

**On some Ramanujan equations (mock theta functions and taxicab numbers) linked to various sectors of String Theory (Brane-World) and to the Black Hole Physics: Further new possible mathematical connections X.**

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**Abstract**

*In this research thesis, we have analyzed and deepened further Ramanujan expressions (mock theta functions and taxicab numbers) applied to some sectors of String Theory (Brane-World) and to the Black Hole Physics. We have therefore described other new possible mathematical connections.*

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<https://www.britannica.com/biography/Srinivasa-Ramanujan>

ff

(i)  $\frac{1+53x+9x^2}{1-82x-82x^2+x^3} = a_0 + a_1x + a_2x^2 + a_3x^3 + \dots$   
 or  $\frac{\alpha_0}{x} + \frac{\alpha_1}{x^2} + \frac{\alpha_2}{x^3} + \dots$

(ii)  $\frac{2-26x-12x^2}{1-82x-82x^2+x^3} = b_0 + b_1x + b_2x^2 + b_3x^3 + \dots$   
 or  $\frac{\beta_0}{x} + \frac{\beta_1}{x^2} + \frac{\beta_2}{x^3} + \dots$

(iii)  $\frac{2+8x-10x^2}{1-82x-82x^2+x^3} = c_0 + c_1x + c_2x^2 + c_3x^3 + \dots$   
 or  $\frac{\gamma_0}{x} + \frac{\gamma_1}{x^2} + \frac{\gamma_2}{x^3} + \dots$

then

$$\left. \begin{aligned} a_n^3 + b_n^3 &= c_n^3 + (-1)^n \\ \text{and } d_n^3 + \beta_n^3 &= \gamma_n^3 + (-1)^n \end{aligned} \right\}$$

Examples

$$135^3 + 138^3 = 172^3 - 1$$

$$11161^3 + 11468^3 = 14258^3 + 1$$

$$791^3 + 812^3 = 1010^3 - 1$$

$$9^3 + 10^3 = 12^3 + 1$$

$$6^3 + 8^3 = 9^3 - 1$$

<https://plus.maths.org/content/ramanujan>

## Ramanujan's manuscript

The representations of 1729 as the sum of two cubes appear in the bottom right corner. The equation expressing the near counter examples to Fermat's last theorem appears further up:  $\alpha^3 + \beta^3 = \gamma^3 + (-1)^n$ .

From Wikipedia

*The **taxicab number**, typically denoted  $Ta(n)$  or  $Taxicab(n)$ , also called the  $n$ th **Hardy–Ramanujan number**, is defined as the smallest integer that can be expressed as a sum of two positive integer cubes in  $n$  distinct ways. The most famous taxicab number is  $1729 = Ta(2) = 1^3 + 12^3 = 9^3 + 10^3$ .*

From

### **Stability of the graviton Bose-Einstein condensate in the brane-world**

*R. Casadio* - Dipartimento di Fisica e Astronomia, Università di Bologna, via Irnerio 46, 40126 Bologna, Italy - INFN, Sezione di Bologna, viale B. Pichat 6, 40127

Bologna, Italy

*Roldao da Rocha* - CMCC, Universidade Federal do ABC, 09210-580, Santo André, SP, Brazil - arXiv: 1610.01572v1 [hep-th] 5 Oct 2016

Now, we have that:

$$\frac{B_\nu(\rho)}{B(\rho)} = 1 - \frac{2c_0}{\sigma \left[ \rho - \frac{3}{4} \tanh^3(\nu\rho) \right] \left[ 5 - 3 \tanh^2(\nu\rho) \right]}, \quad (22)$$

where  $c_0 \simeq 0.275$ . Fig. 1 shows plots of  $B_\nu(\rho)$  for various values of  $\nu$ . It is clear that, for increasing values of  $\nu$ , this

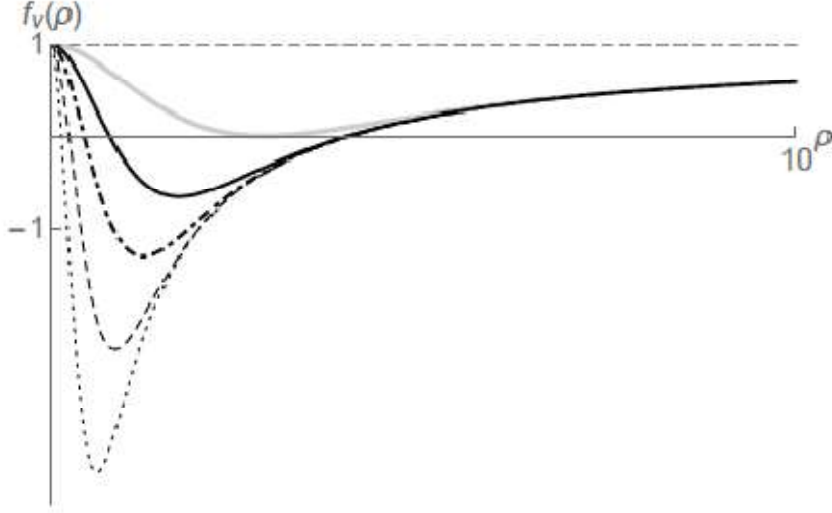


FIG. 1. Plot of  $B_\nu(\rho)$  in Eq. (22), for  $\nu = 0$  (gray dashed line);  $\nu = 0.3$  (thick gray line);  $\nu = 0.5$  (thick black line);  $\nu = \nu_*$  (black dot-dashed line);  $\nu = 1$  (black dashed line);  $\nu = 1.4$  (dotted line).

black hole model rapidly approaches the Schwarzschild black hole. This figure can be compared to Fig. 1 in Ref. [19] for similar parameters.

For any  $\nu$ , the metric component  $B_\nu(\rho)$  has a single local minimum at  $\rho_* = a_*/\nu$ , where  $a_* \approx 1.031$ . Writing  $B_\nu(\rho_*) = 1 - \nu/\nu_*$ , with  $\nu_* \approx 0.694$ , the condition for the existence of an event horizon is  $\nu > \nu_*$ . The case  $\nu = \nu_*$  is extremal [19].

$$\sigma \gtrsim 3.18 \times 10^6 \text{ MeV}^4$$

$$\sigma \gtrsim 5 \times 10^6 \text{ MeV}^4$$

$$\rho = r/M = 2r/r_s$$

For  $M = 1.312806e+40$  and  $R = 1.949322e+13$  (SMBH87 parameters)

$$\frac{B_\nu(\rho)}{B(\rho)} = 1 - \frac{2c_0}{\sigma \left[ \rho - \frac{3}{4} \tanh^3(\nu\rho) \right] \left[ 5 - 3 \tanh^2(\nu\rho) \right]}, \quad (22)$$

$$1 - (2 * 0.275) / [(3.18 * 10^6) (((((1.949322e+13 / 1.312806e+40) - 3/4 \tanh^3(1.4 * (1.949322e+13 / 1.312806e+40)))))) * (((5 - 3 * \tanh^2(1.4 * (1.949322e+13 / 1.312806e+40))))))] ]$$

**Input interpretation:**

$$1 - (2 \times 0.275) / \left( 3.18 \times 10^6 \left( \frac{1.949322 \times 10^{13}}{1.312806 \times 10^{40}} - \frac{3}{4} \tanh^3 \left( 1.4 \times \frac{1.949322 \times 10^{13}}{1.312806 \times 10^{40}} \right) \right) \left( 5 - 3 \tanh^2 \left( 1.4 \times \frac{1.949322 \times 10^{13}}{1.312806 \times 10^{40}} \right) \right) \right)$$

$\tanh(x)$  is the hyperbolic tangent function

**Result:**

$$-2.32961... \times 10^{19}$$

$$-2.32961... * 10^{19}$$

$$((( -1 / ((( (1 - (2 * 0.275) / [(3.18 * 10^6) (((((1.949322e+13 / 1.312806e+40) - 3/4 \tanh^3(1.4 * (1.949322e+13 / 1.312806e+40)))))) * (((5 - 3 * \tanh^2(1.4 * (1.949322e+13 / 1.312806e+40))))))] ])))))) )^{1/4096}$$

**Input interpretation:**

$$\left( - \left( 1 / \left( 1 - (2 \times 0.275) / \left( 3.18 \times 10^6 \left( \frac{1.949322 \times 10^{13}}{1.312806 \times 10^{40}} - \frac{3}{4} \tanh^3 \left( 1.4 \times \frac{1.949322 \times 10^{13}}{1.312806 \times 10^{40}} \right) \right) \left( 5 - 3 \tanh^2 \left( 1.4 \times \frac{1.949322 \times 10^{13}}{1.312806 \times 10^{40}} \right) \right) \right) \right) \right)^{(1/4096)}$$

$\tanh(x)$  is the hyperbolic tangent function

**Result:**

$$0.989171647...$$

0.989171647.... result very near to the value of the following Rogers-Ramanujan continued fraction:

$$\frac{e^{-\frac{\pi}{\sqrt{5}}}}{\sqrt{5}} = 1 - \frac{e^{-\pi\sqrt{5}}}{1 + \frac{e^{-2\pi\sqrt{5}}}{1 + \frac{e^{-3\pi\sqrt{5}}}{1 + \frac{e^{-4\pi\sqrt{5}}}{1 + \dots}}}} \approx 0.9991104684$$

and to the dilaton value **0.989117352243 =  $\phi$**

$$2 * \sqrt{\log_{0.989171647} \left( -1 / \left( 1 - \frac{2 * 0.275}{3.18 * 10^6 \left( \frac{1.949322e+13}{1.312806e+40} - \frac{3}{4} \tanh^3 \left( 1.4 * \frac{1.949322e+13}{1.312806e+40} \right) \right) \right) \right) * \left( 5 - 3 * \tanh^2 \left( 1.4 * \frac{1.949322e+13}{1.312806e+40} \right) \right) \right)}$$

### Input interpretation:

$$2 \sqrt{\log_{0.989171647} \left( -1 / \left( 1 - \frac{2 * 0.275}{3.18 * 10^6 \left( \frac{1.949322 * 10^{13}}{1.312806 * 10^{40}} - \frac{3}{4} \tanh^3 \left( 1.4 * \frac{1.949322 * 10^{13}}{1.312806 * 10^{40}} \right) \right) \right) \right) * \left( 5 - 3 \tanh^2 \left( 1.4 * \frac{1.949322 * 10^{13}}{1.312806 * 10^{40}} \right) \right) \right)}$$

$\tanh(x)$  is the hyperbolic tangent function  
 $\log_b(x)$  is the base- $b$  logarithm

### Result:

128.0000...

128







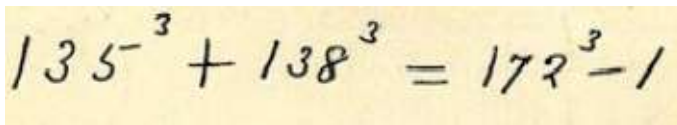
$\tanh(x)$  is the hyperbolic tangent function  
 $\log_b(x)$  is the base- $b$  logarithm

**Result:**

172.000...

172 (Ramanujan taxicab number)

From



A photograph of a piece of aged, yellowed paper with the equation  $135^3 + 138^3 = 172^3 - 1$  written in dark ink in a cursive style.

$$135^3 + 138^3 = 172^3 - 1$$

**Input:**

$$135^3 + 138^3 = 172^3 - 1$$

**Result:**

True

**Left hand side:**

$$135^3 + 138^3 = 5\,088\,447$$

**Right hand side:**

$$172^3 - 1 = 5\,088\,447$$

5088447

$$\ln(135^3 + 138^3)$$

**Input:**

$$\log(135^3 + 138^3)$$

$\log(x)$  is the natural logarithm

**Exact result:**

$$\log(5\,088\,447)$$

**Decimal approximation:**

15.44248323391676327573091977987313063668255249261267663169...

15.4424832339.... result very near to the black hole entropy 15.6730

**Property:**

$\log(5\,088\,447)$  is a transcendental number

**Alternate forms:**

$$3 \log(3) + \log(188\,461)$$

$$3 \log(3) + \log(7) + \log(13) + \log(19) + \log(109)$$

**Alternative representations:**

$$\log(135^3 + 138^3) = \log_e(135^3 + 138^3)$$

$$\log(135^3 + 138^3) = \log(a) \log_a(135^3 + 138^3)$$

$$\log(135^3 + 138^3) = -\text{Li}_1(1 - 135^3 - 138^3)$$

**Integral representations:**

$$\log(135^3 + 138^3) = \int_1^{5\,088\,447} \frac{1}{t} dt$$

$$\log(135^3 + 138^3) = -\frac{i}{2\pi} \int_{-i\infty+\gamma}^{i\infty+\gamma} \frac{5\,088\,446^{-s} \Gamma(-s)^2 \Gamma(1+s)}{\Gamma(1-s)} ds \quad \text{for } -1 < \gamma < 0$$

$\text{Pi} * (135^3 + 138^3)^{1/4} - 11 + \text{golden ratio}$

**Input:**

$$\pi \sqrt[4]{135^3 + 138^3} - 11 + \phi$$

$\phi$  is the golden ratio

**Result:**

$$\phi - 11 + 3^{3/4} \sqrt[4]{188\,461} \pi$$

## Decimal approximation:

139.8274348634976023813964821274141235673143122042902745354...

139.8274348... result practically equal to the rest mass of Pion meson 139.57 MeV

## Property:

$-11 + \phi + 3^{3/4} \sqrt[4]{188461} \pi$  is a transcendental number

## Alternate forms:

$$\frac{1}{2} \left( -21 + \sqrt{5} + 2 \times 3^{3/4} \sqrt[4]{188461} \pi \right)$$

$$-\frac{21}{2} + \frac{\sqrt{5}}{2} + 3^{3/4} \sqrt[4]{188461} \pi$$

$$-11 + \frac{1}{2} \left( 1 + \sqrt{5} \right) + 3^{3/4} \sqrt[4]{188461} \pi$$

## Alternative representations:

$$\pi \sqrt[4]{135^3 + 138^3} - 11 + \phi = -11 - 2 \cos(216^\circ) + \pi \sqrt[4]{135^3 + 138^3}$$

$$\pi \sqrt[4]{135^3 + 138^3} - 11 + \phi = -11 + 2 \cos\left(\frac{\pi}{5}\right) + \pi \sqrt[4]{135^3 + 138^3}$$

$$\pi \sqrt[4]{135^3 + 138^3} - 11 + \phi = -11 - 2 \cos(216^\circ) + 180^\circ \sqrt[4]{135^3 + 138^3}$$

## Series representations:

$$\pi \sqrt[4]{135^3 + 138^3} - 11 + \phi = -\frac{21}{2} + \frac{\sqrt{5}}{2} + 4 \times 3^{3/4} \sqrt[4]{188461} \sum_{k=0}^{\infty} \frac{(-1)^k}{1+2k}$$

$$\begin{aligned} \pi \sqrt[4]{135^3 + 138^3} - 11 + \phi = \\ -\frac{21}{2} + \frac{\sqrt{5}}{2} + \sum_{k=0}^{\infty} -\frac{4(-1)^k 3^{3/4} \times 1195^{-1-2k} \sqrt[4]{188461} (5^{1+2k} - 4 \times 239^{1+2k})}{1+2k} \end{aligned}$$

$$\begin{aligned} \pi \sqrt[4]{135^3 + 138^3} - 11 + \phi = \\ -\frac{21}{2} + \frac{\sqrt{5}}{2} + 3^{3/4} \sqrt[4]{188461} \sum_{k=0}^{\infty} \left(-\frac{1}{4}\right)^k \left(\frac{1}{1+2k} + \frac{2}{1+4k} + \frac{1}{3+4k}\right) \end{aligned}$$

**Integral representations:**

$$\pi \sqrt[4]{135^3 + 138^3} - 11 + \phi = -\frac{21}{2} + \frac{\sqrt{5}}{2} + 4 \times 3^{3/4} \sqrt[4]{188461} \int_0^1 \sqrt{1-t^2} dt$$

$$\pi \sqrt[4]{135^3 + 138^3} - 11 + \phi = -\frac{21}{2} + \frac{\sqrt{5}}{2} + 2 \times 3^{3/4} \sqrt[4]{188461} \int_0^1 \frac{1}{\sqrt{1-t^2}} dt$$

$$\pi \sqrt[4]{135^3 + 138^3} - 11 + \phi = -\frac{21}{2} + \frac{\sqrt{5}}{2} + 2 \times 3^{3/4} \sqrt[4]{188461} \int_0^\infty \frac{1}{1+t^2} dt$$

$\pi \cdot (135^3 + 138^3)^{1/4} - 29 + 4 + \text{golden ratio}$

**Input:**

$$\pi \sqrt[4]{135^3 + 138^3} - 29 + 4 + \phi$$

$\phi$  is the golden ratio

**Result:**

$$\phi - 25 + 3^{3/4} \sqrt[4]{188461} \pi$$

**Decimal approximation:**

125.8274348634976023813964821274141235673143122042902745354...

125.82743486... result very near to the dilaton mass calculated as a type of Higgs boson: 125 GeV for  $T = 0$  and to the Higgs boson mass 125.18 GeV

**Property:**

$-25 + \phi + 3^{3/4} \sqrt[4]{188461} \pi$  is a transcendental number

**Alternate forms:**

$$\frac{1}{2} \left( -49 + \sqrt{5} + 2 \times 3^{3/4} \sqrt[4]{188461} \pi \right)$$

$$-\frac{49}{2} + \frac{\sqrt{5}}{2} + 3^{3/4} \sqrt[4]{188461} \pi$$

$$-25 + \frac{1}{2} \left( 1 + \sqrt{5} \right) + 3^{3/4} \sqrt[4]{188461} \pi$$

### Alternative representations:

$$\pi \sqrt[4]{135^3 + 138^3} - 29 + 4 + \phi = -25 - 2 \cos(216^\circ) + \pi \sqrt[4]{135^3 + 138^3}$$

$$\pi \sqrt[4]{135^3 + 138^3} - 29 + 4 + \phi = -25 + 2 \cos\left(\frac{\pi}{5}\right) + \pi \sqrt[4]{135^3 + 138^3}$$

$$\pi \sqrt[4]{135^3 + 138^3} - 29 + 4 + \phi = -25 - 2 \cos(216^\circ) + 180^\circ \sqrt[4]{135^3 + 138^3}$$

### Series representations:

$$\pi \sqrt[4]{135^3 + 138^3} - 29 + 4 + \phi = -\frac{49}{2} + \frac{\sqrt{5}}{2} + 4 \times 3^{3/4} \sqrt[4]{188461} \sum_{k=0}^{\infty} \frac{(-1)^k}{1+2k}$$

$$\begin{aligned} \pi \sqrt[4]{135^3 + 138^3} - 29 + 4 + \phi = \\ -\frac{49}{2} + \frac{\sqrt{5}}{2} + \sum_{k=0}^{\infty} \frac{4(-1)^k 3^{3/4} \times 1195^{-1-2k} \sqrt[4]{188461} (5^{1+2k} - 4 \times 239^{1+2k})}{1+2k} \end{aligned}$$

$$\begin{aligned} \pi \sqrt[4]{135^3 + 138^3} - 29 + 4 + \phi = \\ -\frac{49}{2} + \frac{\sqrt{5}}{2} + 3^{3/4} \sqrt[4]{188461} \sum_{k=0}^{\infty} \left(-\frac{1}{4}\right)^k \left(\frac{1}{1+2k} + \frac{2}{1+4k} + \frac{1}{3+4k}\right) \end{aligned}$$

### Integral representations:

$$\pi \sqrt[4]{135^3 + 138^3} - 29 + 4 + \phi = -\frac{49}{2} + \frac{\sqrt{5}}{2} + 4 \times 3^{3/4} \sqrt[4]{188461} \int_0^1 \sqrt{1-t^2} dt$$

$$\pi \sqrt[4]{135^3 + 138^3} - 29 + 4 + \phi = -\frac{49}{2} + \frac{\sqrt{5}}{2} + 2 \times 3^{3/4} \sqrt[4]{188461} \int_0^1 \frac{1}{\sqrt{1-t^2}} dt$$

$$\pi \sqrt[4]{135^3 + 138^3} - 29 + 4 + \phi = -\frac{49}{2} + \frac{\sqrt{5}}{2} + 2 \times 3^{3/4} \sqrt[4]{188461} \int_0^{\infty} \frac{1}{1+t^2} dt$$

We have also:

$$(135^3+138^3)^{1/31}$$

**Input:**

$$\sqrt[31]{135^3 + 138^3}$$

**Result:**

$$3^{3/31} \sqrt[31]{188461}$$

**Decimal approximation:**

1.645665103021282483289882047076548993334552545217523451761...

$$1.645665103\dots \approx \zeta(2) = \frac{\pi^2}{6} = 1.644934 \dots$$

**Alternate form:**

root of $x^{31} - 5088447$ near $x = 1.64567$
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Now, we have:

$$f_R^* = \frac{4}{49\pi} \left[ \frac{80 \arctan(y^{1/2})}{(1+y)^2(3y+1)y^{1/2}} + \frac{3y^4 + 41y^3 + 25y^2 - 589y - 240}{3(1+y)^4(1+3y)} \right]$$

$$c \simeq 0.275/R^2.$$

$$y = cR^2$$

$$y = 0.275$$

$$\frac{4}{49\pi} \left( \frac{80 \operatorname{atan}(0.275^{1/2})}{(1+0.275)^2(3 \times 0.275+1)0.275^{1/2}} + \frac{3 \times 0.275^4 + 41 \times 0.275^3 + 25 \times 0.275^2 - 589 \times 0.275 - 240}{3(1+0.275)^4(1+3 \times 0.275)} \right)$$

**Input:**

$$\frac{4}{49\pi} \left( \frac{80 \tan^{-1}(\sqrt{0.275})}{(1+0.275)^2(3 \times 0.275+1)\sqrt{0.275}} + \frac{3 \times 0.275^4 + 41 \times 0.275^3 + 25 \times 0.275^2 - 589 \times 0.275 - 240}{3(1+0.275)^4(1+3 \times 0.275)} \right)$$

$\tan^{-1}(x)$  is the inverse tangent function

**Result:**

-0.0716284...

(result in radians)

-0.0716284...

**Alternative representations:**

$$\frac{\left( \frac{80 \tan^{-1}(\sqrt{0.275})}{(1+0.275)^2 (3 \times 0.275+1) \sqrt{0.275}} + \frac{3 \times 0.275^4 + 41 \times 0.275^3 + 25 \times 0.275^2 - 589 \times 0.275 - 240}{3 (1+0.275)^4 (1+3 \times 0.275)} \right) 4}{49 \pi} =$$

$$4 \left( \frac{80 \sec^{-1}(\sqrt{0.275})}{1.825 \sqrt{0.275} 1.275^2} + \frac{-401.975 + 25 \times 0.275^2 + 41 \times 0.275^3 + 3 \times 0.275^4}{5.475 \times 1.275^4} \right) / 49 \pi$$

$$\frac{\left( \frac{80 \tan^{-1}(\sqrt{0.275})}{(1+0.275)^2 (3 \times 0.275+1) \sqrt{0.275}} + \frac{3 \times 0.275^4 + 41 \times 0.275^3 + 25 \times 0.275^2 - 589 \times 0.275 - 240}{3 (1+0.275)^4 (1+3 \times 0.275)} \right) 4}{49 \pi} =$$

$$4 \left( \frac{80 \tan^{-1}(1, \sqrt{0.275})}{1.825 \sqrt{0.275} 1.275^2} + \frac{-401.975 + 25 \times 0.275^2 + 41 \times 0.275^3 + 3 \times 0.275^4}{5.475 \times 1.275^4} \right) / 49 \pi$$

$$\frac{\left( \frac{80 \tan^{-1}(\sqrt{0.275})}{(1+0.275)^2 (3 \times 0.275+1) \sqrt{0.275}} + \frac{3 \times 0.275^4 + 41 \times 0.275^3 + 25 \times 0.275^2 - 589 \times 0.275 - 240}{3 (1+0.275)^4 (1+3 \times 0.275)} \right) 4}{49 \pi} =$$

$$4 \left( \frac{80 \cot^{-1}\left(\frac{1}{\sqrt{0.275}}\right)}{1.825 \sqrt{0.275} 1.275^2} + \frac{-401.975 + 25 \times 0.275^2 + 41 \times 0.275^3 + 3 \times 0.275^4}{5.475 \times 1.275^4} \right) / 49 \pi$$

**Series representations:**

$$\frac{\left( \frac{80 \tan^{-1}(\sqrt{0.275})}{(1+0.275)^2 (3 \times 0.275+1) \sqrt{0.275}} + \frac{3 \times 0.275^4 + 41 \times 0.275^3 + 25 \times 0.275^2 - 589 \times 0.275 - 240}{3 (1+0.275)^4 (1+3 \times 0.275)} \right) 4}{49 \pi} =$$

$$-\frac{2.2524}{\pi} + \frac{4.19763 \sum_{k=0}^{\infty} \frac{(-1)^k 0.524404^{1+2k}}{1+2k}}{\pi}$$

$$\left( \frac{80 \tan^{-1}(\sqrt{0.275})}{(1+0.275)^2 (3 \times 0.275+1) \sqrt{0.275}} + \frac{3 \times 0.275^4 + 41 \times 0.275^3 + 25 \times 0.275^2 - 589 \times 0.275 - 240}{3 (1+0.275)^4 (1+3 \times 0.275)} \right) 4 =$$

$$-\frac{2.2524}{\pi} + \frac{4.19763 \sum_{k=0}^{\infty} \frac{\left(\frac{-1}{5}\right)^k 1.04881^{1+2k} F_{1+2k} \left(\frac{1}{1+\sqrt{1.22}}\right)^{1+2k}}{1+2k}}{\pi}$$

$$\left( \frac{80 \tan^{-1}(\sqrt{0.275})}{(1+0.275)^2 (3 \times 0.275+1) \sqrt{0.275}} + \frac{3 \times 0.275^4 + 41 \times 0.275^3 + 25 \times 0.275^2 - 589 \times 0.275 - 240}{3 (1+0.275)^4 (1+3 \times 0.275)} \right) 4 =$$

$$-\frac{2.2524}{\pi} + \frac{2.09882 i \log(2)}{\pi} - \frac{2.09882 i \log(-i(-0.524404 + i))}{\pi} -$$

$$\frac{2.09882 i \sum_{k=1}^{\infty} \frac{0.5^k (-i(-0.524404+i))^k}{k}}{\pi}$$

$F_n$  is the  $n^{\text{th}}$  Fibonacci number

$\log(x)$  is the natural logarithm

$i$  is the imaginary unit

### Integral representations:

$$\left( \frac{80 \tan^{-1}(\sqrt{0.275})}{(1+0.275)^2 (3 \times 0.275+1) \sqrt{0.275}} + \frac{3 \times 0.275^4 + 41 \times 0.275^3 + 25 \times 0.275^2 - 589 \times 0.275 - 240}{3 (1+0.275)^4 (1+3 \times 0.275)} \right) 4 =$$

$$-\frac{2.2524}{\pi} + \frac{2.20126}{\pi} \int_0^1 \frac{1}{1+0.275 t^2} dt$$

$$\left( \frac{80 \tan^{-1}(\sqrt{0.275})}{(1+0.275)^2 (3 \times 0.275+1) \sqrt{0.275}} + \frac{3 \times 0.275^4 + 41 \times 0.275^3 + 25 \times 0.275^2 - 589 \times 0.275 - 240}{3 (1+0.275)^4 (1+3 \times 0.275)} \right) 4 =$$

$$-\frac{2.2524}{\pi} - \frac{0.550314 i}{\pi^{5/2}} \int_{-i\infty+\gamma}^{i\infty+\gamma} e^{-0.242946 s} \Gamma\left(\frac{1}{2} - s\right) \Gamma(1-s) \Gamma(s)^2 ds \text{ for } 0 < \gamma < \frac{1}{2}$$

$$\left( \frac{80 \tan^{-1}(\sqrt{0.275})}{(1+0.275)^2 (3 \times 0.275+1) \sqrt{0.275}} + \frac{3 \times 0.275^4 + 41 \times 0.275^3 + 25 \times 0.275^2 - 589 \times 0.275 - 240}{3 (1+0.275)^4 (1+3 \times 0.275)} \right) 4 =$$

$$-\frac{2.2524}{\pi} + \frac{0.550314}{i \pi^2} \int_{-i\infty+\gamma}^{i\infty+\gamma} \frac{e^{1.29098 s} \Gamma\left(\frac{1}{2} - s\right) \Gamma(1-s) \Gamma(s)}{\Gamma\left(\frac{3}{2} - s\right)} ds \text{ for } 0 < \gamma < \frac{1}{2}$$



### Continued fraction representations:

$$\frac{\left( \frac{80 \tan^{-1}(\sqrt{0.275})}{(1+0.275)^2 (3 \times 0.275+1) \sqrt{0.275}} + \frac{3 \times 0.275^4 + 41 \times 0.275^3 + 25 \times 0.275^2 - 589 \times 0.275 - 240}{3(1+0.275)^4 (1+3 \times 0.275)} \right) 4}{49 \pi} =$$

$$\frac{4 \left( -27.5919 + \frac{26.9654}{1 + \sum_{k=1}^{\infty} \frac{0.275 k^2}{1+2k}} \right)}{49 \pi} = \frac{4 \left( -27.5919 + \frac{26.9654}{1 + \frac{0.275}{3 + \frac{1.1}{5 + \frac{2.475}{7 + \frac{4.4}{9 + \dots}}}}}} \right)}{49 \pi}$$

$$\frac{\left( \frac{80 \tan^{-1}(\sqrt{0.275})}{(1+0.275)^2 (3 \times 0.275+1) \sqrt{0.275}} + \frac{3 \times 0.275^4 + 41 \times 0.275^3 + 25 \times 0.275^2 - 589 \times 0.275 - 240}{3(1+0.275)^4 (1+3 \times 0.275)} \right) 4}{49 \pi} =$$

$$\frac{4 \left( -27.5919 + \frac{26.9654}{1 + \sum_{k=1}^{\infty} \frac{0.275 (1-2k)^2}{1.275 + 1.45k}} \right)}{49 \pi} = \frac{4 \left( -27.5919 + \frac{26.9654}{1 + \frac{0.275}{2.725 + \frac{2.475}{4.175 + \frac{6.875}{5.625 + \frac{13.475}{7.075 + \dots}}}}}} \right)}{49 \pi}$$

$$\frac{\left( \frac{80 \tan^{-1}(\sqrt{0.275})}{(1+0.275)^2 (3 \times 0.275+1) \sqrt{0.275}} + \frac{3 \times 0.275^4 + 41 \times 0.275^3 + 25 \times 0.275^2 - 589 \times 0.275 - 240}{3(1+0.275)^4 (1+3 \times 0.275)} \right) 4}{49 \pi} =$$

$$\frac{-0.0511429 - \frac{0.605346}{3 + \sum_{k=1}^{\infty} \frac{0.275 (1+(-1)^{1+k} + k)^2}{3+2k}}}{\pi} = \frac{-0.0511429 - \frac{0.605346}{3 + \frac{2.475}{5 + \frac{1.1}{7 + \frac{6.875}{9 + \frac{4.4}{11 + \dots}}}}}}{\pi}$$

$$\frac{\left( \frac{80 \tan^{-1}(\sqrt{0.275})}{(1+0.275)^2 (3 \times 0.275 + 1) \sqrt{0.275}} + \frac{3 \times 0.275^4 + 41 \times 0.275^3 + 25 \times 0.275^2 - 589 \times 0.275 - 240}{3 (1+0.275)^4 (1+3 \times 0.275)} \right) 4}{49 \pi} =$$

$$4 \left( -27.5919 + \frac{26.9654}{1.275 + \underset{k=1}{\overset{\infty}{\text{K}}} \frac{0.55 \left( 1 - 2 \left| \frac{1+k}{2} \right| \right) \left| \frac{1+k}{2} \right|}{(1.1375 + 0.1375 (-1)^k) (1+2k)}} \right) / 49 \pi =$$

$$4 \left( -27.5919 + \frac{26.9654}{1.275 + \frac{0.55}{3 - \frac{0.55}{6.375 - \frac{3.3}{7 - \frac{3.3}{11.475 + \dots}}}}} \right) / 49 \pi$$

$\underset{k=k_1}{\overset{k_2}{\text{K}}} a_k / b_k$  is a continued fraction

-128/((((4/(49Pi))\*(((80  
atan(0.275^1/2))))/(((1+0.275)^2(3\*0.275+1)0.275^1/2)))+(3\*0.275^4+41\*0.275^3+2  
5\*0.275^2-589\*0.275-240)/(3(1+0.275)^4(1+3\*0.275)))))))-55-3

**Input:**

$$-\frac{128}{49 \pi \left( \frac{80 \tan^{-1}(\sqrt{0.275})}{(1+0.275)^2 (3 \times 0.275 + 1) \sqrt{0.275}} + \frac{3 \times 0.275^4 + 41 \times 0.275^3 + 25 \times 0.275^2 - 589 \times 0.275 - 240}{3 (1+0.275)^4 (1+3 \times 0.275)} \right)} - 55 - 3$$

$\tan^{-1}(x)$  is the inverse tangent function

**Result:**

1729.00...  
(result in radians)

1729

This result is very near to the mass of candidate glueball  $f_0(1710)$  meson. Furthermore, 1728 occurs in the algebraic formula for the j-invariant of an elliptic curve. As a consequence, it is sometimes called a Zagier as a pun on the Gross-

Zagier theorem. The number 1728 is one less than the Hardy–Ramanujan number 1729

**Alternative representations:**

$$\begin{aligned}
 & - \frac{128}{\left( \frac{80 \tan^{-1}(\sqrt{0.275})}{(1+0.275)^2 (3 \times 0.275 + 1) \sqrt{0.275}} + \frac{3 \times 0.275^4 + 41 \times 0.275^3 + 25 \times 0.275^2 - 589 \times 0.275 - 240}{3(1+0.275)^4 (1+3 \times 0.275)} \right)^4} - 55 - 3 = \\
 & -58 - \frac{\frac{49\pi}{128}}{4 \left( \frac{80 \operatorname{sc}^{-1}(\sqrt{0.275} | 0)}{1.825 \sqrt{0.275} \cdot 1.275^2} + \frac{-401.975 + 25 \times 0.275^2 + 41 \times 0.275^3 + 3 \times 0.275^4}{5.475 \times 1.275^4} \right)}
 \end{aligned}$$

$$\begin{aligned}
 & - \frac{128}{\left( \frac{80 \tan^{-1}(\sqrt{0.275})}{(1+0.275)^2 (3 \times 0.275 + 1) \sqrt{0.275}} + \frac{3 \times 0.275^4 + 41 \times 0.275^3 + 25 \times 0.275^2 - 589 \times 0.275 - 240}{3(1+0.275)^4 (1+3 \times 0.275)} \right)^4} - 55 - 3 = \\
 & -58 - \frac{\frac{49\pi}{128}}{4 \left( \frac{80 \tan^{-1}(1, \sqrt{0.275})}{1.825 \sqrt{0.275} \cdot 1.275^2} + \frac{-401.975 + 25 \times 0.275^2 + 41 \times 0.275^3 + 3 \times 0.275^4}{5.475 \times 1.275^4} \right)}
 \end{aligned}$$

$$\begin{aligned}
 & - \frac{128}{\left( \frac{80 \tan^{-1}(\sqrt{0.275})}{(1+0.275)^2 (3 \times 0.275 + 1) \sqrt{0.275}} + \frac{3 \times 0.275^4 + 41 \times 0.275^3 + 25 \times 0.275^2 - 589 \times 0.275 - 240}{3(1+0.275)^4 (1+3 \times 0.275)} \right)^4} - 55 - 3 = \\
 & -58 - \frac{\frac{49\pi}{128}}{4 \left( \frac{80 \cot^{-1}\left(\frac{1}{\sqrt{0.275}}\right)}{1.825 \sqrt{0.275} \cdot 1.275^2} + \frac{-401.975 + 25 \times 0.275^2 + 41 \times 0.275^3 + 3 \times 0.275^4}{5.475 \times 1.275^4} \right)}
 \end{aligned}$$

**Series representations:**

$$\begin{aligned}
 & - \frac{128}{\left( \frac{80 \tan^{-1}(\sqrt{0.275})}{(1+0.275)^2 (3 \times 0.275 + 1) \sqrt{0.275}} + \frac{3 \times 0.275^4 + 41 \times 0.275^3 + 25 \times 0.275^2 - 589 \times 0.275 - 240}{3(1+0.275)^4 (1+3 \times 0.275)} \right)^4} - 55 - 3 = \\
 & -58 - \frac{30.4934 \pi}{-0.536588 + \sum_{k=0}^{\infty} \frac{(-1)^k 0.524404^{1+2k}}{1+2k}}
 \end{aligned}$$

$$\begin{aligned}
& \frac{128}{\left( \frac{80 \tan^{-1}(\sqrt{0.275})}{(1+0.275)^2 (3 \times 0.275+1) \sqrt{0.275}} + \frac{3 \times 0.275^4 + 41 \times 0.275^3 + 25 \times 0.275^2 - 589 \times 0.275 - 240}{3(1+0.275)^4 (1+3 \times 0.275)} \right)^4} - 55 - 3 = \\
& -58 - \frac{30.4934 \pi}{-0.536588 + \sum_{k=0}^{\infty} \frac{\left(-\frac{1}{5}\right)^k 1.04881^{1+2k} F_{1+2k} \left(\frac{1}{1+\sqrt{1.22}}\right)^{1+2k}}{1+2k}}
\end{aligned}$$

$$\begin{aligned}
& \frac{128}{\left( \frac{80 \tan^{-1}(\sqrt{0.275})}{(1+0.275)^2 (3 \times 0.275+1) \sqrt{0.275}} + \frac{3 \times 0.275^4 + 41 \times 0.275^3 + 25 \times 0.275^2 - 589 \times 0.275 - 240}{3(1+0.275)^4 (1+3 \times 0.275)} \right)^4} - 55 - 3 = \\
& \frac{49\pi}{-58 + (30.4934 \pi) / \left( 0.536588 - \tan^{-1}(x) + \pi \left[ \frac{\arg(i(-0.524404 + x))}{2\pi} \right] - \right.} \\
& \left. 0.5 i \sum_{k=1}^{\infty} \frac{\left(-(-i-x)^{-k} + (i-x)^{-k}\right) (0.524404 - x)^k}{k} \right) \text{ for } (ix \in \mathbb{R} \text{ and } ix > 1)
\end{aligned}$$

$F_n$  is the  $n^{\text{th}}$  Fibonacci number

$\arg(z)$  is the complex argument

$[x]$  is the floor function

$i$  is the imaginary unit

$\mathbb{R}$  is the set of real numbers

## Integral representations:

$$\begin{aligned}
& \frac{128}{\left( \frac{80 \tan^{-1}(\sqrt{0.275})}{(1+0.275)^2 (3 \times 0.275+1) \sqrt{0.275}} + \frac{3 \times 0.275^4 + 41 \times 0.275^3 + 25 \times 0.275^2 - 589 \times 0.275 - 240}{3(1+0.275)^4 (1+3 \times 0.275)} \right)^4} - 55 - 3 = \\
& -58 + \frac{58.1486 \pi}{1.02323 - \int_0^1 \frac{1}{1+0.275 t^2} dt}
\end{aligned}$$

$$\begin{aligned}
& \frac{128}{\left( \frac{80 \tan^{-1}(\sqrt{0.275})}{(1+0.275)^2 (3 \times 0.275+1) \sqrt{0.275}} + \frac{3 \times 0.275^4 + 41 \times 0.275^3 + 25 \times 0.275^2 - 589 \times 0.275 - 240}{3(1+0.275)^4 (1+3 \times 0.275)} \right)^4} - 55 - 3 = \\
& -58 + \frac{49\pi}{27.5919 + \frac{6.74135 i}{\pi^{3/2}} \int_{-i\infty+\gamma}^{i\infty+\gamma} e^{-0.242946 s} \Gamma\left(\frac{1}{2} - s\right) \Gamma(1-s) \Gamma(s)^2 ds} \text{ for } 0 < \gamma < \frac{1}{2}
\end{aligned}$$

$$\begin{aligned}
& - \frac{128}{\left( \frac{80 \tan^{-1}(\sqrt{0.275})}{(1+0.275)^2 (3 \times 0.275 + 1) \sqrt{0.275}} + \frac{3 \times 0.275^4 + 41 \times 0.275^3 + 25 \times 0.275^2 - 589 \times 0.275 - 240}{3(1+0.275)^4 (1+3 \times 0.275)} \right)^4} - 55 - 3 = \\
& -58 + \frac{56.8283 i \pi^2}{i \pi - 0.244324 \int_{-i \infty + \gamma}^{i \infty + \gamma} \frac{e^{1.29098 s} \Gamma\left(\frac{1}{2} - s\right) \Gamma(1-s) \Gamma(s)}{\Gamma\left(\frac{3}{2} - s\right)} ds} \quad \text{for } 0 < \gamma < \frac{1}{2}
\end{aligned}$$

**Continued fraction representations:**

$$\begin{aligned}
& - \frac{128}{\left( \frac{80 \tan^{-1}(\sqrt{0.275})}{(1+0.275)^2 (3 \times 0.275 + 1) \sqrt{0.275}} + \frac{3 \times 0.275^4 + 41 \times 0.275^3 + 25 \times 0.275^2 - 589 \times 0.275 - 240}{3(1+0.275)^4 (1+3 \times 0.275)} \right)^4} - 55 - 3 = \\
& -58 + \frac{1568 \pi}{27.5919 - \frac{26.9654}{1 + \mathop{\text{K}}_{k=1}^{\infty} \frac{0.275 k^2}{1+2k}}} = -58 + \frac{1568 \pi}{27.5919 - \frac{26.9654}{1 + \frac{0.275}{3 + \frac{1.1}{5 + \frac{2.475}{7 + \frac{4.4}{9 + \dots}}}}}
\end{aligned}$$

$$\begin{aligned}
& - \frac{128}{\left( \frac{80 \tan^{-1}(\sqrt{0.275})}{(1+0.275)^2 (3 \times 0.275 + 1) \sqrt{0.275}} + \frac{3 \times 0.275^4 + 41 \times 0.275^3 + 25 \times 0.275^2 - 589 \times 0.275 - 240}{3(1+0.275)^4 (1+3 \times 0.275)} \right)^4} - 55 - 3 = \\
& -58 - \frac{1568 \pi}{-27.5919 + \frac{26.9654}{1 + \mathop{\text{K}}_{k=1}^{\infty} \frac{0.275 (1-2k)^2}{1.275 + 1.45k}}} = \\
& -58 - \frac{1568 \pi}{-27.5919 + \frac{26.9654}{1 + \frac{0.275}{2.725 + \frac{2.475}{4.175 + \frac{6.875}{5.625 + \frac{13.475}{7.075 + \dots}}}}}
\end{aligned}$$

$$\begin{aligned}
& - \frac{128}{\left( \frac{80 \tan^{-1}(\sqrt{0.275})}{(1+0.275)^2 (3 \times 0.275 + 1) \sqrt{0.275}} + \frac{3 \times 0.275^4 + 41 \times 0.275^3 + 25 \times 0.275^2 - 589 \times 0.275 - 240}{3(1+0.275)^4 (1+3 \times 0.275)} \right)^4} - 55 - 3 = \\
& -58 - \frac{1568 \pi}{-0.6265 - \frac{7.41548}{3 + \mathop{\text{K}}_{k=1}^{\infty} \frac{0.275 (1+(-1)^{1+k} + k)^2}{3+2k}}} = -58 - \frac{1568 \pi}{-0.6265 - \frac{7.41548}{3 + \frac{2.475}{5 + \frac{1.1}{7 + \frac{6.875}{9 + \frac{4.4}{11 + \dots}}}}}
\end{aligned}$$

$$\begin{aligned}
& - \frac{128}{\left( \frac{80 \tan^{-1}(\sqrt{0.275})}{(1+0.275)^2 (3 \times 0.275 + 1) \sqrt{0.275}} + \frac{3 \times 0.275^4 + 41 \times 0.275^3 + 25 \times 0.275^2 - 589 \times 0.275 - 240}{3(1+0.275)^4 (1+3 \times 0.275)} \right)^4} - 55 - 3 = \\
& -58 - \frac{1568 \pi^{49}}{-27.5919 + \frac{26.9654}{1.275 + \sum_{k=1}^{\infty} \frac{0.55 \left( 1 - 2 \left| \frac{1+k}{2} \right| \right) \left| \frac{1+k}{2} \right|}{\left( 1.1375 + 0.1375 (-1)^k \right) (1+2k)}}} = \\
& -58 - \frac{1568 \pi}{-27.5919 + \frac{26.9654}{1.275 + \frac{0.55}{3 - \frac{0.55}{6.375 - \frac{3.3}{7 - \frac{3.3}{11.475 + \dots}}}}}
\end{aligned}$$

$\sum_{k=k_1}^{k_2} a_k / b_k$  is a continued fraction

$$-128 / \left( \left( \left( \left( \frac{4}{49\pi} \right) \left( \left( \left( \frac{80 \tan^{-1}(\sqrt{0.275})}{(1+0.275)^2 (3 \times 0.275 + 1) \sqrt{0.275}} + \frac{3 \times 0.275^4 + 41 \times 0.275^3 + 25 \times 0.275^2 - 589 \times 0.275 - 240}{3(1+0.275)^4 (1+3 \times 0.275)} \right) \right)^4 \right) - 55 - 3 \right) \right) - \pi$$

**Input:**

$$- \frac{128}{\frac{4}{49\pi} \left( \frac{80 \tan^{-1}(\sqrt{0.275})}{(1+0.275)^2 (3 \times 0.275 + 1) \sqrt{0.275}} + \frac{3 \times 0.275^4 + 41 \times 0.275^3 + 25 \times 0.275^2 - 589 \times 0.275 - 240}{3(1+0.275)^4 (1+3 \times 0.275)} \right)^4} - \pi$$

$\tan^{-1}(x)$  is the inverse tangent function

**Result:**

1783.86...

(result in radians)

1783.86... result in the range of the hypothetical mass of Gluino (gluino = 1785.16 GeV).

### Alternative representations:

$$\frac{128}{\left( \frac{80 \tan^{-1}(\sqrt{0.275})}{(1+0.275)^2 (3 \times 0.275+1) \sqrt{0.275}} + \frac{3 \times 0.275^4 + 41 \times 0.275^3 + 25 \times 0.275^2 - 589 \times 0.275 - 240}{3(1+0.275)^4 (1+3 \times 0.275)} \right)^4} - \pi =$$

$$\frac{40\pi}{128}$$

$$-\pi - \frac{4 \left( \frac{80 \sec^{-1}(\sqrt{0.275})}{1.825 \sqrt{0.275} \cdot 1.275^2} + \frac{-401.975 + 25 \times 0.275^2 + 41 \times 0.275^3 + 3 \times 0.275^4}{5.475 \times 1.275^4} \right)}{49\pi}$$

$$\frac{128}{\left( \frac{80 \tan^{-1}(\sqrt{0.275})}{(1+0.275)^2 (3 \times 0.275+1) \sqrt{0.275}} + \frac{3 \times 0.275^4 + 41 \times 0.275^3 + 25 \times 0.275^2 - 589 \times 0.275 - 240}{3(1+0.275)^4 (1+3 \times 0.275)} \right)^4} - \pi =$$

$$\frac{40\pi}{128}$$

$$-\pi - \frac{4 \left( \frac{80 \tan^{-1}(1, \sqrt{0.275})}{1.825 \sqrt{0.275} \cdot 1.275^2} + \frac{-401.975 + 25 \times 0.275^2 + 41 \times 0.275^3 + 3 \times 0.275^4}{5.475 \times 1.275^4} \right)}{49\pi}$$

$$\frac{128}{\left( \frac{80 \tan^{-1}(\sqrt{0.275})}{(1+0.275)^2 (3 \times 0.275+1) \sqrt{0.275}} + \frac{3 \times 0.275^4 + 41 \times 0.275^3 + 25 \times 0.275^2 - 589 \times 0.275 - 240}{3(1+0.275)^4 (1+3 \times 0.275)} \right)^4} - \pi =$$

$$\frac{40\pi}{128}$$

$$-\pi - \frac{4 \left( \frac{80 \cot^{-1}\left(\frac{1}{\sqrt{0.275}}\right)}{1.825 \sqrt{0.275} \cdot 1.275^2} + \frac{-401.975 + 25 \times 0.275^2 + 41 \times 0.275^3 + 3 \times 0.275^4}{5.475 \times 1.275^4} \right)}{49\pi}$$

### Series representations:

$$\frac{128}{\left( \frac{80 \tan^{-1}(\sqrt{0.275})}{(1+0.275)^2 (3 \times 0.275+1) \sqrt{0.275}} + \frac{3 \times 0.275^4 + 41 \times 0.275^3 + 25 \times 0.275^2 - 589 \times 0.275 - 240}{3(1+0.275)^4 (1+3 \times 0.275)} \right)^4} - \pi =$$

$$\frac{49\pi}{30.4934\pi}$$

$$-\pi - \frac{-0.536588 + \sum_{k=0}^{\infty} \frac{(-1)^k 0.524404^{1+2k}}{1+2k}}{49\pi}$$

$$\frac{128}{\left( \frac{80 \tan^{-1}(\sqrt{0.275})}{(1+0.275)^2 (3 \times 0.275+1) \sqrt{0.275}} + \frac{3 \times 0.275^4 + 41 \times 0.275^3 + 25 \times 0.275^2 - 589 \times 0.275 - 240}{3(1+0.275)^4 (1+3 \times 0.275)} \right)^4} - \pi =$$

$$\frac{49\pi}{60.9868\pi}$$

$$-\pi - \frac{-1.07318 + i \log(2) - i \log(-((-0.524404 + i) i)) - i \sum_{k=1}^{\infty} \frac{0.5^k (-((-0.524404 + i) i))^k}{k}}{49\pi}$$

$$\begin{aligned}
& \frac{128}{\left( \frac{80 \tan^{-1}(\sqrt{0.275})}{(1+0.275)^2 (3 \times 0.275+1) \sqrt{0.275}} + \frac{3 \times 0.275^4 + 41 \times 0.275^3 + 25 \times 0.275^2 - 589 \times 0.275 - 240}{3(1+0.275)^4 (1+3 \times 0.275)} \right)^4} - \pi = \\
& \frac{49\pi}{60.9868 \pi} \\
& -\pi + \frac{1.07318 + i \log(2) - i \log(-i(0.524404 + i)) - i \sum_{k=1}^{\infty} \frac{0.5^k (-i(0.524404 + i))^k}{k}}{1}
\end{aligned}$$

### Integral representations:

$$\begin{aligned}
& \frac{128}{\left( \frac{80 \tan^{-1}(\sqrt{0.275})}{(1+0.275)^2 (3 \times 0.275+1) \sqrt{0.275}} + \frac{3 \times 0.275^4 + 41 \times 0.275^3 + 25 \times 0.275^2 - 589 \times 0.275 - 240}{3(1+0.275)^4 (1+3 \times 0.275)} \right)^4} - \pi = \\
& \frac{49\pi}{58.1486 \pi} \\
& -\pi - \frac{1.02323 + \int_0^1 \frac{1}{1+0.275t^2} dt}{1}
\end{aligned}$$

$$\begin{aligned}
& \frac{128}{\left( \frac{80 \tan^{-1}(\sqrt{0.275})}{(1+0.275)^2 (3 \times 0.275+1) \sqrt{0.275}} + \frac{3 \times 0.275^4 + 41 \times 0.275^3 + 25 \times 0.275^2 - 589 \times 0.275 - 240}{3(1+0.275)^4 (1+3 \times 0.275)} \right)^4} - \pi = \\
& \frac{49\pi}{1568 \pi} \\
& -\pi - \frac{27.5919 - \frac{6.74135i}{\pi^{3/2}} \int_{-i\infty+\gamma}^{i\infty+\gamma} 1.275^{-s} \Gamma\left(\frac{1}{2} - s\right) \Gamma(1-s) \Gamma(s)^2 ds}{1} \text{ for } 0 < \gamma < \frac{1}{2}
\end{aligned}$$

$$\begin{aligned}
& \frac{128}{\left( \frac{80 \tan^{-1}(\sqrt{0.275})}{(1+0.275)^2 (3 \times 0.275+1) \sqrt{0.275}} + \frac{3 \times 0.275^4 + 41 \times 0.275^3 + 25 \times 0.275^2 - 589 \times 0.275 - 240}{3(1+0.275)^4 (1+3 \times 0.275)} \right)^4} - \pi = \\
& \frac{49\pi}{56.8283 i \pi^2} \\
& -\pi + \frac{0.244324 \int_{-i\infty+\gamma}^{i\infty+\gamma} \frac{e^{1.29098s} \Gamma\left(\frac{1}{2}-s\right) \Gamma(1-s) \Gamma(s)}{\Gamma\left(\frac{3}{2}-s\right)} ds}{i \pi} \text{ for } 0 < \gamma < \frac{1}{2}
\end{aligned}$$

### Continued fraction representations:

$$\begin{aligned}
& \frac{128}{\left( \frac{80 \tan^{-1}(\sqrt{0.275})}{(1+0.275)^2 (3 \times 0.275+1) \sqrt{0.275}} + \frac{3 \times 0.275^4 + 41 \times 0.275^3 + 25 \times 0.275^2 - 589 \times 0.275 - 240}{3(1+0.275)^4 (1+3 \times 0.275)} \right)^4} - \pi = \\
& \frac{49\pi}{\pi} \left( -1 + \frac{1568}{27.5919 - \frac{26.9654}{1 + \frac{\infty}{k=1} \frac{0.275 k^2}{1+2k}}} \right) = \pi \left( -1 + \frac{1568}{27.5919 - \frac{26.9654}{1 + \frac{0.275}{3 + \frac{1.1}{5 + \frac{2.475}{7 + \frac{4.4}{9 + \dots}}}}} \right)
\end{aligned}$$



128

$$\begin{aligned}
 & \frac{80 \tan^{-1}(\sqrt{0.275})}{(1+0.275)^2 (3 \times 0.275+1) \sqrt{0.275}} + \frac{3 \times 0.275^4 + 41 \times 0.275^3 + 25 \times 0.275^2 - 589 \times 0.275 - 240}{3(1+0.275)^4 (1+3 \times 0.275)} \Bigg)^4 \quad -\pi = \\
 & \pi \left( -1 - \frac{1568}{-27.5919 + \frac{26.9654}{1 + \sum_{k=1}^{\infty} \frac{0.275(1-2k)^2}{1.275+1.45k}}} \right) = \\
 & \pi \left( -1 - \frac{1568}{-27.5919 + \frac{26.9654}{1 + \frac{2.725 + \frac{2.475}{4.175 + \frac{6.875}{5.625 + \frac{13.475}{7.075 + \dots}}}}}} \right)
 \end{aligned}$$

128

$$\begin{aligned}
 & \frac{80 \tan^{-1}(\sqrt{0.275})}{(1+0.275)^2 (3 \times 0.275+1) \sqrt{0.275}} + \frac{3 \times 0.275^4 + 41 \times 0.275^3 + 25 \times 0.275^2 - 589 \times 0.275 - 240}{3(1+0.275)^4 (1+3 \times 0.275)} \Bigg)^4 \quad -\pi = \\
 & \pi \left( -1 - \frac{1568}{-0.6265 - \frac{7.41548}{3 + \sum_{k=1}^{\infty} \frac{0.275(1+(-1)^{1+k+k})^2}{3+2k}}} \right) = \pi \left( -1 - \frac{1568}{-0.6265 - \frac{7.41548}{3 + \frac{2.475}{5 + \frac{1.1}{7 + \frac{6.875}{9 + \frac{4.4}{11 + \dots}}}}} \right)
 \end{aligned}$$

128

$$\begin{aligned}
 & \frac{80 \tan^{-1}(\sqrt{0.275})}{(1+0.275)^2 (3 \times 0.275+1) \sqrt{0.275}} + \frac{3 \times 0.275^4 + 41 \times 0.275^3 + 25 \times 0.275^2 - 589 \times 0.275 - 240}{3(1+0.275)^4 (1+3 \times 0.275)} \Bigg)^4 \quad -\pi = \\
 & \pi \left( -1 - \frac{1568}{-27.5919 + \frac{26.9654}{1.275 + \sum_{k=1}^{\infty} \frac{0.55(1-2|\frac{1+k}{2}|)|\frac{1+k}{2}|}{(1.1375+0.1375(-1)^k)(1+2k)}}} \right) = \\
 & \pi \left( -1 - \frac{1568}{-27.5919 + \frac{26.9654}{1.275 + \frac{0.55}{3 - \frac{0.55}{6.375 - \frac{3.3}{7 - \frac{3.3}{11.475 + \dots}}}}} \right)
 \end{aligned}$$

$\prod_{k=k_1}^{k_2} a_k/b_k$  is a continued fraction

$$-55/\left(\left(\left(\left(4/(49\pi)\right)\left(\left(80 \operatorname{atan}(0.275^{1/2})\right)\left(\left(\left(1+0.275\right)^2(3 \times 0.275+1)0.275^{1/2}\right)\right)+\left(3 \times 0.275^4+41 \times 0.275^3+25 \times 0.275^2-589 \times 0.275-240\right)\right)\right)\right)/\left(3(1+0.275)^4(1+3 \times 0.275)\right)\right)\right)-34-\pi-\sqrt{7}$$

**Input:**

$$\frac{55}{49\pi \left( \frac{80 \tan^{-1}(\sqrt{0.275})}{(1+0.275)^2 (3 \times 0.275+1) \sqrt{0.275}} + \frac{3 \times 0.275^4 + 41 \times 0.275^3 + 25 \times 0.275^2 - 589 \times 0.275 - 240}{3(1+0.275)^4 (1+3 \times 0.275)} \right) - 34 - \pi - \sqrt{7}}$$

$\tan^{-1}(x)$  is the inverse tangent function

**Result:**

728.064...

(result in radians)

728.064...  $\approx$  728 (Ramanujan taxicab number)

**Alternative representations:**

$$\frac{55}{49\pi \left( \frac{80 \tan^{-1}(\sqrt{0.275})}{(1+0.275)^2 (3 \times 0.275+1) \sqrt{0.275}} + \frac{3 \times 0.275^4 + 41 \times 0.275^3 + 25 \times 0.275^2 - 589 \times 0.275 - 240}{3(1+0.275)^4 (1+3 \times 0.275)} \right) - 34 - \pi - \sqrt{7}}$$

$$\sqrt{7} = -34 - \pi - \frac{55}{49\pi \left( \frac{80 \operatorname{sc}^{-1}(\sqrt{0.275} | 0)}{1.825 \sqrt{0.275} 1.275^2} + \frac{-401.975 + 25 \times 0.275^2 + 41 \times 0.275^3 + 3 \times 0.275^4}{5.475 \times 1.275^4} \right) - \sqrt{7}}$$

$$\frac{55}{49\pi \left( \frac{80 \tan^{-1}(\sqrt{0.275})}{(1+0.275)^2 (3 \times 0.275+1) \sqrt{0.275}} + \frac{3 \times 0.275^4 + 41 \times 0.275^3 + 25 \times 0.275^2 - 589 \times 0.275 - 240}{3(1+0.275)^4 (1+3 \times 0.275)} \right) - 34 - \pi - \sqrt{7}}$$

$$\sqrt{7} = -34 - \pi - \frac{55}{49\pi \left( \frac{80 \tan^{-1}(1, \sqrt{0.275})}{1.825 \sqrt{0.275} 1.275^2} + \frac{-401.975 + 25 \times 0.275^2 + 41 \times 0.275^3 + 3 \times 0.275^4}{5.475 \times 1.275^4} \right) - \sqrt{7}}$$

$$\begin{aligned}
& - \frac{55}{\left( \frac{80 \tan^{-1}(\sqrt{0.275})}{(1+0.275)^2 (3 \times 0.275+1) \sqrt{0.275}} + \frac{3 \times 0.275^4 + 41 \times 0.275^3 + 25 \times 0.275^2 - 589 \times 0.275 - 240}{3(1+0.275)^4 (1+3 \times 0.275)} \right)^4} - 34 - \pi - \\
& \sqrt{7} = -34 - \pi - \frac{49\pi}{55} \frac{55}{4 \left( \frac{80 \cot^{-1}\left(\frac{1}{\sqrt{0.275}}\right)}{1.825 \sqrt{0.275} \cdot 1.275^2} + \frac{-401.975 + 25 \times 0.275^2 + 41 \times 0.275^3 + 3 \times 0.275^4}{5.475 \times 1.275^4} \right)} - \sqrt{7}
\end{aligned}$$

**Series representations:**

$$\begin{aligned}
& - \frac{55}{\left( \frac{80 \tan^{-1}(\sqrt{0.275})}{(1+0.275)^2 (3 \times 0.275+1) \sqrt{0.275}} + \frac{3 \times 0.275^4 + 41 \times 0.275^3 + 25 \times 0.275^2 - 589 \times 0.275 - 240}{3(1+0.275)^4 (1+3 \times 0.275)} \right)^4} - \\
& 34 - \pi - \sqrt{7} = -34 - \pi + \frac{49\pi}{0.536588 - \tan^{-1}(0.524404)} \frac{13.1026 \pi}{-} \\
& \exp\left(i\pi \left\lfloor \frac{\arg(7-x)}{2\pi} \right\rfloor\right) \sqrt{x} \sum_{k=0}^{\infty} \frac{(-1)^k (7-x)^k x^{-k} \left(-\frac{1}{2}\right)_k}{k!} \text{ for } (x \in \mathbb{R} \text{ and } x < 0)
\end{aligned}$$

$$\begin{aligned}
& \frac{\left( \frac{80 \tan^{-1}(\sqrt{0.275})}{(1+0.275)^2 (3 \times 0.275 + 1) \sqrt{0.275}} + \frac{3 \times 0.275^4 + 41 \times 0.275^3 + 25 \times 0.275^2 - 589 \times 0.275 - 240}{3(1+0.275)^4 (1+3 \times 0.275)} \right)^4}{49\pi} \\
34 - \pi - \sqrt{7} &= - \left( \left( -18.244 + 12.566 \pi - 0.536588 \sqrt{6} \sum_{k=0}^{\infty} 6^{-k} \binom{\frac{1}{2}}{k} + \right. \right. \\
& 34 \sum_{k=0}^{\infty} \frac{\left(-\frac{1}{5}\right)^k 1.04881^{1+2k} F_{1+2k} \left(\frac{1}{1+\sqrt{1.22}}\right)^{1+2k}}{1+2k} + \\
& \pi \sum_{k=0}^{\infty} \frac{\left(-\frac{1}{5}\right)^k 1.04881^{1+2k} F_{1+2k} \left(\frac{1}{1+\sqrt{1.22}}\right)^{1+2k}}{1+2k} + \\
& \left. \left. \sqrt{6} \sum_{k_1=0}^{\infty} \sum_{k_2=0}^{\infty} \frac{\left(-\frac{1}{5}\right)^{k_2} 1.04881^{1+2k_2} \times 6^{-k_1} \binom{\frac{1}{2}}{k_1} F_{1+2k_2} \left(\frac{1}{1+\sqrt{1.22}}\right)^{1+2k_2}}{1+2k_2} \right) \right) / \\
& \left( -0.536588 + \sum_{k=0}^{\infty} \frac{\left(-\frac{1}{5}\right)^k 1.04881^{1+2k} F_{1+2k} \left(\frac{1}{1+\sqrt{1.22}}\right)^{1+2k}}{1+2k} \right)
\end{aligned}$$

$$\begin{aligned}
& \frac{\left( \frac{80 \tan^{-1}(\sqrt{0.275})}{(1+0.275)^2 (3 \times 0.275 + 1) \sqrt{0.275}} + \frac{3 \times 0.275^4 + 41 \times 0.275^3 + 25 \times 0.275^2 - 589 \times 0.275 - 240}{3(1+0.275)^4 (1+3 \times 0.275)} \right)^4}{49\pi} \\
34 - \pi - \sqrt{7} &= \\
-34 - \pi - (13.1026 \pi) &/ \left( -0.536588 + \tan^{-1}(x) - \pi \left[ \frac{\arg(i(-0.524404 + x))}{2\pi} \right] + \right. \\
& \left. 0.5 i \sum_{k=1}^{\infty} \frac{\left(-(-i-x)^{-k} + (i-x)^{-k}\right) (0.524404 - x)^k}{k} \right) - \\
& \exp\left(i\pi \left[ \frac{\arg(7-x)}{2\pi} \right]\right) \sqrt{x} \sum_{k=0}^{\infty} \frac{(-1)^k (7-x)^k x^{-k} \left(-\frac{1}{2}\right)_k}{k!}
\end{aligned}$$

for ( $i x \in \mathbb{R}$  and  $i x > 1$  and  $x \in \mathbb{R}$  and  $x < 0$ )

**Integral representations:**

55

$$\frac{\left( \frac{80 \tan^{-1}(\sqrt{0.275})}{(1+0.275)^2 (3 \times 0.275+1) \sqrt{0.275}} + \frac{3 \times 0.275^4 + 41 \times 0.275^3 + 25 \times 0.275^2 - 589 \times 0.275 - 240}{3 (1+0.275)^4 (1+3 \times 0.275)} \right)^4}{49\pi}$$

$$34 - \pi - \sqrt{7} = -34 - \pi + \frac{24.9857\pi}{1.02323 - \int_0^1 \frac{1}{1+0.275t^2} dt} - \sqrt{7}$$

55

$$\frac{\left( \frac{80 \tan^{-1}(\sqrt{0.275})}{(1+0.275)^2 (3 \times 0.275+1) \sqrt{0.275}} + \frac{3 \times 0.275^4 + 41 \times 0.275^3 + 25 \times 0.275^2 - 589 \times 0.275 - 240}{3 (1+0.275)^4 (1+3 \times 0.275)} \right)^4}{49\pi}$$

$$34 - \pi - \sqrt{7} = -34 - \pi + \frac{24.4184\pi^{5/2}}{\pi^{3/2} + 0.244324 i \int_{-i\infty+\gamma}^{i\infty+\gamma} e^{-0.242946s} \Gamma\left(\frac{1}{2}-s\right) \Gamma(1-s) \Gamma(s)^2 ds} - \sqrt{7} \text{ for } 0 < \gamma < \frac{1}{2}$$

55

$$\frac{\left( \frac{80 \tan^{-1}(\sqrt{0.275})}{(1+0.275)^2 (3 \times 0.275+1) \sqrt{0.275}} + \frac{3 \times 0.275^4 + 41 \times 0.275^3 + 25 \times 0.275^2 - 589 \times 0.275 - 240}{3 (1+0.275)^4 (1+3 \times 0.275)} \right)^4}{49\pi}$$

$$34 - \pi - \sqrt{7} = -34 - \pi + \frac{24.4184 i \pi^2}{i \pi - 0.244324 \int_{-i\infty+\gamma}^{i\infty+\gamma} \frac{e^{1.29098s} \Gamma\left(\frac{1}{2}-s\right) \Gamma(1-s) \Gamma(s)}{\Gamma\left(\frac{3}{2}-s\right)} ds} - \sqrt{7} \text{ for } 0 < \gamma < \frac{1}{2}$$

**Continued fraction representations:**

55

$$\frac{\left( \frac{80 \tan^{-1}(\sqrt{0.275})}{(1+0.275)^2 (3 \times 0.275+1) \sqrt{0.275}} + \frac{3 \times 0.275^4 + 41 \times 0.275^3 + 25 \times 0.275^2 - 589 \times 0.275 - 240}{3 (1+0.275)^4 (1+3 \times 0.275)} \right)^4}{49\pi}$$

$$34 - \pi - \sqrt{7} = -34 + \pi \left( -1 + \frac{2695}{110.368 - \frac{107.862}{1 + \frac{0.275 k^2}{1+2k}}} \right) - \sqrt{7} =$$

$$-34 - \sqrt{7} + \pi \left( -1 + \frac{2695}{110.368 - \frac{107.862}{1 + \frac{0.275}{3 + \frac{1.1}{5 + \frac{2.475}{7 + \frac{4.4}{9 + \dots}}}}} \right)$$

$$\frac{\left( \frac{80 \tan^{-1}(\sqrt{0.275})}{(1+0.275)^2 (3 \times 0.275+1) \sqrt{0.275}} + \frac{3 \times 0.275^4 + 41 \times 0.275^3 + 25 \times 0.275^2 - 589 \times 0.275 - 240}{3(1+0.275)^4 (1+3 \times 0.275)} \right)^4}{4}$$

$$34 - \pi - \sqrt{7} = -41 - \mathop{\text{K}}_{k=1}^{\infty} \frac{-42}{14} + \pi \left( -1 + \frac{2695}{110.368 - \frac{107.862}{1 + \mathop{\text{K}}_{k=1}^{\infty} \frac{0.275 k^2}{1+2k}}} \right) =$$

$$-41 - \frac{42}{14 - \frac{42}{14 - \frac{42}{14 - \frac{42}{14 + \dots}}}} + \pi \left( -1 + \frac{2695}{110.368 - \frac{107.862}{1 + \frac{0.275}{3 + \frac{1.1}{5 + \frac{2.475}{7 + \frac{4.4}{9 + \dots}}}}} \right)$$

$$\frac{\left( \frac{80 \tan^{-1}(\sqrt{0.275})}{(1+0.275)^2 (3 \times 0.275+1) \sqrt{0.275}} + \frac{3 \times 0.275^4 + 41 \times 0.275^3 + 25 \times 0.275^2 - 589 \times 0.275 - 240}{3(1+0.275)^4 (1+3 \times 0.275)} \right)^4}{4}$$

$$34 - \pi - \sqrt{7} = -35 - 4 \left( \mathop{\text{K}}_{k=1}^{\infty} \frac{\frac{3}{8}}{\frac{1}{2}} \right) + \pi \left( -1 + \frac{2695}{110.368 - \frac{107.862}{1 + \mathop{\text{K}}_{k=1}^{\infty} \frac{0.275 k^2}{1+2k}}} \right) =$$

$$-35 - 4 \left( \frac{3}{8 \left( \frac{1}{2} + \frac{3}{8 \left( \frac{1}{2} + \frac{3}{8 \left( \frac{1}{2} + \frac{3}{8 \left( \frac{1}{2} + \dots \right)} \right)} \right)} \right)} \right) + \pi \left( -1 + \frac{2695}{110.368 - \frac{107.862}{1 + \frac{0.275}{3 + \frac{1.1}{5 + \frac{2.475}{7 + \frac{4.4}{9 + \dots}}}}} \right)$$

$\mathop{\text{K}}_{k=k_1}^{k_2} a_k/b_k$  is a continued fraction

$$-11/(((4/(49\pi))*((80 \operatorname{atan}(0.275^{1/2}))/((1+0.275)^2(3*0.275+1)0.275^{1/2}))+((3*0.275^4+41*0.275^3+25*0.275^2-589*0.275-240)/(3(1+0.275)^4(1+3*0.275)))))))-13-3+1/2$$

**Input:**

$$\frac{11}{\frac{4}{49\pi} \left( \frac{80 \tan^{-1}(\sqrt{0.275})}{(1+0.275)^2 (3 \times 0.275 + 1) \sqrt{0.275}} + \frac{3 \times 0.275^4 + 41 \times 0.275^3 + 25 \times 0.275^2 - 589 \times 0.275 - 240}{3 (1+0.275)^4 (1+3 \times 0.275)} \right)} - 13 - 3 + \frac{1}{2}$$

$\tan^{-1}(x)$  is the inverse tangent function

**Result:**

138.070...

(result in radians)

138.070...  $\approx$  138 (Ramanujan taxicab number)

**Alternative representations:**

$$\frac{11}{\frac{4}{49\pi} \left( \frac{80 \tan^{-1}(\sqrt{0.275})}{(1+0.275)^2 (3 \times 0.275 + 1) \sqrt{0.275}} + \frac{3 \times 0.275^4 + 41 \times 0.275^3 + 25 \times 0.275^2 - 589 \times 0.275 - 240}{3 (1+0.275)^4 (1+3 \times 0.275)} \right)} - 13 - 3 + \frac{1}{2} = -\frac{31}{2} - \frac{11}{4 \left( \frac{80 \operatorname{sc}^{-1}(\sqrt{0.275})}{1.825 \sqrt{0.275} \cdot 1.275^2} + \frac{-401.975 + 25 \times 0.275^2 + 41 \times 0.275^3 + 3 \times 0.275^4}{5.475 \times 1.275^4} \right)}$$

$$\frac{11}{\frac{4}{49\pi} \left( \frac{80 \tan^{-1}(\sqrt{0.275})}{(1+0.275)^2 (3 \times 0.275 + 1) \sqrt{0.275}} + \frac{3 \times 0.275^4 + 41 \times 0.275^3 + 25 \times 0.275^2 - 589 \times 0.275 - 240}{3 (1+0.275)^4 (1+3 \times 0.275)} \right)} - 13 - 3 + \frac{1}{2} = -\frac{31}{2} - \frac{11}{4 \left( \frac{80 \tan^{-1}(1, \sqrt{0.275})}{1.825 \sqrt{0.275} \cdot 1.275^2} + \frac{-401.975 + 25 \times 0.275^2 + 41 \times 0.275^3 + 3 \times 0.275^4}{5.475 \times 1.275^4} \right)}$$

$$\begin{aligned}
& \frac{11}{\left( \frac{80 \tan^{-1}(\sqrt{0.275})}{(1+0.275)^2 (3 \times 0.275+1) \sqrt{0.275}} + \frac{3 \times 0.275^4 + 41 \times 0.275^3 + 25 \times 0.275^2 - 589 \times 0.275 - 240}{3(1+0.275)^4 (1+3 \times 0.275)} \right)^4} - 13 - 3 + \\
& \frac{1}{2} = -\frac{31}{2} - \frac{49\pi}{4} \frac{11}{\left( \frac{80 \cot^{-1}\left(\frac{1}{\sqrt{0.275}}\right)}{1.825 \sqrt{0.275} \cdot 1.275^2} + \frac{-401.975 + 25 \times 0.275^2 + 41 \times 0.275^3 + 3 \times 0.275^4}{5.475 \times 1.275^4} \right)^4}
\end{aligned}$$

### Series representations:

$$\begin{aligned}
& \frac{11}{\left( \frac{80 \tan^{-1}(\sqrt{0.275})}{(1+0.275)^2 (3 \times 0.275+1) \sqrt{0.275}} + \frac{3 \times 0.275^4 + 41 \times 0.275^3 + 25 \times 0.275^2 - 589 \times 0.275 - 240}{3(1+0.275)^4 (1+3 \times 0.275)} \right)^4} - \\
& 13 - 3 + \frac{1}{2} = -\frac{31}{2} - \frac{49\pi}{2.62053 \pi} - 0.536588 + \sum_{k=0}^{\infty} \frac{(-1)^k 0.524404^{1+2k}}{1+2k}
\end{aligned}$$

$$\begin{aligned}
& \frac{11}{\left( \frac{80 \tan^{-1}(\sqrt{0.275})}{(1+0.275)^2 (3 \times 0.275+1) \sqrt{0.275}} + \frac{3 \times 0.275^4 + 41 \times 0.275^3 + 25 \times 0.275^2 - 589 \times 0.275 - 240}{3(1+0.275)^4 (1+3 \times 0.275)} \right)^4} - \\
& 13 - 3 + \frac{1}{2} = \frac{31}{-1.07318 + i \log(2) - i \log(-((-0.524404 + i) i)) - i \sum_{k=1}^{\infty} \frac{0.5^k (-((-0.524404 + i) i))^k}{k}} \frac{5.24105 \pi}{49\pi}
\end{aligned}$$

$$\begin{aligned}
& \frac{11}{\left( \frac{80 \tan^{-1}(\sqrt{0.275})}{(1+0.275)^2 (3 \times 0.275+1) \sqrt{0.275}} + \frac{3 \times 0.275^4 + 41 \times 0.275^3 + 25 \times 0.275^2 - 589 \times 0.275 - 240}{3(1+0.275)^4 (1+3 \times 0.275)} \right)^4} - \\
& 13 - 3 + \frac{1}{2} = \frac{31}{1.07318 + i \log(2) - i \log(-i(0.524404 + i)) - i \sum_{k=1}^{\infty} \frac{0.5^k (-i(0.524404 + i))^k}{k}} \frac{5.24105 \pi}{49\pi}
\end{aligned}$$

### Integral representations:

$$\begin{aligned}
& \frac{11}{\left( \frac{80 \tan^{-1}(\sqrt{0.275})}{(1+0.275)^2 (3 \times 0.275+1) \sqrt{0.275}} + \frac{3 \times 0.275^4 + 41 \times 0.275^3 + 25 \times 0.275^2 - 589 \times 0.275 - 240}{3(1+0.275)^4 (1+3 \times 0.275)} \right)^4} - \\
& 13 - 3 + \frac{1}{2} = -\frac{31}{2} + \frac{4.99715 \pi}{1.02323 - \int_0^1 \frac{1}{1+0.275 t^2} dt}
\end{aligned}$$



11

$$\begin{aligned}
 & \frac{\left( \frac{80 \tan^{-1}(\sqrt{0.275})}{(1+0.275)^2 (3 \times 0.275 + 1) \sqrt{0.275}} + \frac{3 \times 0.275^4 + 41 \times 0.275^3 + 25 \times 0.275^2 - 589 \times 0.275 - 240}{3(1+0.275)^4 (1+3 \times 0.275)} \right)^4}{49\pi} \\
 & 13 - 3 + \frac{1}{2} = \\
 & -\frac{31}{2} + \frac{4.88368 \pi^{5/2}}{\pi^{3/2} + 0.244324 i \int_{-i\infty+\gamma}^{i\infty+\gamma} e^{-0.242946s} \Gamma\left(\frac{1}{2}-s\right) \Gamma(1-s) \Gamma(s)^2 ds} \quad \text{for } 0 < \gamma < \frac{1}{2}
 \end{aligned}$$

11

$$\begin{aligned}
 & \frac{\left( \frac{80 \tan^{-1}(\sqrt{0.275})}{(1+0.275)^2 (3 \times 0.275 + 1) \sqrt{0.275}} + \frac{3 \times 0.275^4 + 41 \times 0.275^3 + 25 \times 0.275^2 - 589 \times 0.275 - 240}{3(1+0.275)^4 (1+3 \times 0.275)} \right)^4}{49\pi} - 13 - 3 + \\
 & \frac{1}{2} = -\frac{31}{2} + \frac{4.88368 i \pi^2}{i \pi - 0.244324 \int_{-i\infty+\gamma}^{i\infty+\gamma} \frac{e^{1.29098s} \Gamma\left(\frac{1}{2}-s\right) \Gamma(1-s) \Gamma(s)}{\Gamma\left(\frac{3}{2}-s\right)} ds} \quad \text{for } 0 < \gamma < \frac{1}{2}
 \end{aligned}$$

### Continued fraction representations:

11

$$\begin{aligned}
 & \frac{\left( \frac{80 \tan^{-1}(\sqrt{0.275})}{(1+0.275)^2 (3 \times 0.275 + 1) \sqrt{0.275}} + \frac{3 \times 0.275^4 + 41 \times 0.275^3 + 25 \times 0.275^2 - 589 \times 0.275 - 240}{3(1+0.275)^4 (1+3 \times 0.275)} \right)^4}{49\pi} - 13 - \\
 & 3 + \frac{1}{2} = -\frac{31}{2} + \frac{539 \pi}{110.368 - \frac{107.862}{1 + \underset{k=1}{\overset{\infty}{\text{K}} \frac{0.275 k^2}{1+2k}}} = -\frac{31}{2} + \frac{539 \pi}{110.368 - \frac{107.862}{1 + \frac{0.275}{3 + \frac{1.1}{5 + \frac{2.475}{7 + \frac{4.4}{9 + \dots}}}}}
 \end{aligned}$$

11

$$\begin{aligned}
 & \frac{\left( \frac{80 \tan^{-1}(\sqrt{0.275})}{(1+0.275)^2 (3 \times 0.275 + 1) \sqrt{0.275}} + \frac{3 \times 0.275^4 + 41 \times 0.275^3 + 25 \times 0.275^2 - 589 \times 0.275 - 240}{3(1+0.275)^4 (1+3 \times 0.275)} \right)^4}{49\pi} \\
 & 13 - 3 + \frac{1}{2} = -\frac{31}{2} + \frac{539 \pi}{110.368 - \frac{107.862}{1 + \underset{k=1}{\overset{\infty}{\text{K}} \frac{1.1(0.5-k)^2}{1.275+1.45k}}} = \\
 & -\frac{31}{2} + \frac{539 \pi}{110.368 - \frac{107.862}{1 + \frac{0.275}{2.725 + \frac{2.475}{4.175 + \frac{6.875}{5.625 + \frac{13.475}{7.075 + \dots}}}}}
 \end{aligned}$$

$$\begin{aligned}
 & \frac{80 \tan^{-1}(\sqrt{0.275})}{(1+0.275)^2 (3 \times 0.275 + 1) \sqrt{0.275}} + \frac{3 \times 0.275^4 + 41 \times 0.275^3 + 25 \times 0.275^2 - 589 \times 0.275 - 240}{3(1+0.275)^4 (1+3 \times 0.275)} \Bigg|_4 \\
 13 - 3 + \frac{1}{2} &= -\frac{31}{2} - \frac{49\pi}{539\pi} = \\
 & 4 \left( -0.6265 - \frac{7.41548}{3 + \mathop{\text{K}}_{k=1}^{\infty} \frac{0.275(1+(-1)^{1+k}+k)^2}{3+2k}} \right) \\
 \frac{31}{2} & - \frac{539\pi}{539\pi} \\
 & 4 \left( -0.6265 - \frac{7.41548}{3 + \frac{2.475}{5 + \frac{1.1}{7 + \frac{6.875}{9 + \frac{4.4}{11 + \dots}}}}} \right)
 \end{aligned}$$

$$\begin{aligned}
 & \frac{80 \tan^{-1}(\sqrt{0.275})}{(1+0.275)^2 (3 \times 0.275 + 1) \sqrt{0.275}} + \frac{3 \times 0.275^4 + 41 \times 0.275^3 + 25 \times 0.275^2 - 589 \times 0.275 - 240}{3(1+0.275)^4 (1+3 \times 0.275)} \Bigg|_4 \\
 13 - 3 + \frac{1}{2} &= -\frac{31}{2} - \frac{49\pi}{539\pi} = \\
 & 4 \left( -27.5919 + \frac{26.9654}{1.275 + \mathop{\text{K}}_{k=1}^{\infty} \frac{0.55(1-2\lfloor \frac{1+k}{2} \rfloor) \lfloor \frac{1+k}{2} \rfloor}{(1.1375 + 0.1375(-1)^k)(1+2k)}} \right) \\
 \frac{31}{2} & - \frac{539\pi}{539\pi} \\
 & 4 \left( -27.5919 + \frac{26.9654}{1.275 + \frac{0.55}{3 - \frac{0.55}{6.375 - \frac{3.3}{7 - \frac{3.3}{11.475 + \dots}}}}} \right)
 \end{aligned}$$

$\mathop{\text{K}}_{k=k_1}^{k_2} a_k/b_k$  is a continued fraction

$$-11/((((4/(49\pi))*((80 \operatorname{atan}(0.275^{1/2}))/((1+0.275)^2(3*0.275+1)0.275^{1/2}))+((3*0.275^4+41*0.275^3+25*0.275^2-589*0.275-240)/(3(1+0.275)^4(1+3*0.275)))))))-13-5-1/2$$

**Input:**

$$\frac{11}{49\pi \left( \frac{80 \tan^{-1}(\sqrt{0.275})}{(1+0.275)^2 (3 \times 0.275 + 1) \sqrt{0.275}} + \frac{3 \times 0.275^4 + 41 \times 0.275^3 + 25 \times 0.275^2 - 589 \times 0.275 - 240}{3 (1+0.275)^4 (1+3 \times 0.275)} \right) - 13 - 5 - \frac{1}{2}}$$

$\tan^{-1}(x)$  is the inverse tangent function

**Result:**

135.070...

(result in radians)

135.070...  $\approx$  135 (Ramanujan taxicab number)

**Alternative representations:**

$$\frac{11}{49\pi \left( \frac{80 \tan^{-1}(\sqrt{0.275})}{(1+0.275)^2 (3 \times 0.275 + 1) \sqrt{0.275}} + \frac{3 \times 0.275^4 + 41 \times 0.275^3 + 25 \times 0.275^2 - 589 \times 0.275 - 240}{3 (1+0.275)^4 (1+3 \times 0.275)} \right) - 13 - 5 - \frac{1}{2}} = -\frac{37}{2} - \frac{11}{4 \left( \frac{80 \operatorname{sc}^{-1}(\sqrt{0.275} | p)}{1.825 \sqrt{0.275} 1.275^2} + \frac{-401.975 + 25 \times 0.275^2 + 41 \times 0.275^3 + 3 \times 0.275^4}{5.475 \times 1.275^4} \right)}$$

$$\frac{11}{49\pi \left( \frac{80 \tan^{-1}(\sqrt{0.275})}{(1+0.275)^2 (3 \times 0.275 + 1) \sqrt{0.275}} + \frac{3 \times 0.275^4 + 41 \times 0.275^3 + 25 \times 0.275^2 - 589 \times 0.275 - 240}{3 (1+0.275)^4 (1+3 \times 0.275)} \right) - 13 - 5 - \frac{1}{2}} = -\frac{37}{2} - \frac{11}{4 \left( \frac{80 \tan^{-1}(1, \sqrt{0.275})}{1.825 \sqrt{0.275} 1.275^2} + \frac{-401.975 + 25 \times 0.275^2 + 41 \times 0.275^3 + 3 \times 0.275^4}{5.475 \times 1.275^4} \right)}$$

$$\begin{aligned}
& \frac{11}{\left( \frac{80 \tan^{-1}(\sqrt{0.275})}{(1+0.275)^2 (3 \times 0.275+1) \sqrt{0.275}} + \frac{3 \times 0.275^4 + 41 \times 0.275^3 + 25 \times 0.275^2 - 589 \times 0.275 - 240}{3(1+0.275)^4 (1+3 \times 0.275)} \right)^4} - 13 - 5 - \\
& \frac{1}{2} = -\frac{37}{2} - \frac{49\pi}{11} \frac{11}{\left( \frac{80 \cot^{-1}\left(\frac{1}{\sqrt{0.275}}\right)}{1.825 \sqrt{0.275} \cdot 1.275^2} + \frac{-401.975 + 25 \times 0.275^2 + 41 \times 0.275^3 + 3 \times 0.275^4}{5.475 \times 1.275^4} \right)^4}
\end{aligned}$$

### Series representations:

$$\begin{aligned}
& \frac{11}{\left( \frac{80 \tan^{-1}(\sqrt{0.275})}{(1+0.275)^2 (3 \times 0.275+1) \sqrt{0.275}} + \frac{3 \times 0.275^4 + 41 \times 0.275^3 + 25 \times 0.275^2 - 589 \times 0.275 - 240}{3(1+0.275)^4 (1+3 \times 0.275)} \right)^4} - \\
& 13 - 5 - \frac{1}{2} = -\frac{37}{2} - \frac{49\pi}{2.62053 \pi} - 0.536588 + \sum_{k=0}^{\infty} \frac{(-1)^k 0.524404^{1+2k}}{1+2k}
\end{aligned}$$

$$\begin{aligned}
& \frac{11}{\left( \frac{80 \tan^{-1}(\sqrt{0.275})}{(1+0.275)^2 (3 \times 0.275+1) \sqrt{0.275}} + \frac{3 \times 0.275^4 + 41 \times 0.275^3 + 25 \times 0.275^2 - 589 \times 0.275 - 240}{3(1+0.275)^4 (1+3 \times 0.275)} \right)^4} - \\
& 13 - 5 - \frac{1}{2} = \frac{37}{-1.07318 + i \log(2) - i \log(-((-0.524404 + i) i)) - i \sum_{k=1}^{\infty} \frac{0.5^k (-((-0.524404 + i) i))^k}{k}} \frac{5.24105 \pi}{49\pi}
\end{aligned}$$

$$\begin{aligned}
& \frac{11}{\left( \frac{80 \tan^{-1}(\sqrt{0.275})}{(1+0.275)^2 (3 \times 0.275+1) \sqrt{0.275}} + \frac{3 \times 0.275^4 + 41 \times 0.275^3 + 25 \times 0.275^2 - 589 \times 0.275 - 240}{3(1+0.275)^4 (1+3 \times 0.275)} \right)^4} - \\
& 13 - 5 - \frac{1}{2} = \frac{37}{1.07318 + i \log(2) - i \log(-i(0.524404 + i)) - i \sum_{k=1}^{\infty} \frac{0.5^k (-i(0.524404 + i))^k}{k}} \frac{5.24105 \pi}{49\pi}
\end{aligned}$$

### Integral representations:

$$\begin{aligned}
& \frac{11}{\left( \frac{80 \tan^{-1}(\sqrt{0.275})}{(1+0.275)^2 (3 \times 0.275+1) \sqrt{0.275}} + \frac{3 \times 0.275^4 + 41 \times 0.275^3 + 25 \times 0.275^2 - 589 \times 0.275 - 240}{3(1+0.275)^4 (1+3 \times 0.275)} \right)^4} - \\
& 13 - 5 - \frac{1}{2} = -\frac{37}{2} + \frac{4.99715 \pi}{1.02323 - \int_0^1 \frac{1}{1+0.275 t^2} dt}
\end{aligned}$$

$$\begin{aligned}
 & \left( \frac{80 \tan^{-1}(\sqrt{0.275})}{(1+0.275)^2 (3 \times 0.275+1) \sqrt{0.275}} + \frac{3 \times 0.275^4 + 41 \times 0.275^3 + 25 \times 0.275^2 - 589 \times 0.275 - 240}{3(1+0.275)^4 (1+3 \times 0.275)} \right)^4 \\
 & \frac{49\pi}{13 - 5 - \frac{1}{2}} = \\
 & -\frac{37}{2} + \frac{4.88368 \pi^{5/2}}{\pi^{3/2} + 0.244324 i \int_{-i\infty+\gamma}^{i\infty+\gamma} e^{-0.242946s} \Gamma\left(\frac{1}{2}-s\right) \Gamma(1-s) \Gamma(s)^2 ds} \quad \text{for } 0 < \gamma < \frac{1}{2}
 \end{aligned}$$

$$\begin{aligned}
 & \left( \frac{80 \tan^{-1}(\sqrt{0.275})}{(1+0.275)^2 (3 \times 0.275+1) \sqrt{0.275}} + \frac{3 \times 0.275^4 + 41 \times 0.275^3 + 25 \times 0.275^2 - 589 \times 0.275 - 240}{3(1+0.275)^4 (1+3 \times 0.275)} \right)^4 \\
 & \frac{1}{2} = -\frac{37}{2} + \frac{4.88368 i \pi^2}{i \pi - 0.244324 \int_{-i\infty+\gamma}^{i\infty+\gamma} \frac{e^{1.29098s} \Gamma\left(\frac{1}{2}-s\right) \Gamma(1-s) \Gamma(s)}{\Gamma\left(\frac{3}{2}-s\right)} ds} \quad \text{for } 0 < \gamma < \frac{1}{2}
 \end{aligned}$$

### Continued fraction representations:

$$\begin{aligned}
 & \left( \frac{80 \tan^{-1}(\sqrt{0.275})}{(1+0.275)^2 (3 \times 0.275+1) \sqrt{0.275}} + \frac{3 \times 0.275^4 + 41 \times 0.275^3 + 25 \times 0.275^2 - 589 \times 0.275 - 240}{3(1+0.275)^4 (1+3 \times 0.275)} \right)^4 \\
 & 5 - \frac{1}{2} = -\frac{37}{2} + \frac{539 \pi}{110.368 - \frac{107.862}{1 + \underset{k=1}{\overset{\infty}{\text{K}} \frac{0.275 k^2}{1+2k}}} = -\frac{37}{2} + \frac{539 \pi}{110.368 - \frac{107.862}{1 + \frac{0.275}{3 + \frac{1.1}{5 + \frac{2.475}{7 + \frac{4.4}{9 + \dots}}}}}
 \end{aligned}$$

$$\begin{aligned}
 & \left( \frac{80 \tan^{-1}(\sqrt{0.275})}{(1+0.275)^2 (3 \times 0.275+1) \sqrt{0.275}} + \frac{3 \times 0.275^4 + 41 \times 0.275^3 + 25 \times 0.275^2 - 589 \times 0.275 - 240}{3(1+0.275)^4 (1+3 \times 0.275)} \right)^4 \\
 & 13 - 5 - \frac{1}{2} = -\frac{37}{2} + \frac{539 \pi}{110.368 - \frac{107.862}{1 + \underset{k=1}{\overset{\infty}{\text{K}} \frac{1.1(0.5-k)^2}{1.275+1.45k}}} = \\
 & -\frac{37}{2} + \frac{539 \pi}{110.368 - \frac{107.862}{1 + \frac{0.275}{2.725 + \frac{2.475}{4.175 + \frac{6.875}{5.625 + \frac{13.475}{7.075 + \dots}}}}}
 \end{aligned}$$

$$\begin{aligned}
 & \frac{\left( \frac{80 \tan^{-1}(\sqrt{0.275})}{(1+0.275)^2 (3 \times 0.275 + 1) \sqrt{0.275}} + \frac{3 \times 0.275^4 + 41 \times 0.275^3 + 25 \times 0.275^2 - 589 \times 0.275 - 240}{3(1+0.275)^4 (1+3 \times 0.275)} \right)}{4} \\
 13 - 5 - \frac{1}{2} &= -\frac{37}{2} - \frac{49\pi}{539\pi} = \\
 & 4 \left( -0.6265 - \frac{7.41548}{3 + \mathop{\text{K}}_{k=1}^{\infty} \frac{0.275(1+(-1)^{1+k}+k)^2}{3+2k}} \right) \\
 \frac{37}{2} & \frac{539\pi}{4 \left( -0.6265 - \frac{7.41548}{3 + \frac{2.475}{5 + \frac{1.1}{7 + \frac{6.875}{9 + \frac{4.4}{11 + \dots}}}} \right)}
 \end{aligned}$$

$$\begin{aligned}
 & \frac{\left( \frac{80 \tan^{-1}(\sqrt{0.275})}{(1+0.275)^2 (3 \times 0.275 + 1) \sqrt{0.275}} + \frac{3 \times 0.275^4 + 41 \times 0.275^3 + 25 \times 0.275^2 - 589 \times 0.275 - 240}{3(1+0.275)^4 (1+3 \times 0.275)} \right)}{4} \\
 13 - 5 - \frac{1}{2} &= -\frac{37}{2} - \frac{49\pi}{539\pi} = \\
 & 4 \left( -27.5919 + \frac{26.9654}{1.275 + \mathop{\text{K}}_{k=1}^{\infty} \frac{0.55 \left( 1 - 2 \left| \frac{1+k}{2} \right| \right) \left| \frac{1+k}{2} \right|}{(1.1375 + 0.1375(-1)^k)(1+2k)}} \right) \\
 \frac{37}{2} & \frac{539\pi}{4 \left( -27.5919 + \frac{26.9654}{1.275 + \frac{0.55}{3 - \frac{0.55}{6.375 - \frac{3.3}{7 - \frac{3.3}{11.475 + \dots}}}} \right)}
 \end{aligned}$$

$\mathop{\text{K}}_{k=1}^{k_2} a_k/b_k$  is a continued fraction

$$-11/\left(\left(\left(\left(4/(49\pi)\right)\left(\left(\left(80\right)\right)\right)\right)\right)\right)\left(\left(\left(\left(1+0.275\right)^2\left(3\cdot 0.275+1\right)0.275^{1/2}\right)\right)\right)+\left(3\cdot 0.275^4+41\cdot 0.275^3+25\cdot 0.275^2-589\cdot 0.275-240\right)/\left(3\left(1+0.275\right)^4\left(1+3\cdot 0.275\right)\right)\right)\right)+18+1/2$$

**Input:**

$$-\frac{11}{\frac{4}{49\pi} \left( \frac{80 \tan^{-1}(\sqrt{0.275})}{(1+0.275)^2 (3 \times 0.275+1) \sqrt{0.275}} + \frac{3 \times 0.275^4 + 41 \times 0.275^3 + 25 \times 0.275^2 - 589 \times 0.275 - 240}{3 (1+0.275)^4 (1+3 \times 0.275)} \right)} + 18 + \frac{1}{2}$$

$\tan^{-1}(x)$  is the inverse tangent function

**Result:**

172.0703082299606398136043749780507770604998554678811792291...

(result in radians)

172.070308...  $\approx$  172 (Ramanujan taxicab number)

**Alternative representations:**

$$-\frac{11}{\left( \frac{80 \tan^{-1}(\sqrt{0.275})}{(1+0.275)^2 (3 \times 0.275+1) \sqrt{0.275}} + \frac{3 \times 0.275^4 + 41 \times 0.275^3 + 25 \times 0.275^2 - 589 \times 0.275 - 240}{3 (1+0.275)^4 (1+3 \times 0.275)} \right)^4} + 18 + \frac{1}{2} =$$

$$\frac{37}{2} - \frac{\frac{49\pi}{11}}{4 \left( \frac{80 \sec^{-1}(\sqrt{0.275} | 0)}{1.825 \sqrt{0.275} \cdot 1.275^2} + \frac{-401.975 + 25 \times 0.275^2 + 41 \times 0.275^3 + 3 \times 0.275^4}{5.475 \times 1.275^4} \right)}$$

$$-\frac{11}{\left( \frac{80 \tan^{-1}(\sqrt{0.275})}{(1+0.275)^2 (3 \times 0.275+1) \sqrt{0.275}} + \frac{3 \times 0.275^4 + 41 \times 0.275^3 + 25 \times 0.275^2 - 589 \times 0.275 - 240}{3 (1+0.275)^4 (1+3 \times 0.275)} \right)^4} + 18 + \frac{1}{2} =$$

$$\frac{37}{2} - \frac{\frac{49\pi}{11}}{4 \left( \frac{80 \tan^{-1}(1, \sqrt{0.275})}{1.825 \sqrt{0.275} \cdot 1.275^2} + \frac{-401.975 + 25 \times 0.275^2 + 41 \times 0.275^3 + 3 \times 0.275^4}{5.475 \times 1.275^4} \right)}$$

$$\begin{aligned}
& - \frac{11}{\left( \frac{80 \tan^{-1}(\sqrt{0.275})}{(1+0.275)^2 (3 \times 0.275+1) \sqrt{0.275}} + \frac{3 \times 0.275^4 + 41 \times 0.275^3 + 25 \times 0.275^2 - 589 \times 0.275 - 240}{3(1+0.275)^4 (1+3 \times 0.275)} \right)^4} + 18 + \frac{1}{2} = \\
& \frac{37}{2} - \frac{\frac{49\pi}{11}}{4 \left( \frac{80 \cot^{-1}\left(\frac{1}{\sqrt{0.275}}\right)}{1.825 \sqrt{0.275} 1.275^2} + \frac{-401.975 + 25 \times 0.275^2 + 41 \times 0.275^3 + 3 \times 0.275^4}{5.475 \times 1.275^4} \right)}
\end{aligned}$$

### Series representations:

$$\begin{aligned}
& - \frac{11}{\left( \frac{80 \tan^{-1}(\sqrt{0.275})}{(1+0.275)^2 (3 \times 0.275+1) \sqrt{0.275}} + \frac{3 \times 0.275^4 + 41 \times 0.275^3 + 25 \times 0.275^2 - 589 \times 0.275 - 240}{3(1+0.275)^4 (1+3 \times 0.275)} \right)^4} + 18 + \frac{1}{2} = \\
& \frac{37}{2} - \frac{\frac{49\pi}{11}}{-0.536588 + \sum_{k=0}^{\infty} \frac{(-1)^k 0.524404^{1+2k}}{1+2k}}
\end{aligned}$$

$$\begin{aligned}
& - \frac{11}{\left( \frac{80 \tan^{-1}(\sqrt{0.275})}{(1+0.275)^2 (3 \times 0.275+1) \sqrt{0.275}} + \frac{3 \times 0.275^4 + 41 \times 0.275^3 + 25 \times 0.275^2 - 589 \times 0.275 - 240}{3(1+0.275)^4 (1+3 \times 0.275)} \right)^4} + 18 + \frac{1}{2} = \\
& \frac{37}{2} - \frac{\frac{49\pi}{11}}{-0.536588 + \sum_{k=0}^{\infty} \frac{\left(-\frac{1}{5}\right)^k 1.04881^{1+2k} F_{1+2k} \left(\frac{1}{1+\sqrt{1.22}}\right)^{1+2k}}{1+2k}}
\end{aligned}$$

$$\begin{aligned}
& - \frac{11}{\left( \frac{80 \tan^{-1}(\sqrt{0.275})}{(1+0.275)^2 (3 \times 0.275+1) \sqrt{0.275}} + \frac{3 \times 0.275^4 + 41 \times 0.275^3 + 25 \times 0.275^2 - 589 \times 0.275 - 240}{3(1+0.275)^4 (1+3 \times 0.275)} \right)^4} + 18 + \frac{1}{2} = \\
& \frac{37}{2} + (2.62053 \pi) / \left( 0.536588 - \tan^{-1}(x) + \pi \left[ \frac{\arg(i(-0.524404 + x))}{2\pi} \right] - \right. \\
& \left. 0.5 i \sum_{k=1}^{\infty} \frac{(-(-i-x)^{-k} + (i-x)^{-k})(0.524404 - x)^k}{k} \right) \text{ for } (ix \in \mathbb{R} \text{ and } ix > 1)
\end{aligned}$$

$F_n$  is the  $n^{\text{th}}$  Fibonacci number



**Integral representations:**

$$-\frac{11}{\left(\frac{80 \tan^{-1}(\sqrt{0.275})}{(1+0.275)^2 (3 \times 0.275+1) \sqrt{0.275}} + \frac{3 \times 0.275^4 + 41 \times 0.275^3 + 25 \times 0.275^2 - 589 \times 0.275 - 240}{3(1+0.275)^4 (1+3 \times 0.275)}\right)^4} + 18 + \frac{1}{2} =$$

$$\frac{37}{2} + \frac{4.99715 \pi}{1.02323 - \int_0^1 \frac{1}{1+0.275 t^2} dt} \quad 49\pi$$

$$-\frac{11}{\left(\frac{80 \tan^{-1}(\sqrt{0.275})}{(1+0.275)^2 (3 \times 0.275+1) \sqrt{0.275}} + \frac{3 \times 0.275^4 + 41 \times 0.275^3 + 25 \times 0.275^2 - 589 \times 0.275 - 240}{3(1+0.275)^4 (1+3 \times 0.275)}\right)^4} + 18 + \frac{1}{2} =$$

$$\frac{37}{2} + \frac{4.88368 \pi^{5/2}}{\pi^{3/2} + 0.244324 i \int_{-i\infty+\gamma}^{i\infty+\gamma} e^{-0.242946 s} \Gamma\left(\frac{1}{2}-s\right) \Gamma(1-s) \Gamma(s)^2 ds} \quad \text{for } 0 < \gamma < \frac{1}{2}$$

$$-\frac{11}{\left(\frac{80 \tan^{-1}(\sqrt{0.275})}{(1+0.275)^2 (3 \times 0.275+1) \sqrt{0.275}} + \frac{3 \times 0.275^4 + 41 \times 0.275^3 + 25 \times 0.275^2 - 589 \times 0.275 - 240}{3(1+0.275)^4 (1+3 \times 0.275)}\right)^4} + 18 + \frac{1}{2} =$$

$$\frac{37}{2} + \frac{4.88368 i \pi^2}{i \pi - 0.244324 \int_{-i\infty+\gamma}^{i\infty+\gamma} \frac{e^{1.29098 s} \Gamma\left(\frac{1}{2}-s\right) \Gamma(1-s) \Gamma(s)}{\Gamma\left(\frac{3}{2}-s\right)} ds} \quad \text{for } 0 < \gamma < \frac{1}{2}$$

**Continued fraction representations:**

$$-\frac{11}{\left(\frac{80 \tan^{-1}(\sqrt{0.275})}{(1+0.275)^2 (3 \times 0.275+1) \sqrt{0.275}} + \frac{3 \times 0.275^4 + 41 \times 0.275^3 + 25 \times 0.275^2 - 589 \times 0.275 - 240}{3(1+0.275)^4 (1+3 \times 0.275)}\right)^4} + 18 + \frac{1}{2} =$$

$$\frac{37}{2} + \frac{539 \pi}{110.368 - \frac{107.862}{1 + \frac{\infty}{\prod_{k=1}^{\infty} \frac{0.275 k^2}{1+2k}}} = \frac{37}{2} + \frac{539 \pi}{110.368 - \frac{107.862}{1 + \frac{0.275}{3 + \frac{1.1}{5 + \frac{2.475}{7 + \frac{4.4}{9 + \dots}}}}}}$$

$$-\frac{11}{\left(\frac{80 \tan^{-1}(\sqrt{0.275})}{(1+0.275)^2 (3 \times 0.275+1) \sqrt{0.275}} + \frac{3 \times 0.275^4 + 41 \times 0.275^3 + 25 \times 0.275^2 - 589 \times 0.275 - 240}{3(1+0.275)^4 (1+3 \times 0.275)}\right)^4} + 18 + \frac{1}{2} =$$

$$\frac{37}{2} + \frac{539 \pi}{110.368 - \frac{107.862}{1 + \frac{\infty}{\prod_{k=1}^{\infty} \frac{1.1(0.5-k)^2}{1.275+1.45k}}} = \frac{37}{2} + \frac{539 \pi}{110.368 - \frac{107.862}{2.725 + \frac{0.275}{4.175 + \frac{2.475}{5.625 + \frac{6.875}{7.075 + \dots}}}}}}$$

$$\begin{aligned}
& - \frac{11}{\left( \frac{80 \tan^{-1}(\sqrt{0.275})}{(1+0.275)^2 (3 \times 0.275 + 1) \sqrt{0.275}} + \frac{3 \times 0.275^4 + 41 \times 0.275^3 + 25 \times 0.275^2 - 589 \times 0.275 - 240}{3(1+0.275)^4 (1+3 \times 0.275)} \right)^4} + 18 + \frac{1}{2} = \\
& \frac{37}{2} - \frac{539 \pi^{49}}{2} = \frac{37}{2} - \frac{539 \pi}{4 \left( -0.6265 - \frac{7.41548}{3 + \mathop{\text{K}}_{k=1}^{\infty} \frac{0.275 (1+(-1)^{1+k+k})^2}{3+2k}} \right)} = \frac{37}{2} - \frac{539 \pi}{4 \left( -0.6265 - \frac{7.41548}{3 + \frac{2.475}{5 + \frac{1.1}{7 + \frac{6.875}{9 + \frac{4.4}{11 + \dots}}}} \right)}
\end{aligned}$$

$$\begin{aligned}
& - \frac{11}{\left( \frac{80 \tan^{-1}(\sqrt{0.275})}{(1+0.275)^2 (3 \times 0.275 + 1) \sqrt{0.275}} + \frac{3 \times 0.275^4 + 41 \times 0.275^3 + 25 \times 0.275^2 - 589 \times 0.275 - 240}{3(1+0.275)^4 (1+3 \times 0.275)} \right)^4} + 18 + \frac{1}{2} = \\
& \frac{37}{2} - \frac{539 \pi^{49}}{2} = \frac{37}{2} - \frac{539 \pi}{4 \left( -27.5919 + \frac{26.9654}{1.275 + \mathop{\text{K}}_{k=1}^{\infty} \frac{0.55 \left( 1 - 2 \left| \frac{1+k}{2} \right| \right) \left| \frac{1+k}{2} \right|}{(1.1375 + 0.1375 (-1)^k) (1+2k)} \right)} = \\
& \frac{37}{2} - \frac{539 \pi}{4 \left( -27.5919 + \frac{26.9654}{1.275 + \frac{0.55}{3 - \frac{0.55}{6.375 - \frac{3.3}{7 - \frac{3.3}{11.475 + \dots}}}} \right)}
\end{aligned}$$

$\mathop{\text{K}}_{k=k_1}^{k_2} a_k / b_k$  is a continued fraction

From:

## Holographic entanglement entropy under the minimal geometric deformation and extensions

*R. da Rocha, A. A. Tomaz* - arXiv:1905.01548v2 [hep-th] 29 Dec 2019

Now, we have that:

for  $\kappa_1 = \frac{M\chi}{1-M/R}$ . Now, in order to the radial metric component asymptotically approach the Schwarzschild behavior with ADM mass  $M_1 = 2M$ ,  $e^{-\lambda(r)} \sim 1 - \frac{2M_1}{r} + \mathcal{O}(r^{-2})$ , one must necessarily have  $\kappa_1 = -2M$ . In this case, the temporal and spatial components of the metric will be inversely equal to each other (as it is the case of the Schwarzschild solution), containing a tidal charge  $Q_1 = 4M^2$  reproducing a solution that is tidally charged by the Weyl fluid [45]:

$$e^{\nu} = e^{-\lambda} = 1 - \frac{2M_1}{r} + \frac{Q_1}{r^2} \quad (14)$$

It is worth to emphasize that the metric of Eq. (14) has a degenerate event horizon at  $r_h = 2M = M_1$ . Since the degenerate horizon lies behind the Schwarzschild event horizon,  $r_h = M_1 < r_s = 2M_1$ , bulk effects are then responsible for decreasing the gravitational field strength on the brane.

Now the exterior solution for  $k = 2$  can be constructed, making Eq. (12) to yield

$$e^{\nu(r)} = 1 - \frac{2M_2}{r} + \frac{Q_2}{r^2} - \frac{2Q_2M_2}{9r^3}, \quad (15)$$

where  $Q_2 = 12M^2$  and  $M_2 = 3M$ . The radial component, on the other hand, reads

$$e^{-\lambda(r)} = \frac{1}{1 - \frac{2M_2}{3r}} \sum_{m=0}^8 \frac{c_m}{r^m}, \quad (16)$$

where the coefficients  $c_m \equiv c_m(M_2, Q_2, s)$  are

$$c_0 = 1, \quad c_1 = s - \frac{4M_2}{3}, \quad c_2 = \frac{1}{6}(5Q_2 - 7sM_2), \quad (17a)$$

$$c_3 = \frac{M_2}{12}(7sM_2 - 5Q_2), \quad c_4 = \frac{25Q_2^2}{288} - \frac{7}{216}sM_2^3, \quad c_5 = \frac{35}{1296}sM_2^4 - \frac{35}{1728}Q_2^2M_2, \quad (17b)$$

$$c_6 = \frac{5Q_2^3}{20736} - \frac{7sM_2^5}{2592}, \quad c_7 = \frac{28sM_2^6 - 15Q_2^3M_2}{186624}, \quad c_8 = \frac{5Q_2^4}{4644864} - \frac{sM_2^7}{279936}, \quad (17c)$$

and  $s = R\chi(1 - 2M_2/3R)/(2 - M_2/3R)^7$ . The asymptotic Schwarzschild behavior is then assured when  $s = -M_2/96$ . In this case, the degenerate event horizon is at  $r_e \approx 1.12M_2$  [5]. Hence, the bulk Weyl fluid weakens gravitational field effects. The classical tests of GR applied to the EMGD metric provide the following constraints on the value of the deformation parameter,  $k \lesssim 4.2$  for the gravitational redshift of light. The standard MGD corresponds to  $k = 0$ , whereas the Reissner–Nordström solution represents the  $k = 1$  case with the ADM mass  $M_1$ , instead.

For  $M = 1.312806e+40$

$$Q_2 = 12M^2 \text{ and } M_2 = 3M.$$







**Result:**

True

$$-3.438670463997 \times 10^{201}$$

Note that:

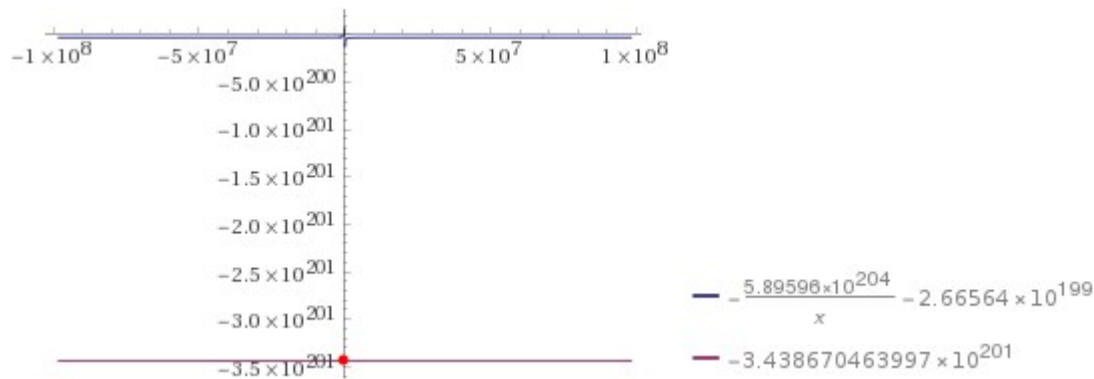
$$35/1296 * (-4.10251875e+38) * (3 * 1.312806e+40)^4 - 35/x * (12 * 1.312806e+40^2)^2 * (3 * 1.312806e+40) = -3.438670463997 * 10^{201}$$

**Input interpretation:**

$$\frac{35}{1296} (-4.10251875 \times 10^{38}) (3 \times 1.312806 \times 10^{40})^4 - \frac{35}{x} (12 (1.312806 \times 10^{40})^2)^2 (3 \times 1.312806 \times 10^{40}) = -(3.438670463997 \times 10^{201})$$

**Result:**

$$-\frac{5.89596 \times 10^{204}}{x} - 2.66564 \times 10^{199} = -3.438670463997 \times 10^{201}$$

**Plot:****Alternate form assuming x is real:**

$$\frac{5.89596 \times 10^{204}}{x} = 3.41201 \times 10^{201}$$

**Alternate form:**

$$\frac{-2.66564 \times 10^{199} x - 5.89596 \times 10^{204}}{x} = -3.438670463997 \times 10^{201}$$

**Alternate form assuming x is positive:**

$$3.41201 \times 10^{201} x = 5.89596 \times 10^{204} \quad (\text{for } x \neq 0)$$

**Solution:** $x \approx 1728.$ 

1728

This result is very near to the mass of candidate glueball  $f_0(1710)$  meson. Furthermore, 1728 occurs in the algebraic formula for the  $j$ -invariant of an elliptic curve. As a consequence, it is sometimes called a Zagier as a pun on the Gross–Zagier theorem. The number 1728 is one less than the Hardy–Ramanujan number 1729

$$c_6 = \frac{5Q_2^3}{20736} - \frac{7sM_2^5}{2592}$$

For  $M = 1.312806e+40$

$$Q_2 = 12M^2 \text{ and } M_2 = 3M.$$

$$-4.10251875e+38 = s$$

(for this expression, we have considered  $12M^2 = 12M$ )

$$5/20736*(12*1.312806e+40)^3-7/2592*(-4.10251875e+38)*(3*1.312806e+40)^5$$

**Input interpretation:**

$$\frac{5}{20736} (12 \times 1.312806 \times 10^{40})^3 - \frac{7}{2592} (-4.10251875 \times 10^{38}) (3 \times 1.312806 \times 10^{40})^5$$

**Result:**

$$1.04984 \times 10^{239}$$

$$1.04984 * 10^{239}$$

**Scientific notation:**

$$1.049838887711270895612631304032544068125000000000000... \times 10^{239}$$





$$(1/4644864 * (5*(12*1.312806e+40^2)^4))-1/279936 * (-4.10251875e+38)*(3*1.312806e+40)^7$$

**Input interpretation:**

$$\frac{1}{4644864} (5 (12 (1.312806 \times 10^{40})^2)^4) - \frac{1}{279936} (-4.10251875 \times 10^{38}) (3 \times 1.312806 \times 10^{40})^7$$

**Result:**

$$1.9909058199156373302495401048779828155564178501577678... \times 10^{319}$$

**Repeating decimal:**

$$1.9909058199156373302495401048779828155564178501577678... \times 10^{319}$$

(period 6)

$$1.990905819... * 10^{319}$$

Now, we have that:

$$(3.72101 * 10^{161}) * 1 / (-3.43097240073758904504375 * 10^{121}) * 1 / (1.74230993294139375 * 10^{81}) * 1 / (-5.2922491875 * 10^{40})$$

**Input interpretation:**

$$(3.72101 \times 10^{161}) \left( -\frac{1}{3.43097240073758904504375 \times 10^{121}} \right) \times \frac{1}{1.74230993294139375 \times 10^{81}} \left( -\frac{1}{5.2922491875 \times 10^{40}} \right)$$

**Result:**

$$1.1761911712325356994330818948413805998749667307530395... \times 10^{-82}$$

$$1.176191171... * 10^{-82}$$

$$(1.990905819 \times 10^{319}) / (-2.82319364691125 \times 10^{280}) * (1.04984 \times 10^{239}) / (-3.438670463997 \times 10^{201}) 1.1761911712325356994330818948413805998749667307530395 \times 10^{-82}$$

**Input interpretation:**

$$-\frac{1.990905819 \times 10^{319}}{2.82319364691125 \times 10^{280}} \left( -\frac{1.04984 \times 10^{239}}{3.438670463997 \times 10^{201}} \right) \times 1.1761911712325356994330818948413805998749667307530395 \times 10^{-82}$$

**Result:**

$$2.5323312858584196635120552564656133384290463417856282... \times 10^{-6}$$

$$2.532331285... \times 10^{-6}$$

$$1 / (((((1.990905819 \times 10^{319}) / (-2.82319364691125 \times 10^{280}) * (1.04984 \times 10^{239}) / (-3.438670463997 \times 10^{201}) 1.1761911712325356994330818948413805998749667307530395 \times 10^{-82}))))))$$

**Input interpretation:**

$$1 / \left( -\frac{1.990905819 \times 10^{319}}{2.82319364691125 \times 10^{280}} \left( -\frac{1.04984 \times 10^{239}}{3.438670463997 \times 10^{201}} \right) \times 1.1761911712325356994330818948413805998749667307530395 \times 10^{-82} \right)$$

**Result:**

$$394893.0400948768633952659389016200041288760404251492327551...$$

$$394893.040094876...$$

Note that, from the formula of coefficients of the '5th order' mock theta function  $\psi_1(q)$ : (A053261 OEIS Sequence)

$$\sqrt{\phi} * \exp(\pi * \sqrt{n/15}) / (2 * 5^{(1/4)} * \sqrt{n})$$

we obtain, for n = 427:

**Input:**

$$\sqrt{\phi} \times \frac{\exp\left(\pi \sqrt{\frac{427}{15}}\right)}{2 \sqrt[4]{5} \sqrt{427}} + 64(2^5 + 2^4) + 55 + \left(\frac{1}{30} (13 \mathcal{W}_{\text{Wad}}) + \pi\right)$$

$\phi$  is the golden ratio

$W_{\text{Wad}}$  is the Wadsworth constant

**Exact result:**

$$\frac{13 W_{\text{Wad}}}{30} + \frac{e^{\sqrt{427/15} \pi} \sqrt{\frac{\phi}{427}}}{2 \sqrt[4]{5}} + 3127 + \pi$$

**Exact form:**

$$\frac{e^{\sqrt{427/15} \pi} \sqrt{\frac{\phi}{427}}}{2 \sqrt[4]{5}} + \frac{312713}{100} + \pi$$

**Decimal approximation:**

394893.04796374744441478828132726070965329840167922103608634...

394893.047963747...

**Alternate forms:**

$$\frac{13 W_{\text{Wad}}}{30} + 3127 + \frac{1}{2} \sqrt{\frac{5 + \sqrt{5}}{4270}} e^{\sqrt{427/15} \pi} + \pi$$

$$\frac{13 W_{\text{Wad}}}{30} + 3127 + \frac{\sqrt{\frac{1}{854} (1 + \sqrt{5})} e^{\sqrt{427/15} \pi}}{2 \sqrt[4]{5}} + \pi$$

$$\frac{11102 W_{\text{Wad}} + 80113740 + 3 \times 5^{3/4} \sqrt{854(1 + \sqrt{5})} e^{\sqrt{427/15} \pi} + 25620 \pi}{25620}$$

We have also:

$$\left( \frac{2\pi}{10^3} + \left( \frac{1}{\left( \frac{1.990905819 \times 10^{319}}{-2.82319364691125 \times 10^{280}} \right) \left( \frac{1.04984 \times 10^{239}}{-3.438670463997 \times 10^{201}} \right) \times 1.1761911712325 \times 10^{-82} \right)} \right)^{1/27}$$

**Input interpretation:**

$$\frac{2\pi}{10^3} + \sqrt[27]{\frac{1}{-\frac{1.990905819 \times 10^{319}}{2.82319364691125 \times 10^{280}} \left( -\frac{1.04984 \times 10^{239}}{3.438670463997 \times 10^{201}} \right) \times 1.1761911712325 \times 10^{-82}}}$$





**Exact result:**

$$\frac{1}{\sqrt[27]{\frac{13 W_{\text{Wad}}}{30} + \frac{e^{\sqrt{427/15} \pi} \sqrt{\frac{\phi}{427}}}{2 \sqrt[4]{5}} + 3127 + \pi + \frac{\pi}{500}}}$$

**Exact form:**

$$\frac{1}{\sqrt[27]{\frac{e^{\sqrt{427/15} \pi} \sqrt{\frac{\phi}{427}}}{2 \sqrt[4]{5}} + \frac{312713}{100} + \pi + \frac{\pi}{500}}}$$

**Decimal approximation:**

0.618063584062091681567447030367761025027834117969898872030...

[0.618063584...](#)

**Alternate forms:**

$$\frac{1}{\sqrt[27]{\frac{13 W_{\text{Wad}}}{30} + 3127 + \frac{1}{2} \sqrt{\frac{5+\sqrt{5}}{4270}} e^{\sqrt{427/15} \pi} + \pi + \frac{\pi}{500}}}$$

$$\frac{1}{\sqrt[27]{\frac{13 W_{\text{Wad}}}{30} + 3127 + \frac{\sqrt{\frac{1}{854} (1+\sqrt{5})} e^{\sqrt{427/15} \pi}}{2 \sqrt[4]{5}} + \pi + \frac{\pi}{500}}}$$

$$640500 \left/ \left( 50 \times 2^{25/27} \times 6405^{26/27} \sqrt[27]{11102 W_{\text{Wad}} + 80113740 + 3 \times 5^{3/4} \sqrt{854(1+\sqrt{5})} e^{\sqrt{427/15} \pi} + 25620 \pi + 1281 \pi} \right) \right.$$

Now, we have that:

Now, the next order reads

$$\begin{aligned} \mathcal{S}_2^{\text{MGD}} &= \frac{A_2}{4} \\ &= \frac{\epsilon^2}{4} \int_{y_0}^0 dy \mathcal{L}_2 = \frac{\pi M^2}{32} \left[ U_1(\xi, y_0) + U_2(\xi) \log\left(\frac{2}{1+y_0}\right) + U_3(\xi) \log(y_0) \right], \end{aligned}$$

with ancillary functions  $U_1(\xi, y_0) = [2\xi(13-3y_0) - (\xi^2+4)(7-y_0)](1-y_0)$ ,  $U_2(\xi) = 16(\xi-2)^2$  and  $U_3(\xi) = 2[(\xi-2)^2 - 2\xi]$ . One can notice the contribution of the MGD parameter, encoding the finite brane tension, as one compares with the HEE for the Schwarzschild spacetime, corresponding to  $\ell \rightarrow 0$  and, hence,  $\xi \rightarrow 0$ . Henceforth, in the general relativistic case of a rigid brane,  $\sigma \rightarrow \infty$ , one recovers the 2<sup>nd</sup>-order correction for Schwarzschild spacetimes. On the other hand, the 2<sup>nd</sup>-order corrections ratio are given by

$$\Phi_2^{\text{MGD}} = \frac{\mathcal{S}_2^{\text{MGD}}}{\mathcal{S}_2^{\text{Schw}}} = 1 + \frac{\xi}{4}(\xi-6) + 4\xi \left[ \frac{1-y_0 - 2\log\left(\frac{2}{1+y_0}\right)}{7-8y_0+y_0^2 - 2\log y_0 - 16\log\left(\frac{2}{1+y_0}\right)} \right]. \quad (37)$$

Both corrections, the 1<sup>st</sup>- and the 2<sup>nd</sup>-order ones, have the MGD parameter as a dominant variable, when considering the minimal surface in large range, correspondly, the lower limit very close to zero. The 1<sup>st</sup>-order ratio does not depend on such range. However, the 2<sup>nd</sup>-order ratio has the limit

$$\Phi_2^{\text{MGD}}|_{y_0 \rightarrow 0} = 1 + \frac{\xi}{4}(\xi-6). \quad (38)$$

As  $\xi < 0$ , it is observed an increment of the value of this order of correction to the HEE. Irrespectively of the limit taken, the limit  $\xi \rightarrow 0$  recovers the 2<sup>nd</sup>-order correction for the HEE in a Schwarzschild spacetime.

From (37), for  $y_0 = 1$ , we obtain (38). From the following Ramanujan mock theta function:

([https://en.wikipedia.org/wiki/Mock\\_modular\\_form#Order\\_6](https://en.wikipedia.org/wiki/Mock_modular_form#Order_6))

$$\sigma(q) = \sum_{n \geq 0} \frac{q^{(n+1)(n+2)/2} (-q; q)_n}{(q; q^2)_{n+1}}$$

That is: (A053271 sequence OEIS)

$$\text{Sum}_{\{n \geq 0\}} q^{((n+1)(n+2)/2)} (1+q)(1+q^2)\dots(1+q^n)/((1-q)(1-q^3)\dots(1-q^{(2n+1)}))$$

From which:

$$\text{sum } q^{((n+1)(n+2)/2)} (1+q)(1+q^2)(1+q^n)/((1-q)(1-q^3)(1-q^{(2n+1)})), n = 0 \text{ to } k$$



$$\sum_{n=0}^k \frac{q^{1/2(n+1)(n+2)} (1+q)(1+q^2)(1+q^n)}{(1-q)(1-q^3)(1-q^{2n+1})}$$

$$\sum_{n=0}^k \frac{q^{1/2(n+1)(n+2)} (1+q)(1+q^2)(1+q^n)}{(1-q)(1-q^3)(1-q^{2n+1})}$$

For  $q = 0.5$  and  $n = 2$ , we develop the above formula in the following way:

$$\frac{(((0.5^{(2+1)(2+2)/2} (1+0.5)(1+0.5^2)(1+0.5^2))))}{(((1-0.5)(1-0.5^3)(1-0.5^{2*2+1}))})}$$

$$\frac{0.5^{(2+1) \times (2+2)/2} (1+0.5)(1+0.5^2)(1+0.5^2)}{(1-0.5)(1-0.5^3)(1-0.5^{2 \times 2+1})}$$

0.086405529953917050691244239631336405529953917050691244239...

**0.0864055...**

For  $\xi = 0.0864055$ , that is the result of above Ramanujan mock theta function, we obtain:

$$1 + (0.0864055/4) * (0.0864055 - 6)$$

**Input interpretation:**

$$1 + \frac{0.0864055}{4} (0.0864055 - 6)$$

**Result:**

0.8722582276075625

0.8722582276075625

From which:

$$(((1 + (0.0864055/4) * (0.0864055 - 6))))^{1/256}$$

**Input interpretation:**

$$\sqrt[256]{1 + \frac{0.0864055}{4} (0.0864055 - 6)}$$

**Result:**

0.999466276...

0.999466276... result very near to the value of the following Rogers-Ramanujan continued fraction:

$$\frac{e^{-\frac{\pi}{\sqrt{5}}}}{\sqrt{5}} = 1 - \frac{e^{-\pi\sqrt{5}}}{1 + \frac{e^{-2\pi\sqrt{5}}}{1 + \frac{e^{-3\pi\sqrt{5}}}{1 + \frac{e^{-4\pi\sqrt{5}}}{1 + \dots}}}} \approx 0.9991104684$$

and to the dilaton value **0.989117352243 =  $\phi$**

1/2 log base 0.999466276(((1+(0.0864055/4)\*(0.0864055-6))))-Pi+1/golden ratio

**Input interpretation:**

$$\frac{1}{2} \log_{0.999466276} \left( 1 + \frac{0.0864055}{4} (0.0864055 - 6) \right) - \pi + \frac{1}{\phi}$$

$\log_b(x)$  is the base- $b$  logarithm

$\phi$  is the golden ratio

**Result:**

125.476...

125.476... result very near to the dilaton mass calculated as a type of Higgs boson: 125 GeV for  $T = 0$  and to the Higgs boson mass 125.18 GeV

**Alternative representation:**

$$\frac{1}{2} \log_{0.999466} \left( 1 + \frac{1}{4} \times 0.0864055 (0.0864055 - 6) \right) - \pi + \frac{1}{\phi} = -\pi + \frac{1}{\phi} + \frac{\log\left(1 - \frac{0.510967}{4}\right)}{2 \log(0.999466)}$$

**Series representations:**

$$\frac{1}{2} \log_{0.999466} \left( 1 + \frac{1}{4} \times 0.0864055 (0.0864055 - 6) \right) - \pi + \frac{1}{\phi} =$$

$$\frac{1}{\phi} - \pi - \frac{\sum_{k=1}^{\infty} \frac{(-1)^k (-0.127742)^k}{k}}{2 \log(0.999466)}$$

$$\frac{1}{2} \log_{0.999466} \left( 1 + \frac{1}{4} \times 0.0864055 (0.0864055 - 6) \right) - \pi + \frac{1}{\phi} =$$

$$\frac{1}{\phi} - \pi - 936.564 \log(0.872258) - \frac{1}{2} \log(0.872258) \sum_{k=0}^{\infty} (-0.000533724)^k G(k)$$

for  $\left( G(0) = 0 \text{ and } G(k) = \frac{(-1)^{1+k} k}{2(1+k)(2+k)} + \sum_{j=1}^k \frac{(-1)^{1+j} G(-j+k)}{1+j} \right)$

1/2log base 0.999466276(((1+(0.0864055/4)\*(0.0864055-6))))+11+1/golden ratio

**Input interpretation:**

$$\frac{1}{2} \log_{0.999466276} \left( 1 + \frac{0.0864055}{4} (0.0864055 - 6) \right) + 11 + \frac{1}{\phi}$$

$\log_b(x)$  is the base- $b$  logarithm

$\phi$  is the golden ratio

**Result:**

139.618...

139.618... result practically equal to the rest mass of Pion meson 139.57 MeV

**Alternative representation:**

$$\frac{1}{2} \log_{0.999466} \left( 1 + \frac{1}{4} \times 0.0864055 (0.0864055 - 6) \right) + 11 + \frac{1}{\phi} = 11 + \frac{1}{\phi} + \frac{\log\left(1 - \frac{0.510967}{4}\right)}{2 \log(0.999466)}$$

**Series representations:**

$$\frac{1}{2} \log_{0.999466} \left( 1 + \frac{1}{4} \times 0.0864055 (0.0864055 - 6) \right) + 11 + \frac{1}{\phi} =$$

$$11 + \frac{1}{\phi} - \frac{\sum_{k=1}^{\infty} \frac{(-1)^k (-0.127742)^k}{k}}{2 \log(0.999466)}$$

$$\frac{1}{2} \log_{0.999466} \left( 1 + \frac{1}{4} \times 0.0864055 (0.0864055 - 6) \right) + 11 + \frac{1}{\phi} =$$

$$11 + \frac{1}{\phi} - 936.564 \log(0.872258) - \frac{1}{2} \log(0.872258) \sum_{k=0}^{\infty} (-0.000533724)^k G(k)$$

for  $\left( G(0) = 0 \text{ and } G(k) = \frac{(-1)^{1+k} k}{2(1+k)(2+k)} + \sum_{j=1}^k \frac{(-1)^{1+j} G(-j+k)}{1+j} \right)$

$$27 \times \frac{1}{4} \log_{\text{base } 0.999466276} \left( \left( 1 + \left( \frac{0.0864055}{4} \right) \times (0.0864055 - 6) \right) \right) + 1$$

**Input interpretation:**

$$27 \times \frac{1}{4} \log_{0.999466276} \left( 1 + \frac{0.0864055}{4} (0.0864055 - 6) \right) + 1$$

$\log_b(x)$  is the base- $b$  logarithm

**Result:**

1729.00...

1729

This result is very near to the mass of candidate glueball  $f_0(1710)$  meson. Furthermore, 1728 occurs in the algebraic formula for the  $j$ -invariant of an elliptic curve. As a consequence, it is sometimes called a Zagier as a pun on the Gross–Zagier theorem. The number 1728 is one less than the Hardy–Ramanujan number 1729

**Alternative representation:**

$$\frac{27}{4} \log_{0.999466} \left( 1 + \frac{1}{4} \times 0.0864055 (0.0864055 - 6) \right) + 1 = 1 + \frac{27 \log \left( 1 - \frac{0.510967}{4} \right)}{4 \log(0.999466)}$$

**Series representations:**

$$\frac{27}{4} \log_{0.999466} \left( 1 + \frac{1}{4} \times 0.0864055 (0.0864055 - 6) \right) + 1 = 1 - \frac{27 \sum_{k=1}^{\infty} \frac{(-1)^k (-0.127742)^k}{k}}{4 \log(0.999466)}$$

$$\begin{aligned} \frac{27}{4} \log_{0.999466} \left( 1 + \frac{1}{4} \times 0.0864055 (0.0864055 - 6) \right) + 1 = \\ 1 - 12\,643.6 \log(0.872258) - 6.75 \log(0.872258) \sum_{k=0}^{\infty} (-0.000533724)^k G(k) \\ \text{for } \left( G(0) = 0 \text{ and } G(k) = \frac{(-1)^{1+k} k}{2(1+k)(2+k)} + \sum_{j=1}^k \frac{(-1)^{1+j} G(-j+k)}{1+j} \right) \end{aligned}$$

$$12 * 1/4 \log \text{ base } 0.999466276(((1+(0.0864055/4)*(0.0864055-6))))-29-11$$

**Input interpretation:**

$$12 \times \frac{1}{4} \log_{0.999466276} \left( 1 + \frac{0.0864055}{4} (0.0864055 - 6) \right) - 29 - 11$$

$\log_b(x)$  is the base- $b$  logarithm

**Result:**

728.000...

728 (Ramanujan taxicab number)

**Alternative representation:**

$$\begin{aligned} \frac{12}{4} \log_{0.999466} \left( 1 + \frac{1}{4} \times 0.0864055 (0.0864055 - 6) \right) - 29 - 11 = \\ -40 + \frac{12 \log \left( 1 - \frac{0.510967}{4} \right)}{4 \log(0.999466)} \end{aligned}$$

**Series representations:**

$$\begin{aligned} \frac{12}{4} \log_{0.999466} \left( 1 + \frac{1}{4} \times 0.0864055 (0.0864055 - 6) \right) - 29 - 11 = \\ -40 - \frac{3 \sum_{k=1}^{\infty} \frac{(-1)^k (-0.127742)^k}{k}}{\log(0.999466)} \end{aligned}$$

$$\frac{12}{4} \log_{0.999466} \left( 1 + \frac{1}{4} \times 0.0864055 (0.0864055 - 6) \right) - 29 - 11 =$$

$$-40 - 5619.38 \log(0.872258) - 3 \log(0.872258) \sum_{k=0}^{\infty} (-0.000533724)^k G(k)$$

$$\text{for } \left( G(0) = 0 \text{ and } G(k) = \frac{(-1)^{1+k} k}{2(1+k)(2+k)} + \sum_{j=1}^k \frac{(-1)^{1+j} G(-j+k)}{1+j} \right)$$

$$12 * 1/4 \log \text{ base } 0.999466276(((1+(0.0864055/4)*(0.0864055-6))))-29-11+47$$

**Input interpretation:**

$$12 \times \frac{1}{4} \log_{0.999466276} \left( 1 + \frac{0.0864055}{4} (0.0864055 - 6) \right) - 29 - 11 + 47$$

$\log_b(x)$  is the base- $b$  logarithm

**Result:**

775.000...

775 result practically equal to the rest mass of Charged rho meson 775.11

**Alternative representation:**

$$\frac{12}{4} \log_{0.999466} \left( 1 + \frac{1}{4} \times 0.0864055 (0.0864055 - 6) \right) - 29 - 11 + 47 =$$

$$7 + \frac{12 \log \left( 1 - \frac{0.510967}{4} \right)}{4 \log(0.999466)}$$

**Series representations:**

$$\frac{12}{4} \log_{0.999466} \left( 1 + \frac{1}{4} \times 0.0864055 (0.0864055 - 6) \right) - 29 - 11 + 47 =$$

$$7 - \frac{3 \sum_{k=1}^{\infty} \frac{(-1)^k (-0.127742)^k}{k}}{\log(0.999466)}$$

$$\frac{12}{4} \log_{0.999466} \left( 1 + \frac{1}{4} \times 0.0864055 (0.0864055 - 6) \right) - 29 - 11 + 47 =$$

$$7 - 5619.38 \log(0.872258) - 3 \log(0.872258) \sum_{k=0}^{\infty} (-0.000533724)^k G(k)$$

$$\text{for } \left( G(0) = 0 \text{ and } G(k) = \frac{(-1)^{1+k} k}{2(1+k)(2+k)} + \sum_{j=1}^k \frac{(-1)^{1+j} G(-j+k)}{1+j} \right)$$

Now, we have that:

$$\Phi_2^{\text{EMGD}_1} = 8 \left[ \frac{y_0^2 - 4y_0 + 3 - 2 \log(y_0) - 8 \log\left(\frac{2}{1+y_0}\right)}{y_0^2 - 8y_0 + 7 - 2 \log(y_0) - 16 \log\left(\frac{2}{1+y_0}\right)} \right] \quad (52)$$

For  $y_0 = 0.99$ , we obtain:

$$8 * \left[ \frac{((0.99^2 - 4 * 0.99 + 3 - 2 \ln(0.99) - 8 \ln(2/1.99)))}{((0.99^2 - 8 * 0.99 + 7 - 2 \ln(0.99) - 16 \ln(2/1.99)))} \right]$$

**Input:**

$$8 \times \frac{0.99^2 - 4 \times 0.99 + 3 - 2 \log(0.99) - 8 \log\left(\frac{2}{1.99}\right)}{0.99^2 - 8 \times 0.99 + 7 - 2 \log(0.99) - 16 \log\left(\frac{2}{1.99}\right)}$$

$\log(x)$  is the natural logarithm

**Result:**

317234.6106478058859430701207273346791451318951095538859066...

317234.6106478...

**Alternative representations:**

$$\frac{8(0.99^2 - 4 \times 0.99 + 3 - 2 \log(0.99) - 8 \log\left(\frac{2}{1.99}\right))}{0.99^2 - 8 \times 0.99 + 7 - 2 \log(0.99) - 16 \log\left(\frac{2}{1.99}\right)} = \frac{\left( \frac{8(-0.96 - 2 \log(a) \log_a(0.99) - 8 \log(a) \log_a\left(\frac{2}{1.99}\right) + 0.99^2)}{-0.92 - 2 \log(a) \log_a(0.99) - 16 \log(a) \log_a\left(\frac{2}{1.99}\right) + 0.99^2} \right)}{\left( \frac{8(0.0201 - 2 \log(a) \log_a(0.99) - 8 \log(a) \log_a(1.00503))}{0.0601 - 2 \log(a) \log_a(0.99) - 16 \log(a) \log_a(1.00503)} \right)}$$

$$\frac{8(0.99^2 - 4 \times 0.99 + 3 - 2 \log(0.99) - 8 \log(\frac{2}{1.99}))}{0.99^2 - 8 \times 0.99 + 7 - 2 \log(0.99) - 16 \log(\frac{2}{1.99})} = \frac{8(-0.96 - 2 \log_e(0.99) - 8 \log_e(\frac{2}{1.99}) + 0.99^2)}{-0.92 - 2 \log_e(0.99) - 16 \log_e(\frac{2}{1.99}) + 0.99^2} = \frac{8(0.0201 - 2 \log_e(0.99) - 8 \log_e(1.00503))}{0.0601 - 2 \log_e(0.99) - 16 \log_e(1.00503)}$$

$$\frac{8(0.99^2 - 4 \times 0.99 + 3 - 2 \log(0.99) - 8 \log(\frac{2}{1.99}))}{0.99^2 - 8 \times 0.99 + 7 - 2 \log(0.99) - 16 \log(\frac{2}{1.99})} = \frac{8(-0.96 + 2 \text{Li}_1(0.01) + 8 \text{Li}_1(1 - \frac{2}{1.99}) + 0.99^2)}{-0.92 + 2 \text{Li}_1(0.01) + 16 \text{Li}_1(1 - \frac{2}{1.99}) + 0.99^2} = \frac{8(0.0201 + 8 \text{Li}_1(-0.00502513) + 2 \text{Li}_1(0.01))}{0.0601 + 16 \text{Li}_1(-0.00502513) + 2 \text{Li}_1(0.01)}$$

### Series representations:

$$\frac{8(0.99^2 - 4 \times 0.99 + 3 - 2 \log(0.99) - 8 \log(\frac{2}{1.99}))}{0.99^2 - 8 \times 0.99 + 7 - 2 \log(0.99) - 16 \log(\frac{2}{1.99})} = \frac{-0.0804 + \sum_{k=1}^{\infty} \frac{-8(-1)^k (-0.01)^k - 32(-0.00502513)^k}{k}}{-0.03005 + \sum_{k=1}^{\infty} \frac{-(-1)^k (-0.01)^k - 8(-0.00502513)^k}{k}}$$

$$\frac{8(0.99^2 - 4 \times 0.99 + 3 - 2 \log(0.99) - 8 \log(\frac{2}{1.99}))}{0.99^2 - 8 \times 0.99 + 7 - 2 \log(0.99) - 16 \log(\frac{2}{1.99})} = \frac{\left( 8 \left[ -0.005025 + i\pi \left[ \frac{\arg(0.99 - x)}{2\pi} \right] + 4 i\pi \left[ \frac{\arg(1.00503 - x)}{2\pi} \right] + 2.5 \log(x) + 0.125 \sum_{k=1}^{\infty} \frac{(-1)^k (-4(0.99 - x)^k - 16(1.00503 - x)^k) x^{-k}}{k} \right)}{\left( -0.015025 + i\pi \left[ \frac{\arg(0.99 - x)}{2\pi} \right] + 8 i\pi \left[ \frac{\arg(1.00503 - x)}{2\pi} \right] + 4.5 \log(x) + \sum_{k=1}^{\infty} \frac{(-1)^k (-0.5(0.99 - x)^k - 4(1.00503 - x)^k) x^{-k}}{k} \right)} \text{ for } x < 0$$



$$\frac{8 \left( 0.99^2 - 4 \times 0.99 + 3 - 2 \log(0.99) - 8 \log\left(\frac{2}{1.99}\right) \right)}{0.99^2 - 8 \times 0.99 + 7 - 2 \log(0.99) - 16 \log\left(\frac{2}{1.99}\right)} =$$

$$\frac{-0.0804 + \sum_{j=1}^{\infty} \left( 8 \left( \text{Res}_{s=-j} \frac{(-0.01)^{-s} \Gamma(-s)^2 \Gamma(1+s)}{\Gamma(1-s)} \right) + 32 \left( \text{Res}_{s=-j} \frac{e^{5.2933 s} \Gamma(-s)^2 \Gamma(1+s)}{\Gamma(1-s)} \right) \right)}{-0.03005 + \sum_{j=1}^{\infty} \left( \text{Res}_{s=-j} \frac{(-0.01)^{-s} \Gamma(-s)^2 \Gamma(1+s)}{\Gamma(1-s)} + 8 \left( \text{Res}_{s=-j} \frac{e^{5.2933 s} \Gamma(-s)^2 \Gamma(1+s)}{\Gamma(1-s)} \right) \right)}$$

$\arg(z)$  is the complex argument

$\lfloor x \rfloor$  is the floor function

$i$  is the imaginary unit

$\Gamma(x)$  is the gamma function

$\text{Res}_f$  is a complex residue  
 $\neq 0$

and:

$$1 + \frac{1}{\sqrt{\left( 8 \times \frac{0.99^2 - 4 \times 0.99 + 3 - 2 \ln(0.99) - 8 \ln(2/1.99)}{0.99^2 - 8 \times 0.99 + 7 - 2 \ln(0.99) - 16 \ln(2/1.99)} \right)}}$$

**Input:**

$$1 + \frac{1}{\sqrt{8 \times \frac{0.99^2 - 4 \times 0.99 + 3 - 2 \log(0.99) - 8 \log\left(\frac{2}{1.99}\right)}{0.99^2 - 8 \times 0.99 + 7 - 2 \log(0.99) - 16 \log\left(\frac{2}{1.99}\right)}}$$

$\log(x)$  is the natural logarithm

**Result:**

1.001775455200579765569659068701249369523822062429397193634...

1.0017754552... result very near to the following Rogers-Ramanujan continued fraction:

$$\frac{e^{\frac{2\pi}{5}}}{\sqrt{\phi\sqrt{5}-\phi}} = 1 + \frac{e^{-2\pi}}{1 + \frac{e^{-4\pi}}{1 + \frac{e^{-6\pi}}{1 + \frac{e^{-8\pi}}{1 + \dots}}}} \approx 1.0018674362$$

**Alternative representations:**

$$1 + \frac{1}{\sqrt{\frac{8(0.99^2 - 4 \times 0.99 + 3 - 2 \log(0.99) - 8 \log(\frac{2}{1.99}))}{0.99^2 - 8 \times 0.99 + 7 - 2 \log(0.99) - 16 \log(\frac{2}{1.99})}}} =$$

$$\left( 1 + \frac{1}{\sqrt{\frac{8(-0.96 - 2 \log_e(0.99) - 8 \log_e(\frac{2}{1.99}) + 0.99^2)}{-0.92 - 2 \log_e(0.99) - 16 \log_e(\frac{2}{1.99}) + 0.99^2}}} = 1 + \frac{1}{\sqrt{\frac{8(0.0201 - 2 \log_e(0.99) - 8 \log_e(1.00503))}{0.0601 - 2 \log_e(0.99) - 16 \log_e(1.00503)}}} \right)$$

$$1 + \frac{1}{\sqrt{\frac{8(0.99^2 - 4 \times 0.99 + 3 - 2 \log(0.99) - 8 \log(\frac{2}{1.99}))}{0.99^2 - 8 \times 0.99 + 7 - 2 \log(0.99) - 16 \log(\frac{2}{1.99})}}} =$$

$$\left( 1 + \frac{1}{\sqrt{\frac{8(-0.96 - 2 \log(a) \log_a(0.99) - 8 \log(a) \log_a(\frac{2}{1.99}) + 0.99^2)}{-0.92 - 2 \log(a) \log_a(0.99) - 16 \log(a) \log_a(\frac{2}{1.99}) + 0.99^2}}} =$$

$$\left( 1 + \frac{1}{\sqrt{\frac{8(0.0201 - 2 \log(a) \log_a(0.99) - 8 \log(a) \log_a(1.00503))}{0.0601 - 2 \log(a) \log_a(0.99) - 16 \log(a) \log_a(1.00503)}}} \right)$$

$$1 + \frac{1}{\sqrt{\frac{8(0.99^2 - 4 \times 0.99 + 3 - 2 \log(0.99) - 8 \log(\frac{2}{1.99}))}{0.99^2 - 8 \times 0.99 + 7 - 2 \log(0.99) - 16 \log(\frac{2}{1.99})}}} =$$

$$\left( 1 + \frac{1}{\sqrt{\frac{8(-0.96 + 2 \operatorname{Li}_1(0.01) + 8 \operatorname{Li}_1(1 - \frac{2}{1.99}) + 0.99^2)}{-0.92 + 2 \operatorname{Li}_1(0.01) + 16 \operatorname{Li}_1(1 - \frac{2}{1.99}) + 0.99^2}}} = \right.$$

$$\left. 1 + \frac{1}{\sqrt{\frac{8(0.0201 + 8 \operatorname{Li}_1(-0.00502513) + 2 \operatorname{Li}_1(0.01))}{0.0601 + 16 \operatorname{Li}_1(-0.00502513) + 2 \operatorname{Li}_1(0.01)}}} \right)$$

### Series representations:

$$1 + \frac{1}{\sqrt{\frac{8(0.99^2 - 4 \times 0.99 + 3 - 2 \log(0.99) - 8 \log(\frac{2}{1.99}))}{0.99^2 - 8 \times 0.99 + 7 - 2 \log(0.99) - 16 \log(\frac{2}{1.99})}}} =$$

$$1 + 1 / \left( \sqrt{\frac{7(-0.00719286 + \log(0.99) + 3.42857 \log(1.00503))}{-0.03005 + \log(0.99) + 8 \log(1.00503)}} \right)$$

$$\sum_{k=0}^{\infty} e^{-1.94591k} \binom{\frac{1}{2}}{k} \left( \frac{-0.00719286 + \log(0.99) + 3.42857 \log(1.00503)}{-0.03005 + \log(0.99) + 8 \log(1.00503)} \right)^{-k}$$

$$1 + \frac{1}{\sqrt{\frac{8(0.99^2 - 4 \times 0.99 + 3 - 2 \log(0.99) - 8 \log(\frac{2}{1.99}))}{0.99^2 - 8 \times 0.99 + 7 - 2 \log(0.99) - 16 \log(\frac{2}{1.99})}}} =$$

$$1 + 1 / \left( \sqrt{\frac{7(-0.00719286 + \log(0.99) + 3.42857 \log(1.00503))}{-0.03005 + \log(0.99) + 8 \log(1.00503)}} \right)$$

$$\sum_{k=0}^{\infty} \frac{(-0.142857)^k \left( \frac{-0.00719286 + \log(0.99) + 3.42857 \log(1.00503)}{-0.03005 + \log(0.99) + 8 \log(1.00503)} \right)^{-k} \left( -\frac{1}{2} \right)_k}{k!}$$

$$1 + \frac{1}{\sqrt{\frac{8 \left( 0.99^2 - 4 \times 0.99 + 3 - 2 \log(0.99) - 8 \log\left(\frac{2}{1.99}\right) \right)}{0.99^2 - 8 \times 0.99 + 7 - 2 \log(0.99) - 16 \log\left(\frac{2}{1.99}\right)}}} =$$

$$1 + \frac{\sqrt{\frac{-0.0804 + \sum_{k=1}^{\infty} \frac{-8(-1)^k (-0.01)^k - 32(-0.00502513)^k}{k}}{-0.03005 + \sum_{k=1}^{\infty} \frac{-(-1)^k (-0.01)^k - 8(-0.00502513)^k}{k}}}}{\sqrt{\frac{-0.0804 + \sum_{k=1}^{\infty} \frac{-8(-1)^k (-0.01)^k - 32(-0.00502513)^k}{k}}{-0.03005 + \sum_{k=1}^{\infty} \frac{-(-1)^k (-0.01)^k - 8(-0.00502513)^k}{k}}}}$$

$$\text{Pi} * \text{sqrt}(\left(\left(\left(8 * \left[\left(\left(0.99^2 - 4 * 0.99 + 3 - 2 \ln(0.99) - 8 \ln(2/1.99)\right)\right)\right]\right)\right) / \left(\left(\left(0.99^2 - 8 * 0.99 + 7 - 2 \ln(0.99) - 16 \ln(2/1.99)\right)\right)\right)\right)) - 29 - 11 - 2/5$$

**Input:**

$$\pi \sqrt{8 \times \frac{0.99^2 - 4 \times 0.99 + 3 - 2 \log(0.99) - 8 \log\left(\frac{2}{1.99}\right)}{0.99^2 - 8 \times 0.99 + 7 - 2 \log(0.99) - 16 \log\left(\frac{2}{1.99}\right)} - 29 - 11 - \frac{2}{5}}$$

log(x) is the natural logarithm

**Result:**

1729.057574915955445991506989948194881041631226369839617527...

1729.05757491...

This result is very near to the mass of candidate glueball  $f_0(1710)$  meson. Furthermore, 1728 occurs in the algebraic formula for the j-invariant of an elliptic curve. As a consequence, it is sometimes called a Zagier as a pun on the Gross–Zagier theorem. The number 1728 is one less than the Hardy–Ramanujan number 1729

**Alternative representations:**

$$\pi \sqrt{\frac{8(0.99^2 - 4 \times 0.99 + 3 - 2 \log(0.99) - 8 \log(\frac{2}{1.99}))}{0.99^2 - 8 \times 0.99 + 7 - 2 \log(0.99) - 16 \log(\frac{2}{1.99})}} - 29 - 11 - \frac{2}{5} =$$

$$\left( -40 - \frac{2}{5} + \pi \sqrt{\frac{8(-0.96 - 2 \log(a) \log_a(0.99) - 8 \log(a) \log_a(\frac{2}{1.99}) + 0.99^2)}{-0.92 - 2 \log(a) \log_a(0.99) - 16 \log(a) \log_a(\frac{2}{1.99}) + 0.99^2}} \right) =$$

$$-\frac{202}{5} + \pi \sqrt{\frac{8(0.0201 - 2 \log(a) \log_a(0.99) - 8 \log(a) \log_a(1.00503))}{0.0601 - 2 \log(a) \log_a(0.99) - 16 \log(a) \log_a(1.00503)}}$$

$$\pi \sqrt{\frac{8(0.99^2 - 4 \times 0.99 + 3 - 2 \log(0.99) - 8 \log(\frac{2}{1.99}))}{0.99^2 - 8 \times 0.99 + 7 - 2 \log(0.99) - 16 \log(\frac{2}{1.99})}} - 29 - 11 - \frac{2}{5} =$$

$$\left( -40 - \frac{2}{5} + \pi \sqrt{\frac{8(-0.96 - 2 \log_e(0.99) - 8 \log_e(\frac{2}{1.99}) + 0.99^2)}{-0.92 - 2 \log_e(0.99) - 16 \log_e(\frac{2}{1.99}) + 0.99^2}} \right) =$$

$$-\frac{202}{5} + \pi \sqrt{\frac{8(0.0201 - 2 \log_e(0.99) - 8 \log_e(1.00503))}{0.0601 - 2 \log_e(0.99) - 16 \log_e(1.00503)}}$$

$$\pi \sqrt{\frac{8(0.99^2 - 4 \times 0.99 + 3 - 2 \log(0.99) - 8 \log(\frac{2}{1.99}))}{0.99^2 - 8 \times 0.99 + 7 - 2 \log(0.99) - 16 \log(\frac{2}{1.99})}} - 29 - 11 - \frac{2}{5} =$$

$$\left( -40 - \frac{2}{5} + \pi \sqrt{\frac{8(-0.96 + 2 \text{Li}_1(0.01) + 8 \text{Li}_1(1 - \frac{2}{1.99}) + 0.99^2)}{-0.92 + 2 \text{Li}_1(0.01) + 16 \text{Li}_1(1 - \frac{2}{1.99}) + 0.99^2}} \right) =$$

$$-\frac{202}{5} + \pi \sqrt{\frac{8(0.0201 + 8 \text{Li}_1(-0.00502513) + 2 \text{Li}_1(0.01))}{0.0601 + 16 \text{Li}_1(-0.00502513) + 2 \text{Li}_1(0.01)}}$$

**Series representations:**

$$\pi \sqrt{\frac{8(0.99^2 - 4 \times 0.99 + 3 - 2 \log(0.99) - 8 \log(\frac{2}{1.99}))}{0.99^2 - 8 \times 0.99 + 7 - 2 \log(0.99) - 16 \log(\frac{2}{1.99})}} - 29 - 11 - \frac{2}{5} =$$

$$\frac{1}{5} \left( -202 + 5 \pi \sqrt{\frac{-0.0804 + \sum_{k=1}^{\infty} \frac{-8(-1)^k (-0.01)^k - 32(-0.00502513)^k}{k}}{-0.03005 + \sum_{k=1}^{\infty} \frac{-(-1)^k (-0.01)^k - 8(-0.00502513)^k}{k}}} \right)$$

$$\pi \sqrt{\frac{8(0.99^2 - 4 \times 0.99 + 3 - 2 \log(0.99) - 8 \log(\frac{2}{1.99}))}{0.99^2 - 8 \times 0.99 + 7 - 2 \log(0.99) - 16 \log(\frac{2}{1.99})}} - 29 - 11 - \frac{2}{5} =$$

$$-\frac{202}{5} + \pi \sqrt{\frac{7(-0.00719286 + \log(0.99) + 3.42857 \log(1.00503))}{-0.03005 + \log(0.99) + 8 \log(1.00503)}}$$

$$\sum_{k=0}^{\infty} e^{-1.94591k} \left(\frac{1}{2}\right)^k \left(\frac{-0.00719286 + \log(0.99) + 3.42857 \log(1.00503)}{-0.03005 + \log(0.99) + 8 \log(1.00503)}\right)^{-k}$$

$$\pi \sqrt{\frac{8(0.99^2 - 4 \times 0.99 + 3 - 2 \log(0.99) - 8 \log(\frac{2}{1.99}))}{0.99^2 - 8 \times 0.99 + 7 - 2 \log(0.99) - 16 \log(\frac{2}{1.99})}} - 29 - 11 - \frac{2}{5} =$$

$$-\frac{202}{5} + \pi \sqrt{\frac{7(-0.00719286 + \log(0.99) + 3.42857 \log(1.00503))}{-0.03005 + \log(0.99) + 8 \log(1.00503)}}$$

$$\sum_{k=0}^{\infty} \frac{(-0.142857)^k \left(\frac{-0.00719286 + \log(0.99) + 3.42857 \log(1.00503)}{-0.03005 + \log(0.99) + 8 \log(1.00503)}\right)^{-k} \left(-\frac{1}{2}\right)^k}{k!}$$

sqrt(((8\*(((0.99^2-4\*0.99+3-2 ln(0.99)-8 ln(2/1.99)))))/(((0.99^2-8\*0.99+7-2 ln(0.99)-16 ln(2/1.99)))))))+123+47-4-2/golden ratio

**Input:**

$$\sqrt{8 \times \frac{0.99^2 - 4 \times 0.99 + 3 - 2 \log(0.99) - 8 \log(\frac{2}{1.99})}{0.99^2 - 8 \times 0.99 + 7 - 2 \log(0.99) - 16 \log(\frac{2}{1.99})}} + 123 + 47 - 4 - \frac{2}{\phi}$$

log(x) is the natural logarithm

φ is the golden ratio

**Result:**

727.9997713010442438496196170375748496892713575184118989732...

727.9997713... ≈ 728 (Ramanujan taxicab number)

**Alternative representations:**

$$\sqrt{\frac{8(0.99^2 - 4 \times 0.99 + 3 - 2 \log(0.99) - 8 \log(\frac{2}{1.99}))}{0.99^2 - 8 \times 0.99 + 7 - 2 \log(0.99) - 16 \log(\frac{2}{1.99})}} + 123 + 47 - 4 - \frac{2}{\phi} =$$

$$\left( 166 - \frac{2}{\phi} + \sqrt{\frac{8(-0.96 - 2 \log(a) \log_a(0.99) - 8 \log(a) \log_a(\frac{2}{1.99}) + 0.99^2)}{-0.92 - 2 \log(a) \log_a(0.99) - 16 \log(a) \log_a(\frac{2}{1.99}) + 0.99^2}} \right) =$$

$$166 - \frac{2}{\phi} + \sqrt{\frac{8(0.0201 - 2 \log(a) \log_a(0.99) - 8 \log(a) \log_a(1.00503))}{0.0601 - 2 \log(a) \log_a(0.99) - 16 \log(a) \log_a(1.00503)}}$$

$$\sqrt{\frac{8(0.99^2 - 4 \times 0.99 + 3 - 2 \log(0.99) - 8 \log(\frac{2}{1.99}))}{0.99^2 - 8 \times 0.99 + 7 - 2 \log(0.99) - 16 \log(\frac{2}{1.99})}} + 123 + 47 - 4 - \frac{2}{\phi} =$$

$$\left( 166 - \frac{2}{\phi} + \sqrt{\frac{8(-0.96 - 2 \log_e(0.99) - 8 \log_e(\frac{2}{1.99}) + 0.99^2)}{-0.92 - 2 \log_e(0.99) - 16 \log_e(\frac{2}{1.99}) + 0.99^2}} \right) =$$

$$166 - \frac{2}{\phi} + \sqrt{\frac{8(0.0201 - 2 \log_e(0.99) - 8 \log_e(1.00503))}{0.0601 - 2 \log_e(0.99) - 16 \log_e(1.00503)}}$$

$$\sqrt{\frac{8(0.99^2 - 4 \times 0.99 + 3 - 2 \log(0.99) - 8 \log(\frac{2}{1.99}))}{0.99^2 - 8 \times 0.99 + 7 - 2 \log(0.99) - 16 \log(\frac{2}{1.99})}} + 123 + 47 - 4 - \frac{2}{\phi} =$$

$$\left( 166 - \frac{2}{\phi} + \sqrt{\frac{8(-0.96 + 2 \text{Li}_1(0.01) + 8 \text{Li}_1(1 - \frac{2}{1.99}) + 0.99^2)}{-0.92 + 2 \text{Li}_1(0.01) + 16 \text{Li}_1(1 - \frac{2}{1.99}) + 0.99^2}} \right) =$$

$$166 - \frac{2}{\phi} + \sqrt{\frac{8(0.0201 + 8 \text{Li}_1(-0.00502513) + 2 \text{Li}_1(0.01))}{0.0601 + 16 \text{Li}_1(-0.00502513) + 2 \text{Li}_1(0.01)}}$$

**Series representations:**

$$\sqrt{\frac{8(0.99^2 - 4 \times 0.99 + 3 - 2 \log(0.99) - 8 \log(\frac{2}{1.99}))}{0.99^2 - 8 \times 0.99 + 7 - 2 \log(0.99) - 16 \log(\frac{2}{1.99})}} + 123 + 47 - 4 - \frac{2}{\phi} =$$

$$166 - \frac{2}{\phi} + \sqrt{\frac{7(-0.00719286 + \log(0.99) + 3.42857 \log(1.00503))}{-0.03005 + \log(0.99) + 8 \log(1.00503)}}$$

$$\sum_{k=0}^{\infty} e^{-1.94591k} \binom{\frac{1}{2}}{k} \left( \frac{-0.00719286 + \log(0.99) + 3.42857 \log(1.00503)}{-0.03005 + \log(0.99) + 8 \log(1.00503)} \right)^k$$

$$\sqrt{\frac{8(0.99^2 - 4 \times 0.99 + 3 - 2 \log(0.99) - 8 \log(\frac{2}{1.99}))}{0.99^2 - 8 \times 0.99 + 7 - 2 \log(0.99) - 16 \log(\frac{2}{1.99})}} + 123 + 47 - 4 - \frac{2}{\phi} =$$

$$\frac{-2 + 166\phi + \phi \sqrt{\frac{-0.0804 + \sum_{k=1}^{\infty} \frac{-8(-1)^k (-0.01)^k - 32(-0.00502513)^k}{k}}{-0.03005 + \sum_{k=1}^{\infty} \frac{-(-1)^k (-0.01)^k - 8(-0.00502513)^k}{k}}}}{\phi}$$

$$\sqrt{\frac{8(0.99^2 - 4 \times 0.99 + 3 - 2 \log(0.99) - 8 \log(\frac{2}{1.99}))}{0.99^2 - 8 \times 0.99 + 7 - 2 \log(0.99) - 16 \log(\frac{2}{1.99})}} + 123 + 47 - 4 - \frac{2}{\phi} =$$

$$166 - \frac{2}{\phi} + \sqrt{\frac{7(-0.00719286 + \log(0.99) + 3.42857 \log(1.00503))}{-0.03005 + \log(0.99) + 8 \log(1.00503)}}$$

$$\sum_{k=0}^{\infty} \frac{(-0.142857)^k \left( \frac{-0.00719286 + \log(0.99) + 3.42857 \log(1.00503)}{-0.03005 + \log(0.99) + 8 \log(1.00503)} \right)^k \left( -\frac{1}{2} \right)_k}{k!}$$

For  $y_0 = 0.01$ , we obtain:

$$8 * [(((0.01^2 - 4 * 0.01 + 3 - 2 \ln(0.01) - 8 \ln(2/1.01)))) / (((0.01^2 - 8 * 0.01 + 7 - 2 \ln(0.01)) - 16 \ln(2/1.01)))]$$

**Input:**

$$8 \times \frac{0.01^2 - 4 \times 0.01 + 3 - 2 \log(0.01) - 8 \log(\frac{2}{1.01})}{0.01^2 - 8 \times 0.01 + 7 - 2 \log(0.01) - 16 \log(\frac{2}{1.01})}$$

$\log(x)$  is the natural logarithm

**Result:**

10.3166...

10.3166...



**Alternative representations:**

$$\frac{8 \left( 0.01^2 - 4 \times 0.01 + 3 - 2 \log(0.01) - 8 \log\left(\frac{2}{1.01}\right) \right)}{0.01^2 - 8 \times 0.01 + 7 - 2 \log(0.01) - 16 \log\left(\frac{2}{1.01}\right)} = \frac{8 \left( 2.96 - 2 \log(a) \log_a(0.01) - 8 \log(a) \log_a\left(\frac{2}{1.01}\right) + 0.01^2 \right)}{6.92 - 2 \log(a) \log_a(0.01) - 16 \log(a) \log_a\left(\frac{2}{1.01}\right) + 0.01^2}$$

$$\frac{8 \left( 0.01^2 - 4 \times 0.01 + 3 - 2 \log(0.01) - 8 \log\left(\frac{2}{1.01}\right) \right)}{0.01^2 - 8 \times 0.01 + 7 - 2 \log(0.01) - 16 \log\left(\frac{2}{1.01}\right)} = \frac{8 \left( 2.96 - 2 \log_e(0.01) - 8 \log_e\left(\frac{2}{1.01}\right) + 0.01^2 \right)}{6.92 - 2 \log_e(0.01) - 16 \log_e\left(\frac{2}{1.01}\right) + 0.01^2}$$

$$\frac{8 \left( 0.01^2 - 4 \times 0.01 + 3 - 2 \log(0.01) - 8 \log\left(\frac{2}{1.01}\right) \right)}{0.01^2 - 8 \times 0.01 + 7 - 2 \log(0.01) - 16 \log\left(\frac{2}{1.01}\right)} = \frac{8 \left( 2.96 + 2 \operatorname{Li}_1(0.99) + 8 \operatorname{Li}_1\left(1 - \frac{2}{1.01}\right) + 0.01^2 \right)}{6.92 + 2 \operatorname{Li}_1(0.99) + 16 \operatorname{Li}_1\left(1 - \frac{2}{1.01}\right) + 0.01^2}$$

**Series representations:**

$$\frac{8 \left( 0.01^2 - 4 \times 0.01 + 3 - 2 \log(0.01) - 8 \log\left(\frac{2}{1.01}\right) \right)}{0.01^2 - 8 \times 0.01 + 7 - 2 \log(0.01) - 16 \log\left(\frac{2}{1.01}\right)} = \frac{-11.8404 + \sum_{k=1}^{\infty} \frac{-8(-1)^k (-0.99)^k - 32(-0.980198)^k}{k}}{-3.46005 + \sum_{k=1}^{\infty} \frac{-(-1)^k (-0.99)^k - 8(-0.980198)^k}{k}}$$

$$\frac{8 \left( 0.01^2 - 4 \times 0.01 + 3 - 2 \log(0.01) - 8 \log\left(\frac{2}{1.01}\right) \right)}{0.01^2 - 8 \times 0.01 + 7 - 2 \log(0.01) - 16 \log\left(\frac{2}{1.01}\right)} = \frac{\left( 8 \left( -0.740025 + i \pi \left[ \frac{\arg(0.01 - x)}{2 \pi} \right] + 4 i \pi \left[ \frac{\arg(1.9802 - x)}{2 \pi} \right] + 2.5 \log(x) + 0.125 \sum_{k=1}^{\infty} \frac{(-1)^k \left( -4(0.01 - x)^k - 16(1.9802 - x)^k \right) x^{-k}}{k} \right)}{\left( -1.73003 + i \pi \left[ \frac{\arg(0.01 - x)}{2 \pi} \right] + 8 i \pi \left[ \frac{\arg(1.9802 - x)}{2 \pi} \right] + 4.5 \log(x) + \sum_{k=1}^{\infty} \frac{(-1)^k \left( -0.5(0.01 - x)^k - 4(1.9802 - x)^k \right) x^{-k}}{k} \right)} \text{ for } x < 0$$

$$\frac{8(0.01^2 - 4 \times 0.01 + 3 - 2 \log(0.01) - 8 \log(\frac{2}{1.01}))}{0.01^2 - 8 \times 0.01 + 7 - 2 \log(0.01) - 16 \log(\frac{2}{1.01})} =$$

$$\frac{-11.8404 + \sum_{j=1}^{\infty} \left( 8 \left( \text{Res}_{s=-j} \frac{(-0.99)^{-s} \Gamma(-s)^2 \Gamma(1+s)}{\Gamma(1-s)} \right) + 32 \left( \text{Res}_{s=-j} \frac{e^{0.0200007s} \Gamma(-s)^2 \Gamma(1+s)}{\Gamma(1-s)} \right) \right)}{-3.46005 + \sum_{j=1}^{\infty} \left( \text{Res}_{s=-j} \frac{(-0.99)^{-s} \Gamma(-s)^2 \Gamma(1+s)}{\Gamma(1-s)} + 8 \left( \text{Res}_{s=-j} \frac{e^{0.0200007s} \Gamma(-s)^2 \Gamma(1+s)}{\Gamma(1-s)} \right) \right)}$$

$$\left( \frac{8 \left( \frac{0.01^2 - 4 \times 0.01 + 3 - 2 \ln(0.01) - 8 \ln(2/1.01)}{0.01^2 - 8 \times 0.01 + 7 - 2 \ln(0.01) - 16 \ln(2/1.01)} \right) \right)^2 + 29 + 4$$

**Input:**

$$\left( 8 \times \frac{0.01^2 - 4 \times 0.01 + 3 - 2 \log(0.01) - 8 \log(\frac{2}{1.01})}{0.01^2 - 8 \times 0.01 + 7 - 2 \log(0.01) - 16 \log(\frac{2}{1.01})} \right)^2 + 29 + 4$$

$\log(x)$  is the natural logarithm

**Result:**

139.432...

139.432... result practically equal to the rest mass of Pion meson 139.57 MeV

**Alternative representations:**

$$\left( \frac{8(0.01^2 - 4 \times 0.01 + 3 - 2 \log(0.01) - 8 \log(\frac{2}{1.01}))}{0.01^2 - 8 \times 0.01 + 7 - 2 \log(0.01) - 16 \log(\frac{2}{1.01})} \right)^2 + 29 + 4 =$$

$$33 + \left( \frac{8(2.96 - 2 \log_e(0.01) - 8 \log_e(\frac{2}{1.01}) + 0.01^2)}{6.92 - 2 \log_e(0.01) - 16 \log_e(\frac{2}{1.01}) + 0.01^2} \right)^2$$

$$\left( \frac{8(0.01^2 - 4 \times 0.01 + 3 - 2 \log(0.01) - 8 \log(\frac{2}{1.01}))}{0.01^2 - 8 \times 0.01 + 7 - 2 \log(0.01) - 16 \log(\frac{2}{1.01})} \right)^2 + 29 + 4 =$$

$$33 + \left( \frac{8(2.96 - 2 \log(a) \log_a(0.01) - 8 \log(a) \log_a(\frac{2}{1.01}) + 0.01^2)}{6.92 - 2 \log(a) \log_a(0.01) - 16 \log(a) \log_a(\frac{2}{1.01}) + 0.01^2} \right)^2$$

$$\left( \frac{8(0.01^2 - 4 \times 0.01 + 3 - 2 \log(0.01) - 8 \log(\frac{2}{1.01}))}{0.01^2 - 8 \times 0.01 + 7 - 2 \log(0.01) - 16 \log(\frac{2}{1.01})} \right)^2 + 29 + 4 =$$

$$33 + \left( \frac{8(2.96 + 2 \text{Li}_1(0.99) + 8 \text{Li}_1(1 - \frac{2}{1.01}) + 0.01^2)}{6.92 + 2 \text{Li}_1(0.99) + 16 \text{Li}_1(1 - \frac{2}{1.01}) + 0.01^2} \right)^2$$

**Series representations:**

$$\left( \frac{8(0.01^2 - 4 \times 0.01 + 3 - 2 \log(0.01) - 8 \log(\frac{2}{1.01}))}{0.01^2 - 8 \times 0.01 + 7 - 2 \log(0.01) - 16 \log(\frac{2}{1.01})} \right)^2 + 29 + 4 =$$

$$\left( 33 \left( 16.2203 - 6.9201 \sum_{k=1}^{\infty} \frac{-(-1)^k (-0.99)^k - 8(-0.980198)^k}{k} + \right. \right.$$

$$\left. \left( \sum_{k=1}^{\infty} \frac{-(-1)^k (-0.99)^k - 8(-0.980198)^k}{k} \right)^2 - \right.$$

$$5.7408 \sum_{k=1}^{\infty} \frac{-(-1)^k (-0.99)^k - 4(-0.980198)^k}{k} +$$

$$1.93939 \left( \sum_{k=1}^{\infty} \frac{-(-1)^k (-0.99)^k - 4(-0.980198)^k}{k} \right)^2 \left. \right) /$$

$$\left( -3.46005 + \sum_{k=1}^{\infty} \frac{-(-1)^k (-0.99)^k - 8(-0.980198)^k}{k} \right)^2$$

$$\left( \frac{8(0.01^2 - 4 \times 0.01 + 3 - 2 \log(0.01) - 8 \log(\frac{2}{1.01}))}{0.01^2 - 8 \times 0.01 + 7 - 2 \log(0.01) - 16 \log(\frac{2}{1.01})} \right)^2 + 29 + 4 =$$

$$33 + \left( 64 \left( 2.9601 - 2 \left( 2i\pi \left[ \frac{\arg(0.01 - x)}{2\pi} \right] + \log(x) - \sum_{k=1}^{\infty} \frac{(-1)^k (0.01 - x)^k x^{-k}}{k} \right) - \right. \right.$$

$$8 \left( 2i\pi \left[ \frac{\arg(1.9802 - x)}{2\pi} \right] + \log(x) - \right.$$

$$\left. \left. \sum_{k=1}^{\infty} \frac{(-1)^k (1.9802 - x)^k x^{-k}}{k} \right) \right)^2 /$$

$$\left( 6.9201 - 2 \left( 2i\pi \left[ \frac{\arg(0.01 - x)}{2\pi} \right] + \log(x) - \sum_{k=1}^{\infty} \frac{(-1)^k (0.01 - x)^k x^{-k}}{k} \right) - \right.$$

$$16 \left( 2i\pi \left[ \frac{\arg(1.9802 - x)}{2\pi} \right] + \log(x) - \right.$$

$$\left. \left. \sum_{k=1}^{\infty} \frac{(-1)^k (1.9802 - x)^k x^{-k}}{k} \right) \right)^2 \text{ for } x < 0$$

$$\begin{aligned}
& \left( \frac{8(0.01^2 - 4 \times 0.01 + 3 - 2 \log(0.01) - 8 \log(\frac{2}{1.01}))}{0.01^2 - 8 \times 0.01 + 7 - 2 \log(0.01) - 16 \log(\frac{2}{1.01})} \right)^2 + 29 + 4 = \\
& 33 + \left[ 64 \left[ -0.740025 + i\pi \left[ -\frac{-\pi + \arg(\frac{0.01}{z_0}) + \arg(z_0)}{2\pi} \right] + \right. \right. \\
& \quad \left. \left. 4i\pi \left[ -\frac{-\pi + \arg(\frac{1.9802}{z_0}) + \arg(z_0)}{2\pi} \right] + 2.5 \log(z_0) + \right. \right. \\
& \quad \left. \left. \sum_{k=1}^{\infty} \frac{(-1)^k (-0.5(0.01 - z_0)^k - 2(1.9802 - z_0)^k) z_0^{-k}}{k} \right]^2 \right] / \\
& \left( -1.73003 + i\pi \left[ -\frac{-\pi + \arg(\frac{0.01}{z_0}) + \arg(z_0)}{2\pi} \right] + 8i\pi \left[ -\frac{-\pi + \arg(\frac{1.9802}{z_0}) + \arg(z_0)}{2\pi} \right] + \right. \\
& \quad \left. 4.5 \log(z_0) + \sum_{k=1}^{\infty} \frac{(-1)^k (-0.5(0.01 - z_0)^k - 4(1.9802 - z_0)^k) z_0^{-k}}{k} \right)^2
\end{aligned}$$

(((8\*(((0.01^2-4\*0.01+3-2 ln(0.01)-8 ln(2/1.01)))))/(((0.01^2-8\*0.01+7-2 ln(0.01)-16 ln(2/1.01))))))^2+18+1/golden ratio

**Input:**

$$\left( 8 \times \frac{0.01^2 - 4 \times 0.01 + 3 - 2 \log(0.01) - 8 \log(\frac{2}{1.01})}{0.01^2 - 8 \times 0.01 + 7 - 2 \log(0.01) - 16 \log(\frac{2}{1.01})} \right)^2 + 18 + \frac{1}{\phi}$$

log(x) is the natural logarithm

φ is the golden ratio

**Result:**

125.050...

125.050... result very near to the dilaton mass calculated as a type of Higgs boson: 125 GeV for T = 0 and to the Higgs boson mass 125.18 GeV

**Alternative representations:**

$$\left( \frac{8(0.01^2 - 4 \times 0.01 + 3 - 2 \log(0.01) - 8 \log(\frac{2}{1.01}))}{0.01^2 - 8 \times 0.01 + 7 - 2 \log(0.01) - 16 \log(\frac{2}{1.01})} \right)^2 + 18 + \frac{1}{\phi} =$$

$$18 + \frac{1}{\phi} + \left( \frac{8(2.96 - 2 \log_e(0.01) - 8 \log_e(\frac{2}{1.01}) + 0.01^2)}{6.92 - 2 \log_e(0.01) - 16 \log_e(\frac{2}{1.01}) + 0.01^2} \right)^2$$

$$\left( \frac{8(0.01^2 - 4 \times 0.01 + 3 - 2 \log(0.01) - 8 \log(\frac{2}{1.01}))}{0.01^2 - 8 \times 0.01 + 7 - 2 \log(0.01) - 16 \log(\frac{2}{1.01})} \right)^2 + 18 + \frac{1}{\phi} =$$

$$18 + \frac{1}{\phi} + \left( \frac{8(2.96 - 2 \log(a) \log_a(0.01) - 8 \log(a) \log_a(\frac{2}{1.01}) + 0.01^2)}{6.92 - 2 \log(a) \log_a(0.01) - 16 \log(a) \log_a(\frac{2}{1.01}) + 0.01^2} \right)^2$$

$$\left( \frac{8(0.01^2 - 4 \times 0.01 + 3 - 2 \log(0.01) - 8 \log(\frac{2}{1.01}))}{0.01^2 - 8 \times 0.01 + 7 - 2 \log(0.01) - 16 \log(\frac{2}{1.01})} \right)^2 + 18 + \frac{1}{\phi} =$$

$$18 + \frac{1}{\phi} + \left( \frac{8(2.96 + 2 \text{Li}_1(0.99) + 8 \text{Li}_1(1 - \frac{2}{1.01}) + 0.01^2)}{6.92 + 2 \text{Li}_1(0.99) + 16 \text{Li}_1(1 - \frac{2}{1.01}) + 0.01^2} \right)^2$$

**Series representations:**

$$\left( \frac{8(0.01^2 - 4 \times 0.01 + 3 - 2 \log(0.01) - 8 \log(\frac{2}{1.01}))}{0.01^2 - 8 \times 0.01 + 7 - 2 \log(0.01) - 16 \log(\frac{2}{1.01})} \right)^2 + 18 + \frac{1}{\phi} =$$

$$\left( 18 \left( 0.665108 + 19.7606 \phi - 0.38445 \sum_{k=1}^{\infty} \frac{-(-1)^k (-0.99)^k - 8(-0.980198)^k}{k} - \right. \right.$$

$$6.9201 \phi \sum_{k=1}^{\infty} \frac{-(-1)^k (-0.99)^k - 8(-0.980198)^k}{k} +$$

$$0.0555556 \left( \sum_{k=1}^{\infty} \frac{-(-1)^k (-0.99)^k - 8(-0.980198)^k}{k} \right)^2 +$$

$$\phi \left( \sum_{k=1}^{\infty} \frac{-(-1)^k (-0.99)^k - 8(-0.980198)^k}{k} \right)^2 -$$

$$10.5248 \phi \sum_{k=1}^{\infty} \frac{-(-1)^k (-0.99)^k - 4(-0.980198)^k}{k} +$$

$$3.55556 \phi \left( \sum_{k=1}^{\infty} \frac{-(-1)^k (-0.99)^k - 4(-0.980198)^k}{k} \right)^2 \left. \right) /$$

$$\left( \phi \left( -3.46005 + \sum_{k=1}^{\infty} \frac{-(-1)^k (-0.99)^k - 8(-0.980198)^k}{k} \right)^2 \right)$$

$$\begin{aligned}
& \left( \frac{8(0.01^2 - 4 \times 0.01 + 3 - 2 \log(0.01) - 8 \log(\frac{2}{1.01}))}{0.01^2 - 8 \times 0.01 + 7 - 2 \log(0.01) - 16 \log(\frac{2}{1.01})} \right)^2 + 18 + \frac{1}{\phi} = \\
& 18 + \frac{1}{\phi} + \left( 64 \left( 2.9601 - 2 \left[ 2 i \pi \left\lfloor \frac{\arg(0.01 - x)}{2 \pi} \right\rfloor + \log(x) - \sum_{k=1}^{\infty} \frac{(-1)^k (0.01 - x)^k x^{-k}}{k} \right] - \right. \right. \\
& \quad \left. \left. 8 \left( 2 i \pi \left\lfloor \frac{\arg(1.9802 - x)}{2 \pi} \right\rfloor + \log(x) - \sum_{k=1}^{\infty} \frac{(-1)^k (1.9802 - x)^k x^{-k}}{k} \right) \right) \right)^2 / \\
& \left( 6.9201 - 2 \left( 2 i \pi \left\lfloor \frac{\arg(0.01 - x)}{2 \pi} \right\rfloor + \log(x) - \sum_{k=1}^{\infty} \frac{(-1)^k (0.01 - x)^k x^{-k}}{k} \right) - \right. \\
& \quad \left. 16 \left( 2 i \pi \left\lfloor \frac{\arg(1.9802 - x)}{2 \pi} \right\rfloor + \log(x) - \sum_{k=1}^{\infty} \frac{(-1)^k (1.9802 - x)^k x^{-k}}{k} \right) \right)^2 \text{ for } x < 0
\end{aligned}$$

$$\begin{aligned}
& \left( \frac{8(0.01^2 - 4 \times 0.01 + 3 - 2 \log(0.01) - 8 \log(\frac{2}{1.01}))}{0.01^2 - 8 \times 0.01 + 7 - 2 \log(0.01) - 16 \log(\frac{2}{1.01})} \right)^2 + 18 + \frac{1}{\phi} = \\
& 18 + \frac{1}{\phi} + \left( 64 \left( 2.9601 - 2 \left( \log(z_0) + \left\lfloor \frac{\arg(0.01 - z_0)}{2 \pi} \right\rfloor \left( \log\left(\frac{1}{z_0}\right) + \log(z_0) \right) - \right. \right. \right. \\
& \quad \left. \left. \sum_{k=1}^{\infty} \frac{(-1)^k (0.01 - z_0)^k z_0^{-k}}{k} \right) - 8 \left( \log(z_0) + \left\lfloor \frac{\arg(1.9802 - z_0)}{2 \pi} \right\rfloor \right. \right. \\
& \quad \left. \left. \left( \log\left(\frac{1}{z_0}\right) + \log(z_0) \right) - \sum_{k=1}^{\infty} \frac{(-1)^k (1.9802 - z_0)^k z_0^{-k}}{k} \right) \right) \right)^2 / \\
& \left( 6.9201 - 2 \left( \log(z_0) + \left\lfloor \frac{\arg(0.01 - z_0)}{2 \pi} \right\rfloor \left( \log\left(\frac{1}{z_0}\right) + \log(z_0) \right) - \right. \right. \\
& \quad \left. \left. \sum_{k=1}^{\infty} \frac{(-1)^k (0.01 - z_0)^k z_0^{-k}}{k} \right) - \right. \\
& \quad \left. 16 \left( \log(z_0) + \left\lfloor \frac{\arg(1.9802 - z_0)}{2 \pi} \right\rfloor \left( \log\left(\frac{1}{z_0}\right) + \log(z_0) \right) - \sum_{k=1}^{\infty} \frac{(-1)^k (1.9802 - z_0)^k z_0^{-k}}{k} \right) \right)^2
\end{aligned}$$



$$\frac{27}{2} \left( \left( \frac{8(0.01^2 - 4 \times 0.01 + 3 - 2 \log(0.01) - 8 \log(\frac{2}{1.01}))}{0.01^2 - 8 \times 0.01 + 7 - 2 \log(0.01) - 16 \log(\frac{2}{1.01})} \right)^2 + 21 \right) + 8 + \frac{1}{\phi} =$$

$$8 + \frac{1}{\phi} + \frac{27}{2} \left( 21 + \left( \frac{8(2.96 + 2 \text{Li}_1(0.99) + 8 \text{Li}_1(1 - \frac{2}{1.01}) + 0.01^2)}{6.92 + 2 \text{Li}_1(0.99) + 16 \text{Li}_1(1 - \frac{2}{1.01}) + 0.01^2} \right)^2 \right)$$

### Series representations:

$$\frac{27}{2} \left( \left( \frac{8(0.01^2 - 4 \times 0.01 + 3 - 2 \log(0.01) - 8 \log(\frac{2}{1.01}))}{0.01^2 - 8 \times 0.01 + 7 - 2 \log(0.01) - 16 \log(\frac{2}{1.01})} \right)^2 + 21 \right) + 8 + \frac{1}{\phi} =$$

$$\left( 291.5 \left( 0.164281 + 73.8587 \phi + 17.5474 \phi \sum_{k=1}^{\infty} \frac{2(-1)^k ((-0.99)^k + 4 \times 0.980198^k)}{k} \right) + \right.$$

$$2.96398 \phi \left( \sum_{k=1}^{\infty} \frac{2(-1)^k ((-0.99)^k + 4 \times 0.980198^k)}{k} \right)^2 +$$

$$0.0474792 \sum_{k=1}^{\infty} \frac{2(-1)^k ((-0.99)^k + 8 \times 0.980198^k)}{k} +$$

$$13.8402 \phi \sum_{k=1}^{\infty} \frac{2(-1)^k ((-0.99)^k + 8 \times 0.980198^k)}{k} +$$

$$0.00343053 \left( \sum_{k=1}^{\infty} \frac{2(-1)^k ((-0.99)^k + 8 \times 0.980198^k)}{k} \right)^2 +$$

$$\left. \phi \left( \sum_{k=1}^{\infty} \frac{2(-1)^k ((-0.99)^k + 8 \times 0.980198^k)}{k} \right)^2 \right) /$$

$$\left( \phi \left( 6.9201 + \sum_{k=1}^{\infty} \frac{2(-1)^k ((-0.99)^k + 8 \times 0.980198^k)}{k} \right)^2 \right)$$



$$\begin{aligned}
& \frac{27}{2} \left( \left( \frac{8(0.01^2 - 4 \times 0.01 + 3 - 2 \log(0.01) - 8 \log(\frac{2}{1.01}))}{0.01^2 - 8 \times 0.01 + 7 - 2 \log(0.01) - 16 \log(\frac{2}{1.01})} \right)^2 + 21 \right) + 8 + \frac{1}{\phi} = \\
& \left( 291.5 \left( 0.0410701 + 18.4647 \phi - 0.0237396 \sum_{k=1}^{\infty} \frac{-(-1)^k (-0.99)^k - 8(-0.980198)^k}{k} - \right. \right. \\
& \quad 6.9201 \phi \sum_{k=1}^{\infty} \frac{-(-1)^k (-0.99)^k - 8(-0.980198)^k}{k} + \\
& \quad \left. 0.00343053 \left( \sum_{k=1}^{\infty} \frac{-(-1)^k (-0.99)^k - 8(-0.980198)^k}{k} \right)^2 + \right. \\
& \quad \left. \phi \left( \sum_{k=1}^{\infty} \frac{-(-1)^k (-0.99)^k - 8(-0.980198)^k}{k} \right)^2 - \right. \\
& \quad 8.77368 \phi \sum_{k=1}^{\infty} \frac{-(-1)^k (-0.99)^k - 4(-0.980198)^k}{k} + \\
& \quad \left. 2.96398 \phi \left( \sum_{k=1}^{\infty} \frac{-(-1)^k (-0.99)^k - 4(-0.980198)^k}{k} \right)^2 \right) / \\
& \left( \phi \left( -3.46005 + \sum_{k=1}^{\infty} \frac{-(-1)^k (-0.99)^k - 8(-0.980198)^k}{k} \right)^2 \right)
\end{aligned}$$

$$\begin{aligned}
& \frac{27}{2} \left( \left( \frac{8(0.01^2 - 4 \times 0.01 + 3 - 2 \log(0.01) - 8 \log(\frac{2}{1.01}))}{0.01^2 - 8 \times 0.01 + 7 - 2 \log(0.01) - 16 \log(\frac{2}{1.01})} \right)^2 + 21 \right) + 8 + \frac{1}{\phi} = 8 + \frac{1}{\phi} + \frac{27}{2} \\
& \left( 21 + \left( 64 \left( 2.9601 - 2 \left( 2 i \pi \left\lfloor \frac{\arg(0.01 - x)}{2 \pi} \right\rfloor + \log(x) - \sum_{k=1}^{\infty} \frac{(-1)^k (0.01 - x)^k x^{-k}}{k} \right) - \right. \right. \right. \\
& \quad \left. \left. 8 \left( 2 i \pi \left\lfloor \frac{\arg(1.9802 - x)}{2 \pi} \right\rfloor + \log(x) - \sum_{k=1}^{\infty} \frac{(-1)^k (1.9802 - x)^k x^{-k}}{k} \right) \right)^2 \right) / \\
& \left( 6.9201 - 2 \left( 2 i \pi \left\lfloor \frac{\arg(0.01 - x)}{2 \pi} \right\rfloor + \log(x) - \sum_{k=1}^{\infty} \frac{(-1)^k (0.01 - x)^k x^{-k}}{k} \right) - \right. \\
& \quad \left. 16 \left( 2 i \pi \left\lfloor \frac{\arg(1.9802 - x)}{2 \pi} \right\rfloor + \log(x) - \sum_{k=1}^{\infty} \frac{(-1)^k (1.9802 - x)^k x^{-k}}{k} \right) \right)^2 \right) \text{ for } x < 0
\end{aligned}$$

$$6 * (((((((8 * (((((0.01^2 - 4 * 0.01 + 3 - 2 \ln(0.01) - 8 \ln(2/1.01)))) / (((0.01^2 - 8 * 0.01 + 7 - 2 \ln(0.01) - 16 \ln(2/1.01)))))))))^2 + 21))) - 34 - 2 - 1/2$$

**Input:**

$$6 \left( \left( 8 \times \frac{0.01^2 - 4 \times 0.01 + 3 - 2 \log(0.01) - 8 \log\left(\frac{2}{1.01}\right)}{0.01^2 - 8 \times 0.01 + 7 - 2 \log(0.01) - 16 \log\left(\frac{2}{1.01}\right)} \right)^2 + 21 \right) - 34 - 2 - \frac{1}{2}$$

$\log(x)$  is the natural logarithm

**Result:**

728.092...

728.092...  $\approx$  728 (Ramanujan taxicab number)

**Alternative representations:**

$$6 \left( \left( \frac{8 \left( 0.01^2 - 4 \times 0.01 + 3 - 2 \log(0.01) - 8 \log\left(\frac{2}{1.01}\right) \right)}{0.01^2 - 8 \times 0.01 + 7 - 2 \log(0.01) - 16 \log\left(\frac{2}{1.01}\right)} \right)^2 + 21 \right) - 34 - 2 - \frac{1}{2} =$$

$$-\frac{73}{2} + 6 \left( 21 + \left( \frac{8 \left( 2.96 - 2 \log_e(0.01) - 8 \log_e\left(\frac{2}{1.01}\right) + 0.01^2 \right)}{6.92 - 2 \log_e(0.01) - 16 \log_e\left(\frac{2}{1.01}\right) + 0.01^2} \right)^2 \right)$$

$$6 \left( \left( \frac{8 \left( 0.01^2 - 4 \times 0.01 + 3 - 2 \log(0.01) - 8 \log\left(\frac{2}{1.01}\right) \right)}{0.01^2 - 8 \times 0.01 + 7 - 2 \log(0.01) - 16 \log\left(\frac{2}{1.01}\right)} \right)^2 + 21 \right) - 34 - 2 - \frac{1}{2} =$$

$$-\frac{73}{2} + 6 \left( 21 + \left( \frac{8 \left( 2.96 - 2 \log(a) \log_a(0.01) - 8 \log(a) \log_a\left(\frac{2}{1.01}\right) + 0.01^2 \right)}{6.92 - 2 \log(a) \log_a(0.01) - 16 \log(a) \log_a\left(\frac{2}{1.01}\right) + 0.01^2} \right)^2 \right)$$

$$6 \left( \left( \frac{8 \left( 0.01^2 - 4 \times 0.01 + 3 - 2 \log(0.01) - 8 \log\left(\frac{2}{1.01}\right) \right)}{0.01^2 - 8 \times 0.01 + 7 - 2 \log(0.01) - 16 \log\left(\frac{2}{1.01}\right)} \right)^2 + 21 \right) - 34 - 2 - \frac{1}{2} =$$

$$-\frac{73}{2} + 6 \left( 21 + \left( \frac{8 \left( 2.96 + 2 \operatorname{Li}_1(0.99) + 8 \operatorname{Li}_1\left(1 - \frac{2}{1.01}\right) + 0.01^2 \right)}{6.92 + 2 \operatorname{Li}_1(0.99) + 16 \operatorname{Li}_1\left(1 - \frac{2}{1.01}\right) + 0.01^2} \right)^2 \right)$$

**Series representations:**

$$6 \left( \left( \frac{8(0.01^2 - 4 \times 0.01 + 3 - 2 \log(0.01)) - 8 \log\left(\frac{2}{1.01}\right)}{0.01^2 - 8 \times 0.01 + 7 - 2 \log(0.01) - 16 \log\left(\frac{2}{1.01}\right)} \right)^2 + 21 \right) - 34 - 2 - \frac{1}{2} =$$

$$\left( 89.5 \left( 85.482 + 25.4006 \sum_{k=1}^{\infty} \frac{2(-1)^k \left( (-0.99)^k + 4 \times 0.980198^k \right)}{k} + \right. \right.$$

$$4.2905 \left( \sum_{k=1}^{\infty} \frac{2(-1)^k \left( (-0.99)^k + 4 \times 0.980198^k \right)}{k} \right)^2 +$$

$$13.8402 \sum_{k=1}^{\infty} \frac{2(-1)^k \left( (-0.99)^k + 8 \times 0.980198^k \right)}{k} +$$

$$\left. \left. \left( \sum_{k=1}^{\infty} \frac{2(-1)^k \left( (-0.99)^k + 8 \times 0.980198^k \right)}{k} \right)^2 \right) \right) /$$

$$\left( 6.9201 + \sum_{k=1}^{\infty} \frac{2(-1)^k \left( (-0.99)^k + 8 \times 0.980198^k \right)}{k} \right)^2$$

$$6 \left( \left( \frac{8(0.01^2 - 4 \times 0.01 + 3 - 2 \log(0.01)) - 8 \log\left(\frac{2}{1.01}\right)}{0.01^2 - 8 \times 0.01 + 7 - 2 \log(0.01) - 16 \log\left(\frac{2}{1.01}\right)} \right)^2 + 21 \right) - 34 - 2 - \frac{1}{2} =$$

$$\left( 89.5 \left( 21.3705 - 6.9201 \sum_{k=1}^{\infty} \frac{-(-1)^k (-0.99)^k - 8(-0.980198)^k}{k} + \right. \right.$$

$$\left. \left( \sum_{k=1}^{\infty} \frac{-(-1)^k (-0.99)^k - 8(-0.980198)^k}{k} \right)^2 - \right.$$

$$12.7003 \sum_{k=1}^{\infty} \frac{-(-1)^k (-0.99)^k - 4(-0.980198)^k}{k} +$$

$$4.2905 \left( \sum_{k=1}^{\infty} \frac{-(-1)^k (-0.99)^k - 4(-0.980198)^k}{k} \right)^2 \left. \right) /$$

$$\left( -3.46005 + \sum_{k=1}^{\infty} \frac{-(-1)^k (-0.99)^k - 8(-0.980198)^k}{k} \right)^2$$

$$\begin{aligned}
& 6 \left( \left( \frac{8(0.01^2 - 4 \times 0.01 + 3 - 2 \log(0.01) - 8 \log(\frac{2}{1.01}))}{0.01^2 - 8 \times 0.01 + 7 - 2 \log(0.01) - 16 \log(\frac{2}{1.01})} \right)^2 + 21 \right) - 34 - 2 - \frac{1}{2} = -\frac{73}{2} + \\
& 6 \left( 21 + \left( 64 \left( 2.9601 - 2 \left( 2 i \pi \left[ \frac{\arg(0.01 - x)}{2 \pi} \right] + \log(x) - \sum_{k=1}^{\infty} \frac{(-1)^k (0.01 - x)^k x^{-k}}{k} \right) - \right. \right. \right. \\
& \quad \left. \left. \left. 8 \left( 2 i \pi \left[ \frac{\arg(1.9802 - x)}{2 \pi} \right] + \log(x) - \sum_{k=1}^{\infty} \frac{(-1)^k (1.9802 - x)^k x^{-k}}{k} \right) \right)^2 \right) \right) / \\
& \left( 6.9201 - 2 \left( 2 i \pi \left[ \frac{\arg(0.01 - x)}{2 \pi} \right] + \log(x) - \sum_{k=1}^{\infty} \frac{(-1)^k (0.01 - x)^k x^{-k}}{k} \right) - \right. \\
& \quad \left. 16 \left( 2 i \pi \left[ \frac{\arg(1.9802 - x)}{2 \pi} \right] + \log(x) - \sum_{k=1}^{\infty} \frac{(-1)^k (1.9802 - x)^k x^{-k}}{k} \right) \right)^2 \right) \text{ for } x < 0
\end{aligned}$$

## Appendix

From:

### Three-dimensional AdS gravity and extremal CFTs at $c = 8m$

*Spyros D. Avramis, Alex Kehagias and Constantina Mattheopoulou*

Received: September 7, 2007 - Accepted: October 28, 2007 - Published: November 9, 2007

$m$	$L_0$	$d$	$S$	$S_{BH}$
3	1	196883	12.1904	12.5664
	2	21296876	16.8741	17.7715
	3	842609326	20.5520	21.7656
4	2/3	139503	11.8458	11.8477
	5/3	69193488	18.0524	18.7328
	8/3	6928824200	22.6589	23.6954
5	1/3	20619	9.9340	9.3664
	4/3	86645620	18.2773	18.7328
	7/3	24157197490	23.9078	24.7812
6	1	42987519	17.5764	17.7715
	2	40448921875	24.4233	25.1327
	3	8463511703277	29.7668	30.7812
7	2/3	7402775	15.8174	15.6730
	5/3	33934039437	24.2477	24.7812
	8/3	16953652012291	30.4615	31.3460
8	1/3	278511	12.5372	11.8477
	4/3	13996384631	23.3621	23.6954
	7/3	19400406113385	30.5963	31.3460

**Table 1:** Degeneracies, microscopic entropies and semiclassical entropies for the first few values of  $m$  and  $L_0$ .

## Conclusion

### Modular equations and approximations to $\pi$

*S. Ramanujan* - Quarterly Journal of Mathematics, XLV, 1914, 350 – 372

Note that:

$$g_{22} = \sqrt{(1 + \sqrt{2})}.$$

Hence

$$\begin{aligned} 64g_{22}^{24} &= e^{\pi\sqrt{22}} - 24 + 276e^{-\pi\sqrt{22}} - \dots, \\ 64g_{22}^{-24} &= 4096e^{-\pi\sqrt{22}} + \dots, \end{aligned}$$

so that

$$64(g_{22}^{24} + g_{22}^{-24}) = e^{\pi\sqrt{22}} - 24 + 4372e^{-\pi\sqrt{22}} + \dots = 64\{(1 + \sqrt{2})^{12} + (1 - \sqrt{2})^{12}\}.$$

Hence

$$e^{\pi\sqrt{22}} = 2508951.9982\dots$$

Thence:

$$64g_{22}^{-24} = 4096e^{-\pi\sqrt{22}} + \dots$$

And

$$64(g_{22}^{24} + g_{22}^{-24}) = e^{\pi\sqrt{22}} - 24 + 4372e^{-\pi\sqrt{22}} + \dots = 64\{(1 + \sqrt{2})^{12} + (1 - \sqrt{2})^{12}\}$$

That are connected with 64, 128, 256, 512, 1024 and  $4096 = 64^2$

All the results of the most important connections are signed in blue throughout the drafting of the paper. We highlight as in the development of the various equations we use always the constants  $\pi$ ,  $\phi$ ,  $1/\phi$ , the Fibonacci and Lucas numbers, linked to the golden ratio, that play a fundamental role in the development, and therefore, in the final results of the analyzed expressions.

## References

### **Stability of the graviton Bose-Einstein condensate in the brane-world**

*R. Casadio* - Dipartimento di Fisica e Astronomia, Università di Bologna, via Iriero 46, 40126 Bologna, Italy - INFN, Sezione di Bologna, viale B. Pichat 6, 40127 Bologna, Italy

*Roldao da Rocha* - CMCC, Universidade Federal do ABC, 09210-580, Santo André, SP, Brazil - arXiv: 1610.01572v1 [hep-th] 5 Oct 2016

### **Holographic entanglement entropy under the minimal geometric deformation and extensions**

*R. da Rocha, A. A. Tomaz* - arXiv:1905.01548v2 [hep-th] 29 Dec 2019