

# **On various Ramanujan's equations of Manuscript Book 1 and some formulas concerning the Eisenstein series: new possible mathematical connections with some parameters of Particle Physics and Cosmology. III**

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## **Abstract**

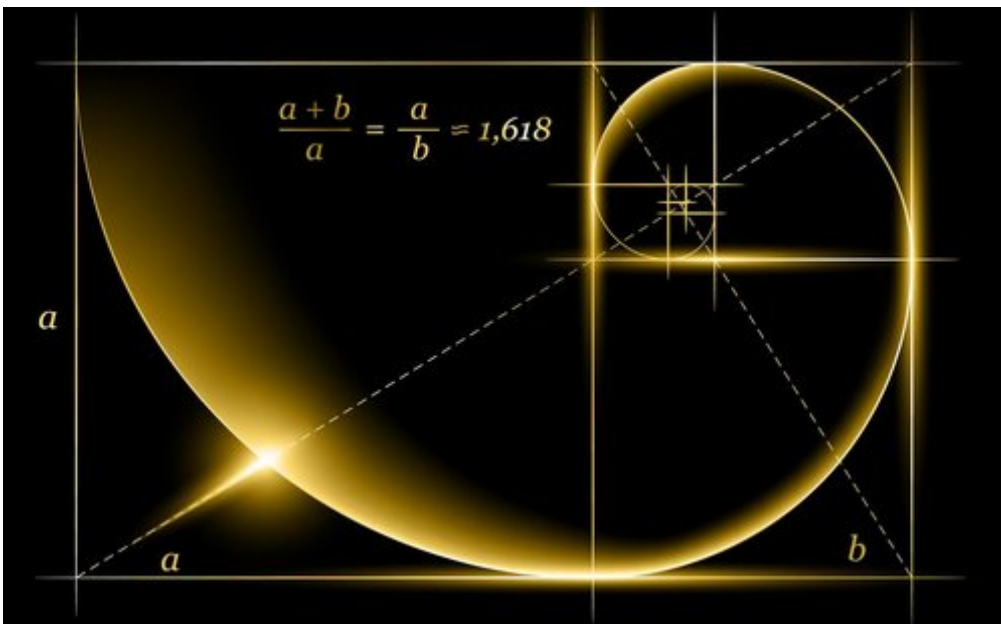
*In this research thesis, we continue to analyze and deepen further Ramanujan's equations of Manuscript Book 1 and some formulas concerning the Eisenstein series and describe new possible mathematical connections with some parameters of Particle Physics and Cosmology.*

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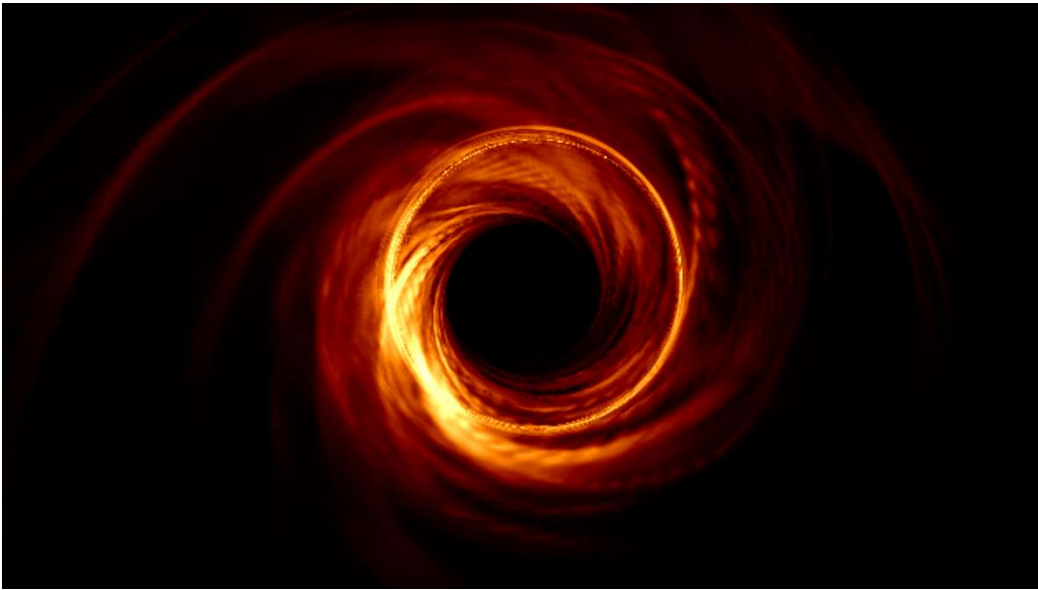
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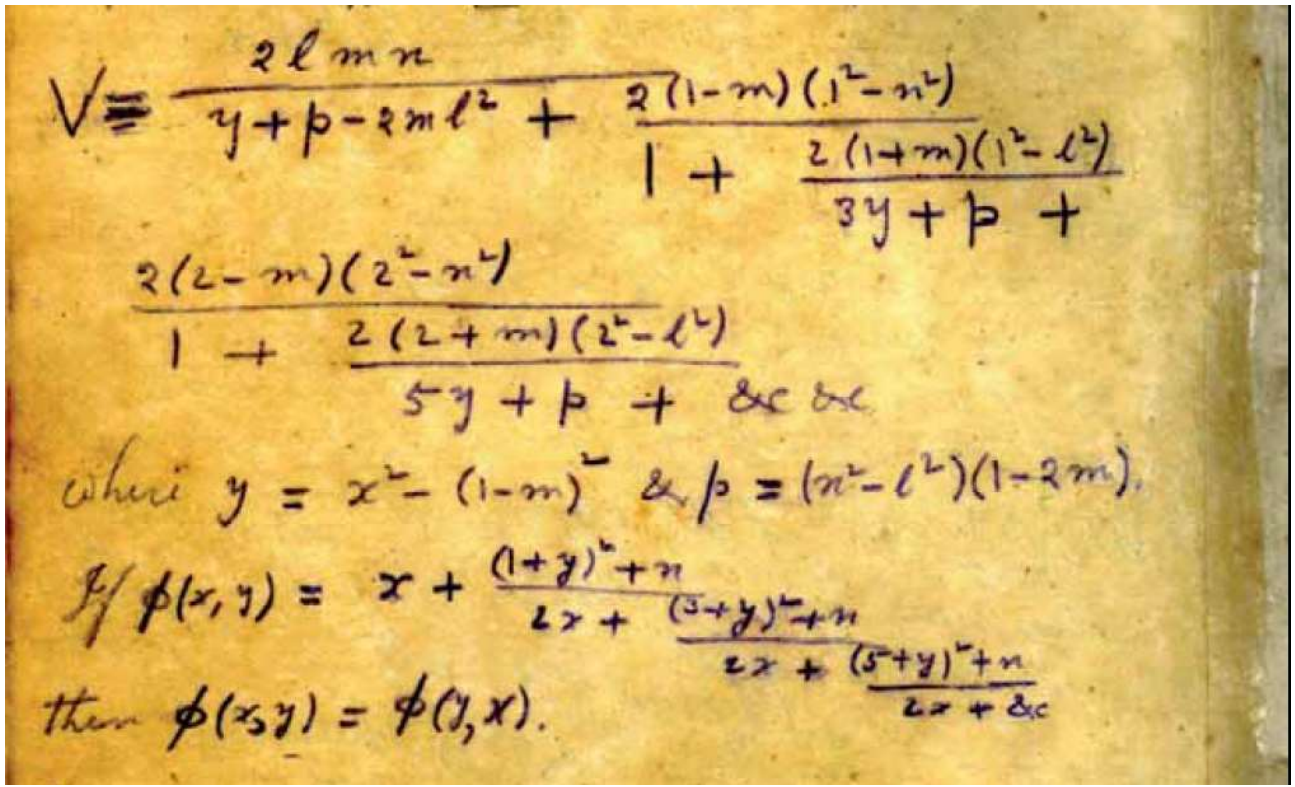
From : <http://scienceofhindu.blogspot.com/2016/04/man-who-knew-infinity-by-ramana.html> (modified by A. Nardelli)



<https://kindtrainer.com/fractalbliss>



<https://www.cambridgesciencefestival.org/event/photographing-black-holes-first-results-from-the-event-horizon-telescope/>



For  $x = 2, l = 3, m = 5, n = 8$

$$y = 2^2 - (1-5)^2 = -12$$

$$p = (8^2 - 3^2)(1 - 2 \cdot 5) = -495$$

$$2 \cdot 3 \cdot 5 \cdot 8 / ((((-12 - 495 - 2 \cdot 5 \cdot 3^2) + (((2(1-5)(1-8^2)) / (1 + (((((-96 / (-36 - 495 + (2(2-5)(4-64)) / (1 + (((((-70) / (-60 - 495))))))))))))))))))$$

**Input:**

$$\frac{2 \times 3 \times 5 \times 8}{(-12 - 495 - 2 \times 5 \times 3^2) + \frac{2(1-5)(1-8^2)}{1 - \frac{96}{-36 - 495 + \frac{2(2-5)(4-64)}{1 - \frac{70}{-60 - 495}}}}$$

**Exact result:**

$$\frac{204880}{213791}$$

**Decimal approximation:**

-0.95831910604281751804332268430382944090256371877207178973...  
 -0.958319106...

**Continued fraction:**

$$\cfrac{1}{-1 + \cfrac{1}{-22 + \cfrac{1}{-1 + \cfrac{1}{-121 + \cfrac{1}{-14 + \cfrac{1}{-1 + \cfrac{1}{-1 + \cfrac{1}{2}}}}}}}}}}$$

$$(-1/(10^{52}))[(2*3*5*8)/(((((-12-495-2*5*3^2)+(((2(1-5)(1-8^2)))/(1+((((((-96/(-36-495+(2(2-5)(4-64)))/(1+((((((-70)/(-60-495)))))))))))))))))))-0.144-0.003-0.0003]$$

Where 144 and 3 are Fibonacci numbers

**Input:**

$$\frac{2 \times 3 \times 5 \times 8}{(-12-495-2 \times 5 \times 3^2) + \frac{2(1-5)(1-8^2)}{1 - \frac{96}{-36-495 + \frac{2(2-5)(4-64)}{1 - \frac{70}{-60-495}}}}} - 0.144 - 0.003 - 0.0003$$

**Result:**

1.1056191060428175180433226843038294409025637187720717... × 10<sup>-52</sup>

1.105619106... \* 10<sup>-52</sup> result practically equal to the value of Cosmological Constant

1.1056 \* 10<sup>-52</sup> m<sup>-2</sup>

$$[-(2*3*5*8)/(((((-12-495-2*5*3^2)+(((2(1-5)(1-8^2)))/(1+((((((-96/(-36-495+(2(2-5)(4-64)))/(1+((((((-70)/(-60-495)))))))))))))))))))]^{1/64}$$

**Input:**

$$\sqrt[64]{\frac{2 \times 3 \times 5 \times 8}{(-12 - 495 - 2 \times 5 \times 3^2) + \frac{2(1-5)(1-8^2)}{1 - \frac{96}{-36-495 + \frac{2(2-5)(4-64)}{1 - \frac{70}{-60-495}}}}}}$$

**Result:**

$$\sqrt[64]{\frac{12\,805}{213\,791}} \sqrt[16]{2}$$

**Decimal approximation:**

0.999334995270014233707606973481877009422036043201135085501...

0.999334995... result very near to the value of the following Rogers-Ramanujan continued fraction:

$$\frac{e^{-\frac{\pi}{\sqrt{5}}}}{\sqrt{5}} = 1 - \frac{e^{-\pi\sqrt{5}}}{1 + \frac{e^{-2\pi\sqrt{5}}}{1 + \frac{e^{-3\pi\sqrt{5}}}{1 + \frac{e^{-4\pi\sqrt{5}}}{1 + \dots}}}} \approx 0.9991104684$$

and to the dilaton value **0.989117352243 =  $\phi$**

**Alternate forms:**

$$\frac{\sqrt[16]{2} \sqrt[64]{12805} \sqrt[63]{213791}}{213791}$$

root of  $213791x^{64} - 204880$  near  $x = 0.999335$

2log base 0.99933499527[-(2\*3\*5\*8)/(((((-12-495-2\*5\*3^2)+(((2(1-5)(1-8^2)))/(1+((((((-96)/(-36-495+(2(2-5)(4-64)))/(1+((((((-70)/(-60-495)))))))))))))))]-Pi+1/golden ratio

**Input interpretation:**

$$2 \log_{0.99933499527} \left( \frac{2 \times 3 \times 5 \times 8}{(-12 - 495 - 2 \times 5 \times 3^2) + \frac{2(1-5)(1-8^2)}{1 - \frac{96}{-36-495 + \frac{2(2-5)(4-64)}{1 - \frac{70}{-60-495}}}}} \right) - \pi + \frac{1}{\phi}$$

$\log_b(x)$  is the base- $b$  logarithm

$\phi$  is the golden ratio

### Result:

125.47644...

125.47644.... result very near to the dilaton mass calculated as a type of Higgs boson: 125 GeV for  $T = 0$  and to the Higgs boson mass 125.18 GeV

### Alternative representation:

$$2 \log_{0.999334995270000} \left( \frac{2 \times 3 \times 5 \times 8}{(-12 - 495 - 2 \times 5 \times 3^2) + \frac{2(1-5)(1-8^2)}{1 - \frac{96}{-36-495 + \frac{2(2-5)(4-64)}{1 - \frac{70}{-60-495}}}}} \right) - \pi + \frac{1}{\phi} =$$

$$- \pi + \frac{1}{\phi} + \frac{2 \log \left( \frac{240}{-597 - \frac{8(1-8^2)}{1 - \frac{96}{-531 + \frac{360}{1 - \frac{70}{555}}}}} \right)}{\log(0.999334995270000)}$$

### Series representations:

$$2 \log_{0.999334995270000} \left( \frac{2 \times 3 \times 5 \times 8}{(-12 - 495 - 2 \times 5 \times 3^2) + \frac{2(1-5)(1-8^2)}{1 - \frac{96}{-36-495 + \frac{2(2-5)(4-64)}{1 - \frac{70}{-60-495}}}}} \right) - \pi + \frac{1}{\phi} =$$

$$\frac{1}{\phi} - \pi - \frac{2 \sum_{k=1}^{\infty} \frac{(-1)^k \left(-\frac{8911}{213791}\right)^k}{k}}{\log(0.999334995270000)}$$

$$2 \log_{0.999334995270000} \left( \frac{2 \times 3 \times 5 \times 8}{(-12 - 495 - 2 \times 5 \times 3^2) + \frac{2(1-5)(1-8^2)}{1 - \frac{96}{-36-495 + \frac{2(2-5)(4-64)}{1 - \frac{70}{-60-495}}}}} \right) - \pi + \frac{1}{\phi} =$$

$$\frac{1}{\phi} - 1.0000000000000000 \pi - 3006.49740532 \log\left(\frac{204880}{213791}\right) -$$

$$2 \log\left(\frac{204880}{213791}\right) \sum_{k=0}^{\infty} (-0.000665004730000)^k G(k)$$

for  $\left( G(0) = 0 \text{ and } G(k) = \frac{(-1)^{1+k} k}{2(1+k)(2+k)} + \sum_{j=1}^k \frac{(-1)^{1+j} G(-j+k)}{1+j} \right)$

2log base 0.99933499527[-(2\*3\*5\*8)/(((((-12-495-2\*5\*3^2)+(((2(1-5)(1-8^2))/(1+((((((-96/(-36-495+(2(2-5)(4-64)))/(1+((((((-70)/(-60-495))))))))))))))))))] + 11 + 1/golden ratio

**Input interpretation:**

$$2 \log_{0.99933499527} \left( \frac{2 \times 3 \times 5 \times 8}{(-12 - 495 - 2 \times 5 \times 3^2) + \frac{2(1-5)(1-8^2)}{1 - \frac{96}{-36-495 + \frac{2(2-5)(4-64)}{1 - \frac{70}{-60-495}}}}} \right) + 11 + \frac{1}{\phi}$$

$\log_b(x)$  is the base- $b$  logarithm

$\phi$  is the golden ratio

**Result:**

139.61803...



139.61803.... result practically equal to the rest mass of Pion meson 139.57 MeV

**Alternative representation:**

$$2 \log_{0.999334995270000} \left( \frac{2 \times 3 \times 5 \times 8}{(-12 - 495 - 2 \times 5 \times 3^2) + \frac{2(1-5)(1-8^2)}{1 - \frac{96}{-36-495 + \frac{2(2-5)(4-64)}{1 - \frac{70}{-60-495}}}}} \right) + 11 + \frac{1}{\phi} =$$

$$11 + \frac{1}{\phi} + \frac{2 \log \left( \frac{240}{-597 - \frac{8(1-8^2)}{1 - \frac{96}{-531 + \frac{360}{1 - \frac{70}{555}}}}} \right)}{\log(0.999334995270000)}$$

**Series representations:**

$$2 \log_{0.999334995270000} \left( \frac{2 \times 3 \times 5 \times 8}{(-12 - 495 - 2 \times 5 \times 3^2) + \frac{2(1-5)(1-8^2)}{1 - \frac{96}{-36-495 + \frac{2(2-5)(4-64)}{1 - \frac{70}{-60-495}}}}} \right) + 11 + \frac{1}{\phi} =$$

$$11 + \frac{1}{\phi} - \frac{2 \sum_{k=1}^{\infty} \frac{(-1)^k \left( -\frac{8911}{213791} \right)^k}{k}}{\log(0.999334995270000)}$$

$$2 \log_{0.999334995270000} \left( \frac{2 \times 3 \times 5 \times 8}{(-12 - 495 - 2 \times 5 \times 3^2) + \frac{2(1-5)(1-8^2)}{1 - \frac{96}{-36-495 + \frac{2(2-5)(4-64)}{1 - \frac{70}{-60-495}}}}} \right) + 11 + \frac{1}{\phi} =$$

$$11 + \frac{1}{\phi} - 3006.49740532 \log \left( \frac{204880}{213791} \right) -$$

$$2 \log \left( \frac{204880}{213791} \right) \sum_{k=0}^{\infty} (-0.000665004730000)^k G(k)$$

for  $\left( G(0) = 0 \text{ and } G(k) = \frac{(-1)^{1+k} k}{2(1+k)(2+k)} + \sum_{j=1}^k \frac{(-1)^{1+j} G(-j+k)}{1+j} \right)$

7log base 0.99933499527[-(2\*3\*5\*8)/(((((-12-495-2\*5\*3^2)+(((2(1-5)(1-8^2))/(1+((((((-96/(-36-495+(2(2-5)(4-64))/(1+((((((-70)/(-60-495))))))))))))))))))] + 34 + golden ratio

**Input interpretation:**

$$7 \log_{0.99933499527} \left( \frac{2 \times 3 \times 5 \times 8}{(-12 - 495 - 2 \times 5 \times 3^2) + \frac{2(1-5)(1-8^2)}{1 - \frac{96}{-36-495 + \frac{2(2-5)(4-64)}{1 - \frac{70}{-60-495}}}}} \right) + 34 + \phi$$

$\log_b(x)$  is the base-  $b$  logarithm

$\phi$  is the golden ratio

**Result:**

483.61803...

483.61803.... result practically equal to Holographic Ricci dark energy model, where

$$\chi_{\text{RDE}}^2 = 483.130.$$

**Alternative representation:**

$$7 \log_{0.999334995270000} \left( \frac{2 \times 3 \times 5 \times 8}{(-12 - 495 - 2 \times 5 \times 3^2) + \frac{2(1-5)(1-8^2)}{1 - \frac{96}{-36-495 + \frac{2(2-5)(4-64)}{1 - \frac{70}{-60-495}}}}} \right) + 34 + \phi =$$

$$7 \log \left( \frac{240}{\frac{-597 - \frac{8(1-8^2)}{1 - \frac{96}{-531 + \frac{360}{1 - \frac{70}{555}}}}}} \right)$$

$$34 + \phi + \frac{7 \log \left( \frac{240}{\frac{-597 - \frac{8(1-8^2)}{1 - \frac{96}{-531 + \frac{360}{1 - \frac{70}{555}}}}}} \right)}{\log(0.999334995270000)}$$

### Series representations:

$$7 \log_{0.999334995270000} \left( \frac{2 \times 3 \times 5 \times 8}{(-12 - 495 - 2 \times 5 \times 3^2) + \frac{2(1-5)(1-8^2)}{1 - \frac{96}{-36-495 + \frac{2(2-5)(4-64)}{1 - \frac{70}{-60-495}}}}} \right) + 34 + \phi =$$

$$34 + \phi - \frac{7 \sum_{k=1}^{\infty} \frac{(-1)^k \left(-\frac{8911}{213791}\right)^k}{k}}{\log(0.999334995270000)}$$

$$7 \log_{0.999334995270000} \left( \frac{2 \times 3 \times 5 \times 8}{(-12 - 495 - 2 \times 5 \times 3^2) + \frac{2(1-5)(1-8^2)}{1 - \frac{96}{-36-495 + \frac{2(2-5)(4-64)}{1 - \frac{70}{-60-495}}}}} \right) + 34 + \phi =$$

$$34.0000000000 + \phi - 10522.7409186 \log\left(\frac{204880}{213791}\right) -$$

$$7.0000000000 \log\left(\frac{204880}{213791}\right) \sum_{k=0}^{\infty} (-0.000665004730000)^k G(k)$$

$$\text{for } \left( G(0) = 0 \text{ and } \frac{(-1)^k k}{2(1+k)(2+k)} + G(k) = \sum_{j=1}^k \frac{(-1)^{1+j} G(-j+k)}{1+j} \right)$$

27log base 0.99933499527[-(2\*3\*5\*8)/(((((-12-495-2\*5\*3^2)+(((2(1-5)(1-8^2))/(1+((((((-96)/(-36-495+(2(2-5)(4-64)))/(1+((((((-70)/(-60-495)))))))))))))))]+1/golden ratio

**Input interpretation:**

$$27 \log_{0.99933499527} \left( \frac{2 \times 3 \times 5 \times 8}{(-12 - 495 - 2 \times 5 \times 3^2) + \frac{2(1-5)(1-8^2)}{1 - \frac{96}{-36-495 + \frac{2(2-5)(4-64)}{1 - \frac{70}{-60-495}}}}} \right) + \frac{1}{\phi}$$

$\log_b(x)$  is the base- $b$  logarithm

$\phi$  is the golden ratio

**Result:**

1728.6180...

1728.6180....

This result is very near to the mass of candidate glueball  $f_0(1710)$  meson. Furthermore, 1728 occurs in the algebraic formula for the  $j$ -invariant of an elliptic curve. As a consequence, it is sometimes called a Zagier as a pun on the Gross–Zagier theorem. The number 1728 is one less than the Hardy–Ramanujan number 1729

**Alternative representation:**

$$27 \log_{0.999334995270000} \left( \frac{2 \times 3 \times 5 \times 8}{(-12 - 495 - 2 \times 5 \times 3^2) + \frac{2(1-5)(1-8^2)}{1 - \frac{96}{-36-495 + \frac{2(2-5)(4-64)}{1 - \frac{70}{-60-495}}}}} \right) + \frac{1}{\phi} =$$

$$\frac{1}{\phi} + \frac{27 \log \left( \frac{240}{-597 - \frac{8(1-8^2)}{1 - \frac{96}{-531 + \frac{360}{1 - \frac{70}{555}}}}} \right)}{\log(0.999334995270000)}$$

**Series representations:**

$$27 \log_{0.999334995270000} \left( \frac{2 \times 3 \times 5 \times 8}{(-12 - 495 - 2 \times 5 \times 3^2) + \frac{2(1-5)(1-8^2)}{1 - \frac{96}{-36-495 + \frac{2(2-5)(4-64)}{1 - \frac{70}{-60-495}}}}} \right) + \frac{1}{\phi} =$$

$$\frac{1}{\phi} - \frac{27 \sum_{k=1}^{\infty} \frac{(-1)^k \left(-\frac{8911}{213791}\right)^k}{k}}{\log(0.999334995270000)}$$

$$27 \log_{0.999334995270000} \left( \frac{2 \times 3 \times 5 \times 8}{(-12 - 495 - 2 \times 5 \times 3^2) + \frac{2(1-5)(1-8^2)}{1 - \frac{96}{-36-495 + \frac{2(2-5)(4-64)}{1 - \frac{70}{-60-495}}}}} \right) + \frac{1}{\phi} =$$

$$\frac{1}{\phi} - 40587.7149718 \log\left(\frac{204880}{213791}\right) -$$

$$27 \log\left(\frac{204880}{213791}\right) \sum_{k=0}^{\infty} (-0.000665004730000)^k G(k)$$

for  $\left( G(0) = 0 \text{ and } G(k) = \frac{(-1)^{1+k} k}{2(1+k)(2+k)} + \sum_{j=1}^k \frac{(-1)^{1+j} G(-j+k)}{1+j} \right)$

$[-(2*3*5*8)/(((((-12-495-2*5*3^2)+(((2(1-5)(1-8^2))/(1+((((((-96)/(-36-495+(2(2-5)(4-64)))/(1+((((((-70)/(-60-495))))))))))))))))))] - 21/10^3 + \text{golden ratio}/10^3$

**Input:**

$$-\frac{2 \times 3 \times 5 \times 8}{(-12 - 495 - 2 \times 5 \times 3^2) + \frac{2(1-5)(1-8^2)}{1 - \frac{96}{-36-495 + \frac{2(2-5)(4-64)}{1 - \frac{70}{-60-495}}}}} - \frac{21}{10^3} + \frac{\phi}{10^3}$$

$\phi$  is the golden ratio

**Result:**

$$\frac{\phi}{1000} + \frac{200390389}{213791000}$$

**Decimal approximation:**

0.938937140031567412891527271138195079020284027951877552600...

0.93893714.... result very near to the rest mass of neutron 0.93956 in GeV

convert 938.937 MeV (megaelectronvolts) to gigaelectronvolts

0.938937 GeV (gigaelectronvolts)

**Alternate forms:**

$$\frac{400\,994\,569 + 213\,791\sqrt{5}}{427\,582\,000}$$

$$\frac{213\,791\phi + 200\,390\,389}{213\,791\,000}$$

$$\frac{400\,994\,569}{427\,582\,000} + \frac{1}{400\sqrt{5}}$$

**Alternative representations:**

$$-\frac{2 \times 3 \times 5 \times 8}{(-12 - 495 - 2 \times 5 \times 3^2) + \frac{2(1-5)(1-8^2)}{1 - \frac{96}{-36-495 + \frac{2(2-5)(4-64)}{1 - \frac{70}{-60-495}}}}} - \frac{21}{10^3} + \frac{\phi}{10^3} =$$

$$-\frac{21}{10^3} - \frac{240}{-597 - \frac{8(1-8^2)}{1 - \frac{96}{-531 + \frac{360}{1 - \frac{70}{555}}}}} + \frac{2 \sin(54^\circ)}{10^3}$$

$$-\frac{2 \times 3 \times 5 \times 8}{(-12 - 495 - 2 \times 5 \times 3^2) + \frac{2(1-5)(1-8^2)}{1 - \frac{96}{-36-495 + \frac{2(2-5)(4-64)}{1 - \frac{70}{-60-495}}}}} - \frac{21}{10^3} + \frac{\phi}{10^3} =$$

$$-\frac{21}{10^3} - \frac{2 \cos(216^\circ)}{10^3} - \frac{240}{-597 - \frac{8(1-8^2)}{1 - \frac{96}{-531 + \frac{360}{1 - \frac{70}{555}}}}}$$

$$-\frac{2 \times 3 \times 5 \times 8}{(-12 - 495 - 2 \times 5 \times 3^2) + \frac{2(1-5)(1-8^2)}{1 - \frac{96}{-36-495 + \frac{2(2-5)(4-64)}{1 - \frac{70}{-60-495}}}}} - \frac{21}{10^3} + \frac{\phi}{10^3} =$$

$$-\frac{21}{10^3} - \frac{240}{-597 - \frac{8(1-8^2)}{1 - \frac{96}{-531 + \frac{360}{1 - \frac{70}{555}}}}} - \frac{2 \sin(666^\circ)}{10^3}$$

Page 189

Handwritten formula on aged paper:

$$x \left\{ 1 + \frac{x^n}{\binom{n}{1}} + \frac{x^{2n}}{\binom{n}{2}} + \frac{x^{3n}}{\binom{n}{3}} + \dots \right\}$$

$$= e^x + e^{x \cos \frac{2\pi}{n}} \cos\left(x \sin \frac{2\pi}{n}\right) + e^{x \cos \frac{4\pi}{n}} \cos\left(x \sin \frac{4\pi}{n}\right) + \dots$$

For  $x = 2$ ,  $n = 8$

$$e^2 + e^{(2 \cos(\pi/4))} \cos(2 \sin(\pi/4)) + e^{(2 \cos(\pi/2))} \cos(2 \sin(\pi/2))$$

**Input:**

$$e^2 + e^{2 \cos(\pi/4)} \cos\left(2 \sin\left(\frac{\pi}{4}\right)\right) + e^{2 \cos(\pi/2)} \cos\left(2 \sin\left(\frac{\pi}{2}\right)\right)$$

**Exact result:**

$$e^2 + \cos(2) + e^{\sqrt{2}} \cos(\sqrt{2})$$

**Decimal approximation:**

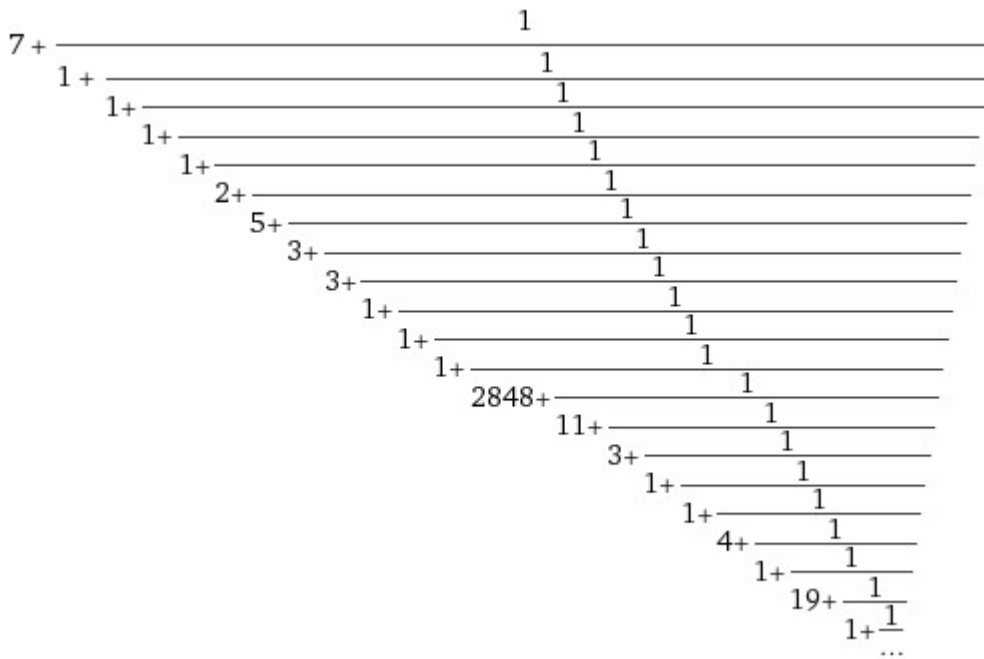
7.614344723945993624243270519851027251262587392333962847858...

7.614344723...

**Alternate form:**

$$\frac{e^{-2i}}{2} + \frac{e^{2i}}{2} + e^2 + \frac{1}{2} e^{(1-i)\sqrt{2}} + \frac{1}{2} e^{(1+i)\sqrt{2}}$$

**Continued fraction:**



**Alternative representations:**

$$e^2 + e^{2 \cos(\pi/4)} \cos\left(2 \sin\left(\frac{\pi}{4}\right)\right) + e^{2 \cos(\pi/2)} \cos\left(2 \sin\left(\frac{\pi}{2}\right)\right) = e^2 + \cosh\left(2 i \sin\left(\frac{\pi}{2}\right)\right) e^{2 \cosh((i \pi)/2)} + \cosh\left(2 i \sin\left(\frac{\pi}{4}\right)\right) e^{2 \cosh((i \pi)/4)}$$

$$e^2 + e^{2 \cos(\pi/4)} \cos\left(2 \sin\left(\frac{\pi}{4}\right)\right) + e^{2 \cos(\pi/2)} \cos\left(2 \sin\left(\frac{\pi}{2}\right)\right) = e^2 + \cosh\left(-2 i \sin\left(\frac{\pi}{2}\right)\right) e^{2 \cosh(-(i \pi)/2)} + \cosh\left(-2 i \sin\left(\frac{\pi}{4}\right)\right) e^{2 \cosh(-(i \pi)/4)}$$

$$e^2 + e^{2 \cos(\pi/4)} \cos\left(2 \sin\left(\frac{\pi}{4}\right)\right) + e^{2 \cos(\pi/2)} \cos\left(2 \sin\left(\frac{\pi}{2}\right)\right) = e^2 + \frac{1}{2} e^{e^{-i \pi/2} + e^{i \pi/2}} \left( e^{-2 i \sin(\pi/2)} + e^{2 i \sin(\pi/2)} \right) + \frac{1}{2} e^{e^{-i \pi/4} + e^{i \pi/4}} \left( e^{-2 i \sin(\pi/4)} + e^{2 i \sin(\pi/4)} \right)$$

**Integral representations:**

$$e^2 + e^{2 \cos(\pi/4)} \cos\left(2 \sin\left(\frac{\pi}{4}\right)\right) + e^{2 \cos(\pi/2)} \cos\left(2 \sin\left(\frac{\pi}{2}\right)\right) = 1 + e^2 + e^{\sqrt{2}} + \int_0^1 \left( -2 \sin(2 t) - \sqrt{2} e^{\sqrt{2}} \sin(\sqrt{2} t) \right) dt$$

$$e^2 + e^{2 \cos(\pi/4)} \cos\left(2 \sin\left(\frac{\pi}{4}\right)\right) + e^{2 \cos(\pi/2)} \cos\left(2 \sin\left(\frac{\pi}{2}\right)\right) = e^2 + \int_{-i \infty + \gamma}^{i \infty + \gamma} - \frac{i e^{-1/s+s} \left( 1 + e^{\sqrt{2} + 1/(2 s)} \right)}{2 \sqrt{\pi} \sqrt{s}} ds \text{ for } \gamma > 0$$



$$e^2 + e^{2\cos(\pi/4)} \cos\left(2 \sin\left(\frac{\pi}{4}\right)\right) + e^{2\cos(\pi/2)} \cos\left(2 \sin\left(\frac{\pi}{2}\right)\right) =$$

$$e^2 + \int_{-i\infty+\gamma}^{i\infty+\gamma} -\frac{i(1+2^s e^{\sqrt{2}})\Gamma(s)}{2\sqrt{\pi}\Gamma\left(\frac{1}{2}-s\right)} ds \text{ for } 0 < \gamma < \frac{1}{2}$$

$$e^2 + e^{2\cos(\pi/4)} \cos\left(2 \sin\left(\frac{\pi}{4}\right)\right) + e^{2\cos(\pi/2)} \cos\left(2 \sin\left(\frac{\pi}{2}\right)\right) =$$

$$e^2 + \int_{\frac{\pi}{2}}^2 \left( -\sin(t) - \frac{e^{\sqrt{2}} \left(\sqrt{2} - \frac{\pi}{2}\right) \sin\left(\frac{-\pi + \frac{\pi}{\sqrt{2}} - \sqrt{2}t + \frac{\pi t}{2}}{-2 + \frac{\pi}{2}}\right)}{2 - \frac{\pi}{2}} \right) dt$$

### Half-argument formula:

$$e^2 + e^{2\cos(\pi/4)} \cos\left(2 \sin\left(\frac{\pi}{4}\right)\right) + e^{2\cos(\pi/2)} \cos\left(2 \sin\left(\frac{\pi}{2}\right)\right) =$$

$$e^2 - \sqrt{\frac{1}{2}(1 + \cos(4))} + e^{\sqrt{2}} \sqrt{\frac{1}{2}(1 + \cos(2\sqrt{2}))}$$

### Multiple-argument formulas:

$$e^2 + e^{2\cos(\pi/4)} \cos\left(2 \sin\left(\frac{\pi}{4}\right)\right) + e^{2\cos(\pi/2)} \cos\left(2 \sin\left(\frac{\pi}{2}\right)\right) =$$

$$-1 + e^2 + 2\cos^2(1) + e^{\sqrt{2}} \left(-1 + 2\cos^2\left(\frac{1}{\sqrt{2}}\right)\right)$$

$$e^2 + e^{2\cos(\pi/4)} \cos\left(2 \sin\left(\frac{\pi}{4}\right)\right) + e^{2\cos(\pi/2)} \cos\left(2 \sin\left(\frac{\pi}{2}\right)\right) =$$

$$1 + e^2 - 2\sin^2(1) + e^{\sqrt{2}} \left(1 - 2\sin^2\left(\frac{1}{\sqrt{2}}\right)\right)$$

$$e^2 + e^{2\cos(\pi/4)} \cos\left(2 \sin\left(\frac{\pi}{4}\right)\right) + e^{2\cos(\pi/2)} \cos\left(2 \sin\left(\frac{\pi}{2}\right)\right) =$$

$$e^2 - 3\cos\left(\frac{2}{3}\right) + 4\cos^3\left(\frac{2}{3}\right) + e^{\sqrt{2}} \left(-3\cos\left(\frac{\sqrt{2}}{3}\right) + 4\cos^3\left(\frac{\sqrt{2}}{3}\right)\right)$$

$$1/10^{27} * (((1/(((e^2 + e^{(2\cos(\pi/4)) \cos(2\sin(\pi/4))+e^{(2\cos(\pi/2)) \cos(2\sin(\pi/2)))-7)+(34+8+3)/10^3))))$$

**Input:**

$$\frac{1}{10^{27}} \left( \frac{1}{\left( e^2 + e^{2 \cos(\pi/4)} \cos\left(2 \sin\left(\frac{\pi}{4}\right)\right) + e^{2 \cos(\pi/2)} \cos\left(2 \sin\left(\frac{\pi}{2}\right)\right) \right) - 7} + \frac{34 + 8 + 3}{10^3} \right)$$

**Exact result:**

$$\frac{\frac{9}{200} + \frac{1}{-7 + e^2 + \cos(2) + e^{\sqrt{2}} \cos(\sqrt{2})}}{1\,000\,000\,000\,000\,000\,000\,000\,000\,000}$$

**Decimal approximation:**

$$1.6727506113780980581129426358897811289548374116773891... \times 10^{-27}$$

1.672750611... \* 10<sup>-27</sup> result very near to the rest mass of proton in kg

**Alternate forms:**

$$\frac{\frac{9}{200\,000\,000\,000\,000\,000\,000\,000\,000} + \frac{1}{1\,000\,000\,000\,000\,000\,000\,000\,000\,000 \left( -7 + e^2 + \cos(2) + e^{\sqrt{2}} \cos(\sqrt{2}) \right)}}{1\,000\,000\,000\,000\,000\,000\,000\,000\,000 \left( -7 + e^2 + \cos(2) + e^{\sqrt{2}} \cos(\sqrt{2}) \right)}$$

$$\frac{137 + 9 e^2 + 9 \cos(2) + 9 e^{\sqrt{2}} \cos(\sqrt{2})}{200\,000\,000\,000\,000\,000\,000\,000\,000 \left( -7 + e^2 + \cos(2) + e^{\sqrt{2}} \cos(\sqrt{2}) \right)}$$

$$\frac{\frac{9}{200\,000\,000\,000\,000\,000\,000\,000\,000} + \frac{1}{1\,000\,000\,000\,000\,000\,000\,000\,000\,000 \left( -7 + \frac{1}{2} \left( e^{-2i} + e^{2i} \right) + e^2 + \frac{1}{2} e^{\sqrt{2}} \left( e^{-i\sqrt{2}} + e^{i\sqrt{2}} \right) \right)}}{1\,000\,000\,000\,000\,000\,000\,000\,000\,000 \left( -7 + \frac{1}{2} \left( e^{-2i} + e^{2i} \right) + e^2 + \frac{1}{2} e^{\sqrt{2}} \left( e^{-i\sqrt{2}} + e^{i\sqrt{2}} \right) \right)}$$

**Alternative representations:**

$$\frac{\frac{1}{\left( e^2 + e^{2 \cos(\pi/4)} \cos\left(2 \sin\left(\frac{\pi}{4}\right)\right) + e^{2 \cos(\pi/2)} \cos\left(2 \sin\left(\frac{\pi}{2}\right)\right) \right) - 7} + \frac{34 + 8 + 3}{10^3}}{10^{27}} = \frac{\frac{45}{10^3} + \frac{1}{-7 + e^2 + \cosh\left(2 i \sin\left(\frac{\pi}{2}\right)\right) e^{2 \cosh\left(i \pi/2\right)} + \cosh\left(2 i \sin\left(\frac{\pi}{4}\right)\right) e^{2 \cosh\left(i \pi/4\right)}}}{10^{27}}$$

$$\frac{\frac{1}{\left( e^2 + e^{2 \cos(\pi/4)} \cos\left(2 \sin\left(\frac{\pi}{4}\right)\right) + e^{2 \cos(\pi/2)} \cos\left(2 \sin\left(\frac{\pi}{2}\right)\right) \right) - 7} + \frac{34 + 8 + 3}{10^3}}{10^{27}} = \frac{\frac{45}{10^3} + \frac{1}{-7 + e^2 + \cosh\left(-2 i \sin\left(\frac{\pi}{2}\right)\right) e^{2 \cosh\left(-i \pi/2\right)} + \cosh\left(-2 i \sin\left(\frac{\pi}{4}\right)\right) e^{2 \cosh\left(-i \pi/4\right)}}}{10^{27}}$$

$$\frac{1}{(e^2 + e^{2 \cos(\pi/4)} \cos(2 \sin(\frac{\pi}{4}))) + e^{2 \cos(\pi/2)} \cos(2 \sin(\frac{\pi}{2})) - 7} + \frac{34+8+3}{10^3} = \frac{1}{10^{27}}$$

$$\left( \frac{45}{10^3} + 1 / \left( -7 + e^2 + \frac{1}{2} e^{e^{-i\pi/2} + e^{i\pi/2}} (e^{-2i \sin(\pi/2)} + e^{2i \sin(\pi/2)}) + \frac{1}{2} e^{e^{-i\pi/4} + e^{i\pi/4}} (e^{-2i \sin(\pi/4)} + e^{2i \sin(\pi/4)}) \right) \right)$$

**Series representations:**

$$\frac{1}{(e^2 + e^{2 \cos(\pi/4)} \cos(2 \sin(\frac{\pi}{4}))) + e^{2 \cos(\pi/2)} \cos(2 \sin(\frac{\pi}{2})) - 7} + \frac{34+8+3}{10^3} =$$

$$\frac{10^{27}}{9} +$$

$$\frac{200\,000\,000\,000\,000\,000\,000\,000\,000\,000\,000\,000\,000\,000\,000}{1} +$$

$$1\,000\,000\,000\,000\,000\,000\,000\,000\,000\,000\,000\,000\,000\,000 \left( -7 + e^2 + \sum_{k=0}^{\infty} \frac{(-4)^k + (-2)^k e^{\sqrt{2}}}{(2k)!} \right)$$

$$\frac{1}{(e^2 + e^{2 \cos(\pi/4)} \cos(2 \sin(\frac{\pi}{4}))) + e^{2 \cos(\pi/2)} \cos(2 \sin(\frac{\pi}{2})) - 7} + \frac{34+8+3}{10^3} =$$

$$\frac{10^{27}}{9} +$$

$$\frac{200\,000\,000\,000\,000\,000\,000\,000\,000\,000\,000\,000\,000\,000}{1} +$$

$$1 / \left( 1\,000\,000\,000\,000\,000\,000\,000\,000\,000\,000\,000\,000\,000\,000 \right.$$

$$\left. \left( -7 + e^2 + \sum_{k=0}^{\infty} \frac{\cos\left(\frac{k\pi}{2} + z_0\right) \left( (2 - z_0)^k + e^{\sqrt{2}} (\sqrt{2} - z_0)^k \right)}{k!} \right) \right)$$

$$\frac{1}{(e^2 + e^{2 \cos(\pi/4)} \cos(2 \sin(\frac{\pi}{4}))) + e^{2 \cos(\pi/2)} \cos(2 \sin(\frac{\pi}{2})) - 7} + \frac{34+8+3}{10^3} =$$

$$\frac{10^{27}}{9} +$$

$$\frac{200\,000\,000\,000\,000\,000\,000\,000\,000\,000\,000\,000\,000\,000}{1} +$$

$$1 / \left( 1\,000\,000\,000\,000\,000\,000\,000\,000\,000\,000\,000\,000\,000\,000 \right.$$

$$\left. \left( -7 + e^2 + \sum_{k=0}^{\infty} \left( \frac{(-1)^{-1+k} \left(2 - \frac{\pi}{2}\right)^{1+2k}}{(1+2k)!} + \frac{(-1)^{-1+k} e^{\sqrt{2}} \left(\sqrt{2} - \frac{\pi}{2}\right)^{1+2k}}{(1+2k)!} \right) \right) \right)$$

**Integral representations:**

$$\frac{\frac{1}{(e^2 + e^{2 \cos(\pi/4)}) \cos(2 \sin(\frac{\pi}{4})) + e^{2 \cos(\pi/2)} \cos(2 \sin(\frac{\pi}{2}))} - 7 + \frac{34+8+3}{10^3}}{10^{27}} =$$

$$\frac{200\,000\,000\,000\,000\,000\,000\,000\,000\,000}{1} +$$

$$1 / \left( 1\,000\,000\,000\,000\,000\,000\,000\,000\,000\,000 \right.$$

$$\left. \left( -7 + e^2 + \int_{-i \infty + \gamma}^{i \infty + \gamma} - \frac{i e^{-1/s+s} (1 + e^{\sqrt{2} + 1/(2s)})}{2 \sqrt{\pi} \sqrt{s}} ds \right) \right) \text{ for } \gamma > 0$$

$$\frac{\frac{1}{(e^2 + e^{2 \cos(\pi/4)}) \cos(2 \sin(\frac{\pi}{4})) + e^{2 \cos(\pi/2)} \cos(2 \sin(\frac{\pi}{2}))} - 7 + \frac{34+8+3}{10^3}}{10^{27}} =$$

$$\frac{200\,000\,000\,000\,000\,000\,000\,000\,000\,000}{1} +$$

$$1\,000\,000\,000\,000\,000\,000\,000\,000\,000\,000 \left( -7 + e^2 + \int_{-i \infty + \gamma}^{i \infty + \gamma} - \frac{i (1 + 2^s e^{\sqrt{2}}) \Gamma(s)}{2 \sqrt{\pi} \Gamma(\frac{1}{2}-s)} ds \right)$$

for  $0 < \gamma < \frac{1}{2}$

$$\frac{\frac{1}{(e^2 + e^{2 \cos(\pi/4)}) \cos(2 \sin(\frac{\pi}{4})) + e^{2 \cos(\pi/2)} \cos(2 \sin(\frac{\pi}{2}))} - 7 + \frac{34+8+3}{10^3}}{9 \cdot 10^{27}} =$$

$$\frac{200\,000\,000\,000\,000\,000\,000\,000\,000\,000}{1} +$$

$$1 / \left( 1\,000\,000\,000\,000\,000\,000\,000\,000\,000\,000 \right.$$

$$\left. \left( -7 + e^2 + \int_{\frac{\pi}{2}}^2 \left( -\sin(t) - \frac{e^{\sqrt{2}} (\sqrt{2} - \frac{\pi}{2}) \sin\left(\frac{-\pi + \frac{\pi}{\sqrt{2}} - \sqrt{2} t + \frac{\pi t}{2}}{-2 + \frac{\pi}{2}}\right)}{2 - \frac{\pi}{2}} \right) dt \right) \right)$$

$((e^2 + e^{(2 \cos(\pi/4)) \cos(2 \sin(\pi/4))} + e^{(2 \cos(\pi/2)) \cos(2 \sin(\pi/2))})^3 + 55 + 1/\text{golden ratio})$

**Input:**

$$\left( e^2 + e^{2 \cos(\pi/4)} \cos\left(2 \sin\left(\frac{\pi}{4}\right)\right) + e^{2 \cos(\pi/2)} \cos\left(2 \sin\left(\frac{\pi}{2}\right)\right) \right)^3 + 55 + \frac{1}{\phi}$$

**Exact result:**

$$\frac{1}{\phi} + 55 + \left( e^2 + \cos(2) + e^{\sqrt{2}} \cos(\sqrt{2}) \right)^3$$

**Decimal approximation:**

497.0843822870373467827192910974907335342370480432952132541...

497.0843822.... result practically equal to the rest mass of Kaon meson 497.614

**Alternate forms:**

$$\frac{1}{2} \left( 109 + \sqrt{5} \right) + \left( e^2 + \cos(2) + e^{\sqrt{2}} \cos(\sqrt{2}) \right)^3$$

$$55 + \frac{2}{1 + \sqrt{5}} + \left( e^2 + \cos(2) + e^{\sqrt{2}} \cos(\sqrt{2}) \right)^3$$

$$\frac{1}{\phi} + 55 + \left( \frac{1}{2} (e^{-2i} + e^{2i}) + e^2 + \frac{1}{2} e^{\sqrt{2}} (e^{-i\sqrt{2}} + e^{i\sqrt{2}}) \right)^3$$

**Alternative representations:**

$$\left( e^2 + e^{2 \cos(\pi/4)} \cos\left(2 \sin\left(\frac{\pi}{4}\right)\right) + e^{2 \cos(\pi/2)} \cos\left(2 \sin\left(\frac{\pi}{2}\right)\right) \right)^3 + 55 + \frac{1}{\phi} =$$

$$55 + \frac{1}{\phi} + \left( e^2 + \cosh\left(2 i \sin\left(\frac{\pi}{2}\right)\right) e^{2 \cosh((i \pi)/2)} + \cosh\left(2 i \sin\left(\frac{\pi}{4}\right)\right) e^{2 \cosh((i \pi)/4)} \right)^3$$

$$\left( e^2 + e^{2 \cos(\pi/4)} \cos\left(2 \sin\left(\frac{\pi}{4}\right)\right) + e^{2 \cos(\pi/2)} \cos\left(2 \sin\left(\frac{\pi}{2}\right)\right) \right)^3 + 55 + \frac{1}{\phi} =$$

$$55 + \frac{1}{\phi} + \left( e^2 + \cosh\left(-2 i \sin\left(\frac{\pi}{2}\right)\right) e^{2 \cosh(-(i \pi)/2)} + \cosh\left(-2 i \sin\left(\frac{\pi}{4}\right)\right) e^{2 \cosh(-(i \pi)/4)} \right)^3$$

$$\left( e^2 + e^{2 \cos(\pi/4)} \cos\left(2 \sin\left(\frac{\pi}{4}\right)\right) + e^{2 \cos(\pi/2)} \cos\left(2 \sin\left(\frac{\pi}{2}\right)\right) \right)^3 + 55 + \frac{1}{\phi} =$$

$$55 + \frac{1}{\phi} + \left( e^2 + \frac{1}{2} e^{e^{-(i \pi)/2} + e^{(i \pi)/2}} \left( e^{-2 i \sin(\pi/2)} + e^{2 i \sin(\pi/2)} \right) + \right.$$

$$\left. \frac{1}{2} e^{e^{-(i \pi)/4} + e^{(i \pi)/4}} \left( e^{-2 i \sin(\pi/4)} + e^{2 i \sin(\pi/4)} \right) \right)^3$$

**Series representations:**

$$\left( e^2 + e^{2 \cos(\pi/4)} \cos\left(2 \sin\left(\frac{\pi}{4}\right)\right) + e^{2 \cos(\pi/2)} \cos\left(2 \sin\left(\frac{\pi}{2}\right)\right) \right)^3 + 55 + \frac{1}{\phi} =$$

$$55 + \frac{1}{\phi} + \left( e^2 + \sum_{k=0}^{\infty} \frac{(-4)^k + (-2)^k e^{\sqrt{2}}}{(2k)!} \right)^3$$

$$\left( e^2 + e^{2 \cos(\pi/4)} \cos\left(2 \sin\left(\frac{\pi}{4}\right)\right) + e^{2 \cos(\pi/2)} \cos\left(2 \sin\left(\frac{\pi}{2}\right)\right) \right)^3 + 55 + \frac{1}{\phi} =$$

$$55 + \frac{1}{\phi} + \left( e^2 + \sum_{k=0}^{\infty} \frac{\cos\left(\frac{k\pi}{2} + z_0\right) \left( (2 - z_0)^k + e^{\sqrt{2}} (\sqrt{2} - z_0)^k \right)}{k!} \right)^3$$

$$\left( e^2 + e^{2 \cos(\pi/4)} \cos\left(2 \sin\left(\frac{\pi}{4}\right)\right) + e^{2 \cos(\pi/2)} \cos\left(2 \sin\left(\frac{\pi}{2}\right)\right) \right)^3 + 55 + \frac{1}{\phi} =$$

$$55 + \frac{1}{\phi} + \left( e^2 + \sum_{k=0}^{\infty} \left( \frac{(-1)^{-1+k} \left(2 - \frac{\pi}{2}\right)^{1+2k}}{(1+2k)!} + \frac{(-1)^{-1+k} e^{\sqrt{2}} \left(\sqrt{2} - \frac{\pi}{2}\right)^{1+2k}}{(1+2k)!} \right) \right)^3$$

4\*(((e^2 + e^(2cos(Pi/4)) cos(2sin(Pi/4))+e^(2cos(Pi/2)) cos(2sin(Pi/2))))^3)-  
47+11-1/golden ratio

**Input:**

$$4 \left( e^2 + e^{2 \cos(\pi/4)} \cos\left(2 \sin\left(\frac{\pi}{4}\right)\right) + e^{2 \cos(\pi/2)} \cos\left(2 \sin\left(\frac{\pi}{2}\right)\right) \right)^3 - 47 + 11 - \frac{1}{\phi}$$

$\phi$  is the golden ratio

**Exact result:**

$$-\frac{1}{\phi} - 36 + 4 \left( e^2 + \cos(2) + e^{\sqrt{2}} \cos(\sqrt{2}) \right)^3$$

**Decimal approximation:**

1729.247359204399912889854230218134743548346646274152038705...

1729.24735....

This result is very near to the mass of candidate glueball  $f_0(1710)$  meson. Furthermore, 1728 occurs in the algebraic formula for the j-invariant of an elliptic curve. As a consequence, it is sometimes called a Zagier as a pun on the Gross–Zagier theorem. The number 1728 is one less than the Hardy–Ramanujan number 1729

**Alternate forms:**

$$-36 - \frac{2}{1 + \sqrt{5}} + 4 \left( e^2 + \cos(2) + e^{\sqrt{2}} \cos(\sqrt{2}) \right)^3$$

$$\frac{1}{2} \left( -71 - \sqrt{5} \right) + 4 \left( e^2 + \cos(2) + e^{\sqrt{2}} \cos(\sqrt{2}) \right)^3$$

$$-\frac{1}{\phi} - 36 + 4 \left( \frac{1}{2} \left( e^{-2i} + e^{2i} \right) + e^2 + \frac{1}{2} e^{\sqrt{2}} \left( e^{-i\sqrt{2}} + e^{i\sqrt{2}} \right) \right)^3$$

### Alternative representations:

$$4 \left( e^2 + e^{2 \cos(\pi/4)} \cos\left(2 \sin\left(\frac{\pi}{4}\right)\right) + e^{2 \cos(\pi/2)} \cos\left(2 \sin\left(\frac{\pi}{2}\right)\right) \right)^3 - 47 + 11 - \frac{1}{\phi} =$$

$$-36 - \frac{1}{\phi} + 4 \left( e^2 + \cosh\left(2 i \sin\left(\frac{\pi}{2}\right)\right) e^{2 \cosh((i \pi)/2)} + \cosh\left(2 i \sin\left(\frac{\pi}{4}\right)\right) e^{2 \cosh((i \pi)/4)} \right)^3$$

$$4 \left( e^2 + e^{2 \cos(\pi/4)} \cos\left(2 \sin\left(\frac{\pi}{4}\right)\right) + e^{2 \cos(\pi/2)} \cos\left(2 \sin\left(\frac{\pi}{2}\right)\right) \right)^3 - 47 + 11 - \frac{1}{\phi} =$$

$$-36 - \frac{1}{\phi} + 4 \left( e^2 + \cosh\left(-2 i \sin\left(\frac{\pi}{2}\right)\right) e^{2 \cosh(-(i \pi)/2)} + \cosh\left(-2 i \sin\left(\frac{\pi}{4}\right)\right) e^{2 \cosh(-(i \pi)/4)} \right)^3$$

$$4 \left( e^2 + e^{2 \cos(\pi/4)} \cos\left(2 \sin\left(\frac{\pi}{4}\right)\right) + e^{2 \cos(\pi/2)} \cos\left(2 \sin\left(\frac{\pi}{2}\right)\right) \right)^3 - 47 + 11 - \frac{1}{\phi} =$$

$$-36 - \frac{1}{\phi} + 4 \left( e^2 + \frac{1}{2} e^{-i \pi/2} + e^{i \pi/2} \left( e^{-2 i \sin(\pi/2)} + e^{2 i \sin(\pi/2)} \right) + \right.$$

$$\left. \frac{1}{2} e^{-i \pi/4} + e^{i \pi/4} \left( e^{-2 i \sin(\pi/4)} + e^{2 i \sin(\pi/4)} \right) \right)^3$$

### Series representations:

$$4 \left( e^2 + e^{2 \cos(\pi/4)} \cos\left(2 \sin\left(\frac{\pi}{4}\right)\right) + e^{2 \cos(\pi/2)} \cos\left(2 \sin\left(\frac{\pi}{2}\right)\right) \right)^3 - 47 + 11 - \frac{1}{\phi} =$$

$$-36 - \frac{1}{\phi} + 4 \left( e^2 + \sum_{k=0}^{\infty} \frac{(-4)^k + (-2)^k e^{\sqrt{2}}}{(2k)!} \right)^3$$

$$4 \left( e^2 + e^{2 \cos(\pi/4)} \cos\left(2 \sin\left(\frac{\pi}{4}\right)\right) + e^{2 \cos(\pi/2)} \cos\left(2 \sin\left(\frac{\pi}{2}\right)\right) \right)^3 - 47 + 11 - \frac{1}{\phi} =$$

$$-36 - \frac{1}{\phi} + 4 \left( e^2 + \sum_{k=0}^{\infty} \frac{\cos\left(\frac{k \pi}{2} + z_0\right) \left( (2 - z_0)^k + e^{\sqrt{2}} (\sqrt{2} - z_0)^k \right)}{k!} \right)^3$$

$$4 \left( e^2 + e^{2 \cos(\pi/4)} \cos\left(2 \sin\left(\frac{\pi}{4}\right)\right) + e^{2 \cos(\pi/2)} \cos\left(2 \sin\left(\frac{\pi}{2}\right)\right) \right)^3 - 47 + 11 - \frac{1}{\phi} =$$

$$-36 - \frac{1}{\phi} + 4 \left( e^2 + \sum_{k=0}^{\infty} \left( \frac{(-1)^{-1+k} \left(2 - \frac{\pi}{2}\right)^{1+2k}}{(1+2k)!} + \frac{(-1)^{-1+k} e^{\sqrt{2}} \left(\sqrt{2} - \frac{\pi}{2}\right)^{1+2k}}{(1+2k)!} \right) \right)^3$$

$$18 * (((e^2 + e^{2 \cos(\pi/4)}) \cos(2 \sin(\pi/4)) + e^{2 \cos(\pi/2)} \cos(2 \sin(\pi/2))))$$

**Input:**

$$18 \left( e^2 + e^{2 \cos(\pi/4)} \cos\left(2 \sin\left(\frac{\pi}{4}\right)\right) + e^{2 \cos(\pi/2)} \cos\left(2 \sin\left(\frac{\pi}{2}\right)\right) \right)$$

**Exact result:**

$$18 \left( e^2 + \cos(2) + e^{\sqrt{2}} \cos(\sqrt{2}) \right)$$

**Decimal approximation:**

137.0582050310278852363788693573184905227265730620113312614...

137.05820503.... this result is practically equal to the inverse of fine-structure constant 137,035

**Alternate forms:**

$$18 e^2 + 18 \cos(2) + 18 e^{\sqrt{2}} \cos(\sqrt{2})$$

$$9 e^{-2i} + 9 e^{2i} + 18 e^2 + 9 e^{(1-i)\sqrt{2}} + 9 e^{(1+i)\sqrt{2}}$$

**Alternative representations:**

$$18 \left( e^2 + e^{2 \cos(\pi/4)} \cos\left(2 \sin\left(\frac{\pi}{4}\right)\right) + e^{2 \cos(\pi/2)} \cos\left(2 \sin\left(\frac{\pi}{2}\right)\right) \right) =$$

$$18 \left( e^2 + \cosh\left(2 i \sin\left(\frac{\pi}{2}\right)\right) e^{2 \cosh((i \pi)/2)} + \cosh\left(2 i \sin\left(\frac{\pi}{4}\right)\right) e^{2 \cosh((i \pi)/4)} \right)$$

$$18 \left( e^2 + e^{2 \cos(\pi/4)} \cos\left(2 \sin\left(\frac{\pi}{4}\right)\right) + e^{2 \cos(\pi/2)} \cos\left(2 \sin\left(\frac{\pi}{2}\right)\right) \right) =$$

$$18 \left( e^2 + \cosh\left(-2 i \sin\left(\frac{\pi}{2}\right)\right) e^{2 \cosh(-(i \pi)/2)} + \cosh\left(-2 i \sin\left(\frac{\pi}{4}\right)\right) e^{2 \cosh(-(i \pi)/4)} \right)$$

$$18 \left( e^2 + e^{2 \cos(\pi/4)} \cos\left(2 \sin\left(\frac{\pi}{4}\right)\right) + e^{2 \cos(\pi/2)} \cos\left(2 \sin\left(\frac{\pi}{2}\right)\right) \right) =$$

$$18 \left( e^2 + \frac{1}{2} e^{e^{-(i \pi)/2} + e^{(i \pi)/2}} \left( e^{-2 i \sin(\pi/2)} + e^{2 i \sin(\pi/2)} \right) + \right.$$

$$\left. \frac{1}{2} e^{e^{-(i \pi)/4} + e^{(i \pi)/4}} \left( e^{-2 i \sin(\pi/4)} + e^{2 i \sin(\pi/4)} \right) \right)$$

**Series representations:**



$$18 \left( e^2 + e^{2 \cos(\pi/4)} \cos\left(2 \sin\left(\frac{\pi}{4}\right)\right) + e^{2 \cos(\pi/2)} \cos\left(2 \sin\left(\frac{\pi}{2}\right)\right) \right) =$$

$$18 e^2 + \sum_{k=0}^{\infty} \frac{9 (-1)^k 2^{1+k} (2^k + e^{\sqrt{2}})}{(2k)!}$$

$$18 \left( e^2 + e^{2 \cos(\pi/4)} \cos\left(2 \sin\left(\frac{\pi}{4}\right)\right) + e^{2 \cos(\pi/2)} \cos\left(2 \sin\left(\frac{\pi}{2}\right)\right) \right) =$$

$$18 e^2 + \sum_{k=0}^{\infty} \frac{18 \cos\left(\frac{k\pi}{2} + z_0\right) \left( (2 - z_0)^k + e^{\sqrt{2}} (\sqrt{2} - z_0)^k \right)}{k!}$$

$$18 \left( e^2 + e^{2 \cos(\pi/4)} \cos\left(2 \sin\left(\frac{\pi}{4}\right)\right) + e^{2 \cos(\pi/2)} \cos\left(2 \sin\left(\frac{\pi}{2}\right)\right) \right) =$$

$$18 e^2 + \sum_{k=0}^{\infty} \left( \frac{18 (-1)^{-1+k} \left(2 - \frac{\pi}{2}\right)^{1+2k}}{(1+2k)!} + \frac{18 (-1)^{-1+k} e^{\sqrt{2}} \left(\sqrt{2} - \frac{\pi}{2}\right)^{1+2k}}{(1+2k)!} \right)$$

### Integral representations:

$$18 \left( e^2 + e^{2 \cos(\pi/4)} \cos\left(2 \sin\left(\frac{\pi}{4}\right)\right) + e^{2 \cos(\pi/2)} \cos\left(2 \sin\left(\frac{\pi}{2}\right)\right) \right) =$$

$$18 + 18 e^2 + 18 e^{\sqrt{2}} + \int_0^1 -18 \left( 2 \sin(2t) + \sqrt{2} e^{\sqrt{2}} \sin(\sqrt{2}t) \right) dt$$

$$18 \left( e^2 + e^{2 \cos(\pi/4)} \cos\left(2 \sin\left(\frac{\pi}{4}\right)\right) + e^{2 \cos(\pi/2)} \cos\left(2 \sin\left(\frac{\pi}{2}\right)\right) \right) =$$

$$18 e^2 + \int_{-i\infty+\gamma}^{i\infty+\gamma} -\frac{9 i e^{-1/s+s} \left(1 + e^{\sqrt{2}+1/(2s)}\right)}{\sqrt{\pi} \sqrt{s}} ds \text{ for } \gamma > 0$$

$$18 \left( e^2 + e^{2 \cos(\pi/4)} \cos\left(2 \sin\left(\frac{\pi}{4}\right)\right) + e^{2 \cos(\pi/2)} \cos\left(2 \sin\left(\frac{\pi}{2}\right)\right) \right) =$$

$$18 e^2 + \int_{-i\infty+\gamma}^{i\infty+\gamma} -\frac{9 i \left(1 + 2^s e^{\sqrt{2}}\right) \Gamma(s)}{\sqrt{\pi} \Gamma\left(\frac{1}{2} - s\right)} ds \text{ for } 0 < \gamma < \frac{1}{2}$$

$$18 \left( e^2 + e^{2 \cos(\pi/4)} \cos\left(2 \sin\left(\frac{\pi}{4}\right)\right) + e^{2 \cos(\pi/2)} \cos\left(2 \sin\left(\frac{\pi}{2}\right)\right) \right) =$$

$$18 e^2 + \int_{\frac{\pi}{2}}^2 \left( -18 \sin(t) - \frac{18 e^{\sqrt{2}} \left(\sqrt{2} - \frac{\pi}{2}\right) \sin\left(\frac{-\pi + \frac{\pi}{\sqrt{2}} - \sqrt{2}t + \frac{\pi t}{2}}{-2 + \frac{\pi}{2}}\right)}{2 - \frac{\pi}{2}} \right) dt$$

$18 * (((e^2 + e^{(2\cos(\pi/4)) \cos(2\sin(\pi/4))} + e^{(2\cos(\pi/2)) \cos(2\sin(\pi/2))})) + \text{golden ratio}^2$

**Input:**

$$18 \left( e^2 + e^{2\cos(\pi/4)} \cos\left(2 \sin\left(\frac{\pi}{4}\right)\right) + e^{2\cos(\pi/2)} \cos\left(2 \sin\left(\frac{\pi}{2}\right)\right) \right) + \phi^2$$

$\phi$  is the golden ratio

**Exact result:**

$$\phi^2 + 18 \left( e^2 + \cos(2) + e^{\sqrt{2}} \cos(\sqrt{2}) \right)$$

**Decimal approximation:**

139.6762390197777800845834561916841286404468822418170941235...

139.676239.... result practically equal to the rest mass of Pion meson 139.57 MeV

**Alternate forms:**

$$\frac{1}{2} \left( 3 + \sqrt{5} + 36 e^2 + 36 \cos(2) + 36 e^{\sqrt{2}} \cos(\sqrt{2}) \right)$$

$$\frac{1}{2} \left( 3 + \sqrt{5} \right) + 18 \left( e^2 + \cos(2) + e^{\sqrt{2}} \cos(\sqrt{2}) \right)$$

$$\frac{3}{2} + \frac{\sqrt{5}}{2} + 18 e^2 + 18 \cos(2) + 18 e^{\sqrt{2}} \cos(\sqrt{2})$$

**Alternative representations:**

$$18 \left( e^2 + e^{2\cos(\pi/4)} \cos\left(2 \sin\left(\frac{\pi}{4}\right)\right) + e^{2\cos(\pi/2)} \cos\left(2 \sin\left(\frac{\pi}{2}\right)\right) \right) + \phi^2 =$$

$$18 \left( e^2 + \cosh\left(2 i \sin\left(\frac{\pi}{2}\right)\right) e^{2\cosh((i \pi)/2)} + \cosh\left(2 i \sin\left(\frac{\pi}{4}\right)\right) e^{2\cosh((i \pi)/4)} \right) + \phi^2$$

$$18 \left( e^2 + e^{2\cos(\pi/4)} \cos\left(2 \sin\left(\frac{\pi}{4}\right)\right) + e^{2\cos(\pi/2)} \cos\left(2 \sin\left(\frac{\pi}{2}\right)\right) \right) + \phi^2 =$$

$$18 \left( e^2 + \cosh\left(-2 i \sin\left(\frac{\pi}{2}\right)\right) e^{2\cosh(-(i \pi)/2)} + \cosh\left(-2 i \sin\left(\frac{\pi}{4}\right)\right) e^{2\cosh(-(i \pi)/4)} \right) + \phi^2$$

$$18 \left( e^2 + e^{2\cos(\pi/4)} \cos\left(2 \sin\left(\frac{\pi}{4}\right)\right) + e^{2\cos(\pi/2)} \cos\left(2 \sin\left(\frac{\pi}{2}\right)\right) \right) + \phi^2 =$$

$$18 \left( e^2 + \frac{1}{2} e^{-i \pi/2} + e^{i \pi/2} \left( e^{-2 i \sin(\pi/2)} + e^{2 i \sin(\pi/2)} \right) + \frac{1}{2} e^{-i \pi/4} + e^{i \pi/4} \left( e^{-2 i \sin(\pi/4)} + e^{2 i \sin(\pi/4)} \right) \right) + \phi^2$$

### Series representations:

$$18 \left( e^2 + e^{2 \cos(\pi/4)} \cos\left(2 \sin\left(\frac{\pi}{4}\right)\right) + e^{2 \cos(\pi/2)} \cos\left(2 \sin\left(\frac{\pi}{2}\right)\right) \right) + \phi^2 =$$

$$18 e^2 + \phi^2 + \sum_{k=0}^{\infty} \frac{9 (-1)^k 2^{1+k} (2^k + e^{\sqrt{2}})}{(2k)!}$$

$$18 \left( e^2 + e^{2 \cos(\pi/4)} \cos\left(2 \sin\left(\frac{\pi}{4}\right)\right) + e^{2 \cos(\pi/2)} \cos\left(2 \sin\left(\frac{\pi}{2}\right)\right) \right) + \phi^2 =$$

$$18 e^2 + \phi^2 + \sum_{k=0}^{\infty} \frac{18 \cos\left(\frac{k\pi}{2} + z_0\right) \left( (2 - z_0)^k + e^{\sqrt{2}} (\sqrt{2} - z_0)^k \right)}{k!}$$

$$18 \left( e^2 + e^{2 \cos(\pi/4)} \cos\left(2 \sin\left(\frac{\pi}{4}\right)\right) + e^{2 \cos(\pi/2)} \cos\left(2 \sin\left(\frac{\pi}{2}\right)\right) \right) + \phi^2 =$$

$$18 e^2 + \phi^2 + \sum_{k=0}^{\infty} \left( \frac{18 (-1)^{-1+k} \left(2 - \frac{\pi}{2}\right)^{1+2k}}{(1+2k)!} + \frac{18 (-1)^{-1+k} e^{\sqrt{2}} \left(\sqrt{2} - \frac{\pi}{2}\right)^{1+2k}}{(1+2k)!} \right)$$

### Integral representations:

$$18 \left( e^2 + e^{2 \cos(\pi/4)} \cos\left(2 \sin\left(\frac{\pi}{4}\right)\right) + e^{2 \cos(\pi/2)} \cos\left(2 \sin\left(\frac{\pi}{2}\right)\right) \right) + \phi^2 =$$

$$\frac{39}{2} + \frac{\sqrt{5}}{2} + 18 e^2 + 18 e^{\sqrt{2}} + \int_0^1 -18 \left( 2 \sin(2t) + \sqrt{2} e^{\sqrt{2}} \sin(\sqrt{2} t) \right) dt$$

$$18 \left( e^2 + e^{2 \cos(\pi/4)} \cos\left(2 \sin\left(\frac{\pi}{4}\right)\right) + e^{2 \cos(\pi/2)} \cos\left(2 \sin\left(\frac{\pi}{2}\right)\right) \right) + \phi^2 =$$

$$\frac{3}{2} + \frac{\sqrt{5}}{2} + 18 e^2 + \int_{-i\infty+\gamma}^{i\infty+\gamma} - \frac{9 i e^{-1/s+s} \left(1 + e^{\sqrt{2} + 1/(2s)}\right)}{\sqrt{\pi} \sqrt{s}} ds \quad \text{for } \gamma > 0$$

$$18 \left( e^2 + e^{2 \cos(\pi/4)} \cos\left(2 \sin\left(\frac{\pi}{4}\right)\right) + e^{2 \cos(\pi/2)} \cos\left(2 \sin\left(\frac{\pi}{2}\right)\right) \right) + \phi^2 =$$

$$\frac{3}{2} + \frac{\sqrt{5}}{2} + 18 e^2 + \int_{-i\infty+\gamma}^{i\infty+\gamma} - \frac{9 i \left(1 + 2^s e^{\sqrt{2}}\right) \Gamma(s)}{\sqrt{\pi} \Gamma\left(\frac{1}{2} - s\right)} ds \quad \text{for } 0 < \gamma < \frac{1}{2}$$

$$18 \left( e^2 + e^{2 \cos(\pi/4)} \cos\left(2 \sin\left(\frac{\pi}{4}\right)\right) + e^{2 \cos(\pi/2)} \cos\left(2 \sin\left(\frac{\pi}{2}\right)\right) \right) + \phi^2 =$$

$$18 e^2 + \phi^2 + \int_{\frac{\pi}{2}}^2 \left( -18 \sin(t) - \frac{18 e^{\sqrt{2}} \left(\sqrt{2} - \frac{\pi}{2}\right) \sin\left(\frac{-\pi + \frac{\pi}{\sqrt{2}} - \sqrt{2} t + \frac{\pi t}{2}}{-2 + \frac{\pi}{2}}\right)}{2 - \frac{\pi}{2}} \right) dt$$

Page 223

~~If  $\alpha + \beta = \delta$ , then~~ If  $\alpha + \beta + 1 = \gamma + \delta$ .

$$\int \frac{x^{n-2} \left\{ 1 + \frac{\alpha}{L} \cdot \frac{\beta}{r} x + \frac{\alpha(\alpha+1)}{L^2} \cdot \frac{\beta(\beta+1)}{r(r+1)} x^2 + \dots \right\} dx}{x^\gamma (1-x)^\delta \left\{ 1 + \frac{\alpha}{L} \cdot \frac{\beta}{r} x + \frac{\alpha(\alpha+1)}{L^2} \cdot \frac{\beta(\beta+1)}{r(r+1)} x^2 + \dots \right\}^2}$$

$$= \frac{x^{n-\gamma} (1-x)^{\delta-1}}{(n-\gamma)(n-1)} \left\{ 1 + \frac{(n-\alpha)(n-\beta)}{n(n-\gamma+1)} x + \frac{(n-\alpha)(n-\alpha+1)(n-\beta)(n-\beta+1)}{n(n+1)(n-\gamma+1)(n-\gamma+2)} x^2 + \dots \right\}$$

$$\div \left\{ 1 + \frac{\alpha}{L} \cdot \frac{\beta}{r} x + \frac{\alpha(\alpha+1)}{L^2} \cdot \frac{\beta(\beta+1)}{r(r+1)} x^2 + \dots \right\}$$

$$\alpha\beta = 4\pi^2$$

$$x = 2, n = 8, \gamma = 3\pi, \delta = 4\pi$$

$$\frac{((2^{8-3\pi})(1-2)^{1-4\pi})/((8-3\pi)(8-1))*(((1+((8-2\pi)(8-2\pi)*2)/(8(8-3\pi+1)))+((8-2\pi)(8-2\pi+1)(8-2\pi)(8-2\pi+2))/((8(8+1)(8-3\pi+1)(8-3\pi+2))))))}{(8-3\pi)(8-1)}$$

**Input:**

$$\frac{2^{8-3\pi} (1-2)^{1-4\pi}}{(8-3\pi)(8-1)} \left( 1 + \frac{(8-2\pi)(8-2\pi) \times 2}{8(8-3\pi+1)} + \frac{(8-2\pi)(8-2\pi+1)(8-2\pi)(8-2\pi+2)}{8(8+1)(8-3\pi+1)(8-3\pi+2)} \right)$$

**Exact result:**

$$\frac{(-1)^{1-4\pi} 2^{8-3\pi} \left( 1 + \frac{(8-2\pi)^2}{4(9-3\pi)} + \frac{(8-2\pi)^2(9-2\pi)(10-2\pi)}{72(9-3\pi)(10-3\pi)} \right)}{7(8-3\pi)}$$

**Decimal approximation:**

0.01875892211413696177519775509698509915174472429118961333... +  
0.08865917977421401172539221406115005684444827578969843829... i

**Polar coordinates:**

$r \approx 0.090622$  (radius),  $\theta \approx 78.0533^\circ$  (angle)

0.090622

**Alternate forms:**

$$\frac{(-1)^{-4\pi} 2^{8-3\pi} (2970 + \pi(2\pi(308 + (\pi - 31)\pi) - 2329))}{189(\pi - 3)(3\pi - 10)(3\pi - 8)}$$

$$\frac{(-1)^{1-4\pi} 2^{8-3\pi} (2970 - 2329\pi + 616\pi^2 - 62\pi^3 + 2\pi^4)}{189(\pi - 3)(3\pi - 10)(3\pi - 8)}$$

$$\begin{aligned}
& i \left( \frac{2^{8-3\pi} \sin((1-4\pi)\pi)}{7(8-3\pi)} + \frac{2^{12-3\pi} \sin((1-4\pi)\pi)}{7(8-3\pi)(9-3\pi)} + \frac{5 \times 2^{12-3\pi} \sin((1-4\pi)\pi)}{7(8-3\pi)(9-3\pi)(10-3\pi)} - \right. \\
& \quad \frac{2^{11-3\pi} \pi \sin((1-4\pi)\pi)}{7(8-3\pi)(9-3\pi)} - \frac{83 \times 2^{11-3\pi} \pi \sin((1-4\pi)\pi)}{63(8-3\pi)(9-3\pi)(10-3\pi)} + \\
& \quad \frac{2^{8-3\pi} \pi^2 \sin((1-4\pi)\pi)}{7(8-3\pi)(9-3\pi)} + \frac{229 \times 2^{8-3\pi} \pi^2 \sin((1-4\pi)\pi)}{63(8-3\pi)(9-3\pi)(10-3\pi)} - \\
& \quad \left. \frac{5 \times 2^{8-3\pi} \pi^3 \sin((1-4\pi)\pi)}{9(8-3\pi)(9-3\pi)(10-3\pi)} + \frac{2^{9-3\pi} \pi^4 \sin((1-4\pi)\pi)}{63(8-3\pi)(9-3\pi)(10-3\pi)} \right) + \\
& \frac{2^{8-3\pi} \cos((1-4\pi)\pi)}{7(8-3\pi)} + \frac{2^{12-3\pi} \cos((1-4\pi)\pi)}{7(8-3\pi)(9-3\pi)} + \\
& \frac{5 \times 2^{12-3\pi} \cos((1-4\pi)\pi)}{7(8-3\pi)(9-3\pi)(10-3\pi)} - \\
& \frac{2^{11-3\pi} \pi \cos((1-4\pi)\pi)}{7(8-3\pi)(9-3\pi)} - \\
& \frac{83 \times 2^{11-3\pi} \pi \cos((1-4\pi)\pi)}{63(8-3\pi)(9-3\pi)(10-3\pi)} + \\
& \frac{2^{8-3\pi} \pi^2 \cos((1-4\pi)\pi)}{7(8-3\pi)(9-3\pi)} + \\
& \frac{229 \times 2^{8-3\pi} \pi^2 \cos((1-4\pi)\pi)}{63(8-3\pi)(9-3\pi)(10-3\pi)} - \\
& \frac{5 \times 2^{8-3\pi} \pi^3 \cos((1-4\pi)\pi)}{9(8-3\pi)(9-3\pi)(10-3\pi)} + \frac{2^{9-3\pi} \pi^4 \cos((1-4\pi)\pi)}{63(8-3\pi)(9-3\pi)(10-3\pi)}
\end{aligned}$$

**Expanded form:**

$$\begin{aligned}
& \frac{(-1)^{1-4\pi} 2^{8-3\pi}}{7(8-3\pi)} + \frac{(-1)^{1-4\pi} 2^{12-3\pi}}{7(8-3\pi)(9-3\pi)} + \frac{5(-1)^{1-4\pi} 2^{12-3\pi}}{7(8-3\pi)(9-3\pi)(10-3\pi)} + \\
& \frac{(-1)^{2-4\pi} 2^{11-3\pi} \pi}{7(8-3\pi)(9-3\pi)} - \frac{83(-1)^{1-4\pi} 2^{11-3\pi} \pi}{63(8-3\pi)(9-3\pi)(10-3\pi)} + \\
& \frac{(-1)^{1-4\pi} 2^{8-3\pi} \pi^2}{7(8-3\pi)(9-3\pi)} + \frac{229(-1)^{1-4\pi} 2^{8-3\pi} \pi^2}{63(8-3\pi)(9-3\pi)(10-3\pi)} - \\
& \frac{5(-1)^{1-4\pi} 2^{8-3\pi} \pi^3}{9(8-3\pi)(9-3\pi)(10-3\pi)} + \frac{(-1)^{1-4\pi} 2^{9-3\pi} \pi^4}{63(8-3\pi)(9-3\pi)(10-3\pi)}
\end{aligned}$$

**Alternative representations:**

$$\frac{\left(1 + \frac{(8-2\pi)(8-2\pi)2}{8(8-3\pi+1)} + \frac{(8-2\pi)(8-2\pi+1)(8-2\pi)(8-2\pi+2)}{8(8+1)(8-3\pi+1)(8-3\pi+2)}\right)(2^{8-3\pi}(1-2)^{1-4\pi})}{(8-3\pi)(8-1)} =$$

$$\frac{1}{7(8-3\cos^{-1}(-1))}(-1)^{1-4\cos^{-1}(-1)}2^{8-3\cos^{-1}(-1)}$$

$$\left(1 + \frac{2(8-2\cos^{-1}(-1))^2}{8(9-3\cos^{-1}(-1))} + \frac{(8-2\cos^{-1}(-1))^2(9-2\cos^{-1}(-1))(10-2\cos^{-1}(-1))}{72(9-3\cos^{-1}(-1))(10-3\cos^{-1}(-1))}\right)$$

$$\frac{\left(1 + \frac{(8-2\pi)(8-2\pi)2}{8(8-3\pi+1)} + \frac{(8-2\pi)(8-2\pi+1)(8-2\pi)(8-2\pi+2)}{8(8+1)(8-3\pi+1)(8-3\pi+2)}\right)(2^{8-3\pi}(1-2)^{1-4\pi})}{(8-3\pi)(8-1)} =$$

$$\frac{(-1)^{1-8E(0)}2^{8-6E(0)}\left(1 + \frac{2(8-4E(0))^2}{8(9-6E(0))} + \frac{(8-4E(0))^2(9-4E(0))(10-4E(0))}{72(9-6E(0))(10-6E(0))}\right)}{7(8-6E(0))}$$

$$\frac{\left(1 + \frac{(8-2\pi)(8-2\pi)2}{8(8-3\pi+1)} + \frac{(8-2\pi)(8-2\pi+1)(8-2\pi)(8-2\pi+2)}{8(8+1)(8-3\pi+1)(8-3\pi+2)}\right)(2^{8-3\pi}(1-2)^{1-4\pi})}{(8-3\pi)(8-1)} =$$

$$\frac{(-1)^{1-8K(0)}2^{8-6K(0)}\left(1 + \frac{2(8-4K(0))^2}{8(9-6K(0))} + \frac{(8-4K(0))^2(9-4K(0))(10-4K(0))}{72(9-6K(0))(10-6K(0))}\right)}{7(8-6K(0))}$$

### Series representations:

$$\frac{\left(1 + \frac{(8-2\pi)(8-2\pi)2}{8(8-3\pi+1)} + \frac{(8-2\pi)(8-2\pi+1)(8-2\pi)(8-2\pi+2)}{8(8+1)(8-3\pi+1)(8-3\pi+2)}\right)(2^{8-3\pi}(1-2)^{1-4\pi})}{(8-3\pi)(8-1)} =$$

$$\left((-1)^{-16\sum_{k=0}^{\infty}(-1)^k/(1+2k)}2^{6-12\sum_{k=0}^{\infty}(-1)^k/(1+2k)}\left(1485 - 4658\sum_{k=0}^{\infty}\frac{(-1)^k}{1+2k} +\right.\right.$$

$$\left.4928\left(\sum_{k=0}^{\infty}\frac{(-1)^k}{1+2k}\right)^2 - 1984\left(\sum_{k=0}^{\infty}\frac{(-1)^k}{1+2k}\right)^3 + 256\left(\sum_{k=0}^{\infty}\frac{(-1)^k}{1+2k}\right)^4\right)/$$

$$\left(189\left(-2 + 3\sum_{k=0}^{\infty}\frac{(-1)^k}{1+2k}\right)\left(-3 + 4\sum_{k=0}^{\infty}\frac{(-1)^k}{1+2k}\right)\left(-5 + 6\sum_{k=0}^{\infty}\frac{(-1)^k}{1+2k}\right)\right)$$

$$\begin{aligned}
& \frac{\left(1 + \frac{(8-2\pi)(8-2\pi)2}{8(8-3\pi+1)} + \frac{(8-2\pi)(8-2\pi+1)(8-2\pi)(8-2\pi+2)}{8(8+1)(8-3\pi+1)(8-3\pi+2)}\right) (2^{8-3\pi} (1-2)^{1-4\pi})}{(8-3\pi)(8-1)} = \\
& \left( (-1)^{-4 \sum_{k=0}^{\infty} (-1/4)^k (1/(1+2k)+2/(1+4k)+1/(3+4k))} 2^{8-3 \sum_{k=0}^{\infty} (-1/4)^k (1/(1+2k)+2/(1+4k)+1/(3+4k))} \right. \\
& \quad \left( 2970 - 2329 \sum_{k=0}^{\infty} \left(-\frac{1}{4}\right)^k \left(\frac{1}{1+2k} + \frac{2}{1+4k} + \frac{1}{3+4k}\right) + \right. \\
& \quad 616 \left( \sum_{k=0}^{\infty} \left(-\frac{1}{4}\right)^k \left(\frac{1}{1+2k} + \frac{2}{1+4k} + \frac{1}{3+4k}\right) \right)^2 - \\
& \quad 62 \left( \sum_{k=0}^{\infty} \left(-\frac{1}{4}\right)^k \left(\frac{1}{1+2k} + \frac{2}{1+4k} + \frac{1}{3+4k}\right) \right)^3 + \\
& \quad \left. \left. 2 \left( \sum_{k=0}^{\infty} \left(-\frac{1}{4}\right)^k \left(\frac{1}{1+2k} + \frac{2}{1+4k} + \frac{1}{3+4k}\right) \right)^4 \right) \right) / \\
& \left( 189 \left( -3 + \sum_{k=0}^{\infty} \left(-\frac{1}{4}\right)^k \left(\frac{1}{1+2k} + \frac{2}{1+4k} + \frac{1}{3+4k}\right) \right) \right. \\
& \quad \left( -10 + 3 \sum_{k=0}^{\infty} \left(-\frac{1}{4}\right)^k \left(\frac{1}{1+2k} + \frac{2}{1+4k} + \frac{1}{3+4k}\right) \right) \\
& \quad \left. \left( -8 + 3 \sum_{k=0}^{\infty} \left(-\frac{1}{4}\right)^k \left(\frac{1}{1+2k} + \frac{2}{1+4k} + \frac{1}{3+4k}\right) \right) \right)
\end{aligned}$$



$$\begin{aligned}
& \frac{\left(1 + \frac{(8-2\pi)(8-2\pi)2}{8(8-3\pi+1)} + \frac{(8-2\pi)(8-2\pi+1)(8-2\pi)(8-2\pi+2)}{8(8+1)(8-3\pi+1)(8-3\pi+2)}\right) (2^{8-3\pi} (1-2)^{1-4\pi})}{(8-3\pi)(8-1)} = \\
& \left( (-1)^{-4 \sum_{k=0}^{\infty} -4(-1)^k 1195^{-1-2k} (5^{1+2k} - 4 \times 239^{1+2k})} / (1+2k) \right. \\
& \quad 2^{8-3 \sum_{k=0}^{\infty} -4(-1)^k 1195^{-1-2k} (5^{1+2k} - 4 \times 239^{1+2k})} / (1+2k) \\
& \quad \left. \left( 2970 - 2329 \sum_{k=0}^{\infty} -\frac{4(-1)^k 1195^{-1-2k} (5^{1+2k} - 4 \times 239^{1+2k})}{1+2k} \right) + \right. \\
& \quad 616 \left( \sum_{k=0}^{\infty} -\frac{4(-1)^k 1195^{-1-2k} (5^{1+2k} - 4 \times 239^{1+2k})^2}{1+2k} \right) - \\
& \quad 62 \left( \sum_{k=0}^{\infty} -\frac{4(-1)^k 1195^{-1-2k} (5^{1+2k} - 4 \times 239^{1+2k})^3}{1+2k} \right) + \\
& \quad \left. \left. 2 \left( \sum_{k=0}^{\infty} -\frac{4(-1)^k 1195^{-1-2k} (5^{1+2k} - 4 \times 239^{1+2k})^4}{1+2k} \right) \right) \right) / \\
& \left( 189 \left( -3 + \sum_{k=0}^{\infty} -\frac{4(-1)^k 1195^{-1-2k} (5^{1+2k} - 4 \times 239^{1+2k})}{1+2k} \right) \right. \\
& \quad \left( -10 + 3 \sum_{k=0}^{\infty} -\frac{4(-1)^k 1195^{-1-2k} (5^{1+2k} - 4 \times 239^{1+2k})}{1+2k} \right) \\
& \quad \left. \left( -8 + 3 \sum_{k=0}^{\infty} -\frac{4(-1)^k 1195^{-1-2k} (5^{1+2k} - 4 \times 239^{1+2k})}{1+2k} \right) \right)
\end{aligned}$$

$$1 + ((2\pi * 2\pi) * 2) / (3\pi) + (2\pi(2\pi+1)) / 2 * ((2\pi(2\pi+1))^4) / ((3\pi(3\pi+1)))$$

**Input:**

$$1 + \frac{(2\pi \times 2\pi) \times 2}{3\pi} + \left( \frac{1}{2} (2\pi(2\pi+1)) \right) \times \frac{(2\pi(2\pi+1)) \times 4}{3\pi(3\pi+1)}$$

**Result:**

$$1 + \frac{8\pi}{3} + \frac{8\pi(1+2\pi)^2}{3(1+3\pi)}$$

**Decimal approximation:**

52.00553663879689783174615357371493805751758702273571947459...

52.0055366...

**Property:**

$$1 + \frac{8\pi}{3} + \frac{8\pi(1+2\pi)^2}{3(1+3\pi)} \text{ is a transcendental number}$$

**Alternate forms:**

$$\frac{3 + 25\pi + 56\pi^2 + 32\pi^3}{3(1 + 3\pi)}$$

$$\frac{3 + \pi(25 + 8\pi(7 + 4\pi))}{3 + 9\pi}$$

$$1 + \frac{8\pi}{3} + \frac{8\pi(1 + 2\pi)^2}{3 + 9\pi}$$

**Alternative representations:**

$$1 + \frac{(2\pi \cdot 2\pi) \cdot 2}{3\pi} + \frac{((2\pi(2\pi + 1)) \cdot 4) \cdot (2\pi(2\pi + 1))}{(3\pi(3\pi + 1)) \cdot 2} =$$

$$1 + \frac{8 \cos^{-1}(-1)^2}{3 \cos^{-1}(-1)} + \frac{8 \cos^{-1}(-1)^2 (1 + 2 \cos^{-1}(-1))^2}{3 \cos^{-1}(-1) (1 + 3 \cos^{-1}(-1))}$$

$$1 + \frac{(2\pi \cdot 2\pi) \cdot 2}{3\pi} + \frac{((2\pi(2\pi + 1)) \cdot 4) \cdot (2\pi(2\pi + 1))}{(3\pi(3\pi + 1)) \cdot 2} = 1 + \frac{32 E(0)^2}{6 E(0)} + \frac{32 E(0)^2 (1 + 4 E(0))^2}{6 E(0) (1 + 6 E(0))}$$

$$1 + \frac{(2\pi \cdot 2\pi) \cdot 2}{3\pi} + \frac{((2\pi(2\pi + 1)) \cdot 4) \cdot (2\pi(2\pi + 1))}{(3\pi(3\pi + 1)) \cdot 2} = 1 + \frac{32 K(0)^2}{6 K(0)} + \frac{32 K(0)^2 (1 + 4 K(0))^2}{6 K(0) (1 + 6 K(0))}$$

**Series representations:**

$$1 + \frac{(2\pi \cdot 2\pi) \cdot 2}{3\pi} + \frac{((2\pi(2\pi + 1)) \cdot 4) \cdot (2\pi(2\pi + 1))}{(3\pi(3\pi + 1)) \cdot 2} =$$

$$\frac{3 + 100 \sum_{k=0}^{\infty} \frac{(-1)^k}{1+2k} + 896 \left( \sum_{k=0}^{\infty} \frac{(-1)^k}{1+2k} \right)^2 + 2048 \left( \sum_{k=0}^{\infty} \frac{(-1)^k}{1+2k} \right)^3}{3 \left( 1 + 12 \sum_{k=0}^{\infty} \frac{(-1)^k}{1+2k} \right)}$$

$$1 + \frac{(2\pi \cdot 2\pi) \cdot 2}{3\pi} + \frac{((2\pi(2\pi + 1)) \cdot 4) \cdot (2\pi(2\pi + 1))}{(3\pi(3\pi + 1)) \cdot 2} =$$

$$\left( 3 + 25 \sum_{k=0}^{\infty} \left( -\frac{1}{4} \right)^k \left( \frac{1}{1+2k} + \frac{2}{1+4k} + \frac{1}{3+4k} \right) + \right.$$

$$56 \left( \sum_{k=0}^{\infty} \left( -\frac{1}{4} \right)^k \left( \frac{1}{1+2k} + \frac{2}{1+4k} + \frac{1}{3+4k} \right) \right)^2 +$$

$$32 \left( \sum_{k=0}^{\infty} \left( -\frac{1}{4} \right)^k \left( \frac{1}{1+2k} + \frac{2}{1+4k} + \frac{1}{3+4k} \right) \right)^3 \Bigg) /$$

$$\left( 3 \left( 1 + 3 \sum_{k=0}^{\infty} \left( -\frac{1}{4} \right)^k \left( \frac{1}{1+2k} + \frac{2}{1+4k} + \frac{1}{3+4k} \right) \right) \right)$$

$$1 + \frac{(2\pi)^2}{3\pi} + \frac{((2\pi(2\pi+1))^4)(2\pi(2\pi+1))}{(3\pi(3\pi+1))^2} =$$

$$\left( 3 + 25 \sum_{k=0}^{\infty} -\frac{4(-1)^k 1195^{-1-2k} (5^{1+2k} - 4 \times 239^{1+2k})}{1+2k} + \right.$$

$$56 \left( \sum_{k=0}^{\infty} -\frac{4(-1)^k 1195^{-1-2k} (5^{1+2k} - 4 \times 239^{1+2k})}{1+2k} \right)^2 +$$

$$32 \left( \sum_{k=0}^{\infty} -\frac{4(-1)^k 1195^{-1-2k} (5^{1+2k} - 4 \times 239^{1+2k})}{1+2k} \right)^3 \Bigg) /$$

$$\left( 3 \left( 1 + 3 \sum_{k=0}^{\infty} -\frac{4(-1)^k 1195^{-1-2k} (5^{1+2k} - 4 \times 239^{1+2k})}{1+2k} \right) \right)$$

$$[\frac{(((-1)^{(1-4\pi)} 2^{(8-3\pi)} (1 + (8-2\pi)^2 / (4(9-3\pi)) + ((8-2\pi)^2 (9-2\pi)(10-2\pi)) / (72(9-3\pi)(10-3\pi)))) / (7(8-3\pi)))}{((1 + (8\pi)/3 + (8\pi(1+2\pi)^2) / (3(1+3\pi))))}]$$

**Input:**

$$\frac{(-1)^{1-4\pi} 2^{8-3\pi} \left( 1 + \frac{(8-2\pi)^2}{4(9-3\pi)} + \frac{(8-2\pi)^2 (9-2\pi)(10-2\pi)}{72(9-3\pi)(10-3\pi)} \right)}{7(8-3\pi)}$$

$$1 + \frac{8\pi}{3} + \frac{8\pi(1+2\pi)^2}{3(1+3\pi)}$$

**Exact result:**

$$\frac{(-1)^{1-4\pi} 2^{8-3\pi} \left( 1 + \frac{(8-2\pi)^2}{4(9-3\pi)} + \frac{(8-2\pi)^2 (9-2\pi)(10-2\pi)}{72(9-3\pi)(10-3\pi)} \right)}{7(8-3\pi) \left( 1 + \frac{8\pi}{3} + \frac{8\pi(1+2\pi)^2}{3(1+3\pi)} \right)}$$

**Decimal approximation:**

0.00036071009601201824794439554838021483755196146584145089... +  
0.0017048027095652149309638420175202877341450192383332704... i

**Polar coordinates:**

$r \approx 0.00174255$  (radius),  $\theta \approx 78.0533^\circ$  (angle)

0.00174255

**Alternate forms:**

$$\frac{(-1)^{-4\pi} 2^{8-3\pi} (1+3\pi)(2970 + \pi(2\pi(308 + (\pi-31)\pi) - 2329))}{63(\pi-3)(3\pi-10)(3\pi-8)(3+\pi(25+8\pi(7+4\pi)))}$$

$$-\frac{(-1)^{1-4\pi} 2^{8-3\pi} (1+3\pi)(2970 - 2329\pi + 616\pi^2 - 62\pi^3 + 2\pi^4)}{63(\pi-3)(3\pi-10)(3\pi-8)(3+25\pi+56\pi^2+32\pi^3)}$$

$$\begin{aligned}
& \frac{2^{8-3\pi} \cos((1-4\pi)\pi)}{7(8-3\pi)\left(1 + \frac{8\pi}{3} + \frac{8\pi(1+2\pi)^2}{3(1+3\pi)}\right)} + \frac{2^{12-3\pi} \cos((1-4\pi)\pi)}{7(8-3\pi)(9-3\pi)\left(1 + \frac{8\pi}{3} + \frac{8\pi(1+2\pi)^2}{3(1+3\pi)}\right)} + \\
& \frac{5 \times 2^{12-3\pi} \cos((1-4\pi)\pi)}{7(8-3\pi)(9-3\pi)(10-3\pi)\left(1 + \frac{8\pi}{3} + \frac{8\pi(1+2\pi)^2}{3(1+3\pi)}\right)} - \\
& \frac{2^{11-3\pi} \pi \cos((1-4\pi)\pi)}{7(8-3\pi)(9-3\pi)\left(1 + \frac{8\pi}{3} + \frac{8\pi(1+2\pi)^2}{3(1+3\pi)}\right)} - \\
& \frac{83 \times 2^{11-3\pi} \pi \cos((1-4\pi)\pi)}{63(8-3\pi)(9-3\pi)(10-3\pi)\left(1 + \frac{8\pi}{3} + \frac{8\pi(1+2\pi)^2}{3(1+3\pi)}\right)} + \\
& \frac{2^{8-3\pi} \pi^2 \cos((1-4\pi)\pi)}{7(8-3\pi)(9-3\pi)\left(1 + \frac{8\pi}{3} + \frac{8\pi(1+2\pi)^2}{3(1+3\pi)}\right)} + \\
& \frac{229 \times 2^{8-3\pi} \pi^2 \cos((1-4\pi)\pi)}{63(8-3\pi)(9-3\pi)(10-3\pi)\left(1 + \frac{8\pi}{3} + \frac{8\pi(1+2\pi)^2}{3(1+3\pi)}\right)} - \\
& \frac{5 \times 2^{8-3\pi} \pi^3 \cos((1-4\pi)\pi)}{9(8-3\pi)(9-3\pi)(10-3\pi)\left(1 + \frac{8\pi}{3} + \frac{8\pi(1+2\pi)^2}{3(1+3\pi)}\right)} + \\
& \frac{2^{9-3\pi} \pi^4 \cos((1-4\pi)\pi)}{63(8-3\pi)(9-3\pi)(10-3\pi)\left(1 + \frac{8\pi}{3} + \frac{8\pi(1+2\pi)^2}{3(1+3\pi)}\right)} + \\
& i \left( \frac{2^{8-3\pi} \sin((1-4\pi)\pi)}{7(8-3\pi)\left(1 + \frac{8\pi}{3} + \frac{8\pi(1+2\pi)^2}{3(1+3\pi)}\right)} + \frac{2^{12-3\pi} \sin((1-4\pi)\pi)}{7(8-3\pi)(9-3\pi)\left(1 + \frac{8\pi}{3} + \frac{8\pi(1+2\pi)^2}{3(1+3\pi)}\right)} + \right. \\
& \frac{5 \times 2^{12-3\pi} \sin((1-4\pi)\pi)}{7(8-3\pi)(9-3\pi)(10-3\pi)\left(1 + \frac{8\pi}{3} + \frac{8\pi(1+2\pi)^2}{3(1+3\pi)}\right)} - \\
& \frac{2^{11-3\pi} \pi \sin((1-4\pi)\pi)}{7(8-3\pi)(9-3\pi)\left(1 + \frac{8\pi}{3} + \frac{8\pi(1+2\pi)^2}{3(1+3\pi)}\right)} - \\
& \frac{83 \times 2^{11-3\pi} \pi \sin((1-4\pi)\pi)}{63(8-3\pi)(9-3\pi)(10-3\pi)\left(1 + \frac{8\pi}{3} + \frac{8\pi(1+2\pi)^2}{3(1+3\pi)}\right)} + \\
& \frac{2^{8-3\pi} \pi^2 \sin((1-4\pi)\pi)}{7(8-3\pi)(9-3\pi)\left(1 + \frac{8\pi}{3} + \frac{8\pi(1+2\pi)^2}{3(1+3\pi)}\right)} + \\
& \frac{229 \times 2^{8-3\pi} \pi^2 \sin((1-4\pi)\pi)}{63(8-3\pi)(9-3\pi)(10-3\pi)\left(1 + \frac{8\pi}{3} + \frac{8\pi(1+2\pi)^2}{3(1+3\pi)}\right)} - \\
& \frac{5 \times 2^{8-3\pi} \pi^3 \sin((1-4\pi)\pi)}{9(8-3\pi)(9-3\pi)(10-3\pi)\left(1 + \frac{8\pi}{3} + \frac{8\pi(1+2\pi)^2}{3(1+3\pi)}\right)} + \\
& \left. \frac{2^{9-3\pi} \pi^4 \sin((1-4\pi)\pi)}{63(8-3\pi)(9-3\pi)(10-3\pi)\left(1 + \frac{8\pi}{3} + \frac{8\pi(1+2\pi)^2}{3(1+3\pi)}\right)} \right)
\end{aligned}$$

**Expanded form:**

$$\begin{aligned}
& \frac{(-1)^{1-4\pi} 2^{8-3\pi}}{7(8-3\pi) \left(1 + \frac{8\pi}{3} + \frac{8\pi(1+2\pi)^2}{3(1+3\pi)}\right)} + \frac{(-1)^{1-4\pi} 2^{12-3\pi}}{7(8-3\pi)(9-3\pi) \left(1 + \frac{8\pi}{3} + \frac{8\pi(1+2\pi)^2}{3(1+3\pi)}\right)} + \\
& \frac{5(-1)^{1-4\pi} 2^{12-3\pi}}{7(8-3\pi)(9-3\pi)(10-3\pi) \left(1 + \frac{8\pi}{3} + \frac{8\pi(1+2\pi)^2}{3(1+3\pi)}\right)} + \\
& \frac{(-1)^{2-4\pi} 2^{11-3\pi} \pi}{7(8-3\pi)(9-3\pi) \left(1 + \frac{8\pi}{3} + \frac{8\pi(1+2\pi)^2}{3(1+3\pi)}\right)} - \\
& \frac{83(-1)^{1-4\pi} 2^{11-3\pi} \pi}{7(8-3\pi)(9-3\pi) \left(1 + \frac{8\pi}{3} + \frac{8\pi(1+2\pi)^2}{3(1+3\pi)}\right)} + \\
& \frac{63(8-3\pi)(9-3\pi)(10-3\pi) \left(1 + \frac{8\pi}{3} + \frac{8\pi(1+2\pi)^2}{3(1+3\pi)}\right)}{(-1)^{1-4\pi} 2^{8-3\pi} \pi^2} + \\
& \frac{7(8-3\pi)(9-3\pi) \left(1 + \frac{8\pi}{3} + \frac{8\pi(1+2\pi)^2}{3(1+3\pi)}\right)}{229(-1)^{1-4\pi} 2^{8-3\pi} \pi^2} + \\
& \frac{63(8-3\pi)(9-3\pi)(10-3\pi) \left(1 + \frac{8\pi}{3} + \frac{8\pi(1+2\pi)^2}{3(1+3\pi)}\right)}{5(-1)^{1-4\pi} 2^{8-3\pi} \pi^3} - \\
& \frac{9(8-3\pi)(9-3\pi)(10-3\pi) \left(1 + \frac{8\pi}{3} + \frac{8\pi(1+2\pi)^2}{3(1+3\pi)}\right)}{(-1)^{1-4\pi} 2^{9-3\pi} \pi^4} + \\
& \frac{63(8-3\pi)(9-3\pi)(10-3\pi) \left(1 + \frac{8\pi}{3} + \frac{8\pi(1+2\pi)^2}{3(1+3\pi)}\right)}{(-1)^{1-4\pi} 2^{8-3\pi} \pi^2}
\end{aligned}$$

**Alternative representations:**

$$\begin{aligned}
& \frac{(-1)^{1-4\pi} 2^{8-3\pi} \left(1 + \frac{(8-2\pi)^2}{4(9-3\pi)} + \frac{(8-2\pi)^2(9-2\pi)(10-2\pi)}{72(9-3\pi)(10-3\pi)}\right)}{\left(1 + \frac{8\pi}{3} + \frac{8\pi(1+2\pi)^2}{3(1+3\pi)}\right) (7(8-3\pi))} = \\
& \left( (-1)^{1-4\cos^{-1}(-1)} 2^{8-3\cos^{-1}(-1)} \left(1 + \frac{(8-2\cos^{-1}(-1))^2}{4(9-3\cos^{-1}(-1))} + \frac{(9-2\cos^{-1}(-1))(10-2\cos^{-1}(-1))(8-2\cos^{-1}(-1))^2}{72(9-3\cos^{-1}(-1))(10-3\cos^{-1}(-1))}\right) \right) / \\
& \left( (7(8-3\cos^{-1}(-1))) \left(1 + \frac{8}{3} \cos^{-1}(-1) + \frac{8\cos^{-1}(-1)(1+2\cos^{-1}(-1))^2}{3(1+3\cos^{-1}(-1))}\right) \right) \\
& \frac{(-1)^{1-4\pi} 2^{8-3\pi} \left(1 + \frac{(8-2\pi)^2}{4(9-3\pi)} + \frac{(8-2\pi)^2(9-2\pi)(10-2\pi)}{72(9-3\pi)(10-3\pi)}\right)}{\left(1 + \frac{8\pi}{3} + \frac{8\pi(1+2\pi)^2}{3(1+3\pi)}\right) (7(8-3\pi))} = \\
& \frac{(-1)^{1-8E(0)} 2^{8-6E(0)} \left(1 + \frac{(8-4E(0))^2}{4(9-6E(0))} + \frac{(9-4E(0))(10-4E(0))(8-4E(0))^2}{72(9-6E(0))(10-6E(0))}\right)}{(7(8-6E(0))) \left(1 + \frac{16E(0)}{3} + \frac{16E(0)(1+4E(0))^2}{3(1+6E(0))}\right)}
\end{aligned}$$

$$\frac{(-1)^{1-4\pi} 2^{8-3\pi} \left(1 + \frac{(8-2\pi)^2}{4(9-3\pi)} + \frac{(8-2\pi)^2(9-2\pi)(10-2\pi)}{72(9-3\pi)(10-3\pi)}\right)}{\left(1 + \frac{8\pi}{3} + \frac{8\pi(1+2\pi)^2}{3(1+3\pi)}\right)(7(8-3\pi))} =$$

$$\frac{(-1)^{1-8K(0)} 2^{8-6K(0)} \left(1 + \frac{(8-4K(0))^2}{4(9-6K(0))} + \frac{(9-4K(0))(10-4K(0))(8-4K(0))^2}{72(9-6K(0))(10-6K(0))}\right)}{(7(8-6K(0))) \left(1 + \frac{16K(0)}{3} + \frac{16K(0)(1+4K(0))^2}{3(1+6K(0))}\right)}$$

### Series representations:

$$\frac{(-1)^{1-4\pi} 2^{8-3\pi} \left(1 + \frac{(8-2\pi)^2}{4(9-3\pi)} + \frac{(8-2\pi)^2(9-2\pi)(10-2\pi)}{72(9-3\pi)(10-3\pi)}\right)}{\left(1 + \frac{8\pi}{3} + \frac{8\pi(1+2\pi)^2}{3(1+3\pi)}\right)(7(8-3\pi))} =$$

$$\left[ (-1)^{-16 \sum_{k=0}^{\infty} (-1)^k / (1+2k)} 2^{6-12 \sum_{k=0}^{\infty} (-1)^k / (1+2k)} \right.$$

$$\left. \left( 1 + 12 \sum_{k=0}^{\infty} \frac{(-1)^k}{1+2k} \right) \left( 1485 - 4658 \sum_{k=0}^{\infty} \frac{(-1)^k}{1+2k} + \right.$$

$$\left. 4928 \left( \sum_{k=0}^{\infty} \frac{(-1)^k}{1+2k} \right)^2 - 1984 \left( \sum_{k=0}^{\infty} \frac{(-1)^k}{1+2k} \right)^3 + 256 \left( \sum_{k=0}^{\infty} \frac{(-1)^k}{1+2k} \right)^4 \right) \Bigg] /$$

$$\left( 63 \left( -2 + 3 \sum_{k=0}^{\infty} \frac{(-1)^k}{1+2k} \right) \left( -3 + 4 \sum_{k=0}^{\infty} \frac{(-1)^k}{1+2k} \right) \left( -5 + 6 \sum_{k=0}^{\infty} \frac{(-1)^k}{1+2k} \right) \right.$$

$$\left. \left( 3 + 100 \sum_{k=0}^{\infty} \frac{(-1)^k}{1+2k} + 896 \left( \sum_{k=0}^{\infty} \frac{(-1)^k}{1+2k} \right)^2 + 2048 \left( \sum_{k=0}^{\infty} \frac{(-1)^k}{1+2k} \right)^3 \right) \right)$$

$$\begin{aligned}
& \frac{(-1)^{1-4\pi} 2^{8-3\pi} \left( 1 + \frac{(8-2\pi)^2}{4(9-3\pi)} + \frac{(8-2\pi)^2(9-2\pi)(10-2\pi)}{72(9-3\pi)(10-3\pi)} \right)}{\left( 1 + \frac{8\pi}{3} + \frac{8\pi(1+2\pi)^2}{3(1+3\pi)} \right) (7(8-3\pi))} = \\
& \left( (-1)^{-4 \sum_{k=0}^{\infty} (-1/4)^k (1/(1+2k)+2/(1+4k)+1/(3+4k))} 2^{8-3 \sum_{k=0}^{\infty} (-1/4)^k (1/(1+2k)+2/(1+4k)+1/(3+4k))} \right. \\
& \quad \left( 1 + 3 \sum_{k=0}^{\infty} \left(-\frac{1}{4}\right)^k \left( \frac{1}{1+2k} + \frac{2}{1+4k} + \frac{1}{3+4k} \right) \right) \\
& \quad \left( 2970 - 2329 \sum_{k=0}^{\infty} \left(-\frac{1}{4}\right)^k \left( \frac{1}{1+2k} + \frac{2}{1+4k} + \frac{1}{3+4k} \right) + \right. \\
& \quad 616 \left( \sum_{k=0}^{\infty} \left(-\frac{1}{4}\right)^k \left( \frac{1}{1+2k} + \frac{2}{1+4k} + \frac{1}{3+4k} \right) \right)^2 - \\
& \quad 62 \left( \sum_{k=0}^{\infty} \left(-\frac{1}{4}\right)^k \left( \frac{1}{1+2k} + \frac{2}{1+4k} + \frac{1}{3+4k} \right) \right)^3 + \\
& \quad \left. \left. 2 \left( \sum_{k=0}^{\infty} \left(-\frac{1}{4}\right)^k \left( \frac{1}{1+2k} + \frac{2}{1+4k} + \frac{1}{3+4k} \right) \right)^4 \right) \right) / \\
& \quad \left( 63 \left( -3 + \sum_{k=0}^{\infty} \left(-\frac{1}{4}\right)^k \left( \frac{1}{1+2k} + \frac{2}{1+4k} + \frac{1}{3+4k} \right) \right) \right. \\
& \quad \left( -10 + 3 \sum_{k=0}^{\infty} \left(-\frac{1}{4}\right)^k \left( \frac{1}{1+2k} + \frac{2}{1+4k} + \frac{1}{3+4k} \right) \right) \\
& \quad \left( -8 + 3 \sum_{k=0}^{\infty} \left(-\frac{1}{4}\right)^k \left( \frac{1}{1+2k} + \frac{2}{1+4k} + \frac{1}{3+4k} \right) \right) \\
& \quad \left( 3 + 25 \sum_{k=0}^{\infty} \left(-\frac{1}{4}\right)^k \left( \frac{1}{1+2k} + \frac{2}{1+4k} + \frac{1}{3+4k} \right) + \right. \\
& \quad 56 \left( \sum_{k=0}^{\infty} \left(-\frac{1}{4}\right)^k \left( \frac{1}{1+2k} + \frac{2}{1+4k} + \frac{1}{3+4k} \right) \right)^2 + \\
& \quad \left. \left. 32 \left( \sum_{k=0}^{\infty} \left(-\frac{1}{4}\right)^k \left( \frac{1}{1+2k} + \frac{2}{1+4k} + \frac{1}{3+4k} \right) \right)^3 \right) \right)
\end{aligned}$$

$$\begin{aligned}
& \frac{(-1)^{1-4\pi} 2^{8-3\pi} \left( 1 + \frac{(8-2\pi)^2}{4(9-3\pi)} + \frac{(8-2\pi)^2(9-2\pi)(10-2\pi)}{72(9-3\pi)(10-3\pi)} \right)}{\left( 1 + \frac{8\pi}{3} + \frac{8\pi(1+2\pi)^2}{3(1+3\pi)} \right) (7(8-3\pi))} = \\
& \left( (-1)^{-4 \sum_{k=0}^{\infty} -4(-1)^k 1195^{-1-2k} (5^{1+2k} - 4 \times 239^{1+2k})} / (1+2k) \right. \\
& \quad 2^{8-3 \sum_{k=0}^{\infty} -4(-1)^k 1195^{-1-2k} (5^{1+2k} - 4 \times 239^{1+2k})} / (1+2k) \\
& \quad \left( 1 + 3 \sum_{k=0}^{\infty} - \frac{4(-1)^k 1195^{-1-2k} (5^{1+2k} - 4 \times 239^{1+2k})}{1+2k} \right) \\
& \quad \left( 2970 - 2329 \sum_{k=0}^{\infty} - \frac{4(-1)^k 1195^{-1-2k} (5^{1+2k} - 4 \times 239^{1+2k})}{1+2k} \right) + \\
& \quad 616 \left( \sum_{k=0}^{\infty} - \frac{4(-1)^k 1195^{-1-2k} (5^{1+2k} - 4 \times 239^{1+2k})^2}{1+2k} \right) - \\
& \quad 62 \left( \sum_{k=0}^{\infty} - \frac{4(-1)^k 1195^{-1-2k} (5^{1+2k} - 4 \times 239^{1+2k})^3}{1+2k} \right) + \\
& \quad \left. \left. \left. 2 \left( \sum_{k=0}^{\infty} - \frac{4(-1)^k 1195^{-1-2k} (5^{1+2k} - 4 \times 239^{1+2k})^4}{1+2k} \right) \right) \right) \right) / \\
& \left( 63 \left( -3 + \sum_{k=0}^{\infty} - \frac{4(-1)^k 1195^{-1-2k} (5^{1+2k} - 4 \times 239^{1+2k})}{1+2k} \right) \right. \\
& \quad \left( -10 + 3 \sum_{k=0}^{\infty} - \frac{4(-1)^k 1195^{-1-2k} (5^{1+2k} - 4 \times 239^{1+2k})}{1+2k} \right) \\
& \quad \left( -8 + 3 \sum_{k=0}^{\infty} - \frac{4(-1)^k 1195^{-1-2k} (5^{1+2k} - 4 \times 239^{1+2k})}{1+2k} \right) \\
& \quad \left( 3 + 25 \sum_{k=0}^{\infty} - \frac{4(-1)^k 1195^{-1-2k} (5^{1+2k} - 4 \times 239^{1+2k})}{1+2k} \right) + \\
& \quad 56 \left( \sum_{k=0}^{\infty} - \frac{4(-1)^k 1195^{-1-2k} (5^{1+2k} - 4 \times 239^{1+2k})^2}{1+2k} \right) + \\
& \quad \left. \left. \left. 32 \left( \sum_{k=0}^{\infty} - \frac{4(-1)^k 1195^{-1-2k} (5^{1+2k} - 4 \times 239^{1+2k})^3}{1+2k} \right) \right) \right) \right)
\end{aligned}$$



$$\left( \left( \left( \left( (-1)^{1-4\pi} 2^{8-3\pi} \left( 1 + \frac{(8-2\pi)^2}{4(9-3\pi)} + \frac{(8-2\pi)^2(9-2\pi)(10-2\pi)}{72(9-3\pi)(10-3\pi)} \right) \right) \right) \right) / (7(8-3\pi) \left( 1 + \frac{8\pi}{3} + \frac{8\pi(1+2\pi)^2}{3(1+3\pi)} \right) \right) \right)^{1/1024}$$

**Input:**

$$1024 \sqrt[1024]{\frac{(-1)^{1-4\pi} \times 2^{8-3\pi} \left( 1 + \frac{(8-2\pi)^2}{4(9-3\pi)} + \frac{(8-2\pi)^2(9-2\pi)(10-2\pi)}{72(9-3\pi)(10-3\pi)} \right)}{7(8-3\pi) \left( 1 + \frac{8\pi}{3} + \frac{8\pi(1+2\pi)^2}{3(1+3\pi)} \right)}}$$

**Exact result:**

$$2^{1/128-(3\pi)/1024} \sqrt[1024]{\frac{(-1)^{1-4\pi} \left( 1 + \frac{(8-2\pi)^2}{4(9-3\pi)} + \frac{(8-2\pi)^2(9-2\pi)(10-2\pi)}{72(9-3\pi)(10-3\pi)} \right)}{7(8-3\pi) \left( 1 + \frac{8\pi}{3} + \frac{8\pi(1+2\pi)^2}{3(1+3\pi)} \right)}}$$

**Decimal approximation:**

$$0.993814798815275548636762641576142803111011546220407793... + 0.00132213053957212386859110982261870597465951684458599512... i$$

**Polar coordinates:**

$$r \approx 0.993816 \text{ (radius), } \theta \approx 0.0762239^\circ \text{ (angle)}$$

0.993816... result very near to the value of the following Rogers-Ramanujan continued fraction:

$$\frac{e^{-\frac{\pi}{\sqrt{5}}}}{\sqrt{5}} = 1 - \frac{e^{-\pi\sqrt{5}}}{1 + \frac{e^{-2\pi\sqrt{5}}}{1 + \frac{e^{-3\pi\sqrt{5}}}{1 + \frac{e^{-4\pi\sqrt{5}}}{1 + \dots}}}} \approx 0.9991104684$$

$$\frac{1 + \sqrt[5]{\sqrt{\varphi^5 4\sqrt{5^3} - 1}}}{\sqrt{5}} - \varphi + 1$$

and to the dilaton value **0.989117352243 =  $\phi$**

**Alternate forms:**

$$\frac{2^{(8-3\pi)/1024} 1024 \sqrt[1024]{\frac{(-1)^{-4\pi} (2970+6581\pi-6371\pi^2+1786\pi^3-184\pi^4+6\pi^5)}{7(-720-5274\pi-7633\pi^2+3874\pi^3+3433\pi^4-2088\pi^5+288\pi^6)}}}{512\sqrt[3]{3}}$$

$$\frac{2^{(8-3\pi)/1024} 1024 \sqrt[1024]{\frac{(-1)^{-4\pi} (1+3\pi)(2970+\pi(2\pi(308+(\pi-31)\pi)-2329))}{7(\pi-3)(3\pi-10)(3\pi-8)(3+\pi(25+8\pi(7+4\pi)))}}}{512\sqrt[3]{3}}$$

$$\frac{2^{1/128-(3\pi)/1024} 1024 \sqrt[1024]{\frac{(-1)^{2-4\pi} (1+3\pi)(2970-2329\pi+616\pi^2-62\pi^3+2\pi^4)}{7(\pi-3)(3\pi-10)(3\pi-8)(3+25\pi+56\pi^2+32\pi^3)}}}{512\sqrt[3]{3}}$$

**All 1024th roots of  $((-1)^{(1-4\pi)} 2^{(8-3\pi)} (1+(8-2\pi)^2/(4(9-3\pi)) + ((8-2\pi)^2(9-2\pi)(10-2\pi))/(72(9-3\pi)(10-3\pi)))/(7(8-3\pi)(1+(8\pi)/3+(8\pi(1+2\pi)^2)/(3(1+3\pi))))$ :**

$$\frac{2^{1/128-(3\pi)/1024} e^{(i \operatorname{arg}((-1)^{1-4\pi}))/1024}}{\approx 0.99381 + 0.001322 i \text{ (principal root)}}$$

$$1024 \sqrt[1024]{\frac{7(3\pi-8) \left(1 + \frac{8\pi}{3} + \frac{8\pi(1+2\pi)^2}{3(1+3\pi)}\right)}{-1 - \frac{(8-2\pi)^2}{4(9-3\pi)} - \frac{(8-2\pi)^2(9-2\pi)(10-2\pi)}{72(9-3\pi)(10-3\pi)}}$$

$$\frac{2^{1/128-(3\pi)/1024} e^{(i(2\pi + \operatorname{arg}((-1)^{1-4\pi}))/1024}}{\approx 0.99379 + 0.00742 i}$$

$$1024 \sqrt[1024]{\frac{7(3\pi-8) \left(1 + \frac{8\pi}{3} + \frac{8\pi(1+2\pi)^2}{3(1+3\pi)}\right)}{-1 - \frac{(8-2\pi)^2}{4(9-3\pi)} - \frac{(8-2\pi)^2(9-2\pi)(10-2\pi)}{72(9-3\pi)(10-3\pi)}}$$

$$\frac{2^{1/128-(3\pi)/1024} e^{(i(4\pi + \operatorname{arg}((-1)^{1-4\pi}))/1024}}{\approx 0.99372 + 0.01352 i}$$

$$1024 \sqrt[1024]{\frac{7(3\pi-8) \left(1 + \frac{8\pi}{3} + \frac{8\pi(1+2\pi)^2}{3(1+3\pi)}\right)}{-1 - \frac{(8-2\pi)^2}{4(9-3\pi)} - \frac{(8-2\pi)^2(9-2\pi)(10-2\pi)}{72(9-3\pi)(10-3\pi)}}$$

$$\frac{2^{1/128-(3\pi)/1024} e^{(i(6\pi + \operatorname{arg}((-1)^{1-4\pi}))/1024}}{\approx 0.99362 + 0.01961 i}$$

$$1024 \sqrt[1024]{\frac{7(3\pi-8) \left(1 + \frac{8\pi}{3} + \frac{8\pi(1+2\pi)^2}{3(1+3\pi)}\right)}{-1 - \frac{(8-2\pi)^2}{4(9-3\pi)} - \frac{(8-2\pi)^2(9-2\pi)(10-2\pi)}{72(9-3\pi)(10-3\pi)}}$$

$$2^{1/128-3\pi/1024} e^{(i(8\pi+\arg((-1)^{1-4\pi}))/1024)} \approx 0.99348 + 0.02571 i$$

$$\sqrt[1024]{\frac{7(3\pi-8)\left(1+\frac{8\pi}{3}+\frac{8\pi(1+2\pi)^2}{3(1+3\pi)}\right)}{-1-\frac{(8-2\pi)^2}{4(9-3\pi)}-\frac{(8-2\pi)^2(9-2\pi)(10-2\pi)}{72(9-3\pi)(10-3\pi)}}$$

$\arg(z)$  is the complex argument

### Alternative representations:

$$\sqrt[1024]{\frac{(-1)^{1-4\pi}\left(2^{8-3\pi}\left(1+\frac{(8-2\pi)^2}{4(9-3\pi)}+\frac{(8-2\pi)^2(9-2\pi)(10-2\pi)}{72(9-3\pi)(10-3\pi)}\right)\right)}{7(8-3\pi)\left(1+\frac{8\pi}{3}+\frac{8\pi(1+2\pi)^2}{3(1+3\pi)}\right)}} =$$

$$\left(\left(\left(-1\right)^{1-4\cos^{-1}(-1)} 2^{8-3\cos^{-1}(-1)}\left(1+\frac{(8-2\cos^{-1}(-1))^2}{4(9-3\cos^{-1}(-1))}+\frac{(9-2\cos^{-1}(-1))(10-2\cos^{-1}(-1))(8-2\cos^{-1}(-1))^2}{72(9-3\cos^{-1}(-1))(10-3\cos^{-1}(-1))}\right)\right)\right)/$$

$$\left(7(8-3\cos^{-1}(-1))\left(1+\frac{8}{3}\cos^{-1}(-1)+\frac{8\cos^{-1}(-1)(1+2\cos^{-1}(-1))^2}{3(1+3\cos^{-1}(-1))}\right)\right)\right)^{\wedge(1/1024)}$$

$$\sqrt[1024]{\frac{(-1)^{1-4\pi}\left(2^{8-3\pi}\left(1+\frac{(8-2\pi)^2}{4(9-3\pi)}+\frac{(8-2\pi)^2(9-2\pi)(10-2\pi)}{72(9-3\pi)(10-3\pi)}\right)\right)}{7(8-3\pi)\left(1+\frac{8\pi}{3}+\frac{8\pi(1+2\pi)^2}{3(1+3\pi)}\right)}} =$$

$$\sqrt[1024]{\frac{(-1)^{1-8E(0)} 2^{8-6E(0)}\left(1+\frac{(8-4E(0))^2}{4(9-6E(0))}+\frac{(9-4E(0))(10-4E(0))(8-4E(0))^2}{72(9-6E(0))(10-6E(0))}\right)}{7(8-6E(0))\left(1+\frac{16E(0)}{3}+\frac{16E(0)(1+4E(0))^2}{3(1+6E(0))}\right)}}$$

$$\sqrt[1024]{\frac{(-1)^{1-4\pi}\left(2^{8-3\pi}\left(1+\frac{(8-2\pi)^2}{4(9-3\pi)}+\frac{(8-2\pi)^2(9-2\pi)(10-2\pi)}{72(9-3\pi)(10-3\pi)}\right)\right)}{7(8-3\pi)\left(1+\frac{8\pi}{3}+\frac{8\pi(1+2\pi)^2}{3(1+3\pi)}\right)}} =$$

$$\sqrt[1024]{\frac{(-1)^{1-8K(0)} 2^{8-6K(0)}\left(1+\frac{(8-4K(0))^2}{4(9-6K(0))}+\frac{(9-4K(0))(10-4K(0))(8-4K(0))^2}{72(9-6K(0))(10-6K(0))}\right)}{7(8-6K(0))\left(1+\frac{16K(0)}{3}+\frac{16K(0)(1+4K(0))^2}{3(1+6K(0))}\right)}}$$

**Series representations:**

$$\begin{aligned}
 & \sqrt[1024]{\frac{(-1)^{1-4\pi} \left( 2^{8-3\pi} \left( 1 + \frac{(8-2\pi)^2}{4(9-3\pi)} + \frac{(8-2\pi)^2(9-2\pi)(10-2\pi)}{72(9-3\pi)(10-3\pi)} \right) \right)}{7(8-3\pi) \left( 1 + \frac{8\pi}{3} + \frac{8\pi(1+2\pi)^2}{3(1+3\pi)} \right)}} = \\
 & \frac{1}{\sqrt[512]{3} \sqrt[1024]{7}} 2^{3/512-3/256 \sum_{k=0}^{\infty} (-1)^k / (1+2k)} \\
 & \left( (-1)^{-16 \sum_{k=0}^{\infty} (-1)^k / (1+2k)} \left( 1485 + 13162 \sum_{k=0}^{\infty} \frac{(-1)^k}{1+2k} - 50968 \left( \sum_{k=0}^{\infty} \frac{(-1)^k}{1+2k} \right)^2 + \right. \right. \\
 & \quad \left. \left. 57152 \left( \sum_{k=0}^{\infty} \frac{(-1)^k}{1+2k} \right)^3 - 23552 \left( \sum_{k=0}^{\infty} \frac{(-1)^k}{1+2k} \right)^4 + 3072 \left( \sum_{k=0}^{\infty} \frac{(-1)^k}{1+2k} \right)^5 \right) \right) / \\
 & \left( -90 - 2637 \sum_{k=0}^{\infty} \frac{(-1)^k}{1+2k} - 15266 \left( \sum_{k=0}^{\infty} \frac{(-1)^k}{1+2k} \right)^2 + 30992 \left( \sum_{k=0}^{\infty} \frac{(-1)^k}{1+2k} \right)^3 + \right. \\
 & \quad \left. 109856 \left( \sum_{k=0}^{\infty} \frac{(-1)^k}{1+2k} \right)^4 - 267264 \left( \sum_{k=0}^{\infty} \frac{(-1)^k}{1+2k} \right)^5 + \right. \\
 & \quad \left. 147456 \left( \sum_{k=0}^{\infty} \frac{(-1)^k}{1+2k} \right)^6 \right) \right)^{(1/1024)}
 \end{aligned}$$

$$\begin{aligned}
& \sqrt[1024]{\frac{(-1)^{1-4\pi} \left( 2^{8-3\pi} \left( 1 + \frac{(8-2\pi)^2}{4(9-3\pi)} + \frac{(8-2\pi)^2(9-2\pi)(10-2\pi)}{72(9-3\pi)(10-3\pi)} \right) \right)}{7(8-3\pi) \left( 1 + \frac{8\pi}{3} + \frac{8\pi(1+2\pi)^2}{3(1+3\pi)} \right)}} = \frac{1}{512\sqrt{3} \sqrt[1024]{7}} \\
& 2^{1/128} \left( 3 \sum_{k=0}^{\infty} \left(-\frac{1}{4}\right)^k \left( \frac{1}{1+2k} + \frac{2}{1+4k} + \frac{1}{3+4k} \right) \right) / \sqrt[1024]{\left( (-1)^{-4 \sum_{k=0}^{\infty} (-1/4)^k (1/(1+2k)+2/(1+4k)+1/(3+4k))} \right.} \\
& \quad \left( 2970 + 6581 \sum_{k=0}^{\infty} \left(-\frac{1}{4}\right)^k \left( \frac{1}{1+2k} + \frac{2}{1+4k} + \frac{1}{3+4k} \right) - \right. \\
& \quad 6371 \left( \sum_{k=0}^{\infty} \left(-\frac{1}{4}\right)^k \left( \frac{1}{1+2k} + \frac{2}{1+4k} + \frac{1}{3+4k} \right) \right)^2 + \\
& \quad 1786 \left( \sum_{k=0}^{\infty} \left(-\frac{1}{4}\right)^k \left( \frac{1}{1+2k} + \frac{2}{1+4k} + \frac{1}{3+4k} \right) \right)^3 - \\
& \quad 184 \left( \sum_{k=0}^{\infty} \left(-\frac{1}{4}\right)^k \left( \frac{1}{1+2k} + \frac{2}{1+4k} + \frac{1}{3+4k} \right) \right)^4 + \\
& \quad \left. \left. 6 \left( \sum_{k=0}^{\infty} \left(-\frac{1}{4}\right)^k \left( \frac{1}{1+2k} + \frac{2}{1+4k} + \frac{1}{3+4k} \right) \right)^5 \right) \right) / \\
& \quad \left( -720 - 5274 \sum_{k=0}^{\infty} \left(-\frac{1}{4}\right)^k \left( \frac{1}{1+2k} + \frac{2}{1+4k} + \frac{1}{3+4k} \right) - \right. \\
& \quad 7633 \left( \sum_{k=0}^{\infty} \left(-\frac{1}{4}\right)^k \left( \frac{1}{1+2k} + \frac{2}{1+4k} + \frac{1}{3+4k} \right) \right)^2 + \\
& \quad 3874 \left( \sum_{k=0}^{\infty} \left(-\frac{1}{4}\right)^k \left( \frac{1}{1+2k} + \frac{2}{1+4k} + \frac{1}{3+4k} \right) \right)^3 + \\
& \quad 3433 \left( \sum_{k=0}^{\infty} \left(-\frac{1}{4}\right)^k \left( \frac{1}{1+2k} + \frac{2}{1+4k} + \frac{1}{3+4k} \right) \right)^4 - \\
& \quad 2088 \left( \sum_{k=0}^{\infty} \left(-\frac{1}{4}\right)^k \left( \frac{1}{1+2k} + \frac{2}{1+4k} + \frac{1}{3+4k} \right) \right)^5 + \\
& \quad \left. \left. 288 \left( \sum_{k=0}^{\infty} \left(-\frac{1}{4}\right)^k \left( \frac{1}{1+2k} + \frac{2}{1+4k} + \frac{1}{3+4k} \right) \right)^6 \right) \right) \wedge (1/1024)
\end{aligned}$$

$$\begin{aligned}
& \sqrt[1024]{\frac{(-1)^{1-4\pi} \left( 2^{8-3\pi} \left( 1 + \frac{(8-2\pi)^2}{4(9-3\pi)} + \frac{(8-2\pi)^2(9-2\pi)(10-2\pi)}{72(9-3\pi)(10-3\pi)} \right) \right)}{7(8-3\pi) \left( 1 + \frac{8\pi}{3} + \frac{8\pi(1+2\pi)^2}{3(1+3\pi)} \right)}} = \\
& \frac{1}{512\sqrt{3} \ 1024\sqrt{7}} 2^{1/128 + \sum_{k=0}^{\infty} (3(-1)^k 1195^{-1-2k} (5^{1+2k} - 4 \times 239^{1+2k})) / (256(1+2k))} \\
& \left( (-1)^{\sum_{k=0}^{\infty} (16(-1)^k 1195^{-1-2k} (5^{1+2k} - 4 \times 239^{1+2k})) / (1+2k)} \right. \\
& \left( 2970 + 6581 \sum_{k=0}^{\infty} - \frac{4(-1)^k 1195^{-1-2k} (5^{1+2k} - 4 \times 239^{1+2k})}{1+2k} - \right. \\
& 6371 \left( \sum_{k=0}^{\infty} - \frac{4(-1)^k 1195^{-1-2k} (5^{1+2k} - 4 \times 239^{1+2k})}{1+2k} \right)^2 + \\
& 1786 \left( \sum_{k=0}^{\infty} - \frac{4(-1)^k 1195^{-1-2k} (5^{1+2k} - 4 \times 239^{1+2k})}{1+2k} \right)^3 - \\
& 184 \left( \sum_{k=0}^{\infty} - \frac{4(-1)^k 1195^{-1-2k} (5^{1+2k} - 4 \times 239^{1+2k})}{1+2k} \right)^4 + \\
& \left. \left. \left. 6 \left( \sum_{k=0}^{\infty} - \frac{4(-1)^k 1195^{-1-2k} (5^{1+2k} - 4 \times 239^{1+2k})}{1+2k} \right)^5 \right) \right) \right) / \\
& \left( -720 - 5274 \sum_{k=0}^{\infty} - \frac{4(-1)^k 1195^{-1-2k} (5^{1+2k} - 4 \times 239^{1+2k})}{1+2k} - \right. \\
& 7633 \left( \sum_{k=0}^{\infty} - \frac{4(-1)^k 1195^{-1-2k} (5^{1+2k} - 4 \times 239^{1+2k})}{1+2k} \right)^2 + \\
& 3874 \left( \sum_{k=0}^{\infty} - \frac{4(-1)^k 1195^{-1-2k} (5^{1+2k} - 4 \times 239^{1+2k})}{1+2k} \right)^3 + \\
& 3433 \left( \sum_{k=0}^{\infty} - \frac{4(-1)^k 1195^{-1-2k} (5^{1+2k} - 4 \times 239^{1+2k})}{1+2k} \right)^4 - \\
& 2088 \left( \sum_{k=0}^{\infty} - \frac{4(-1)^k 1195^{-1-2k} (5^{1+2k} - 4 \times 239^{1+2k})}{1+2k} \right)^5 + \\
& \left. \left. \left. 288 \left( \sum_{k=0}^{\infty} - \frac{4(-1)^k 1195^{-1-2k} (5^{1+2k} - 4 \times 239^{1+2k})}{1+2k} \right)^6 \right) \right) \right) \wedge (1/1024)
\end{aligned}$$

1/8 log base 0.993816(0.00174255) - Pi + 1/golden ratio

**Input interpretation:**

$$\frac{1}{8} \log_{0.993816}(0.00174255) - \pi + \frac{1}{\phi}$$

$\log_b(x)$  is the base-  $b$  logarithm

$\phi$  is the golden ratio

**Result:**

125.483...

125.483.... result very near to the dilaton mass calculated as a type of Higgs boson: 125 GeV for  $T = 0$  and to the Higgs boson mass 125.18 GeV

**Alternative representation:**

$$\frac{\log_{0.993816}(0.00174255)}{8} - \pi + \frac{1}{\phi} = -\pi + \frac{1}{\phi} + \frac{\log(0.00174255)}{8 \log(0.993816)}$$

**Series representations:**

$$\frac{\log_{0.993816}(0.00174255)}{8} - \pi + \frac{1}{\phi} = \frac{1}{\phi} - \pi - \frac{\sum_{k=1}^{\infty} \frac{(-1)^k (-0.998257)^k}{k}}{8 \log(0.993816)}$$

$$\begin{aligned} &\frac{\log_{0.993816}(0.00174255)}{8} - \pi + \frac{1}{\phi} = \\ &\frac{1}{\phi} - \pi - 20.151 \log(0.00174255) - \frac{1}{8} \log(0.00174255) \sum_{k=0}^{\infty} (-0.006184)^k G(k) \\ &\text{for } \left( G(0) = 0 \text{ and } G(k) = \frac{(-1)^{1+k} k}{2(1+k)(2+k)} + \sum_{j=1}^k \frac{(-1)^{1+j} G(-j+k)}{1+j} \right) \end{aligned}$$

1/8 log base 0.993816(0.00174255)+11 + 1/golden ratio

**Input interpretation:**

$$\frac{1}{8} \log_{0.993816}(0.00174255) + 11 + \frac{1}{\phi}$$

$\log_b(x)$  is the base-  $b$  logarithm

$\phi$  is the golden ratio

**Result:**

139.625...

139.625.... result practically equal to the rest mass of Pion meson 139.57 MeV

**Alternative representation:**

$$\frac{\log_{0.993816}(0.00174255)}{8} + 11 + \frac{1}{\phi} = 11 + \frac{1}{\phi} + \frac{\log(0.00174255)}{8 \log(0.993816)}$$

**Series representations:**

$$\frac{\log_{0.993816}(0.00174255)}{8} + 11 + \frac{1}{\phi} = 11 + \frac{1}{\phi} - \frac{\sum_{k=1}^{\infty} \frac{(-1)^k (-0.998257)^k}{k}}{8 \log(0.993816)}$$

$$\begin{aligned} &\frac{\log_{0.993816}(0.00174255)}{8} + 11 + \frac{1}{\phi} = \\ &11 + \frac{1}{\phi} - 20.151 \log(0.00174255) - \frac{1}{8} \log(0.00174255) \sum_{k=0}^{\infty} (-0.006184)^k G(k) \\ &\text{for } \left( G(0) = 0 \text{ and } G(k) = \frac{(-1)^{1+k} k}{2(1+k)(2+k)} + \sum_{j=1}^k \frac{(-1)^{1+j} G(-j+k)}{1+j} \right) \end{aligned}$$

27/16 log base 0.993816(0.00174255)

**Input interpretation:**

$$\frac{27}{16} \log_{0.993816}(0.00174255)$$

log<sub>b</sub>(x) is the base- b logarithm

**Result:**

1728.09...

1728.09...

This result is very near to the mass of candidate glueball f<sub>0</sub>(1710) meson. Furthermore, 1728 occurs in the algebraic formula for the j-invariant of an elliptic curve. As a consequence, it is sometimes called a Zagier as a pun on the Gross-



Zagier theorem. The number 1728 is one less than the Hardy–Ramanujan number 1729

**Alternative representation:**

$$\frac{\log_{0.993816}(0.00174255) 27}{16} = \frac{27 \log(0.00174255)}{16 \log(0.993816)}$$

**Series representations:**

$$\frac{\log_{0.993816}(0.00174255) 27}{16} = - \frac{27 \sum_{k=1}^{\infty} \frac{(-1)^k (-0.998257)^k}{k}}{16 \log(0.993816)}$$

$$\frac{\log_{0.993816}(0.00174255) 27}{16} = -272.038 \log(0.00174255) - 1.6875 \log(0.00174255) \sum_{k=0}^{\infty} (-0.006184)^k G(k)$$

for  $\left( G(0) = 0 \text{ and } G(k) = \frac{(-1)^{1+k} k}{2(1+k)(2+k)} + \sum_{j=1}^k \frac{(-1)^{1+j} G(-j+k)}{1+j} \right)$

Now, we have that:

$$1 + \left[ \frac{(((-1)^{(1-4\pi)} 2^{(8-3\pi)} (1 + (8-2\pi)^2 / (4(9-3\pi)) + ((8-2\pi)^2 (9-2\pi)(10-2\pi) - 2\pi)) / (72(9-3\pi)(10-3\pi)))) / (7(8-3\pi))}{((1 + (8\pi)/3 + (8\pi(1+2\pi)^2) / (3(1+3\pi))))} \right]$$

**Input:**

$$1 + \frac{(-1)^{1-4\pi} 2^{8-3\pi} \left( 1 + \frac{(8-2\pi)^2}{4(9-3\pi)} + \frac{(8-2\pi)^2 (9-2\pi)(10-2\pi)}{72(9-3\pi)(10-3\pi)} \right)}{7(8-3\pi) \left( 1 + \frac{8\pi}{3} + \frac{8\pi(1+2\pi)^2}{3(1+3\pi)} \right)}$$

**Exact result:**

$$1 + \frac{(-1)^{1-4\pi} 2^{8-3\pi} \left( 1 + \frac{(8-2\pi)^2}{4(9-3\pi)} + \frac{(8-2\pi)^2 (9-2\pi)(10-2\pi)}{72(9-3\pi)(10-3\pi)} \right)}{7(8-3\pi) \left( 1 + \frac{8\pi}{3} + \frac{8\pi(1+2\pi)^2}{3(1+3\pi)} \right)}$$

**Decimal approximation:**

1.00036071009601201824794439554838021483755196146584145... +  
0.00170480270956521493096384201752028773414501923833327046... i

**Input interpretation:**

1.00036071009601201824794439554838021483755196146584145 +  
 0.00170480270956521493096384201752028773414501923833327046  $i$

$i$  is the imaginary unit

**Result:**

1.00036071009601201824794439554838021483755196146584145... +  
 0.00170480270956521493096384201752028773414501923833327046...  $i$

**Polar coordinates:**

$r = 1.000362162747110273966814649274736824940907572439012973$

(radius),

$\theta = 0.097642684897811785528347228172101171638324615872035323^\circ$  (angle)

1.00036216274711...

**Alternate forms:**

$$1 - \frac{(-1)^{1-4\pi} 2^{8-3\pi} (1+3\pi)(2970 - 2329\pi + 616\pi^2 - 62\pi^3 + 2\pi^4)}{63(\pi-3)(3\pi-10)(3\pi-8)(3+25\pi+56\pi^2+32\pi^3)}$$

$$\begin{aligned} & -\left( (45\,360 + 1485(-1)^{1-4\pi} 2^{9-3\pi} + \right. \\ & \quad (332\,262 + 6581(-1)^{1-4\pi} 2^{8-3\pi})\pi + (480\,879 - 6371(-1)^{1-4\pi} 2^{8-3\pi})\pi^2 + \\ & \quad (893(-1)^{1-4\pi} 2^{9-3\pi} - 244\,062)\pi^3 + (-216\,279 - 23(-1)^{1-4\pi} 2^{11-3\pi})\pi^4 + \\ & \quad \left. (131\,544 + 3(-1)^{1-4\pi} 2^{9-3\pi})\pi^5 - 18\,144\pi^6 \right) / \\ & (63(\pi-3)(3\pi-10)(3\pi-8)(3+25\pi+56\pi^2+32\pi^3)) \end{aligned}$$

$$\begin{aligned} & \left( (-1)^{-4\pi} 2^{-3\pi} (760\,320 - 2835(-1)^{4\pi} 2^{4+3\pi} + 1\,684\,736\pi - 166\,131(-1)^{4\pi} 2^{1+3\pi}\pi - \right. \\ & \quad 1\,630\,976\pi^2 - 480\,879(-1)^{4\pi} 2^{3\pi}\pi^2 + 457\,216\pi^3 + \\ & \quad 122\,031(-1)^{4\pi} 2^{1+3\pi}\pi^3 - 47\,104\pi^4 + 216\,279(-1)^{4\pi} 2^{3\pi}\pi^4 + \\ & \quad \left. 1536\pi^5 - 16\,443(-1)^{4\pi} 2^{3+3\pi}\pi^5 + 567(-1)^{4\pi} 2^{5+3\pi}\pi^6 \right) / \\ & (63(\pi-3)(3\pi-10)(3\pi-8)(3+25\pi+56\pi^2+32\pi^3)) \end{aligned}$$

**Expanded form:**

$$\begin{aligned}
& 1 + \frac{(-1)^{1-4\pi} 2^{8-3\pi}}{7(8-3\pi) \left(1 + \frac{8\pi}{3} + \frac{8\pi(1+2\pi)^2}{3(1+3\pi)}\right)} + \frac{(-1)^{1-4\pi} 2^{12-3\pi}}{7(8-3\pi)(9-3\pi) \left(1 + \frac{8\pi}{3} + \frac{8\pi(1+2\pi)^2}{3(1+3\pi)}\right)} + \\
& \frac{5(-1)^{1-4\pi} 2^{12-3\pi}}{7(8-3\pi)(9-3\pi)(10-3\pi) \left(1 + \frac{8\pi}{3} + \frac{8\pi(1+2\pi)^2}{3(1+3\pi)}\right)} + \\
& \frac{(-1)^{2-4\pi} 2^{11-3\pi} \pi}{7(8-3\pi)(9-3\pi) \left(1 + \frac{8\pi}{3} + \frac{8\pi(1+2\pi)^2}{3(1+3\pi)}\right)} - \\
& \frac{83(-1)^{1-4\pi} 2^{11-3\pi} \pi}{7(8-3\pi)(9-3\pi) \left(1 + \frac{8\pi}{3} + \frac{8\pi(1+2\pi)^2}{3(1+3\pi)}\right)} + \\
& \frac{63(8-3\pi)(9-3\pi)(10-3\pi) \left(1 + \frac{8\pi}{3} + \frac{8\pi(1+2\pi)^2}{3(1+3\pi)}\right)}{(-1)^{1-4\pi} 2^{8-3\pi} \pi^2} + \\
& \frac{7(8-3\pi)(9-3\pi) \left(1 + \frac{8\pi}{3} + \frac{8\pi(1+2\pi)^2}{3(1+3\pi)}\right)}{229(-1)^{1-4\pi} 2^{8-3\pi} \pi^2} - \\
& \frac{63(8-3\pi)(9-3\pi)(10-3\pi) \left(1 + \frac{8\pi}{3} + \frac{8\pi(1+2\pi)^2}{3(1+3\pi)}\right)}{5(-1)^{1-4\pi} 2^{8-3\pi} \pi^3} + \\
& \frac{9(8-3\pi)(9-3\pi)(10-3\pi) \left(1 + \frac{8\pi}{3} + \frac{8\pi(1+2\pi)^2}{3(1+3\pi)}\right)}{(-1)^{1-4\pi} 2^{9-3\pi} \pi^4} + \\
& \frac{63(8-3\pi)(9-3\pi)(10-3\pi) \left(1 + \frac{8\pi}{3} + \frac{8\pi(1+2\pi)^2}{3(1+3\pi)}\right)}{(-1)^{1-4\pi} 2^{8-3\pi} \pi^4}
\end{aligned}$$

### Alternative representations:

$$\begin{aligned}
& 1 + \frac{(-1)^{1-4\pi} 2^{8-3\pi} \left(1 + \frac{(8-2\pi)^2}{4(9-3\pi)} + \frac{(8-2\pi)^2(9-2\pi)(10-2\pi)}{72(9-3\pi)(10-3\pi)}\right)}{\left(1 + \frac{8\pi}{3} + \frac{8\pi(1+2\pi)^2}{3(1+3\pi)}\right) (7(8-3\pi))} = \\
& 1 + \left( (-1)^{1-4\cos^{-1}(-1)} 2^{8-3\cos^{-1}(-1)} \left( 1 + \frac{(8-2\cos^{-1}(-1))^2}{4(9-3\cos^{-1}(-1))} + \right. \right. \\
& \left. \left. \frac{(9-2\cos^{-1}(-1))(10-2\cos^{-1}(-1))(8-2\cos^{-1}(-1))^2}{72(9-3\cos^{-1}(-1))(10-3\cos^{-1}(-1))} \right) \right) / \\
& \left( (7(8-3\cos^{-1}(-1))) \left( 1 + \frac{8}{3} \cos^{-1}(-1) + \frac{8\cos^{-1}(-1)(1+2\cos^{-1}(-1))^2}{3(1+3\cos^{-1}(-1))} \right) \right) \\
& 1 + \frac{(-1)^{1-4\pi} 2^{8-3\pi} \left(1 + \frac{(8-2\pi)^2}{4(9-3\pi)} + \frac{(8-2\pi)^2(9-2\pi)(10-2\pi)}{72(9-3\pi)(10-3\pi)}\right)}{\left(1 + \frac{8\pi}{3} + \frac{8\pi(1+2\pi)^2}{3(1+3\pi)}\right) (7(8-3\pi))} = \\
& 1 + \frac{(-1)^{1-8E(0)} 2^{8-6E(0)} \left(1 + \frac{(8-4E(0))^2}{4(9-6E(0))} + \frac{(9-4E(0))(10-4E(0))(8-4E(0))^2}{72(9-6E(0))(10-6E(0))}\right)}{(7(8-6E(0))) \left(1 + \frac{16E(0)}{3} + \frac{16E(0)(1+4E(0))^2}{3(1+6E(0))}\right)}
\end{aligned}$$

$$1 + \frac{(-1)^{1-4\pi} 2^{8-3\pi} \left(1 + \frac{(8-2\pi)^2}{4(9-3\pi)} + \frac{(8-2\pi)^2(9-2\pi)(10-2\pi)}{72(9-3\pi)(10-3\pi)}\right)}{\left(1 + \frac{8\pi}{3} + \frac{8\pi(1+2\pi)^2}{3(1+3\pi)}\right)(7(8-3\pi))} =$$

$$1 + \frac{(-1)^{1-8K(0)} 2^{8-6K(0)} \left(1 + \frac{(8-4K(0))^2}{4(9-6K(0))} + \frac{(9-4K(0))(10-4K(0))(8-4K(0))^2}{72(9-6K(0))(10-6K(0))}\right)}{(7(8-6K(0))) \left(1 + \frac{16K(0)}{3} + \frac{16K(0)(1+4K(0))^2}{3(1+6K(0))}\right)}$$

### Series representations:

$$1 + \frac{(-1)^{1-4\pi} 2^{8-3\pi} \left(1 + \frac{(8-2\pi)^2}{4(9-3\pi)} + \frac{(8-2\pi)^2(9-2\pi)(10-2\pi)}{72(9-3\pi)(10-3\pi)}\right)}{\left(1 + \frac{8\pi}{3} + \frac{8\pi(1+2\pi)^2}{3(1+3\pi)}\right)(7(8-3\pi))} =$$

$$\left( (-1)^{-16 \sum_{k=0}^{\infty} (-1)^k / (1+2k)} 2^{-12 \sum_{k=0}^{\infty} (-1)^k / (1+2k)} \right.$$

$$\left( 95\,040 - 2835 (-1)^{16 \sum_{k=0}^{\infty} (-1)^k / (1+2k)} 2^{1+12 \sum_{k=0}^{\infty} (-1)^k / (1+2k)} + \right.$$

$$842\,368 \sum_{k=0}^{\infty} \frac{(-1)^k}{1+2k} - 166\,131 (-1)^{16 \sum_{k=0}^{\infty} (-1)^k / (1+2k)} 2^{12 \sum_{k=0}^{\infty} (-1)^k / (1+2k)}$$

$$\sum_{k=0}^{\infty} \frac{(-1)^k}{1+2k} - 3\,261\,952 \left( \sum_{k=0}^{\infty} \frac{(-1)^k}{1+2k} \right)^2 - 480\,879 (-1)^{16 \sum_{k=0}^{\infty} (-1)^k / (1+2k)}$$

$$2^{1+12 \sum_{k=0}^{\infty} (-1)^k / (1+2k)} \left( \sum_{k=0}^{\infty} \frac{(-1)^k}{1+2k} \right)^2 + 3\,657\,728 \left( \sum_{k=0}^{\infty} \frac{(-1)^k}{1+2k} \right)^3 +$$

$$122\,031 (-1)^{16 \sum_{k=0}^{\infty} (-1)^k / (1+2k)} 2^{4+12 \sum_{k=0}^{\infty} (-1)^k / (1+2k)} \left( \sum_{k=0}^{\infty} \frac{(-1)^k}{1+2k} \right)^3 -$$

$$1507\,328 \left( \sum_{k=0}^{\infty} \frac{(-1)^k}{1+2k} \right)^4 + 216\,279 (-1)^{16 \sum_{k=0}^{\infty} (-1)^k / (1+2k)}$$

$$2^{5+12 \sum_{k=0}^{\infty} (-1)^k / (1+2k)} \left( \sum_{k=0}^{\infty} \frac{(-1)^k}{1+2k} \right)^4 + 196\,608 \left( \sum_{k=0}^{\infty} \frac{(-1)^k}{1+2k} \right)^5 -$$

$$16\,443 (-1)^{16 \sum_{k=0}^{\infty} (-1)^k / (1+2k)} 2^{10+12 \sum_{k=0}^{\infty} (-1)^k / (1+2k)} \left( \sum_{k=0}^{\infty} \frac{(-1)^k}{1+2k} \right)^5 +$$

$$567 (-1)^{16 \sum_{k=0}^{\infty} (-1)^k / (1+2k)} 2^{14+12 \sum_{k=0}^{\infty} (-1)^k / (1+2k)} \left( \sum_{k=0}^{\infty} \frac{(-1)^k}{1+2k} \right)^6 \Big) /$$

$$\left( 63 \left( -2 + 3 \sum_{k=0}^{\infty} \frac{(-1)^k}{1+2k} \right) \left( -3 + 4 \sum_{k=0}^{\infty} \frac{(-1)^k}{1+2k} \right) \left( -5 + 6 \sum_{k=0}^{\infty} \frac{(-1)^k}{1+2k} \right) \right.$$

$$\left. \left( 3 + 100 \sum_{k=0}^{\infty} \frac{(-1)^k}{1+2k} + 896 \left( \sum_{k=0}^{\infty} \frac{(-1)^k}{1+2k} \right)^2 + 2048 \left( \sum_{k=0}^{\infty} \frac{(-1)^k}{1+2k} \right)^3 \right) \right)$$

$$\begin{aligned}
& 1 + \frac{(-1)^{1-4\pi} 2^{8-3\pi} \left( 1 + \frac{(8-2\pi)^2}{4(9-3\pi)} + \frac{(8-2\pi)^2(9-2\pi)(10-2\pi)}{72(9-3\pi)(10-3\pi)} \right)}{\left( 1 + \frac{8\pi}{3} + \frac{8\pi(1+2\pi)^2}{3(1+3\pi)} \right) (7(8-3\pi))} = \\
& \left( (-1)^{-4 \sum_{k=0}^{\infty} (-1/4)^k (1/(1+2k)+2/(1+4k)+1/(3+4k))} 2^{-3 \sum_{k=0}^{\infty} (-1/4)^k (1/(1+2k)+2/(1+4k)+1/(3+4k))} \right. \\
& \quad \left( 760320 - 2835 (-1)^4 \sum_{k=0}^{\infty} (-1/4)^k (1/(1+2k)+2/(1+4k)+1/(3+4k)) \right. \\
& \quad \quad 2^{4+3 \sum_{k=0}^{\infty} (-1/4)^k (1/(1+2k)+2/(1+4k)+1/(3+4k))} + \\
& \quad 1684736 \sum_{k=0}^{\infty} \left( -\frac{1}{4} \right)^k \left( \frac{1}{1+2k} + \frac{2}{1+4k} + \frac{1}{3+4k} \right) - \\
& \quad 166131 (-1)^4 \sum_{k=0}^{\infty} (-1/4)^k (1/(1+2k)+2/(1+4k)+1/(3+4k)) \\
& \quad \quad 2^{1+3 \sum_{k=0}^{\infty} (-1/4)^k (1/(1+2k)+2/(1+4k)+1/(3+4k))} \\
& \quad \quad \sum_{k=0}^{\infty} \left( -\frac{1}{4} \right)^k \left( \frac{1}{1+2k} + \frac{2}{1+4k} + \frac{1}{3+4k} \right) - \\
& \quad 1630976 \left( \sum_{k=0}^{\infty} \left( -\frac{1}{4} \right)^k \left( \frac{1}{1+2k} + \frac{2}{1+4k} + \frac{1}{3+4k} \right) \right)^2 - \\
& \quad 480879 (-1)^4 \sum_{k=0}^{\infty} (-1/4)^k (1/(1+2k)+2/(1+4k)+1/(3+4k)) \\
& \quad \quad 2^3 \sum_{k=0}^{\infty} (-1/4)^k (1/(1+2k)+2/(1+4k)+1/(3+4k)) \\
& \quad \quad \left( \sum_{k=0}^{\infty} \left( -\frac{1}{4} \right)^k \left( \frac{1}{1+2k} + \frac{2}{1+4k} + \frac{1}{3+4k} \right) \right)^2 + \\
& \quad 457216 \left( \sum_{k=0}^{\infty} \left( -\frac{1}{4} \right)^k \left( \frac{1}{1+2k} + \frac{2}{1+4k} + \frac{1}{3+4k} \right) \right)^3 + \\
& \quad 122031 (-1)^4 \sum_{k=0}^{\infty} (-1/4)^k (1/(1+2k)+2/(1+4k)+1/(3+4k)) \\
& \quad \quad 2^{1+3 \sum_{k=0}^{\infty} (-1/4)^k (1/(1+2k)+2/(1+4k)+1/(3+4k))} \\
& \quad \quad \left( \sum_{k=0}^{\infty} \left( -\frac{1}{4} \right)^k \left( \frac{1}{1+2k} + \frac{2}{1+4k} + \frac{1}{3+4k} \right) \right)^3 - \\
& \quad 47104 \left( \sum_{k=0}^{\infty} \left( -\frac{1}{4} \right)^k \left( \frac{1}{1+2k} + \frac{2}{1+4k} + \frac{1}{3+4k} \right) \right)^4 + \\
& \quad 216279 (-1)^4 \sum_{k=0}^{\infty} (-1/4)^k (1/(1+2k)+2/(1+4k)+1/(3+4k)) \\
& \quad \quad 2^3 \sum_{k=0}^{\infty} (-1/4)^k (1/(1+2k)+2/(1+4k)+1/(3+4k)) \\
& \quad \quad \left( \sum_{k=0}^{\infty} \left( -\frac{1}{4} \right)^k \left( \frac{1}{1+2k} + \frac{2}{1+4k} + \frac{1}{3+4k} \right) \right)^4 + \\
& \quad 1536 \left( \sum_{k=0}^{\infty} \left( -\frac{1}{4} \right)^k \left( \frac{1}{1+2k} + \frac{2}{1+4k} + \frac{1}{3+4k} \right) \right)^5 - \\
& \quad 16443 (-1)^4 \sum_{k=0}^{\infty} (-1/4)^k (1/(1+2k)+2/(1+4k)+1/(3+4k)) \\
& \quad \quad 2^{3+3 \sum_{k=0}^{\infty} (-1/4)^k (1/(1+2k)+2/(1+4k)+1/(3+4k))} \\
& \quad \quad \left( \sum_{k=0}^{\infty} \left( -\frac{1}{4} \right)^k \left( \frac{1}{1+2k} + \frac{2}{1+4k} + \frac{1}{3+4k} \right) \right)^5 + \\
& \quad 567 (-1)^4 \sum_{k=0}^{\infty} (-1/4)^k (1/(1+2k)+2/(1+4k)+1/(3+4k)) \\
& \quad \quad 2^{5+3 \sum_{k=0}^{\infty} (-1/4)^k (1/(1+2k)+2/(1+4k)+1/(3+4k))} \\
& \quad \quad \left. \left( \sum_{k=0}^{\infty} \left( -\frac{1}{4} \right)^k \left( \frac{1}{1+2k} + \frac{2}{1+4k} + \frac{1}{3+4k} \right) \right)^6 \right) / \\
& \quad \left( 63 \left( 2 \sum_{k=0}^{\infty} \left( -\frac{1}{4} \right)^k \left( \frac{1}{1+2k} + \frac{2}{1+4k} + \frac{1}{3+4k} \right) \right) \right)
\end{aligned}$$

$$\begin{aligned}
& 1 + \frac{(-1)^{1-4\pi} 2^{8-3\pi} \left( 1 + \frac{(8-2\pi)^2}{4(9-3\pi)} + \frac{(8-2\pi)^2 (9-2\pi)(10-2\pi)}{72(9-3\pi)(10-3\pi)} \right)}{\left( 1 + \frac{8\pi}{3} + \frac{8\pi(1+2\pi)^2}{3(1+3\pi)} \right) (7(8-3\pi))} = \\
& \left( (-1)^{-4 \sum_{k=0}^{\infty} (4(-1)^k 1195^{-1-2k} (5^{1+2k-4} \times 239^{1+2k})) / (1+2k)} \right. \\
& \quad 2^{-3 \sum_{k=0}^{\infty} (4(-1)^k 1195^{-1-2k} (5^{1+2k-4} \times 239^{1+2k})) / (1+2k)} \\
& \quad \left. \left( 760320 - 2835 (-1)^4 \sum_{k=0}^{\infty} (4(-1)^k 1195^{-1-2k} (5^{1+2k-4} \times 239^{1+2k})) / (1+2k) \right. \right. \\
& \quad \quad 2^{4+3 \sum_{k=0}^{\infty} (4(-1)^k 1195^{-1-2k} (5^{1+2k-4} \times 239^{1+2k})) / (1+2k)} + \\
& \quad \quad 1684736 \sum_{k=0}^{\infty} \frac{4(-1)^k 1195^{-1-2k} (5^{1+2k} - 4 \times 239^{1+2k})}{1+2k} - \\
& \quad \quad 166131 (-1)^4 \sum_{k=0}^{\infty} (4(-1)^k 1195^{-1-2k} (5^{1+2k-4} \times 239^{1+2k})) / (1+2k) \\
& \quad \quad 2^{1+3 \sum_{k=0}^{\infty} (4(-1)^k 1195^{-1-2k} (5^{1+2k-4} \times 239^{1+2k})) / (1+2k)} \\
& \quad \quad \left. \sum_{k=0}^{\infty} \frac{4(-1)^k 1195^{-1-2k} (5^{1+2k} - 4 \times 239^{1+2k})}{1+2k} - \right. \\
& \quad \quad 1630976 \left( \sum_{k=0}^{\infty} \frac{4(-1)^k 1195^{-1-2k} (5^{1+2k} - 4 \times 239^{1+2k})}{1+2k} \right)^2 - \\
& \quad \quad 480879 (-1)^4 \sum_{k=0}^{\infty} (4(-1)^k 1195^{-1-2k} (5^{1+2k-4} \times 239^{1+2k})) / (1+2k) \\
& \quad \quad 2^3 \sum_{k=0}^{\infty} (4(-1)^k 1195^{-1-2k} (5^{1+2k-4} \times 239^{1+2k})) / (1+2k) \\
& \quad \quad \left( \sum_{k=0}^{\infty} \frac{4(-1)^k 1195^{-1-2k} (5^{1+2k} - 4 \times 239^{1+2k})}{1+2k} \right)^2 + \\
& \quad \quad 457216 \left( \sum_{k=0}^{\infty} \frac{4(-1)^k 1195^{-1-2k} (5^{1+2k} - 4 \times 239^{1+2k})}{1+2k} \right)^3 + \\
& \quad \quad 122031 (-1)^4 \sum_{k=0}^{\infty} (4(-1)^k 1195^{-1-2k} (5^{1+2k-4} \times 239^{1+2k})) / (1+2k) \\
& \quad \quad 2^{1+3 \sum_{k=0}^{\infty} (4(-1)^k 1195^{-1-2k} (5^{1+2k-4} \times 239^{1+2k})) / (1+2k)} \\
& \quad \quad \left( \sum_{k=0}^{\infty} \frac{4(-1)^k 1195^{-1-2k} (5^{1+2k} - 4 \times 239^{1+2k})}{1+2k} \right)^3 - \\
& \quad \quad 47104 \left( \sum_{k=0}^{\infty} \frac{4(-1)^k 1195^{-1-2k} (5^{1+2k} - 4 \times 239^{1+2k})}{1+2k} \right)^4 + \\
& \quad \quad 216279 (-1)^4 \sum_{k=0}^{\infty} (4(-1)^k 1195^{-1-2k} (5^{1+2k-4} \times 239^{1+2k})) / (1+2k) \\
& \quad \quad 2^3 \sum_{k=0}^{\infty} (4(-1)^k 1195^{-1-2k} (5^{1+2k-4} \times 239^{1+2k})) / (1+2k) \\
& \quad \quad \left( \sum_{k=0}^{\infty} \frac{4(-1)^k 1195^{-1-2k} (5^{1+2k} - 4 \times 239^{1+2k})}{1+2k} \right)^4 + \\
& \quad \quad 1536 \left( \sum_{k=0}^{\infty} \frac{4(-1)^k 1195^{-1-2k} (5^{1+2k} - 4 \times 239^{1+2k})}{1+2k} \right)^5 - \\
& \quad \quad 16443 (-1)^4 \sum_{k=0}^{\infty} (4(-1)^k 1195^{-1-2k} (5^{1+2k-4} \times 239^{1+2k})) / (1+2k) \\
& \quad \quad 2^{3+3 \sum_{k=0}^{\infty} (4(-1)^k 1195^{-1-2k} (5^{1+2k-4} \times 239^{1+2k})) / (1+2k)} \\
& \quad \quad \left( \sum_{k=0}^{\infty} \frac{4(-1)^k 1195^{-1-2k} (5^{1+2k} - 4 \times 239^{1+2k})}{1+2k} \right)^5 + \\
& \quad \quad 567 (-1)^4 \sum_{k=0}^{\infty} (4(-1)^k 1195^{-1-2k} (5^{1+2k-4} \times 239^{1+2k})) / (1+2k) \\
& \quad \quad 2^{5+3 \sum_{k=0}^{\infty} (4(-1)^k 1195^{-1-2k} (5^{1+2k-4} \times 239^{1+2k})) / (1+2k)}
\end{aligned}$$

**Integral representation:**

$$\begin{aligned}
 & 1 + \frac{(-1)^{1-4\pi} 2^{8-3\pi} \left( 1 + \frac{(8-2\pi)^2}{4(9-3\pi)} + \frac{(8-2\pi)^2(9-2\pi)(10-2\pi)}{72(9-3\pi)(10-3\pi)} \right)}{\left( 1 + \frac{8\pi}{3} + \frac{8\pi(1+2\pi)^2}{3(1+3\pi)} \right) (7(8-3\pi))} = \\
 & 1 + \left( (-1)^{1-4\left(\frac{3\sqrt{3}}{4} + 24 \int_0^{\frac{1}{4}} \sqrt{t-t^2} dt\right)} 2^{8-3\left(\frac{3\sqrt{3}}{4} + 24 \int_0^{\frac{1}{4}} \sqrt{t-t^2} dt\right)} \right. \\
 & \quad \left( 1 + \frac{\left( 8-2\left(\frac{3\sqrt{3}}{4} + 24 \int_0^{\frac{1}{4}} \sqrt{t-t^2} dt\right)\right)^2}{4\left(9-3\left(\frac{3\sqrt{3}}{4} + 24 \int_0^{\frac{1}{4}} \sqrt{t-t^2} dt\right)\right)} + \right. \\
 & \quad \left. \left( \left( 8-2\left(\frac{3\sqrt{3}}{4} + 24 \int_0^{\frac{1}{4}} \sqrt{t-t^2} dt\right)\right)^2 \right. \right. \\
 & \quad \quad \left. \left( 9-2\left(\frac{3\sqrt{3}}{4} + 24 \int_0^{\frac{1}{4}} \sqrt{t-t^2} dt\right)\right) \right. \\
 & \quad \quad \left. \left. \left( 10-2\left(\frac{3\sqrt{3}}{4} + 24 \int_0^{\frac{1}{4}} \sqrt{t-t^2} dt\right)\right) \right) \right) / \\
 & \quad \left. \left. \left( 72\left(9-3\left(\frac{3\sqrt{3}}{4} + 24 \int_0^{\frac{1}{4}} \sqrt{t-t^2} dt\right)\right) \right. \right. \right. \\
 & \quad \quad \left. \left. \left. \left( 10-3\left(\frac{3\sqrt{3}}{4} + 24 \int_0^{\frac{1}{4}} \sqrt{t-t^2} dt\right)\right) \right) \right) \right) \right) / \\
 & \quad \left( 7\left(8-3\left(\frac{3\sqrt{3}}{4} + 24 \int_0^{\frac{1}{4}} \sqrt{t-t^2} dt\right)\right) \left( 1 + \frac{8}{3}\left(\frac{3\sqrt{3}}{4} + 24 \int_0^{\frac{1}{4}} \sqrt{t-t^2} dt\right) + \right. \right. \\
 & \quad \left. \left. \frac{8\left(\frac{3\sqrt{3}}{4} + 24 \int_0^{\frac{1}{4}} \sqrt{t-t^2} dt\right) \left( 1 + 2\left(\frac{3\sqrt{3}}{4} + 24 \int_0^{\frac{1}{4}} \sqrt{t-t^2} dt\right)\right)^2}{3\left(1+3\left(\frac{3\sqrt{3}}{4} + 24 \int_0^{\frac{1}{4}} \sqrt{t-t^2} dt\right)\right)} \right) \right) \right)
 \end{aligned}$$

From which:

$$1/10^{52}(((1.000360710096 + 0.001704802709i)+(76+29)/10^3+3/10^4)))$$

**Input interpretation:**

$$\frac{1}{10^{52}} \left( (1.000360710096 + 0.001704802709i) + \frac{76+29}{10^3} + \frac{3}{10^4} \right)$$

$i$  is the imaginary unit

**Result:**

$$1.105660710096... \times 10^{-52} + 1.704802709 \times 10^{-55} i$$

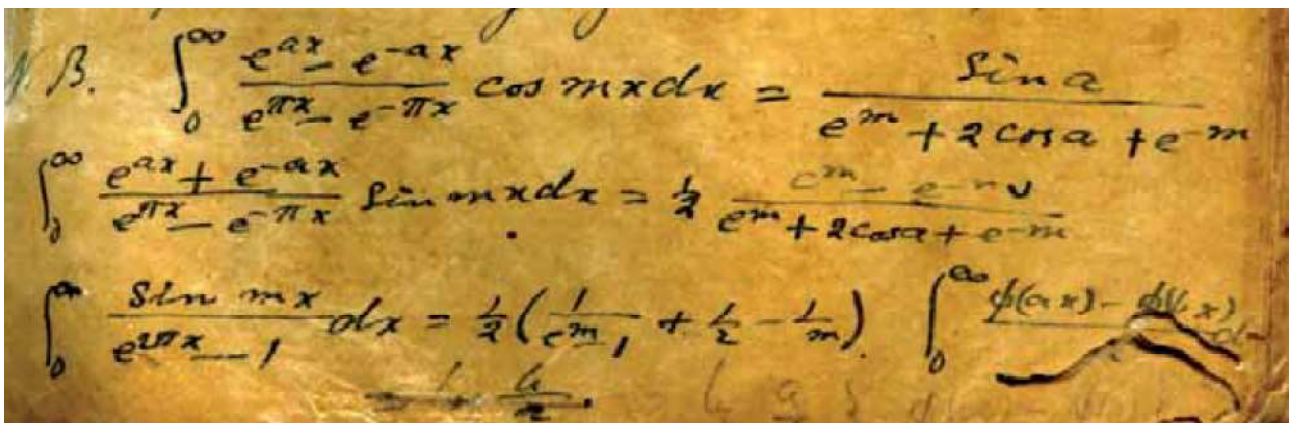
**Polar coordinates:**

$$r = 1.10566 \times 10^{-52} \text{ (radius), } \theta = 0.0883435^\circ \text{ (angle)}$$

1.10566\*10<sup>-52</sup> result practically equal to the value of Cosmological Constant

$$1.1056 \times 10^{-52} \text{ m}^{-2}$$

Page 254



$$a = 2, m = 3$$

$$\sin 2 / ((e^3 + 2 \cos 2 + e^{-3}))$$

**Input:**

$$\frac{\sin(2)}{e^3 + 2 \cos(2) + \frac{1}{e^3}}$$

**Decimal approximation:**

0.047106460064772198232953943913892893797971808627911778686...

0.04710646006...

**Alternate forms:**

$$\frac{\sin(2)}{2 (\cos(2) + \cosh(3))}$$



$$\frac{e^3 \sin(2)}{1 + e^6 + 2 e^3 \cos(2)}$$

$$\frac{i(e^{-2i} - e^{2i})}{2\left(\frac{1}{e^3} + e^{-2i} + e^{2i} + e^3\right)}$$

$\cosh(x)$  is the hyperbolic cosine function

### Alternative representations:

$$\frac{\sin(2)}{e^3 + 2 \cos(2) + \frac{1}{e^3}} = \frac{\cos\left(-2 + \frac{\pi}{2}\right)}{2 \cosh(-2i) + \frac{1}{e^3} + e^3}$$

$$\frac{\sin(2)}{e^3 + 2 \cos(2) + \frac{1}{e^3}} = -\frac{\cos\left(2 + \frac{\pi}{2}\right)}{2 \cosh(-2i) + \frac{1}{e^3} + e^3}$$

$$\frac{\sin(2)}{e^3 + 2 \cos(2) + \frac{1}{e^3}} = \frac{\cos\left(-2 + \frac{\pi}{2}\right)}{2 \cosh(2i) + \frac{1}{e^3} + e^3}$$

### Series representations:

$$\frac{\sin(2)}{e^3 + 2 \cos(2) + \frac{1}{e^3}} = \frac{e^3 \sum_{k=0}^{\infty} \frac{(-1)^k \left(2 - \frac{\pi}{2}\right)^{2k}}{(2k)!}}{1 + e^6 + 2 e^3 \sum_{k=0}^{\infty} \frac{(-4)^k}{(2k)!}}$$

$$\frac{\sin(2)}{e^3 + 2 \cos(2) + \frac{1}{e^3}} = \frac{e^3 \sum_{k=0}^{\infty} \frac{(-1)^k 2^{1+2k}}{(1+2k)!}}{1 + e^6 + 2 e^3 \sum_{k=0}^{\infty} \frac{(-4)^k}{(2k)!}}$$

$$\frac{\sin(2)}{e^3 + 2 \cos(2) + \frac{1}{e^3}} = \frac{e^3 \sum_{k=0}^{\infty} \frac{(-1)^k 2^{1+2k}}{(1+2k)!}}{1 + e^6 - 2 e^3 \sum_{k=0}^{\infty} \frac{(-1)^k \left(2 - \frac{\pi}{2}\right)^{1+2k}}{(1+2k)!}}$$

$$1/2 * ((e^3 - e^{-3})) / ((e^3 + 2 \cos 2 + e^{-3}))$$

**Input:**

$$\frac{1}{2} \times \frac{e^3 - \frac{1}{e^3}}{e^3 + 2 \cos(2) + \frac{1}{e^3}}$$

**Exact result:**

$$\frac{e^3 - \frac{1}{e^3}}{2 \left( \frac{1}{e^3} + e^3 + 2 \cos(2) \right)}$$

**Decimal approximation:**

0.518979391428966218461989305055755202918649445135042499799...

0.5189793914...

**Alternate forms:**

$$\frac{e^6 - 1}{2(1 + e^6 + 2e^3 \cos(2))}$$

$$\frac{e^6 - 1}{2 + 2e^6 + 4e^3 \cos(2)}$$

$$-\frac{1 - e^6}{2(1 + e^6 + 2e^3 \cos(2))}$$

**Alternative representations:**

$$\frac{e^3 - \frac{1}{e^3}}{\left(e^3 + 2 \cos(2) + \frac{1}{e^3}\right) 2} = \frac{-\frac{1}{e^3} + e^3}{2 \left(2 \cosh(-2i) + \frac{1}{e^3} + e^3\right)}$$

$$\frac{e^3 - \frac{1}{e^3}}{\left(e^3 + 2 \cos(2) + \frac{1}{e^3}\right) 2} = \frac{-\frac{1}{e^3} + e^3}{2 \left(\frac{1}{e^3} + e^3 + \frac{2}{\sec(2)}\right)}$$

$$\frac{e^3 - \frac{1}{e^3}}{\left(e^3 + 2 \cos(2) + \frac{1}{e^3}\right) 2} = \frac{-\frac{1}{e^3} + e^3}{2 \left(2 \cosh(2i) + \frac{1}{e^3} + e^3\right)}$$

**Series representations:**

$$\frac{e^3 - \frac{1}{e^3}}{\left(e^3 + 2 \cos(2) + \frac{1}{e^3}\right) 2} = \frac{-1 + e^6}{2 \left(1 + e^6 + 2 e^3 \sum_{k=0}^{\infty} \frac{(-4)^k}{(2k)!}\right)}$$

$$\frac{e^3 - \frac{1}{e^3}}{\left(e^3 + 2 \cos(2) + \frac{1}{e^3}\right) 2} = \frac{-\frac{1}{e^3} + e^3}{2 \left(\frac{1}{e^3} + e^3 - 2 \sum_{k=0}^{\infty} \frac{(-1)^k \left(2 - \frac{\pi}{2}\right)^{1+2k}}{(1+2k)!}\right)}$$

$$\frac{e^3 - \frac{1}{e^3}}{\left(e^3 + 2 \cos(2) + \frac{1}{e^3}\right) 2} = \frac{-1 + e^6}{2 + 2 e^6 + 4 e^3 \sum_{k=0}^{\infty} \frac{\cos\left(\frac{k\pi}{2} + z_0\right) (2 - z_0)^k}{k!}}$$

**Integral representations:**

$$\frac{e^3 - \frac{1}{e^3}}{\left(e^3 + 2 \cos(2) + \frac{1}{e^3}\right) 2} = \frac{(-1 + e^6) \sqrt{\pi}}{2 (1 + e^6) \sqrt{\pi} - 2 i e^3 \int_{-i\infty+\gamma}^{i\infty+\gamma} \frac{e^{-1/s+s}}{\sqrt{s}} ds} \quad \text{for } \gamma > 0$$

$$\frac{e^3 - \frac{1}{e^3}}{\left(e^3 + 2 \cos(2) + \frac{1}{e^3}\right) 2} = \frac{(-1 + e^6) \sqrt{\pi}}{2 (1 + e^6) \sqrt{\pi} - 2 i e^3 \int_{-i\infty+\gamma}^{i\infty+\gamma} \frac{\Gamma(s)}{\Gamma\left(\frac{1}{2}-s\right)} ds} \quad \text{for } 0 < \gamma < \frac{1}{2}$$

$$1/2 (1/(e^3-1)+1/2-1/3)$$

**Input:**

$$\frac{1}{2} \left( \frac{1}{e^3 - 1} + \frac{1}{2} - \frac{1}{3} \right)$$

**Result:**

$$\frac{1}{2} \left( \frac{1}{6} + \frac{1}{e^3 - 1} \right)$$

**Decimal approximation:**

0.109531181578961309319333030633210860935284582735748268842...

0.1095311815...

**Property:**

$\frac{1}{2} \left( \frac{1}{6} + \frac{1}{-1+e^3} \right)$  is a transcendental number

**Alternate forms:**

$$\frac{5+e^3}{12(e^3-1)}$$

$$\frac{1}{12} + \frac{1}{2(e^3-1)}$$

$$\frac{5+e^3}{12(e-1)(1+e+e^2)}$$

**Alternative representation:**

$$\frac{1}{2} \left( \frac{1}{e^3-1} + \frac{1}{2} - \frac{1}{3} \right) = \frac{1}{2} \left( \frac{1}{\exp^3(z)-1} + \frac{1}{2} - \frac{1}{3} \right) \text{ for } z=1$$

**Series representations:**

$$\frac{1}{2} \left( \frac{1}{e^3-1} + \frac{1}{2} - \frac{1}{3} \right) = \frac{1}{12} + \frac{1}{2 \left( -1 + \sum_{k=0}^{\infty} \frac{3^k}{k!} \right)}$$

$$\frac{1}{2} \left( \frac{1}{e^3-1} + \frac{1}{2} - \frac{1}{3} \right) = \frac{1}{12} + \frac{1}{2 \left( -1 + \left( \sum_{k=0}^{\infty} \frac{1}{k!} \right)^3 \right)}$$

$$\frac{1}{2} \left( \frac{1}{e^3-1} + \frac{1}{2} - \frac{1}{3} \right) = \frac{1}{12} + \frac{4}{-8 + \left( \sum_{k=0}^{\infty} \frac{1+k}{k!} \right)^3}$$

Now, we have that:

$$\left[ \left( \frac{\sin 2}{(e^3 + 2 \cos 2 + e^{-3})} \right) \right] + \left[ \left( \frac{1}{2} \left( \frac{1}{(e^3-1)} + \frac{1}{2} - \frac{1}{3} \right) \right) \right] + \left[ \left( \frac{1}{2} \cdot \frac{(e^3 - e^{-3})}{(e^3 + 2 \cos 2 + e^{-3})} \right) \right]$$

**Input:**

$$\frac{\sin(2)}{e^3 + 2 \cos(2) + \frac{1}{e^3}} + \frac{1}{2} \left( \frac{1}{e^3-1} + \frac{1}{2} - \frac{1}{3} \right) + \frac{1}{2} \times \frac{e^3 - \frac{1}{e^3}}{e^3 + 2 \cos(2) + \frac{1}{e^3}}$$

**Exact result:**

$$\frac{1}{2} \left( \frac{1}{6} + \frac{1}{e^3 - 1} \right) + \frac{e^3 - \frac{1}{e^3}}{2 \left( \frac{1}{e^3} + e^3 + 2 \cos(2) \right)} + \frac{\sin(2)}{e^3 + e^3 + 2 \cos(2)}$$

**Decimal approximation:**

0.675617033072699726014276279602858957651905836498702547328...

0.675617033...

**Alternate forms:**

$$\frac{7}{12} + \frac{1}{2(e^3 - 1)} - \frac{1 + e^3 (\cos(2) - \sin(2))}{1 + e^6 + 2 e^3 \cos(2)}$$

$$\frac{1}{2} \left( \frac{1}{6} + \frac{1}{e^3 - 1} + \frac{e^6 - 1}{1 + e^6 + 2 e^3 \cos(2)} + \frac{\sin(2)}{\cos(2) + \cosh(3)} \right)$$

$$\frac{1}{2} \left( \frac{1}{6} + \frac{1}{e^3 - 1} + \frac{e^6 - 1}{1 + e^6 + 2 e^3 \cos(2)} + \frac{2 \sin(2)}{\frac{1}{e^3} + e^3 + 2 \cos(2)} \right)$$

**Alternative representations:**

$$\begin{aligned} & \frac{\sin(2)}{e^3 + 2 \cos(2) + \frac{1}{e^3}} + \frac{1}{2} \left( \frac{1}{e^3 - 1} + \frac{1}{2} - \frac{1}{3} \right) + \frac{e^3 - \frac{1}{e^3}}{\left( e^3 + 2 \cos(2) + \frac{1}{e^3} \right) 2} = \\ & \frac{1}{2} \left( \frac{1}{6} + \frac{1}{-1 + e^3} \right) + \frac{\cos\left(-2 + \frac{\pi}{2}\right)}{2 \cosh(-2i) + \frac{1}{e^3} + e^3} + \frac{-\frac{1}{e^3} + e^3}{2 \left( 2 \cosh(-2i) + \frac{1}{e^3} + e^3 \right)} \end{aligned}$$

$$\begin{aligned} & \frac{\sin(2)}{e^3 + 2 \cos(2) + \frac{1}{e^3}} + \frac{1}{2} \left( \frac{1}{e^3 - 1} + \frac{1}{2} - \frac{1}{3} \right) + \frac{e^3 - \frac{1}{e^3}}{\left( e^3 + 2 \cos(2) + \frac{1}{e^3} \right) 2} = \\ & \frac{1}{2} \left( \frac{1}{6} + \frac{1}{-1 + e^3} \right) - \frac{\cos\left(2 + \frac{\pi}{2}\right)}{2 \cosh(-2i) + \frac{1}{e^3} + e^3} + \frac{-\frac{1}{e^3} + e^3}{2 \left( 2 \cosh(-2i) + \frac{1}{e^3} + e^3 \right)} \end{aligned}$$

$$\begin{aligned} & \frac{\sin(2)}{e^3 + 2 \cos(2) + \frac{1}{e^3}} + \frac{1}{2} \left( \frac{1}{e^3 - 1} + \frac{1}{2} - \frac{1}{3} \right) + \frac{e^3 - \frac{1}{e^3}}{\left( e^3 + 2 \cos(2) + \frac{1}{e^3} \right) 2} = \\ & \frac{1}{2} \left( \frac{1}{6} + \frac{1}{-1 + e^3} \right) + \frac{\cos\left(-2 + \frac{\pi}{2}\right)}{2 \cosh(2i) + \frac{1}{e^3} + e^3} + \frac{-\frac{1}{e^3} + e^3}{2 \left( 2 \cosh(2i) + \frac{1}{e^3} + e^3 \right)} \end{aligned}$$

**Series representations:**

$$\frac{\sin(2)}{e^3 + 2 \cos(2) + \frac{1}{e^3}} + \frac{1}{2} \left( \frac{1}{e^3 - 1} + \frac{1}{2} - \frac{1}{3} \right) + \frac{e^3 - \frac{1}{e^3}}{\left( e^3 + 2 \cos(2) + \frac{1}{e^3} \right) 2} =$$

$$\frac{11 - 5e^3 - e^6 + 7e^9 + \sum_{k=0}^{\infty} \frac{(-1)^k 2^{1+2k} e^3 (12(-1+e^3)(2k)! + (5+e^3)(1+2k)!)}{(2k)!(1+2k)!}}{12(-1+e)(1+e+e^2) \left( 1 + e^6 + 2e^3 \sum_{k=0}^{\infty} \frac{(-4)^k}{(2k)!} \right)}$$

$$\frac{\sin(2)}{e^3 + 2 \cos(2) + \frac{1}{e^3}} + \frac{1}{2} \left( \frac{1}{e^3 - 1} + \frac{1}{2} - \frac{1}{3} \right) + \frac{e^3 - \frac{1}{e^3}}{\left( e^3 + 2 \cos(2) + \frac{1}{e^3} \right) 2} =$$

$$\frac{11 - 5e^3 - e^6 + 7e^9 + \sum_{k=0}^{\infty} \left( \frac{(-1)^k 2^{1+2k} e^3 (5+e^3)}{(2k)!} + \frac{12(-1)^k e^3 (-1+e^3) \left(2 - \frac{\pi}{2}\right)^{2k}}{(2k)!} \right)}{12(-1+e)(1+e+e^2) \left( 1 + e^6 + 2e^3 \sum_{k=0}^{\infty} \frac{(-4)^k}{(2k)!} \right)}$$

$$\frac{\sin(2)}{e^3 + 2 \cos(2) + \frac{1}{e^3}} + \frac{1}{2} \left( \frac{1}{e^3 - 1} + \frac{1}{2} - \frac{1}{3} \right) + \frac{e^3 - \frac{1}{e^3}}{\left( e^3 + 2 \cos(2) + \frac{1}{e^3} \right) 2} =$$

$$\left( 11 - 5e^3 - e^6 + 7e^9 - 12e^3 \sum_{k=0}^{\infty} \frac{(-1)^k 2^{1+2k}}{(1+2k)!} + 12e^6 \sum_{k=0}^{\infty} \frac{(-1)^k 2^{1+2k}}{(1+2k)!} - \right.$$

$$\left. 10e^3 \sum_{k=0}^{\infty} \frac{(-1)^k \left(2 - \frac{\pi}{2}\right)^{1+2k}}{(1+2k)!} - 2e^6 \sum_{k=0}^{\infty} \frac{(-1)^k \left(2 - \frac{\pi}{2}\right)^{1+2k}}{(1+2k)!} \right) /$$

$$\left( 12(-1+e)(1+e+e^2) \left( 1 + e^6 - 2e^3 \sum_{k=0}^{\infty} \frac{(-1)^k \left(2 - \frac{\pi}{2}\right)^{1+2k}}{(1+2k)!} \right) \right)$$

**Integral representations:**

$$\frac{\sin(2)}{e^3 + 2 \cos(2) + \frac{1}{e^3}} + \frac{1}{2} \left( \frac{1}{e^3 - 1} + \frac{1}{2} - \frac{1}{3} \right) + \frac{e^3 - \frac{1}{e^3}}{\left( e^3 + 2 \cos(2) + \frac{1}{e^3} \right) 2} =$$

$$\left( 11 \sqrt{\pi} - 5 e^3 \sqrt{\pi} - e^6 \sqrt{\pi} + 7 e^9 \sqrt{\pi} + 6 i e^3 \int_{-i \infty + \gamma}^{i \infty + \gamma} \frac{e^{-1/s+s}}{s^{3/2}} ds - \right.$$

$$\left. 6 i e^6 \int_{-i \infty + \gamma}^{i \infty + \gamma} \frac{e^{-1/s+s}}{s^{3/2}} ds - 10 e^3 \sqrt{\pi} \int_{\frac{\pi}{2}}^2 \sin(t) dt - 2 e^6 \sqrt{\pi} \int_{\frac{\pi}{2}}^2 \sin(t) dt \right) /$$

$$\left( 12(-1+e)(1+e+e^2) \sqrt{\pi} \left( 1 + e^6 - 2 e^3 \int_{\frac{\pi}{2}}^2 \sin(t) dt \right) \right) \text{ for } \gamma > 0$$

$$\frac{\sin(2)}{e^3 + 2 \cos(2) + \frac{1}{e^3}} + \frac{1}{2} \left( \frac{1}{e^3 - 1} + \frac{1}{2} - \frac{1}{3} \right) + \frac{e^3 - \frac{1}{e^3}}{\left( e^3 + 2 \cos(2) + \frac{1}{e^3} \right) 2} =$$

$$\left( 11 \sqrt{\pi} - 5 e^3 \sqrt{\pi} - e^6 \sqrt{\pi} + 7 e^9 \sqrt{\pi} - 5 i e^3 \int_{-i \infty + \gamma}^{i \infty + \gamma} \frac{e^{-1/s+s}}{\sqrt{s}} ds - \right.$$

$$\left. i e^6 \int_{-i \infty + \gamma}^{i \infty + \gamma} \frac{e^{-1/s+s}}{\sqrt{s}} ds - 24 e^3 \sqrt{\pi} \int_0^1 \cos(2t) dt + 24 e^6 \sqrt{\pi} \int_0^1 \cos(2t) dt \right) /$$

$$\left( 12(-1+e)(1+e+e^2) \left( \sqrt{\pi} + e^6 \sqrt{\pi} - i e^3 \int_{-i \infty + \gamma}^{i \infty + \gamma} \frac{e^{-1/s+s}}{\sqrt{s}} ds \right) \right) \text{ for } \gamma > 0$$

$$\frac{\sin(2)}{e^3 + 2 \cos(2) + \frac{1}{e^3}} + \frac{1}{2} \left( \frac{1}{e^3 - 1} + \frac{1}{2} - \frac{1}{3} \right) + \frac{e^3 - \frac{1}{e^3}}{\left( e^3 + 2 \cos(2) + \frac{1}{e^3} \right) 2} =$$

$$\left( 11 \sqrt{\pi} - 5 e^3 \sqrt{\pi} - e^6 \sqrt{\pi} + 7 e^9 \sqrt{\pi} + 6 i e^3 \int_{-i \infty + \gamma}^{i \infty + \gamma} \frac{e^{-1/s+s}}{s^{3/2}} ds - \right.$$

$$\left. 6 i e^6 \int_{-i \infty + \gamma}^{i \infty + \gamma} \frac{e^{-1/s+s}}{s^{3/2}} ds - 5 i e^3 \int_{-i \infty + \gamma}^{i \infty + \gamma} \frac{e^{-1/s+s}}{\sqrt{s}} ds - i e^6 \int_{-i \infty + \gamma}^{i \infty + \gamma} \frac{e^{-1/s+s}}{\sqrt{s}} ds \right) /$$

$$\left( 12(-1+e)(1+e+e^2) \left( \sqrt{\pi} + e^6 \sqrt{\pi} - i e^3 \int_{-i \infty + \gamma}^{i \infty + \gamma} \frac{e^{-1/s+s}}{\sqrt{s}} ds \right) \right) \text{ for } \gamma > 0$$

$$\left[ \left( \left( \left( \sin 2 / \left( e^3 + 2 \cos 2 + e^{-3} \right) \right) \right) + \left( \left( \left( 1/2 \left( 1/(e^3-1) + 1/2 - 1/3 \right) \right) \right) + \left( \left( 1/2 * \left( e^3 - e^{-3} \right) / \left( e^3 + 2 \cos 2 + e^{-3} \right) \right) \right) \right) \right]^{1/64}$$

**Input:**

$$\sqrt[64]{\frac{\sin(2)}{e^3 + 2 \cos(2) + \frac{1}{e^3}} + \frac{1}{2} \left( \frac{1}{e^3 - 1} + \frac{1}{2} - \frac{1}{3} \right) + \frac{1}{2} \times \frac{e^3 - \frac{1}{e^3}}{e^3 + 2 \cos(2) + \frac{1}{e^3}}}$$

**Exact result:**

$$\sqrt[64]{\frac{1}{2} \left( \frac{1}{6} + \frac{1}{e^3 - 1} \right) + \frac{e^3 - \frac{1}{e^3}}{2 \left( \frac{1}{e^3} + e^3 + 2 \cos(2) \right)} + \frac{\sin(2)}{\frac{1}{e^3} + e^3 + 2 \cos(2)}}$$

**Decimal approximation:**

0.993891718082108684738449466431730354003644145648880225618...

0.993891718... result very near to the value of the following Rogers-Ramanujan continued fraction:

$$\frac{e^{-\frac{\pi}{\sqrt{5}}}}{\sqrt{5}} = 1 - \frac{e^{-\pi\sqrt{5}}}{1 + \frac{e^{-2\pi\sqrt{5}}}{1 + \frac{e^{-3\pi\sqrt{5}}}{1 + \frac{e^{-4\pi\sqrt{5}}}{1 + \dots}}}} \approx 0.9991104684$$

and to the dilaton value **0.989117352243 =  $\phi$**

**Alternate forms:**

$$\sqrt[64]{\frac{1}{\frac{1}{6 + \frac{1}{e^3 - 1} + \frac{e^6 - 1}{1 + e^6 + 2 e^3 \cos(2)} + \frac{2 \sin(2)}{e^3 + e^3 + 2 \cos(2)}}}}$$

$$\sqrt[32]{2} \sqrt[64]{\frac{1}{\frac{3(e^3 - 1)(1 + e^6 + 2 e^3 \cos(2))}{11 - 5 e^3 - e^6 + 7 e^9 - 12 e^3 \sin(2) + 12 e^6 \sin(2) + 10 e^3 \cos(2) + 2 e^6 \cos(2)}}}}$$

$$\sqrt[64]{\frac{i(e^{-2i} - e^{2i})}{2 \left( \frac{1}{e^3} + e^{-2i} + e^{2i} + e^3 \right)} + \frac{e^3 - \frac{1}{e^3}}{2 \left( \frac{1}{e^3} + e^{-2i} + e^{2i} + e^3 \right)} + \frac{1}{2} \left( \frac{1}{6} + \frac{1}{e^3 - 1} \right)}$$

**All 64th roots of  $\frac{1}{2} \left( \frac{1}{6} + \frac{1}{e^3 - 1} \right) + \frac{e^3 - 1/e^3}{2 \left( \frac{1}{e^3} + e^3 + 2 \cos(2) \right)} + \frac{\sin(2)}{\left( \frac{1}{e^3} + e^3 + 2 \cos(2) \right)}$ :**

$$e^0 \sqrt[64]{\frac{1}{2} \left( \frac{1}{6} + \frac{1}{e^3 - 1} \right) + \frac{e^3 - \frac{1}{e^3}}{2 \left( \frac{1}{e^3} + e^3 + 2 \cos(2) \right)} + \frac{\sin(2)}{\frac{1}{e^3} + e^3 + 2 \cos(2)}} \approx 0.993892$$

(real, principal root)



$$e^{(i\pi)/32} \sqrt[64]{\frac{1}{2} \left( \frac{1}{6} + \frac{1}{e^3 - 1} \right) + \frac{e^3 - \frac{1}{e^3}}{2 \left( \frac{1}{e^3} + e^3 + 2 \cos(2) \right)} + \frac{\sin(2)}{\frac{1}{e^3} + e^3 + 2 \cos(2)}} \\ \approx 0.98911 + 0.09742 i$$

$$e^{(i\pi)/16} \sqrt[64]{\frac{1}{2} \left( \frac{1}{6} + \frac{1}{e^3 - 1} \right) + \frac{e^3 - \frac{1}{e^3}}{2 \left( \frac{1}{e^3} + e^3 + 2 \cos(2) \right)} + \frac{\sin(2)}{\frac{1}{e^3} + e^3 + 2 \cos(2)}} \\ \approx 0.97479 + 0.19390 i$$

$$e^{(3i\pi)/32} \sqrt[64]{\frac{1}{2} \left( \frac{1}{6} + \frac{1}{e^3 - 1} \right) + \frac{e^3 - \frac{1}{e^3}}{2 \left( \frac{1}{e^3} + e^3 + 2 \cos(2) \right)} + \frac{\sin(2)}{\frac{1}{e^3} + e^3 + 2 \cos(2)}} \\ \approx 0.95110 + 0.28851 i$$

$$e^{(i\pi)/8} \sqrt[64]{\frac{1}{2} \left( \frac{1}{6} + \frac{1}{e^3 - 1} \right) + \frac{e^3 - \frac{1}{e^3}}{2 \left( \frac{1}{e^3} + e^3 + 2 \cos(2) \right)} + \frac{\sin(2)}{\frac{1}{e^3} + e^3 + 2 \cos(2)}} \\ \approx 0.91824 + 0.38035 i$$

### Alternative representations:

$$\sqrt[64]{\frac{\sin(2)}{e^3 + 2 \cos(2) + \frac{1}{e^3}} + \frac{1}{2} \left( \frac{1}{e^3 - 1} + \frac{1}{2} - \frac{1}{3} \right) + \frac{e^3 - \frac{1}{e^3}}{\left( e^3 + 2 \cos(2) + \frac{1}{e^3} \right) 2}} = \\ \sqrt[64]{\frac{1}{2} \left( \frac{1}{6} + \frac{1}{-1 + e^3} \right) + \frac{\cos(-2 + \frac{\pi}{2})}{2 \cosh(-2i) + \frac{1}{e^3} + e^3} + \frac{-\frac{1}{e^3} + e^3}{2 \left( 2 \cosh(-2i) + \frac{1}{e^3} + e^3 \right)}}$$

$$\sqrt[64]{\frac{\sin(2)}{e^3 + 2 \cos(2) + \frac{1}{e^3}} + \frac{1}{2} \left( \frac{1}{e^3 - 1} + \frac{1}{2} - \frac{1}{3} \right) + \frac{e^3 - \frac{1}{e^3}}{\left( e^3 + 2 \cos(2) + \frac{1}{e^3} \right) 2}} = \\ \sqrt[64]{\frac{1}{2} \left( \frac{1}{6} + \frac{1}{-1 + e^3} \right) - \frac{\cos(2 + \frac{\pi}{2})}{2 \cosh(-2i) + \frac{1}{e^3} + e^3} + \frac{-\frac{1}{e^3} + e^3}{2 \left( 2 \cosh(-2i) + \frac{1}{e^3} + e^3 \right)}}$$

$$\sqrt[64]{\frac{\sin(2)}{e^3 + 2 \cos(2) + \frac{1}{e^3}} + \frac{1}{2} \left( \frac{1}{e^3 - 1} + \frac{1}{2} - \frac{1}{3} \right) + \frac{e^3 - \frac{1}{e^3}}{\left( e^3 + 2 \cos(2) + \frac{1}{e^3} \right) 2}} =$$

$$\sqrt[64]{\frac{1}{2} \left( \frac{1}{6} + \frac{1}{-1 + e^3} \right) + \frac{\cos(-2 + \frac{\pi}{2})}{2 \cosh(2i) + \frac{1}{e^3} + e^3} + \frac{-\frac{1}{e^3} + e^3}{2 \left( 2 \cosh(2i) + \frac{1}{e^3} + e^3 \right)}}$$

### Series representations:

$$\sqrt[64]{\frac{\sin(2)}{e^3 + 2 \cos(2) + \frac{1}{e^3}} + \frac{1}{2} \left( \frac{1}{e^3 - 1} + \frac{1}{2} - \frac{1}{3} \right) + \frac{e^3 - \frac{1}{e^3}}{\left( e^3 + 2 \cos(2) + \frac{1}{e^3} \right) 2}} =$$

$$\sqrt[64]{\frac{11 - 5e^3 - e^6 + 7e^9 + 10e^3 \sum_{k=0}^{\infty} \frac{(-4)^k}{(2k)!} + 2e^6 \sum_{k=0}^{\infty} \frac{(-4)^k}{(2k)!} - 12e^3 \sum_{k=0}^{\infty} \frac{(-1)^k 2^{1+2k}}{(1+2k)!} + 12e^6 \sum_{k=0}^{\infty} \frac{(-1)^k 2^{1+2k}}{(1+2k)!}}{1 + e^6 + 2e^3 \sum_{k=0}^{\infty} \frac{(-4)^k}{(2k)!}}}$$

$$\frac{1}{\sqrt[32]{2} \sqrt[64]{3(-1 + e^3)}}$$

$$\sqrt[64]{\frac{\sin(2)}{e^3 + 2 \cos(2) + \frac{1}{e^3}} + \frac{1}{2} \left( \frac{1}{e^3 - 1} + \frac{1}{2} - \frac{1}{3} \right) + \frac{e^3 - \frac{1}{e^3}}{\left( e^3 + 2 \cos(2) + \frac{1}{e^3} \right) 2}} =$$

$$\frac{1}{\sqrt[32]{2} \sqrt[64]{3(-1 + e^3)}}$$

$$\left( \left( \left( 11 - 5e^3 - e^6 + 7e^9 + 10e^3 \sum_{k=0}^{\infty} \frac{(-4)^k}{(2k)!} + 2e^6 \sum_{k=0}^{\infty} \frac{(-4)^k}{(2k)!} - 12e^3 \sum_{k=0}^{\infty} \frac{(-1)^k \left( 2 - \frac{\pi}{2} \right)^{2k}}{(2k)!} + \right. \right. \right.$$

$$\left. \left. \left. 12e^6 \sum_{k=0}^{\infty} \frac{(-1)^k \left( 2 - \frac{\pi}{2} \right)^{2k}}{(2k)!} \right) \right) / \left( 1 + e^6 + 2e^3 \sum_{k=0}^{\infty} \frac{(-4)^k}{(2k)!} \right) \right)^{\wedge (1/64)}$$

$$\sqrt[64]{\frac{\sin(2)}{e^3 + 2 \cos(2) + \frac{1}{e^3}} + \frac{1}{2} \left( \frac{1}{e^3 - 1} + \frac{1}{2} - \frac{1}{3} \right) + \frac{e^3 - \frac{1}{e^3}}{\left( e^3 + 2 \cos(2) + \frac{1}{e^3} \right) 2}} =$$

$$\frac{1}{\sqrt[32]{2} \sqrt[64]{3(-1 + e^3)}}$$

$$\left( \left( \left( 11 - 5e^3 - e^6 + 7e^9 - 12e^3 \sum_{k=0}^{\infty} \frac{(-1)^k 2^{1+2k}}{(1+2k)!} + 12e^6 \sum_{k=0}^{\infty} \frac{(-1)^k 2^{1+2k}}{(1+2k)!} - \right. \right. \right.$$

$$\left. \left. \left. 10e^3 \sum_{k=0}^{\infty} \frac{(-1)^k \left( 2 - \frac{\pi}{2} \right)^{1+2k}}{(1+2k)!} - 2e^6 \sum_{k=0}^{\infty} \frac{(-1)^k \left( 2 - \frac{\pi}{2} \right)^{1+2k}}{(1+2k)!} \right) \right) / \right.$$

$$\left. \left( 1 + e^6 - 2e^3 \sum_{k=0}^{\infty} \frac{(-1)^k \left( 2 - \frac{\pi}{2} \right)^{1+2k}}{(1+2k)!} \right) \right)^{\wedge (1/64)}$$

### Integral representations:

$$\sqrt[64]{\frac{\sin(2)}{e^3 + 2 \cos(2) + \frac{1}{e^3}} + \frac{1}{2} \left( \frac{1}{e^3 - 1} + \frac{1}{2} - \frac{1}{3} \right) + \frac{e^3 - \frac{1}{e^3}}{\left( e^3 + 2 \cos(2) + \frac{1}{e^3} \right) 2}} =$$

$$\left( \left( 11 \sqrt{\pi} - 5 e^3 \sqrt{\pi} - e^6 \sqrt{\pi} + 7 e^9 \sqrt{\pi} + 6 i e^3 \int_{-i \infty + \gamma}^{i \infty + \gamma} \frac{e^{-1/s+s}}{s^{3/2}} ds - \right. \right.$$

$$6 i e^6 \int_{-i \infty + \gamma}^{i \infty + \gamma} \frac{e^{-1/s+s}}{s^{3/2}} ds - 10 e^3 \sqrt{\pi} \int_{\frac{\pi}{2}}^2 \sin(t) dt -$$

$$\left. \left. 2 e^6 \sqrt{\pi} \int_{\frac{\pi}{2}}^2 \sin(t) dt \right) / \left( 1 + e^6 - 2 e^3 \int_{\frac{\pi}{2}}^2 \sin(t) dt \right) \right)^{\wedge}$$

$$(1/64) / \left( \sqrt[32]{2} \sqrt[64]{3(-1+e^3)} \sqrt[128]{\pi} \right) \text{ for } \gamma > 0$$

$$\sqrt[64]{\frac{\sin(2)}{e^3 + 2 \cos(2) + \frac{1}{e^3}} + \frac{1}{2} \left( \frac{1}{e^3 - 1} + \frac{1}{2} - \frac{1}{3} \right) + \frac{e^3 - \frac{1}{e^3}}{\left( e^3 + 2 \cos(2) + \frac{1}{e^3} \right) 2}} =$$

$$\left( \left( 11 \sqrt{\pi} - 5 e^3 \sqrt{\pi} - e^6 \sqrt{\pi} + 7 e^9 \sqrt{\pi} + 6 i e^3 \int_{-i \infty + \gamma}^{i \infty + \gamma} \frac{\Gamma(s)}{\Gamma\left(\frac{3}{2} - s\right)} ds - \right. \right.$$

$$6 i e^6 \int_{-i \infty + \gamma}^{i \infty + \gamma} \frac{\Gamma(s)}{\Gamma\left(\frac{3}{2} - s\right)} ds - 10 e^3 \sqrt{\pi} \int_{\frac{\pi}{2}}^2 \sin(t) dt -$$

$$\left. \left. 2 e^6 \sqrt{\pi} \int_{\frac{\pi}{2}}^2 \sin(t) dt \right) / \left( 1 + e^6 - 2 e^3 \int_{\frac{\pi}{2}}^2 \sin(t) dt \right) \right)^{\wedge}$$

$$(1/64) / \left( \sqrt[32]{2} \sqrt[64]{3(-1+e^3)} \sqrt[128]{\pi} \right) \text{ for } 0 < \gamma < 1$$

$$\sqrt[64]{\frac{\sin(2)}{e^3 + 2 \cos(2) + \frac{1}{e^3}} + \frac{1}{2} \left( \frac{1}{e^3 - 1} + \frac{1}{2} - \frac{1}{3} \right) + \frac{e^3 - \frac{1}{e^3}}{\left( e^3 + 2 \cos(2) + \frac{1}{e^3} \right) 2}} =$$

$$\frac{1}{\sqrt[32]{2} \sqrt[64]{3(-1+e^3)}} \left( \left( 11 \sqrt{\pi} - 5 e^3 \sqrt{\pi} - e^6 \sqrt{\pi} + 7 e^9 \sqrt{\pi} - 5 i e^3 \int_{-i \infty + \gamma}^{i \infty + \gamma} \frac{e^{-1/s+s}}{\sqrt{s}} ds - \right. \right.$$

$$i e^6 \int_{-i \infty + \gamma}^{i \infty + \gamma} \frac{e^{-1/s+s}}{\sqrt{s}} ds - 24 e^3 \sqrt{\pi} \int_0^1 \cos(2t) dt +$$

$$\left. \left. 24 e^6 \sqrt{\pi} \int_0^1 \cos(2t) dt \right) / \right.$$

$$\left. \left( \sqrt{\pi} + e^6 \sqrt{\pi} - i e^3 \int_{-i \infty + \gamma}^{i \infty + \gamma} \frac{e^{-1/s+s}}{\sqrt{s}} ds \right) \right)^{\wedge} (1/64) \text{ for } \gamma > 0$$

2log base 0.993891718[(((sin 2 / ((e^3 + 2 cos 2 + e^(-3)))))) + (((1/2 (1/(e^3-1)+1/2-1/3)))) + (((1/2 \* ((e^3 - e^(-3)))/(e^3 + 2 cos 2 + e^(-3)))))]-Pi+1/golden ratio

**Input interpretation:**

$$2 \log_{0.993891718} \left( \frac{\sin(2)}{e^3 + 2 \cos(2) + \frac{1}{e^3}} + \frac{1}{2} \left( \frac{1}{e^3 - 1} + \frac{1}{2} - \frac{1}{3} \right) + \frac{1}{2} \times \frac{e^3 - \frac{1}{e^3}}{e^3 + 2 \cos(2) + \frac{1}{e^3}} \right) - \pi + \frac{1}{\phi}$$

$\log_b(x)$  is the base- $b$  logarithm

$\phi$  is the golden ratio

**Result:**

125.4764...

125.4764... result very near to the dilaton mass calculated as a type of Higgs boson: 125 GeV for T = 0 and to the Higgs boson mass 125.18 GeV

**Alternative representations:**

$$2 \log_{0.993892} \left( \frac{\sin(2)}{e^3 + 2 \cos(2) + \frac{1}{e^3}} + \frac{1}{2} \left( \frac{1}{e^3 - 1} + \frac{1}{2} - \frac{1}{3} \right) + \frac{e^3 - \frac{1}{e^3}}{(e^3 + 2 \cos(2) + \frac{1}{e^3}) 2} \right) - \pi + \frac{1}{\phi} =$$

$$- \pi + \frac{1}{\phi} + \frac{2 \log \left( \frac{1}{2} \left( \frac{1}{6} + \frac{1}{-1+e^3} \right) + \frac{-\frac{1}{e^3}+e^3}{2(2 \cos(2)+\frac{1}{e^3}+e^3)} + \frac{\sin(2)}{2 \cos(2)+\frac{1}{e^3}+e^3} \right)}{\log(0.993892)}$$

$$2 \log_{0.993892} \left( \frac{\sin(2)}{e^3 + 2 \cos(2) + \frac{1}{e^3}} + \frac{1}{2} \left( \frac{1}{e^3 - 1} + \frac{1}{2} - \frac{1}{3} \right) + \frac{e^3 - \frac{1}{e^3}}{(e^3 + 2 \cos(2) + \frac{1}{e^3}) 2} \right) - \pi + \frac{1}{\phi} =$$

$$- \pi + 2 \log_{0.993892} \left( \frac{1}{2} \left( \frac{1}{6} + \frac{1}{-1+e^3} \right) + \frac{\cos\left(-2 + \frac{\pi}{2}\right)}{2 \cosh(-2i) + \frac{1}{e^3} + e^3} + \frac{-\frac{1}{e^3} + e^3}{2(2 \cosh(-2i) + \frac{1}{e^3} + e^3)} \right) + \frac{1}{\phi}$$

$$2 \log_{0.993892} \left( \frac{\sin(2)}{e^3 + 2 \cos(2) + \frac{1}{e^3}} + \frac{1}{2} \left( \frac{1}{e^3 - 1} + \frac{1}{2} - \frac{1}{3} \right) + \frac{e^3 - \frac{1}{e^3}}{\left( e^3 + 2 \cos(2) + \frac{1}{e^3} \right) 2} \right) - \pi + \frac{1}{\phi} =$$

$$-\pi + 2 \log_{0.993892} \left( \frac{1}{2} \left( \frac{1}{6} + \frac{1}{-1 + e^3} \right) - \frac{\cos\left(2 + \frac{\pi}{2}\right)}{2 \cosh(-2i) + \frac{1}{e^3} + e^3} + \frac{-\frac{1}{e^3} + e^3}{2 \left( 2 \cosh(-2i) + \frac{1}{e^3} + e^3 \right)} \right) + \frac{1}{\phi}$$

### Series representations:

$$2 \log_{0.993892} \left( \frac{\sin(2)}{e^3 + 2 \cos(2) + \frac{1}{e^3}} + \frac{1}{2} \left( \frac{1}{e^3 - 1} + \frac{1}{2} - \frac{1}{3} \right) + \frac{e^3 - \frac{1}{e^3}}{\left( e^3 + 2 \cos(2) + \frac{1}{e^3} \right) 2} \right) - \pi + \frac{1}{\phi} =$$

$$\frac{1}{\phi} - \pi - \frac{2 \sum_{k=1}^{\infty} \frac{(-1)^k \left( -1 + \frac{1}{2} \left( \frac{1}{6} + \frac{1}{-1 + e^3} \right) + \frac{-\frac{1}{e^3} + e^3}{2 \left( \frac{1}{e^3} + e^3 + 2 \cos(2) \right)} + \frac{\sin(2)}{e^3 + e^3 + 2 \cos(2)} \right)^k}{k \log(0.993892)}$$

$$2 \log_{0.993892} \left( \frac{\sin(2)}{e^3 + 2 \cos(2) + \frac{1}{e^3}} + \frac{1}{2} \left( \frac{1}{e^3 - 1} + \frac{1}{2} - \frac{1}{3} \right) + \frac{e^3 - \frac{1}{e^3}}{\left( e^3 + 2 \cos(2) + \frac{1}{e^3} \right) 2} \right) - \pi + \frac{1}{\phi} =$$

$$\frac{1}{\phi} - \pi - \frac{2 \sum_{k=1}^{\infty} \frac{\left( -\frac{1}{2} \right)^k \left( -\frac{11}{6} + \frac{1}{-1 + e^3} + \frac{-1 + e^6}{1 + e^6 + 2 e^3 \cos(2)} + \frac{2 \sin(2)}{e^3 + e^3 + 2 \cos(2)} \right)^k}{k \log(0.993892)}$$

$$2 \log_{0.993892} \left( \frac{\sin(2)}{e^3 + 2 \cos(2) + \frac{1}{e^3}} + \frac{1}{2} \left( \frac{1}{e^3 - 1} + \frac{1}{2} - \frac{1}{3} \right) + \frac{e^3 - \frac{1}{e^3}}{\left( e^3 + 2 \cos(2) + \frac{1}{e^3} \right) 2} \right) - \pi + \frac{1}{\phi} =$$

$$\frac{-1 + \phi \pi - 2 \phi \log_{0.993892} \left( \frac{11 - 5 e^3 - e^6 + 7 e^9 + \sum_{k=0}^{\infty} \frac{(-1)^k 2^{1+2k} e^3 (12(-1+e^3)(2k)! + (5+e^3)(1+2k)!)}{(2k)!(1+2k)!}}{12(-1+e^3) \left( 1 + e^6 + 2 e^3 \sum_{k=0}^{\infty} \frac{(-4)^k}{(2k)!} \right)} \right)}{\phi}$$

**Integral representations:**

$$2 \log_{0.993892} \left( \frac{\sin(2)}{e^3 + 2 \cos(2) + \frac{1}{e^3}} + \frac{1}{2} \left( \frac{1}{e^3 - 1} + \frac{1}{2} - \frac{1}{3} \right) + \frac{e^3 - \frac{1}{e^3}}{\left( e^3 + 2 \cos(2) + \frac{1}{e^3} \right) 2} \right) - \pi + \frac{1}{\phi} =$$

$$-\frac{1}{\phi} \left( -1 + \phi \pi - 2 \phi \log_{0.993892} \left( \frac{1}{2} \left( \frac{1}{6} + \frac{1}{-1 + e^3} \right) + \frac{-\frac{1}{e^3} + e^3}{2 \left( \frac{1}{e^3} + e^3 + 2 \left( 1 - 2 \int_0^1 \sin(2t) dt \right) \right)} + \frac{2}{\frac{1}{e^3} + e^3 + 2 \left( 1 - 2 \int_0^1 \sin(2t) dt \right)} \int_0^1 \cos(2t) dt \right) \right)$$

$$2 \log_{0.993892} \left( \frac{\sin(2)}{e^3 + 2 \cos(2) + \frac{1}{e^3}} + \frac{1}{2} \left( \frac{1}{e^3 - 1} + \frac{1}{2} - \frac{1}{3} \right) + \frac{e^3 - \frac{1}{e^3}}{\left( e^3 + 2 \cos(2) + \frac{1}{e^3} \right) 2} \right) - \pi + \frac{1}{\phi} =$$

$$-\frac{1}{\phi} \left( -1 + \phi \pi - 2 \phi \log_{0.993892} \left( \frac{1}{2} \left( \frac{1}{6} + \frac{1}{-1 + e^3} \right) + \frac{-\frac{1}{e^3} + e^3}{2 \left( \frac{1}{e^3} + e^3 - 2 \int_{\frac{\pi}{2}}^2 \sin(t) dt \right)} + \frac{2}{\frac{1}{e^3} + e^3 - 2 \int_{\frac{\pi}{2}}^2 \sin(t) dt} \int_0^1 \cos(2t) dt \right) \right)$$

$$2 \log_{0.993892} \left( \frac{\sin(2)}{e^3 + 2 \cos(2) + \frac{1}{e^3}} + \frac{1}{2} \left( \frac{1}{e^3 - 1} + \frac{1}{2} - \frac{1}{3} \right) + \frac{e^3 - \frac{1}{e^3}}{\left( e^3 + 2 \cos(2) + \frac{1}{e^3} \right) 2} \right) - \pi + \frac{1}{\phi} =$$

$$-\frac{1}{\phi} \left( -1 + \phi \pi - 2 \phi \log_{0.993892} \left( \frac{1}{2} \left( \frac{1}{6} + \frac{1}{-1 + e^3} \right) + \frac{-\frac{1}{e^3} + e^3}{2 \left( \frac{1}{e^3} + e^3 + 2 \left( 1 - 2 \int_0^1 \sin(2t) dt \right) \right)} + \frac{\sqrt{\pi}}{2 i \pi \left( \frac{1}{e^3} + e^3 + 2 \left( 1 - 2 \int_0^1 \sin(2t) dt \right) \right)} \int_{-i \infty + \gamma}^{i \infty + \gamma} \frac{\mathcal{A}^{-1/s+s}}{s^{3/2}} ds \right) \right) \text{ for } \gamma > 0$$

2log base 0.993891718[(((sin 2 / ((e^3 + 2 cos 2 + e^(-3)))))) + (((1/2 (1/(e^3-1)+1/2-1/3)))) + (((1/2 \* ((e^3 - e^(-3)))/((e^3 + 2 cos 2 + e^(-3))))))] + 11 + 1/golden ratio

**Input interpretation:**

$$2 \log_{0.993891718} \left( \frac{\sin(2)}{e^3 + 2 \cos(2) + \frac{1}{e^3}} + \frac{1}{2} \left( \frac{1}{e^3 - 1} + \frac{1}{2} - \frac{1}{3} \right) + \frac{1}{2} \times \frac{e^3 - \frac{1}{e^3}}{e^3 + 2 \cos(2) + \frac{1}{e^3}} \right) + 11 + \frac{1}{\phi}$$

$\log_b(x)$  is the base- $b$  logarithm

$\phi$  is the golden ratio

**Result:**

139.6180...

139.618... result practically equal to the rest mass of Pion meson 139.57 MeV

**Alternative representations:**

$$2 \log_{0.993892} \left( \frac{\sin(2)}{e^3 + 2 \cos(2) + \frac{1}{e^3}} + \frac{1}{2} \left( \frac{1}{e^3 - 1} + \frac{1}{2} - \frac{1}{3} \right) + \frac{e^3 - \frac{1}{e^3}}{\left( e^3 + 2 \cos(2) + \frac{1}{e^3} \right) 2} \right) + 11 + \frac{1}{\phi} =$$

$$11 + \frac{1}{\phi} + \frac{2 \log \left( \frac{1}{2} \left( \frac{1}{6} + \frac{1}{-1+e^3} \right) + \frac{-\frac{1}{e^3} + e^3}{2 \left( 2 \cos(2) + \frac{1}{e^3} + e^3 \right)} + \frac{\sin(2)}{2 \cos(2) + \frac{1}{e^3} + e^3} \right)}{\log(0.993892)}$$

$$2 \log_{0.993892} \left( \frac{\sin(2)}{e^3 + 2 \cos(2) + \frac{1}{e^3}} + \frac{1}{2} \left( \frac{1}{e^3 - 1} + \frac{1}{2} - \frac{1}{3} \right) + \frac{e^3 - \frac{1}{e^3}}{\left( e^3 + 2 \cos(2) + \frac{1}{e^3} \right) 2} \right) + 11 + \frac{1}{\phi} =$$

$$11 + 2 \log_{0.993892} \left( \frac{1}{2} \left( \frac{1}{6} + \frac{1}{-1+e^3} \right) + \frac{\cos(-2 + \frac{\pi}{2})}{2 \cosh(-2i) + \frac{1}{e^3} + e^3} + \frac{-\frac{1}{e^3} + e^3}{2 \left( 2 \cosh(-2i) + \frac{1}{e^3} + e^3 \right)} \right) + \frac{1}{\phi}$$

$$2 \log_{0.993892} \left( \frac{\sin(2)}{e^3 + 2 \cos(2) + \frac{1}{e^3}} + \frac{1}{2} \left( \frac{1}{e^3 - 1} + \frac{1}{2} - \frac{1}{3} \right) + \frac{e^3 - \frac{1}{e^3}}{\left( e^3 + 2 \cos(2) + \frac{1}{e^3} \right) 2} \right) + 11 + \frac{1}{\phi} =$$

$$11 + 2 \log_{0.993892} \left( \frac{1}{2} \left( \frac{1}{6} + \frac{1}{-1+e^3} \right) - \frac{\cos(2 + \frac{\pi}{2})}{2 \cosh(-2i) + \frac{1}{e^3} + e^3} + \frac{-\frac{1}{e^3} + e^3}{2 \left( 2 \cosh(-2i) + \frac{1}{e^3} + e^3 \right)} \right) + \frac{1}{\phi}$$

### Series representations:

$$2 \log_{0.993892} \left( \frac{\sin(2)}{e^3 + 2 \cos(2) + \frac{1}{e^3}} + \frac{1}{2} \left( \frac{1}{e^3 - 1} + \frac{1}{2} - \frac{1}{3} \right) + \frac{e^3 - \frac{1}{e^3}}{\left( e^3 + 2 \cos(2) + \frac{1}{e^3} \right) 2} \right) + 11 + \frac{1}{\phi} =$$

$$11 + \frac{1}{\phi} - \frac{2 \sum_{k=1}^{\infty} \frac{(-1)^k \left( -1 + \frac{1}{2} \left( \frac{1}{6} + \frac{1}{-1+e^3} \right) + \frac{-\frac{1}{e^3} + e^3}{2 \left( \frac{1}{e^3} + e^3 + 2 \cos(2) \right)} + \frac{\sin(2)}{e^3 + e^3 + 2 \cos(2)} \right)^k}{k \log(0.993892)}$$

$$2 \log_{0.993892} \left( \frac{\sin(2)}{e^3 + 2 \cos(2) + \frac{1}{e^3}} + \frac{1}{2} \left( \frac{1}{e^3 - 1} + \frac{1}{2} - \frac{1}{3} \right) + \frac{e^3 - \frac{1}{e^3}}{\left( e^3 + 2 \cos(2) + \frac{1}{e^3} \right) 2} \right) + 11 + \frac{1}{\phi} =$$

$$11 + \frac{1}{\phi} - \frac{2 \sum_{k=1}^{\infty} \frac{\left( \frac{-1}{2} \right)^k \left( -\frac{11}{6} + \frac{1}{-1+e^3} + \frac{-1+e^6}{1+e^6+2e^3 \cos(2)} + \frac{2 \sin(2)}{e^3 + e^3 + 2 \cos(2)} \right)^k}{k \log(0.993892)}$$

$$2 \log_{0.993892} \left( \frac{\sin(2)}{e^3 + 2 \cos(2) + \frac{1}{e^3}} + \frac{1}{2} \left( \frac{1}{e^3 - 1} + \frac{1}{2} - \frac{1}{3} \right) + \frac{e^3 - \frac{1}{e^3}}{\left( e^3 + 2 \cos(2) + \frac{1}{e^3} \right) 2} \right) + 11 + \frac{1}{\phi} =$$

$$\frac{1 + 11 \phi + 2 \phi \log_{0.993892} \left( \frac{\sum_{k=0}^{\infty} \frac{(-1)^k 2^{1+2k} e^3 (12(-1+e^3)(2k)!(5+e^3)(1+2k)!)}{(2k)!(1+2k)!}}{12(-1+e^3) \left( 1+e^6+2e^3 \sum_{k=0}^{\infty} \frac{(-4)^k}{(2k)!} \right)} \right)}{\phi}$$

### Integral representations:

$$2 \log_{0.993892} \left( \frac{\sin(2)}{e^3 + 2 \cos(2) + \frac{1}{e^3}} + \frac{1}{2} \left( \frac{1}{e^3 - 1} + \frac{1}{2} - \frac{1}{3} \right) + \frac{e^3 - \frac{1}{e^3}}{\left( e^3 + 2 \cos(2) + \frac{1}{e^3} \right) 2} \right) + 11 + \frac{1}{\phi} =$$

$$\frac{1}{\phi} \left( 1 + 11 \phi + 2 \phi \log_{0.993892} \left( \frac{1}{2} \left( \frac{1}{6} + \frac{1}{-1+e^3} \right) + \frac{-\frac{1}{e^3} + e^3}{2 \left( \frac{1}{e^3} + e^3 + 2 \left( 1 - 2 \int_0^1 \sin(2t) dt \right) \right)} + \frac{2}{\frac{1}{e^3} + e^3 + 2 \left( 1 - 2 \int_0^1 \sin(2t) dt \right)} \int_0^1 \cos(2t) dt \right) \right)$$



$$2 \log_{0.993892} \left( \frac{\sin(2)}{e^3 + 2 \cos(2) + \frac{1}{e^3}} + \frac{1}{2} \left( \frac{1}{e^3 - 1} + \frac{1}{2} - \frac{1}{3} \right) + \frac{e^3 - \frac{1}{e^3}}{(e^3 + 2 \cos(2) + \frac{1}{e^3})^2} \right) + 11 + \frac{1}{\phi} =$$

$$\frac{1}{\phi} \left( 1 + 11 \phi + 2 \phi \log_{0.993892} \left( \frac{1}{2} \left( \frac{1}{6} + \frac{1}{-1 + e^3} \right) + \frac{-\frac{1}{e^3} + e^3}{2 \left( \frac{1}{e^3} + e^3 - 2 \int_{\frac{\pi}{2}}^2 \sin(t) dt \right)} + \frac{2}{e^3 + e^3 - 2 \int_{\frac{\pi}{2}}^2 \sin(t) dt} \int_0^1 \cos(2t) dt \right) \right)$$

$$2 \log_{0.993892} \left( \frac{\sin(2)}{e^3 + 2 \cos(2) + \frac{1}{e^3}} + \frac{1}{2} \left( \frac{1}{e^3 - 1} + \frac{1}{2} - \frac{1}{3} \right) + \frac{e^3 - \frac{1}{e^3}}{(e^3 + 2 \cos(2) + \frac{1}{e^3})^2} \right) + 11 + \frac{1}{\phi} =$$

$$\frac{1}{\phi} \left( 1 + 11 \phi + 2 \phi \log_{0.993892} \left( \frac{1}{2} \left( \frac{1}{6} + \frac{1}{-1 + e^3} \right) + \frac{-\frac{1}{e^3} + e^3}{2 \left( \frac{1}{e^3} + e^3 + 2 \left( 1 - 2 \int_0^1 \sin(2t) dt \right) \right)} + \frac{\sqrt{\pi}}{2 i \pi \left( \frac{1}{e^3} + e^3 + 2 \left( 1 - 2 \int_0^1 \sin(2t) dt \right) \right)} \int_{-i \infty + \gamma}^{i \infty + \gamma} \frac{\mathcal{A}^{-1/s+s}}{s^{3/2}} ds \right) \right) \text{ for } \gamma > 0$$

$$27 \log \text{ base } 0.993891718 [(((\sin 2 / ((e^3 + 2 \cos 2 + e^{(-3)})))))) + (((1/2 (1/(e^3 - 1) + 1/2 - 1/3)))) + (((1/2 * ((e^3 - e^{(-3)})) / ((e^3 + 2 \cos 2 + e^{(-3)}))))))] ]$$

**Input interpretation:**

$$27 \log_{0.993891718} \left( \frac{\sin(2)}{e^3 + 2 \cos(2) + \frac{1}{e^3}} + \frac{1}{2} \left( \frac{1}{e^3 - 1} + \frac{1}{2} - \frac{1}{3} \right) + \frac{1}{2} \times \frac{e^3 - \frac{1}{e^3}}{e^3 + 2 \cos(2) + \frac{1}{e^3}} \right)$$

$\log_b(x)$  is the base- $b$  logarithm

**Result:**

1728.000...

1728

This result is very near to the mass of candidate glueball  $f_0(1710)$  meson. Furthermore, 1728 occurs in the algebraic formula for the  $j$ -invariant of an elliptic curve. As a consequence, it is sometimes called a Zagier as a pun on the Gross–Zagier theorem. The number 1728 is one less than the Hardy–Ramanujan number 1729

### Alternative representations:

$$27 \log_{0.993892} \left( \frac{\sin(2)}{e^3 + 2 \cos(2) + \frac{1}{e^3}} + \frac{1}{2} \left( \frac{1}{e^3 - 1} + \frac{1}{2} - \frac{1}{3} \right) + \frac{e^3 - \frac{1}{e^3}}{\left( e^3 + 2 \cos(2) + \frac{1}{e^3} \right) 2} \right) =$$

$$\frac{27 \log \left( \frac{1}{2} \left( \frac{1}{6} + \frac{1}{-1+e^3} \right) + \frac{-\frac{1}{e^3}+e^3}{2 \left( 2 \cos(2) + \frac{1}{e^3} + e^3 \right)} + \frac{\sin(2)}{2 \cos(2) + \frac{1}{e^3} + e^3} \right)}{\log(0.993892)}$$

$$27 \log_{0.993892} \left( \frac{\sin(2)}{e^3 + 2 \cos(2) + \frac{1}{e^3}} + \frac{1}{2} \left( \frac{1}{e^3 - 1} + \frac{1}{2} - \frac{1}{3} \right) + \frac{e^3 - \frac{1}{e^3}}{\left( e^3 + 2 \cos(2) + \frac{1}{e^3} \right) 2} \right) =$$

$$27 \log_{0.993892} \left( \frac{1}{2} \left( \frac{1}{6} + \frac{1}{-1+e^3} \right) + \frac{\cos(-2 + \frac{\pi}{2})}{2 \cosh(-2i) + \frac{1}{e^3} + e^3} + \frac{-\frac{1}{e^3} + e^3}{2 \left( 2 \cosh(-2i) + \frac{1}{e^3} + e^3 \right)} \right)$$

$$27 \log_{0.993892} \left( \frac{\sin(2)}{e^3 + 2 \cos(2) + \frac{1}{e^3}} + \frac{1}{2} \left( \frac{1}{e^3 - 1} + \frac{1}{2} - \frac{1}{3} \right) + \frac{e^3 - \frac{1}{e^3}}{\left( e^3 + 2 \cos(2) + \frac{1}{e^3} \right) 2} \right) =$$

$$27 \log_{0.993892} \left( \frac{1}{2} \left( \frac{1}{6} + \frac{1}{-1+e^3} \right) - \frac{\cos(2 + \frac{\pi}{2})}{2 \cosh(-2i) + \frac{1}{e^3} + e^3} + \frac{-\frac{1}{e^3} + e^3}{2 \left( 2 \cosh(-2i) + \frac{1}{e^3} + e^3 \right)} \right)$$

### Series representations:

$$27 \log_{0.993892} \left( \frac{\sin(2)}{e^3 + 2 \cos(2) + \frac{1}{e^3}} + \frac{1}{2} \left( \frac{1}{e^3 - 1} + \frac{1}{2} - \frac{1}{3} \right) + \frac{e^3 - \frac{1}{e^3}}{\left( e^3 + 2 \cos(2) + \frac{1}{e^3} \right) 2} \right) =$$

$$\frac{27 \sum_{k=1}^{\infty} \frac{(-1)^k \left( -1 + \frac{1}{2} \left( \frac{1}{6} + \frac{1}{-1+e^3} \right) + \frac{-\frac{1}{e^3}+e^3}{2 \left( \frac{1}{e^3}+e^3+2 \cos(2) \right)} + \frac{\sin(2)}{e^3+e^3+2 \cos(2)} \right)^k}{k}}{\log(0.993892)}$$

$$27 \log_{0.993892} \left( \frac{\sin(2)}{e^3 + 2 \cos(2) + \frac{1}{e^3}} + \frac{1}{2} \left( \frac{1}{e^3 - 1} + \frac{1}{2} - \frac{1}{3} \right) + \frac{e^3 - \frac{1}{e^3}}{\left( e^3 + 2 \cos(2) + \frac{1}{e^3} \right) 2} \right) =$$

$$\frac{27 \sum_{k=1}^{\infty} \frac{\left( -\frac{1}{2} \right)^k \left( -\frac{11}{6} + \frac{1}{-1+e^3} + \frac{-1+e^6}{1+e^6+2 e^3 \cos(2)} + \frac{2 \sin(2)}{e^3+e^3+2 \cos(2)} \right)^k}{k}}{\log(0.993892)}$$

$$27 \log_{0.993892} \left( \frac{\sin(2)}{e^3 + 2 \cos(2) + \frac{1}{e^3}} + \frac{1}{2} \left( \frac{1}{e^3 - 1} + \frac{1}{2} - \frac{1}{3} \right) + \frac{e^3 - \frac{1}{e^3}}{\left( e^3 + 2 \cos(2) + \frac{1}{e^3} \right) 2} \right) =$$

$$27 \log_{0.993892} \left( \frac{11 - 5 e^3 - e^6 + 7 e^9 + \sum_{k=0}^{\infty} \frac{(-1)^k 2^{1+2k} e^3 (12(-1+e^3)(2k)! + (5+e^3)(1+2k)!)}{(2k)!(1+2k)!}}{12(-1+e^3) \left( 1 + e^6 + 2 e^3 \sum_{k=0}^{\infty} \frac{(-4)^k}{(2k)!} \right)} \right)$$

### Integral representations:

$$27 \log_{0.993892} \left( \frac{\sin(2)}{e^3 + 2 \cos(2) + \frac{1}{e^3}} + \frac{1}{2} \left( \frac{1}{e^3 - 1} + \frac{1}{2} - \frac{1}{3} \right) + \frac{e^3 - \frac{1}{e^3}}{\left( e^3 + 2 \cos(2) + \frac{1}{e^3} \right) 2} \right) =$$

$$27 \log_{0.993892} \left( \frac{11 + 5 e^3 + e^6 + 7 e^9 + \int_0^1 4 e^3 (6(-1+e^3) \cos(2t) - (5+e^3) \sin(2t)) dt}{12(-1+e^3) \left( (1+e^3)^2 - 4 e^3 \int_0^1 \sin(2t) dt \right)} \right)$$

$$27 \log_{0.993892} \left( \frac{\sin(2)}{e^3 + 2 \cos(2) + \frac{1}{e^3}} + \frac{1}{2} \left( \frac{1}{e^3 - 1} + \frac{1}{2} - \frac{1}{3} \right) + \frac{e^3 - \frac{1}{e^3}}{\left( e^3 + 2 \cos(2) + \frac{1}{e^3} \right) 2} \right) =$$

$$27 \log_{0.993892} \left( \frac{1}{2} \left( \frac{1}{6} + \frac{1}{-1+e^3} \right) + \frac{-\frac{1}{e^3} + e^3}{2 \left( \frac{1}{e^3} + e^3 - 2 \int_{\frac{\pi}{2}}^2 \sin(t) dt \right)} + \frac{2}{e^3 + e^3 - 2 \int_{\frac{\pi}{2}}^2 \sin(t) dt} \int_0^1 \cos(2t) dt \right)$$

$$27 \log_{0.993892} \left( \frac{\sin(2)}{e^3 + 2 \cos(2) + \frac{1}{e^3}} + \frac{1}{2} \left( \frac{1}{e^3 - 1} + \frac{1}{2} - \frac{1}{3} \right) + \frac{e^3 - \frac{1}{e^3}}{\left( e^3 + 2 \cos(2) + \frac{1}{e^3} \right) 2} \right) =$$

$$27 \log_{0.993892} \left( \frac{1}{2} \left( \frac{1}{6} + \frac{1}{-1+e^3} \right) + \frac{-\frac{1}{e^3} + e^3}{2 \left( \frac{1}{e^3} + e^3 + 2(1 - 2 \int_0^1 \sin(2t) dt) \right)} + \frac{\sqrt{\pi}}{2 i \pi \left( \frac{1}{e^3} + e^3 + 2(1 - 2 \int_0^1 \sin(2t) dt) \right)} \int_{-i\infty+\gamma}^{i\infty+\gamma} \frac{\mathcal{A}^{-1/s+s}}{s^{3/2}} ds \right) \text{ for } \gamma > 0$$



### Alternative representations:

$$\frac{\frac{1}{e^3+2\cos(2)+\frac{1}{e^3}} + \frac{1}{2}\left(\frac{1}{e^3-1} + \frac{1}{2-\frac{1}{3}}\right) + \frac{e^3-\frac{1}{e^3}}{2\left(e^3+2\cos(2)+\frac{1}{e^3}\right)} - \frac{29+7}{10^2} - \frac{11+3}{10^3} - \frac{5}{10^4}}{10^{52}} =$$

$$-\frac{36}{10^2} - \frac{14}{10^3} - \frac{5}{10^4} + \frac{10^{52}}{2\left(\frac{1}{6} + \frac{1}{-1+e^3}\right) + \frac{\cos(-2+\frac{\pi}{2})}{2\cosh(-2i)+\frac{1}{e^3}+e^3} + \frac{-\frac{1}{e^3}+e^3}{2\left(2\cosh(-2i)+\frac{1}{e^3}+e^3\right)}}$$

$$\frac{\frac{1}{e^3+2\cos(2)+\frac{1}{e^3}} + \frac{1}{2}\left(\frac{1}{e^3-1} + \frac{1}{2-\frac{1}{3}}\right) + \frac{e^3-\frac{1}{e^3}}{2\left(e^3+2\cos(2)+\frac{1}{e^3}\right)} - \frac{29+7}{10^2} - \frac{11+3}{10^3} - \frac{5}{10^4}}{10^{52}} =$$

$$-\frac{36}{10^2} - \frac{14}{10^3} - \frac{5}{10^4} + \frac{10^{52}}{2\left(\frac{1}{6} + \frac{1}{-1+e^3}\right) - \frac{\cos(2+\frac{\pi}{2})}{2\cosh(-2i)+\frac{1}{e^3}+e^3} + \frac{-\frac{1}{e^3}+e^3}{2\left(2\cosh(-2i)+\frac{1}{e^3}+e^3\right)}}$$

$$\frac{\frac{1}{e^3+2\cos(2)+\frac{1}{e^3}} + \frac{1}{2}\left(\frac{1}{e^3-1} + \frac{1}{2-\frac{1}{3}}\right) + \frac{e^3-\frac{1}{e^3}}{2\left(e^3+2\cos(2)+\frac{1}{e^3}\right)} - \frac{29+7}{10^2} - \frac{11+3}{10^3} - \frac{5}{10^4}}{10^{52}} =$$

$$-\frac{36}{10^2} - \frac{14}{10^3} - \frac{5}{10^4} + \frac{10^{52}}{2\left(\frac{1}{6} + \frac{1}{-1+e^3}\right) + \frac{\cos(-2+\frac{\pi}{2})}{2\cosh(2i)+\frac{1}{e^3}+e^3} + \frac{-\frac{1}{e^3}+e^3}{2\left(2\cosh(2i)+\frac{1}{e^3}+e^3\right)}}$$









From:

**RAMANUJAN'S CONTRIBUTIONS TO EISENSTEIN SERIES,  
ESPECIALLY IN HIS LOST NOTEBOOK**  
*BRUCE C. BERNDT AND AE JA YEE*

Now, we have that:

$$(a; q)_{\infty} := \prod_{n=0}^{\infty} (1 - aq^n), \quad |q| < 1.$$

Define, after Ramanujan,

$$f(-q) := (q; q)_{\infty} =: e^{-2\pi i\tau/24} \eta(\tau), \quad q = e^{2\pi i\tau}, \quad \text{Im } \tau > 0, \quad (1.6)$$

where  $\eta$  denotes the Dedekind eta-function.

$$R(q) := 1 - 504 \sum_{k=1}^{\infty} \frac{k^5 q^k}{1 - q^k}.$$

$$R(q) = \left( \frac{f^{15}(-q)}{f^3(-q^5)} - 500q f^9(-q) f^3(-q^5) - 15625q^2 f^3(-q) f^9(-q^5) \right) \\ \times \sqrt{1 + 22q \frac{f^6(-q^5)}{f^6(-q)} + 125q^2 \frac{f^{12}(-q^5)}{f^{12}(-q)}}$$

and

$$R(q^5) = \left( \frac{f^{15}(-q)}{f^3(-q^5)} + 4q f^9(-q) f^3(-q^5) - q^2 f^3(-q) f^9(-q^5) \right) \\ \times \sqrt{1 + 22q \frac{f^6(-q^5)}{f^6(-q)} + 125q^2 \frac{f^{12}(-q^5)}{f^{12}(-q)}}.$$

We calculated:

$$\left( (e^{-2\pi/24} e^{\pi/12}) \right)^{15} \prod_{n=1}^4 (1 - e^{2\pi n})$$

**Input interpretation:**

$$\left( e^{-2\pi/24} e^{\pi/12} \right)^{15} \prod_{n=1}^4 (1 - e^{2\pi n})$$

**Result:**

$$(1 - e^{2\pi})(1 - e^{4\pi})(1 - e^{6\pi})(1 - e^{8\pi}) \approx 1.93515 \times 10^{27}$$

$$1.93515 * 10^{27}$$

**Alternate forms:**

$$128 e^{10\pi} \sinh^4(\pi) \cosh^2(\pi) \cosh(2\pi) (1 + 2 \cosh(2\pi))$$

$$(e^{2\pi} - 1)(e^{4\pi} - 1)(e^{6\pi} - 1)(e^{8\pi} - 1)$$

$$1 - e^{2\pi} - e^{4\pi} + 2e^{10\pi} - e^{16\pi} - e^{18\pi} + e^{20\pi}$$

and

$$((e^{-2\pi/24} e^{\pi/12}))^{12} (((\text{product } 1 - e^{(2\pi * n)}, n=1 \text{ to } 4)))^5$$

**Input interpretation:**

$$(e^{-2\pi/24} e^{\pi/12})^{12} \left( \prod_{n=1}^4 (1 - e^{2\pi n}) \right)^5$$

**Result:**

$$(1 - e^{2\pi})^5 (1 - e^{4\pi})^5 (1 - e^{6\pi})^5 (1 - e^{8\pi})^5 \approx 2.71374 \times 10^{136}$$

$$2.71374 * 10^{136}$$

**Alternate forms:**

$$(e^{2\pi} - 1)^5 (e^{4\pi} - 1)^5 (e^{6\pi} - 1)^5 (e^{8\pi} - 1)^5$$

$$(e^\pi - 1)^{20} (1 + e^\pi)^{20} (1 + e^{2\pi})^{10} (1 - e^\pi + e^{2\pi})^5 (1 + e^\pi + e^{2\pi})^5 (1 + e^{4\pi})^5$$

$$1 - 5e^{2\pi} + 5e^{4\pi} + 10e^{6\pi} - 15e^{8\pi} - e^{10\pi} - 25e^{12\pi} + 30e^{14\pi} + 70e^{16\pi} - 40e^{18\pi} - 26e^{20\pi} - 170e^{22\pi} + 35e^{24\pi} + 275e^{26\pi} + 85e^{28\pi} + 50e^{30\pi} - 525e^{32\pi} - 275e^{34\pi} + 295e^{36\pi} + 420e^{38\pi} + 650e^{40\pi} - 530e^{42\pi} - 730e^{44\pi} - 340e^{46\pi} + 180e^{48\pi} + 1152e^{50\pi} + 180e^{52\pi} - 340e^{54\pi} - 730e^{56\pi} - 530e^{58\pi} + 650e^{60\pi} + 420e^{62\pi} + 295e^{64\pi} - 275e^{66\pi} - 525e^{68\pi} + 50e^{70\pi} + 85e^{72\pi} + 275e^{74\pi} + 35e^{76\pi} - 170e^{78\pi} - 26e^{80\pi} - 40e^{82\pi} + 70e^{84\pi} + 30e^{86\pi} - 25e^{88\pi} - e^{90\pi} - 15e^{92\pi} + 10e^{94\pi} + 5e^{96\pi} - 5e^{98\pi} + e^{100\pi}$$

These two values are always valid. That's why we only calculated two. Mainly, we wanted to highlight how we developed the calculations. Now, we have, after some other calculation:

$$R(q) = \left( \frac{f^{15}(-q)}{f^3(-q^5)} - 500qf^9(-q)f^3(-q^5) - 15625q^2f^3(-q)f^9(-q^5) \right) \\ \times \sqrt{1 + 22q \frac{f^6(-q^5)}{f^6(-q)} + 125q^2 \frac{f^{12}(-q^5)}{f^{12}(-q)}}$$

$$(((7.13093369298458953e-110 - 500 * e^{(2\pi)} * 1.93515e+27 * 2.71374e+136 - 15625 * (e^{(2\pi)})^2 * 1.93515e+27 * 2.71374e+136))) * \text{sqrt}(((1 + 22 * e^{(2\pi)} * 1.4023409038e+109 + 125 * (e^{(2\pi)})^2 * 1.4023409038e+109)))$$

**Input interpretation:**

$$\left( \frac{7.13093369298458953}{10^{110}} - 500 e^{2\pi} \times 1.93515 \times 10^{27} (2.71374 \times 10^{136}) - 15625 (e^{2\pi})^2 \times 1.93515 \times 10^{27} (2.71374 \times 10^{136}) \right) \\ \sqrt{1 + 22 e^{2\pi} (1.4023409038 \times 10^{109}) + 125 (e^{2\pi})^2 (1.4023409038 \times 10^{109})}$$

**Result:**

$$-5.27643... \times 10^{231}$$

$$-5.27643... * 10^{231}$$

And:

$$R(q^5) = \left( \frac{f^{15}(-q)}{f^3(-q^5)} + 4qf^9(-q)f^3(-q^5) - q^2f^3(-q)f^9(-q^5) \right) \\ \times \sqrt{1 + 22q \frac{f^6(-q^5)}{f^6(-q)} + 125q^2 \frac{f^{12}(-q^5)}{f^{12}(-q)}}$$

$$(((7.13093369298458953e-110 + 4 * e^{(2\pi)} * 1.93515e+27 * 2.71374e+136 - (e^{(2\pi)})^2 * 1.93515e+27 * 2.71374e+136))) * \text{sqrt}(((1 + 22 * e^{(2\pi)} * 1.4023409038e+109 + 125 * (e^{(2\pi)})^2 * 1.4023409038e+109)))$$

**Input interpretation:**

$$\left( \frac{7.13093369298458953}{10^{110}} + 4 e^{2\pi} \times 1.93515 \times 10^{27} (2.71374 \times 10^{136}) - (e^{2\pi})^2 \times 1.93515 \times 10^{27} (2.71374 \times 10^{136}) \right) \\ \sqrt{1 + 22 e^{2\pi} (1.4023409038 \times 10^{109}) + 125 (e^{2\pi})^2 (1.4023409038 \times 10^{109})}$$

**Result:**

$$-3.35149... \times 10^{227}$$

$$-3.35149... * 10^{227}$$

Now, we have that:

$$(-5.27643 * 10^{231}) / (-3.35149 * 10^{227})$$

**Input interpretation:**

$$\frac{-5.27643 \times 10^{231}}{3.35149 \times 10^{227}}$$

**Result:**

$$15743.53496504539771862664068817152967784477948613900086230...$$

$$15743.534965...$$

And:

$$1/9((( -5.27643 * 10^{231}) / (-3.35149 * 10^{227}))) - 21$$

**Input interpretation:**

$$\frac{1}{9} \left( \frac{-5.27643 \times 10^{231}}{3.35149 \times 10^{227}} \right) - 21$$

**Result:**

$$1728.281662782821968736293409796836630871642165126555651367...$$

$$1728.28166278....$$

This result is very near to the mass of candidate glueball  $f_0(1710)$  meson. Furthermore, 1728 occurs in the algebraic formula for the  $j$ -invariant of an elliptic curve. As a consequence, it is sometimes called a Zagier as a pun on the Gross–Zagier theorem. The number 1728 is one less than the Hardy–Ramanujan number 1729

$$1/12((( -5.27643 * 10^{231}) / (-3.35149 * 10^{227}))) - 76 - \text{Pi}$$

**Input interpretation:**

$$\frac{1}{12} \left( -\frac{5.27643 \times 10^{231}}{3.35149 \times 10^{227}} \right) - 76 - \pi$$

**Result:**

1232.82...

1232.82... result practically equal to the rest mass of Delta baryon 1232

**Alternative representations:**

$$\frac{-5.27643 \times 10^{231}}{(-3.35149 \times 10^{227}) 12} - 76 - \pi = -76 - 180^\circ - \frac{5.27643 \times 10^{231}}{12(-3.35149 \times 10^{227})}$$

$$\frac{-5.27643 \times 10^{231}}{(-3.35149 \times 10^{227}) 12} - 76 - \pi = -76 + i \log(-1) - \frac{5.27643 \times 10^{231}}{12(-3.35149 \times 10^{227})}$$

$$\frac{-5.27643 \times 10^{231}}{(-3.35149 \times 10^{227}) 12} - 76 - \pi = -76 - \cos^{-1}(-1) - \frac{5.27643 \times 10^{231}}{12(-3.35149 \times 10^{227})}$$

**Series representations:**

$$\frac{-5.27643 \times 10^{231}}{(-3.35149 \times 10^{227}) 12} - 76 - \pi = 1235.96 - 4 \sum_{k=0}^{\infty} \frac{(-1)^k}{1+2k}$$

$$\frac{-5.27643 \times 10^{231}}{(-3.35149 \times 10^{227}) 12} - 76 - \pi = 1237.96 - 2 \sum_{k=1}^{\infty} \frac{2^k}{\binom{2k}{k}}$$

$$\frac{-5.27643 \times 10^{231}}{(-3.35149 \times 10^{227}) 12} - 76 - \pi = 1235.96 - \sum_{k=0}^{\infty} \frac{2^{-k} (-6+50k)}{\binom{3k}{k}}$$

**Integral representations:**

$$\frac{-5.27643 \times 10^{231}}{(-3.35149 \times 10^{227}) 12} - 76 - \pi = 1235.96 - 2 \int_0^{\infty} \frac{1}{1+t^2} dt$$

$$\frac{-5.27643 \times 10^{231}}{(-3.35149 \times 10^{227}) 12} - 76 - \pi = 1235.96 - 4 \int_0^1 \sqrt{1-t^2} dt$$

$$\frac{-5.27643 \times 10^{231}}{(-3.35149 \times 10^{227})^{12}} - 76 - \pi = 1235.96 - 2 \int_0^\infty \frac{\sin(t)}{t} dt$$

$$1/29((( -5.27643 * 10^231) / (-3.35149 * 10^227))) + 5$$

**Input interpretation:**

$$\frac{1}{29} \left( -\frac{5.27643 \times 10^{231}}{3.35149 \times 10^{227}} \right) + 5$$

**Result:**

547.8805160360481971940220926955699888911992926254827883552...

547.880516... result practically equal to the rest mass of Eta meson 547.862

$$1/76((( -5.27643 * 10^231) / (-3.35149 * 10^227))) - 64 - 4 + 1/\text{golden ratio}$$

**Input interpretation:**

$$\frac{1}{76} \left( -\frac{5.27643 \times 10^{231}}{3.35149 \times 10^{227}} \right) - 64 - 4 + \frac{1}{\phi}$$

$\phi$  is the golden ratio

**Result:**

139.770...

139.770... result practically equal to the rest mass of Pion meson 139.57 MeV

**Alternative representations:**

$$\frac{-5.27643 \times 10^{231}}{(-3.35149 \times 10^{227})^{76}} - 64 - 4 + \frac{1}{\phi} = -68 - \frac{5.27643 \times 10^{231}}{76(-3.35149 \times 10^{227})} + \frac{1}{2 \sin(54^\circ)}$$

$$\frac{-5.27643 \times 10^{231}}{(-3.35149 \times 10^{227})^{76}} - 64 - 4 + \frac{1}{\phi} = -68 + \frac{1}{2 \cos(216^\circ)} - \frac{5.27643 \times 10^{231}}{76(-3.35149 \times 10^{227})}$$

$$\frac{-5.27643 \times 10^{231}}{(-3.35149 \times 10^{227}) 76} - 64 - 4 + \frac{1}{\phi} = -68 - \frac{5.27643 \times 10^{231}}{76(-3.35149 \times 10^{227})} + -\frac{1}{2 \sin(666^\circ)}$$

But, we have also:

$$R(q^5) = \left( \frac{f^{15}(-q)}{f^3(-q^5)} + 4qf^9(-q)f^3(-q^5) - q^2f^3(-q)f^9(-q^5) \right) \times \sqrt{1 + 22q \frac{f^6(-q^5)}{f^6(-q)} + 125q^2 \frac{f^{12}(-q^5)}{f^{12}(-q)}}$$

$$\left( (7.13093369298458953e-110 + 4 * e^{(2\pi)} * 1.93515e+27 * 2.71374e+136 - (e^{(2\pi)})^2 * 1.93515e+27 * 2.71374e+136) \right) * \text{sqrt} \left( (1 + 22 * e^{(2\pi)} * 1.4023409038e+109 + x * (e^{(2\pi)})^2 * 1.4023409038e+109) \right) = -3.35149e+227$$

**Input interpretation:**

$$\left( \frac{7.13093369298458953}{10^{110}} + 4 e^{2\pi} \times 1.93515 \times 10^{27} (2.71374 \times 10^{136}) - (e^{2\pi})^2 \times 1.93515 \times 10^{27} (2.71374 \times 10^{136}) \right) \times \sqrt{1 + 22 e^{2\pi} (1.4023409038 \times 10^{109}) + x (e^{2\pi})^2 (1.4023409038 \times 10^{109})} = - (3.35149 \times 10^{227})$$

**Result:**

$$-1.49462 \times 10^{169} \sqrt{4.0212309563 \times 10^{114} x + 1.6520720748 \times 10^{113}} = -3.35149 \times 10^{227}$$

**Alternate form assuming x is positive:**

$$1.49462 \times 10^{169} \sqrt{4.0212309563 \times 10^{114} x + 1.6520720748 \times 10^{113}} = 3.35149 \times 10^{227}$$

**Solution:**

$$x \approx 125.$$

125 result very near to the dilaton mass calculated as a type of Higgs boson: 125 GeV for T = 0 and to the Higgs boson mass 125.18 GeV

## Ramanujan mathematics applied to Black Hole Physics

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Regular Article - Theoretical Physics

**The golden ratio in Schwarzschild–Kottler black holes**

*Norman Cruz, Marco Olivares, J. R. Villanueva*

We have that:

Therefore, when massless particles are close to having a maximum radial acceleration, their impact parameter  $b \rightarrow b_\phi$ , and then we obtain the identity

$$\Phi = \lim_{b \rightarrow b_\phi} \zeta(b, M) = \frac{\sqrt{3}}{2} \tan \left[ \frac{1}{3} \arccos \left( -\sqrt{\frac{27}{32}} \right) \right] - \frac{1}{2},$$

$$\left( \frac{du}{d\phi} \right)^2 = 2M \left( \frac{1}{4M} - u \right) \left( u + \frac{\Phi}{4M} \right) \left( \frac{1 + \Phi}{4M} - u \right)$$

$$u_p = 1/r_p$$

$$r_p = 4M$$

From

$$\Phi = \lim_{b \rightarrow b_\phi} \zeta(b, M) = \frac{\sqrt{3}}{2} \tan \left[ \frac{1}{3} \arccos \left( -\sqrt{\frac{27}{32}} \right) \right] - \frac{1}{2},$$



where  $\Phi = 0.618034\dots = 1/(1+\Phi)$  is the golden ratio. An important corollary of the previous statement is obtained in the Schwarzschild–de Sitter case. From Eq. (17),  $b_\Phi \rightarrow \infty$  when  $\Lambda = 3\mathcal{B}_\Phi^{-2}$ , and therefore, it is not hard to see from Eqs. (7) and (8) that  $r_{++} = 4M$  and  $r_+ = 4M\Phi$ , i.e., the horizons are in the golden ratio.

We obtain:

$$\sqrt{3}/2 \tan(1/3 \arccos(-\sqrt{27/32})) - 1/2$$

**Input:**

$$\frac{\sqrt{3}}{2} \tan\left(\frac{1}{3} \cos^{-1}\left(-\sqrt{\frac{27}{32}}\right)\right) - \frac{1}{2}$$

$\cos^{-1}(x)$  is the inverse cosine function

**Exact Result:**

$$\frac{1}{2} \sqrt{3} \tan\left(\frac{1}{3} \cos^{-1}\left(-\frac{3\sqrt{\frac{3}{2}}}{4}\right)\right) - \frac{1}{2}$$

(result in radians)

**Decimal approximation:**

0.618033988749894848204586834365638117720309179805762862135...

(result in radians)

[0.6180339887498....](#)

**Alternate forms:**

$$\frac{1}{2}(\sqrt{5} - 1)$$

$$\frac{\sqrt{5}}{2} - \frac{1}{2}$$

$$\frac{1}{2} \left( \sqrt{3} \tan\left(\frac{1}{3} \cos^{-1}\left(-\frac{3\sqrt{\frac{3}{2}}}{4}\right)\right) - 1 \right)$$

**Continued fraction:**



$$\frac{1}{2} \tan\left(\frac{1}{3} \cos^{-1}\left(-\sqrt{\frac{27}{32}}\right)\right) \sqrt{3} - \frac{1}{2} =$$

$$\frac{1}{2} i \left( i + \sqrt{3} \sum_{k=1}^{\infty} (-1)^k e^{2/3 i k \cos^{-1}\left(-\left(3\sqrt{\frac{3}{2}}\right)/4\right)} - \sqrt{3} \sum_{k=-\infty}^{-1} (-1)^k e^{2/3 i k \cos^{-1}\left(-\left(3\sqrt{\frac{3}{2}}\right)/4\right)} \right)$$

$$\frac{1}{2} \tan\left(\frac{1}{3} \cos^{-1}\left(-\sqrt{\frac{27}{32}}\right)\right) \sqrt{3} - \frac{1}{2} =$$

$$-\frac{1}{2} + \frac{4 \cos^{-1}\left(-\frac{3\sqrt{\frac{3}{2}}}{4}\right) \sum_{k=1}^{\infty} \frac{1}{(1-2k)^2 \pi^2 - \frac{4}{9} \cos^{-1}\left(-\frac{3\sqrt{\frac{3}{2}}}{4}\right)^2}}{\sqrt{3}}$$

$$\frac{1}{2} \tan\left(\frac{1}{3} \cos^{-1}\left(-\sqrt{\frac{27}{32}}\right)\right) \sqrt{3} - \frac{1}{2} =$$

$$-\frac{1}{2} + \frac{4 \cos^{-1}\left(-\frac{3\sqrt{\frac{3}{2}}}{4}\right) \sum_{k=1}^{\infty} \frac{1}{(-1+2k)^2 \pi^2 - \frac{4}{9} \cos^{-1}\left(-\frac{3\sqrt{\frac{3}{2}}}{4}\right)^2}}{\sqrt{3}}$$

**Integral representations:**

$$\frac{1}{2} \tan\left(\frac{1}{3} \cos^{-1}\left(-\sqrt{\frac{27}{32}}\right)\right) \sqrt{3} - \frac{1}{2} = -\frac{1}{2} + \frac{\sqrt{3}}{2} \int_0^{\frac{1}{3} \cos^{-1}\left(-\frac{3\sqrt{\frac{3}{2}}}{4}\right)} \sec^2(t) dt$$

$$\frac{1}{2} \tan\left(\frac{1}{3} \cos^{-1}\left(-\sqrt{\frac{27}{32}}\right)\right) \sqrt{3} - \frac{1}{2} = -\frac{1}{2} + \frac{\sqrt{3}}{\pi} \int_0^{\infty} \frac{-1+t \left(2 \cos^{-1}\left(-\frac{3\sqrt{\frac{3}{2}}}{4}\right)\right) / (3\pi)}{-1+t^2} dt$$

and:

$$1/((((\sqrt{3}/2 \tan (1/3 \arccos(-\sqrt{27/32}))-1/2))))$$

**Input:**

$$\frac{1}{\frac{\sqrt{3}}{2} \tan\left(\frac{1}{3} \cos^{-1}\left(-\sqrt{\frac{27}{32}}\right)\right) - \frac{1}{2}}$$

$\cos^{-1}(x)$  is the inverse cosine function

**Exact Result:**

$$\frac{1}{\frac{1}{2} \sqrt{3} \tan\left(\frac{1}{3} \cos^{-1}\left(-\frac{\sqrt[3]{\frac{3}{2}}}{4}\right)\right) - \frac{1}{2}}$$

(result in radians)

**Decimal approximation:**

$$1.618033988749894848204586834365638117720309179805762862135...$$

(result in radians)

[1.61803398874989...](#)

**Alternate forms:**

$$\frac{1}{2} (1 + \sqrt{5})$$

$$\frac{1}{\frac{\sqrt{5}}{2} - \frac{1}{2}}$$

$$\frac{2}{\sqrt{3} \tan\left(\frac{1}{3} \cos^{-1}\left(-\frac{\sqrt[3]{\frac{3}{2}}}{4}\right)\right) - 1}$$

**Alternative representations:**

$$\frac{1}{\frac{1}{2} \tan\left(\frac{1}{3} \cos^{-1}\left(-\sqrt{\frac{27}{32}}\right)\right) \sqrt{3} - \frac{1}{2}} = \frac{1}{-\frac{1}{2} - \frac{1}{2} \cot\left(-\frac{\pi}{2} + \frac{1}{3} \cos^{-1}\left(-\sqrt{\frac{27}{32}}\right)\right) \sqrt{3}}$$

$$\frac{1}{\frac{1}{2} \tan\left(\frac{1}{3} \cos^{-1}\left(-\sqrt{\frac{27}{32}}\right)\right) \sqrt{3} - \frac{1}{2}} = \frac{1}{-\frac{1}{2} + \frac{1}{2} \left(-i + \frac{2i}{1 + e^{\frac{2/3 i \cos^{-1}\left(-\sqrt{27/32}\right)}}}\right) \sqrt{3}}$$

$$\frac{1}{\frac{1}{2} \tan\left(\frac{1}{3} \cos^{-1}\left(-\sqrt{\frac{27}{32}}\right)\right) \sqrt{3} - \frac{1}{2}} = \frac{1}{i \left( e^{-1/3 i \cos^{-1}\left(-\sqrt{27/32}\right)} - e^{1/3 i \cos^{-1}\left(-\sqrt{27/32}\right)} \right) \sqrt{3} - \frac{1}{2} + \frac{1}{2 \left( e^{-1/3 i \cos^{-1}\left(-\sqrt{27/32}\right)} + e^{1/3 i \cos^{-1}\left(-\sqrt{27/32}\right)} \right)}}$$

**Series representations:**

$$\frac{1}{\frac{1}{2} \tan\left(\frac{1}{3} \cos^{-1}\left(-\sqrt{\frac{27}{32}}\right)\right) \sqrt{3} - \frac{1}{2}} = \frac{1}{-\frac{1}{2} + \frac{1}{2} i \sqrt{3} \left(1 + 2 \sum_{k=1}^{\infty} (-1)^k q^{2k}\right)}$$

for  $q = e^{1/3 i \cos^{-1}\left(-3\sqrt{\frac{3}{2}}/4\right)}$

$$\frac{1}{\frac{1}{2} \tan\left(\frac{1}{3} \cos^{-1}\left(-\sqrt{\frac{27}{32}}\right)\right) \sqrt{3} - \frac{1}{2}} = \frac{-3 + 8 \sqrt{3} \cos^{-1}\left(-\frac{3\sqrt{\frac{3}{2}}}{4}\right) \sum_{k=1}^{\infty} \frac{1}{(1-2k)^2 \pi^2 - \frac{4}{9} \cos^{-1}\left(-\frac{3\sqrt{\frac{3}{2}}}{4}\right)^2}}$$

$$\frac{1}{\frac{1}{2} \tan\left(\frac{1}{3} \cos^{-1}\left(-\sqrt{\frac{27}{32}}\right)\right) \sqrt{3} - \frac{1}{2}} = 2 / \left( -1 + i \sqrt{3} \sum_{k=1}^{\infty} (-1)^k e^{2/3 i k \cos^{-1}\left(-3\sqrt{\frac{3}{2}}/4\right)} - i \sqrt{3} \sum_{k=-\infty}^{-1} (-1)^k e^{2/3 i k \cos^{-1}\left(-3\sqrt{\frac{3}{2}}/4\right)} \right)$$

From

$$\Lambda < 1/9M^2.$$

$$\Theta = \arccos(-3M\sqrt{\Lambda})/3$$



Input interpretation:

$$1 + 0.449329 (1 + 0.449329) + 0.449329^3 (1 + 0.449329) (1 + 0.449329^2) + 0.449329^6 (1 + 0.449329) (1 + 0.449329^2) (1 + 0.449329^3)$$

Result:

$$1.823668114519603459609150422857916769816281700020239762302...$$

$$\psi(q) = 1.8236681145196...$$

We put  $\Lambda = 1.8236681145196 * 10^{35}$

$$\Theta = \arccos(-3M\sqrt{\Lambda})/3$$

We obtain:

$$r_+ = \frac{1}{\sqrt{\Lambda}} \left( \sqrt{3} \sin \Theta - \cos \Theta \right) \quad (7)$$

$$1/(\text{sqrt}(1.8236681145196\text{e}+35))((\text{sqrt}3 * \sin(((\text{arccos}(-3*13.12806\text{e}+39*\text{sqrt}(1.8236681145196\text{e}+35))/3)) - \cos(((\text{arccos}(3*13.12806\text{e}+39*\text{sqrt}(1.8236681145196\text{e}+35))/3))))))$$

Input interpretation:

$$\frac{1}{\sqrt{1.8236681145196 \times 10^{35}}} \left( \sqrt{3} \sin \left( \frac{1}{3} \cos^{-1} \left( -3 \times 13.12806 \times 10^{39} \sqrt{1.8236681145196 \times 10^{35}} \right) - \cos \left( \frac{1}{3} \cos^{-1} \left( 3 \times 13.12806 \times 10^{39} \sqrt{1.8236681145196 \times 10^{35}} \right) \right) \right) \right)$$

$\cos^{-1}(x)$  is the inverse cosine function

Result:

$$31.1298... + 57.5883... i$$

(result in radians)

Input interpretation:

$$31.1298 + 57.5883 i$$

$i$  is the imaginary unit

Result:

31.1298... +  
57.5883... *i*

**Polar coordinates:**

$r = 65.4636$  (radius),  $\theta = 61.6062^\circ$  (angle)

65.4636

From:

$$r_{++} = \frac{2}{\sqrt{\Lambda}} \cos \Theta. \tag{8}$$

$$\Theta = \arccos(-3M\sqrt{\Lambda})/3.$$

We obtain:

$$2/(\text{sqrt}(1.8236681145196\text{e}+35))* \cos (((\arccos(-3*13.12806\text{e}+39*\text{sqrt}(1.8236681145196\text{e}+35))/3))))$$

**Input interpretation:**

$$\frac{2}{\sqrt{1.8236681145196 \times 10^{35}} \cos\left(\frac{1}{3} \cos^{-1}\left(-3 \times 13.12806 \times 10^{39} \sqrt{1.8236681145196 \times 10^{35}}\right)\right)}$$

$\cos^{-1}(x)$  is the inverse cosine function

**Result:**

37.7954... +  
65.4635... *i*

(result in radians)

**Input interpretation:**

37.7954 + 65.4635 *i*

*i* is the imaginary unit

**Result:**

37.7954... +  
65.4635... *i*

**Polar coordinates:**



$r = 75.5908$  (radius),  $\theta = 60.^\circ$  (angle)

75.5908

From

$$u = u_p \left[ 1 - \frac{\Phi}{2} (1 + \cos \chi) \right] \quad (u = u_p \text{ when } \chi = \pi),$$

$$u_p = 1/r_p$$

$$r_p = 4M$$

We obtain:

$$1/(4*13.12806e+39)((1-0.618034/2(1+\cos(\text{Pi})))$$

**Input interpretation:**

$$\frac{1}{4 \times 13.12806 \times 10^{39}} \left( 1 - \frac{0.618034}{2} (1 + \cos(\pi)) \right)$$

**Result:**

$$1.9043179266395796484781452857467135281222054134426564... \times 10^{-41}$$

$$1.90431792663957... * 10^{-41}$$

From

$$\left( \frac{du}{d\phi} \right)^2 = 2M \left( \frac{1}{4M} - u \right) \left( u + \frac{\Phi}{4M} \right) \left( \frac{1 + \Phi}{4M} - u \right)$$

We obtain:

$$(2*13.12806e+39)(1/(4*13.12806e+39)-1.90431792663957e-41)($$

$$1.90431792663957e-$$

$$41+0.618034/(4*13.12806e+39))((1+0.618034)/(4*13.12806e+39)-$$

$$1.90431792663957e-41)$$

**Input interpretation:**

$$(2 \times 13.12806 \times 10^{39}) \left( \frac{1}{4 \times 13.12806 \times 10^{39}} - 1.90431792663957 \times 10^{-41} \right) \\ \left( 1.90431792663957 \times 10^{-41} + \frac{0.618034}{4 \times 13.12806 \times 10^{39}} \right) \\ \left( \frac{1 + 0.618034}{4 \times 13.12806 \times 10^{39}} - 1.90431792663957 \times 10^{-41} \right)$$

**Result:**

9.1868851795342493689323562812142880851065478478393520...  $\times 10^{-97}$   
 9.186885179534249...  $\times 10^{-97}$

Thence:

$$\left( \frac{du}{d\phi} \right)^2 = 2M \left( u - \frac{1}{4M} \right) \left( u + \frac{\Phi}{4M} \right) \left( u - \frac{1}{4M\Phi} \right). \tag{34}$$

$(2 \times 13.12806 \times 10^{39})(1.90431792663957 \times 10^{-41} - 1/(4 \times 13.12806 \times 10^{39}))$   
 $(1.90431792663957 \times 10^{-41} + (0.618034/(4 \times 13.12806 \times 10^{39})))$   
 $((1.90431792663957 \times 10^{-41} - (1/(0.618034 \times 4 \times 13.12806 \times 10^{39}))))$

**Input interpretation:**

$$(2 \times 13.12806 \times 10^{39}) \left( 1.90431792663957 \times 10^{-41} - \frac{1}{4 \times 13.12806 \times 10^{39}} \right) \\ \left( 1.90431792663957 \times 10^{-41} + \frac{0.618034}{4 \times 13.12806 \times 10^{39}} \right) \\ \left( 1.90431792663957 \times 10^{-41} - \frac{1}{0.618034 \times 4 \times 13.12806 \times 10^{39}} \right)$$

**Result:**

9.1868845744927840135379493249433700677955443087684383...  $\times 10^{-97}$   
 9.186884574492784...  $\times 10^{-97}$

And:

$$u = \frac{1}{4M} \left( 1 + \Phi \sec \frac{\chi}{2} \right)$$

$$1/(4*13.12806e+39)*(1+0.618034*\sec(\text{Pi}/2))$$

**Input interpretation:**

$$\frac{1}{4 \times 13.12806 \times 10^{39}} \left( 1 + 0.618034 \sec\left(\frac{\pi}{2}\right) \right)$$

sec(x) is the secant function

**Result:**

$\infty$

Where  $u \rightarrow \infty$

If instead we insert  $\cos(\pi/2)$  in the above formula, we obtain:

$$1/(4*13.12806e+39)*(1+0.618034*\cos(\text{Pi}/2))$$

**Input interpretation:**

$$\frac{1}{4 \times 13.12806 \times 10^{39}} \left( 1 + 0.618034 \cos\left(\frac{\pi}{2}\right) \right)$$

**Result:**

$$1.9043179266395796484781452857467135281222054134426564... \times 10^{-41}$$

$$1.9043179266395... * 10^{-41}$$

From the results

$$9.186885179534249... * 10^{-97}, 1.90431792663957... * 10^{-41}, 75.5908, 65.4636$$

we obtain:

$$1+(65.4636/75.5908)^3$$

**Input interpretation:**

$$1 + \left( \frac{65.4636}{75.5908} \right)^3$$

**Result:**

$$1.649520449069099790692806866740575929971370927518853525758...$$

$$1.649520449... \approx \zeta(2) = \frac{\pi^2}{6} = 1.644934 ...$$

And:

$$1/10^{35}(((1*1/(((9.186885179534249e-97 / (1.90431792663957e-41)^2))))^{1/(7*10))))$$

**Input interpretation:**

$$\frac{1}{10^{35}} \left( 1 \times \frac{1}{7 \times 10 \sqrt{\frac{9.186885179534249 \times 10^{-97}}{(1.90431792663957 \times 10^{-41})^2}}} \right)$$

**Result:**

$$1.6162879055703348... \times 10^{-35}$$

1.6162879... \* 10<sup>-35</sup> result practically equal to the value of Planck length

$$1.616252 * 10^{-35} \text{ m}$$

We observe that:

$$(((9.186885179534249e-97 + (1.90431792663957e-41))))x = \text{golden ratio}$$

**Input interpretation:**

$$(9.186885179534249 \times 10^{-97} + 1.90431792663957 \times 10^{-41})x = \phi$$

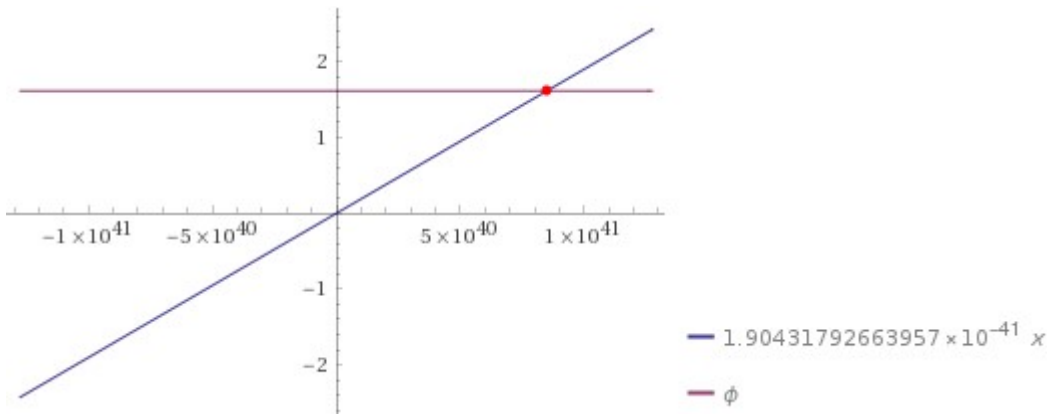
$\phi$  is the golden ratio

**Result:**

$$1.90431792663957 \times 10^{-41} x = \phi$$

$$1.90431792663957 * 10^{-41}$$

**Plot:**



**Alternate forms:**

$$1.90431792663957 \times 10^{-41} x - \phi = 0$$

$$1.90431792663957 \times 10^{-41} x = \frac{1}{2} (1 + \sqrt{5})$$

**Solution:**

$$x = 84966589145392208738122327843865979858962$$

**Integer solution:**

$$x = 84966589145392207802148593827401618316685$$

**Scientific notation:**

$$8.4966589145392207802148593827401618316685 \times 10^{40}$$

$$8.496658914539... * 10^{40}$$

From

golden ratio/ $1.90431792663957e-41$

we obtain:

**Input interpretation:**

$$\frac{\phi}{1.90431792663957 \times 10^{-41}}$$

$\phi$  is the golden ratio

**Result:**

$$8.49665891453922... \times 10^{40}$$

$$8.4966589... * 10^{40}$$

From which:

$$1.90431792663957e-41 * 8.49665891453922 \times 10^{40}$$

**Input interpretation:**

$$1.90431792663957 \times 10^{-41} \times 8.49665891453922 \times 10^{40}$$

**Result:**

$$1.61803398874989468180295689354$$

$$1.61803398874989468180295689354$$

Furthermore:

$$4(13.12806e+39/8.49665891453922 \times 10^{40})$$

**Input interpretation:**

$$4 \times \frac{13.12806 \times 10^{39}}{8.49665891453922 \times 10^{40}}$$

**Result:**

$$0.618033988749891780413569200521328025583832386618583981031...$$

$$0.61803398874989...$$

From which:

$$4(13.12806e+39/x) = 1/\text{golden ratio}$$

**Input interpretation:**

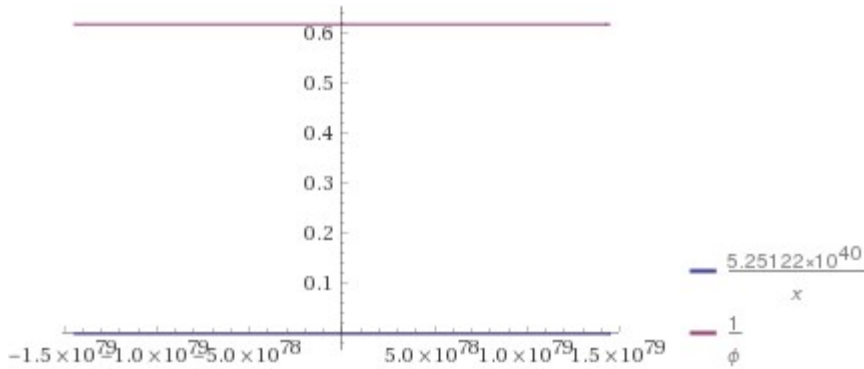
$$4 \times \frac{13.12806 \times 10^{39}}{x} = \frac{1}{\phi}$$

$\phi$  is the golden ratio

**Result:**

$$\frac{5.25122 \times 10^{40}}{x} = \frac{1}{\phi}$$

### Plot:



### Alternate forms:

$$\frac{5.25122 \times 10^{40}}{x} = \frac{1}{2} (\sqrt{5} - 1)$$

$$\frac{5.25122 \times 10^{40}}{x} = \frac{2}{1 + \sqrt{5}}$$

### Alternate form assuming x is positive:

$$x = 8.49666 \times 10^{40} \quad (\text{for } x \neq 0)$$

### Alternate forms assuming x is real:

$$\frac{5.25122 \times 10^{40}}{x} + 0 = \frac{1}{\phi}$$

$$\frac{8.49666 \times 10^{40}}{x} = 1$$

### Solution:

$$x = 84966589145391774200783268573727005081600$$

### Integer solution:

$$x = 84966589145391774200783268573727005081600$$

### Scientific notation:

$$8.49665891453917742007832685737270050816 \times 10^{40}$$

$$8.49665891453917742007832685737270050816 * 10^{40}$$

that is the result previously obtained

## Appendix

This is the Ramanujan fundamental formula for obtain a beautiful and highly precise golden ratio:

$$\sqrt[5]{\left(\frac{1}{\frac{1}{32}(-1+\sqrt{5})^5+5e^{(-\sqrt{5}\pi)^5}} - \frac{11 \times 5e^{(-\sqrt{5}\pi)^5}}{2\left(\frac{1}{32}(-1+\sqrt{5})^5+5e^{(-\sqrt{5}\pi)^5}\right)} - \frac{5\sqrt{5} \times 5e^{(-\sqrt{5}\pi)^5}}{2\left(\frac{1}{32}(-1+\sqrt{5})^5+5e^{(-\sqrt{5}\pi)^5}\right)}\right)}$$

$$1/\left(\left(\frac{1}{32}(-1+\sqrt{5})^5+5e^{(-\sqrt{5}\pi)^5}\right)\right)$$

**Input:**

$$\frac{1}{\frac{1}{32}(-1+\sqrt{5})^5+5e^{(-\sqrt{5}\pi)^5}}$$

**Exact result:**

$$\frac{1}{\frac{1}{32}(\sqrt{5}-1)^5+5e^{-25\sqrt{5}\pi^5}}$$

**Decimal approximation:**

11.09016994374947424102293417182819058860154589902881431067...

11.09016994374947424102293417182819058860154589902881431067

$$\left(\frac{11 \times 5 \times (e^{(-\sqrt{5}\pi)^5})}{\left(\frac{1}{32}(-1+\sqrt{5})^5+5e^{(-\sqrt{5}\pi)^5}\right)}\right) / \left(\frac{2 \times \left(\frac{1}{32}(-1+\sqrt{5})^5+5e^{(-\sqrt{5}\pi)^5}\right)}{\left(\frac{1}{32}(-1+\sqrt{5})^5+5e^{(-\sqrt{5}\pi)^5}\right)}\right)$$

**Input:**

$$\frac{11 \times 5 e^{(-\sqrt{5}\pi)^5}}{2\left(\frac{1}{32}(-1+\sqrt{5})^5+5e^{(-\sqrt{5}\pi)^5}\right)}$$



**Exact result:**

$$\frac{55 e^{-25 \sqrt{5} \pi^5}}{2 \left( \frac{1}{32} (\sqrt{5} - 1)^5 + 5 e^{-25 \sqrt{5} \pi^5} \right)}$$

**Decimal approximation:**

$$9.99290225070718723070536304129457122742436976265255... \times 10^{-7428}$$

$$9.99290225070718723070536304129457122742436976265255 \times 10^{-7428}$$

$$(5\sqrt{5} * 5 * (e^{(-\sqrt{5} * \pi)^5})) / (((2 * (((1/32 * (-1 + \sqrt{5})^5 + 5 * (e^{(-\sqrt{5} * \pi)^5}))))))$$

**Input:**

$$\frac{5 \sqrt{5} \times 5 e^{(-\sqrt{5} \pi)^5}}{2 \left( \frac{1}{32} (-1 + \sqrt{5})^5 + 5 e^{(-\sqrt{5} \pi)^5} \right)}$$

**Exact result:**

$$\frac{25 \sqrt{5} e^{-25 \sqrt{5} \pi^5}}{2 \left( \frac{1}{32} (\sqrt{5} - 1)^5 + 5 e^{-25 \sqrt{5} \pi^5} \right)}$$

**Decimal approximation:**

$$1.01567312386781438874777576295646917898823529098784... \times 10^{-7427}$$

$$1.01567312386781438874777576295646917898823529098784 \times 10^{-7427}$$

**Input interpretation:**

$$\left( \frac{1}{\left( \frac{1}{32} (-1 + \sqrt{5})^5 + 5 e^{(-\sqrt{5} \pi)^5} \right)} \right) \frac{9.99290225070718723070536304129457122742436976265255}{10^{7428}} \frac{1.01567312386781438874777576295646917898823529098784}{10^{7427}} \Big)^{(1/5)}$$

**Result:**

$$1.618033988749894848204586834365638117720309179805762862135...$$

$$1.6180339887498....$$



From:

$$\left( \left( \left( \frac{1}{\left( \left( \frac{1}{32}(-1 + \sqrt{5})^5 + 5 * (e^{(-\sqrt{5} * \pi)^5}) \right) \right) \right) + (1.6382898797095665677239458827012056245798314722584 \times 10^{-7429}) \right) \right)^{1/5}$$

**Input interpretation:**

$$\sqrt[5]{\frac{1}{\left( \frac{1}{32}(-1 + \sqrt{5})^5 + 5 e^{(-\sqrt{5} \pi)^5} \right) + \frac{1.6382898797095665677239458827012056245798314722584}{10^{7429}}}}$$

**Result:**

1.618033988749894848204586834365638117720309179805762862135...  
 1.618033988749...

we obtain:

$$\left( \left( \left( \frac{1}{\left( \left( \frac{1}{32}(-1 + \sqrt{5})^5 + 5 * (e^{(-\sqrt{5} * \pi)^5}) \right) \right) \right) + (x) \right) \right)^{1/5} = \text{golden ratio}$$

**Input:**

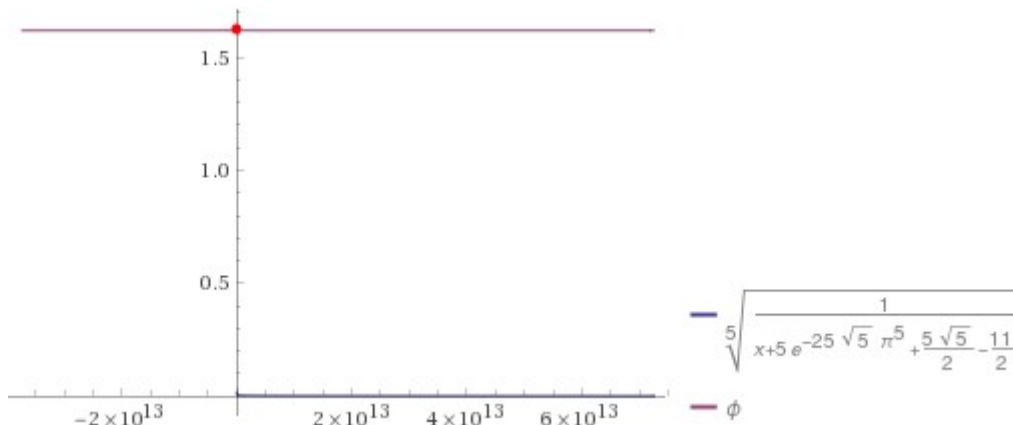
$$\sqrt[5]{\frac{1}{\left( \frac{1}{32}(-1 + \sqrt{5})^5 + 5 e^{(-\sqrt{5} \pi)^5} \right) + x}} = \phi$$

$\phi$  is the golden ratio

**Exact result:**

$$\sqrt[5]{\frac{1}{x + 5 e^{-25 \sqrt{5} \pi^5} + \frac{1}{32} (\sqrt{5} - 1)^5}} = \phi$$

**Plot:**



**Alternate forms:**

$$\sqrt[5]{\frac{1}{x+5e^{-25\sqrt{5}\pi^5} + \frac{5\sqrt{5}}{2} - \frac{11}{2}}} = \phi$$

$$\sqrt[5]{\frac{1}{x+5e^{-25\sqrt{5}\pi^5} + \frac{1}{2}(5\sqrt{5}-11)}} = \frac{1}{2}(1+\sqrt{5})$$

$$\sqrt[5]{\frac{1}{x+5e^{-25\sqrt{5}\pi^5} + \frac{1}{32}(\sqrt{5}-1)^5}} = \frac{1}{2} + \frac{\sqrt{5}}{2}$$

**Solution:**

$$x = -5e^{-25\sqrt{5}\pi^5}$$

$$-5e^{(-25\sqrt{5}\pi^5)}$$

**Input:**

$$-5e^{-25\sqrt{5}\pi^5}$$

**Decimal approximation:**

$$-1.6382898797095665677239458827012056245798314722589... \times 10^{-7429}$$

$$-1.6382898797... * 10^{-7429}$$

**Property:**

$-5e^{-25\sqrt{5}\pi^5}$  is a transcendental number

**Series representations:**

$$-5e^{-25\sqrt{5}\pi^5} = -5e^{-25\pi^5\sqrt{4}\sum_{k=0}^{\infty}4^{-k}\binom{1/2}{k}}$$

$$-5e^{-25\sqrt{5}\pi^5} = -5\exp\left(-25\pi^5\sqrt{4}\sum_{k=0}^{\infty}\frac{\left(-\frac{1}{4}\right)^k\left(-\frac{1}{2}\right)_k}{k!}\right)$$

$$-5e^{-25\sqrt{5}\pi^5} = -5\exp\left(-\frac{25\pi^5\sum_{j=0}^{\infty}\operatorname{Res}_{s=-\frac{1}{2}+j}4^{-s}\Gamma\left(-\frac{1}{2}-s\right)\Gamma(s)}{2\sqrt{\pi}}\right)$$

$$\left(\left(-5 e^{(-25 \sqrt{5} \pi^5)}\right)\right)^{1/(199 \cdot 7428)}$$

**Input:**

$$199 \times 7428 \sqrt[199 \times 7428]{-(-5 e^{-25 \sqrt{5} \pi^5})}$$

**Exact result:**

$$\sqrt[1478172]{5} e^{-(25 \sqrt{5} \pi^5)/1478172}$$

**Decimal approximation:**

0.988494694978618089881968949379155050394187268476546782511...

0.988494694... result very near to the dilaton value **0.989117352243 =  $\phi$**

**Property:**

$\sqrt[1478172]{5} e^{-(25 \sqrt{5} \pi^5)/1478172}$  is a transcendental number

**All 1478172nd roots of  $5 e^{(-25 \sqrt{5} \pi^5)}$ :**

$$\sqrt[1478172]{5} e^{-(25 \sqrt{5} \pi^5)/1478172} e^0 \approx 0.988495 \text{ (real, principal root)}$$

$$\sqrt[1478172]{5} e^{-(25 \sqrt{5} \pi^5)/1478172} e^{(i \pi)/739086} \approx 0.988495 + 4.2017 \times 10^{-6} i$$

$$\sqrt[1478172]{5} e^{-(25 \sqrt{5} \pi^5)/1478172} e^{(i \pi)/369543} \approx 0.988495 + 8.403 \times 10^{-6} i$$

$$\sqrt[1478172]{5} e^{-(25 \sqrt{5} \pi^5)/1478172} e^{(i \pi)/246362} \approx 0.988495 + 0.000012605 i$$

$$\sqrt[1478172]{5} e^{-(25 \sqrt{5} \pi^5)/1478172} e^{(2 i \pi)/369543} \approx 0.988495 + 0.000016807 i$$

**Series representations:**

$$199 \times 7428 \sqrt[199 \times 7428]{-(-5) e^{-25 \sqrt{5} \pi^5}} = \sqrt[1478172]{5} \sqrt[1478172]{e^{-25 \pi^5 \sqrt{4} \sum_{k=0}^{\infty} 4^{-k} \binom{1/2}{k}}}$$

$$199 \times 7428 \sqrt{-(-5) e^{-25 \sqrt{5} \pi^5}} = 1478172 \sqrt{5} \sqrt{1478172 \exp\left(-25 \pi^5 \sqrt{4} \sum_{k=0}^{\infty} \frac{\left(-\frac{1}{4}\right)^k \left(-\frac{1}{2}\right)_k}{k!}\right)}$$

$$199 \times 7428 \sqrt{-(-5) e^{-25 \sqrt{5} \pi^5}} = 1478172 \sqrt{5} \sqrt{1478172 \exp\left(-\frac{25 \pi^5 \sum_{j=0}^{\infty} \operatorname{Res}_{s=-\frac{1}{2}+j} 4^{-s} \Gamma\left(-\frac{1}{2}-s\right) \Gamma(s)}{2 \sqrt{\pi}}\right)}$$

### Integral representation:

$$(1+z)^\alpha = \frac{\int_{-i\infty+\gamma}^{i\infty+\gamma} \frac{\Gamma(s)\Gamma(-\alpha-s)}{z^s} ds}{(2\pi i)\Gamma(-\alpha)} \quad \text{for } (0 < \gamma < -\operatorname{Re}(\alpha) \text{ and } |\arg(z)| < \pi)$$

$$4 * (((((\log \text{ base } 0.988494694978618((-5 e^{(-25 \sqrt{5} \pi^5)}))))))))^{1/4}$$

### Input interpretation:

$$4 \sqrt[4]{\log_{0.988494694978618}\left(-(-5 e^{-25 \sqrt{5} \pi^5})\right)}$$

$\log_b(x)$  is the base- $b$  logarithm

### Result:

139.47335060456...

139.47335..... result practically equal to the rest mass of Pion meson 139.57 MeV

### Alternative representation:

$$4 \sqrt[4]{\log_{0.9884946949786180000}\left(-(-5) e^{-25 \sqrt{5} \pi^5}\right)} = 4 \sqrt[4]{\frac{\log\left(5 e^{-25 \pi^5 \sqrt{5}}\right)}{\log(0.9884946949786180000)}}$$

**Series representations:**

$$4 \sqrt[4]{\log_{0.9884946949786180000} \left( -(-5) e^{-25 \sqrt{5} \pi^5} \right)} =$$

$$4 \sqrt[4]{\log_{0.9884946949786180000} \left( 5 e^{-25 \pi^5 \sqrt{4} \sum_{k=0}^{\infty} 4^{-k} \binom{1/2}{k}} \right)}$$

$$4 \sqrt[4]{\log_{0.9884946949786180000} \left( -(-5) e^{-25 \sqrt{5} \pi^5} \right)} = 4 \sqrt[4]{-\frac{\sum_{k=1}^{\infty} \frac{(-1)^k \left( -1+5 e^{-25 \pi^5 \sqrt{5}} \right)^k}{k}}{\log(0.9884946949786180000)}}$$

$$4 \sqrt[4]{\log_{0.9884946949786180000} \left( -(-5) e^{-25 \sqrt{5} \pi^5} \right)} =$$

$$4 \sqrt[4]{\log_{0.9884946949786180000} \left( 5 \exp \left( -25 \pi^5 \sqrt{4} \sum_{k=0}^{\infty} \frac{\left( -\frac{1}{4} \right)^k \left( -\frac{1}{2} \right)_k}{k!} \right) \right)}$$

$$4 * (((((\log \text{ base } 0.988494694978618((-5 e^{(-25 \text{ sqrt}(5) \pi^5)}))))))))^{1/4} - 11 - \text{Pi}$$

**Input interpretation:**

$$4 \sqrt[4]{\log_{0.988494694978618} \left( -(-5 e^{-25 \sqrt{5} \pi^5}) \right)} - 11 - \pi$$

$\log_b(x)$  is the base- $b$  logarithm

**Result:**

125.33175795097...

125.33175.... result very near to the dilaton mass calculated as a type of Higgs boson: 125 GeV for  $T = 0$  and to the Higgs boson mass 125.18 GeV









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## References

### Manuscript Book 1 of Srinivasa Ramanujan

**RAMANUJAN'S CONTRIBUTIONS TO EISENSTEIN SERIES,  
ESPECIALLY IN HIS LOST NOTEBOOK**

*BRUCE C. BERNDT AND AE JA YEE*