)}} = 256\sqrt{\frac{1440^\circ}{672}}$$

$$256\sqrt{\frac{8\pi}{(3 \times 2 + 1)(3 \times 2 + 2)(3 \times 3 + 3)}} = 256\sqrt{-\frac{8}{672} i \log(-1)}$$

$$256\sqrt{\frac{8\pi}{(3 \times 2 + 1)(3 \times 2 + 2)(3 \times 3 + 3)}} = 256\sqrt{\frac{8}{672} \cos^{-1}(-1)}$$

Series representations:

$$\sqrt[256]{\frac{8\pi}{(3 \times 2 + 1)(3 \times 2 + 2)(3 \times 3 + 3)}} = \frac{\sqrt[256]{\sum_{k=0}^{\infty} \frac{(-1)^k}{1+2k}}}{\sqrt[256]{21}}$$

$$\sqrt[256]{\frac{8\pi}{(3 \times 2 + 1)(3 \times 2 + 2)(3 \times 3 + 3)}} = \frac{\sqrt[256]{\sum_{k=0}^{\infty} \frac{(-1)^{1+k} 1195^{-1-2k} (5^{1+2k} - 4 \times 239^{1+2k})}{1+2k}}}{\sqrt[256]{21}}$$

$$\sqrt[256]{\frac{8\pi}{(3 \times 2 + 1)(3 \times 2 + 2)(3 \times 3 + 3)}} = \frac{\sqrt[256]{\sum_{k=0}^{\infty} \left(-\frac{1}{4}\right)^k \left(\frac{1}{1+2k} + \frac{2}{1+4k} + \frac{1}{3+4k}\right)}}{\sqrt[128]{2} \sqrt[256]{21}}$$

Integral representations:

$$\sqrt[256]{\frac{8\pi}{(3 \times 2 + 1)(3 \times 2 + 2)(3 \times 3 + 3)}} = \frac{\sqrt[256]{\int_0^1 \sqrt{1-t^2} dt}}{\sqrt[256]{21}}$$

$$\sqrt[256]{\frac{8\pi}{(3 \times 2 + 1)(3 \times 2 + 2)(3 \times 3 + 3)}} = \frac{\sqrt[256]{\int_0^{\infty} \frac{1}{1+t^2} dt}}{\sqrt[256]{42}}$$

$$\sqrt[256]{\frac{8\pi}{(3 \times 2 + 1)(3 \times 2 + 2)(3 \times 3 + 3)}} = \frac{\sqrt[256]{\int_0^1 \frac{1}{\sqrt{1-t^2}} dt}}{\sqrt[256]{42}}$$

And:

$$\left(\left(\left(\left(\left(\left(\frac{\pi}{4} \left(\cos(0.45418) - \cos(3 \times 0.45418) + \cos(5 \times 0.45418)\right)\right)\right)\right)\right)\right)\right)^{1/256}$$

Input:

$$\sqrt[256]{\frac{\pi}{4} (\cos(0.45418) - \cos(3 \times 0.45418) + \cos(5 \times 0.45418))}$$

Result:

0.987241773569774971127901758550986993748276054428144565796...

0.9872417735697.....

Thence, we obtain the following mathematical connection:

$$\left(\sqrt[256]{\frac{8\pi}{(3 \times 2 + 1)(3 \times 2 + 2)(3 \times 3 + 3)}} \right) = 0.9872457566225 \dots \Rightarrow$$

$$\Rightarrow \left(\sqrt[256]{\frac{\pi}{4} (\cos(0.45418) - \cos(3 \times 0.45418) + \cos(5 \times 0.45418))} \right) = 0.9872417735697 \dots$$

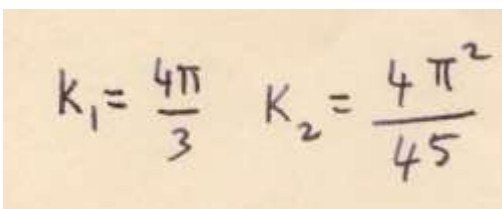
$0.9872457566225 \approx 0.9872417735697 \dots$ results also very near to the value of the following Rogers-Ramanujan continued fraction:

$$\frac{e^{-\frac{\pi}{\sqrt{5}}}}{\sqrt{5}} = 1 - \frac{e^{-\pi\sqrt{5}}}{1 + \frac{e^{-2\pi\sqrt{5}}}{1 + \frac{e^{-3\pi\sqrt{5}}}{1 + \frac{e^{-4\pi\sqrt{5}}}{1 + \dots}}}} \approx 0.9991104684$$

$$\frac{1 + \sqrt[5]{\sqrt{\varphi^5 \sqrt[4]{5^3}} - 1}}{\sqrt{5}} - \varphi + 1$$

and to the dilaton value $0.989117352243 = \phi$

Now:



$$K_1 = \frac{4\pi}{3} \quad K_2 = \frac{4\pi^2}{45}$$

$$4\pi/3 + (4\pi^2)/45$$

Input:

$$4 \times \frac{\pi}{3} + \frac{1}{45} (4\pi^2)$$

Result:

$$\frac{4\pi}{3} + \frac{4\pi^2}{45}$$

Decimal approximation:

5.066088373772111750735479266583883946512999146477100261425...

5.066088373...

Property:

$\frac{4\pi}{3} + \frac{4\pi^2}{45}$ is a transcendental number

Alternate form:

$$\frac{4}{45} \pi (15 + \pi)$$

Alternative representations:

$$\frac{4\pi}{3} + \frac{4\pi^2}{45} = 240^\circ + \frac{4}{45} (180^\circ)^2$$

$$\frac{4\pi}{3} + \frac{4\pi^2}{45} = \frac{4}{3} \cos^{-1}(-1) + \frac{4}{45} \cos^{-1}(-1)^2$$

$$\frac{4\pi}{3} + \frac{4\pi^2}{45} = -\frac{4}{3} i \log(-1) + \frac{4}{45} (-i \log(-1))^2$$

Series representations:

$$\frac{4\pi}{3} + \frac{4\pi^2}{45} = \frac{4\pi}{3} + \frac{8}{15} \sum_{k=1}^{\infty} \frac{1}{k^2}$$

$$\frac{4\pi}{3} + \frac{4\pi^2}{45} = \frac{4\pi}{3} - \frac{16}{15} \sum_{k=1}^{\infty} \frac{(-1)^k}{k^2}$$

$$\frac{4\pi}{3} + \frac{4\pi^2}{45} = \frac{4\pi}{3} + \frac{32}{45} \sum_{k=0}^{\infty} \frac{1}{(1+2k)^2}$$

Integral representations:

$$\frac{4\pi}{3} + \frac{4\pi^2}{45} = \frac{16}{45} \left(\int_0^1 \sqrt{1-t^2} dt \right) \left(15 + 4 \int_0^1 \sqrt{1-t^2} dt \right)$$

$$\frac{4\pi}{3} + \frac{4\pi^2}{45} = \frac{8}{45} \left(\int_0^1 \frac{1}{\sqrt{1-t^2}} dt \right) \left(15 + 2 \int_0^1 \frac{1}{\sqrt{1-t^2}} dt \right)$$

$$\frac{4\pi}{3} + \frac{4\pi^2}{45} = \frac{8}{45} \left(\int_0^\infty \frac{1}{1+t^2} dt \right) \left(15 + 2 \int_0^\infty \frac{1}{1+t^2} dt \right)$$

And:

$$\left(\left(\frac{4\pi}{3} + \frac{4\pi^2}{45} \right) \right)^3 + 5$$

Input:

$$\left(4 \times \frac{\pi}{3} + \frac{1}{45} (4\pi^2) \right)^3 + 5$$

Result:

$$5 + \left(\frac{4\pi}{3} + \frac{4\pi^2}{45} \right)^3$$

Decimal approximation:

135.0224317825415255515361914289691258146464964409024574509...

135.02243178.... result very near to the rest mass of Pion meson 134.9766

Property:

$5 + \left(\frac{4\pi}{3} + \frac{4\pi^2}{45} \right)^3$ is a transcendental number

Alternate forms:

$$\frac{216\,000\pi^3 + 43\,200\pi^4 + 2880\pi^5 + 64\pi^6 + 455\,625}{91\,125}$$

$$5 + \frac{64\pi^3}{27} + \frac{64\pi^4}{135} + \frac{64\pi^5}{2025} + \frac{64\pi^6}{91\,125}$$

Alternative representations:

$$\left(\frac{4\pi}{3} + \frac{4\pi^2}{45}\right)^3 + 5 = 5 + \left(240^\circ + \frac{4}{45}(180^\circ)^2\right)^3$$

$$\left(\frac{4\pi}{3} + \frac{4\pi^2}{45}\right)^3 + 5 = 5 + \left(\frac{4\pi}{3} + \frac{24\zeta(2)}{45}\right)^3$$

$$\left(\frac{4\pi}{3} + \frac{4\pi^2}{45}\right)^3 + 5 = 5 + \left(\frac{4}{3}\cos^{-1}(-1) + \frac{4}{45}\cos^{-1}(-1)^2\right)^3$$

Series representations:

$$\left(\frac{4\pi}{3} + \frac{4\pi^2}{45}\right)^3 + 5 = 5 + \left(\frac{4\pi}{3} + \frac{8}{15}\sum_{k=1}^{\infty}\frac{1}{k^2}\right)^3$$

$$\left(\frac{4\pi}{3} + \frac{4\pi^2}{45}\right)^3 + 5 = 5 + \left(\frac{4\pi}{3} - \frac{16}{15}\sum_{k=1}^{\infty}\frac{(-1)^k}{k^2}\right)^3$$

$$\left(\frac{4\pi}{3} + \frac{4\pi^2}{45}\right)^3 + 5 = 5 + \left(\frac{4\pi}{3} + \frac{32}{45}\sum_{k=0}^{\infty}\frac{1}{(1+2k)^2}\right)^3$$

Integral representations:

$$\left(\frac{4\pi}{3} + \frac{4\pi^2}{45}\right)^3 + 5 = \frac{\left(\sqrt{3} + 32\int_0^{\frac{1}{4}}\sqrt{-(-1+t)t} dt\right)^3 \left(20 + \sqrt{3} + 32\int_0^{\frac{1}{4}}\sqrt{-(-1+t)t} dt\right)^3}{8000} + 5$$

$$\left(\frac{4\pi}{3} + \frac{4\pi^2}{45}\right)^3 + 5 = \frac{1}{91125} \left(455625 + 1728000\left(\int_0^{\infty}\frac{1}{1+t^2} dt\right)^3 + 691200\left(\int_0^{\infty}\frac{1}{1+t^2} dt\right)^4 + 92160\left(\int_0^{\infty}\frac{1}{1+t^2} dt\right)^5 + 4096\left(\int_0^{\infty}\frac{1}{1+t^2} dt\right)^6\right)$$

$$\left(\frac{4\pi}{3} + \frac{4\pi^2}{45}\right)^3 + 5 = \frac{1}{91125} \left(455625 + 1728000\left(\int_0^{\infty}\frac{\sin(t)}{t} dt\right)^3 + 691200\left(\int_0^{\infty}\frac{\sin(t)}{t} dt\right)^4 + 92160\left(\int_0^{\infty}\frac{\sin(t)}{t} dt\right)^5 + 4096\left(\int_0^{\infty}\frac{\sin(t)}{t} dt\right)^6\right)$$

$$4\pi^2/45 * 1/((4\pi)/3) =$$

$$= 4\pi^2/45 * 3/((4\pi))$$

Input:

$$4 \times \frac{\pi^2}{45} \times \frac{3}{4\pi}$$

Result:

$$\frac{\pi}{15}$$

Decimal approximation:

0.209439510239319549230842892218633525613144626625007054731...

0.209439510239....

Property:

$\frac{\pi}{15}$ is a transcendental number

Alternative representations:

$$\frac{(4 \times 3) \pi^2}{(4 \pi) 45} = \frac{12 (180^\circ)^2}{45 (720^\circ)}$$

$$\frac{(4 \times 3) \pi^2}{(4 \pi) 45} = \frac{12 \cos^{-1}(-1)^2}{45 (4 \cos^{-1}(-1))}$$

$$\frac{(4 \times 3) \pi^2}{(4 \pi) 45} = \frac{12 (-i \log(-1))^2}{45 (-4 i \log(-1))}$$

Series representations:

$$\frac{(4 \times 3) \pi^2}{(4 \pi) 45} = \frac{4}{15} \sum_{k=0}^{\infty} \frac{(-1)^k}{1 + 2k}$$

$$\frac{(4 \times 3) \pi^2}{(4 \pi) 45} = \sum_{k=0}^{\infty} \frac{4 (-1)^k (956 \times 5^{-2k} - 5 \times 239^{-2k})}{17925 (1 + 2k)}$$

$$\frac{(4 \times 3) \pi^2}{(4 \pi) 45} = \frac{1}{15} \sum_{k=0}^{\infty} \left(-\frac{1}{4}\right)^k \left(\frac{1}{1+2k} + \frac{2}{1+4k} + \frac{1}{3+4k}\right)$$

Integral representations:

$\tan^{-1}(x)$ is the inverse tangent function

Result:

0.988589744523409512...

(result in radians)

0.9885897445234.....

And:

$$\left(\left(\left(4\pi^2/45 * 3/(4\pi)\right)\right)\right)^{1/128}$$

Input:

$$\sqrt[128]{4 \times \frac{\pi^2}{45} \times \frac{3}{4\pi}}$$

Exact result:

$$\sqrt[128]{\frac{\pi}{15}}$$

Decimal approximation:

0.987860841377183814071620011690442378654802904475647523726...

0.987860841377....

Property:

$\sqrt[128]{\frac{\pi}{15}}$ is a transcendental number

All 128th roots of $\pi/15$:

$$\sqrt[128]{\frac{\pi}{15}} e^0 \approx 0.987861 \text{ (real, principal root)}$$

$$\sqrt[128]{\frac{\pi}{15}} e^{(i\pi)/64} \approx 0.986671 + 0.048472 i$$

$$\sqrt[128]{\frac{\pi}{15}} e^{(i\pi)/32} \approx 0.983104 + 0.09683 i$$

$$\sqrt[128]{\frac{\pi}{15}} e^{(3i\pi)/64} \approx 0.977169 + 0.14495 i$$

$${}^{128}\sqrt{\frac{\pi}{15}} e^{(i\pi)/16} \approx 0.968879 + 0.19272 i$$

Alternative representations:

$${}^{128}\sqrt{\frac{(4 \times 3) \pi^2}{(4 \pi) 45}} = {}^{128}\sqrt{\frac{12 (180^\circ)^2}{45 (720^\circ)}}$$

$${}^{128}\sqrt{\frac{(4 \times 3) \pi^2}{(4 \pi) 45}} = {}^{128}\sqrt{\frac{72 \zeta(2)}{45 (4 \pi)}}$$

$${}^{128}\sqrt{\frac{(4 \times 3) \pi^2}{(4 \pi) 45}} = {}^{128}\sqrt{\frac{12 \cos^{-1}(-1)^2}{45 (4 \cos^{-1}(-1))}}$$

Series representations:

$${}^{128}\sqrt{\frac{(4 \times 3) \pi^2}{(4 \pi) 45}} = \frac{{}^{64}\sqrt{2} {}^{128}\sqrt{\sum_{k=0}^{\infty} \frac{(-1)^k}{1+2k}}}{{}^{128}\sqrt{15}}$$

$${}^{128}\sqrt{\frac{(4 \times 3) \pi^2}{(4 \pi) 45}} = \frac{{}^{128}\sqrt{\sum_{k=0}^{\infty} \frac{4(-1)^k 1195^{-1-2k} (5^{1+2k} - 4 \times 239^{1+2k})}{1+2k}}}{{}^{128}\sqrt{15}}$$

$${}^{128}\sqrt{\frac{(4 \times 3) \pi^2}{(4 \pi) 45}} = \frac{{}^{128}\sqrt{\sum_{k=0}^{\infty} \left(-\frac{1}{4}\right)^k \left(\frac{1}{1+2k} + \frac{2}{1+4k} + \frac{1}{3+4k}\right)}}{{}^{128}\sqrt{15}}$$

Integral representations:

$${}^{128}\sqrt{\frac{(4 \times 3) \pi^2}{(4 \pi) 45}} = {}^{128}\sqrt{\frac{2}{15}} {}^{128}\sqrt{\int_0^{\infty} \frac{1}{1+t^2} dt}$$

$${}^{128}\sqrt{\frac{(4 \times 3) \pi^2}{(4 \pi) 45}} = {}^{128}\sqrt{\frac{2}{15}} {}^{128}\sqrt{\int_0^1 \frac{1}{\sqrt{1-t^2}} dt}$$

$$\sqrt[128]{\frac{(4 \times 3) \pi^2}{(4 \pi) 45}} = \frac{64\sqrt[2]{128} \sqrt{\int_0^1 \sqrt{1-t^2} dt}}{\sqrt[128]{15}}$$

Thence, the following mathematical connection:

$$\left(\left(0.1562756303129776 + \frac{1}{10} \sqrt{10 - 2\sqrt{5}} \tan^{-1} \left(\frac{2\sqrt{10 - 2\sqrt{5}}}{4 - 2(\sqrt{5} + 1)} \right) + \left(\frac{1}{10} \sqrt{10 + 2\sqrt{5}} \right) \tan^{-1} \left(\frac{2\sqrt{10 + 2\sqrt{5}}}{4 + 2(\sqrt{5} - 1)} \right) \right)^{1/128} \right) = 0.9885897445234 \dots \Rightarrow$$

$$\Rightarrow \left(\sqrt[128]{4 \times \frac{\pi^2}{45} \times \frac{3}{4\pi}} \right) = 0.987860841377 \dots$$

$0.9885897445234 \dots \approx 0.987860841377 \dots$ results also very near to the value of the following Rogers-Ramanujan continued fraction:

$$\frac{e^{-\frac{\pi}{\sqrt{5}}}}{\sqrt{5}} = 1 - \frac{e^{-\pi\sqrt{5}}}{1 + \frac{e^{-2\pi\sqrt{5}}}{1 + \frac{e^{-3\pi\sqrt{5}}}{1 + \frac{e^{-4\pi\sqrt{5}}}{1 + \dots}}}} \approx 0.9991104684$$

$$\frac{1 + \sqrt[5]{\sqrt{\varphi^5 \sqrt[4]{5^3} - 1}}}{\sqrt{5}} - \varphi + 1$$

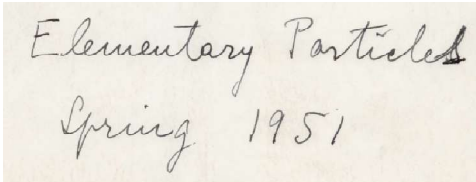
and to the dilaton value $0.989117352243 = \phi$

Acknowledgments

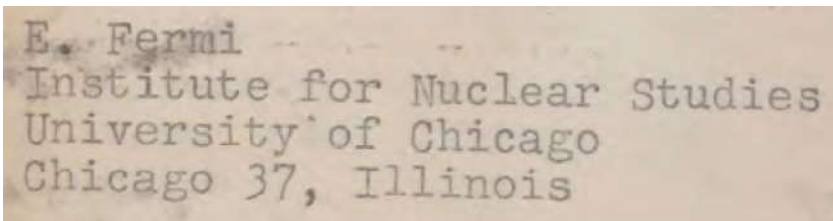
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Block notes – *Enrico Fermi*, University of Chicago - 1951



Bruce C. Berndt – **Ramanujan's Notebooks** (paper) – University of Illinois

Ramanujan's Notebooks

Working mostly in isolation, Ramanujan noted striking and sometimes still unproved results in series, special functions and number theory.

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