

The Quasicrystal Rosetta Stone

Ark Thoth*

*Quantum Gravity Research
90290 Topanga, California, USA
(Dated: January 27, 2020)*

The standard model is unified with gravity in a F_4 gauge theory where the spacetime is a quasicrystalline compactification of an E_9 Lorentzian lattice. The Higgs is played by a neutrino condensate.

INTRODUCTION

It was proposed by Freeman Dyson[1] that quasicrystal order is a language. The Rosetta stone was found in Trashit, Egypt in 1799 by french soldiers, and used by Champollion to decipher[2] the Egyptian hieroglyphs. Similarly, using three languages where the first is a Hilbert loop (see figure 1), the second is a list of vertices in the E_8 Lattice, and the third is a two-dimensional quasicrystal exhibiting five-fold symmetry, the longtime sought generation formula for general five-fold symmetric quasicrystals is revealed.

The natural quasicrystals exhibit icosahedral or decagonal symmetry[3–9]. Quasicrystal dynamics is also studied with dodecagonal symmetry, especially with graphene[10–12]. Soft quasicrystal dynamics has recently been explored [9, 13–15]. They become to be used for quantum computation[16].

The Lagrangian of the standard model is well known[3]. We propose to simplify and explain its structure as a Yang-Mills theory over the exceptional f_4 Lie algebra with 52 generators acting as the derivation algebra of the exceptional Jordan algebra of dimension 27. The branching rules of F_4 are classified[3] and we can choose between $F_4 \rightarrow SO_9$ and $F_4 \rightarrow SO_{3x}G_2$.

RESULTS

Geometry, algebra and information encoding complements each other. The most symmetrical geometric object in any dimension is the icositetrachoron, a regular polytope occurring only in four dimensions, whose vertices are the Hurwitz integer quaternions, and also form the root polytope of the Lie algebra \mathfrak{d}_4 . The ultimate exceptional algebra is the Lie algebra \mathfrak{e}_8 , whose root polytope are the Cayley integral octonions. The basic information unit needed to elucidate and encode this two objects and their relationship is not the bit, but the trit, a ternary information unit. It has three states -1, 0 and +1. Its quantum declination, the qutrit plays also a role in quantum computation.

F_4

The f_4 Lie algebra is $f_4 = \mathfrak{der}(\mathfrak{h}_3(\mathbb{O}))$ and the Lie group is $F_4 = Aut(\mathfrak{h}_3(\mathbb{O}))$

This section reviews the procedure presented in [A08] to build the F_4 roots polytope from a ring of 4 by 4 trits matrices $\mathcal{M}_4(\mathbb{T})$ where $\mathbb{T} = \{-1, 0, 1\}$ and enhances it by complexification to build the E_8 roots polytope as the convex hull of a compound of 5 rotated F_4 polytopes.

The icositetrachoron is the convex hull of the compound of a 4D hypercube and its rescaled dual, the 4-orthoplex or hyperoctahedron. Also named 24-cell, because made of 24 octahedrons, the coordinates of its 24 vertices are the 24 permutations of $\{\pm 1, \pm 1, 0, 0\}$, and it is the roots polytope of $D_4 = SO(8)$.

Using quaternions represented as trits matrices, it was shown in [A08] and [IA16] that only two generators $e_1 = i$ and $e_2 = j$ generates by multiplication the hyperoctahedron, root polytope of the Lie group $B_2 = Sp(1)$ (see table II), $\sqrt{B_2} = \{e_0 := 1 := e_1^4, e_1 = i, e_2 = j, e_3 := k := e_1 e_2, -e_0 := -1 := e_1^2, -e_1 := -i := e_1^3, -e_2 := -j := e_2^3, -e_3 := -k := -e_1^3 e_2\}$ where we use the notation $\sqrt{X_n}$ for the set of root vectors of the Lie group X_n .

Using biquaternions the generators and the root vectors can be expressed in a ring of $\mathcal{M}_2(\mathbb{C} \otimes \mathbb{T})$ of the algebra $\mathcal{M}_2(\mathbb{C})$. The biquaternion w where w, x, y and z are complex numbers, is represented by the matrix $\begin{pmatrix} w + ix & y + iz \\ -y + iz & w - ix \end{pmatrix}$ A third generator, $e_4 := i$ (representing the complex imaginary unit, while i is the quaternionic e_1 imaginary unit), is required, and commutes with the two first. Consistently with [A18], the generators are indexed by a power of two, but here e_4 is not an octonion but a biquaternion. The biquaternion algebra is isomorphic to several Clifford algebras, and to the Geometric Algebra: $Cl_3^0(\mathbb{C})$, $Cl_2(\mathbb{C})$, $Cl_{1,2}(\mathbb{R})$, $Cl_{3,0}(\mathbb{R})$, $Cl_{1,3}(\mathbb{R})$ and $Cl_{3,1}^0(\mathbb{R})$ (see [F16] and references therein).

| Generator | i | j | $\frac{1}{2}(1-i-j-k)$ | $\frac{1}{2}(1+i+j+k)$ | i | | | |
|-----------|-------|-------|------------------------|------------------------|-------|-------|-------|--|
| Unit | e_1 | e_2 | t | t^2 | e_4 | e_5 | e_6 | |
| Matrix | | | | | | | | |
| B | 10 | 0.01 | | | | | | |

TABLE I. . The generators of B_2 and D_4 .

* <http://www.quantumgravityresearch.org>

| | | | | | | | | |
|--------------|-----------------------|-----------------------|-----------------------|-----------------------|-------------------------|-------------------------|-------------------------|-------------------------|
| Biquaternion | 1 | i | j | k | -1 | - i | - j | - k |
| Unit | e ₀ | e ₁ | e ₂ | e ₃ | - e ₀ | - e ₁ | - e ₂ | - e ₃ |
| Matrix | | | | | | | | |
| B | 10 | 0.01 | | | | | | |

TABLE II. . The roots of B_2 .

DISCUSSION

The simulation provided proposes to use methods from Material science associated with Group theory from theoretical physics to discuss the possibility of a unification theory where a crystal lattice high dimensional discrete space (spanned by integer coordinates) contains a subset isomorphic to a quasi-crystal of half its dimension.

THEORY AND METHODS

The rise of awareness of Nature

The truth is not observable. The observation is not the truth.

The most powerful mathematical structure that science and physicists use to describe and understand Nature are 'deformation' of pure abstract concepts.

A simple example is the structure of group, and its deformation is named quantum group. Quantum mechanics is a deformation of classical mechanics. The deformation of zero is the Planck constant \hbar , a very small number. Mathematicians and group theorists are fascinated by pure concepts where more and more complex objects emanates in successive layers from the simplest one. As a general paradigm this is known as theory of complex systems, and has been developed with the rise of the computers to understand Nature's phenomena that we can even not model with equations. Modelization is based on linearization, or first order approximation of observed and reproducible behaviors.

Grassmann variables θ are used to represent the fermion creation operator, implementing automatically the Pauli exclusion principle because they square to zero. They satisfy the anticommutation rules:

$$\{ \theta_m, \theta_n \} := \theta_m \theta_n + \theta_n \theta_m = 0 \quad (1)$$

Together with their dual momentum π they satisfy the

anticommutation duality:

$$\{ \theta_m, \pi_n \} := \theta_m \pi_n + \pi_n \theta_m = \delta_{mn} \quad (2)$$

We have the five deformed Grassmann variables θ_1 to θ_5 , and their conjugate variables π_1 to π_5 , deduced by transposition: $\pi_m :=^T \theta_m$, where m goes from 1 to k, from which we can build 5 pairs of deforme Clifford gamma matrices γ_m^+, γ_m^- by

$$\gamma_m^+ = \theta_m + \pi_m, \quad \gamma_m^- = -i(\theta_m - \pi_m). \quad (3)$$

CONCLUSIONS

This allows quasicrystalline compactification of quantum gravity theories. The author, Ark Thoth, expect to become to quasicrystal physics who Nicolas Bourbaki is for mathematics.

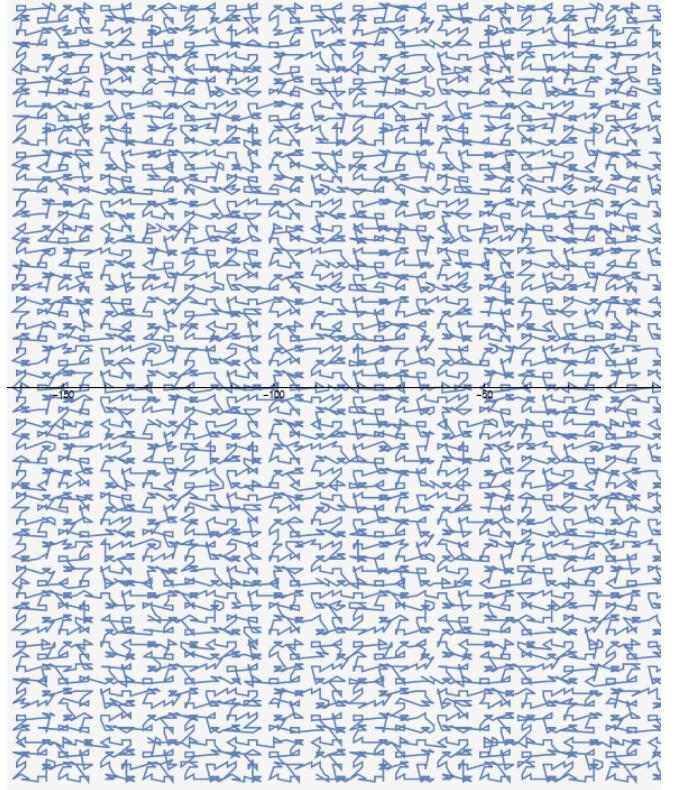


FIG. 1. Rosetta quasicrystal language, we see clearly the complex alphabet

-
- [1] F. J. Dyson, *Birds and Frogs: Selected Papers, 1990-2014* (World Scientific Publishing Company Pte Limited, 2015).
 - [2] J.-F. Champollion, Lettre à M. Dacier relative à l'alphabet des hiéroglyphes phonétiques (Firmin-Didot, Paris, 1822) pp. 1-Pl. IV.
 - [3] L. Bindi, N. Yao, C. Lin, L. S. Hollister, C. L. Andronicos, V. V. Distler, M. P. Eddy, A. Kostin, V. Kryachko, G. J. MacPherson, W. M. Steinhardt, M. Yudovskaya, and P. J. Steinhardt, Natural quasicrystal with decagonal symmetry, *Sci Rep* 5, 1 (2015).

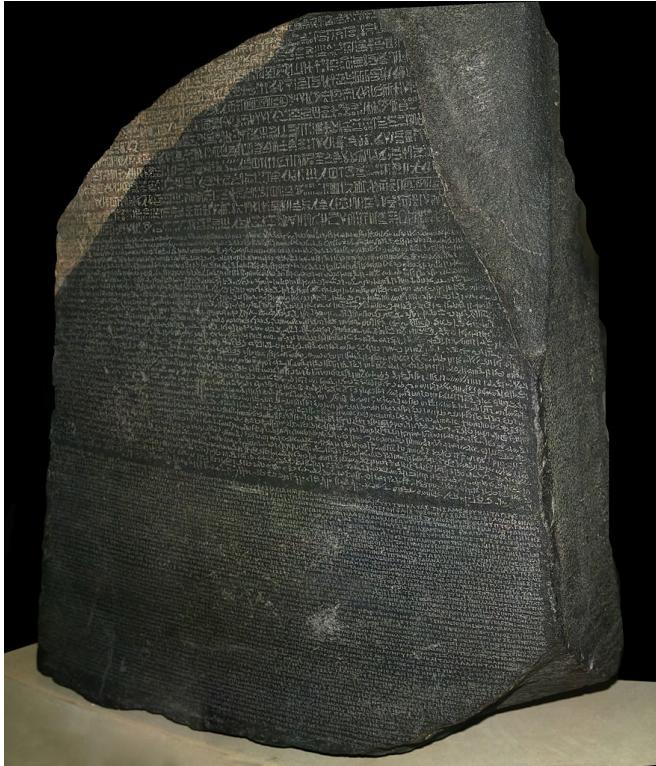


FIG. 2. Rosetta stone, [17]

- [4] L. Bindi, C. Lin, C. Ma, and P. J. Steinhardt, Collisions in outer space produced an icosahedral phase in the Khatyrka meteorite never observed previously in the laboratory, *Scientific Reports* **6**, 10.1038/srep38117 (2016).
- [5] J. Mikhael, J. Roth, L. Helden, and C. Bechinger, Archimedean-like tiling on decagonal quasicrystalline surfaces, *Nature* **454**, 501 (2008).
- [6] S.-Y. Jeon, H. Kwon, and K. Hur, Intrinsic photonic wave localization in a three-dimensional icosahedral quasicrystal, *Nature Phys* **13**, 363 (2017).
- [7] M. Renner and G. v. Freymann, Spatial correlations and optical properties in three-dimensional deterministic aperiodic structures, *Sci Rep* **5**, 1 (2015).
- [8] L. C. Collins, T. G. Witte, R. Silverman, D. B. Green, and K. K. Gomes, Imaging quasiperiodic electronic states in a synthetic Penrose tiling, *Nat Commun* **8**, 1 (2017).
- [9] T. Dotera, T. Oshiro, and P. Zicherl, Mosaic two-lengthscale quasicrystals, *Nature advance online publication*, 10.1038/nature12938 (2014).
- [10] G. Yu, Z. Wu, Z. Zhan, M. I. Katsnelson, and S. Yuan, Dodecagonal bilayer graphene quasicrystal and its approximants, *npj Comput Mater* **5**, 1 (2019).
- [11] A. Haji-Akbari, M. Engel, A. S. Keys, X. Zheng, R. G. Petschek, P. Palffy-Muhoray, and S. C. Glotzer, Disordered, quasicrystalline and crystalline phases of densely packed tetrahedra, *Nature* **462**, 773 (2009).
- [12] S. J. Ahn, P. Moon, T.-H. Kim, H.-W. Kim, H.-C. Shin, E. H. Kim, H. W. Cha, S.-J. Kahng, P. Kim, M. Koshino, Y.-W. Son, C.-W. Yang, and J. R. Ahn, Dirac electrons in a dodecagonal graphene quasicrystal, *Science* **361**, 782 (2018).

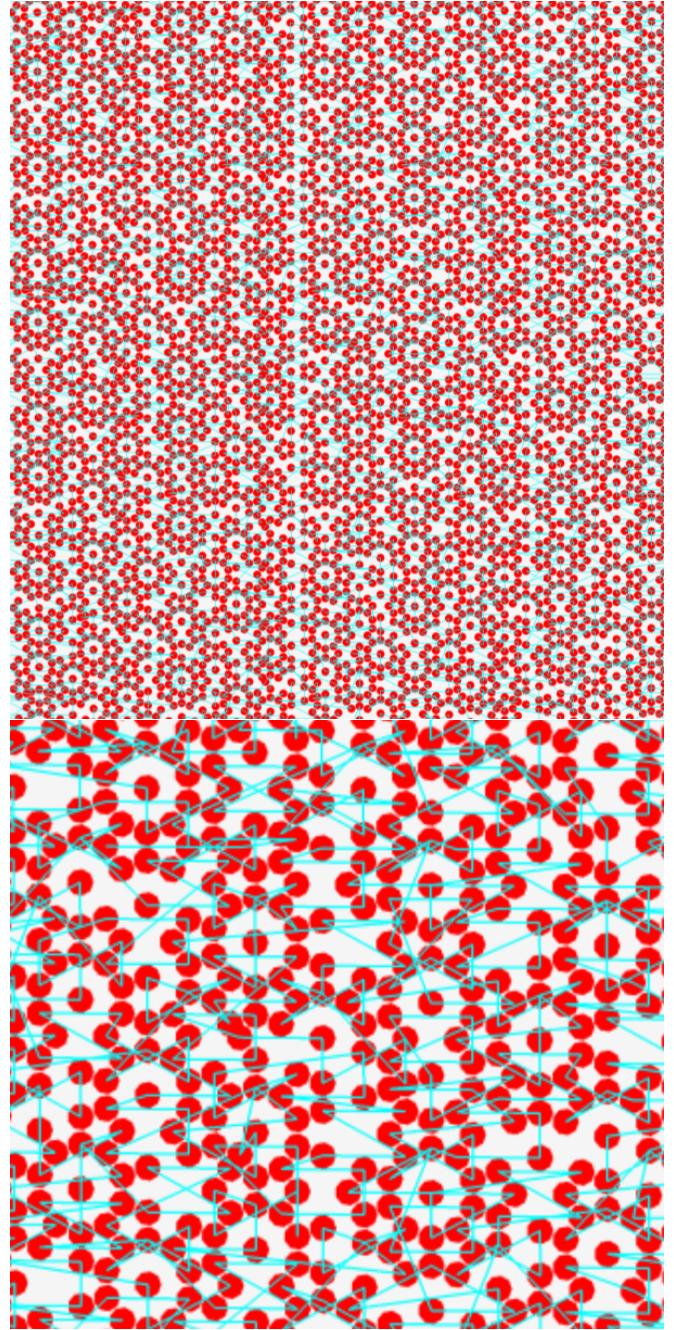


FIG. 3. Rosetta quasicrystal, a two dimensional slice from the Elser-Sloane E8 quasicrystal, with the Hilbert loop walking along all the quasicrystal, and at the bottom, a zoomed view

- [13] M. Engel, P. F. Damasceno, C. L. Phillips, and S. C. Glotzer, Computational self-assembly of a one-component icosahedral quasicrystal, *Nat Mater* **14**, 109 (2015).
- [14] N. A. Wasio, R. C. Quardokus, R. P. Forrest, C. S. Lent, S. A. Corcelli, J. A. Christie, K. W. Henderson, and S. A. Kandel, Self-assembly of hydrogen-bonded two-dimensional quasicrystals, *Nature* **507**, 86 (2014).
- [15] P.-Y. Wang and T. G. Mason, A Brownian quasi-crystal of pre-assembled colloidal Penrose tiles, *Nature* **561**, 94

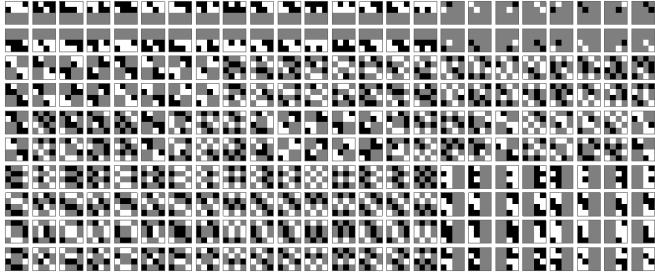


FIG. 4. Gosset polytope vertices partitioned into ten icosite-trachorons, coordinated in M4R

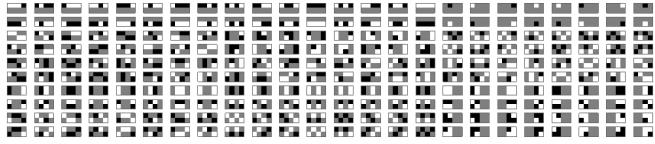


FIG. 5. Gosset polytope vertices partitioned into ten icosite-trachorons, coordinated in HC

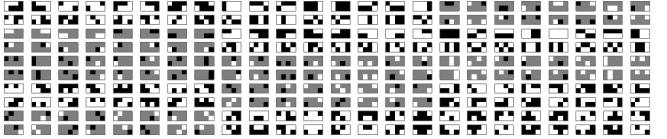


FIG. 6. Gosset polytope vertices partitioned into ten icosite-trachorons, coordinated in Z8

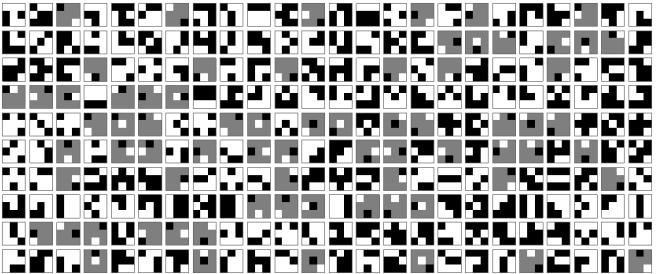


FIG. 7. Gosset polytope vertices partitioned into ten icosite-trachorons, coordinated in Z9

- (2018).
- [16] S. Slizovskiy, E. McCann, M. Koshino, and V. I. Fal'ko, Films of rhombohedral graphite as two-dimensional topological semimetals, *Commun Phys* **2**, 1 (2019).
 - [17] H. Hillewaert, Deutsch: Der Stein von Rosette im Britischen Museum. (2007).
 - [18] X. Zeng, G. Ungar, Y. Liu, V. Percec, A. E. Dulcey, and J. K. Hobbs, Supramolecular dendritic liquid quasicrystals, *Nature* **428**, 157 (2004).
 - [19] D. V. Talapin, E. V. Shevchenko, M. I. Bodnarchuk, X. Ye, J. Chen, and C. B. Murray, Quasicrystalline order in self-assembled binary nanoparticle superlattices, *Nature* **461**, 964 (2009).
 - [20] Y. Cao, V. Fatemi, S. Fang, K. Watanabe, T. Taniguchi, E. Kaxiras, and P. Jarillo-Herrero, Unconventional superconductivity in magic-angle graphene superlattices, *Nature* **556**, 43 (2018).
 - [21] D. Poland and D. Simmons-Duffin, The conformal bootstrap, *Nat Phys* , 535 (2016).
 - [22] M. W. Reimann, M. Nolte, M. Scolamiero, K. Turner, R. Perin, G. Chindemi, P. Dłotko, R. Levi, K. Hess, and H. Markram, Cliques of Neurons Bound into Cavities Provide a Missing Link between Structure and Function, *Front. Comput. Neurosci.* **11**, 10.3389/fncom.2017.00048 (2017).
 - [23] A. Hankey, A complexity basis for phenomenology: How information states at criticality offer a new approach to understanding experience of self, being and time, *Progress in Biophysics and Molecular Biology* **119**, 288 (2015).
 - [24] H. Hoffmann and D. W. Payton, Optimization by Self-Organized Criticality, *Scientific Reports* **8**, 2358 (2018).
 - [25] J. Hesse and T. Gross, Self-organized criticality as a fundamental property of neural systems, *Front. Syst. Neurosci.* **8**, 10.3389/fnsys.2014.00166 (2014).
 - [26] M. Piattelli-Palmarini and G. Vitiello, Quantum field theory and the linguistic Minimalist Program: a remarkable isomorphism, *J. Phys.: Conf. Ser.* **880**, 012016 (2017).
 - [27] P. Lévay and F. Holweck, A finite geometric toy model of space-time as an error correcting code, *Phys. Rev. D* **99**, 086015 (2019), arXiv: 1812.07242.
 - [28] M. Cremonesi, *Development of a multivariate tool to reject background in a WZ diboson search for the CDF experiment*, Tech. Rep. CDF-11181; FERMILAB-MASTERS-2011-09 (Fermi National Accelerator Lab. (FNAL), Batavia, IL (United States), 2015).
 - [29] C. Cao, S. M. Carroll, and S. Michalakis, Space from Hilbert Space: Recovering Geometry from Bulk Entanglement, *Phys. Rev. D* **95**, 024031 (2017), arXiv: 1606.08444.
 - [30] D. George and E. A. Huerta, Deep neural networks to enable real-time multimessenger astrophysics, *Physical Review D* **97**, 10.1103/PhysRevD.97.044039 (2018).
 - [31] D. George and E. Huerta, Deep Learning for real-time gravitational wave detection and parameter estimation: Results with Advanced LIGO data, *Physics Letters B* **778**, 64 (2018).
 - [32] D. George, H. Shen, and E. A. Huerta, Classification and unsupervised clustering of LIGO data with Deep Transfer Learning, *Physical Review D* **97**, 10.1103/PhysRevD.97.101501 (2018).
 - [33] E. A. Huerta, C. J. Moore, P. Kumar, D. George, A. J. K. Chua, R. Haas, E. Wessel, D. Johnson, D. Glennon, A. Rebei, A. M. Holgado, J. R. Gair, and H. P. Pfeiffer, Eccentric, nonspinning, inspiral, Gaussian-process merger approximant for the detection and characterization of eccentric binary black hole mergers, *Physical Review D* **97**, 10.1103/PhysRevD.97.024031 (2018).
 - [34] D. George, H. Shen, and E. A. Huerta, Glitch Classification and Clustering for LIGO with Deep Transfer Learning, *Phys. Rev. D* **97**, 101501 (2018), arXiv: 1711.07468.
 - [35] S. Wu, China: How science made a superpower, *Nature* **574**, 25 (2019).
 - [36] I. M. Comsa, M. Firsching, and T. Fischbacher, SO(8) Supergravity and the Magic of Machine Learning, *J. High Energ. Phys.* **2019** (8), 57, arXiv: 1906.00207.

- [37] M. J. Rozenberg, O. Schneegans, and P. Stolar, An ultra-compact leaky-integrate-and-fire model for building spiking neural networks, *Sci Rep* **9**, 1 (2019).
- [38] J.-P. Luminet, J. R. Weeks, A. Riazuelo, R. Lehoucq, and J.-P. Uzan, Dodecahedral space topology as an explanation for weak wide-angle temperature correlations in the cosmic microwave background, *Nature* **425**, 593 (2003).
- [39] P. A. M. Dirac, Is there an æther?, *Nature* **169**, 702 (1952).
- [40] L. Piazza, T. T. A. Lummen, E. Quiñonez, Y. Murooka, B. W. Reed, B. Barwick, and F. Carbone, Simultaneous observation of the quantization and the interference pattern of a plasmonic near-field, *Nature Communications* **6**, 6407 (2015).
- [41] P. Ball, Two constants to rule us all, *Nature News* 10.1038/news.2007.389 (2007).
- [42] K. Deguchi, S. Matsukawa, N. K. Sato, T. Hattori, K. Ishida, H. Takakura, and T. Ishimasa, Quantum critical state in a magnetic quasicrystal, *Nature Materials* 10.1038/Nmat3432 (2012), arXiv: 1210.3160.
- [43] W. McCutcheon, A. Pappa, B. A. Bell, A. McMillan, A. Chailloux, T. Lawson, M. Mafu, D. Markham, E. Diamanti, I. Kerenidis, J. G. Rarity, and M. S. Tame, Experimental verification of multipartite entanglement in quantum networks, *Nature Communications* **7**, 13251 (2016).
- [44] H. Takakura, C. P. Gómez, A. Yamamoto, M. De Boissieu, and A. P. Tsai, Atomic structure of the binary icosahedral Yb–Cd quasicrystal, *Nat Mater* **6**, 58 (2007).
- [45] Y. Xu, I. Miotkowski, C. Liu, J. Tian, H. Nam, N. Alidoust, J. Hu, C.-K. Shih, M. Z. Hasan, and Y. P. Chen, Observation of topological surface state quantum Hall effect in an intrinsic three-dimensional topological insulator, *Nat Phys* **10**, 956 (2014).
- [46] Y. E. Kraus and O. Zilberberg, Quasiperiodicity and topology transcend dimensions, *Nat Phys* **12**, 624 (2016).
- [47] The Serial Universe : Abstract : Nature, *Nature* **134**, 729 (1934).
- [48] L. S. Hollister, L. Bindi, N. Yao, G. R. Poirier, C. L. Andronicos, G. J. MacPherson, C. Lin, V. V. Distler, M. P. Eddy, A. Kostin, V. Kryachko, W. M. Steinhardt, M. Yudovskaya, J. M. Eiler, Y. Guan, J. J. Clarke, and P. J. Steinhardt, Impact-induced shock and the formation of natural quasicrystals in the early solar system, *Nature Communications* **5**, 10.1038/ncomms5040 (2014).
- [49] X. Z. Yu, N. Kanazawa, Y. Onose, K. Kimoto, W. Z. Zhang, S. Ishiwata, Y. Matsui, and Y. Tokura, Near room-temperature formation of a skyrmion crystal in thin-films of the helimagnet FeGe, *Nat Mater* **10**, 106 (2011).
- [50] K. A. Modic, T. E. Smidt, I. Kimchi, N. P. Breznay, A. Biffin, S. Choi, R. D. Johnson, R. Coldea, P. Watkins-Curry, G. T. McCandless, F. Gandara, Z. Islam, A. Vishwanath, J. Y. Chan, A. Shekhter, R. D. McDonald, and J. G. Analytis, A new spin-anisotropic harmonic honeycomb iridate, *Nature Communications* **5**, 10.1038/ncomms5203 (2014), arXiv: 1402.3254.
- [51] P. J. Ackerman and I. I. Smalyukh, Static three-dimensional topological solitons in fluid chiral ferromagnets and colloids, *Nature Materials* **16**, nmat4826 (2016).
- [52] R. Cowen, The quantum source of space-time, *Nature News* **527**, 290 (2015).
- [53] K. Kamiya, T. Takeuchi, N. Kabeya, N. Wada, T. Ishimasa, A. Ochiai, K. Deguchi, K. Imura, and N. K. Sato, Discovery of superconductivity in quasicrystal, *Nature Communications* **9**, 154 (2018).
- [54] M. A. Broome, S. K. Gorman, M. G. House, S. J. Hile, J. G. Keizer, D. Keith, C. D. Hill, T. F. Watson, W. J. Baker, L. C. L. Hollenberg, and M. Y. Simmons, Two-electron spin correlations in precision placed donors in silicon, *Nature Communications* **9**, 980 (2018).
- [55] F. Fröwis, P. C. Strassmann, A. Tirnov, C. Gut, J. Lavoie, N. Brunner, F. Bussières, M. Afzelius, and N. Gisin, Experimental certification of millions of genuinely entangled atoms in a solid, *Nature Communications* **8**, 907 (2017).
- [56] M. Fuwa, S. Takeda, M. Zwierz, H. M. Wiseman, and A. Furusawa, Experimental Proof of Nonlocal Wavefunction Collapse for a Single Particle Using Homodyne Measurement, *Nature Communications* **6**, 10.1038/ncomms7665 (2015), arXiv: 1412.7790.
- [57] K. Nagata, K. Kuramitani, Y. Sekiguchi, and H. Kosaka, Universal holonomic quantum gates over geometric spin qubits with polarised microwaves, *Nature Communications* **9**, 10.1038/s41467-018-05664-w (2018).
- [58] J. R. Ellis, The Superstring: Theory of Everything, or of Nothing?, *Nature* **323**, 595 (1986).
- [59] D. V. Nanopoulos and K. A. Olive, A SUPERSTRING NEUTRINO DOMINATED UNIVERSE?, *Nature* **327**, 487 (1987).
- [60] L. Lu, J. D. Joannopoulos, and M. Soljačić, Topological photonics, *Nature Photonics* **8**, 821 (2014).
- [61] Q. Q. Gao, W. E. Putzbach, A. E. Murmann, S. Chen, A. A. Sarshad, J. M. Peter, E. T. Bartom, M. Hafner, and M. E. Peter, 6mer seed toxicity in tumor suppressive microRNAs, *Nature Communications* **9**, 4504 (2018).
- [62] D. Fujita, Y. Ueda, S. Sato, N. Mizuno, T. Kumasaki, and M. Fujita, Self-assembly of tetravalent Goldberg polyhedra from 144 small components, *Nature* **540**, 563 (2016).
- [63] A. J. Kollár, M. Fitzpatrick, and A. A. Houck, Hyperbolic lattices in circuit quantum electrodynamics, *Nature* **571**, 45 (2019).
- [64] Quasicrystals: the thrill of the chase () .
- [65] The LIGO Scientific Collaboration and The Virgo Collaboration, The 1M2H Collaboration, The Dark Energy Camera GW-EM Collaboration and the DES Collaboration, The DLT40 Collaboration, The Las Cumbres Observatory Collaboration, The VINROUGE Collaboration, and The MASTER Collaboration, A gravitational-wave standard siren measurement of the Hubble constant, *Nature* **551**, 85 (2017).
- [66] C. King, The Central Enigma of Consciousness, *Nature Precedings* 10.1038/npre.2008.2465.1 (2008).
- [67] C. Dutreix, H. González-Herrero, I. Brihuega, M. I. Katnelson, C. Chapelier, and V. T. Renard, Measuring the Berry phase of graphene from wavefront dislocations in Friedel oscillations, *Nature* , 1 (2019).
- [68] Y. P. Kandel, H. Qiao, S. Fallahi, G. C. Gardner, M. J. Manfra, and J. M. Nichol, Coherent spin-state transfer via Heisenberg exchange, *Nature* **573**, 553 (2019).
- [69] R. Heyrovská, Structures at the Atomic Level of Cobalt, Zinc and Lead Niobates (with an Appendix: Atomic structure of cobalt niobate crystal), *Nat Prec* , 1 (2011).
- [70] M. B. Green, Unification of forces and particles in superstring theories, *Nature* **314**, 409 (1985).
- [71] Superstrings () .

- [72] X.-C. Yao, T.-X. Wang, H.-Z. Chen, W.-B. Gao, A. G. Fowler, R. Raussendorf, Z.-B. Chen, N.-L. Liu, C.-Y. Lu, Y.-J. Deng, Y.-A. Chen, and J.-W. Pan, Experimental demonstration of topological error correction, *Nature* **482**, 489 (2012).
- [73] H. Nicolai, A beauty and a beast, *Nature* **447**, 41 (2007).
- [74] H. Nicolai, Back to basics, *Nature* **449**, 797 (2007).
- [75] G. F. R. Ellis, K. A. Meissner, and H. Nicolai, The physics of infinity, *Nature Phys* **14**, 770 (2018).