Particle wave duality

Ilija Barukčić[1]

[1] Internist, Horandstrasse, DE-26441 Jever, Germany

Correspondence: Ilija Barukčić, Horandstrasse, DE-26441 Jever, Germany. Tel: 49-4466-333.

E-mail: Barukcic@t-online.de

Received: December 22th, 2019; Accepted December 22th, 2019; Published: December 22th, 2019

Abstract

Objective: In spite of countless attempts, the considerable efforts on relativistic wave equations for particles and waves were not crowned with an ultimate success, the book on this topic is still not closed completely.

Methods: In this publication, Einstein's relativistic energy momentum relation has been re-analyzed again. An entirely novel approach has been adopted to solve the problem of relativistic wave equations.

Results: In particular, a normalized relativistic energy momentum relation is derived. The derived normalized relativistic energy momentum relation has been combined together with the Schrödinger equation to establish a wave equation consistent with special relativity theory.

Conclusions: A special relativity theory consistent wave equation as the mathematical foundation of relativistic quantum theory has been derived.

Keywords: ***Special theory of relativity, Schrödinger equation, Relativistic quantum theory.***

1. Introduction

What is nature and where does nature might ultimately have originated from? Has nature originated from something at all? What causes changes in nature? How can something be preserved or what is preserved through such changes? Any alteration in nature and of nature itself may raise subtle problems. How can nature itself be different from itself or non-identical to itself, to possess even incompatible properties and yet remain that what it is, nature itself? The problem of the consistency of alteration or changes in nature is associated with a change in place or with motion too. In a very general way, for the one author the answer to questions like these is clear while for another it is not. For René Descartes the answer was clear and he proposed in the year 1644 a law of the quantity of motion in his book *Principia philosophiae* (Des-Cartes, 1677), measured as the product of mass (m) and speed (v) and now called momentum (mv). In 1686, *Gottfried Wilhelm Leibniz* published his article *Short demonstration of a remarkable error by Descartes* (G. W. F. von Leibniz, 1686) and pointed out that in contrast to René Descartes *vis viva* or (mvv) is the true law of the quantity of motion which is preserved through changes. In so doing, Leibniz initiated the famous and long-lasting dispute known as *the vis viva controversy*. In fact, it is v = at and mv = mat = Ft or the momentum of an object is at the end the Newtonian force F (Newton, 1687) acting through time t. The highly controversial and heated dispute between Leibniz and Newton (Alexander, 1977; Barukčić, 2011b) which arose around momentum (mv) and vis viva (mvv) culminated by Jean d'Alembert's conclusion in his book *Trait de Dynamique* (Alembert, 1743) that *vis viva* (mvv) and *momentum* (mv) were equally valid (Iltis, 1970). However, even today it is not clear, what is preserved through changes. To date, the highly deterministic *Schrödinger equation* (Schrödinger, 1926a) as the foundation of the description of the Copenhagen interpretation of quantum mechanics dominated non-relativistic and indeterministic (Heisenberg, 1927) objective reality is itself non-relativistic but determined by the momentum too. In quantum mechanics, things are claimed to be different from our real life, even several relativistic wave equations like the Klein–Gordon equation (Gordon, 1926; Klein, 1926), the Dirac equation (Dirac, 1930) and other, which tried to apply special relativity with quantum mechanics together, did not succeed to reconcile relativity theory and quantum mechanics. However, a relativity theory and quantum theory consistent solution of *the particle-wave duality* is already published (Barukčić, 2013, 2016c) and can serve as the foundation to solve the problem of a generally valid and invariant form of a relativity theory consistent wave equation or a fully self-consistent relativistic quantum theory.

2. Methods and material

In the absence of a clear definition of the notions used, contradictions are more or less virtually preprogrammed. For this and other reasons, in contrast to other possible approaches to the matter itself, it is generally more appropriate and desirable to clear the notions, the methods and axioms used as precise as possible.

2.1. Methods

2.1.1. Definitions

Definition 1. (The number + 1)

Let c denote *the speed of light in vacuum* (Drude, 1894; Tombe, 2015; W. E. Weber & Kohlrausch, 1856; W. Weber & Kohlrausch, 1857), let ε_0 denote the electric constant and let μ_0 the magnetic constant. Let *i* denote the imaginary number (Bombelli, 1579). The number +1 is defined as the expression

$$+(c^2 \times \varepsilon_0 \times \mu_0) \equiv \quad +1+0 \equiv \quad -i^2 \quad = \quad +1 \tag{1}$$

while "=" denotes the equals sign (Recorde, 1557) or equality sign (Rolle, 1690) used to indicate equality and "-" (Pacioli, 1494; Widmann, 1489) denotes minus signs used to represent the operations of subtraction and the notions of negative as well and "+" denotes the plus (Recorde, 1557) signs used to represent the operations of addition and the notions of positive as well.

Definition 2. (The number + 0)

Let c denote the speed of light in vacuum (Drude, 1894; Tombe, 2015; W. E. Weber & Kohlrausch, 1856; W. Weber & Kohlrausch, 1857), let ε_0 denote the electric constant and let μ_0 the magnetic constant. Let *i* denote the imaginary number (Bombelli, 1579). The number +0 is defined as the expression

$$+(c^2 \times \varepsilon_0 \times \mu_0) - (c^2 \times \varepsilon_0 \times \mu_0) \equiv \quad +1-1 \equiv \quad -i^2 + i^2 \quad = \quad +0 \tag{2}$$

while "=" denotes the equals sign (Recorde, 1557) or equality sign (Rolle, 1690) used to indicate equality and "-" (Pacioli, 1494; Widmann, 1489) denotes minus signs used to represent the operations of subtraction and the notions of negative as well and "+" denotes the plus (Recorde, 1557) signs used to represent the operations of addition and the notions of positive as well.

Remark 1.

The definition of the basic numbers +1 and +0 (G. W. F. von Leibniz, 1703) in terms of physical "constants" provides among other the possibility too, to unify Physics and Number Theory (Irwin, 2019).

Definition 3. (The sample space)

Let ${}_{R}C_{t}$ denote the set of all the possible outcomes of a random experiment, a phenomenon in nature, at a (random) Bernoulli trial t. Let ${}_{0}x_{t}$ denote an event, a subset of the sample space ${}_{R}C_{t}$. Let ${}_{0}x_{t}$ denote the negation of an event ${}_{0}\underline{x}_{t}$, another, complementary subset of the sample space ${}_{R}C_{t}$. In general, we define the sample space ${}_{R}C_{t}$ as

$$ {}_{R}C_{t} \equiv \{ {}_{0}x_{t} , {}_{0}\underline{x}_{t} \} \tag{3} $$

or equally as

$$ {}_{R}C_{t} \equiv {}_{0}x_{t} + {}_{0}\underline{x}_{t} \tag{4} $$

In other words, and according to quantum theory, the sample space ${}_{R}C_{t}$ at one certain Bernoulli trial t is in a state of superposition of ${}_{0}x_{t}$ and ${}_{0}\underline{x}_{t}$. Under conditions of classical logic, it is $({}_{0}x_{t} + {}_{0}\underline{x}_{t}) = {}_{R}C_{t} = +1$.

Definition 4. (The Eigen-Value of ${}_{0}x_{t}$)

Under conditions of classical logic, ${}_{0}x_{t}$ can take only one of the values

$$ {}_{0}x_{t} \equiv \{+0 , +1\} \tag{5} $$

Definition 5. (The Eigen-Value of ${}_{0}\underline{x}_{t}$)

Under conditions of classical logic, ${}_{0}\underline{x}_{t}$ can take only one of the values

$$ {}_{0}\underline{x}_{t} \equiv \{+0 , +1\} \tag{6} $$

Definition 6. (The general form of negation)

Let $_0\underline{x}_t$ denote the negation of an event/outcome/eigenvalue $_0x_t$ (i. e. anti $_0x_t$). In general, we define the simple mathematical form of negation $_0\underline{x}_t$ of an event/outcome/eigenvalue $_0x_t$ as

$$_0\underline{x}_t \equiv {}_RC_t - {}_0x_t \tag{7}$$

Under conditions of classical logic 'anti $_0x_t$' passes over to 'not $_0x_t$'. Negation is a very important concept in philosophy (Newstadt, 2015) and classical logic. In classical logic, negation converts false to true and true to false. In other words, it is

$$_0\underline{x}_t \equiv (\sim {}_0x_t) \times ({}_RC_t = 1) \tag{8}$$

where ˜ denotes the sign of negation of classical logic. So, if $_0x_t$= +1 (or true), then (˜ $_0x_t$)×1= $_0\underline{x}_t$ (pronounced 'not $_0x_t$' or equally 'anti $_0x_t$') would therefore be $_0\underline{x}_t$ = +0 (false); and conversely, if $_0\underline{x}_t$ = +1 (true) then (˜ $_0\underline{x}_t$)×1= $_0x_t$ = +0 would be false.

Table 1. The relationship between $_0x_t$ and $_0\underline{x}_t$

Bernoulli trial t	$_0x_t = (\tilde{}\,_0\underline{x}_t)\times 1$	$_0\underline{x}_t = (\tilde{}\,_0x_t)\times 1$	$_0\underline{x}_t + {}_0\underline{x}_t = {}_RC_t = 1$	$_RC_t$
1	+1	+0	+1 +0 = +1	+1
2	+1	+0	+1 +0 = +1	+1
3	+0	+1	+0 + 1 = +1	+1
4	+0	+1	+0 +1 = +1	+1
…	…	…	…	…

The first and very simple mathematical or algebraical formulation of *the notion negation* was published by Georg Boole. In general, following Boole, negation in terms of algebra, can be expressed something as $_0\underline{x}_t$ =1-$_0x_t$. According to Boole, "… *in general, whatever … is represented by the symbol x, the contrary … will be expressed by 1 - x*" (Boole, 1854, p. 48). In other words, according to Boole, "*If x represent any … objects, then … 1 - x represent the contrary or supplementary …*" (Boole, 1854, p. 48). Under conditions of classical logic, it is $_RC_t$ = 1, and Boole's most simple form of negation can be abbreviated as "**1-**" too. **The *double negation*** would be (**1-**(**1-** $_0x_t$)) = $_0x_t$ and is sometimes identical with negatio negationis or **the negation of negation**. In a slightly different way, it is necessary to generalize Boole's simple form of negation to a general form of Boole's negation as

$$ {}_{0}\underline{x}_{t} \equiv {}_{R}C_{t} - {}_{0}x_{t} \tag{9}$$

Equally, it is in the same respect that

$$ {}_{0}x_{t} \equiv {}_{R}C_{t} - {}_{0}\underline{x}_{t} \tag{10}$$

Normalizing, we obtain (Barukčić, 2019) ***the general normalized form of negation*** as

$$ {}_{0}x_{t} \equiv \left(\mathbf{1} - \left(\frac{{}_{0}\underline{\boldsymbol{x}}_{t}}{{}_{R}\boldsymbol{C}_{t}}\right)\right) \times {}_{R}C_{t} \tag{11}$$

Under conditions of classical logic, it is $\mathbf{{}_{R}C_{t} = +1}$ and we obtain

$$ {}_{0}x_{t} \equiv \left(1 - \left(\frac{{}_{0}\underline{x}_{t}}{+1}\right)\right) \times +1 \tag{12}$$

or

$$ {}_{0}x_{t} \equiv \left(1 - \left({}_{0}\underline{x}_{t} \right)\right) \times +1 \tag{13}$$

and extremely simplified

$$ {}_{0}x_{t} \equiv \left(\sim\left({}_{0}\underline{x}_{t} \right)\right) \times 1 \tag{14}$$

Definition 7. (The relationship between $_{0}x_{t}$ and anti $_{0}x_{t}$)

In general, we define

$$ \left({}_{0}x_{t} \right) + \left({}_{0}\underline{x}_{t} \right) \equiv {}_{R}C_{t} \tag{15}$$

and

$$ a_{t}^{2} \equiv {}_{0}x_{t} \times {}_{R}C_{t} \tag{16}$$

and

$$ b_{t}^{2} \equiv {}_{0}\underline{x}_{t} \times {}_{R}C_{t}^{2} \tag{17}$$

Remark 2.

The equation ${}_{R}C_{t} = {}_{0}x_{t} + {}_{0}\underline{x}_{t}$ is valid even under conditions of classical (bivalent) logic. Under conditions of classical bivalent logic, it is ${}_{R}C_{t} = +1$ while ${}_{0}x_{t}$ takes the only values *either* +0 *or* +1. Since ${}_{0}\underline{x}_{t} = {}_{R}C_{t} - {}_{0}x_{t}$, ${}_{0}\underline{x}_{t}$ itself takes also only the values *either* +0 *or* +1. However, if ${}_{0}x_{t} = 0$ then ${}_{0}\underline{x}_{t} = 1$and vice versa. If ${}_{0}x_{t} = 1$ then ${}_{0}\underline{x}_{t} = 0$.

Definition 8. (Euclid's theorem)

Euclid's (ca. 360 - 280 BC) derived his geometric mean theorem or *right triangle altitude theorem* or Euclid's theorem and published the same in his book Elements (Euclid & Taylor, 1893) in a corollary to proposition 8 in Book VI, used in proposition 14 of Book II to square a rectangle too, as

$$\Delta_t^2 \equiv \frac{(a_t^2) \times (b_t^2)}{{}_R C_t^2} = {}_R C_t^2 \times \left(\sin^2(\alpha) \times \cos^2(\alpha)\right) = \left({}_0 x_t \right) \times \left({}_0 \underline{x}_t \right) \quad (18)$$

Definition 9. (Pythagorean theorem)

The Pythagorean theorem is defined as

$$\left({}_R C_t \times \left(\left({}_0 x_t \right) + \left({}_0 \underline{x}_t \right) \right) \right) = \left({}_R C_t \times {}_0 x_t \right) + \left({}_R C_t \times {}_0 \underline{x}_t \right) = (a_t^2 + b_t^2) \equiv {}_R C_t^2 \quad (19)$$

Definition 10. (The normalization of the Pythagorean theorem)

The *normalization* (Barukčić, 2013, 2016c) of the Pythagorean theorem is defined as

$$\left(\frac{a_t^2}{{}_R C_t^2} \right) + \left(\frac{b_t^2}{{}_R C_t^2} \right) = \sin^2(\alpha) + cos^2(\alpha) \equiv +1 \quad (20)$$

Definition 11. (The variance of the Pythagorean theorem)

The variance σ^2 of a right-angled triangle (Barukčić, 2013, 2016c) is defined as

$$\sigma_t^2 \equiv \frac{(a_t^2) \times (b_t^2)}{\left({}_R C_t^2 \right) \times \left({}_R C_t^2 \right)} \equiv \sin^2(\alpha) \times \sin^2(\beta) = \sin^2(\alpha) \times \cos^2(\alpha) = \frac{\Delta_t^2}{\left({}_R C_t^2 \right)} \quad (21)$$

Definition 12. (The right-angled triangle)

General relativity but special relativity too are also profoundly geometrical theories. To put it concisely, Einstein's special theory of relativity is more or less the application of Pythagorean theorem in physics (Okun, 2008). A right-angled triangle is a triangle in which one angle is 90-degree angle. Let ${}_R C_t$ denote the *hypotenuse*, the side opposite the right angle (side ${}_R C_t$ in the figure 1). The sides a_t and b_t are called legs. In a right-angled triangle ABC, the side AC, which is abbreviated as b_t, is the side which is *adjacent* to the angle α, while the side CB, denoted as

a_t, is the side *opposite* to angle α. The following figure 1 ((Bettinger & Englund, 1960), p. 117) may illustrate a right-angled triangle.

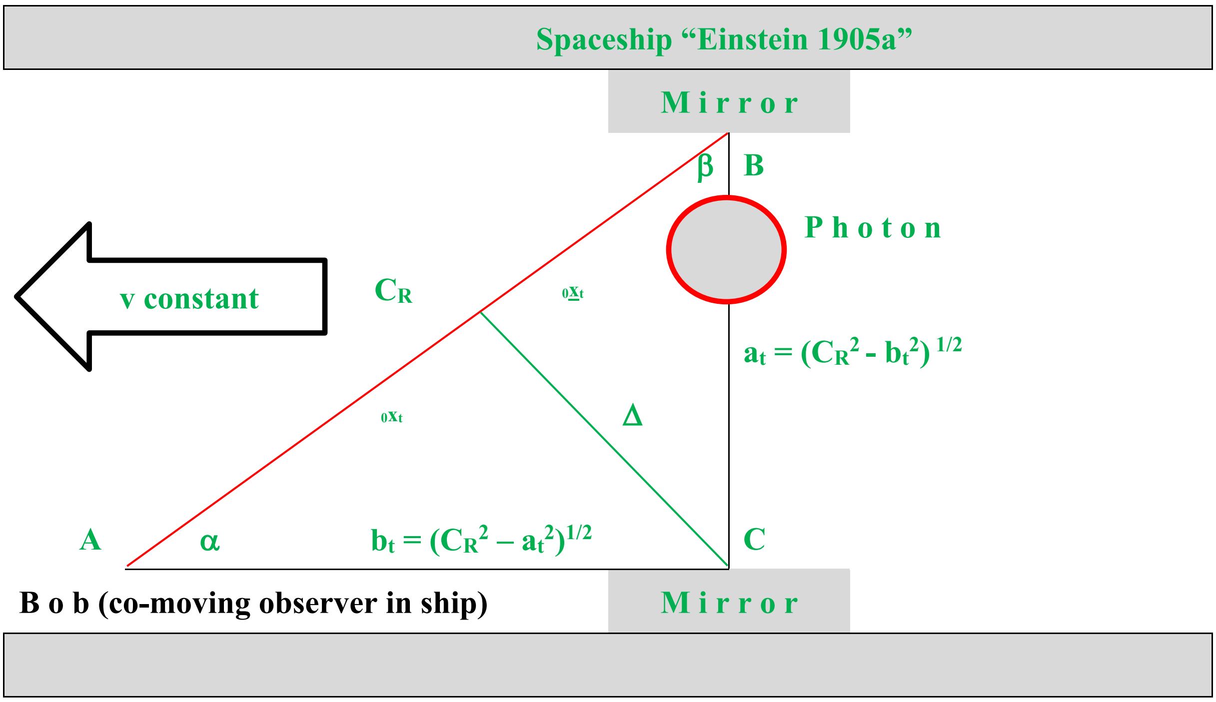

Figure 1. A right-angled triangle

Alice (stationary observer in deep space outside the ship above)

Definition 13. (The Lorentz factor)

The **Lorentz factor** (A. Einstein, 1905b; FitzGerald (1851-1901), 1889; Larmor (1857-1942), 1897; Lorentz (1853-1928), 1892; Poincaré (1854-1912), 1905; Voigt (1850-1919), 1887) **squared** or Lorentz term squared denoted as γ^2 is defined as

$$\gamma^2 = \frac{+1}{\left(1 - \left(\frac{v^2}{c^2}\right)\right)} \tag{22}$$

where v is the relative velocity between inertial reference frames and c is the speed of light (Rømer (1644–1710) & Huygens (1629-1695), 1888) in a vacuum. In general, it is

$$\left(1-\left(\frac{v^2}{c^2}\right)\right) \times \gamma^2 = +1 \tag{23}$$

Definition 14. (Einstein's mass-energy equivalence relation)

Einstein discovered the equivalence (A. Einstein, 1905b) of mass and energy. "Gibt ein Körper die Energie L in From von Strahlung ab, so verkleinert sich seine Masse um L/V^2" (A. Einstein, 1905a). Under conditions of Einstein's special theory of relativity, it is

$$m_0 = \sqrt[2]{1-\frac{v^2}{c^2}} \times m_R \tag{24}$$

were m_0 denotes the "rest-mass" as measured by the co-moving observer at a certain (period or point in) time t, m_R denotes the "relativistic-mass" as measured by the stationary observer at a same or simultaneous (period or point in) time t, v is the relative velocity between the co-moving and the stationary observer, c is the speed of the light in vacuum. Multiplying the equation above by the speed of the light in vacuum c squared, we obtain

$$\begin{aligned} \underbrace{m_0 \times c^2} &= \sqrt[2]{1-\frac{v^2}{c^2}} \times \underbrace{m_R \times c^2} \\ E_0 &= \sqrt[2]{1-\frac{v^2}{c^2}} \times E_R \\ E_0^2 &= \left(1-\frac{v^2}{c^2}\right) \times E_R^2 \end{aligned} \tag{25}$$

were $E_0 = m_0 \times c \times c$ denotes the rest-energy (A. Einstein, 1935) as measured by i. e. by a co-moving observer Bob (B), an observer at rest in the moving system, moving with constant velocity v relatively to the stationary system were Alice (A) is located. Let $E_R = m_R \times c \times c$ denote ***the total relativistic energy*** (Lewis & Tolman, 1909; Tolman, 1912) of the same entity as measured by in the stationary system by Alice (A) at the same (period of) time. Furthermore, let $\underline{E}_0 = E_R - E_0$ denote the local hidden variable.

Definition 15. (The normalized Einstein's mass-energy equivalence relation)

Einstein's mass–energy equivalence (A. Einstein, 1935) can be normalized (Barukčić, 2013, 2016c) as

$$\frac{m_0^2}{m_R^2} + \frac{v^2}{c^2} = +1 \tag{26}$$

Definition 16. (The (time dependent) Schrödinger equation)

Erwin Rudolf Josef Alexander Schrödinger (1887 – 1961), a Nobel Prize of physics winning Austrian physicist, developed a wave function (Schrödinger, 1926a) of a quantum-mechanical system (Schrödinger, 1926c) based on Newtonian physics with (classic *kinetic energy*) + (classical *potential energy*) rather than (relativistic kinetic energy) + (relativistic potential energy) and therefore more or less not valid for relativistic particles. The time-dependent Schrödinger equation (Schrödinger, 1926e, 1926b, 1926d) is defined as

$$i\hbar \frac{\partial \psi(t)}{\partial t} = H \times \psi(t) \tag{27}$$

where i is the imaginary unit, $\hbar$ is the reduced Planck constant, Ψ is the state vector of the quantum system, t is time, and H is the Hamiltonian operator (the quantum mechanical operator corresponding to the *total energy* of a (quantum mechanical) system).

Definition 17. (The relativistic energy of an electromagnetic wave)

Let E_{EMW} denote the energy of an electro-magnetic wave. Let $p_R = (m_R \times v)$ denote the relativistic momentum. In general, the energy of an electro-magnetic wave (i. e. photon) is defined as

$$E_{EMW} = m_R \times v \times c = p_R \times c \tag{28}$$

The energy of an electro-magnetic wave (i. e. photon) squared is defined as

$$
\begin{aligned}
\left(E_{EMW}\right)^2 &= \left(p_R \times c\right)^2 \\
&= \left(m_R \times v \times c\right)^2 \\
&= m_R \times m_R \times v \times v \times c \times c \\
&= m_R \times v \times v \times m_R \times c \times c \\
&= \left(m_R \times v \times v\right) \times \left(m_R \times c \times c\right) \\
&= \left(E_{RK}\right) \times \left(E_R\right)
\end{aligned} \tag{29}
$$

Definition 18. (The relativistic kinetic energy ("vis viva") E_{RK})

The **relativistic kinetic energy** is related to the historical concept of ***vis viva*** (G. W. F. von Leibniz, 1686, 1695) as originally proposed by Gottfried Wilhelm Leibniz (1646-1716). The relativistic kinetic energy E_{RK} is defined as

$$E_{RK} = m_R \times v \times v = p_R \times v = \frac{\left(E_{EMW}\right)^2}{\left(E_R\right)} \tag{30}$$

while *the mass-equivalent* m_{RK} of the relativistic kinetic energy is

$$m_{RK} = \frac{E_{RK}}{c^2} = m_R \times \frac{v \times v}{c^2} = p_R \times \frac{v}{c^2} = \frac{\left(E_{EMW}\right)^2}{c^2 \times \left(E_R\right)} \tag{31}$$

which does not imply that the relativistic kinetic energy itself must possess mass.

Definition 19. (The quantum mechanical operator of relativistic kinetic energy)

The quantum mechanical operator of the relativistic kinetic energy, denoted as $\mathbf{E}_{RK}$, follows as

$$\mathbf{E}_{RK} = \left(\frac{E_{EMW}^2 \times H}{E_R^2}\right) \times E_R = i\hbar \frac{\partial\left(\frac{v^2}{c^2}\right) \times c^2 \times m_R}{\partial t} \tag{32}$$

with definitions as above in this publication.

Definition 20. (The relativistic potential energy E_{RP})

The definition of ***the relativistic potential energy*** denoted as E_{RP} has been proposed by Einstein's himself to as "**Jeglicher Energie E kommt also im Gravitationsfelde eine Energie der Lage zu, die ebenso groß ist, wie die Energie der Lage einer 'ponderablen' Masse von der Größe E/c².**" (Albert Einstein, 1908) Translated into English: "Thus, to each energy E in the gravitational field there corresponds an energy of position that equals the potential energy of a 'ponderable' mass of magnitude E/c^2." The relativistic potential energy E_{RP} is defined as

$$E_{RP} = (E_R) - (E_{RK}) = (m_R \times c^2) - (m_R \times v^2) = (m_R \times (c^2 - v^2)) = \frac{(E_0)^2}{(E_R)} \quad (33)$$

while ***the mass* of the relativistic potential energy** m_{RP} is

$$m_{RP} = \frac{E_{RP}}{c^2} = \frac{((c^2 - v^2) \times m_R)}{c^2} = \left(1 - \frac{(v^2)}{c^2}\right) \times m_R = \frac{(E_0)^2}{c^2 \times (E_R)} \quad (34)$$

Definition 21. (The quantum mechanical operator of relativistic potential energy)

The quantum mechanical operator of the relativistic potential energy, denoted as $\mathbf{E}_{RP}$, is defined

$$\boldsymbol{E}_{RP} = \left(\frac{E_0^2 \times H}{E_R^2}\right) \times E_R = i\hbar \frac{\partial \left(1 - \frac{v^2}{c^2}\right) \times c^2 \times m_R}{\partial t} \quad (35)$$

with definitions as above in this publication.

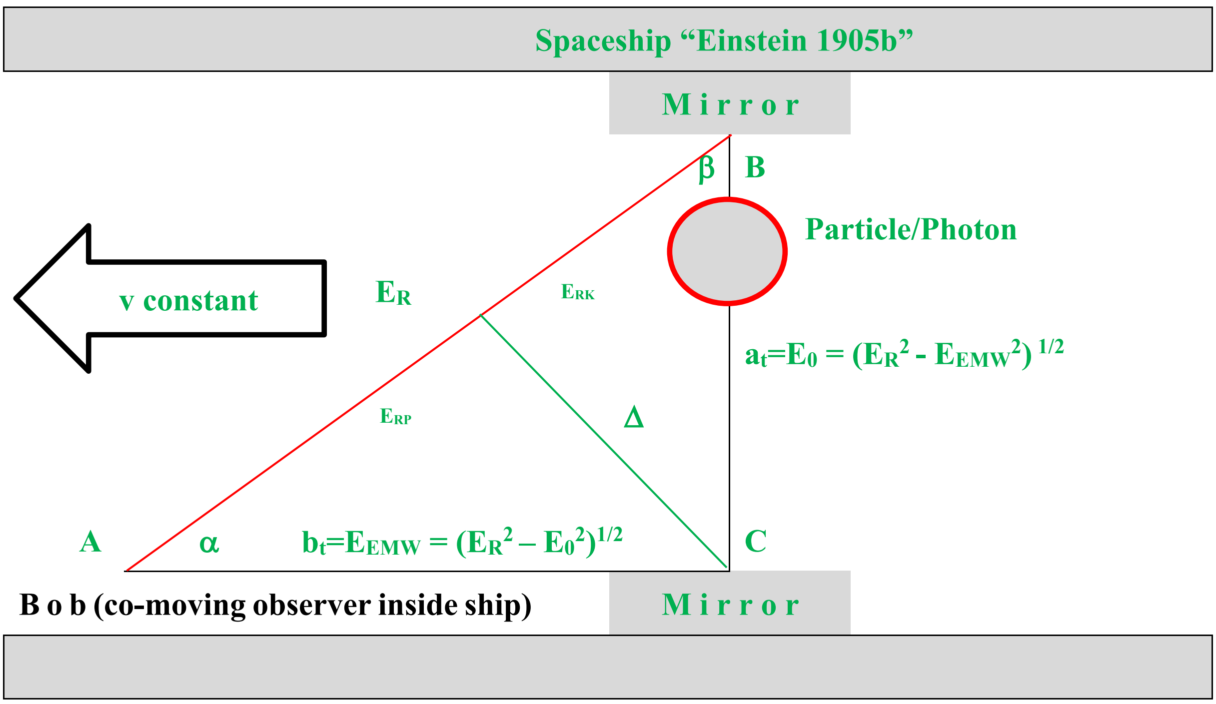

Figure 2. Pythagorean theorem and Einstein's special theory of realtivity

Alice (stationary observer in deep space outside the ship above)

Remark 3.

According to Okun's quite straightforward and persuading remarks, it is possible to "**present the main formulas of the theory of relativity in the simplest possible way, using mostly the Pythagorean theorem**." (Okun, 2008). Briefly summarized, Einstein's special theory of relativity is more or less just a concretization of the Pythagorean theorem to special fields of physics. In short, if we keep on formulating it to its extreme, "**The Pythagorean theorem graphically relates energy, momentum and mass**." (Okun, 2008) as illustrated by Figure 2.

2.1.2. Axioms

Axioms (Hilbert, 1917) and rules which are chosen carefully can be of use to avoid logical inconsistency and equally preventing science from supporting particular ideologies. Rightly or wrongly, long lasting advances in our knowledge of nature are enabled by suitable axioms (Easwaran, 2008) too. Einstein himself brings it to the point. "**Die wahrhaft großen Fortschritte der Naturerkenntnis sind auf einem der Induktion fast diametral entgegengesetzten Wege entstanden**." (Albert Einstein, 1919, p. 17). In general, Einstein himself advocated basic law (axioms) and conclusions derived from the same as a logical foundation of any 'theory'. "**Grundgesetz (Axiome) und Folgerungen zusammen bilden das was man eine 'Theorie' nennt**." (Albert Einstein, 1919, p. 17). *Lex identitatis* i.e. "**Chaque chose est ce qu'elle est. Et dans autant d'exemples qu'on voudra A est A, B est B**" (G. W. Leibniz Freiherr von, 1765, p. 327) or A = A, B=B or +1=+1, *lex contradictionis* (Boole, 1854; Hessen, 1928; Korch, 1965) and may be *lex negationis* (Hegel, 1812; Hegel, Di Giovanni, & Hegel, 2010; Newstadt, 2015) too have the theoretical potential to be the most simple, the most basic, the most general and the most far reaching axioms of science.

Axiom 1. (Lex identitatis)

$$+1 \equiv +1 \tag{36}$$

Axiom 2. (Lex contradictionis)

$$+0 \equiv +1 \tag{37}$$

3. Results

THEOREM 3.1 (PARTICLE WAVE DUALITY)

After several famous and brilliant breakthroughs, Einstein unified Newtonian mechanics (Newton, 1687) and James Clerk Maxwell's electromagnetic theory (Maxwell, 1861) into one single theory, **the theory of special relativity** (A. Einstein, 1905b), and resolved equally **the conflict between a particle and a wave** too. Later, Albert Einstein spent decades of his life and wrote more than forty technical papers (Sauer, 2019) on the unification of the electromagnetic and, possibly, other fields with the gravitational filed (A. Einstein, 1916) into a single elegant theory the unified field theory (Goenner, 2004, 2014). It is a pity that Einstein's work on a unified field theory has not been crowned with success.

CLAIM.

In addressing the issue of a particle and its own wave and vice versa, Einstein's special theory of relativity defines the duality between a particle and a wave as

$$\frac{(E_0^2)}{(E_R^2)} + \frac{(E_{EMW}^2)}{(E_R^2)} = +1 \tag{38}$$

In other words, to more something manifests itself as a particle, the less it is equally a wave et vice versa.

PROOF.

In general, taking axiom 1 to be true, it is

$$+1 \equiv +1 \tag{39}$$

Multiplying by m_0, the "rest-mass" as measured by the co-moving observer at a certain (period or point in) time t, it is

$$m_0 = m_0 \tag{40}$$

or according to Einstein's special theory of relativity, this equation can be rearranged as

$$m_0 = \sqrt[2]{1 - \frac{v^2}{c^2}} \times m_R \tag{41}$$

were m_R denotes the "relativistic-mass" as measured by the stationary observer at a same or simultaneous (period or point in) time t, v is the relative velocity between the co-moving observer B and the stationary observer A, c is the speed of the light in vacuum. Simplifying it is

$$m_0^2 = \left(1 - \frac{v^2}{c^2}\right) \times m_R^2 \tag{42}$$

or

$$\frac{m_0^2}{m_R^2} = \left(1 - \frac{v^2}{c^2}\right) \tag{43}$$

The normalized relativistic energy-momentum relation (Barukčić, 2013, 2016c) follows as

$$\frac{m_0^2}{m_R^2} + \frac{v^2}{c^2} = +1 \tag{44}$$

It is

$$\frac{m_0^2}{m_R^2} = +1 - \frac{v^2}{c^2} \tag{45}$$

and thus far equally as

$$\left(1 - \left(\frac{v^2}{c^2}\right)\right) + \frac{v^2}{c^2} = +1 \tag{46}$$

Applying Schrödinger's equation, we obtain *the simple form of the relativistic wave equation* as

$$\left(\left(1 - \left(\frac{v^2}{c^2}\right)\right) \times H\right) \times \Psi(t) + \left(\frac{v^2}{c^2} \times H\right) \times \Psi(t) = H \times \Psi(t) \tag{47}$$

or

$$\left(\left(1 - \left(\frac{v^2}{c^2}\right)\right) \times H\right) \times \Psi(t) + \left(\frac{v^2}{c^2} \times H\right) \times \Psi(t) = i\hbar \frac{\partial \psi(t)}{\partial t} \tag{48}$$

We shall here not consider even briefly alternative ways of expressing this relativistic wave equation. Rearranging equation above $\mathbf{((m_0^2/m_R^2)+(v^2/c^2)=+1)}$, it is

$$\frac{(m_0^2)\times c^4}{(m_R^2)\times c^4} + \frac{(m_R^2)\times (v^2)\times (c^2)}{(m_R^2)\times (c^2)\times (c^2)} = +1 \tag{49}$$

or

$$\frac{(E_0^2)}{(E_R^2)} + \frac{(p_R^2)\times (c^2)}{(E_R^2)} = +1 \tag{50}$$

or

$$\frac{(E_0^2)}{(E_R^2)} + \frac{(E_{EMW}^2)}{(E_R^2)} = +1 \tag{51}$$

with definitions as above in this publication where E_0^2 denotes the rest energy of a particle and E_{EMV}^2 denote at the same time, *the local hidden variable*, the energy of an electromagnetic wave as associated with the same particle in the same context.

QUOD ERAT DEMONSTRANDUM.

Remark 4.

A particle as such, "comparable" with a black hole, is more or less localized, while something like a wavelength and frequency requires a kind of an extension in space and in time. Thus far and for a long time, a particle and a wave were treated as mutually exclusive. However, as demonstrated before, a fundamental and universal description of the foundations of quantum mechanics can be grounded on the attribution of particle and wave properties to any object without the need to differ drastically from the picture of classical physics. One striking aspect of such an approach is that exact simultaneous values can be assigned even to all quantum entities. In brief, the nightmare stemmed from todays to some extent restricted and invalid concept of measurement of the quantum theory loses its fright. In other words, **the more precisely something is given or measured (i.e. the relative velocity v), the more precisely can one calculate what its other, its complementary et cetera is** (**The quantum mechanical certainty principle**). A particle which has zero rest-mass is determined by its momentum and energy as $\mathbf{E_{EMW}^2 = E_R^2}$.

THEOREM 3.2 (RELATIVISTIC WAVE EQUATION)

Before proceeding further, let us point again to the necessary background to derive a relativistic wave equation. In 1913, **Niels Bohr** succeeded in deriving the equation $M_0=h/2\pi$ (Bohr, 1913, p. 14). Soon **Louis de Broglie** suggested that light which consists of waves of electromagnetic fields has particle characteristics too and vice versa. Classical particles, i. e. such as electrons, have wave characteristics too (Broglie, 1925). Louis de Broglie associated the particle momentum with the wavelength λ of these waves through the relation $p=h/\lambda$, known as the de Broglie relation, where h denotes Planck's constant (Planck, 1901). The **Davisson–Germer experiment** (Davisson & Germer, 1928) was the first of experiments to confirm the matter-wave hypothesis of wave-particle duality as advanced by Louis de Broglie.

CLAIM.

The relativistic wave equation (Barukčić, 2013) can be derived as

$$(E_{RP}) \times \Psi(t) \quad + \quad (E_{RK}) \times \Psi(t) \quad = \quad i\hbar \frac{\partial \psi(t)}{\partial t} \tag{52}$$

PROOF.

In general, taking axiom 1 to be true, it is

$$+1 \quad \equiv \quad +1 \tag{53}$$

Multiplying by *v*, the relative velocity, it is

$$v \quad = \quad v \tag{54}$$

or

$$v^2 \quad = \quad v^2 \tag{55}$$

or

$$v^2 \quad \times \quad m_R^2 \quad \times \quad c^2 \quad = \quad m_R^2 \times v^2 \times c^2 \tag{56}$$

or

$$m_R \times v^2 \quad \times \quad m_R \times c^2 \quad = \quad p_R^2 \times c^2 \tag{57}$$

or

$$E_{RK} \quad \times \quad E_R \quad = \quad E_{EMW}^2 \tag{58}$$

or

$$\frac{(E_{RK} \times E_R)}{E_R^2} = \frac{(E_{EMW}^2)}{E_R^2} \tag{59}$$

or

$$\frac{(E_0^2)}{(E_R^2)} + \frac{(E_{RK} \times E_R)}{E_R^2} = \frac{(E_0^2)}{(E_R^2)} + \frac{(E_{EMW}^2)}{(E_R^2)} = +1 \tag{60}$$

or

$$\frac{(E_{RP} \times E_R)}{(E_R^2)} + \frac{(E_{RK} \times E_R)}{E_R^2} = \frac{(E_0^2)}{(E_R^2)} + \frac{(E_{EMW}^2)}{(E_R^2)} = +1 \tag{61}$$

or

$$\left(\frac{(E_{RP} \times E_R)}{(E_R^2)} + \frac{(E_{RK} \times E_R)}{E_R^2}\right) \times H \times \Psi(t) = \left(\frac{(E_0^2)}{(E_R^2)} + \frac{(E_{EMW}^2)}{(E_R^2)}\right) \times H \times \Psi(t) = H \times \Psi(t) \tag{62}$$

or

$$\left(\frac{(E_{RP} \times H)}{(E_R)} + \frac{(E_{RK} \times H)}{E_R}\right) \times \Psi(t) = \left(\frac{(E_0^2 \times H)}{(E_R^2)} + \frac{(E_{EMW}^2 \times H)}{(E_R^2)}\right) \times \Psi(t) = i\hbar \frac{\partial \psi(t)}{\partial t} \tag{63}$$

Especially **under conditions where $E_R = H$**, it follows that

$$\left((E_{RP}) + (E_{RK})\right) \times \Psi(t) = \left(\frac{(E_0^2)}{(E_R)} + \frac{(E_{EMW}^2)}{(E_R)}\right) \times \Psi(t) = i\hbar \frac{\partial \psi(t)}{\partial t} \tag{64}$$

and at the end the relativistic wave equation as

$$(E_{RP}) \times \Psi(t) + (E_{RK}) \times \Psi(t) = \left(\frac{(E_0^2) \times \Psi(t)}{(E_R)}\right) + \left(\frac{(E_{EMW}^2) \times \Psi(t)}{(E_R)}\right) = i\hbar \frac{\partial \psi(t)}{\partial t} \tag{65}$$

QUOD ERAT DEMONSTRANDUM.

Remark 5.

The generally valid and invariant form of a relativity theory consistent wave equation has been derived as

$$\left(\left(\frac{E_0^2 \times H}{E_R^2}\right) + \left(\frac{E_{EMW}^2 \times H}{E_R^2}\right)\right) \times \psi(x,t) = i\hbar \frac{\partial \psi(x,t)}{\partial t} \tag{66}$$

Multiplying by minus 1, we obtain the equation for negative particles as

$$-\left(\left(\left(\frac{E_0^2 \times H}{E_R^2}\right)+\left(\frac{E_{EMW}^2 \times H}{E_R^2}\right)\right)\times \psi(x,t)\right) = -\left(i\hbar\frac{\partial\psi(x,t)}{\partial t}\right) \qquad (67)$$

Especially under conditions where $\mathbf{E_R^2 = H}$, we obtain

$$\left(\left(\frac{E_0^2}{E_R}\right)+\left(\frac{E_{EMW}^2}{E_R}\right)\right)\times \psi(x,t) = i\hbar\frac{\partial\psi(x,t)}{\partial t} \qquad (68)$$

while the equation for negative particles follows as

$$-\left(\left(\left(\frac{E_0^2}{E_R}\right)+\left(\frac{E_{EMW}^2}{E_R}\right)\right)\times \psi(x,t)\right) = -\left(i\hbar\frac{\partial\psi(x,t)}{\partial t}\right) \qquad (69)$$

In other words, the relativistic wave equation enables negative particles too.

4. Discussion

In contrast to the special theory of relativity, quantum mechanics prefers to describe objective reality which is existing independently and outside of human mind and consciousness by waves (the wave function) rather than with discrete particles. However, such a different operational approach to objective reality doesn't justify any contradictions in science. Formally, it is possible to express quantum mechanical systems completely and without any contradiction by the notions of the theory of special relativity too.

Nonetheless, the attentive and critical reader who is familiar with the matter in detail will be able to become more critical, more pessimistic and sometimes even more aggressive when one considers that total relativistic energy (E_R) and the Hamiltonian operator (**H**) have been equated. It will, of course, be a question of the further research whether such an approach is generally valid. However, and beyond any doubt, there are for sure such circumstances where E_R = **H**.

5. Conclusion

The historical dominance of the indeterminate, non-relativistic and causal Copenhagen interpretation of quantum mechanics is no longer justified. From a different but equivalent point of view, the main principles of today's Copenhagen interpretation dominated quantum mechanics like **Heisenberg's uncertainty principle** (Barukčić, 2010, 2011a, 2014, 2016b; Heisenberg, 1927), **Bell's theorem** (Barukčić, 2012, 2015, 2016a; Bell, 1964) and the **CHSH inequality** (Barukčić, 2012, 2015, 2016a; Bell, 1964) are already refuted with the consequence that a fully, relativity theory consistent quantum theory is possible.

This publication has re-solved the historical problem of the wave-particle duality and derived a relativity theory consistent wave equation.

Acknowledgements

The open source, independent and non-profit **Zotero Citation Manager** was used to create and manage references and bibliographies. The public domain software GnuPlot is used frequently, to draw some figures or pictures.

Author Contributions

The author confirms being the sole contributor of this work and has approved it for publication.

Conflict of Interest Statement

The author declares that the research was conducted in the absence of any commercial or financial relationships that could be construed as a potential conflict of interest. Furthermore, there are no conflict of interest exists according to the guidelines of the International Committee of Medical Journal Editors.

Financial support and sponsorship

Nil.

Erratum.

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The right tailed critical Chi square value for alpha level 5% is 3.84145882.

...

The value of the test statistic is 8.28534801 and exceeds the critical value 3.84145882.

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$$ {}_{0}\omega_{\mu v} \equiv 2_{\mu v} \cap {}_{R}\pi_{\mu v} \cap {}_{R}f_{\mu v} \equiv \left(\frac{1_{\mu v}}{{}_{R}\hbar_{\mu v}}\right) \cap \left(G_{\mu v} + \left(\Lambda \times g_{\mu v}\right)\right) \quad (76) $$

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The number +1 is defined as the expression

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$$ 0^2 = +0 - 0 \quad (254) $$

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