# **Analyzing some Ramanujan formulas: mathematical connections with various sectors of Black Hole Physics**

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## Abstract

*The purpose of this paper is to show how using certain mathematical values and / or constants from various Ramanujan expressions, we obtain some mathematical connections with several sectors of Black Hole Physics*

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Monster black hole 100,000 times more massive than the sun is found in the heart of our galaxy (SMBH Sagittarius A = $1,9891*10^{35}$)
https://www.dailymail.co.uk/sciencetech/article-4850546/Mini-black-hole-25-000-light-years-Earth.html

https://wssrmnn.net/index.php/2017/01/23/man-saw-number-pi-dreams/

From:

**Manuscript Book 2 of Srinivasa Ramanujan**

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1/1^3 + 1/5^3 + 1/9^3 + …

**Input interpretation:**

$$\frac{1}{1^3} + \frac{1}{5^3} + \frac{1}{9^3} + \cdots$$

**Infinite sum:**

$$\sum_{n=1}^{\infty} \frac{1}{(4n-3)^3} = \frac{1}{64}\left(28\,\zeta(3) + \pi^3\right)$$

$\zeta(s)$ is the Riemann zeta function

**Decimal approximation:**

1.010372968262007190104202868584718670994451636740923068505...

1.010372968262…..

**Convergence tests:**

The ratio test is inconclusive.

The root test is inconclusive.

By the comparison test, the series converges.

**Partial sum formula:**

$$\sum_{n=1}^{m} \frac{1}{(-3+4n)^3} = \frac{1}{128}\left(\psi^{(2)}\left(m+\frac{1}{4}\right) - \psi^{(2)}\left(\frac{1}{4}\right)\right)$$

$\psi^{(n)}(x)$ is the $n^{th}$ derivative of the digamma function

**Alternate form:**

$$\frac{7\,\zeta(3)}{16} + \frac{\pi^3}{64}$$

**Series representations:**

$$\frac{1}{64}\left(\pi^3 + 28\,\zeta(3)\right) = \frac{\pi^3}{64} + \frac{7}{16}\sum_{k=1}^{\infty}\frac{1}{k^3}$$

$$\frac{1}{64}\left(\pi^3 + 28\,\zeta(3)\right) = \frac{\pi^3}{64} + \frac{1}{2}\sum_{k=0}^{\infty}\frac{1}{(1+2\,k)^3}$$

$$\frac{1}{64}\left(\pi^3 + 28\,\zeta(3)\right) = \frac{7}{16}\,e^{\sum_{k=1}^{\infty} P(3\,k)/k} + \frac{\pi^3}{64}$$

$$\frac{1}{64}\left(\pi^3 + 28\,\zeta(3)\right) = \frac{1}{64}\left(\pi^3 + 14\sum_{n=0}^{\infty}\frac{\sum_{k=0}^{n}\frac{(-1)^k\binom{n}{k}}{(1+k)^2}}{1+n}\right)$$

(Pi^3)/64 + 7/16 zeta(3)      (Note that $S_3$ is ζ(3))

**Input:**

$$\frac{\pi^3}{64} + \frac{7}{16}\,\zeta(3)$$

$\zeta(s)$ is the Riemann zeta function

**Decimal approximation:**

1.010372968262007190104202868584718670994451636740923068505...

1.010372968262….

**Alternate form:**

$$\frac{1}{64}\left(28\,\zeta(3) + \pi^3\right)$$

**Alternative representations:**

$$\frac{\pi^3}{64} + \frac{\zeta(3)\,7}{16} = \frac{\pi^3}{64} + \frac{7\,\zeta(3,1)}{16}$$

$$\frac{\pi^3}{64} + \frac{\zeta(3)\,7}{16} = \frac{7\,S_{2,1}(1)}{16} + \frac{\pi^3}{64}$$

$$\frac{\pi^3}{64} + \frac{\zeta(3)\,7}{16} = -\frac{7\,\mathrm{Li}_3(-1)}{\frac{3\times 16}{4}} + \frac{\pi^3}{64}$$

**Series representations:**

$$\frac{\pi^3}{64} + \frac{\zeta(3)\,7}{16} = \frac{\pi^3}{64} + \frac{7}{16}\sum_{k=1}^{\infty}\frac{1}{k^3}$$

$$\frac{\pi^3}{64} + \frac{\zeta(3)\,7}{16} = \frac{\pi^3}{64} + \frac{1}{2}\sum_{k=0}^{\infty}\frac{1}{(1+2\,k)^3}$$

$$\frac{\pi^3}{64} + \frac{\zeta(3)\,7}{16} = \frac{7}{16}\,e^{\sum_{k=1}^{\infty} P(3\,k)/k} + \frac{\pi^3}{64}$$

**Integral representations:**

$$\frac{\pi^3}{64} + \frac{\zeta(3)\,7}{16} = \frac{\pi^3}{64} - \frac{7}{48}\int_0^1 \frac{\log^3(1-t^2)}{t^3}\,dt$$

$$\frac{\pi^3}{64} + \frac{\zeta(3)\,7}{16} = \frac{\pi^3}{64} + \frac{1}{8}\int_0^{\infty} t^2\,\mathrm{csch}(t)\,dt$$

$$\frac{\pi^3}{64} + \frac{\zeta(3)\,7}{16} = \frac{\pi^3}{64} + \frac{7}{32}\int_0^{\infty}\frac{t^2}{-1+e^t}\,dt$$

Thence:

1/1^3 + 1/5^3 + 1/9^3 + … = (Pi^3)/64 + 7/16 zeta(3)

**Input interpretation:**

$$\frac{1}{1^3} + \frac{1}{5^3} + \frac{1}{9^3} + \cdots = \frac{\pi^3}{64} + \frac{7}{16}\,\zeta(3)$$

$\zeta(s)$ is the Riemann zeta function

**Result:**

$$\frac{1}{64}\left(28\,\zeta(3) + \pi^3\right) = \frac{7\,\zeta(3)}{16} + \frac{\pi^3}{64}$$

**Alternate form:**

True

From the right-hand side of the expression, we obtain:

(((1/ ((((Pi^3)/64 + 7/16 zeta(3)))))))^1/12

**Input:**

$$\sqrt[12]{\frac{1}{\frac{\pi^3}{64}+\frac{7}{16}\zeta(3)}}$$

$\zeta(s)$ is the Riemann zeta function

**Exact result:**

$$\frac{1}{\sqrt[12]{\frac{7\zeta(3)}{16}+\frac{\pi^3}{64}}}$$

**Decimal approximation:**

0.999140408144708492742501571872941269617856182995634489415...

0.999140408144… result very near to the value of the following Rogers-Ramanujan continued fraction:

$$\frac{e^{-\frac{\pi}{\sqrt{5}}}}{\frac{\sqrt{5}}{1+\sqrt[5]{\sqrt{\varphi^5\sqrt[4]{5^3}}-1}}-\varphi+1}=1-\frac{e^{-\pi\sqrt{5}}}{1+\frac{e^{-2\pi\sqrt{5}}}{1+\frac{e^{-3\pi\sqrt{5}}}{1+\frac{e^{-4\pi\sqrt{5}}}{1+\ldots}}}}\approx 0.9991104684$$

and to the dilaton value $\mathbf{0.989117352243} = \boldsymbol{\phi}$

**Alternate form:**

$$\frac{\sqrt{2}}{\sqrt[12]{28\,\zeta(3)+\pi^3}}$$

**All 12th roots of 1/((7 ζ(3))/16 + π^3/64):**

$$\frac{e^0}{\sqrt[12]{\frac{7\zeta(3)}{16}+\frac{\pi^3}{64}}}\approx 0.99914 \text{ (real, principal root)}$$

$$\frac{e^{(i\pi)/6}}{\sqrt[12]{\frac{7\zeta(3)}{16}+\frac{\pi^3}{64}}}\approx 0.8653+0.49957\,i$$

$$\frac{e^{(i\pi)/3}}{\sqrt[12]{\frac{7\zeta(3)}{16}+\frac{\pi^3}{64}}} \approx 0.49957 + 0.8653\, i$$

$$\frac{e^{(i\pi)/2}}{\sqrt[12]{\frac{7\zeta(3)}{16}+\frac{\pi^3}{64}}} \approx 0.99914\, i$$

$$\frac{e^{(2i\pi)/3}}{\sqrt[12]{\frac{7\zeta(3)}{16}+\frac{\pi^3}{64}}} \approx -0.49957 + 0.8653\, i$$

**Alternative representations:**

$$\sqrt[12]{\frac{1}{\frac{\pi^3}{64}+\frac{\zeta(3)\,7}{16}}} = \sqrt[12]{\frac{1}{\frac{\pi^3}{64}+\frac{7\zeta(3,1)}{16}}}$$

$$\sqrt[12]{\frac{1}{\frac{\pi^3}{64}+\frac{\zeta(3)\,7}{16}}} = \sqrt[12]{\frac{1}{\frac{7 S_{2,1}(1)}{16}+\frac{\pi^3}{64}}}$$

$$\sqrt[12]{\frac{1}{\frac{\pi^3}{64}+\frac{\zeta(3)\,7}{16}}} = \sqrt[12]{\frac{1}{-\frac{7\,\mathrm{Li}_3(-1)}{\frac{3\times 16}{4}}+\frac{\pi^3}{64}}}$$

**Series representations:**

$$\sqrt[12]{\frac{1}{\frac{\pi^3}{64}+\frac{\zeta(3)\,7}{16}}} = \frac{\sqrt{2}}{\sqrt[12]{\pi^3+28\sum_{k=1}^{\infty}\frac{1}{k^3}}}$$

$$\sqrt[12]{\frac{1}{\frac{\pi^3}{64}+\frac{\zeta(3)\,7}{16}}} = \frac{\sqrt{2}}{\sqrt[12]{\pi^3+32\sum_{k=0}^{\infty}\frac{1}{(1+2k)^3}}}$$

$$\sqrt[12]{\frac{1}{\frac{\pi^3}{64}+\frac{\zeta(3)\,7}{16}}} = \frac{\sqrt{2}}{\sqrt[12]{28\, e^{\sum_{k=1}^{\infty} P(3k)/k}+\pi^3}}$$

**Integral representations:**

$$\sqrt[12]{\frac{1}{\frac{\pi^3}{64}+\frac{\zeta(3)\,7}{16}}} = \frac{\sqrt{2}}{\sqrt[12]{\pi^3+8\int_0^\infty t^2\operatorname{csch}(t)\,dt}}$$

$$\sqrt[12]{\frac{1}{\frac{\pi^3}{64}+\frac{\zeta(3)\,7}{16}}} = \frac{\sqrt{2}}{\sqrt[12]{\pi^3+14\int_0^\infty \frac{t^2}{-1+e^t}\,dt}}$$

$$\sqrt[12]{\frac{1}{\frac{\pi^3}{64}+\frac{\zeta(3)\,7}{16}}} = \frac{1}{\sqrt[12]{\frac{\pi^3}{64}-\frac{7}{48}\int_0^1 \frac{\log^3(1-t^2)}{t^3}\,dt}}$$

From which:

11*log base 0.9991404081447(((1/ ((((Pi^3)/64 + 7/16 zeta(3)))))))+7+1/golden ratio

**Input interpretation:**

$$11\log_{0.9991404081447}\left(\frac{1}{\frac{\pi^3}{64}+\frac{7}{16}\zeta(3)}\right)+7+\frac{1}{\phi}$$

$\zeta(s)$ is the Riemann zeta function

$\log_b(x)$ is the base- $b$ logarithm

$\phi$ is the golden ratio

**Result:**

139.6180340...

139.618034… result practically equal to the rest mass of Pion meson 139.57 MeV

**Alternative representations:**

$$11\log_{0.99914040814470000}\left(\frac{1}{\frac{\pi^3}{64}+\frac{\zeta(3)\,7}{16}}\right)+7+\frac{1}{\phi} = 7+\frac{1}{\phi}+\frac{11\log\left(\frac{1}{\frac{\pi^3}{64}+\frac{7\zeta(3)}{16}}\right)}{\log(0.99914040814470000)}$$

$$11\log_{0.99914040814470000}\left(\frac{1}{\frac{\pi^3}{64}+\frac{\zeta(3)\,7}{16}}\right)+7+\frac{1}{\phi} =$$

$$7+11\log_{0.99914040814470000}\left(\frac{1}{\frac{\pi^3}{64}+\frac{7\zeta(3,1)}{16}}\right)+\frac{1}{\phi}$$

$$11 \log_{0.99914040814470000}\left(\frac{1}{\frac{\pi^3}{64}+\frac{\zeta(3)\,7}{16}}\right)+7+\frac{1}{\phi} = 7+11 \log_{0.99914040814470000}\left(\frac{1}{\frac{7 S_{2,1}(1)}{16}+\frac{\pi^3}{64}}\right)+\frac{1}{\phi}$$

## Series representations:

$$11 \log_{0.99914040814470000}\left(\frac{1}{\frac{\pi^3}{64}+\frac{\zeta(3)\,7}{16}}\right)+7+\frac{1}{\phi} = 7+\frac{1}{\phi}+11 \log_{0.99914040814470000}\left(\frac{64}{\pi^3+28 \sum_{k=1}^{\infty} \frac{1}{k^3}}\right)$$

$$11 \log_{0.99914040814470000}\left(\frac{1}{\frac{\pi^3}{64}+\frac{\zeta(3)\,7}{16}}\right)+7+\frac{1}{\phi} = 7+\frac{1}{\phi}-\frac{11 \sum_{k=1}^{\infty} \frac{(-1)^k \left(-1+\frac{64}{\pi^3+28\,\zeta(3)}\right)^k}{k}}{\log(0.99914040814470000)}$$

$$11 \log_{0.99914040814470000}\left(\frac{1}{\frac{\pi^3}{64}+\frac{\zeta(3)\,7}{16}}\right)+7+\frac{1}{\phi} = 7+\frac{1}{\phi}+11 \log_{0.99914040814470000}\left(\frac{64}{\pi^3+28 \exp\left(-\sum_{k=1}^{\infty} \log\left(1-\frac{1}{(p_k)^3}\right)\right)}\right)$$

## Integral representations:

$$11 \log_{0.99914040814470000}\left(\frac{1}{\frac{\pi^3}{64}+\frac{\zeta(3)\,7}{16}}\right)+7+\frac{1}{\phi} = 7+\frac{1}{\phi}+11 \log_{0.99914040814470000}\left(\frac{64\,\Gamma(3)}{\pi^3\,\Gamma(3)+16 \int_0^{\infty} t^2 \operatorname{csch}(t)\,dt}\right)$$

$$11 \log_{0.99914040814470000}\left(\frac{1}{\frac{\pi^3}{64}+\frac{\zeta(3)\,7}{16}}\right)+7+\frac{1}{\phi} = 7+\frac{1}{\phi}+11 \log_{0.99914040814470000}\left(\frac{64\,\Gamma(3)}{\pi^3\,\Gamma(3)+28 \int_0^{\infty} \frac{t^2}{-1+e^t}\,dt}\right)$$

$$11\log_{0.99914040814470000}\left(\frac{1}{\frac{\pi^3}{64}+\frac{\zeta(3)\,7}{16}}\right)+7+\frac{1}{\phi}=$$

$$7+\frac{1}{\phi}+11\log_{0.99914040814470000}\left(\frac{64\,\Gamma(4)}{\pi^3\,\Gamma(4)+112\int_0^\infty t^3\operatorname{csch}^2(t)\,dt}\right)$$

And:

11*log base 0.99914040081447(((1/ ((((Pi^3)/64 + 7/16 zeta(3)))))))-7+1/golden ratio

**Input interpretation:**

$$11\log_{0.9991404081447}\left(\frac{1}{\frac{\pi^3}{64}+\frac{7}{16}\zeta(3)}\right)-7+\frac{1}{\phi}$$

$\zeta(s)$ is the Riemann zeta function

$\log_b(x)$ is the base- $b$ logarithm

$\phi$ is the golden ratio

**Result:**

125.6180340...

125.618034… result very near to the dilaton mass calculated as a type of Higgs boson: 125 GeV for T = 0 and to the Higgs boson mass 125.18 GeV

**Alternative representations:**

$$11\log_{0.99914040814470000}\left(\frac{1}{\frac{\pi^3}{64}+\frac{\zeta(3)\,7}{16}}\right)-7+\frac{1}{\phi}=-7+\frac{1}{\phi}+\frac{11\log\left(\frac{1}{\frac{\pi^3}{64}+\frac{7\zeta(3)}{16}}\right)}{\log(0.99914040814470000)}$$

$$11\log_{0.99914040814470000}\left(\frac{1}{\frac{\pi^3}{64}+\frac{\zeta(3)\,7}{16}}\right)-7+\frac{1}{\phi}=$$

$$-7+11\log_{0.99914040814470000}\left(\frac{1}{\frac{\pi^3}{64}+\frac{7\zeta(3,1)}{16}}\right)+\frac{1}{\phi}$$

$$11\log_{0.99914040814470000}\left(\frac{1}{\frac{\pi^3}{64}+\frac{\zeta(3)\,7}{16}}\right)-7+\frac{1}{\phi}=$$
$$-7+11\log_{0.99914040814470000}\left(\frac{1}{\frac{7S_{2,1}(1)}{16}+\frac{\pi^3}{64}}\right)+\frac{1}{\phi}$$

## Series representations:

$$11\log_{0.99914040814470000}\left(\frac{1}{\frac{\pi^3}{64}+\frac{\zeta(3)\,7}{16}}\right)-7+\frac{1}{\phi}=$$
$$-7+\frac{1}{\phi}+11\log_{0.99914040814470000}\left(\frac{64}{\pi^3+28\sum_{k=1}^{\infty}\frac{1}{k^3}}\right)$$

$$11\log_{0.99914040814470000}\left(\frac{1}{\frac{\pi^3}{64}+\frac{\zeta(3)\,7}{16}}\right)-7+\frac{1}{\phi}=-7+\frac{1}{\phi}-\frac{11\sum_{k=1}^{\infty}\frac{(-1)^k\left(-1+\frac{64}{\pi^3+28\zeta(3)}\right)^k}{k}}{\log(0.99914040814470000)}$$

$$11\log_{0.99914040814470000}\left(\frac{1}{\frac{\pi^3}{64}+\frac{\zeta(3)\,7}{16}}\right)-7+\frac{1}{\phi}=$$
$$-7+\frac{1}{\phi}+11\log_{0.99914040814470000}\left(\frac{64}{\pi^3+28\exp\left(-\sum_{k=1}^{\infty}\log\left(1-\frac{1}{(p_k)^3}\right)\right)}\right)$$

## Integral representations:

$$11\log_{0.99914040814470000}\left(\frac{1}{\frac{\pi^3}{64}+\frac{\zeta(3)\,7}{16}}\right)-7+\frac{1}{\phi}=$$
$$-7+\frac{1}{\phi}+11\log_{0.99914040814470000}\left(\frac{64\,\Gamma(3)}{\pi^3\,\Gamma(3)+16\int_0^{\infty}t^2\operatorname{csch}(t)\,dt}\right)$$

$$11\log_{0.99914040814470000}\left(\frac{1}{\frac{\pi^3}{64}+\frac{\zeta(3)\,7}{16}}\right)-7+\frac{1}{\phi}=$$
$$-7+\frac{1}{\phi}+11\log_{0.99914040814470000}\left(\frac{64\,\Gamma(3)}{\pi^3\,\Gamma(3)+28\int_0^{\infty}\frac{t^2}{-1+e^t}\,dt}\right)$$

$$11\log_{0.99914040814470000}\left(\frac{1}{\frac{\pi^3}{64}+\frac{\zeta(3)\,7}{16}}\right)-7+\frac{1}{\phi}=$$
$$-7+\frac{1}{\phi}+11\log_{0.99914040814470000}\left(\frac{64\,\Gamma(4)}{\pi^3\,\Gamma(4)+112\int_0^{\infty}t^3\operatorname{csch}^2(t)\,dt}\right)$$

Now, we have that:

1/(1^3) + 1/(4^3) + 1/(7^3) +... = (2Pi^3)/81sqrt2 + 13/27 zeta(3)

1/(1^3) + 1/(4^3) + 1/(7^3) +...

**Input interpretation:**

$$\frac{1}{1^3} + \frac{1}{4^3} + \frac{1}{7^3} + \cdots$$

**Infinite sum:**

$$\sum_{n=1}^{\infty} \frac{1}{(3n-2)^3} = \frac{1}{243}\left(117\,\zeta(3) + 2\sqrt{3}\,\pi^3\right)$$

$\zeta(s)$ is the Riemann zeta function

**Decimal approximation:**

1.020780044433363102823254739903981825353410937519069669735...

1.020780044433363…

**Convergence tests:**

The ratio test is inconclusive.

The root test is inconclusive.

By the comparison test, the series converges.

**Partial sum formula:**

$$\sum_{n=1}^{m} \frac{1}{(-2+3n)^3} = \frac{1}{54}\left(\psi^{(2)}\left(m+\frac{1}{3}\right) - \psi^{(2)}\left(\frac{1}{3}\right)\right)$$

$\psi^{(n)}(x)$ is the $n^{\text{th}}$ derivative of the digamma function

**Alternate form:**

$$\frac{13\,\zeta(3)}{27} + \frac{2\pi^3}{81\sqrt{3}}$$

**Series representations:**

$$\frac{1}{243}\left(2\sqrt{3}\,\pi^3 + 117\,\zeta(3)\right) = \frac{2\pi^3}{81\sqrt{3}} + \frac{13}{27}\sum_{k=1}^{\infty}\frac{1}{k^3}$$

$$\frac{1}{243}\left(2\sqrt{3}\,\pi^3 + 117\,\zeta(3)\right) = \frac{2\pi^3}{81\sqrt{3}} + \frac{104}{189}\sum_{k=0}^{\infty}\frac{1}{(1+2k)^3}$$

$$\frac{1}{243}\left(2\sqrt{3}\,\pi^3+117\,\zeta(3)\right)=\frac{13}{27}\,e^{\sum_{k=1}^{\infty} P(3k)/k}+\frac{2\,\pi^3}{81\sqrt{3}}$$

$$\frac{1}{243}\left(2\sqrt{3}\,\pi^3+117\,\zeta(3)\right)=\frac{2}{243}\left(\sqrt{3}\,\pi^3+78\times\sum_{n=0}^{\infty}2^{-1-n}\sum_{k=0}^{n}\frac{(-1)^k\binom{n}{k}}{(1+k)^3}\right)$$

(2Pi^3)/(81sqrt2) + 13/27 zeta(3)

**Input:**

$$\frac{2\,\pi^3}{81\sqrt{2}}+\frac{13}{27}\,\zeta(3)$$

$\zeta(s)$ is the Riemann zeta function

**Exact result:**

$$\frac{13\,\zeta(3)}{27}+\frac{\sqrt{2}\,\pi^3}{81}$$

**Decimal approximation:**

1.120119953372800115556848609058141510791754061631991953629...

1.1201199533728….

**Alternate form:**

$$\frac{1}{81}\left(39\,\zeta(3)+\sqrt{2}\,\pi^3\right)$$

**Alternative representations:**

$$\frac{2\,\pi^3}{81\sqrt{2}}+\frac{\zeta(3)\,13}{27}=\frac{2\,\pi^3}{81\sqrt{2}}+\frac{13\,\zeta(3,1)}{27}$$

$$\frac{2\,\pi^3}{81\sqrt{2}}+\frac{\zeta(3)\,13}{27}=\frac{13\,S_{2,1}(1)}{27}+\frac{2\,\pi^3}{81\sqrt{2}}$$

$$\frac{2\,\pi^3}{81\sqrt{2}}+\frac{\zeta(3)\,13}{27}=-\frac{13\,\mathrm{Li}_3(-1)}{\frac{3\times27}{4}}+\frac{2\,\pi^3}{81\sqrt{2}}$$

**Series representations:**

$$\frac{2\,\pi^3}{81\sqrt{2}}+\frac{\zeta(3)\,13}{27}=\frac{\sqrt{2}\,\pi^3}{81}+\frac{13}{27}\sum_{k=1}^{\infty}\frac{1}{k^3}$$

$$\frac{2\pi^3}{81\sqrt{2}} + \frac{\zeta(3)\,13}{27} = \frac{\sqrt{2}\,\pi^3}{81} + \frac{104}{189}\sum_{k=0}^{\infty}\frac{1}{(1+2k)^3}$$

$$\frac{2\pi^3}{81\sqrt{2}} + \frac{\zeta(3)\,13}{27} = \frac{13}{27}\,e^{\sum_{k=1}^{\infty} P(3k)/k} + \frac{\sqrt{2}\,\pi^3}{81}$$

**Integral representations:**

$$\frac{2\pi^3}{81\sqrt{2}} + \frac{\zeta(3)\,13}{27} = \frac{\sqrt{2}\,\pi^3}{81} - \frac{13}{81}\int_0^1 \frac{\log^3(1-t^2)}{t^3}\,dt$$

$$\frac{2\pi^3}{81\sqrt{2}} + \frac{\zeta(3)\,13}{27} = \frac{\sqrt{2}\,\pi^3}{81} + \frac{13}{54}\int_0^{\infty} \frac{t^2}{-1+e^t}\,dt$$

$$\frac{2\pi^3}{81\sqrt{2}} + \frac{\zeta(3)\,13}{27} = \frac{\sqrt{2}\,\pi^3}{81} + \frac{26}{81}\int_0^{\infty} \frac{t^2}{1+e^t}\,dt$$

From which:

(((1/((((2Pi^3)/(81sqrt2) + 13/27 zeta(3)))))))^1/128

**Input:**

$$\sqrt[128]{\frac{1}{\frac{2\pi^3}{81\sqrt{2}} + \frac{13}{27}\zeta(3)}}$$

$\zeta(s)$ is the Riemann zeta function

**Exact result:**

$$\frac{1}{\sqrt[128]{\frac{13\,\zeta(3)}{27} + \frac{\sqrt{2}\,\pi^3}{81}}}$$

**Decimal approximation:**

0.999114175536858768080401697435111237630999529642565743801...

0.999114175536… result very near to the value of the following Rogers-Ramanujan continued fraction:

$$\frac{e^{-\frac{\pi}{\sqrt{5}}}}{\frac{\sqrt{5}}{1+\sqrt[5]{\sqrt{\varphi^5\sqrt[4]{5^3}}-1}}-\varphi+1}=1-\cfrac{e^{-\pi\sqrt{5}}}{1+\cfrac{e^{-2\pi\sqrt{5}}}{1+\cfrac{e^{-3\pi\sqrt{5}}}{1+\cfrac{e^{-4\pi\sqrt{5}}}{1+\ldots}}}}\approx 0.9991104684$$

and to the dilaton value $\mathbf{0.989117352243} = \boldsymbol{\phi}$

**Alternate form:**

$$\frac{\sqrt[32]{3}}{\sqrt[128]{39\,\zeta(3)+\sqrt{2}\,\pi^3}}$$

**All 128th roots of 1/((13 ζ(3))/27 + (sqrt(2) π^3)/81):**

$$\frac{e^0}{\sqrt[128]{\frac{13\,\zeta(3)}{27}+\frac{\sqrt{2}\,\pi^3}{81}}}\approx 0.999114 \text{ (real, principal root)}$$

$$\frac{e^{(i\pi)/64}}{\sqrt[128]{\frac{13\,\zeta(3)}{27}+\frac{\sqrt{2}\,\pi^3}{81}}}\approx 0.997911+0.049024\,i$$

$$\frac{e^{(i\pi)/32}}{\sqrt[128]{\frac{13\,\zeta(3)}{27}+\frac{\sqrt{2}\,\pi^3}{81}}}\approx 0.994303+0.09793\,i$$

$$\frac{e^{(3i\pi)/64}}{\sqrt[128]{\frac{13\,\zeta(3)}{27}+\frac{\sqrt{2}\,\pi^3}{81}}}\approx 0.988300+0.14660\,i$$

$$\frac{e^{(i\pi)/16}}{\sqrt[128]{\frac{13\,\zeta(3)}{27}+\frac{\sqrt{2}\,\pi^3}{81}}}\approx 0.97992+0.19492\,i$$

**Alternative representations:**

$$\sqrt[128]{\frac{1}{\frac{2\pi^3}{81\sqrt{2}}+\frac{\zeta(3)\,13}{27}}}=\sqrt[128]{\frac{1}{\frac{2\pi^3}{81\sqrt{2}}+\frac{13\,\zeta(3,1)}{27}}}$$

$$\sqrt[128]{\frac{1}{\frac{2\pi^3}{81\sqrt{2}} + \frac{\zeta(3)\,13}{27}}} = \sqrt[128]{\frac{1}{\frac{13\,S_{2,1}(1)}{27} + \frac{2\pi^3}{81\sqrt{2}}}}$$

$$\sqrt[128]{\frac{1}{\frac{2\pi^3}{81\sqrt{2}} + \frac{\zeta(3)\,13}{27}}} = \sqrt[128]{\frac{1}{-\frac{13\,\mathrm{Li}_3(-1)}{\frac{3\times 27}{4}} + \frac{2\pi^3}{81\sqrt{2}}}}$$

## Series representations:

$$\sqrt[128]{\frac{1}{\frac{2\pi^3}{81\sqrt{2}} + \frac{\zeta(3)\,13}{27}}} = \frac{\sqrt[32]{3}}{\sqrt[128]{\sqrt{2}\,\pi^3 + 39\sum_{k=1}^{\infty}\frac{1}{k^3}}}$$

$$\sqrt[128]{\frac{1}{\frac{2\pi^3}{81\sqrt{2}} + \frac{\zeta(3)\,13}{27}}} = \frac{1}{\sqrt[128]{\frac{\sqrt{2}\,\pi^3}{81} + \frac{104}{189}\sum_{k=0}^{\infty}\frac{1}{(1+2k)^3}}}$$

$$\sqrt[128]{\frac{1}{\frac{2\pi^3}{81\sqrt{2}} + \frac{\zeta(3)\,13}{27}}} = \frac{\sqrt[32]{3}}{\sqrt[128]{39\,e^{\sum_{k=1}^{\infty} P(3k)/k} + \sqrt{2}\,\pi^3}}$$

## Integral representations:

$$\sqrt[128]{\frac{1}{\frac{2\pi^3}{81\sqrt{2}} + \frac{\zeta(3)\,13}{27}}} = \frac{\sqrt[32]{3}}{\sqrt[128]{\sqrt{2}\,\pi^3 - 13\int_0^1 \frac{\log^3(1-t^2)}{t^3}\,dt}}$$

$$\sqrt[128]{\frac{1}{\frac{2\pi^3}{81\sqrt{2}} + \frac{\zeta(3)\,13}{27}}} = \frac{\sqrt[32]{3}}{\sqrt[128]{\sqrt{2}\,\pi^3 + 26\int_0^{\infty} \frac{t^2}{1+e^t}\,dt}}$$

$$\sqrt[128]{\frac{1}{\frac{2\pi^3}{81\sqrt{2}} + \frac{\zeta(3)\,13}{27}}} = \frac{\sqrt[32]{3}}{\sqrt[128]{\sqrt{2}\,\pi^3 + 26\int_0^{\infty} t^3\,\mathrm{csch}^2(t)\,dt}}$$

From which:

log base 0.999114175536(((1/((((2Pi^3)/(81sqrt2) + 13/27 zeta(3)))))))-Pi+1/golden ratio

**Input interpretation:**

$$\log_{0.999114175536}\left(\frac{1}{\frac{2\pi^3}{81\sqrt{2}}+\frac{13}{27}\zeta(3)}\right)-\pi+\frac{1}{\phi}$$

$\zeta(s)$ is the Riemann zeta function

$\log_b(x)$ is the base- $b$ logarithm

$\phi$ is the golden ratio

**Result:**

125.476441...

125.476441... result very near to the dilaton mass calculated as a type of Higgs boson: 125 GeV for T = 0 and to the Higgs boson mass 125.18 GeV

**Alternative representations:**

$$\log_{0.9991141755360000}\left(\frac{1}{\frac{2\pi^3}{81\sqrt{2}}+\frac{\zeta(3)\,13}{27}}\right)-\pi+\frac{1}{\phi}=$$
$$-\pi+\log_{0.9991141755360000}\left(\frac{1}{\frac{2\pi^3}{81\sqrt{2}}+\frac{13\,\zeta(3,1)}{27}}\right)+\frac{1}{\phi}$$

$$\log_{0.9991141755360000}\left(\frac{1}{\frac{2\pi^3}{81\sqrt{2}}+\frac{\zeta(3)\,13}{27}}\right)-\pi+\frac{1}{\phi}=-\pi+\frac{1}{\phi}+\frac{\log\left(\frac{1}{\frac{2\pi^3}{81\sqrt{2}}+\frac{13\,\zeta(3)}{27}}\right)}{\log(0.9991141755360000)}$$

$$\log_{0.9991141755360000}\left(\frac{1}{\frac{2\pi^3}{81\sqrt{2}}+\frac{\zeta(3)\,13}{27}}\right)-\pi+\frac{1}{\phi}=$$
$$-\pi+\log_{0.9991141755360000}\left(\frac{1}{\frac{13\,S_{2,1}(1)}{27}+\frac{2\pi^3}{81\sqrt{2}}}\right)+\frac{1}{\phi}$$

## Series representations:

$$\log_{0.9991141755360000}\left(\frac{1}{\frac{2\pi^3}{81\sqrt{2}}+\frac{\zeta(3)\,13}{27}}\right)-\pi+\frac{1}{\phi}=\frac{1}{\phi}-\pi-\frac{\sum_{k=1}^{\infty}\frac{(-1)^k\left(-1+\frac{81\sqrt{2}}{2\pi^3+39\sqrt{2}\,\zeta(3)}\right)^k}{k}}{\log(0.9991141755360000)}$$

$$\log_{0.9991141755360000}\left(\frac{1}{\frac{2\pi^3}{81\sqrt{2}}+\frac{\zeta(3)\,13}{27}}\right)-\pi+\frac{1}{\phi}=\frac{1.000000000000}{\phi}-$$
$$1.000000000000\,\pi-1128.391829747\log\left(\frac{81\sqrt{2}}{2\pi^3+39\sqrt{2}\,\zeta(3)}\right)-$$
$$1.000000000000\log\left(\frac{81\sqrt{2}}{2\pi^3+39\sqrt{2}\,\zeta(3)}\right)\sum_{k=0}^{\infty}(-0.0008858244640000)^k\,G(k)$$
$$\text{for }\left(G(0)=0\text{ and }\frac{(-1)^k\,k}{2(1+k)(2+k)}+G(k)=\sum_{j=1}^{k}\frac{(-1)^{1+j}\,G(-j+k)}{1+j}\right)$$

$$\log_{0.9991141755360000}\left(\frac{1}{\frac{2\pi^3}{81\sqrt{2}}+\frac{\zeta(3)\,13}{27}}\right)-\pi+\frac{1}{\phi}=\frac{1.000000000000}{\phi}-$$
$$1.000000000000\,\pi-1128.391829747\log\left(\frac{81\sqrt{2}}{2\pi^3+39\sqrt{2}\,\zeta(3)}\right)-$$
$$1.000000000000\log\left(\frac{81\sqrt{2}}{2\pi^3+39\sqrt{2}\,\zeta(3)}\right)\sum_{k=0}^{\infty}(-0.0008858244640000)^k\,G(k)$$
$$\text{for }\left(G(0)=0\text{ and }G(k)=\frac{(-1)^{1+k}\,k}{2(1+k)(2+k)}+\sum_{j=1}^{k}\frac{(-1)^{1+j}\,G(-j+k)}{1+j}\right)$$

## Integral representations:

$$\log_{0.9991141755360000}\left(\frac{1}{\frac{2\pi^3}{81\sqrt{2}}+\frac{\zeta(3)\,13}{27}}\right)-\pi+\frac{1}{\phi}=$$
$$\frac{1}{\phi}-\pi+\log_{0.9991141755360000}\left(\frac{567\,\Gamma(3)\sqrt{2}}{14\pi^3\,\Gamma(3)+156\sqrt{2}\int_0^{\infty}t^2\,\operatorname{csch}(t)\,dt}\right)$$

$$\log_{0.9991141755360000}\left(\frac{1}{\frac{2\pi^3}{81\sqrt{2}}+\frac{\zeta(3)\,13}{27}}\right)-\pi+\frac{1}{\phi}=$$
$$\frac{1}{\phi}-\pi+\log_{0.9991141755360000}\left(\frac{81\,\Gamma(3)\sqrt{2}}{2\pi^3\,\Gamma(3)+39\sqrt{2}\int_0^{\infty}\frac{t^2}{-1+e^t}\,dt}\right)$$

$$\log_{0.9991141755360000}\left(\frac{1}{\frac{2\pi^3}{81\sqrt{2}}+\frac{\zeta(3)\,13}{27}}\right)-\pi+\frac{1}{\phi}=$$

$$\frac{1}{\phi}-\pi+\log_{0.9991141755360000}\left(\frac{81\,\Gamma(3)\sqrt{2}}{2\pi^3\,\Gamma(3)+52\sqrt{2}\int_0^\infty \frac{t^2}{1+e^t}\,dt}\right)$$

log base 0.999114175536(((1/((((2Pi^3)/(81sqrt2) + 13/27 zeta(3)))))))+11+1/golden ratio

**Input interpretation:**

$$\log_{0.999114175536}\left(\frac{1}{\frac{2\pi^3}{81\sqrt{2}}+\frac{13}{27}\zeta(3)}\right)+11+\frac{1}{\phi}$$

$\zeta(s)$ is the Riemann zeta function

$\log_b(x)$ is the base- $b$ logarithm

$\phi$ is the golden ratio

**Result:**

139.618034...

139.618034… result practically equal to the rest mass of Pion meson 139.57 MeV

**Alternative representations:**

$$\log_{0.9991141755360000}\left(\frac{1}{\frac{2\pi^3}{81\sqrt{2}}+\frac{\zeta(3)\,13}{27}}\right)+11+\frac{1}{\phi}=$$

$$11+\log_{0.9991141755360000}\left(\frac{1}{\frac{2\pi^3}{81\sqrt{2}}+\frac{13\,\zeta(3,1)}{27}}\right)+\frac{1}{\phi}$$

$$\log_{0.9991141755360000}\left(\frac{1}{\frac{2\pi^3}{81\sqrt{2}}+\frac{\zeta(3)\,13}{27}}\right)+11+\frac{1}{\phi}=11+\frac{1}{\phi}+\frac{\log\left(\frac{1}{\frac{2\pi^3}{81\sqrt{2}}+\frac{13\,\zeta(3)}{27}}\right)}{\log(0.9991141755360000)}$$

$$\log_{0.9991141755360000}\left(\frac{1}{\frac{2\pi^3}{81\sqrt{2}}+\frac{\zeta(3)\,13}{27}}\right)+11+\frac{1}{\phi}=$$
$$11+\log_{0.9991141755360000}\left(\frac{1}{\frac{13\,S_{2,1}(1)}{27}+\frac{2\pi^3}{81\sqrt{2}}}\right)+\frac{1}{\phi}$$

**Series representations:**

$$\log_{0.9991141755360000}\left(\frac{1}{\frac{2\pi^3}{81\sqrt{2}}+\frac{\zeta(3)\,13}{27}}\right)+11+\frac{1}{\phi}=11+\frac{1}{\phi}-\frac{\sum_{k=1}^{\infty}\frac{(-1)^k\left(-1+\frac{81\sqrt{2}}{2\pi^3+39\sqrt{2}\,\zeta(3)}\right)^k}{k}}{\log(0.9991141755360000)}$$

$$\log_{0.9991141755360000}\left(\frac{1}{\frac{2\pi^3}{81\sqrt{2}}+\frac{\zeta(3)\,13}{27}}\right)+11+\frac{1}{\phi}=$$
$$11.00000000000+\frac{1.000000000000}{\phi}-1128.391829747\log\left(\frac{81\sqrt{2}}{2\pi^3+39\sqrt{2}\,\zeta(3)}\right)-$$
$$1.000000000000\log\left(\frac{81\sqrt{2}}{2\pi^3+39\sqrt{2}\,\zeta(3)}\right)\sum_{k=0}^{\infty}(-0.0008858244640000)^k\,G(k)$$
$$\text{for}\left(G(0)=0\text{ and }\frac{(-1)^k\,k}{2(1+k)(2+k)}+G(k)=\sum_{j=1}^{k}\frac{(-1)^{1+j}\,G(-j+k)}{1+j}\right)$$

$$\log_{0.9991141755360000}\left(\frac{1}{\frac{2\pi^3}{81\sqrt{2}}+\frac{\zeta(3)\,13}{27}}\right)+11+\frac{1}{\phi}=$$
$$11.00000000000+\frac{1.000000000000}{\phi}-1128.391829747\log\left(\frac{81\sqrt{2}}{2\pi^3+39\sqrt{2}\,\zeta(3)}\right)-$$
$$1.000000000000\log\left(\frac{81\sqrt{2}}{2\pi^3+39\sqrt{2}\,\zeta(3)}\right)\sum_{k=0}^{\infty}(-0.0008858244640000)^k\,G(k)$$
$$\text{for}\left(G(0)=0\text{ and }G(k)=\frac{(-1)^{1+k}\,k}{2(1+k)(2+k)}+\sum_{j=1}^{k}\frac{(-1)^{1+j}\,G(-j+k)}{1+j}\right)$$

**Integral representations:**

$$\log_{0.9991141755360000}\left(\frac{1}{\frac{2\pi^3}{81\sqrt{2}}+\frac{\zeta(3)\,13}{27}}\right)+11+\frac{1}{\phi}=$$

$$11+\frac{1}{\phi}+\log_{0.9991141755360000}\left(\frac{567\,\Gamma(3)\sqrt{2}}{14\pi^3\,\Gamma(3)+156\sqrt{2}\int_0^\infty t^2\operatorname{csch}(t)\,dt}\right)$$

$$\log_{0.9991141755360000}\left(\frac{1}{\frac{2\pi^3}{81\sqrt{2}}+\frac{\zeta(3)\,13}{27}}\right)+11+\frac{1}{\phi}=$$

$$11+\frac{1}{\phi}+\log_{0.9991141755360000}\left(\frac{81\,\Gamma(3)\sqrt{2}}{2\pi^3\,\Gamma(3)+39\sqrt{2}\int_0^\infty \frac{t^2}{-1+e^t}\,dt}\right)$$

$$\log_{0.9991141755360000}\left(\frac{1}{\frac{2\pi^3}{81\sqrt{2}}+\frac{\zeta(3)\,13}{27}}\right)+11+\frac{1}{\phi}=$$

$$11+\frac{1}{\phi}+\log_{0.9991141755360000}\left(\frac{81\,\Gamma(3)\sqrt{2}}{2\pi^3\,\Gamma(3)+52\sqrt{2}\int_0^\infty \frac{t^2}{1+e^t}\,dt}\right)$$

Now, we have that:

(Pi^3)/36sqrt3 + 91/216 zeta(3)

1/(1^3) + 1/7^3 + 1/13^3 + ...

**Input interpretation:**

$$\frac{1}{1^3}+\frac{1}{7^3}+\frac{1}{13^3}+\cdots$$

**Infinite sum:**

$$\sum_{n=1}^{\infty}\frac{1}{(6n-5)^3}=\frac{1}{216}\left(91\,\zeta(3)+2\sqrt{3}\,\pi^3\right)$$

$\zeta(s)$ is the Riemann zeta function

**Decimal approximation:**

1.003685515347952697063230137024860573152727843593893327866...

1.00368551534….

**Convergence tests:**

The ratio test is inconclusive.

The root test is inconclusive.

By the comparison test, the series converges.

**Partial sum formula:**

$$\sum_{n=1}^{m} \frac{1}{(-5+6n)^3} = \frac{1}{432}\left(\psi^{(2)}\left(m+\frac{1}{6}\right) - \psi^{(2)}\left(\frac{1}{6}\right)\right)$$

$\psi^{(n)}(x)$ is the $n^{\text{th}}$ derivative of the digamma function

**Alternate form:**

$$\frac{91\,\zeta(3)}{216} + \frac{\pi^3}{36\sqrt{3}}$$

**Series representations:**

$$\frac{1}{216}\left(2\sqrt{3}\,\pi^3 + 91\,\zeta(3)\right) = \frac{\pi^3}{36\sqrt{3}} + \frac{91}{216}\sum_{k=1}^{\infty}\frac{1}{k^3}$$

$$\frac{1}{216}\left(2\sqrt{3}\,\pi^3 + 91\,\zeta(3)\right) = \frac{\pi^3}{36\sqrt{3}} + \frac{13}{27}\sum_{k=0}^{\infty}\frac{1}{(1+2k)^3}$$

$$\frac{1}{216}\left(2\sqrt{3}\,\pi^3 + 91\,\zeta(3)\right) = \frac{91}{216}\,e^{\sum_{k=1}^{\infty} P(3k)/k} + \frac{\pi^3}{36\sqrt{3}}$$

$$\frac{1}{216}\left(2\sqrt{3}\,\pi^3 + 91\,\zeta(3)\right) = \frac{1}{432}\left(4\sqrt{3}\,\pi^3 + 91\sum_{n=0}^{\infty}\frac{\sum_{k=0}^{n}\frac{(-1)^k\binom{n}{k}}{(1+k)^2}}{1+n}\right)$$

(Pi^3)/(36sqrt3) + 91/216 zeta(3)

**Input:**

$$\frac{\pi^3}{36\sqrt{3}} + \frac{91}{216}\,\zeta(3)$$

$\zeta(s)$ is the Riemann zeta function

**Exact result:**

$$\frac{91\,\zeta(3)}{216} + \frac{\pi^3}{36\sqrt{3}}$$

**Decimal approximation:**

1.0036855153479526970632301370248605731527278435938933278 66...

1.003685515347933333

**Alternate forms:**

$$\frac{1}{216}\left(91\,\zeta(3)+2\sqrt{3}\,\pi^3\right)$$

$$\frac{91\sqrt{3}\,\zeta(3)+6\,\pi^3}{216\sqrt{3}}$$

**Alternative representations:**

$$\frac{\pi^3}{36\sqrt{3}}+\frac{\zeta(3)\,91}{216}=\frac{\pi^3}{36\sqrt{3}}+\frac{91\,\zeta(3,1)}{216}$$

$$\frac{\pi^3}{36\sqrt{3}}+\frac{\zeta(3)\,91}{216}=\frac{91\,S_{2,1}(1)}{216}+\frac{\pi^3}{36\sqrt{3}}$$

$$\frac{\pi^3}{36\sqrt{3}}+\frac{\zeta(3)\,91}{216}=-\frac{91\,\mathrm{Li}_3(-1)}{\frac{3\times 216}{4}}+\frac{\pi^3}{36\sqrt{3}}$$

**Series representations:**

$$\frac{\pi^3}{36\sqrt{3}}+\frac{\zeta(3)\,91}{216}=\frac{\pi^3}{36\sqrt{3}}+\frac{91}{216}\sum_{k=1}^{\infty}\frac{1}{k^3}$$

$$\frac{\pi^3}{36\sqrt{3}}+\frac{\zeta(3)\,91}{216}=\frac{\pi^3}{36\sqrt{3}}+\frac{13}{27}\sum_{k=0}^{\infty}\frac{1}{(1+2\,k)^3}$$

$$\frac{\pi^3}{36\sqrt{3}}+\frac{\zeta(3)\,91}{216}=\frac{91}{216}\,e^{\sum_{k=1}^{\infty}P(3\,k)/k}+\frac{\pi^3}{36\sqrt{3}}$$

**Integral representations:**

$$\frac{\pi^3}{36\sqrt{3}}+\frac{\zeta(3)\,91}{216}=\frac{\pi^3}{36\sqrt{3}}-\frac{91}{648}\int_0^1\frac{\log^3(1-t^2)}{t^3}\,dt$$

$$\frac{\pi^3}{36\sqrt{3}}+\frac{\zeta(3)\,91}{216}=\frac{\pi^3}{36\sqrt{3}}+\frac{91}{432}\int_0^{\infty}\frac{t^2}{-1+e^t}\,dt$$

$$\frac{\pi^3}{36\sqrt{3}}+\frac{\zeta(3)\,91}{216}=\frac{\pi^3}{36\sqrt{3}}+\frac{91}{324}\int_0^{\infty}\frac{t^2}{1+e^t}\,dt$$

1/(1^3) + 1/(3^3) + 1/(5^3) +...

**Input interpretation:**

$$\frac{1}{1^3}+\frac{1}{3^3}+\frac{1}{5^3}+\cdots$$

**Infinite sum:**

$$\sum_{n=1}^{\infty}\frac{1}{(2n-1)^3}=\frac{7\zeta(3)}{8}$$

$\zeta(s)$ is the Riemann zeta function

**Decimal approximation:**

1.051799790264644999724770891322518741919363005797936521568...

1.05179979026…

**Convergence tests:**

The ratio test is inconclusive.

The root test is inconclusive.

By the comparison test, the series converges.

**Partial sum formula:**

$$\sum_{n=1}^{m}\frac{1}{(-1+2n)^3}=\frac{1}{16}\left(\psi^{(2)}\left(m+\frac{1}{2}\right)-\psi^{(2)}\left(\frac{1}{2}\right)\right)$$

$\psi^{(n)}(x)$ is the $n^{\text{th}}$ derivative of the digamma function

**Series representations:**

$$\frac{7\zeta(3)}{8}=\frac{7}{8}\sum_{k=1}^{\infty}\frac{1}{k^3}$$

$$\frac{7\zeta(3)}{8}=\sum_{k=0}^{\infty}\frac{1}{(1+2k)^3}$$

$$\frac{7\zeta(3)}{8}=\frac{7}{8}e^{\sum_{k=1}^{\infty}P(3k)/k}$$

$$\frac{7\zeta(3)}{8} = \frac{7}{6} \times \sum_{n=0}^{\infty} 2^{-1-n} \sum_{k=0}^{n} \frac{(-1)^k \binom{n}{k}}{(1+k)^3}$$

7/8 zeta(3)

**Input:**

$$\frac{7}{8}\zeta(3)$$

$\zeta(s)$ is the Riemann zeta function

**Exact result:**

$$\frac{7\zeta(3)}{8}$$

**Decimal approximation:**

1.051799790264644999724770891322518741919363005797936521568...

1.0517997902646…

**Alternative representations:**

$$\frac{\zeta(3)\,7}{8} = \frac{7\zeta(3,1)}{8}$$

$$\frac{\zeta(3)\,7}{8} = \frac{7\,S_{2,1}(1)}{8}$$

$$\frac{\zeta(3)\,7}{8} = -\frac{7\,\mathrm{Li}_3(-1)}{\frac{3\times 8}{4}}$$

**Series representations:**

$$\frac{\zeta(3)\,7}{8} = \frac{7}{8}\sum_{k=1}^{\infty}\frac{1}{k^3}$$

$$\frac{\zeta(3)\,7}{8} = \sum_{k=0}^{\infty}\frac{1}{(1+2k)^3}$$

$$\frac{\zeta(3)\,7}{8} = \frac{7}{8}\,e^{\sum_{k=1}^{\infty} P(3k)/k}$$

**Integral representations:**

$$\frac{\zeta(3)\,7}{8} = -\frac{7}{24}\int_0^1 \frac{\log^3(1-t^2)}{t^3}\,dt$$

$$\frac{\zeta(3)\,7}{8} = \frac{1}{4}\int_0^\infty t^2 \operatorname{csch}(t)\,dt$$

$$\frac{\zeta(3)\,7}{8} = \frac{7}{16}\int_0^\infty \frac{t^2}{-1+e^t}\,dt$$

Now, we perform the sum of the four expressions:

7/8 zeta(3) (Note that $S_3$ is ζ(3))

(2Pi^3)/(81sqrt2) + 13/27 zeta(3)

(Pi^3)/64 + 7/16 zeta(3)

(Pi^3)/(36sqrt3) + 91/216 zeta(3)

We obtain:

7/8 zeta(3) + (2Pi^3)/(81sqrt2) + 13/27 zeta(3) + (Pi^3)/64 + 7/16 zeta(3) + (Pi^3)/(36sqrt3) + 91/216 zeta(3)

**Input:**

$$\frac{7}{8}\zeta(3) + \frac{2\pi^3}{81\sqrt{2}} + \frac{13}{27}\zeta(3) + \frac{\pi^3}{64} + \frac{7}{16}\zeta(3) + \frac{\pi^3}{36\sqrt{3}} + \frac{91}{216}\zeta(3)$$

$\zeta(s)$ is the Riemann zeta function

**Exact result:**

$$\frac{319\,\zeta(3)}{144} + \frac{\pi^3}{64} + \frac{\sqrt{2}\,\pi^3}{81} + \frac{\pi^3}{36\sqrt{3}}$$

**Decimal approximation:**

4.185978227247405002449052505990239496858296547764744871569...

4.185978227247…

**Alternate forms:**

$$\frac{319\,\zeta(3)}{144} + \frac{(81+64\sqrt{2}+48\sqrt{3})\pi^3}{5184}$$

$$\frac{11484\,\zeta(3) + 81\pi^3 + 64\sqrt{2}\,\pi^3 + 48\sqrt{3}\,\pi^3}{5184}$$

$$\frac{11484\sqrt{3}\,\zeta(3)+\left(144+81\sqrt{3}+64\sqrt{6}\right)\pi^3}{5184\sqrt{3}}$$

**Alternative representations:**

$$\frac{\zeta(3)\,7}{8}+\frac{2\pi^3}{81\sqrt{2}}+\frac{\zeta(3)\,13}{27}+\frac{\pi^3}{64}+\frac{\zeta(3)\,7}{16}+\frac{\pi^3}{36\sqrt{3}}+\frac{\zeta(3)\,91}{216}=$$
$$\frac{\pi^3}{64}+\frac{2\pi^3}{81\sqrt{2}}+\frac{\pi^3}{36\sqrt{3}}+\frac{7\zeta(3,1)}{8}+\frac{7\zeta(3,1)}{16}+\frac{13\zeta(3,1)}{27}+\frac{91\zeta(3,1)}{216}$$

$$\frac{\zeta(3)\,7}{8}+\frac{2\pi^3}{81\sqrt{2}}+\frac{\zeta(3)\,13}{27}+\frac{\pi^3}{64}+\frac{\zeta(3)\,7}{16}+\frac{\pi^3}{36\sqrt{3}}+\frac{\zeta(3)\,91}{216}=$$
$$\frac{7S_{2,1}(1)}{8}+\frac{7S_{2,1}(1)}{16}+\frac{13S_{2,1}(1)}{27}+\frac{91S_{2,1}(1)}{216}+\frac{\pi^3}{64}+\frac{2\pi^3}{81\sqrt{2}}+\frac{\pi^3}{36\sqrt{3}}$$

$$\frac{\zeta(3)\,7}{8}+\frac{2\pi^3}{81\sqrt{2}}+\frac{\zeta(3)\,13}{27}+\frac{\pi^3}{64}+\frac{\zeta(3)\,7}{16}+\frac{\pi^3}{36\sqrt{3}}+\frac{\zeta(3)\,91}{216}=$$
$$-\frac{7\,\mathrm{Li}_3(-1)}{\frac{3\times 8}{4}}-\frac{7\,\mathrm{Li}_3(-1)}{\frac{3\times 16}{4}}-\frac{13\,\mathrm{Li}_3(-1)}{\frac{3\times 27}{4}}-\frac{91\,\mathrm{Li}_3(-1)}{\frac{3\times 216}{4}}+\frac{\pi^3}{64}+\frac{2\pi^3}{81\sqrt{2}}+\frac{\pi^3}{36\sqrt{3}}$$

**Series representations:**

$$\frac{\zeta(3)\,7}{8}+\frac{2\pi^3}{81\sqrt{2}}+\frac{\zeta(3)\,13}{27}+\frac{\pi^3}{64}+\frac{\zeta(3)\,7}{16}+\frac{\pi^3}{36\sqrt{3}}+\frac{\zeta(3)\,91}{216}=$$
$$\frac{\pi^3}{64}+\frac{\sqrt{2}\,\pi^3}{81}+\frac{\pi^3}{36\sqrt{3}}+\frac{319}{144}\sum_{k=1}^{\infty}\frac{1}{k^3}$$

$$\frac{\zeta(3)\,7}{8}+\frac{2\pi^3}{81\sqrt{2}}+\frac{\zeta(3)\,13}{27}+\frac{\pi^3}{64}+\frac{\zeta(3)\,7}{16}+\frac{\pi^3}{36\sqrt{3}}+\frac{\zeta(3)\,91}{216}=$$
$$\frac{\pi^3}{64}+\frac{\sqrt{2}\,\pi^3}{81}+\frac{\pi^3}{36\sqrt{3}}+\frac{319}{126}\sum_{k=0}^{\infty}\frac{1}{(1+2k)^3}$$

$$\frac{\zeta(3)\,7}{8}+\frac{2\pi^3}{81\sqrt{2}}+\frac{\zeta(3)\,13}{27}+\frac{\pi^3}{64}+\frac{\zeta(3)\,7}{16}+\frac{\pi^3}{36\sqrt{3}}+\frac{\zeta(3)\,91}{216}=$$
$$\frac{81\pi^3+64\sqrt{2}\,\pi^3+48\sqrt{3}\,\pi^3+5742\sum_{n=0}^{\infty}\frac{\sum_{k=0}^{n}\frac{(-1)^k\binom{n}{k}}{(1+k)^2}}{1+n}}{5184}$$

**Integral representations:**

$$\frac{\zeta(3)\,7}{8}+\frac{2\pi^3}{81\sqrt{2}}+\frac{\zeta(3)\,13}{27}+\frac{\pi^3}{64}+\frac{\zeta(3)\,7}{16}+\frac{\pi^3}{36\sqrt{3}}+\frac{\zeta(3)\,91}{216}=$$
$$\frac{\pi^3}{64}+\frac{\sqrt{2}\,\pi^3}{81}+\frac{\pi^3}{36\sqrt{3}}-\frac{319}{432}\int_0^1\frac{\log^3(1-t^2)}{t^3}\,dt$$

$$\frac{\zeta(3)\,7}{8}+\frac{2\pi^3}{81\sqrt{2}}+\frac{\zeta(3)\,13}{27}+\frac{\pi^3}{64}+\frac{\zeta(3)\,7}{16}+\frac{\pi^3}{36\sqrt{3}}+\frac{\zeta(3)\,91}{216}=\frac{\pi^3}{64}+\frac{\sqrt{2}\,\pi^3}{81}+\frac{\pi^3}{36\sqrt{3}}+\frac{319}{288}\int_0^\infty\frac{t^2}{-1+e^t}\,dt$$

$$\frac{\zeta(3)\,7}{8}+\frac{2\pi^3}{81\sqrt{2}}+\frac{\zeta(3)\,13}{27}+\frac{\pi^3}{64}+\frac{\zeta(3)\,7}{16}+\frac{\pi^3}{36\sqrt{3}}+\frac{\zeta(3)\,91}{216}=\frac{\pi^3}{64}+\frac{\sqrt{2}\,\pi^3}{81}+\frac{\pi^3}{36\sqrt{3}}+\frac{319}{216}\int_0^\infty\frac{t^2}{1+e^t}\,dt$$

From which:

((81 + 64 sqrt(2) + 48 sqrt(3)) x^3)/5184 + (319 ζ(3))/144 = 4.1859782272474

**Input interpretation:**

$$\frac{(81+64\sqrt{2}+48\sqrt{3})x^3}{5184}+\frac{319\,\zeta(3)}{144}=4.1859782272474$$

$\zeta(s)$ is the Riemann zeta function

**Result:**

$$\frac{(81+64\sqrt{2}+48\sqrt{3})x^3}{5184}+\frac{319\,\zeta(3)}{144}=4.1859782272474$$

**Alternate forms:**

$$\frac{(81+64\sqrt{2}+48\sqrt{3})x^3}{5184}-1.5230882820536=0$$

$$\frac{x^3}{36\sqrt{3}}+\frac{\sqrt{2}\,x^3}{81}+\frac{x^3}{64}-1.5230882820536=0$$

$$\frac{\left(81+16\sqrt{59+24\sqrt{6}}\right)x^3}{5184}+\frac{319\,\zeta(3)}{144}=4.1859782272474$$

**Expanded form:**

$$\frac{x^3}{36\sqrt{3}}+\frac{\sqrt{2}\,x^3}{81}+\frac{x^3}{64}+\frac{319\,\zeta(3)}{144}=4.1859782272474$$

**Real solution:**

$x \approx 3.14159265359$

3.14159265359 ≈ π

**Complex solutions:**

$x \approx -1.57079632679 - 2.72069904635\, i$

$x \approx -1.57079632679 + 2.72069904635\, i$

((81 + 64 sqrt(2) + 48 sqrt(3)) π^3)/5184 + (319 ζ(3))/((x-1)/12) = 4.1859782272474

**Input interpretation:**

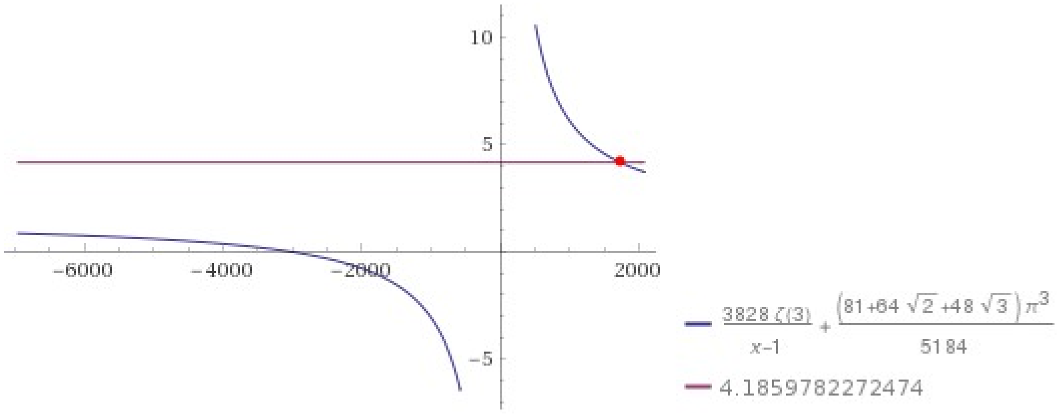
$$\frac{(81 + 64\sqrt{2} + 48\sqrt{3})\pi^3}{5184} + \frac{319\,\zeta(3)}{\frac{x-1}{12}} = 4.1859782272474$$

$\zeta(s)$ is the Riemann zeta function

**Result:**

$$\frac{3828\,\zeta(3)}{x-1} + \frac{(81 + 64\sqrt{2} + 48\sqrt{3})\pi^3}{5184} = 4.1859782272474$$

**Plot:**




**Alternate form assuming x is real:**

$$-\frac{1728.0000000}{1.000000000000 - 1.00000000000\,x} = 1.0000000000$$

**Alternate form:**

$$\frac{48\sqrt{3}\,\pi^3 x + 64\sqrt{2}\,\pi^3 x + 81\pi^3 x + 19\,844\,352\,\zeta(3) - 48\sqrt{3}\,\pi^3 - 64\sqrt{2}\,\pi^3 - 81\pi^3}{5184(x-1)} = 4.1859782272474$$

**Solution:**

$x \approx 1729.000000000$

1729

This result is very near to the mass of candidate glueball $f_0(1710)$ meson. Furthermore, 1728 occurs in the algebraic formula for the j-invariant of an elliptic curve. As a consequence, it is sometimes called a Zagier as a pun on the Gross–Zagier theorem. The number 1728 is one less than the Hardy–Ramanujan number 1729 (taxicab number)

(((((81 + 64 sqrt(2) + 48 sqrt(3)) π^3)/5184 + (319 ζ(3))/144)))^1/3

**Input:**

$$\sqrt[3]{\frac{(81+64\sqrt{2}+48\sqrt{3})\pi^3}{5184}+\frac{319\,\zeta(3)}{144}}$$

$\zeta(s)$ is the Riemann zeta function

**Decimal approximation:**

1.611631157728558233010611244286714690400108716561115072185...

1.6116311577… result that is near to the value of the golden ratio 1,618033988749...

**Alternate forms:**

$$\sqrt[3]{\frac{319\,\zeta(3)}{144}+\frac{\left(81+16\sqrt{59+24\sqrt{6}}\right)\pi^3}{5184}}$$

$$\frac{1}{12}\sqrt[3]{\frac{1}{3}\left(11484\,\zeta(3)+\left(81+64\sqrt{2}+48\sqrt{3}\right)\pi^3\right)}$$

$$\frac{1}{12\sqrt[3]{\frac{3}{11484\,\zeta(3)+81\pi^3+64\sqrt{2}\,\pi^3+48\sqrt{3}\,\pi^3}}}$$

**All 3rd roots of (319 ζ(3))/144 + ((81 + 64 sqrt(2) + 48 sqrt(3)) π^3)/5184:**

$$e^0\sqrt[3]{\frac{319\,\zeta(3)}{144}+\frac{(81+64\sqrt{2}+48\sqrt{3})\pi^3}{5184}}\approx 1.6116 \text{ (real, principal root)}$$

$$e^{(2i\pi)/3}\sqrt[3]{\frac{319\,\zeta(3)}{144}+\frac{(81+64\sqrt{2}+48\sqrt{3})\pi^3}{5184}}\approx -0.8058+1.3957\,i$$

$$e^{-(2i\pi)/3}\sqrt[3]{\frac{319\,\zeta(3)}{144}+\frac{(81+64\sqrt{2}+48\sqrt{3})\pi^3}{5184}}\approx -0.8058-1.3957\,i$$

**Alternative representations:**

$$\sqrt[3]{\frac{(81+64\sqrt{2}+48\sqrt{3})\pi^3}{5184}+\frac{319\zeta(3)}{144}} = \sqrt[3]{\frac{\pi^3(81+64\sqrt{2}+48\sqrt{3})}{5184}+\frac{319\zeta(3,1)}{144}}$$

$$\sqrt[3]{\frac{(81+64\sqrt{2}+48\sqrt{3})\pi^3}{5184}+\frac{319\zeta(3)}{144}} = \sqrt[3]{\frac{319 S_{2,1}(1)}{144}+\frac{\pi^3(81+64\sqrt{2}+48\sqrt{3})}{5184}}$$

$$\sqrt[3]{\frac{(81+64\sqrt{2}+48\sqrt{3})\pi^3}{5184}+\frac{319\zeta(3)}{144}} = \sqrt[3]{-\frac{319\,\text{Li}_3(-1)}{\frac{3\times 144}{4}}+\frac{\pi^3(81+64\sqrt{2}+48\sqrt{3})}{5184}}$$

**Series representations:**

$$\sqrt[3]{\frac{(81+64\sqrt{2}+48\sqrt{3})\pi^3}{5184}+\frac{319\zeta(3)}{144}} = \sqrt[3]{\frac{(81+64\sqrt{2}+48\sqrt{3})\pi^3}{5184}+\frac{319}{144}\sum_{k=1}^{\infty}\frac{1}{k^3}}$$

$$\sqrt[3]{\frac{(81+64\sqrt{2}+48\sqrt{3})\pi^3}{5184}+\frac{319\zeta(3)}{144}} = \sqrt[3]{\frac{(81+64\sqrt{2}+48\sqrt{3})\pi^3}{5184}+\frac{319}{126}\sum_{k=0}^{\infty}\frac{1}{(1+2k)^3}}$$

$$\sqrt[3]{\frac{(81+64\sqrt{2}+48\sqrt{3})\pi^3}{5184}+\frac{319\zeta(3)}{144}} = \sqrt[3]{\frac{319}{144}e^{\sum_{k=1}^{\infty}P(3k)/k}+\frac{(81+64\sqrt{2}+48\sqrt{3})\pi^3}{5184}}$$

**Integral representations:**

$$\sqrt[3]{\frac{(81+64\sqrt{2}+48\sqrt{3})\pi^3}{5184}+\frac{319\zeta(3)}{144}} = \frac{1}{12}\sqrt[3]{\frac{1}{3}\left(81+64\sqrt{2}+48\sqrt{3}\right)\pi^3+1914\int_0^{\infty}\frac{t^2}{-1+e^t}\,dt}$$

$$\sqrt[3]{\frac{(81+64\sqrt{2}+48\sqrt{3})\pi^3}{5184}+\frac{319\zeta(3)}{144}} = \sqrt[3]{\frac{(81+64\sqrt{2}+48\sqrt{3})\pi^3}{5184}+\frac{319}{504}\int_0^{\infty}t^2\,\text{csch}(t)\,dt}$$

$$\sqrt[3]{\frac{(81+64\sqrt{2}+48\sqrt{3})\pi^3}{5184}+\frac{319\zeta(3)}{144}} = \sqrt[3]{\frac{(81+64\sqrt{2}+48\sqrt{3})\pi^3}{5184}-\frac{319}{432}\int_0^1 \frac{\log^3(1-t^2)}{t^3}\,dt}$$

((81 + 64 sqrt(2) + 48 sqrt(3)) π^3)/(x-99+4) + (319 ζ(3))/144 = 4.1859782272474

**Input interpretation:**

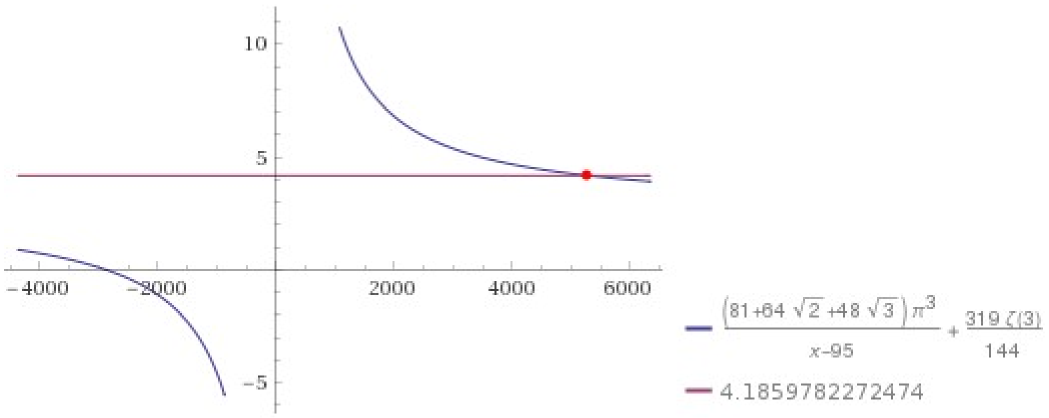
$$\frac{(81+64\sqrt{2}+48\sqrt{3})\pi^3}{x-99+4}+\frac{319\zeta(3)}{144} = 4.1859782272474$$

$\zeta(s)$ is the Riemann zeta function

**Result:**

$$\frac{(81+64\sqrt{2}+48\sqrt{3})\pi^3}{x-95}+\frac{319\zeta(3)}{144} = 4.1859782272474$$

**Plot:**




**Alternate form:**

$$\frac{319\,x\,\zeta(3)-30\,305\,\zeta(3)+6912\sqrt{3}\,\pi^3+9216\sqrt{2}\,\pi^3+11\,664\,\pi^3}{144\,(x-95)} = 4.1859782272474$$

**Alternate form assuming x is positive:**

$$-\frac{5184.000000}{95.000000000-1.0000000000\,x} = 1.0000000000$$

**Expanded form:**

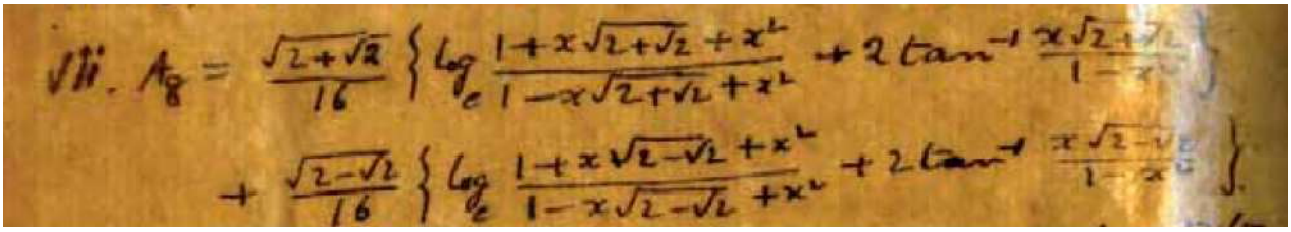
$$\frac{48\sqrt{3}\,\pi^3}{x-95}+\frac{64\sqrt{2}\,\pi^3}{x-95}+\frac{81\,\pi^3}{x-95}+\frac{319\zeta(3)}{144} = 4.1859782272474$$

**Solution:**

$x \approx 5279.00000000$

5279 result practically equal to the rest mass of B meson 5279.15

Now, we have that:



$$\text{vii. } A_8 = \frac{\sqrt{2+\sqrt{2}}}{16}\left\{\log_e \frac{1+x\sqrt{2+\sqrt{2}}+x^2}{1-x\sqrt{2+\sqrt{2}}+x^2} + 2\tan^{-1}\frac{x\sqrt{2+\sqrt{2}}}{1-x^2}\right\} + \frac{\sqrt{2-\sqrt{2}}}{16}\left\{\log_e \frac{1+x\sqrt{2-\sqrt{2}}+x^2}{1-x\sqrt{2-\sqrt{2}}+x^2} + 2\tan^{-1}\frac{x\sqrt{2-\sqrt{2}}}{1-x^2}\right\}.$$

1/16*(2+sqrt2)^(1/2) [ln(((((1+2(2+sqrt2)^(1/2)+4)))/(((1-2(2+sqrt2)^(1/2)+4))))) + 2 tan^-1 ((2(2+sqrt2)^(1/2)/(1-4)))]

**Input:**

$$\frac{1}{16}\sqrt{2+\sqrt{2}}\left(\log\left(\frac{1+2\sqrt{2+\sqrt{2}}+4}{1-2\sqrt{2+\sqrt{2}}+4}\right)+2\tan^{-1}\left(2\times\frac{\sqrt{2+\sqrt{2}}}{1-4}\right)\right)$$

log(x) is the natural logarithm

$\tan^{-1}(x)$ is the inverse tangent function

**Exact Result:**

$$\frac{1}{16}\sqrt{2+\sqrt{2}}\left(\log\left(\frac{5+2\sqrt{2+\sqrt{2}}}{5-2\sqrt{2+\sqrt{2}}}\right)-2\tan^{-1}\left(\frac{2\sqrt{2+\sqrt{2}}}{3}\right)\right)$$

(result in radians)

**Decimal approximation:**

0.013764838311382013868966278430595886004523852083036857721...

(result in radians)

0.013764838311...

**Alternate forms:**

$$\frac{1}{8}\sqrt{2+\sqrt{2}}\left(\tanh^{-1}\left(\frac{2\sqrt{2+\sqrt{2}}}{5}\right)-\tan^{-1}\left(\frac{2\sqrt{2+\sqrt{2}}}{3}\right)\right)$$

$$\frac{1}{16}\sqrt{2+\sqrt{2}}\left(\log\left(\frac{1}{514}\left(1186+400\sqrt{2}+257\sqrt{\frac{1462400}{66049}+\frac{948800\sqrt{2}}{66049}}\right)\right)-2\tan^{-1}\left(\frac{2\sqrt{2+\sqrt{2}}}{3}\right)\right)$$

$$-\frac{\left(\sqrt{1-i}+\sqrt{1+i}\right)\left(2\tan^{-1}\left(\frac{2\sqrt{2+\sqrt{2}}}{3}\right)-\log\left(\frac{5+2\sqrt{2+\sqrt{2}}}{5-2\sqrt{2+\sqrt{2}}}\right)\right)}{16\sqrt[4]{2}}$$

$\tanh^{-1}(x)$ is the inverse hyperbolic tangent function

**Alternative representations:**

$$\frac{1}{16}\sqrt{2+\sqrt{2}}\left(\log\left(\frac{1+2\sqrt{2+\sqrt{2}}+4}{1-2\sqrt{2+\sqrt{2}}+4}\right)+2\tan^{-1}\left(\frac{2\sqrt{2+\sqrt{2}}}{1-4}\right)\right)=$$
$$\frac{1}{16}\left(2\tan^{-1}\left(1,-\frac{2}{3}\sqrt{2+\sqrt{2}}\right)+\log\left(\frac{5+2\sqrt{2+\sqrt{2}}}{5-2\sqrt{2+\sqrt{2}}}\right)\right)\sqrt{2+\sqrt{2}}$$

$$\frac{1}{16}\sqrt{2+\sqrt{2}}\left(\log\left(\frac{1+2\sqrt{2+\sqrt{2}}+4}{1-2\sqrt{2+\sqrt{2}}+4}\right)+2\tan^{-1}\left(\frac{2\sqrt{2+\sqrt{2}}}{1-4}\right)\right)=$$
$$\frac{1}{16}\left(2\tan^{-1}\left(-\frac{2}{3}\sqrt{2+\sqrt{2}}\right)+\log_e\left(\frac{5+2\sqrt{2+\sqrt{2}}}{5-2\sqrt{2+\sqrt{2}}}\right)\right)\sqrt{2+\sqrt{2}}$$

$$\frac{1}{16}\sqrt{2+\sqrt{2}}\left(\log\left(\frac{1+2\sqrt{2+\sqrt{2}}+4}{1-2\sqrt{2+\sqrt{2}}+4}\right)+2\tan^{-1}\left(\frac{2\sqrt{2+\sqrt{2}}}{1-4}\right)\right)=$$
$$\frac{1}{16}\left(2\tan^{-1}\left(-\frac{2}{3}\sqrt{2+\sqrt{2}}\right)+\log(a)\log_a\left(\frac{5+2\sqrt{2+\sqrt{2}}}{5-2\sqrt{2+\sqrt{2}}}\right)\right)\sqrt{2+\sqrt{2}}$$

**Series representations:**

$$\frac{1}{16}\sqrt{2+\sqrt{2}}\left(\log\left(\frac{1+2\sqrt{2+\sqrt{2}}+4}{1-2\sqrt{2+\sqrt{2}}+4}\right)+2\tan^{-1}\left(\frac{2\sqrt{2+\sqrt{2}}}{1-4}\right)\right)=$$

$$-\frac{1}{8}\sqrt{2+\sqrt{2}}\tan^{-1}\left(\frac{2\sqrt{2+\sqrt{2}}}{3}\right)+$$

$$\frac{1}{16}\sqrt{2+\sqrt{2}}\log\left(-1+\frac{5+2\sqrt{2+\sqrt{2}}}{5-2\sqrt{2+\sqrt{2}}}\right)-\frac{1}{16}\sqrt{2+\sqrt{2}}\sum_{k=1}^{\infty}\frac{\left(\frac{1}{2}-\frac{5}{4\sqrt{2+\sqrt{2}}}\right)^k}{k}$$

$$\frac{1}{16}\sqrt{2+\sqrt{2}}\left(\log\left(\frac{1+2\sqrt{2+\sqrt{2}}+4}{1-2\sqrt{2+\sqrt{2}}+4}\right)+2\tan^{-1}\left(\frac{2\sqrt{2+\sqrt{2}}}{1-4}\right)\right)=$$

$$-\frac{1}{8}\sqrt{2+\sqrt{2}}\tan^{-1}\left(\frac{2\sqrt{2+\sqrt{2}}}{3}\right)+$$

$$\frac{1}{32}\sqrt{2+\sqrt{2}}\log\left(2+\sqrt{2}\right)+\frac{1}{16}\sqrt{2+\sqrt{2}}\log\left(\frac{4}{5-2\sqrt{2+\sqrt{2}}}\right)-$$

$$\frac{1}{16}\sqrt{2+\sqrt{2}}\sum_{k=1}^{\infty}\frac{4^{-k}\left(2+\sqrt{2}\right)^{-k/2}\left(-5+2\sqrt{2+\sqrt{2}}\right)^k}{k}$$

$$\frac{1}{16}\sqrt{2+\sqrt{2}}\left(\log\left(\frac{1+2\sqrt{2+\sqrt{2}}+4}{1-2\sqrt{2+\sqrt{2}}+4}\right)+2\tan^{-1}\left(\frac{2\sqrt{2+\sqrt{2}}}{1-4}\right)\right)=$$

$$-\frac{1}{8}\sqrt{2+\sqrt{2}}\tan^{-1}(z_0)+\frac{1}{16}\sqrt{2+\sqrt{2}}\log\left(-1+\frac{5+2\sqrt{2+\sqrt{2}}}{5-2\sqrt{2+\sqrt{2}}}\right)+$$

$$\sum_{k=1}^{\infty}\left(\frac{(-1)^{-1+k}\sqrt{2+\sqrt{2}}\left(-1+\frac{5+2\sqrt{2+\sqrt{2}}}{5-2\sqrt{2+\sqrt{2}}}\right)^{-k}}{16k}-\right.$$

$$\left.\frac{i\sqrt{2+\sqrt{2}}\left(-(-i-z_0)^{-k}+(i-z_0)^{-k}\right)\left(\frac{2\sqrt{2+\sqrt{2}}}{3}-z_0\right)^k}{16k}\right)$$

for $(i z_0 \notin \mathbb{R}$ or ( not $(1 \leq i z_0 < \infty)$ and not $(-\infty < i z_0 \leq -1)))$

**Integral representations:**

$$\frac{1}{16}\sqrt{2+\sqrt{2}}\left(\log\left(\frac{1+2\sqrt{2+\sqrt{2}}+4}{1-2\sqrt{2+\sqrt{2}}+4}\right)+2\tan^{-1}\left(\frac{2\sqrt{2+\sqrt{2}}}{1-4}\right)\right)=$$
$$\int_0^1 -\frac{3\left(2+\sqrt{2}\right)}{4\left(9+4\left(2+\sqrt{2}\right)t^2\right)}\,dt+\frac{1}{16}\sqrt{2+\sqrt{2}}\,\log\left(\frac{5+2\sqrt{2+\sqrt{2}}}{5-2\sqrt{2+\sqrt{2}}}\right)$$

$$\frac{1}{16}\sqrt{2+\sqrt{2}}\left(\log\left(\frac{1+2\sqrt{2+\sqrt{2}}+4}{1-2\sqrt{2+\sqrt{2}}+4}\right)+2\tan^{-1}\left(\frac{2\sqrt{2+\sqrt{2}}}{1-4}\right)\right)=$$
$$\int_{-i\infty+\gamma}^{i\infty+\gamma}\frac{i\left(2+\sqrt{2}\right)\left(1+\frac{4}{9}\left(2+\sqrt{2}\right)\right)^{-s}\Gamma\left(\frac{1}{2}-s\right)\Gamma(1-s)\Gamma(s)^2}{48\pi^{3/2}}\,ds+$$
$$\frac{1}{16}\sqrt{2+\sqrt{2}}\,\log\left(\frac{5+2\sqrt{2+\sqrt{2}}}{5-2\sqrt{2+\sqrt{2}}}\right)\ \text{for } 0<\gamma<\frac{1}{2}$$

$$\frac{1}{16}\sqrt{2+\sqrt{2}}\left(\log\left(\frac{1+2\sqrt{2+\sqrt{2}}+4}{1-2\sqrt{2+\sqrt{2}}+4}\right)+2\tan^{-1}\left(\frac{2\sqrt{2+\sqrt{2}}}{1-4}\right)\right)=$$
$$\int_{-i\infty+\gamma}^{i\infty+\gamma}\frac{i\,2^{-7/2-2s}\times 3^{-1+2s}\left(1+\sqrt{2}\right)\left(2+\sqrt{2}\right)^{-s}\Gamma\left(\frac{1}{2}-s\right)\Gamma(1-s)\Gamma(s)}{\pi\,\Gamma\left(\frac{3}{2}-s\right)}\,ds+$$
$$\frac{1}{16}\sqrt{2+\sqrt{2}}\,\log\left(\frac{5+2\sqrt{2+\sqrt{2}}}{5-2\sqrt{2+\sqrt{2}}}\right)\ \text{for } 0<\gamma<\frac{1}{2}$$

1/16*(2-sqrt2)^(1/2) [ln(((((1+2(2-sqrt2)^(1/2)+4)))/(((1-2(2-sqrt2)^(1/2)+4))))) + 2 tan^-1 ((2(2-sqrt2)^(1/2)/(1-4)))]

**Input:**

$$\frac{1}{16}\sqrt{2-\sqrt{2}}\left(\log\left(\frac{1+2\sqrt{2-\sqrt{2}}+4}{1-2\sqrt{2-\sqrt{2}}+4}\right)+2\tan^{-1}\left(2\times\frac{\sqrt{2-\sqrt{2}}}{1-4}\right)\right)$$

$\log(x)$ is the natural logarithm

$\tan^{-1}(x)$ is the inverse tangent function

**Exact Result:**

$$\frac{1}{16}\sqrt{2-\sqrt{2}}\left(\log\left(\frac{5+2\sqrt{2-\sqrt{2}}}{5-2\sqrt{2-\sqrt{2}}}\right)-2\tan^{-1}\left(\frac{2\sqrt{2-\sqrt{2}}}{3}\right)\right)$$

(result in radians)

**Decimal approximation:**

-0.01487888040278285650039035025666952617526559293627054867...

(result in radians)

-0.014878880402782…

**Alternate forms:**

$$\frac{1}{8}\sqrt{2-\sqrt{2}}\left(\tanh^{-1}\left(\frac{2\sqrt{2-\sqrt{2}}}{5}\right)-\tan^{-1}\left(\frac{2\sqrt{2-\sqrt{2}}}{3}\right)\right)$$

$$-\frac{\left(\sqrt{-1-i}+\sqrt{-1+i}\right)\left(2\tan^{-1}\left(\frac{2\sqrt{2-\sqrt{2}}}{3}\right)-\log\left(\frac{5+2\sqrt{2-\sqrt{2}}}{5-2\sqrt{2-\sqrt{2}}}\right)\right)}{16\sqrt[4]{2}}$$

$$-\frac{1}{16}i\sqrt{2-\sqrt{2}}\log\left(1-\frac{2}{3}i\sqrt{2-\sqrt{2}}\right)+\frac{1}{16}i\sqrt{2-\sqrt{2}}\log\left(1+\frac{2}{3}i\sqrt{2-\sqrt{2}}\right)+\frac{1}{16}\sqrt{2-\sqrt{2}}\log\left(\frac{5+2\sqrt{2-\sqrt{2}}}{5-2\sqrt{2-\sqrt{2}}}\right)$$

$\tanh^{-1}(x)$ is the inverse hyperbolic tangent function

**Alternative representations:**

$$\frac{1}{16}\sqrt{2-\sqrt{2}}\left(\log\left(\frac{1+2\sqrt{2-\sqrt{2}}+4}{1-2\sqrt{2-\sqrt{2}}+4}\right)+2\tan^{-1}\left(\frac{2\sqrt{2-\sqrt{2}}}{1-4}\right)\right)=\frac{1}{16}\left(2\tan^{-1}\left(1,-\frac{2}{3}\sqrt{2-\sqrt{2}}\right)+\log\left(\frac{5+2\sqrt{2-\sqrt{2}}}{5-2\sqrt{2-\sqrt{2}}}\right)\right)\sqrt{2-\sqrt{2}}$$

$$\frac{1}{16}\sqrt{2-\sqrt{2}}\left(\log\left(\frac{1+2\sqrt{2-\sqrt{2}}+4}{1-2\sqrt{2-\sqrt{2}}+4}\right)+2\tan^{-1}\left(\frac{2\sqrt{2-\sqrt{2}}}{1-4}\right)\right)=\frac{1}{16}\left(2\tan^{-1}\left(-\frac{2}{3}\sqrt{2-\sqrt{2}}\right)+\log_e\left(\frac{5+2\sqrt{2-\sqrt{2}}}{5-2\sqrt{2-\sqrt{2}}}\right)\right)\sqrt{2-\sqrt{2}}$$

$$\frac{1}{16}\sqrt{2-\sqrt{2}}\left(\log\left(\frac{1+2\sqrt{2-\sqrt{2}}+4}{1-2\sqrt{2-\sqrt{2}}+4}\right)+2\tan^{-1}\left(\frac{2\sqrt{2-\sqrt{2}}}{1-4}\right)\right)=\frac{1}{16}\left(2\tan^{-1}\left(-\frac{2}{3}\sqrt{2-\sqrt{2}}\right)+\log(a)\log_a\left(\frac{5+2\sqrt{2-\sqrt{2}}}{5-2\sqrt{2-\sqrt{2}}}\right)\right)\sqrt{2-\sqrt{2}}$$

**Series representations:**

$$\frac{1}{16}\sqrt{2-\sqrt{2}}\left(\log\left(\frac{1+2\sqrt{2-\sqrt{2}}+4}{1-2\sqrt{2-\sqrt{2}}+4}\right)+2\tan^{-1}\left(\frac{2\sqrt{2-\sqrt{2}}}{1-4}\right)\right)= -\frac{1}{8}\sqrt{2-\sqrt{2}}\tan^{-1}\left(\frac{2\sqrt{2-\sqrt{2}}}{3}\right)- \frac{1}{16}\sqrt{2-\sqrt{2}}\sum_{k=1}^{\infty}\frac{4^k\left(2-\sqrt{2}\right)^{k/2}\left(\frac{1}{-5+2\sqrt{2-\sqrt{2}}}\right)^k}{k}$$

$$\frac{1}{16}\sqrt{2-\sqrt{2}}\left(\log\left(\frac{1+2\sqrt{2-\sqrt{2}}+4}{1-2\sqrt{2-\sqrt{2}}+4}\right)+2\tan^{-1}\left(\frac{2\sqrt{2-\sqrt{2}}}{1-4}\right)\right)=-\frac{1}{16}\sqrt{2-\sqrt{2}}\left(\sum_{k=1}^{\infty}\frac{4^k\left(2-\sqrt{2}\right)^{k/2}\left(\frac{1}{-5+2\sqrt{2-\sqrt{2}}}\right)^k}{k}+2\sum_{k=0}^{\infty}\frac{(-1)^k\,2^{1+2k}\times 3^{-1-2k}\left(2-\sqrt{2}\right)^{1/2+k}}{1+2k}\right)$$

$$\frac{1}{16}\sqrt{2-\sqrt{2}}\left(\log\left(\frac{1+2\sqrt{2-\sqrt{2}}+4}{1-2\sqrt{2-\sqrt{2}}+4}\right)+2\tan^{-1}\left(\frac{2\sqrt{2-\sqrt{2}}}{1-4}\right)\right)= -\frac{1}{8}\sqrt{2-\sqrt{2}}\tan^{-1}(z_0)+\sum_{k=1}^{\infty}\left(\frac{(-1)^{1+k}\,4^{-2+k}\left(2-\sqrt{2}\right)^{1/2+k/2}\left(5-2\sqrt{2-\sqrt{2}}\right)^{-k}}{k}-\frac{i\sqrt{2-\sqrt{2}}\left(-(-i-z_0)^{-k}+(i-z_0)^{-k}\right)\left(\frac{2\sqrt{2-\sqrt{2}}}{3}-z_0\right)^k}{16k}\right)$$

for $(i z_0 \notin \mathbb{R}$ or ( not $(1 \le i z_0 < \infty)$ and not $(-\infty < i z_0 \le -1)))$

$$\frac{1}{16}\sqrt{2-\sqrt{2}}\left(\log\left(\frac{1+2\sqrt{2-\sqrt{2}}+4}{1-2\sqrt{2-\sqrt{2}}+4}\right)+2\tan^{-1}\left(\frac{2\sqrt{2-\sqrt{2}}}{1-4}\right)\right)=$$

$$-\frac{1}{8}\sqrt{2-\sqrt{2}}\tan^{-1}(z_0)+\sum_{k=1}^{\infty}\left(\frac{(-1)^{-1+k}\sqrt{2-\sqrt{2}}\left(-1+\frac{5+2\sqrt{2-\sqrt{2}}}{5-2\sqrt{2-\sqrt{2}}}\right)^k}{16k}-\frac{i\sqrt{2-\sqrt{2}}\left(-(-i-z_0)^{-k}+(i-z_0)^{-k}\right)\left(\frac{2\sqrt{2-\sqrt{2}}}{3}-z_0\right)^k}{16k}\right)$$

for ($i z_0 \notin \mathbb{R}$ or ( not ($1 \leq i z_0 < \infty$) and not ($-\infty < i z_0 \leq -1$)))

**Integral representations:**

$$\frac{1}{16}\sqrt{2-\sqrt{2}}\left(\log\left(\frac{1+2\sqrt{2-\sqrt{2}}+4}{1-2\sqrt{2-\sqrt{2}}+4}\right)+2\tan^{-1}\left(\frac{2\sqrt{2-\sqrt{2}}}{1-4}\right)\right)=$$

$$\int_0^1\frac{6-3\sqrt{2}}{4\left(-9+4\left(-2+\sqrt{2}\right)t^2\right)}\,dt+\frac{1}{16}\sqrt{2-\sqrt{2}}\log\left(\frac{5+2\sqrt{2-\sqrt{2}}}{5-2\sqrt{2-\sqrt{2}}}\right)$$

$$\frac{1}{16}\sqrt{2-\sqrt{2}}\left(\log\left(\frac{1+2\sqrt{2-\sqrt{2}}+4}{1-2\sqrt{2-\sqrt{2}}+4}\right)+2\tan^{-1}\left(\frac{2\sqrt{2-\sqrt{2}}}{1-4}\right)\right)=$$

$$\int_{-i\infty+\gamma}^{i\infty+\gamma}-\frac{i\left(\frac{17}{9}-\frac{4\sqrt{2}}{9}\right)^{-s}\left(-2+\sqrt{2}\right)\Gamma\left(\frac{1}{2}-s\right)\Gamma(1-s)\Gamma(s)^2}{48\pi^{3/2}}\,ds+\frac{1}{16}\sqrt{2-\sqrt{2}}\log\left(\frac{5+2\sqrt{2-\sqrt{2}}}{5-2\sqrt{2-\sqrt{2}}}\right)\quad\text{for } 0<\gamma<\frac{1}{2}$$

$$\frac{1}{16}\sqrt{2-\sqrt{2}}\left(\log\left(\frac{1+2\sqrt{2-\sqrt{2}}+4}{1-2\sqrt{2-\sqrt{2}}+4}\right)+2\tan^{-1}\left(\frac{2\sqrt{2-\sqrt{2}}}{1-4}\right)\right)=$$

$$\int_{-i\infty+\gamma}^{i\infty+\gamma}\frac{i\,2^{-7/2-3s}\times 3^{-1+2s}\left(-1+\sqrt{2}\right)\left(2+\sqrt{2}\right)^s\Gamma\left(\frac{1}{2}-s\right)\Gamma(1-s)\Gamma(s)}{\pi\,\Gamma\left(\frac{3}{2}-s\right)}\,ds+\frac{1}{16}\sqrt{2-\sqrt{2}}\log\left(\frac{5+2\sqrt{2-\sqrt{2}}}{5-2\sqrt{2-\sqrt{2}}}\right)\quad\text{for } 0<\gamma<\frac{1}{2}$$

(0.0137648383113820138-0.0148788804027828565)

**Input interpretation:**

0.0137648383113820138 − 0.0148788804027828565

**Result:**

−0.0011140420914008427

-0.0011140420914008427

Thence, we obtain:

(-(0.0137648383113820138-0.0148788804027828565))^1/1024

**Input interpretation:**

$$\sqrt[1024]{-(0.0137648383113820138 - 0.0148788804027828565)}$$

**Result:**

0.993381607705052362 56...

0.9933816077... result very near to the value of the following Rogers-Ramanujan continued fraction:

$$\frac{e^{-\frac{\pi}{\sqrt{5}}}}{\frac{\sqrt{5}}{1+\sqrt[5]{\sqrt{\varphi^5\sqrt[4]{5^3}}-1}}-\varphi+1}=1-\frac{e^{-\pi\sqrt{5}}}{1+\frac{e^{-2\pi\sqrt{5}}}{1+\frac{e^{-3\pi\sqrt{5}}}{1+\frac{e^{-4\pi\sqrt{5}}}{1+\dots}}}}\approx 0.9991104684$$

and to the dilaton value $\mathbf{0.989117352243 = \phi}$

1/8 log base 0.993381607705 (-(0.0137648383-0.0148788804))-Pi+1/golden ratio

**Input interpretation:**

$$\frac{1}{8}\log_{0.993381607705}(-(0.0137648383 - 0.0148788804)) - \pi + \frac{1}{\phi}$$

$\log_b(x)$ is the base- $b$ logarithm

$\phi$ is the golden ratio

**Result:**

125.47644...

125.47644… result very near to the dilaton mass calculated as a type of Higgs boson: 125 GeV for T = 0 and to the Higgs boson mass 125.18 GeV

**Alternative representation:**

$$\frac{1}{8}\log_{0.9933816077050000}(-(0.0137648-0.0148789))-\pi+\frac{1}{\phi}=-\pi+\frac{1}{\phi}+\frac{\log(0.00111404)}{8\log(0.9933816077050000)}$$

**Series representations:**

$$\frac{1}{8}\log_{0.9933816077050000}(-(0.0137648-0.0148789))-\pi+\frac{1}{\phi}=\frac{1}{\phi}-\pi-\frac{\sum_{k=1}^{\infty}\frac{(-1)^k(-0.998886)^k}{k}}{8\log(0.9933816077050000)}$$

$$\frac{1}{8}\log_{0.9933816077050000}(-(0.0137648-0.0148789))-\pi+\frac{1}{\phi}=\frac{1}{\phi}-\pi-18.824261985148\log(0.00111404)-\frac{1}{8}\log(0.00111404)\sum_{k=0}^{\infty}(-0.0066183922950000)^k G(k)$$

$$\text{for }\left(G(0)=0\text{ and }G(k)=\frac{(-1)^{1+k}k}{2(1+k)(2+k)}+\sum_{j=1}^{k}\frac{(-1)^{1+j}G(-j+k)}{1+j}\right)$$

1/8 log base 0.993381607705 (-(0.0137648383-0.0148788804))+11+1/golden ratio

**Input interpretation:**

$$\frac{1}{8}\log_{0.993381607705}(-(0.0137648383-0.0148788804))+11+\frac{1}{\phi}$$

$\log_b(x)$ is the base- $b$ logarithm

$\phi$ is the golden ratio

**Result:**

139.61803...

139.61803… result practically equal to the rest mass of Pion meson 139.57 MeV

**Alternative representation:**

$$\frac{1}{8}\log_{0.9933816077050000}(-(0.0137648-0.0148789))+11+\frac{1}{\phi}=11+\frac{1}{\phi}+\frac{\log(0.00111404)}{8\log(0.9933816077050000)}$$

**Series representations:**

$$\frac{1}{8}\log_{0.9933816077050000}(-(0.0137648-0.0148789))+11+\frac{1}{\phi}=11+\frac{1}{\phi}-\frac{\sum_{k=1}^{\infty}\frac{(-1)^k(-0.998886)^k}{k}}{8\log(0.9933816077050000)}$$

$$\frac{1}{8}\log_{0.9933816077050000}(-(0.0137648-0.0148789))+11+\frac{1}{\phi}=11+\frac{1}{\phi}-18.824261985148\log(0.00111404)-\frac{1}{8}\log(0.00111404)\sum_{k=0}^{\infty}(-0.0066183922950000)^k G(k)$$

$$\text{for}\left(G(0)=0\text{ and }G(k)=\frac{(-1)^{1+k}k}{2(1+k)(2+k)}+\sum_{j=1}^{k}\frac{(-1)^{1+j}G(-j+k)}{1+j}\right)$$

1/10^52(((1+(-(0.0137648383-0.0148788804))+0.08+0.02+0.0047-0.0002)))

**Input interpretation:**

$$\frac{1}{10^{52}}(1-(0.0137648383-0.0148788804)+0.08+0.02+0.0047-0.0002)$$

**Result:**

$1.1056140421\times10^{-52}$

$1.1056140421*10^{-52}$ result practically equal to the value of Cosmological Constant $1.1056*10^{-52}$ $m^{-2}$

Now, we have that:

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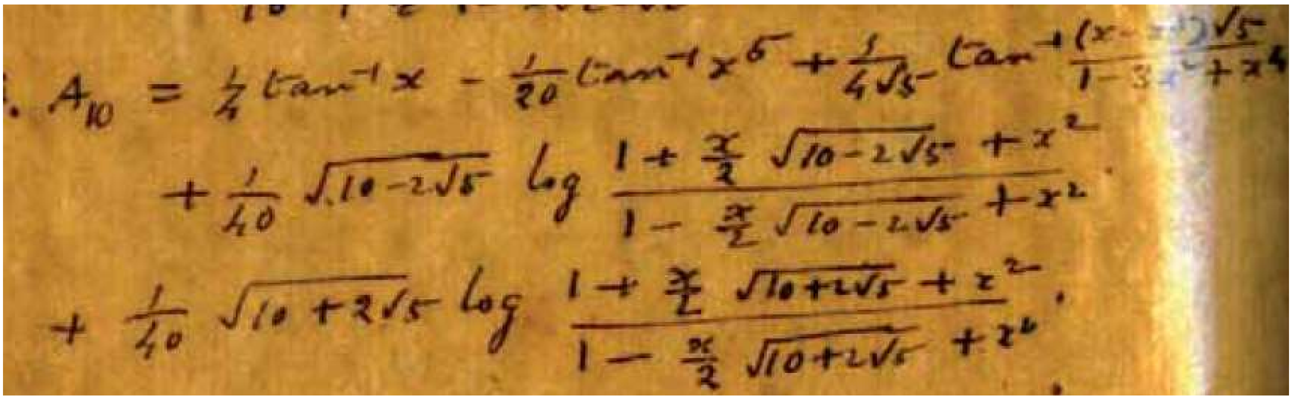


$$A_{10} = \frac{1}{4}\tan^{-1}x - \frac{1}{20}\tan^{-1}x^5 + \frac{1}{4\sqrt{5}}\tan^{-1}\frac{(x-x^3)\sqrt{5}}{1-3x^2+x^4} + \frac{1}{40}\sqrt{10-2\sqrt{5}}\log\frac{1+\frac{x}{2}\sqrt{10-2\sqrt{5}}+x^2}{1-\frac{x}{2}\sqrt{10-2\sqrt{5}}+x^2} + \frac{1}{40}\sqrt{10+2\sqrt{5}}\log\frac{1+\frac{x}{2}\sqrt{10+2\sqrt{5}}+x^2}{1-\frac{x}{2}\sqrt{10+2\sqrt{5}}+x^2}.$$

((1/4 tan^-1 (2))) – ((1/20 tan^-1 (2)^5)) + 1/(4sqrt5) tan^-1(((((2-2^3)sqrt5)) /((1-3*2^2+2^4)))) + 1/40 (10-2sqrt5)^(1/2)* ln (((1+1(10-2sqrt5)^(1/2)+4))/(((1-1(10-2sqrt5)^(1/2)+4))))

**Input:**

$$\frac{1}{4}\tan^{-1}(2) - \frac{1}{20}\tan^{-1}(2)^5 + \frac{1}{4\sqrt{5}}\tan^{-1}\left(\frac{(2-2^3)\sqrt{5}}{1-3\times 2^2+2^4}\right) + \frac{1}{40}\sqrt{10-2\sqrt{5}}\,\log\left(\frac{1+1\sqrt{10-2\sqrt{5}}+4}{1-1\sqrt{10-2\sqrt{5}}+4}\right)$$

$\tan^{-1}(x)$ is the inverse tangent function

$\log(x)$ is the natural logarithm

**Exact Result:**

$$\frac{1}{40}\sqrt{10-2\sqrt{5}}\,\log\left(\frac{5+\sqrt{10-2\sqrt{5}}}{5-\sqrt{10-2\sqrt{5}}}\right) + \frac{1}{4}\tan^{-1}(2) - \frac{1}{20}\tan^{-1}(2)^5 - \frac{\tan^{-1}\left(\frac{6}{\sqrt{5}}\right)}{4\sqrt{5}}$$

(result in radians)

**Decimal approximation:**

0.117871277524338220859857341320591906495581624687993036863...

(result in radians)

0.1178712775243382208598…

## Alternate forms:

$$\frac{1}{20}\sqrt{\frac{1}{2}\left(5-\sqrt{5}\right)}\log\left(\frac{1}{41}\left(109-20\sqrt{5}+2\sqrt{10\left(305-109\sqrt{5}\right)}\right)\right)+\frac{1}{4}\tan^{-1}(2)-\frac{1}{20}\tan^{-1}(2)^5-\frac{\tan^{-1}\left(\frac{6}{\sqrt{5}}\right)}{4\sqrt{5}}$$

$$\frac{1}{8}i(\log(1-2i)-\log(1+2i))-\frac{1}{640}i(\log(1-2i)-\log(1+2i))^5-\frac{i\left(\log\left(1-\frac{6i}{\sqrt{5}}\right)-\log\left(1+\frac{6i}{\sqrt{5}}\right)\right)}{8\sqrt{5}}+\frac{1}{40}\sqrt{10-2\sqrt{5}}\log\left(\frac{5+\sqrt{10-2\sqrt{5}}}{5-\sqrt{10-2\sqrt{5}}}\right)$$

$$\frac{1}{40}\left(\sqrt{10-2\sqrt{5}}\left(\log\left(5+\sqrt{10-2\sqrt{5}}\right)-\log\left(5-\sqrt{10-2\sqrt{5}}\right)\right)+10\tan^{-1}(2)-2\tan^{-1}(2)^5-2\sqrt{5}\tan^{-1}\left(\frac{6}{\sqrt{5}}\right)\right)$$

## Alternative representations:

$$\frac{1}{4}\tan^{-1}(2)-\frac{1}{20}\tan^{-1}(2)^5+\frac{\tan^{-1}\left(\frac{(2-2^3)\sqrt{5}}{1-3\times 2^2+2^4}\right)}{4\sqrt{5}}+\frac{1}{40}\sqrt{10-2\sqrt{5}}\log\left(\frac{1+1\sqrt{10-2\sqrt{5}}+4}{1-1\sqrt{10-2\sqrt{5}}+4}\right)=\frac{1}{4}\tan^{-1}(2)-\frac{1}{20}\tan^{-1}(2)^5+\frac{1}{40}\log_e\left(\frac{5+\sqrt{10-2\sqrt{5}}}{5-\sqrt{10-2\sqrt{5}}}\right)\sqrt{10-2\sqrt{5}}+\frac{\tan^{-1}\left(-\frac{6\sqrt{5}}{-11+2^4}\right)}{4\sqrt{5}}$$

$$\frac{1}{4}\tan^{-1}(2)-\frac{1}{20}\tan^{-1}(2)^5+\frac{\tan^{-1}\left(\frac{(2-2^3)\sqrt{5}}{1-3\times 2^2+2^4}\right)}{4\sqrt{5}}+\frac{1}{40}\sqrt{10-2\sqrt{5}}\log\left(\frac{1+1\sqrt{10-2\sqrt{5}}+4}{1-1\sqrt{10-2\sqrt{5}}+4}\right)=\frac{1}{4}\tan^{-1}(1,2)-\frac{1}{20}\tan^{-1}(1,2)^5+\frac{1}{40}\log\left(\frac{5+\sqrt{10-2\sqrt{5}}}{5-\sqrt{10-2\sqrt{5}}}\right)\sqrt{10-2\sqrt{5}}+\frac{\tan^{-1}\left(1,-\frac{6\sqrt{5}}{-11+2^4}\right)}{4\sqrt{5}}$$

$$\frac{1}{4}\tan^{-1}(2)-\frac{1}{20}\tan^{-1}(2)^5+\frac{\tan^{-1}\left(\frac{(2-2^3)\sqrt{5}}{1-3\times 2^2+2^4}\right)}{4\sqrt{5}}+\frac{1}{40}\sqrt{10-2\sqrt{5}}\log\left(\frac{1+1\sqrt{10-2\sqrt{5}}+4}{1-1\sqrt{10-2\sqrt{5}}+4}\right)=\frac{1}{4}\tan^{-1}(2)-\frac{1}{20}\tan^{-1}(2)^5+\frac{1}{40}\log(a)\log_a\left(\frac{5+\sqrt{10-2\sqrt{5}}}{5-\sqrt{10-2\sqrt{5}}}\right)\sqrt{10-2\sqrt{5}}+\frac{\tan^{-1}\left(-\frac{6\sqrt{5}}{-11+2^4}\right)}{4\sqrt{5}}$$

**Series representations:**

$$\frac{1}{4}\tan^{-1}(2)-\frac{1}{20}\tan^{-1}(2)^5+\frac{\tan^{-1}\left(\frac{(2-2^3)\sqrt{5}}{1-3\times 2^2+2^4}\right)}{4\sqrt{5}}+\frac{1}{40}\sqrt{10-2\sqrt{5}}\log\left(\frac{1+1\sqrt{10-2\sqrt{5}}+4}{1-1\sqrt{10-2\sqrt{5}}+4}\right)=\frac{1}{4}\tan^{-1}(2)-\frac{1}{20}\tan^{-1}(2)^5-\frac{\tan^{-1}\left(\frac{6}{\sqrt{5}}\right)}{4\sqrt{5}}+\frac{1}{40}\sqrt{10-2\sqrt{5}}\log\left(-1+\frac{5+\sqrt{10-2\sqrt{5}}}{5-\sqrt{10-2\sqrt{5}}}\right)-\frac{1}{40}\sqrt{10-2\sqrt{5}}\sum_{k=1}^{\infty}\frac{\left(\frac{1}{2}-\frac{5}{2\sqrt{10-2\sqrt{5}}}\right)^k}{k}$$

$$\frac{1}{4}\tan^{-1}(2)-\frac{1}{20}\tan^{-1}(2)^5+\frac{\tan^{-1}\left(\frac{(2-2^3)\sqrt{5}}{1-3\times 2^2+2^4}\right)}{4\sqrt{5}}+\frac{1}{40}\sqrt{10-2\sqrt{5}}\log\left(\frac{1+1\sqrt{10-2\sqrt{5}}+4}{1-1\sqrt{10-2\sqrt{5}}+4}\right)=$$

$$\frac{1}{640}\left(160\tan^{-1}(z_0)-32\sqrt{5}\tan^{-1}(z_0)-32\tan^{-1}(z_0)^5+16\sqrt{2\left(5-\sqrt{5}\right)}\log\left(-1+\frac{5+\sqrt{10-2\sqrt{5}}}{5-\sqrt{10-2\sqrt{5}}}\right)-16\sqrt{2\left(5-\sqrt{5}\right)}\sum_{k=1}^{\infty}\frac{\left(-\frac{1}{-1+\frac{5+\sqrt{10-2\sqrt{5}}}{5-\sqrt{10-2\sqrt{5}}}}\right)^k}{k}+80\,i\sum_{k=1}^{\infty}\frac{\left(-(-i-z_0)^{-k}+(i-z_0)^{-k}\right)(2-z_0)^k}{k}-80\,i\tan^{-1}(z_0)^4\sum_{k=1}^{\infty}\frac{\left(-(-i-z_0)^{-k}+(i-z_0)^{-k}\right)(2-z_0)^k}{k}+80\tan^{-1}(z_0)^3\left(\sum_{k=1}^{\infty}\frac{\left(-(-i-z_0)^{-k}+(i-z_0)^{-k}\right)(2-z_0)^k}{k}\right)^2+40\,i\tan^{-1}(z_0)^2\left(\sum_{k=1}^{\infty}\frac{\left(-(-i-z_0)^{-k}+(i-z_0)^{-k}\right)(2-z_0)^k}{k}\right)^3-10\tan^{-1}(z_0)\left(\sum_{k=1}^{\infty}\frac{\left(-(-i-z_0)^{-k}+(i-z_0)^{-k}\right)(2-z_0)^k}{k}\right)^4-i\left(\sum_{k=1}^{\infty}\frac{\left(-(-i-z_0)^{-k}+(i-z_0)^{-k}\right)(2-z_0)^k}{k}\right)^5-16\,i\sqrt{5}\sum_{k=1}^{\infty}\frac{\left(-(-i-z_0)^{-k}+(i-z_0)^{-k}\right)\left(\frac{6}{\sqrt{5}}-z_0\right)^k}{k}\right)$$

for $(i z_0 \notin \mathbb{R}$ or ( not $(1 \le i z_0 < \infty)$ and not $(-\infty < i z_0 \le -1))$

1/40 (10+2sqrt5)^(1/2)* ln (((1+1(10+2sqrt5)^(1/2)+4))/(((1-1(10+2sqrt5)^(1/2)+4))))

**Input:**

$$\frac{1}{40}\sqrt{10+2\sqrt{5}}\,\log\left(\frac{1+1\sqrt{10+2\sqrt{5}}+4}{1-1\sqrt{10+2\sqrt{5}}+4}\right)$$

log(x) is the natural logarithm

**Exact result:**

$$\frac{1}{40}\sqrt{10+2\sqrt{5}}\,\log\left(\frac{5+\sqrt{10+2\sqrt{5}}}{5-\sqrt{10+2\sqrt{5}}}\right)$$

**Decimal approximation:**

0.189872557940113444479006186860777045433398567588140907800...

0.18987255794…

**Property:**

$$\frac{1}{40}\sqrt{10+2\sqrt{5}}\,\log\left(\frac{5+\sqrt{10+2\sqrt{5}}}{5-\sqrt{10+2\sqrt{5}}}\right)$$ is a transcendental number

**Alternate forms:**

$$\frac{1}{20}\sqrt{\frac{1}{2}\left(5+\sqrt{5}\right)}\,\log\left(\frac{1}{82}\left(218+40\sqrt{5}+41\sqrt{\frac{48800}{1681}+\frac{17440\sqrt{5}}{1681}}\right)\right)$$

$$\frac{\left(\sqrt{1-2i}+\sqrt{1+2i}\right)\log\left(\frac{5+\sqrt{2\left(5+\sqrt{5}\right)}}{5-\sqrt{2\left(5+\sqrt{5}\right)}}\right)}{8\times 5^{3/4}}$$

$$\frac{1}{20}\sqrt{\frac{1}{2}\left(5+\sqrt{5}\right)}\left(\log\left(5+\sqrt{2\left(5+\sqrt{5}\right)}\right)-\log\left(5-\sqrt{2\left(5+\sqrt{5}\right)}\right)\right)$$

## Alternative representations:

$$\frac{1}{40}\sqrt{10+2\sqrt{5}}\,\log\left(\frac{1+1\sqrt{10+2\sqrt{5}}+4}{1-1\sqrt{10+2\sqrt{5}}+4}\right)=\frac{1}{40}\log_e\left(\frac{5+\sqrt{10+2\sqrt{5}}}{5-\sqrt{10+2\sqrt{5}}}\right)\sqrt{10+2\sqrt{5}}$$

$$\frac{1}{40}\sqrt{10+2\sqrt{5}}\,\log\left(\frac{1+1\sqrt{10+2\sqrt{5}}+4}{1-1\sqrt{10+2\sqrt{5}}+4}\right)=\frac{1}{40}\log(a)\log_a\left(\frac{5+\sqrt{10+2\sqrt{5}}}{5-\sqrt{10+2\sqrt{5}}}\right)\sqrt{10+2\sqrt{5}}$$

$$\frac{1}{40}\sqrt{10+2\sqrt{5}}\,\log\left(\frac{1+1\sqrt{10+2\sqrt{5}}+4}{1-1\sqrt{10+2\sqrt{5}}+4}\right)=-\frac{1}{40}\text{Li}_1\left(1-\frac{5+\sqrt{10+2\sqrt{5}}}{5-\sqrt{10+2\sqrt{5}}}\right)\sqrt{10+2\sqrt{5}}$$

## Series representations:

$$\frac{1}{40}\sqrt{10+2\sqrt{5}}\,\log\left(\frac{1+1\sqrt{10+2\sqrt{5}}+4}{1-1\sqrt{10+2\sqrt{5}}+4}\right)=\frac{1}{40}\sqrt{10+2\sqrt{5}}\,\log\left(-1+\frac{5+\sqrt{10+2\sqrt{5}}}{5-\sqrt{10+2\sqrt{5}}}\right)-\frac{1}{40}\sqrt{10+2\sqrt{5}}\sum_{k=1}^{\infty}\frac{\left(\frac{1}{2}-\frac{5}{2\sqrt{2\left(5+\sqrt{5}\right)}}\right)^k}{k}$$

$$\frac{1}{40}\sqrt{10+2\sqrt{5}}\,\log\left(\frac{1+1\sqrt{10+2\sqrt{5}}+4}{1-1\sqrt{10+2\sqrt{5}}+4}\right)=\frac{1}{20}\sqrt{\frac{1}{2}\left(5+\sqrt{5}\right)}\log\left(-\frac{2\sqrt{2\left(5+\sqrt{5}\right)}}{-5+\sqrt{2\left(5+\sqrt{5}\right)}}\right)-\frac{1}{20}\sqrt{\frac{1}{2}\left(5+\sqrt{5}\right)}\sum_{k=1}^{\infty}\frac{2^{-(3k)/2}\left(5+\sqrt{5}\right)^{-k/2}\left(-5+\sqrt{2\left(5+\sqrt{5}\right)}\right)^k}{k}$$

$$\frac{1}{40}\sqrt{10+2\sqrt{5}}\,\log\left(\frac{1+1\sqrt{10+2\sqrt{5}}+4}{1-1\sqrt{10+2\sqrt{5}}+4}\right)=$$

$$\frac{1}{40}\sqrt{10+2\sqrt{5}}\,\log\left(-1+\frac{5+\sqrt{10+2\sqrt{5}}}{5-\sqrt{10+2\sqrt{5}}}\right)-$$

$$\frac{1}{40}\sqrt{10+2\sqrt{5}}\sum_{k=1}^{\infty}\frac{\left(-\frac{1}{-1+\frac{5+\sqrt{10+2\sqrt{5}}}{5-\sqrt{10+2\sqrt{5}}}}\right)^k}{k}$$

**Integral representations:**

$$\frac{1}{40}\sqrt{10+2\sqrt{5}}\,\log\left(\frac{1+1\sqrt{10+2\sqrt{5}}+4}{1-1\sqrt{10+2\sqrt{5}}+4}\right)=\frac{1}{20}\sqrt{\frac{1}{2}\left(5+\sqrt{5}\right)}\int_1^{\frac{5+\sqrt{2\left(5+\sqrt{5}\right)}}{5-\sqrt{2\left(5+\sqrt{5}\right)}}}\frac{1}{t}\,dt$$

$$\frac{1}{40}\sqrt{10+2\sqrt{5}}\,\log\left(\frac{1+1\sqrt{10+2\sqrt{5}}+4}{1-1\sqrt{10+2\sqrt{5}}+4}\right)=$$

$$-\frac{i\sqrt{10+2\sqrt{5}}}{80\pi}\int_{-i\infty+\gamma}^{i\infty+\gamma}\frac{\left(-1+\frac{5+\sqrt{10+2\sqrt{5}}}{5-\sqrt{10+2\sqrt{5}}}\right)^{-s}\Gamma(-s)^2\,\Gamma(1+s)}{\Gamma(1-s)}\,ds \quad \text{for } -1<\gamma<0$$

((1/4 tan^-1 (2))) – ((1/20 tan^-1 (2)^5)) + 1/(4sqrt5) tan^-1[(((2-2^3)sqrt5)) /((1-3*2^2+2^4))] + 1/40 (10-2sqrt5)^(1/2)* ln [((1+1(10-2sqrt5)^(1/2)+4))/((1-1(10-2sqrt5)^(1/2)+4))] + 0.18987255794

**Input interpretation:**

$$\frac{1}{4}\tan^{-1}(2)-\frac{1}{20}\tan^{-1}(2)^5+\frac{1}{4\sqrt{5}}\tan^{-1}\left(\frac{(2-2^3)\sqrt{5}}{1-3\times 2^2+2^4}\right)+$$

$$\frac{1}{40}\sqrt{10-2\sqrt{5}}\,\log\left(\frac{1+1\sqrt{10-2\sqrt{5}}+4}{1-1\sqrt{10-2\sqrt{5}}+4}\right)+0.18987255794$$

$\tan^{-1}(x)$ is the inverse tangent function

$\log(x)$ is the natural logarithm

**Result:**

0.30774383546...

(result in radians)

0.30774383546…

**Alternative representations:**

$$\frac{1}{4}\tan^{-1}(2)-\frac{1}{20}\tan^{-1}(2)^5+\frac{\tan^{-1}\left(\frac{(2-2^3)\sqrt{5}}{1-3\times 2^2+2^4}\right)}{4\sqrt{5}}+\frac{1}{40}\sqrt{10-2\sqrt{5}}\log\left(\frac{1+1\sqrt{10-2\sqrt{5}}+4}{1-1\sqrt{10-2\sqrt{5}}+4}\right)+0.189872557940000=0.189872557940000+\frac{1}{4}\tan^{-1}(2)-\frac{1}{20}\tan^{-1}(2)^5+\frac{1}{40}\log_e\left(\frac{5+\sqrt{10-2\sqrt{5}}}{5-\sqrt{10-2\sqrt{5}}}\right)\sqrt{10-2\sqrt{5}}+\frac{\tan^{-1}\left(-\frac{6\sqrt{5}}{-11+2^4}\right)}{4\sqrt{5}}$$

$$\frac{1}{4}\tan^{-1}(2)-\frac{1}{20}\tan^{-1}(2)^5+\frac{\tan^{-1}\left(\frac{(2-2^3)\sqrt{5}}{1-3\times 2^2+2^4}\right)}{4\sqrt{5}}+\frac{1}{40}\sqrt{10-2\sqrt{5}}\log\left(\frac{1+1\sqrt{10-2\sqrt{5}}+4}{1-1\sqrt{10-2\sqrt{5}}+4}\right)+0.189872557940000=0.189872557940000+\frac{1}{4}\tan^{-1}(1,2)-\frac{1}{20}\tan^{-1}(1,2)^5+\frac{1}{40}\log\left(\frac{5+\sqrt{10-2\sqrt{5}}}{5-\sqrt{10-2\sqrt{5}}}\right)\sqrt{10-2\sqrt{5}}+\frac{\tan^{-1}\left(1,-\frac{6\sqrt{5}}{-11+2^4}\right)}{4\sqrt{5}}$$

$$\frac{1}{4}\tan^{-1}(2)-\frac{1}{20}\tan^{-1}(2)^5+\frac{\tan^{-1}\left(\frac{(2-2^3)\sqrt{5}}{1-3\times 2^2+2^4}\right)}{4\sqrt{5}}+\frac{1}{40}\sqrt{10-2\sqrt{5}}\log\left(\frac{1+1\sqrt{10-2\sqrt{5}}+4}{1-1\sqrt{10-2\sqrt{5}}+4}\right)+0.189872557940000=0.189872557940000+\frac{1}{4}\tan^{-1}(2)-\frac{1}{20}\tan^{-1}(2)^5+\frac{1}{40}\log(a)\log_a\left(\frac{5+\sqrt{10-2\sqrt{5}}}{5-\sqrt{10-2\sqrt{5}}}\right)\sqrt{10-2\sqrt{5}}+\frac{\tan^{-1}\left(-\frac{6\sqrt{5}}{-11+2^4}\right)}{4\sqrt{5}}$$

## Continued fraction representations:

$$\frac{1}{4}\tan^{-1}(2)-\frac{1}{20}\tan^{-1}(2)^5+\frac{\tan^{-1}\left(\frac{(2-2^3)\sqrt{5}}{1-3\times 2^2+2^4}\right)}{4\sqrt{5}}+\frac{1}{40}\sqrt{10-2\sqrt{5}}\log\left(\frac{1+1\sqrt{10-2\sqrt{5}}+4}{1-1\sqrt{10-2\sqrt{5}}+4}\right)+0.189872557940000=$$

$$0.189872557940000-\frac{8}{5\left(1+\underset{k=1}{\overset{\infty}{\mathrm{K}}}\frac{4k^2}{1+2k}\right)^5}+\frac{1}{2\left(1+\underset{k=1}{\overset{\infty}{\mathrm{K}}}\frac{4k^2}{1+2k}\right)}-\frac{3}{10\left(1+\underset{k=1}{\overset{\infty}{\mathrm{K}}}\frac{\frac{36}{25}k^2\sqrt{5}^2}{1+2k}\right)}+\frac{\left(-1+\frac{5+\sqrt{10-2\sqrt{5}}}{5-\sqrt{10-2\sqrt{5}}}\right)\sqrt{10-2\sqrt{5}}}{40\left(1+\underset{k=1}{\overset{\infty}{\mathrm{K}}}\frac{\left\lfloor\frac{1+k}{2}\right\rfloor^2\left(-1+\frac{5+\sqrt{10-2\sqrt{5}}}{5-\sqrt{10-2\sqrt{5}}}\right)}{1+k}\right)}=$$

$$0.189872557940000-\frac{8}{5\left(1+\frac{4}{3+\frac{16}{5+\frac{36}{7+\frac{64}{9+\dots}}}}\right)^5}+\frac{1}{2\left(1+\frac{4}{3+\frac{16}{5+\frac{36}{7+\frac{64}{9+\dots}}}}\right)}-\frac{3}{10\left(1+\frac{36}{5\left(3+\frac{144}{5\left(5+\frac{324}{5\left(7+\frac{576}{5(9+\dots)}\right)}\right)}\right)}\right)}+\frac{\sqrt{10-2\sqrt{5}}\left(-1+\frac{5+\sqrt{10-2\sqrt{5}}}{5-\sqrt{10-2\sqrt{5}}}\right)}{40\left(1+\frac{-1+\frac{5+\sqrt{10-2\sqrt{5}}}{5-\sqrt{10-2\sqrt{5}}}}{2+\frac{-1+\frac{5+\sqrt{10-2\sqrt{5}}}{5-\sqrt{10-2\sqrt{5}}}}{3+\frac{4\left(-1+\frac{5+\sqrt{10-2\sqrt{5}}}{5-\sqrt{10-2\sqrt{5}}}\right)}{4+\frac{4\left(-1+\frac{5+\sqrt{10-2\sqrt{5}}}{5-\sqrt{10-2\sqrt{5}}}\right)}{5+\dots}}}}\right)}$$

$$\frac{1}{4}\tan^{-1}(2)-\frac{1}{20}\tan^{-1}(2)^5+\frac{\tan^{-1}\left(\frac{(2-2^3)\sqrt{5}}{1-3\times 2^2+2^4}\right)}{4\sqrt{5}}+\frac{1}{40}\sqrt{10-2\sqrt{5}}\log\left(\frac{1+1\sqrt{10-2\sqrt{5}}+4}{1-1\sqrt{10-2\sqrt{5}}+4}\right)+0.189872557940000=$$

$$0.189872557940000-\frac{8}{5\left(1+\underset{k=1}{\overset{\infty}{\mathrm{K}}}\frac{4k^2}{1+2k}\right)^5}+\frac{1}{2\left(1+\underset{k=1}{\overset{\infty}{\mathrm{K}}}\frac{4k^2}{1+2k}\right)}-\frac{3}{10\left(1+\underset{k=1}{\overset{\infty}{\mathrm{K}}}\frac{\frac{36}{25}k^2\sqrt{5}^2}{1+2k}\right)}+\frac{\left(-1+\frac{5+\sqrt{10-2\sqrt{5}}}{5-\sqrt{10-2\sqrt{5}}}\right)\sqrt{10-2\sqrt{5}}}{40\left(1+\underset{k=1}{\overset{\infty}{\mathrm{K}}}\frac{\left\lfloor\frac{1+k}{2}\right\rfloor\left(-1+\frac{5+\sqrt{10-2\sqrt{5}}}{5-\sqrt{10-2\sqrt{5}}}\right)}{\frac{1}{2}\left(3+(-1)^k(-1+k)+k\right)}\right)}=$$

$$0.189872557940000-\frac{8}{5\left(1+\frac{4}{3+\frac{16}{5+\frac{36}{7+\frac{64}{9+\dots}}}}\right)^5}+\frac{1}{2\left(1+\frac{4}{3+\frac{16}{5+\frac{36}{7+\frac{64}{9+\dots}}}}\right)}-\frac{3}{10\left(1+\frac{36}{5\left(3+\frac{144}{5\left(5+\frac{324}{5\left(7+\frac{576}{5(9+\dots)}\right)}\right)}\right)}\right)}+\frac{\sqrt{10-2\sqrt{5}}\left(-1+\frac{5+\sqrt{10-2\sqrt{5}}}{5-\sqrt{10-2\sqrt{5}}}\right)}{40\left(1+\frac{-1+\frac{5+\sqrt{10-2\sqrt{5}}}{5-\sqrt{10-2\sqrt{5}}}}{2+\frac{-1+\frac{5+\sqrt{10-2\sqrt{5}}}{5-\sqrt{10-2\sqrt{5}}}}{3+\frac{2\left(-1+\frac{5+\sqrt{10-2\sqrt{5}}}{5-\sqrt{10-2\sqrt{5}}}\right)}{2+\frac{2\left(-1+\frac{5+\sqrt{10-2\sqrt{5}}}{5-\sqrt{10-2\sqrt{5}}}\right)}{5+\dots}}}}\right)}$$

42/[1/4 tan^(-1)(2)-1/20 tan^(-1)(2)^5 + (tan^(-1)(((2-2^3) sqrt(5))/(1-3 2^2 + 2^4)))/(4 sqrt(5))+1/40 sqrt(10-2 sqrt(5)) log((1+1 sqrt(10-2 sqrt(5))+4)/(1-1 sqrt(10-2 sqrt(5))+4))+0.18987255794] +Pi

**Input interpretation:**

$$42 \Bigg/ \left( \frac{1}{4} \tan^{-1}(2) - \frac{1}{20} \tan^{-1}(2)^5 + \frac{\tan^{-1}\left(\frac{(2-2^3)\sqrt{5}}{1-3\times 2^2+2^4}\right)}{4\sqrt{5}} + \frac{1}{40} \sqrt{10-2\sqrt{5}} \log\left(\frac{1+1\sqrt{10-2\sqrt{5}}+4}{1-1\sqrt{10-2\sqrt{5}}+4}\right) + 0.18987255794 \right) + \pi$$

$\tan^{-1}(x)$ is the inverse tangent function

$\log(x)$ is the natural logarithm

**Result:**

139.61873747...

(result in radians)

139.61873747… result practically equal to the rest mass of Pion meson 139.57 MeV

**Alternative representations:**

$$42 \Bigg/ \left( \frac{1}{4} \tan^{-1}(2) - \frac{1}{20} \tan^{-1}(2)^5 + \frac{\tan^{-1}\left(\frac{(2-2^3)\sqrt{5}}{1-3\times 2^2+2^4}\right)}{4\sqrt{5}} + \frac{1}{40} \sqrt{10-2\sqrt{5}} \log\left(\frac{1+1\sqrt{10-2\sqrt{5}}+4}{1-1\sqrt{10-2\sqrt{5}}+4}\right) + 0.189872557940000 \right) + \pi =$$

$$\pi + 42 \Bigg/ \left( 0.189872557940000 + \frac{1}{4} \tan^{-1}(2) - \frac{1}{20} \tan^{-1}(2)^5 + \frac{\tan^{-1}\left(-\frac{6\sqrt{5}}{-11+2^4}\right)}{4\sqrt{5}} + \frac{1}{40} \log_e\left(\frac{5+\sqrt{10-2\sqrt{5}}}{5-\sqrt{10-2\sqrt{5}}}\right) \sqrt{10-2\sqrt{5}} \right)$$

$$42 \Bigg/ \left( \frac{1}{4} \tan^{-1}(2) - \frac{1}{20} \tan^{-1}(2)^5 + \frac{\tan^{-1}\left( \frac{(2-2^3)\sqrt{5}}{1-3\times 2^2+2^4} \right)}{4\sqrt{5}} + \frac{1}{40} \sqrt{10-2\sqrt{5}} \log\left( \frac{1+1\sqrt{10-2\sqrt{5}}+4}{1-1\sqrt{10-2\sqrt{5}}+4} \right) + 0.189872557940000 \right) + \pi =$$

$$\pi + 42 \Bigg/ \left( 0.189872557940000 + \frac{1}{4} \tan^{-1}(1, 2) - \frac{1}{20} \tan^{-1}(1, 2)^5 + \frac{\tan^{-1}\left(1, -\frac{6\sqrt{5}}{-11+2^4}\right)}{4\sqrt{5}} + \frac{1}{40} \log\left( \frac{5+\sqrt{10-2\sqrt{5}}}{5-\sqrt{10-2\sqrt{5}}} \right) \sqrt{10-2\sqrt{5}} \right)$$

$$42 \Bigg/ \left( \frac{1}{4} \tan^{-1}(2) - \frac{1}{20} \tan^{-1}(2)^5 + \frac{\tan^{-1}\left( \frac{(2-2^3)\sqrt{5}}{1-3\times 2^2+2^4} \right)}{4\sqrt{5}} + \frac{1}{40} \sqrt{10-2\sqrt{5}} \log\left( \frac{1+1\sqrt{10-2\sqrt{5}}+4}{1-1\sqrt{10-2\sqrt{5}}+4} \right) + 0.189872557940000 \right) + \pi =$$

$$\pi + 42 \Bigg/ \left( 0.189872557940000 + \frac{1}{4} \tan^{-1}(2) - \frac{1}{20} \tan^{-1}(2)^5 + \frac{\tan^{-1}\left(-\frac{6\sqrt{5}}{-11+2^4}\right)}{4\sqrt{5}} + \frac{1}{40} \log(a) \log_a\left( \frac{5+\sqrt{10-2\sqrt{5}}}{5-\sqrt{10-2\sqrt{5}}} \right) \sqrt{10-2\sqrt{5}} \right)$$

**Series representations:**

$$42\Bigg/\left(\frac{1}{4}\tan^{-1}(2)-\frac{1}{20}\tan^{-1}(2)^5+\frac{\tan^{-1}\left(\frac{(2-2^3)\sqrt{5}}{1-3\times 2^2+2^4}\right)}{4\sqrt{5}}+\frac{1}{40}\sqrt{10-2\sqrt{5}}\,\log\left(\frac{1+1\sqrt{10-2\sqrt{5}}+4}{1-1\sqrt{10-2\sqrt{5}}+4}\right)+0.189872557940000\right)+$$

$$\pi=\pi+42\Bigg/\left(0.189872557940000+\frac{1}{4}\sum_{k=0}^{\infty}\frac{\left(-\frac{1}{5}\right)^k 4^{1+2k}F_{1+2k}\left(\frac{1}{1+\sqrt{\frac{21}{5}}}\right)^{1+2k}}{1+2k}-\frac{1}{20}\left(\sum_{k=0}^{\infty}\frac{\left(-\frac{1}{5}\right)^k 4^{1+2k}F_{1+2k}\left(\frac{1}{1+\sqrt{\frac{21}{5}}}\right)^{1+2k}}{1+2k}\right)^5+\frac{1}{40}\exp\left(i\pi\left\lfloor\frac{\arg(10-x-2\sqrt{5})}{2\pi}\right\rfloor\right)\sqrt{x}\left(\sum_{k=0}^{\infty}\frac{(-1)^k x^{-k}\left(-\frac{1}{2}\right)_k(10-x-2\sqrt{5})^k}{k!}\right)\left(\log\left(-1+\frac{5+\sqrt{10-2\sqrt{5}}}{5-\sqrt{10-2\sqrt{5}}}\right)-\sum_{k=1}^{\infty}\frac{\left(-\frac{1}{2}\right)^k\left(-\frac{\sqrt{10-2\sqrt{5}}}{-5+\sqrt{10-2\sqrt{5}}}\right)^{-k}}{k}\right)+\frac{\sum_{k=0}^{\infty}\frac{(-1)^k 5^{-1-3k}\times 12^{1+2k}F_{1+2k}\left(-\frac{\sqrt{5}}{1+\sqrt{1+\frac{144\sqrt{5}^2}{125}}}\right)^{1+2k}}{1+2k}}{4\exp\left(i\pi\left\lfloor\frac{\arg(5-x)}{2\pi}\right\rfloor\right)\sqrt{x}\sum_{k=0}^{\infty}\frac{(-1)^k(5-x)^k x^{-k}\left(-\frac{1}{2}\right)_k}{k!}}\right)$$

for ($x \in \mathbb{R}$ and $x < 0$)

$$42 \Big/ \left( \frac{1}{4}\tan^{-1}(2) - \frac{1}{20}\tan^{-1}(2)^5 + \frac{\tan^{-1}\left(\frac{(2-2^3)\sqrt{5}}{1-3\times 2^2+2^4}\right)}{4\sqrt{5}} + \frac{1}{40}\sqrt{10-2\sqrt{5}}\,\log\left(\frac{1+1\sqrt{10-2\sqrt{5}}+4}{1-1\sqrt{10-2\sqrt{5}}+4}\right) + 0.189872557940000 \right) + \pi =$$

$$\pi + 42 \Big/ \left( 0.189872557940000 + \frac{1}{4}\sum_{k=0}^{\infty} \frac{\left(-\frac{1}{5}\right)^k 4^{1+2k} F_{1+2k}\left(\frac{1}{1+\sqrt{\frac{21}{5}}}\right)^{1+2k}}{1+2k} - \frac{1}{20}\left(\sum_{k=0}^{\infty} \frac{\left(-\frac{1}{5}\right)^k 4^{1+2k} F_{1+2k}\left(\frac{1}{1+\sqrt{\frac{21}{5}}}\right)^{1+2k}}{1+2k}\right)^5 + \right.$$

$$\left( \left(\frac{1}{z_0}\right)^{-1/2\lfloor \arg(5-z_0)/(2\pi)\rfloor} z_0^{1/2(-1-\lfloor \arg(5-z_0)/(2\pi)\rfloor)} \sum_{k=0}^{\infty} \frac{(-1)^k 5^{-1-3k}\times 12^{1+2k} F_{1+2k}\left(-\frac{\sqrt{5}}{1+\sqrt{1+\frac{144\sqrt{5}^2}{125}}}\right)^{1+2k}}{1+2k} \right) \Big/$$

$$\left(4\sum_{k=0}^{\infty} \frac{(-1)^k \left(-\frac{1}{2}\right)_k (5-z_0)^k z_0^{-k}}{k!}\right) +$$

$$\frac{1}{40}\left(\frac{1}{z_0}\right)^{1/2\left\lfloor \arg\left(10-2\sqrt{5}-z_0\right)/(2\pi)\right\rfloor} z_0^{1/2\left(1+\left\lfloor \arg\left(10-2\sqrt{5}-z_0\right)/(2\pi)\right\rfloor\right)}$$

$$\left( \log\left(-1 + \frac{5+\sqrt{10-2\sqrt{5}}}{5-\sqrt{10-2\sqrt{5}}}\right) - \sum_{k=1}^{\infty} \frac{\left(-\frac{1}{2}\right)^k \left(-\frac{\sqrt{10-2\sqrt{5}}}{-5+\sqrt{10-2\sqrt{5}}}\right)^{-k}}{k} \right)$$

$$\sum_{k=0}^{\infty} \frac{(-1)^k \left(-\frac{1}{2}\right)_k \left(10-2\sqrt{5}-z_0\right)^k z_0^{-k}}{k!}$$

$$42 \Bigg/ \left( \frac{1}{4} \tan^{-1}(2) - \frac{1}{20} \tan^{-1}(2)^5 + \frac{\tan^{-1}\left(\frac{(2-2^3)\sqrt{5}}{1-3\times 2^2+2^4}\right)}{4\sqrt{5}} + \frac{1}{40} \sqrt{10-2\sqrt{5}} \log\left(\frac{1+1\sqrt{10-2\sqrt{5}}+4}{1-1\sqrt{10-2\sqrt{5}}+4}\right) + 0.189872557940000 \right) +$$

$$\pi = \pi + 42 \Bigg/ \Bigg( 0.189872557940000 + \frac{1}{4}\left(\tan^{-1}(x) + \pi \left\lfloor \frac{\arg(i(2-x))}{2\pi} \right\rfloor + \frac{1}{2} i \sum_{k=1}^{\infty} \frac{\left(-(-i-x)^{-k} + (i-x)^{-k}\right)(2-x)^k}{k}\right) - \frac{1}{20}\left(\tan^{-1}(x) + \pi \left\lfloor \frac{\arg(i(2-x))}{2\pi} \right\rfloor + \frac{1}{2} i \sum_{k=1}^{\infty} \frac{\left(-(-i-x)^{-k} + (i-x)^{-k}\right)(2-x)^k}{k}\right)^5 + \frac{\tan^{-1}(x) + \pi \left\lfloor \frac{\arg\left(i\left(-x-\frac{6\sqrt{5}}{5}\right)\right)}{2\pi} \right\rfloor + \frac{1}{2} i \sum_{k=1}^{\infty} \frac{\left(-(-i-x)^{-k}+(i-x)^{-k}\right)\left(-x-\frac{6\sqrt{5}}{5}\right)^k}{k}}{4 \exp\left(i\pi \left\lfloor \frac{\arg(5-x)}{2\pi} \right\rfloor\right)\sqrt{x} \sum_{k=0}^{\infty} \frac{(-1)^k (5-x)^k x^{-k} \left(-\frac{1}{2}\right)_k}{k!}} + \frac{1}{40} \exp\left(i\pi \left\lfloor \frac{\arg(10-x-2\sqrt{5})}{2\pi} \right\rfloor\right)\sqrt{x} \left(\sum_{k=0}^{\infty} \frac{(-1)^k x^{-k} \left(-\frac{1}{2}\right)_k (10-x-2\sqrt{5})^k}{k!}\right) \left(\log\left(-1 + \frac{5+\sqrt{10-2\sqrt{5}}}{5-\sqrt{10-2\sqrt{5}}}\right) - \sum_{k=1}^{\infty} \frac{\left(-\frac{1}{2}\right)^k \left(-\frac{\sqrt{10-2\sqrt{5}}}{-5+\sqrt{10-2\sqrt{5}}}\right)^{-k}}{k}\right)\Bigg)$$

for ($i x \in \mathbb{R}$ and $i x < -1$ and $x \in \mathbb{R}$ and $x < 0$)

42/[1/4 tan^(-1)(2)-1/20 tan^(-1)(2)^5 + (tan^(-1)(((2-2^3) sqrt(5))/(1-3 2^2 + 2^4)))/(4 sqrt(5))+1/40 sqrt(10-2 sqrt(5)) log((1+1 sqrt(10-2 sqrt(5))+4)/(1-1 sqrt(10-2 sqrt(5))+4))+0.18987255794]-11

**Input interpretation:**

$$42 \Big/ \left( \frac{1}{4} \tan^{-1}(2) - \frac{1}{20} \tan^{-1}(2)^5 + \frac{\tan^{-1}\left(\frac{(2-2^3)\sqrt{5}}{1-3\times 2^2+2^4}\right)}{4\sqrt{5}} + \frac{1}{40} \sqrt{10-2\sqrt{5}} \log\left(\frac{1+1\sqrt{10-2\sqrt{5}}+4}{1-1\sqrt{10-2\sqrt{5}}+4}\right) + 0.18987255794 \right) - 11$$

$\tan^{-1}(x)$ is the inverse tangent function

$\log(x)$ is the natural logarithm

**Result:**

125.47714482...

(result in radians)

125.47714482… result very near to the dilaton mass calculated as a type of Higgs boson: 125 GeV for T = 0 and to the Higgs boson mass 125.18 GeV

**Alternative representations:**

$$42 \Big/ \left( \frac{1}{4} \tan^{-1}(2) - \frac{1}{20} \tan^{-1}(2)^5 + \frac{\tan^{-1}\left(\frac{(2-2^3)\sqrt{5}}{1-3\times 2^2+2^4}\right)}{4\sqrt{5}} + \frac{1}{40} \sqrt{10-2\sqrt{5}} \log\left(\frac{1+1\sqrt{10-2\sqrt{5}}+4}{1-1\sqrt{10-2\sqrt{5}}+4}\right) + 0.189872557940000 \right) - 11 = -11 + 42 \Big/ \left( 0.189872557940000 + \frac{1}{4} \tan^{-1}(2) - \frac{1}{20} \tan^{-1}(2)^5 + \frac{\tan^{-1}\left(-\frac{6\sqrt{5}}{-11+2^4}\right)}{4\sqrt{5}} + \frac{1}{40} \log_e\left(\frac{5+\sqrt{10-2\sqrt{5}}}{5-\sqrt{10-2\sqrt{5}}}\right) \sqrt{10-2\sqrt{5}} \right)$$

$$42 \Big/ \left( \frac{1}{4} \tan^{-1}(2) - \frac{1}{20} \tan^{-1}(2)^5 + \frac{\tan^{-1}\left( \frac{(2-2^3)\sqrt{5}}{1-3\times 2^2+2^4} \right)}{4\sqrt{5}} + \frac{1}{40} \sqrt{10-2\sqrt{5}}\, \log\left( \frac{1+1\sqrt{10-2\sqrt{5}}+4}{1-1\sqrt{10-2\sqrt{5}}+4} \right) + 0.189872557940000 \right) -$$

$$11 = -11 + 42 \Big/ \left( 0.189872557940000 + \frac{1}{4} \tan^{-1}(1, 2) - \frac{1}{20} \tan^{-1}(1, 2)^5 + \frac{\tan^{-1}\left(1, -\frac{6\sqrt{5}}{-11+2^4}\right)}{4\sqrt{5}} + \frac{1}{40} \log\left( \frac{5+\sqrt{10-2\sqrt{5}}}{5-\sqrt{10-2\sqrt{5}}} \right) \sqrt{10-2\sqrt{5}} \right)$$

$$42 \Big/ \left( \frac{1}{4} \tan^{-1}(2) - \frac{1}{20} \tan^{-1}(2)^5 + \frac{\tan^{-1}\left( \frac{(2-2^3)\sqrt{5}}{1-3\times 2^2+2^4} \right)}{4\sqrt{5}} + \frac{1}{40} \sqrt{10-2\sqrt{5}}\, \log\left( \frac{1+1\sqrt{10-2\sqrt{5}}+4}{1-1\sqrt{10-2\sqrt{5}}+4} \right) + 0.189872557940000 \right) -$$

$$11 = -11 + 42 \Big/ \left( 0.189872557940000 + \frac{1}{4} \tan^{-1}(2) - \frac{1}{20} \tan^{-1}(2)^5 + \frac{\tan^{-1}\left(-\frac{6\sqrt{5}}{-11+2^4}\right)}{4\sqrt{5}} + \frac{1}{40} \log(a) \log_a\left( \frac{5+\sqrt{10-2\sqrt{5}}}{5-\sqrt{10-2\sqrt{5}}} \right) \sqrt{10-2\sqrt{5}} \right)$$

## Series representations:

$$42\Big/\left(\frac{1}{4}\tan^{-1}(2)-\frac{1}{20}\tan^{-1}(2)^5+\frac{\tan^{-1}\left(\frac{(2-2^3)\sqrt{5}}{1-3\times 2^2+2^4}\right)}{4\sqrt{5}}+\frac{1}{40}\sqrt{10-2\sqrt{5}}\log\left(\frac{1+1\sqrt{10-2\sqrt{5}}+4}{1-1\sqrt{10-2\sqrt{5}}+4}\right)+0.189872557940000\right)-11=$$

$$-11+42\Big/\left(0.189872557940000+\frac{1}{4}\sum_{k=0}^{\infty}\frac{\left(-\frac{1}{5}\right)^k 4^{1+2k}F_{1+2k}\left(\frac{1}{1+\sqrt{\frac{21}{5}}}\right)^{1+2k}}{1+2k}-\frac{1}{20}\left(\sum_{k=0}^{\infty}\frac{\left(-\frac{1}{5}\right)^k 4^{1+2k}F_{1+2k}\left(\frac{1}{1+\sqrt{\frac{21}{5}}}\right)^{1+2k}}{1+2k}\right)^5+\frac{1}{40}\exp\left(i\pi\left\lfloor\frac{\arg(10-x-2\sqrt{5})}{2\pi}\right\rfloor\right)\sqrt{x}\left(\sum_{k=0}^{\infty}\frac{(-1)^k x^{-k}\left(-\frac{1}{2}\right)_k(10-x-2\sqrt{5})^k}{k!}\right)\left(\log\left(-1+\frac{5+\sqrt{10-2\sqrt{5}}}{5-\sqrt{10-2\sqrt{5}}}\right)-\sum_{k=1}^{\infty}\frac{\left(-\frac{1}{2}\right)^k\left(-\frac{\sqrt{10-2\sqrt{5}}}{-5+\sqrt{10-2\sqrt{5}}}\right)^{-k}}{k}\right)+\frac{\sum_{k=0}^{\infty}\frac{(-1)^k 5^{-1-3k}\times 12^{1+2k}F_{1+2k}\left(-\frac{\sqrt{5}}{1+\sqrt{1+\frac{144\sqrt{5}^2}{125}}}\right)^{1+2k}}{1+2k}}{4\exp\left(i\pi\left\lfloor\frac{\arg(5-x)}{2\pi}\right\rfloor\right)\sqrt{x}\sum_{k=0}^{\infty}\frac{(-1)^k(5-x)^k x^{-k}\left(-\frac{1}{2}\right)_k}{k!}}\right)$$

for ($x\in\mathbb{R}$ and $x<0$)

$$42 \Big/ \left( \frac{1}{4} \tan^{-1}(2) - \frac{1}{20} \tan^{-1}(2)^5 + \frac{\tan^{-1}\left(\frac{(2-2^3)\sqrt{5}}{1-3\times 2^2+2^4}\right)}{4\sqrt{5}} + \frac{1}{40}\sqrt{10-2\sqrt{5}} \log\left(\frac{1+1\sqrt{10-2\sqrt{5}}+4}{1-1\sqrt{10-2\sqrt{5}}+4}\right) + 0.189872557940000 \right) -$$

$$11 = -11 + 42 \Big/ \left( 0.189872557940000 + \frac{1}{4} \sum_{k=0}^{\infty} \frac{\left(-\frac{1}{5}\right)^k 4^{1+2k} F_{1+2k} \left(\frac{1}{1+\sqrt{\frac{21}{5}}}\right)^{1+2k}}{1+2k} - \frac{1}{20} \left( \sum_{k=0}^{\infty} \frac{\left(-\frac{1}{5}\right)^k 4^{1+2k} F_{1+2k} \left(\frac{1}{1+\sqrt{\frac{21}{5}}}\right)^{1+2k}}{1+2k} \right)^5 + \right.$$

$$\left( \left(\frac{1}{z_0}\right)^{-1/2 \lfloor \arg(5-z_0)/(2\pi) \rfloor} z_0^{1/2(-1-\lfloor \arg(5-z_0)/(2\pi) \rfloor)} \sum_{k=0}^{\infty} \frac{(-1)^k 5^{-1-3k} \times 12^{1+2k} F_{1+2k} \left(-\frac{\sqrt{5}}{1+\sqrt{1+\frac{144\sqrt{5}^2}{125}}}\right)^{1+2k}}{1+2k} \right) \Big/$$

$$\left( 4 \sum_{k=0}^{\infty} \frac{(-1)^k \left(-\frac{1}{2}\right)_k (5-z_0)^k z_0^{-k}}{k!} \right) +$$

$$\frac{1}{40} \left(\frac{1}{z_0}\right)^{1/2 \left\lfloor \arg\left(10-2\sqrt{5}-z_0\right)/(2\pi) \right\rfloor} z_0^{1/2 \left(1+\left\lfloor \arg\left(10-2\sqrt{5}-z_0\right)/(2\pi) \right\rfloor\right)}$$

$$\left( \log\left(-1 + \frac{5+\sqrt{10-2\sqrt{5}}}{5-\sqrt{10-2\sqrt{5}}}\right) - \sum_{k=1}^{\infty} \frac{\left(-\frac{1}{2}\right)^k \left(-\frac{\sqrt{10-2\sqrt{5}}}{-5+\sqrt{10-2\sqrt{5}}}\right)^{-k}}{k} \right)$$

$$\sum_{k=0}^{\infty} \frac{(-1)^k \left(-\frac{1}{2}\right)_k \left(10-2\sqrt{5}-z_0\right)^k z_0^{-k}}{k!}$$

$$42 \Big/ \left( \frac{1}{4} \tan^{-1}(2) - \frac{1}{20} \tan^{-1}(2)^5 + \frac{\tan^{-1}\left( \frac{(2-2^3)\sqrt{5}}{1-3\times 2^2+2^4} \right)}{4\sqrt{5}} + \frac{1}{40} \sqrt{10-2\sqrt{5}} \log\left( \frac{1+1\sqrt{10-2\sqrt{5}}+4}{1-1\sqrt{10-2\sqrt{5}}+4} \right) + 0.189872557940000 \right) -$$

$$11 = -11 + 42 \Big/ \left( 0.189872557940000 + \frac{1}{4} \left( \tan^{-1}(x) + \pi \left\lfloor \frac{\arg(i(2-x))}{2\pi} \right\rfloor + \frac{1}{2} i \sum_{k=1}^{\infty} \frac{\left(-(-i-x)^{-k} + (i-x)^{-k}\right)(2-x)^k}{k} \right) - \frac{1}{20} \left( \tan^{-1}(x) + \pi \left\lfloor \frac{\arg(i(2-x))}{2\pi} \right\rfloor + \frac{1}{2} i \sum_{k=1}^{\infty} \frac{\left(-(-i-x)^{-k} + (i-x)^{-k}\right)(2-x)^k}{k} \right)^5 + \frac{\tan^{-1}(x) + \pi \left\lfloor \frac{\arg\left(i\left(-x-\frac{6\sqrt{5}}{5}\right)\right)}{2\pi} \right\rfloor + \frac{1}{2} i \sum_{k=1}^{\infty} \frac{\left(-(-i-x)^{-k}+(i-x)^{-k}\right)\left(-x-\frac{6\sqrt{5}}{5}\right)^k}{k}}{4 \exp\left(i\pi \left\lfloor \frac{\arg(5-x)}{2\pi} \right\rfloor\right) \sqrt{x} \sum_{k=0}^{\infty} \frac{(-1)^k (5-x)^k x^{-k} \left(-\frac{1}{2}\right)_k}{k!}} + \frac{1}{40} \exp\left(i\pi \left\lfloor \frac{\arg(10-x-2\sqrt{5})}{2\pi} \right\rfloor\right) \sqrt{x} \left( \sum_{k=0}^{\infty} \frac{(-1)^k x^{-k} \left(-\frac{1}{2}\right)_k (10-x-2\sqrt{5})^k}{k!} \right) \left( \log\left( -1 + \frac{5+\sqrt{10-2\sqrt{5}}}{5-\sqrt{10-2\sqrt{5}}} \right) - \sum_{k=1}^{\infty} \frac{\left(-\frac{1}{2}\right)^k \left( -\frac{\sqrt{10-2\sqrt{5}}}{-5+\sqrt{10-2\sqrt{5}}} \right)^{-k}}{k} \right) \right)$$

for ($i x \in \mathbb{R}$ and $i x < -1$ and $x \in \mathbb{R}$ and $x < 0$)

$A_6 = \frac{1}{2}\tan^{-1}x + \frac{1}{6}\tan^{-1}x^3 + \frac{1}{4\sqrt{3}}\log\frac{1+x\sqrt{3}+x^2}{1-x\sqrt{3}+x^2}$

1/2 tan^-1 (2) + 1/6 tan^-1 (8) + 1/(4sqrt3) ln (((1+2sqrt3+4)/(1-2sqrt3+4)))

**Input:**

$$\frac{1}{2}\tan^{-1}(2) + \frac{1}{6}\tan^{-1}(8) + \frac{1}{4\sqrt{3}}\log\left(\frac{1+2\sqrt{3}+4}{1-2\sqrt{3}+4}\right)$$

$\tan^{-1}(x)$ is the inverse tangent function

$\log(x)$ is the natural logarithm

**Exact Result:**

$$\frac{\log\left(\frac{5+2\sqrt{3}}{5-2\sqrt{3}}\right)}{4\sqrt{3}} + \frac{1}{2}\tan^{-1}(2) + \frac{1}{6}\tan^{-1}(8)$$

(result in radians)

**Decimal approximation:**

1.040991496732833639573748611915498201204183344336196931089...

(result in radians)

1.040991496…

**Alternate forms:**

$$\frac{1}{12}\left(\sqrt{3}\log\left(\frac{1}{13}\left(37+20\sqrt{3}\right)\right) + 6\tan^{-1}(2) + 2\tan^{-1}(8)\right)$$

$$\frac{\log\left(\frac{1}{13}\left(37+20\sqrt{3}\right)\right)}{4\sqrt{3}} + \frac{1}{2}\tan^{-1}(2) + \frac{1}{6}\tan^{-1}(8)$$

$$\frac{1}{12}\left(\sqrt{3}\log\left(\frac{5+2\sqrt{3}}{5-2\sqrt{3}}\right) + 6\tan^{-1}(2) + 2\tan^{-1}(8)\right)$$

**Alternative representations:**

$$\frac{1}{2}\tan^{-1}(2) + \frac{1}{6}\tan^{-1}(8) + \frac{\log\left(\frac{1+2\sqrt{3}+4}{1-2\sqrt{3}+4}\right)}{4\sqrt{3}} = \frac{1}{2}\tan^{-1}(2) + \frac{1}{6}\tan^{-1}(8) + \frac{\log_e\left(\frac{5+2\sqrt{3}}{5-2\sqrt{3}}\right)}{4\sqrt{3}}$$

$$\frac{1}{2}\tan^{-1}(2) + \frac{1}{6}\tan^{-1}(8) + \frac{\log\left(\frac{1+2\sqrt{3}+4}{1-2\sqrt{3}+4}\right)}{4\sqrt{3}} = \frac{1}{2}\tan^{-1}(1, 2) + \frac{1}{6}\tan^{-1}(1, 8) + \frac{\log\left(\frac{5+2\sqrt{3}}{5-2\sqrt{3}}\right)}{4\sqrt{3}}$$

$$\frac{1}{2}\tan^{-1}(2)+\frac{1}{6}\tan^{-1}(8)+\frac{\log\left(\frac{1+2\sqrt{3}+4}{1-2\sqrt{3}+4}\right)}{4\sqrt{3}}=$$

$$\frac{1}{2}\tan^{-1}(2)+\frac{1}{6}\tan^{-1}(8)+\frac{\log(a)\log_a\left(\frac{5+2\sqrt{3}}{5-2\sqrt{3}}\right)}{4\sqrt{3}}$$

**Series representations:**

$$\frac{1}{2}\tan^{-1}(2)+\frac{1}{6}\tan^{-1}(8)+\frac{\log\left(\frac{1+2\sqrt{3}+4}{1-2\sqrt{3}+4}\right)}{4\sqrt{3}}=$$

$$\frac{1}{2}\tan^{-1}(2)+\frac{1}{6}\tan^{-1}(8)+\frac{\log\left(\frac{4}{13}\left(6+5\sqrt{3}\right)\right)}{4\sqrt{3}}-\frac{\sum_{k=1}^{\infty}\frac{\left(\frac{1}{12}\left(6-5\sqrt{3}\right)\right)^k}{k}}{4\sqrt{3}}$$

$$\frac{1}{2}\tan^{-1}(2)+\frac{1}{6}\tan^{-1}(8)+\frac{\log\left(\frac{1+2\sqrt{3}+4}{1-2\sqrt{3}+4}\right)}{4\sqrt{3}}=$$

$$\frac{1}{2}\tan^{-1}(2)+\frac{1}{6}\tan^{-1}(8)+\frac{\log\left(-1+\frac{5+2\sqrt{3}}{5-2\sqrt{3}}\right)}{4\sqrt{3}}-\frac{\sum_{k=1}^{\infty}\frac{\left(\frac{1}{12}\left(6-5\sqrt{3}\right)\right)^k}{k}}{4\sqrt{3}}$$

$$\frac{1}{2}\tan^{-1}(2)+\frac{1}{6}\tan^{-1}(8)+\frac{\log\left(\frac{1+2\sqrt{3}+4}{1-2\sqrt{3}+4}\right)}{4\sqrt{3}}=$$

$$\frac{2}{3}\tan^{-1}(z_0)+\frac{\log\left(-1+\frac{5+2\sqrt{3}}{5-2\sqrt{3}}\right)}{4\sqrt{3}}+\sum_{k=1}^{\infty}\left(\frac{(-1)^{-1+k}\left(-1+\frac{5+2\sqrt{3}}{5-2\sqrt{3}}\right)^{-k}}{4\sqrt{3}\,k}+\frac{i\left(-(-i-z_0)^{-k}+(i-z_0)^{-k}\right)(2-z_0)^k}{4k}+\frac{i\left(-(-i-z_0)^{-k}+(i-z_0)^{-k}\right)(8-z_0)^k}{12k}\right)$$

for $(i z_0 \notin \mathbb{R}$ or ( not $(1 \le i z_0 < \infty)$ and not $(-\infty < i z_0 \le -1)))$

**Integral representations:**

$$\frac{1}{2}\tan^{-1}(2)+\frac{1}{6}\tan^{-1}(8)+\frac{\log\left(\frac{1+2\sqrt{3}+4}{1-2\sqrt{3}+4}\right)}{4\sqrt{3}}=$$

$$\int_0^1\left(\frac{1}{1+4t^2}+\frac{4}{3+192t^2}\right)dt+\frac{\log\left(\frac{1}{13}\left(37+20\sqrt{3}\right)\right)}{4\sqrt{3}}$$

$$\frac{1}{2}\tan^{-1}(2)+\frac{1}{6}\tan^{-1}(8)+\frac{\log\left(\frac{1+2\sqrt{3}+4}{1-2\sqrt{3}+4}\right)}{4\sqrt{3}}=$$

$$\int_1^{\frac{1}{13}\left(37+20\sqrt{3}\right)}\left(\frac{\frac{1}{1+\frac{4(1-t)^2}{\left(1+\frac{1}{13}\left(-37-20\sqrt{3}\right)\right)^2}}+\frac{4}{3\left(1+\frac{64(1-t)^2}{\left(1+\frac{1}{13}\left(-37-20\sqrt{3}\right)\right)^2}\right)}}{-1+\frac{1}{13}\left(37+20\sqrt{3}\right)}+\frac{1}{4\sqrt{3}\,t}\right)dt$$

$$\frac{1}{2}\tan^{-1}(2)+\frac{1}{6}\tan^{-1}(8)+\frac{\log\left(\frac{1+2\sqrt{3}+4}{1-2\sqrt{3}+4}\right)}{4\sqrt{3}}=$$

$$\int_{-i\infty+\gamma}^{i\infty+\gamma}-\frac{i\,65^{-s}\left(4+3\times 13^s\right)\Gamma\left(\frac{1}{2}-s\right)\Gamma(1-s)\,\Gamma(s)^2}{12\,\pi^{3/2}}\,ds+\frac{\log\left(\frac{1}{13}\left(37+20\sqrt{3}\right)\right)}{4\sqrt{3}}\quad\text{for}\quad 0<\gamma<\frac{1}{2}$$

## Continued fraction representations:

$$\frac{1}{2}\tan^{-1}(2)+\frac{1}{6}\tan^{-1}(8)+\frac{\log\left(\frac{1+2\sqrt{3}+4}{1-2\sqrt{3}+4}\right)}{4\sqrt{3}}=$$

$$\frac{\log\left(\frac{1}{13}\left(37+20\sqrt{3}\right)\right)}{4\sqrt{3}}+\frac{1}{1+\underset{k=1}{\overset{\infty}{\mathrm{K}}}\frac{4k^2}{1+2k}}+\frac{4}{3\left(1+\underset{k=1}{\overset{\infty}{\mathrm{K}}}\frac{64k^2}{1+2k}\right)}=$$

$$\frac{\log\left(\frac{1}{13}\left(37+20\sqrt{3}\right)\right)}{4\sqrt{3}}+\frac{1}{1+\frac{4}{3+\frac{16}{5+\frac{36}{7+\frac{64}{9+\dots}}}}}+\frac{4}{3\left(1+\frac{64}{3+\frac{256}{5+\frac{576}{7+\frac{1024}{9+\dots}}}}\right)}$$

$$\frac{1}{2}\tan^{-1}(2)+\frac{1}{6}\tan^{-1}(8)+\frac{\log\left(\frac{1+2\sqrt{3}+4}{1-2\sqrt{3}+4}\right)}{4\sqrt{3}}=$$

$$\frac{\log\left(\frac{5+2\sqrt{3}}{5-2\sqrt{3}}\right)}{4\sqrt{3}}+\frac{1}{1+\underset{k=1}{\overset{\infty}{\mathrm{K}}}\frac{4k^2}{1+2k}}+\frac{4}{3\left(1+\underset{k=1}{\overset{\infty}{\mathrm{K}}}\frac{64k^2}{1+2k}\right)}=$$

$$\frac{\log\left(\frac{5+2\sqrt{3}}{5-2\sqrt{3}}\right)}{4\sqrt{3}}+\frac{1}{1+\frac{4}{3+\frac{16}{5+\frac{36}{7+\frac{64}{9+\dots}}}}}+\frac{4}{3\left(1+\frac{64}{3+\frac{256}{5+\frac{576}{7+\frac{1024}{9+\dots}}}}\right)}$$

$$\frac{1}{2}\tan^{-1}(2)+\frac{1}{6}\tan^{-1}(8)+\frac{\log\left(\frac{1+2\sqrt{3}+4}{1-2\sqrt{3}+4}\right)}{4\sqrt{3}}=$$

$$\frac{\log\left(\frac{1}{13}\left(37+20\sqrt{3}\right)\right)}{4\sqrt{3}}+\frac{1}{1+\underset{k=1}{\overset{\infty}{\mathrm{K}}}\frac{4(1-2k)^2}{5-6k}}+\frac{4}{3\left(1+\underset{k=1}{\overset{\infty}{\mathrm{K}}}\frac{64(1-2k)^2}{65-126k}\right)}=$$

$$\frac{\log\left(\frac{1}{13}\left(37+20\sqrt{3}\right)\right)}{4\sqrt{3}}+\frac{1}{1+\frac{4}{-1+\frac{36}{-7+\frac{100}{-13+\frac{196}{-19+\dots}}}}}+$$

$$\frac{4}{3\left(1+\frac{64}{-61+\frac{576}{-187+\frac{1600}{-313+\frac{3136}{-439+\dots}}}}\right)}$$

((((1/2 tan^-1 (2) + 1/6 tan^-1 (8) + 1/(4sqrt3) ln (((1+2sqrt3+4)/(1-2sqrt3+4)))))))^12

**Input:**

$$\left(\frac{1}{2}\tan^{-1}(2)+\frac{1}{6}\tan^{-1}(8)+\frac{1}{4\sqrt{3}}\log\left(\frac{1+2\sqrt{3}+4}{1-2\sqrt{3}+4}\right)\right)^{12}$$

$\tan^{-1}(x)$ is the inverse tangent function

$\log(x)$ is the natural logarithm

**Exact Result:**

$$\left(\frac{\log\left(\frac{5+2\sqrt{3}}{5-2\sqrt{3}}\right)}{4\sqrt{3}} + \frac{1}{2}\tan^{-1}(2) + \frac{1}{6}\tan^{-1}(8)\right)^{12}$$

(result in radians)

**Decimal approximation:**

1.619444930152370038737329829009437718851016351898044916404...

(result in radians)

1.619444930152… result that is a good approximation to the value of the golden ratio 1,618033988749...

**Alternate forms:**

$$\left(\frac{\log\left(\frac{1}{13}\left(37+20\sqrt{3}\right)\right)}{4\sqrt{3}} + \frac{1}{2}\tan^{-1}(2) + \frac{1}{6}\tan^{-1}(8)\right)^{12}$$

$$\frac{\left(\sqrt{3}\log\left(\frac{5+2\sqrt{3}}{5-2\sqrt{3}}\right) + 6\tan^{-1}(2) + 2\tan^{-1}(8)\right)^{12}}{8\,916\,100\,448\,256}$$

$$\frac{\left(3\log\left(-\frac{5+2\sqrt{3}}{2\sqrt{3}-5}\right) + 2\sqrt{3}\left(3\tan^{-1}(2) + \tan^{-1}(8)\right)\right)^{12}}{6\,499\,837\,226\,778\,624}$$

**Alternative representations:**

$$\left(\frac{1}{2}\tan^{-1}(2) + \frac{1}{6}\tan^{-1}(8) + \frac{\log\left(\frac{1+2\sqrt{3}+4}{1-2\sqrt{3}+4}\right)}{4\sqrt{3}}\right)^{12} = \left(\frac{1}{2}\tan^{-1}(2) + \frac{1}{6}\tan^{-1}(8) + \frac{\log_e\left(\frac{5+2\sqrt{3}}{5-2\sqrt{3}}\right)}{4\sqrt{3}}\right)^{12}$$

$$\left(\frac{1}{2}\tan^{-1}(2)+\frac{1}{6}\tan^{-1}(8)+\frac{\log\left(\frac{1+2\sqrt{3}+4}{1-2\sqrt{3}+4}\right)}{4\sqrt{3}}\right)^{12} = \left(\frac{1}{2}\tan^{-1}(1,2)+\frac{1}{6}\tan^{-1}(1,8)+\frac{\log\left(\frac{5+2\sqrt{3}}{5-2\sqrt{3}}\right)}{4\sqrt{3}}\right)^{12}$$

$$\left(\frac{1}{2}\tan^{-1}(2)+\frac{1}{6}\tan^{-1}(8)+\frac{\log\left(\frac{1+2\sqrt{3}+4}{1-2\sqrt{3}+4}\right)}{4\sqrt{3}}\right)^{12} = \left(\frac{1}{2}\tan^{-1}(2)+\frac{1}{6}\tan^{-1}(8)+\frac{\log(a)\log_a\left(\frac{5+2\sqrt{3}}{5-2\sqrt{3}}\right)}{4\sqrt{3}}\right)^{12}$$

**Series representations:**

$$\left(\frac{1}{2}\tan^{-1}(2)+\frac{1}{6}\tan^{-1}(8)+\frac{\log\left(\frac{1+2\sqrt{3}+4}{1-2\sqrt{3}+4}\right)}{4\sqrt{3}}\right)^{12} = \left(\frac{1}{2}\tan^{-1}(2)+\frac{1}{6}\tan^{-1}(8)+\frac{\log\left(-1+\frac{5+2\sqrt{3}}{5-2\sqrt{3}}\right)-\sum_{k=1}^{\infty}\frac{\left(\frac{1}{12}\left(6-5\sqrt{3}\right)\right)^k}{k}}{4\sqrt{3}}\right)^{12}$$

$$\left(\frac{1}{2}\tan^{-1}(2)+\frac{1}{6}\tan^{-1}(8)+\frac{\log\left(\frac{1+2\sqrt{3}+4}{1-2\sqrt{3}+4}\right)}{4\sqrt{3}}\right)^{12} = \frac{1}{8916100448256}\left(8\tan^{-1}(z_0)+\sqrt{3}\log\left(-1+\frac{5+2\sqrt{3}}{5-2\sqrt{3}}\right)-\sqrt{3}\sum_{k=1}^{\infty}\frac{\left(\frac{1}{12}\left(6-5\sqrt{3}\right)\right)^k}{k}+3i\sum_{k=1}^{\infty}\frac{\left(-(-i-z_0)^{-k}+(i-z_0)^{-k}\right)(2-z_0)^k}{k}+i\sum_{k=1}^{\infty}\frac{\left(-(-i-z_0)^{-k}+(i-z_0)^{-k}\right)(8-z_0)^k}{k}\right)^{12}$$

for $(i z_0 \notin \mathbb{R}$ or ( not $(1 \leq i z_0 < \infty)$ and not $(-\infty < i z_0 \leq -1)))$

## Continued fraction representations:

$$\left(\frac{1}{2}\tan^{-1}(2)+\frac{1}{6}\tan^{-1}(8)+\frac{\log\left(\frac{1+2\sqrt{3}+4}{1-2\sqrt{3}+4}\right)}{4\sqrt{3}}\right)^{12} = \left(\frac{\log\left(\frac{1}{13}\left(37+20\sqrt{3}\right)\right)}{4\sqrt{3}}+\frac{1}{1+\underset{k=1}{\overset{\infty}{\mathrm{K}}}\frac{4k^2}{1+2k}}+\frac{4}{3\left(1+\underset{k=1}{\overset{\infty}{\mathrm{K}}}\frac{64k^2}{1+2k}\right)}\right)^{12} = \left(\frac{\log\left(\frac{1}{13}\left(37+20\sqrt{3}\right)\right)}{4\sqrt{3}}+\frac{1}{1+\frac{4}{3+\frac{16}{5+\frac{36}{7+\frac{64}{9+\dots}}}}}+\frac{4}{3\left(1+\frac{64}{3+\frac{256}{5+\frac{576}{7+\frac{1024}{9+\dots}}}}\right)}\right)^{12}$$

$$\left(\frac{1}{2}\tan^{-1}(2)+\frac{1}{6}\tan^{-1}(8)+\frac{\log\left(\frac{1+2\sqrt{3}+4}{1-2\sqrt{3}+4}\right)}{4\sqrt{3}}\right)^{12} = \left(\frac{\log\left(\frac{5+2\sqrt{3}}{5-2\sqrt{3}}\right)}{4\sqrt{3}}+\frac{1}{1+\underset{k=1}{\overset{\infty}{\mathrm{K}}}\frac{4k^2}{1+2k}}+\frac{4}{3\left(1+\underset{k=1}{\overset{\infty}{\mathrm{K}}}\frac{64k^2}{1+2k}\right)}\right)^{12} = \left(\frac{\log\left(\frac{5+2\sqrt{3}}{5-2\sqrt{3}}\right)}{4\sqrt{3}}+\frac{1}{1+\frac{4}{3+\frac{16}{5+\frac{36}{7+\frac{64}{9+\dots}}}}}+\frac{4}{3\left(1+\frac{64}{3+\frac{256}{5+\frac{576}{7+\frac{1024}{9+\dots}}}}\right)}\right)^{12}$$

$$\left(\frac{1}{2}\tan^{-1}(2)+\frac{1}{6}\tan^{-1}(8)+\frac{\log\left(\frac{1+2\sqrt{3}+4}{1-2\sqrt{3}+4}\right)}{4\sqrt{3}}\right)^{12}=$$

$$\left(\frac{\log\left(\frac{1}{13}\left(37+20\sqrt{3}\right)\right)}{4\sqrt{3}}+\frac{1}{1+\underset{k=1}{\overset{\infty}{\mathrm{K}}}\frac{4(1-2k)^2}{5-6k}}+\frac{4}{3\left(1+\underset{k=1}{\overset{\infty}{\mathrm{K}}}\frac{64(1-2k)^2}{65-126k}\right)}\right)^{12}=$$

$$\left(\frac{\log\left(\frac{1}{13}\left(37+20\sqrt{3}\right)\right)}{4\sqrt{3}}+\frac{1}{1+\frac{4}{-1+\frac{36}{-7+\frac{100}{-13+\frac{196}{-19+\dots}}}}}+\frac{4}{3\left(1+\frac{64}{-61+\frac{576}{-187+\frac{1600}{-313+\frac{3136}{-439+\dots}}}}\right)}\right)^{12}$$

1/10^27(((((((1/2 tan^-1 (2) + 1/6 tan^-1 (8) + 1/(4sqrt3) ln (((1+2sqrt3+4)/(1-2sqrt3+4)))))))^12 +(55-2)*1/10^3)))

**Input:**

$$\frac{1}{10^{27}}\left(\left(\frac{1}{2}\tan^{-1}(2)+\frac{1}{6}\tan^{-1}(8)+\frac{1}{4\sqrt{3}}\log\left(\frac{1+2\sqrt{3}+4}{1-2\sqrt{3}+4}\right)\right)^{12}+(55-2)\times\frac{1}{10^3}\right)$$

$\tan^{-1}(x)$ is the inverse tangent function

$\log(x)$ is the natural logarithm

**Exact Result:**

$$\frac{\frac{53}{1000}+\left(\frac{\log\left(\frac{5+2\sqrt{3}}{5-2\sqrt{3}}\right)}{4\sqrt{3}}+\frac{1}{2}\tan^{-1}(2)+\frac{1}{6}\tan^{-1}(8)\right)^{12}}{1\,000\,000\,000\,000\,000\,000\,000\,000\,000\,000}$$

(result in radians)

**Decimal approximation:**

$$1.672444930152370038737329829009437718851016351898044 9\ldots\times 10^{-27}$$

(result in radians)

$1.6724449301523\ldots*10^{-27}$ result practically equal to the proton mass

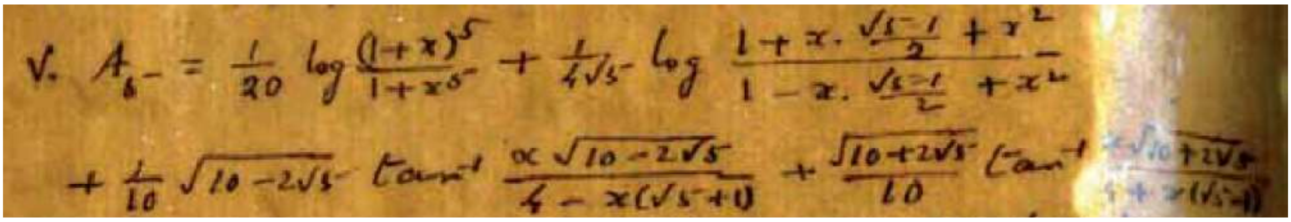
**Alternate forms:**

$$\frac{\frac{53}{1000}+\left(\frac{\log\left(\frac{1}{13}\left(37+20\sqrt{3}\right)\right)}{4\sqrt{3}}+\frac{1}{2}\tan^{-1}(2)+\frac{1}{6}\tan^{-1}(8)\right)^{12}}{1\,000\,000\,000\,000\,000\,000\,000\,000\,000\,000}$$

$$\frac{\frac{53}{1000}+\left(\frac{\pi}{3}+\frac{\log\left(5+2\sqrt{3}\right)-\log\left(5-2\sqrt{3}\right)}{4\sqrt{3}}+\frac{1}{12}\left(\tan^{-1}\left(\frac{36}{323}\right)-\pi\right)\right)^{12}}{1\,000\,000\,000\,000\,000\,000\,000\,000\,000\,000}$$

$$\frac{\frac{53}{1000}+\left(\frac{1}{4}i\left(\log(1-2i)-\log(1+2i)\right)+\frac{1}{12}i\left(\log(1-8i)-\log(1+8i)\right)+\frac{\log\left(\frac{5+2\sqrt{3}}{5-2\sqrt{3}}\right)}{4\sqrt{3}}\right)^{12}}{1\,000\,000\,000\,000\,000\,000\,000\,000\,000\,000}$$

We have that:



$$V.\ A_5 = \frac{1}{20}\log\frac{(1+x)^5}{1+x^5}+\frac{1}{4\sqrt{5}}\log\frac{1+x.\frac{\sqrt{5}-1}{2}+x^2}{1-x.\frac{\sqrt{5}-1}{2}+x^2}+\frac{1}{10}\sqrt{10-2\sqrt{5}}\tan^{-1}\frac{x\sqrt{10-2\sqrt{5}}}{4-x(\sqrt{5}+1)}+\frac{\sqrt{10+2\sqrt{5}}}{10}\tan^{-1}\frac{x\sqrt{10+2\sqrt{5}}}{4+x(\sqrt{5}-1)}$$

1/20 ln((((1+2)^5)/(1+2^5))) + 1/(4sqrt5) ln ((((((1+2*((sqrt5-1)/2)+4))) / (((1-2*((sqrt5-1)/2)+4)))))) + 1/20 (10-2sqrt5)^(1/2) tan^-1 (((((2*(10-2sqrt5)^(1/2)))/((4-2(sqrt5+1))))))

**Input:**

$$\frac{1}{20}\log\left(\frac{(1+2)^5}{1+2^5}\right)+\frac{1}{4\sqrt{5}}\log\left(\frac{1+2\left(\frac{1}{2}\left(\sqrt{5}-1\right)\right)+4}{1-2\left(\frac{1}{2}\left(\sqrt{5}-1\right)\right)+4}\right)+\frac{1}{20}\sqrt{10-2\sqrt{5}}\tan^{-1}\left(\frac{2\sqrt{10-2\sqrt{5}}}{4-2\left(\sqrt{5}+1\right)}\right)$$

log(x) is the natural logarithm

$\tan^{-1}(x)$ is the inverse tangent function

**Exact Result:**

$$\frac{1}{20}\log\left(\frac{81}{11}\right)+\frac{\log\left(\frac{4+\sqrt{5}}{6-\sqrt{5}}\right)}{4\sqrt{5}}+\frac{1}{20}\sqrt{10-2\sqrt{5}}\tan^{-1}\left(\frac{2\sqrt{10-2\sqrt{5}}}{4-2\left(1+\sqrt{5}\right)}\right)$$

(result in radians)

**Decimal approximation:**

0.028517407231721521731978720428288813074858647677244607539...

(result in radians)

0.0285174072…

**Alternate forms:**

$$\frac{1}{20}\log\left(\frac{81}{11}\right)+\frac{\log\left(\frac{1}{31}\left(29+10\sqrt{5}\right)\right)}{4\sqrt{5}}-\frac{1}{10}\sqrt{\frac{1}{2}\left(5-\sqrt{5}\right)}\tan^{-1}\left(\sqrt{\frac{1}{2}\left(5+\sqrt{5}\right)}\right)$$

$$\frac{1}{20}\left(\log\left(\frac{81}{11}\right)+\sqrt{5}\log\left(\frac{4+\sqrt{5}}{6-\sqrt{5}}\right)+\sqrt{2\left(5-\sqrt{5}\right)}\tan^{-1}\left(\frac{2\sqrt{10-2\sqrt{5}}}{4-2\left(1+\sqrt{5}\right)}\right)\right)$$

$$\frac{1}{20}\log\left(\frac{81}{11}\right)+\frac{\log\left(\frac{4+\sqrt{5}}{6-\sqrt{5}}\right)}{4\sqrt{5}}+\frac{1}{40}i\sqrt{10-2\sqrt{5}}\log\left(1-\frac{2i\sqrt{10-2\sqrt{5}}}{4-2\left(1+\sqrt{5}\right)}\right)-\frac{1}{40}i\sqrt{10-2\sqrt{5}}\log\left(1+\frac{2i\sqrt{10-2\sqrt{5}}}{4-2\left(1+\sqrt{5}\right)}\right)$$

## Alternative representations:

$$\frac{1}{20}\log\left(\frac{(1+2)^5}{1+2^5}\right)+\frac{\log\left(\frac{1+\frac{2}{2}\left(\sqrt{5}-1\right)+4}{1-\frac{2}{2}\left(\sqrt{5}-1\right)+4}\right)}{4\sqrt{5}}+\frac{1}{20}\sqrt{10-2\sqrt{5}}\tan^{-1}\left(\frac{2\sqrt{10-2\sqrt{5}}}{4-2\left(\sqrt{5}+1\right)}\right)=$$

$$\frac{1}{20}\log\left(\frac{3^5}{1+2^5}\right)+\frac{1}{20}\tan^{-1}\left(1,\ \frac{2\sqrt{10-2\sqrt{5}}}{4-2\left(1+\sqrt{5}\right)}\right)\sqrt{10-2\sqrt{5}}+\frac{\log\left(\frac{4+\sqrt{5}}{6-\sqrt{5}}\right)}{4\sqrt{5}}$$

$$\frac{1}{20}\log\left(\frac{(1+2)^5}{1+2^5}\right)+\frac{\log\left(\frac{1+\frac{2}{2}\left(\sqrt{5}-1\right)+4}{1-\frac{2}{2}\left(\sqrt{5}-1\right)+4}\right)}{4\sqrt{5}}+\frac{1}{20}\sqrt{10-2\sqrt{5}}\tan^{-1}\left(\frac{2\sqrt{10-2\sqrt{5}}}{4-2\left(\sqrt{5}+1\right)}\right)=$$

$$\frac{1}{20}\log(a)\log_a\left(\frac{3^5}{1+2^5}\right)+$$

$$\frac{1}{20}\tan^{-1}\left(\frac{2\sqrt{10-2\sqrt{5}}}{4-2\left(1+\sqrt{5}\right)}\right)\sqrt{10-2\sqrt{5}}+\frac{\log(a)\log_a\left(\frac{4+\sqrt{5}}{6-\sqrt{5}}\right)}{4\sqrt{5}}$$

$$\frac{1}{20}\log\left(\frac{(1+2)^5}{1+2^5}\right)+\frac{\log\left(\frac{1+\frac{2}{2}\left(\sqrt{5}-1\right)+4}{1-\frac{2}{2}\left(\sqrt{5}-1\right)+4}\right)}{4\sqrt{5}}+\frac{1}{20}\sqrt{10-2\sqrt{5}}\tan^{-1}\left(\frac{2\sqrt{10-2\sqrt{5}}}{4-2\left(\sqrt{5}+1\right)}\right)=$$

$$\frac{1}{20}\log_e\left(\frac{3^5}{1+2^5}\right)+\frac{1}{20}\tan^{-1}\left(\frac{2\sqrt{10-2\sqrt{5}}}{4-2\left(1+\sqrt{5}\right)}\right)\sqrt{10-2\sqrt{5}}+\frac{\log_e\left(\frac{4+\sqrt{5}}{6-\sqrt{5}}\right)}{4\sqrt{5}}$$

## Integral representations:

$$\frac{1}{20}\log\left(\frac{(1+2)^5}{1+2^5}\right)+\frac{\log\left(\frac{1+\frac{2}{2}\left(\sqrt{5}-1\right)+4}{1-\frac{2}{2}\left(\sqrt{5}-1\right)+4}\right)}{4\sqrt{5}}+\frac{1}{20}\sqrt{10-2\sqrt{5}}\tan^{-1}\left(\frac{2\sqrt{10-2\sqrt{5}}}{4-2\left(\sqrt{5}+1\right)}\right)=$$

$$\int_0^1 \frac{-5+3\sqrt{5}}{10\left(-3+\sqrt{5}+\left(-5+\sqrt{5}\right)t^2\right)}\,dt+\frac{1}{20}\log\left(\frac{81}{11}\right)+\frac{\log\left(\frac{4+\sqrt{5}}{6-\sqrt{5}}\right)}{4\sqrt{5}}$$

$$\frac{1}{20}\log\left(\frac{(1+2)^5}{1+2^5}\right)+\frac{\log\left(\frac{1+\frac{2}{2}\left(\sqrt{5}-1\right)+4}{1-\frac{2}{2}\left(\sqrt{5}-1\right)+4}\right)}{4\sqrt{5}}+\frac{1}{20}\sqrt{10-2\sqrt{5}}\tan^{-1}\left(\frac{2\sqrt{10-2\sqrt{5}}}{4-2\left(\sqrt{5}+1\right)}\right)=$$

$$\int_1^{\frac{81}{11}}\left(\frac{11}{70}\left(\frac{1}{\left(4-2\left(1+\sqrt{5}\right)\right)\left(1+\frac{121\left(10-2\sqrt{5}\right)(1-t)^2}{1225\left(4-2\left(1+\sqrt{5}\right)\right)^2}\right)}-\frac{1}{\sqrt{5}\left(4-2\left(1+\sqrt{5}\right)\right)\left(1+\frac{121\left(10-2\sqrt{5}\right)(1-t)^2}{1225\left(4-2\left(1+\sqrt{5}\right)\right)^2}\right)}\right)+\frac{1}{20\,t}-\frac{-1+\frac{4+\sqrt{5}}{6-\sqrt{5}}}{4\sqrt{5}\left(-\frac{81}{11}+\frac{4+\sqrt{5}}{6-\sqrt{5}}+t-\frac{\left(4+\sqrt{5}\right)t}{6-\sqrt{5}}\right)}\right)dt$$

$$\frac{1}{20}\log\left(\frac{(1+2)^5}{1+2^5}\right)+\frac{\log\left(\frac{1+\frac{2}{2}\left(\sqrt{5}-1\right)+4}{1-\frac{2}{2}\left(\sqrt{5}-1\right)+4}\right)}{4\sqrt{5}}+\frac{1}{20}\sqrt{10-2\sqrt{5}}\tan^{-1}\left(\frac{2\sqrt{10-2\sqrt{5}}}{4-2\left(\sqrt{5}+1\right)}\right)=$$

$$-\frac{i\left(10-2\sqrt{5}\right)}{40\left(4-2\left(1+\sqrt{5}\right)\right)\pi^{3/2}}\int_{-i\infty+\gamma}^{i\infty+\gamma}\left(1+\frac{4\left(10-2\sqrt{5}\right)}{\left(4-2\left(1+\sqrt{5}\right)\right)^2}\right)^{-s}\Gamma\left(\frac{1}{2}-s\right)\Gamma(1-s)\,\Gamma(s)^2\,ds+\frac{1}{20}\log\left(\frac{81}{11}\right)+\frac{\log\left(\frac{4+\sqrt{5}}{6-\sqrt{5}}\right)}{4\sqrt{5}}\quad\text{for } 0<\gamma<\frac{1}{2}$$

(((10+2sqrt5)^(1/2)))/20 tan^-1 ((((2*(10+2sqrt5)^(1/2))/((4+2(sqrt5-1))))))

**Input:**

$$\left(\frac{1}{20}\sqrt{10+2\sqrt{5}}\right)\tan^{-1}\left(\frac{2\sqrt{10+2\sqrt{5}}}{4+2\left(\sqrt{5}-1\right)}\right)$$

$\tan^{-1}(x)$ is the inverse tangent function

**Exact Result:**

$$\frac{1}{20}\sqrt{10+2\sqrt{5}}\tan^{-1}\left(\frac{2\sqrt{10+2\sqrt{5}}}{4+2\left(\sqrt{5}-1\right)}\right)$$

(result in radians)

**Decimal approximation:**

0.164708638338231507885004448413669921250834714283698623665...

(result in radians)

0.164708638…

**Alternate forms:**

$$\frac{1}{10}\sqrt{\frac{1}{2}\left(5+\sqrt{5}\right)}\cot^{-1}\left(\sqrt{\frac{1}{10}\left(5+\sqrt{5}\right)}\right)$$

$$\frac{1}{10}\sqrt{\frac{1}{2}\left(5+\sqrt{5}\right)}\tan^{-1}\left(\sqrt{\frac{1}{2}\left(5-\sqrt{5}\right)}\right)$$

$$\frac{\left(\sqrt{1-2i}+\sqrt{1+2i}\right)\tan^{-1}\left(\frac{\sqrt{2\left(5+\sqrt{5}\right)}}{1+\sqrt{5}}\right)}{4\times 5^{3/4}}$$

$\cot^{-1}(x)$ is the inverse cotangent function

**Alternative representations:**

$$\frac{1}{20}\tan^{-1}\left(\frac{2\sqrt{10+2\sqrt{5}}}{4+2\left(\sqrt{5}-1\right)}\right)\sqrt{10+2\sqrt{5}} = \frac{1}{20}\,\mathrm{sc}^{-1}\left(\frac{2\sqrt{10+2\sqrt{5}}}{4+2\left(-1+\sqrt{5}\right)}\,\middle|\,0\right)\sqrt{10+2\sqrt{5}}$$

$$\frac{1}{20}\tan^{-1}\left(\frac{2\sqrt{10+2\sqrt{5}}}{4+2\left(\sqrt{5}-1\right)}\right)\sqrt{10+2\sqrt{5}} = \frac{1}{20}\tan^{-1}\left(1,\frac{2\sqrt{10+2\sqrt{5}}}{4+2\left(-1+\sqrt{5}\right)}\right)\sqrt{10+2\sqrt{5}}$$

$$\frac{1}{20}\tan^{-1}\left(\frac{2\sqrt{10+2\sqrt{5}}}{4+2\left(\sqrt{5}-1\right)}\right)\sqrt{10+2\sqrt{5}} = \frac{1}{20}\,i\tanh^{-1}\left(-\frac{2i\sqrt{10+2\sqrt{5}}}{4+2\left(-1+\sqrt{5}\right)}\right)\sqrt{10+2\sqrt{5}}$$

## Series representations:

$$\frac{1}{20}\tan^{-1}\left(\frac{2\sqrt{10+2\sqrt{5}}}{4+2\left(\sqrt{5}-1\right)}\right)\sqrt{10+2\sqrt{5}} = \frac{1}{40}\sqrt{10+2\sqrt{5}}\,\pi - \frac{1}{20}\sqrt{10+2\sqrt{5}}\sum_{k=0}^{\infty}\frac{(-1)^k\,2^{-1-2k}\left(10+2\sqrt{5}\right)^{1/2(-1-2k)}\left(4+2\left(-1+\sqrt{5}\right)\right)^{1+2k}}{1+2k}$$

$$\frac{1}{20}\tan^{-1}\left(\frac{2\sqrt{10+2\sqrt{5}}}{4+2\left(\sqrt{5}-1\right)}\right)\sqrt{10+2\sqrt{5}} = -\frac{1}{20}\,i\sqrt{\frac{1}{2}\left(5+\sqrt{5}\right)}\left(\log(2)+\log\left(1+\sqrt{5}\right)-\log\left(1+\sqrt{5}-i\sqrt{2\left(5+\sqrt{5}\right)}\right)-\sum_{k=1}^{\infty}\frac{\left(\frac{1+\sqrt{5}-i\sqrt{2\left(5+\sqrt{5}\right)}}{2+2\sqrt{5}}\right)^k}{k}\right)$$

$$\frac{1}{20}\tan^{-1}\left(\frac{2\sqrt{10+2\sqrt{5}}}{4+2\left(\sqrt{5}-1\right)}\right)\sqrt{10+2\sqrt{5}} = -\frac{1}{40}\,i\sqrt{10+2\sqrt{5}}\,\log(2)+\frac{1}{40}\,i\sqrt{10+2\sqrt{5}}\,\log\left(-i\left(i+\frac{2\sqrt{10+2\sqrt{5}}}{4+2\left(-1+\sqrt{5}\right)}\right)\right)+\frac{1}{40}\,i\sqrt{10+2\sqrt{5}}\sum_{k=1}^{\infty}\frac{\left(\frac{1+\sqrt{5}-i\sqrt{2\left(5+\sqrt{5}\right)}}{2+2\sqrt{5}}\right)^k}{k}$$

## Integral representations:

$$\frac{1}{20}\tan^{-1}\left(\frac{2\sqrt{10+2\sqrt{5}}}{4+2\left(\sqrt{5}-1\right)}\right)\sqrt{10+2\sqrt{5}} = \frac{\left(3+\sqrt{5}\right)\left(5+\sqrt{5}\right)}{10\left(1+\sqrt{5}\right)}\int_0^1\frac{1}{3+\sqrt{5}+\left(5+\sqrt{5}\right)t^2}\,dt$$

$$\frac{1}{20}\tan^{-1}\left(\frac{2\sqrt{10+2\sqrt{5}}}{4+2\left(\sqrt{5}-1\right)}\right)\sqrt{10+2\sqrt{5}} = -\frac{i\left(10+2\sqrt{5}\right)}{40\left(4+2\left(-1+\sqrt{5}\right)\right)\pi^{3/2}}\int_{-i\infty+\gamma}^{i\infty+\gamma}\left(1+\frac{4\left(10+2\sqrt{5}\right)}{\left(4+2\left(-1+\sqrt{5}\right)\right)^2}\right)^{-s}\Gamma\left(\frac{1}{2}-s\right)\Gamma(1-s)\,\Gamma(s)^2\,ds \quad \text{for } 0<\gamma<\frac{1}{2}$$

$$\frac{1}{20}\tan^{-1}\left(\frac{2\sqrt{10+2\sqrt{5}}}{4+2\left(\sqrt{5}-1\right)}\right)\sqrt{10+2\sqrt{5}} = -\frac{i\left(10+2\sqrt{5}\right)}{40\left(4+2\left(-1+\sqrt{5}\right)\right)\pi} \int_{-i\,\infty+\gamma}^{i\,\infty+\gamma} \frac{\left(4\left(10+2\sqrt{5}\right)\right)^{-s}\left(4+2\left(-1+\sqrt{5}\right)\right)^{2s}\,\Gamma\left(\frac{1}{2}-s\right)\Gamma(1-s)\,\Gamma(s)}{\Gamma\left(\frac{3}{2}-s\right)}\,ds \quad \text{for } 0<\gamma<\frac{1}{2}$$

## Continued fraction representations:

$$\frac{1}{20}\tan^{-1}\left(\frac{2\sqrt{10+2\sqrt{5}}}{4+2\left(\sqrt{5}-1\right)}\right)\sqrt{10+2\sqrt{5}} = \frac{5+\sqrt{5}}{10\left(1+\sqrt{5}\right)\left(1+\underset{k=1}{\overset{\infty}{\mathrm{K}}}\frac{\frac{\left(5+\sqrt{5}\right)k^2}{3+\sqrt{5}}}{1+2k}\right)} =$$

$$\frac{5+\sqrt{5}}{10\left(1+\sqrt{5}\right)\left(1+\frac{5+\sqrt{5}}{\left(3+\sqrt{5}\right)\left(3+\frac{4\left(5+\sqrt{5}\right)}{\left(3+\sqrt{5}\right)\left(5+\frac{9\left(5+\sqrt{5}\right)}{\left(3+\sqrt{5}\right)\left(7+\frac{16\left(5+\sqrt{5}\right)}{\left(3+\sqrt{5}\right)(9+\ldots)}\right)}\right)}\right)}\right)}$$

$$\frac{1}{20}\tan^{-1}\left(\frac{2\sqrt{10+2\sqrt{5}}}{4+2\left(\sqrt{5}-1\right)}\right)\sqrt{10+2\sqrt{5}} =$$

$$\frac{30+14\sqrt{5}+4\left(5+2\sqrt{5}\right)\left(\underset{k=1}{\overset{\infty}{\mathrm{K}}}\frac{\frac{\left(5+\sqrt{5}\right)\left(1+(-1)^{1+k}+k\right)^2}{3+\sqrt{5}}}{3+2k}\right)}{5\left(1+\sqrt{5}\right)^3\left(3+\underset{k=1}{\overset{\infty}{\mathrm{K}}}\frac{\frac{\left(5+\sqrt{5}\right)\left(1+(-1)^{1+k}+k\right)^2}{3+\sqrt{5}}}{3+2k}\right)} =$$

$$\frac{30+14\sqrt{5}+4\left(5+2\sqrt{5}\right)\frac{9\left(5+\sqrt{5}\right)}{\left(3+\sqrt{5}\right)\left(5+\frac{4\left(5+\sqrt{5}\right)}{\left(3+\sqrt{5}\right)\left(7+\frac{25\left(5+\sqrt{5}\right)}{\left(3+\sqrt{5}\right)\left(9+\frac{16\left(5+\sqrt{5}\right)}{\left(3+\sqrt{5}\right)\left(11+\ldots\right)}\right)}\right)}\right)}}{5\left(1+\sqrt{5}\right)^3\left(3+\frac{9\left(5+\sqrt{5}\right)}{\left(3+\sqrt{5}\right)\left(5+\frac{4\left(5+\sqrt{5}\right)}{\left(3+\sqrt{5}\right)\left(7+\frac{25\left(5+\sqrt{5}\right)}{\left(3+\sqrt{5}\right)\left(9+\frac{16\left(5+\sqrt{5}\right)}{\left(3+\sqrt{5}\right)\left(11+\ldots\right)}\right)}\right)}\right)}\right)}$$

$$\frac{1}{20}\tan^{-1}\left(\frac{2\sqrt{10+2\sqrt{5}}}{4+2(\sqrt{5}-1)}\right)\sqrt{10+2\sqrt{5}} =$$

$$\frac{5+\sqrt{5}}{10(1+\sqrt{5})\left(1+\underset{k=1}{\overset{\infty}{\mathrm{K}}}\frac{\frac{(5+\sqrt{5})(1-2k)^2}{3+\sqrt{5}}}{\frac{4(4+\sqrt{5}-2k)}{(1+\sqrt{5})^2}}\right)} = (5+\sqrt{5}) \Bigg/ \left(10(1+\sqrt{5})\left(1+(5+\sqrt{5}) \Bigg/ \left((3+\sqrt{5})\left(\frac{4(2+\sqrt{5})}{(1+\sqrt{5})^2}+(9(5+\sqrt{5})) \Bigg/ \left((3+\sqrt{5})\left(\frac{4\sqrt{5}}{(1+\sqrt{5})^2}+\frac{25(5+\sqrt{5})}{(3+\sqrt{5})\left(\frac{4(-2+\sqrt{5})}{(1+\sqrt{5})^2}+\frac{49(5+\sqrt{5})}{(3+\sqrt{5})\left(\frac{4(-4+\sqrt{5})}{(1+\sqrt{5})^2}+\ldots\right)}\right)}\right)\right)\right)\right)\right)\right)$$

thence, we obtain:

1/20 ln((((1+2)^5)/(1+2^5))) + 1/(4sqrt5) ln ((((((1+2*((sqrt5-1)/2)+4))) / (((1-2*((sqrt5-1)/2)+4)))))) + 1/20 (10-2sqrt5)^(1/2) tan^-1 (((((2*(10-2sqrt5)^(1/2)))/((4-2(sqrt5+1)))))) + 0.164708638338

**Input interpretation:**

$$\frac{1}{20}\log\left(\frac{(1+2)^5}{1+2^5}\right)+\frac{1}{4\sqrt{5}}\log\left(\frac{1+2\left(\frac{1}{2}\left(\sqrt{5}-1\right)\right)+4}{1-2\left(\frac{1}{2}\left(\sqrt{5}-1\right)\right)+4}\right)+\frac{1}{20}\sqrt{10-2\sqrt{5}}\tan^{-1}\left(\frac{2\sqrt{10-2\sqrt{5}}}{4-2\left(\sqrt{5}+1\right)}\right)+0.164708638338$$

log(x) is the natural logarithm

$\tan^{-1}(x)$ is the inverse tangent function

**Result:**

0.193226045570...

(result in radians)

0.19322604557…

**Alternative representations:**

$$\frac{1}{20}\log\left(\frac{(1+2)^5}{1+2^5}\right)+\frac{\log\left(\frac{1+\frac{2}{2}\left(\sqrt{5}-1\right)+4}{1-\frac{2}{2}\left(\sqrt{5}-1\right)+4}\right)}{4\sqrt{5}}+\frac{1}{20}\sqrt{10-2\sqrt{5}}\tan^{-1}\left(\frac{2\sqrt{10-2\sqrt{5}}}{4-2\left(\sqrt{5}+1\right)}\right)+0.1647086383380000=0.1647086383380000+\frac{1}{20}\log\left(\frac{3^5}{1+2^5}\right)+\frac{1}{20}\tan^{-1}\left(1,\frac{2\sqrt{10-2\sqrt{5}}}{4-2\left(1+\sqrt{5}\right)}\right)\sqrt{10-2\sqrt{5}}+\frac{\log\left(\frac{4+\sqrt{5}}{6-\sqrt{5}}\right)}{4\sqrt{5}}$$

$$\frac{1}{20}\log\left(\frac{(1+2)^5}{1+2^5}\right)+\frac{\log\left(\frac{1+\frac{2}{2}\left(\sqrt{5}-1\right)+4}{1-\frac{2}{2}\left(\sqrt{5}-1\right)+4}\right)}{4\sqrt{5}}+\frac{1}{20}\sqrt{10-2\sqrt{5}}\tan^{-1}\left(\frac{2\sqrt{10-2\sqrt{5}}}{4-2\left(\sqrt{5}+1\right)}\right)+$$
$$0.1647086383380000 = 0.1647086383380000+\frac{1}{20}\log(a)\log_a\left(\frac{3^5}{1+2^5}\right)+$$
$$\frac{1}{20}\tan^{-1}\left(\frac{2\sqrt{10-2\sqrt{5}}}{4-2\left(1+\sqrt{5}\right)}\right)\sqrt{10-2\sqrt{5}}+\frac{\log(a)\log_a\left(\frac{4+\sqrt{5}}{6-\sqrt{5}}\right)}{4\sqrt{5}}$$

$$\frac{1}{20}\log\left(\frac{(1+2)^5}{1+2^5}\right)+\frac{\log\left(\frac{1+\frac{2}{2}\left(\sqrt{5}-1\right)+4}{1-\frac{2}{2}\left(\sqrt{5}-1\right)+4}\right)}{4\sqrt{5}}+\frac{1}{20}\sqrt{10-2\sqrt{5}}\tan^{-1}\left(\frac{2\sqrt{10-2\sqrt{5}}}{4-2\left(\sqrt{5}+1\right)}\right)+$$
$$0.1647086383380000 = 0.1647086383380000+\frac{1}{20}\log_e\left(\frac{3^5}{1+2^5}\right)+$$
$$\frac{1}{20}\tan^{-1}\left(\frac{2\sqrt{10-2\sqrt{5}}}{4-2\left(1+\sqrt{5}\right)}\right)\sqrt{10-2\sqrt{5}}+\frac{\log_e\left(\frac{4+\sqrt{5}}{6-\sqrt{5}}\right)}{4\sqrt{5}}$$

## Integral representations:

$$\frac{1}{20}\log\left(\frac{(1+2)^5}{1+2^5}\right)+\frac{\log\left(\frac{1+\frac{2}{2}\left(\sqrt{5}-1\right)+4}{1-\frac{2}{2}\left(\sqrt{5}-1\right)+4}\right)}{4\sqrt{5}}+\frac{1}{20}\sqrt{10-2\sqrt{5}}\tan^{-1}\left(\frac{2\sqrt{10-2\sqrt{5}}}{4-2\left(\sqrt{5}+1\right)}\right)+$$
$$0.1647086383380000 = 0.1647086383380000+$$
$$\int_0^1 -\frac{\left(-5+\sqrt{5}\right)\left(-1+\sqrt{5}\right)}{20t^2\left(-5+\sqrt{5}\right)-10\left(-1+\sqrt{5}\right)^2}\,dt+\frac{1}{20}\log\left(\frac{81}{11}\right)+\frac{\log\left(\frac{4+\sqrt{5}}{6-\sqrt{5}}\right)}{4\sqrt{5}}$$

$$\frac{1}{20}\log\left(\frac{(1+2)^5}{1+2^5}\right)+\frac{\log\left(\frac{1+\frac{2}{2}\left(\sqrt{5}-1\right)+4}{1-\frac{2}{2}\left(\sqrt{5}-1\right)+4}\right)}{4\sqrt{5}}+\frac{1}{20}\sqrt{10-2\sqrt{5}}\tan^{-1}\left(\frac{2\sqrt{10-2\sqrt{5}}}{4-2(\sqrt{5}+1)}\right)+0.1647086383380000=$$

$$0.1647086383380000+\int_1^{\frac{81}{11}}\left(\frac{1}{20t}-\frac{-1+\frac{4+\sqrt{5}}{6-\sqrt{5}}}{4\sqrt{5}\left(-\frac{81}{11}+t+\frac{4+\sqrt{5}}{6-\sqrt{5}}-\frac{t\left(4+\sqrt{5}\right)}{6-\sqrt{5}}\right)}+\frac{11}{70}\left(\frac{1}{(4-2(1+\sqrt{5}))\left(1+\frac{121(1-t)^2\left(10-2\sqrt{5}\right)}{1225\left(4-2\left(1+\sqrt{5}\right)\right)^2}\right)}-\frac{\sqrt{5}}{5(4-2(1+\sqrt{5}))\left(1+\frac{121(1-t)^2\left(10-2\sqrt{5}\right)}{1225\left(4-2\left(1+\sqrt{5}\right)\right)^2}\right)}\right)\right)dt$$

$$\frac{1}{20}\log\left(\frac{(1+2)^5}{1+2^5}\right)+\frac{\log\left(\frac{1+\frac{2}{2}\left(\sqrt{5}-1\right)+4}{1-\frac{2}{2}\left(\sqrt{5}-1\right)+4}\right)}{4\sqrt{5}}+\frac{1}{20}\sqrt{10-2\sqrt{5}}\tan^{-1}\left(\frac{2\sqrt{10-2\sqrt{5}}}{4-2(\sqrt{5}+1)}\right)+0.1647086383380000=$$

$$0.1647086383380000+\frac{1}{20}\log\left(\frac{81}{11}\right)+\frac{\log\left(\frac{4+\sqrt{5}}{6-\sqrt{5}}\right)}{4\sqrt{5}}-\frac{i(10-2\sqrt{5})}{40\pi^{3/2}(4-2(1+\sqrt{5}))}\int_{-i\infty+\gamma}^{i\infty+\gamma}\Gamma\left(\frac{1}{2}-s\right)\Gamma(1-s)\Gamma(s)^2\left(1+\frac{4(10-2\sqrt{5})}{(4-2(1+\sqrt{5}))^2}\right)^{-s}ds \quad \text{for } 0<\gamma<\frac{1}{2}$$

## Continued fraction representations:

$$\frac{1}{20}\log\left(\frac{(1+2)^5}{1+2^5}\right)+\frac{\log\left(\frac{1+\frac{2}{2}\left(\sqrt{5}-1\right)+4}{1-\frac{2}{2}\left(\sqrt{5}-1\right)+4}\right)}{4\sqrt{5}}+\frac{1}{20}\sqrt{10-2\sqrt{5}}\tan^{-1}\left(\frac{2\sqrt{10-2\sqrt{5}}}{4-2\left(\sqrt{5}+1\right)}\right)+$$

$$0.1647086383380000 = 0.1647086383380000+\frac{7}{22\left(1+\underset{k=1}{\overset{\infty}{\mathrm{K}}}\frac{\frac{70}{11}\left\lfloor\frac{1+k}{2}\right\rfloor^2}{1+k}\right)}+$$

$$\frac{10-2\sqrt{5}}{10\left(1+\underset{k=1}{\overset{\infty}{\mathrm{K}}}\frac{\frac{4k^2\left(10-2\sqrt{5}\right)}{\left(4-2\left(1+\sqrt{5}\right)\right)^2}}{1+2k}\right)\left(4-2\left(1+\sqrt{5}\right)\right)}+\frac{-1+\frac{4+\sqrt{5}}{6-\sqrt{5}}}{4\left(1+\underset{k=1}{\overset{\infty}{\mathrm{K}}}\frac{\left\lfloor\frac{1+k}{2}\right\rfloor^2\left(-1+\frac{4+\sqrt{5}}{6-\sqrt{5}}\right)}{1+k}\right)\sqrt{5}} =$$

$$0.1647086383380000+\left(10-2\sqrt{5}\right)\Big/\left(10\left(4-2\left(1+\sqrt{5}\right)\right)\left(1+\left(4\left(10-2\sqrt{5}\right)\right)\Big/\left(\left(4-2\left(1+\sqrt{5}\right)\right)^2\left(3+\left(16\left(10-2\sqrt{5}\right)\right)\Big/\left(\left(4-2\left(1+\sqrt{5}\right)\right)^2\left(5+\frac{36\left(10-2\sqrt{5}\right)}{\left(4-2\left(1+\sqrt{5}\right)\right)^2\left(7+\frac{64\left(10-2\sqrt{5}\right)}{\left(4-2\left(1+\sqrt{5}\right)\right)^2\left(9+\ldots\right)}\right)}\right)\right)\right)\right)\right)\right)$$

$$+\frac{7}{22\left(1+\frac{70}{11\left(2+\frac{70}{11\left(3+\frac{280}{11\left(4+\frac{280}{11(5+\ldots)}\right)}\right)}\right)}\right)}+$$

$$\frac{-1+\frac{4+\sqrt{5}}{6-\sqrt{5}}}{4\sqrt{5}\left(1+\frac{-1+\frac{4+\sqrt{5}}{6-\sqrt{5}}}{2+\frac{-1+\frac{4+\sqrt{5}}{6-\sqrt{5}}}{3+\frac{4\left(-1+\frac{4+\sqrt{5}}{6-\sqrt{5}}\right)}{4+\frac{4\left(-1+\frac{4+\sqrt{5}}{6-\sqrt{5}}\right)}{5+\ldots}}}}\right)}$$

$$\frac{1}{20}\log\left(\frac{(1+2)^5}{1+2^5}\right)+\frac{\log\left(\frac{1+\frac{2}{2}\left(\sqrt{5}-1\right)+4}{1-\frac{2}{2}\left(\sqrt{5}-1\right)+4}\right)}{4\sqrt{5}}+\frac{1}{20}\sqrt{10-2\sqrt{5}}\tan^{-1}\left(\frac{2\sqrt{10-2\sqrt{5}}}{4-2\left(\sqrt{5}+1\right)}\right)+$$

$$0.1647086383380000 = 0.1647086383380000+\frac{7}{22\left(1+\underset{k=1}{\overset{\infty}{\mathrm{K}}}\frac{\frac{70\left\lfloor\frac{1+k}{2}\right\rfloor}{11}}{\frac{1}{2}\left(3+(-1)^k(-1+k)+k\right)}\right)}+$$

$$\frac{10-2\sqrt{5}}{10\left(1+\underset{k=1}{\overset{\infty}{\mathrm{K}}}\frac{\frac{4k^2\left(10-2\sqrt{5}\right)}{\left(4-2\left(1+\sqrt{5}\right)\right)^2}}{1+2k}\right)\left(4-2\left(1+\sqrt{5}\right)\right)}+\frac{-1+\frac{4+\sqrt{5}}{6-\sqrt{5}}}{4\left(1+\underset{k=1}{\overset{\infty}{\mathrm{K}}}\frac{\left\lfloor\frac{1+k}{2}\right\rfloor\left(-1+\frac{4+\sqrt{5}}{6-\sqrt{5}}\right)}{\frac{1}{2}\left(3+(-1)^k(-1+k)+k\right)}\right)\sqrt{5}}=$$

$$0.1647086383380000+\left(10-2\sqrt{5}\right)\Bigg/\left(10\left(4-2\left(1+\sqrt{5}\right)\right)\left(1+\left(4\left(10-2\sqrt{5}\right)\right)\Bigg/\left(\left(4-2\left(1+\sqrt{5}\right)\right)^2\left(3+\left(16\left(10-2\sqrt{5}\right)\right)\Bigg/\left(\left(4-2\left(1+\sqrt{5}\right)\right)^2\left(5+\frac{36\left(10-2\sqrt{5}\right)}{\left(4-2\left(1+\sqrt{5}\right)\right)^2\left(7+\frac{64\left(10-2\sqrt{5}\right)}{\left(4-2\left(1+\sqrt{5}\right)\right)^2(9+\ldots)}\right)}\right)\right)\right)\right)\right)\right)$$

$$+\frac{7}{22\left(1+\frac{70}{11\left(2+\frac{70}{11\left(3+\frac{140}{11\left(2+\frac{140}{11(5+\ldots)}\right)}\right)}\right)}\right)}+$$

$$\frac{-1+\frac{4+\sqrt{5}}{6-\sqrt{5}}}{4\sqrt{5}\left(1+\frac{-1+\frac{4+\sqrt{5}}{6-\sqrt{5}}}{2+\frac{-1+\frac{4+\sqrt{5}}{6-\sqrt{5}}}{3+\frac{2\left(-1+\frac{4+\sqrt{5}}{6-\sqrt{5}}\right)}{2+\frac{2\left(-1+\frac{4+\sqrt{5}}{6-\sqrt{5}}\right)}{5+\ldots}}}}\right)}$$

$$\frac{1}{20}\log\left(\frac{(1+2)^5}{1+2^5}\right)+\frac{\log\left(\frac{1+\frac{2}{2}\left(\sqrt{5}-1\right)+4}{1-\frac{2}{2}\left(\sqrt{5}-1\right)+4}\right)}{4\sqrt{5}}+\frac{1}{20}\sqrt{10-2\sqrt{5}}\tan^{-1}\left(\frac{2\sqrt{10-2\sqrt{5}}}{4-2\left(\sqrt{5}+1\right)}\right)+$$

$$0.1647086383380000=0.1647086383380000+\frac{7}{22\left(1+\underset{k=1}{\overset{\infty}{\mathrm{K}}}\frac{\frac{70}{11}\left\lfloor\frac{1+k}{2}\right\rfloor^2}{1+k}\right)}+$$

$$\frac{-1+\frac{4+\sqrt{5}}{6-\sqrt{5}}}{4\left(1+\underset{k=1}{\overset{\infty}{\mathrm{K}}}\frac{\left\lfloor\frac{1+k}{2}\right\rfloor^2\left(-1+\frac{4+\sqrt{5}}{6-\sqrt{5}}\right)}{1+k}\right)\sqrt{5}}+\frac{1}{20}\sqrt{10-2\sqrt{5}}$$

$$\left(-\frac{8\left(10-2\sqrt{5}\right)^{3/2}}{\left(3+\underset{k=1}{\overset{\infty}{\mathrm{K}}}\frac{\frac{4\left(1+(-1)^{1+k}+k\right)^2\left(10-2\sqrt{5}\right)}{\left(4-2\left(1+\sqrt{5}\right)\right)^2}}{3+2k}\right)\left(4-2\left(1+\sqrt{5}\right)\right)^3}+\frac{2\sqrt{10-2\sqrt{5}}}{4-2\left(1+\sqrt{5}\right)}\right)=$$

$$0.1647086383380000+\frac{1}{20}\sqrt{10-2\sqrt{5}}\left(\frac{2\sqrt{10-2\sqrt{5}}}{4-2\left(1+\sqrt{5}\right)}-\right.$$

$$\left(8\left(10-2\sqrt{5}\right)^{3/2}\right)\Big/\left(\left(4-2\left(1+\sqrt{5}\right)\right)^3\left(3+\left(36\left(10-2\sqrt{5}\right)\right)\Big/\right.\right.$$

$$\left(\left(4-2\left(1+\sqrt{5}\right)\right)^2\left(5+\left(16\left(10-2\sqrt{5}\right)\right)\Big/\left(\left(4-2\left(1+\sqrt{5}\right)\right)^2\right.\right.\right.$$

$$\left(7+\left(100\left(10-2\sqrt{5}\right)\right)\Big/\left(\left(4-2\left(1+\sqrt{5}\right)\right)^2\right.\right.$$

$$\left.\left(9+\frac{64\left(10-2\sqrt{5}\right)}{\left(4-2\left(1+\sqrt{5}\right)\right)^2(11+\ldots)}\right)\right)\right)\right)\right)\right)\right)\right)\Bigg)+$$

$$\frac{7}{22\left(1+\frac{70}{11\left(2+\frac{70}{11\left(3+\frac{280}{11\left(4+\frac{280}{11(5+\ldots)}\right)}\right)}\right)}\right)}+\frac{-1+\frac{4+\sqrt{5}}{6-\sqrt{5}}}{4\sqrt{5}\left(1+\frac{-1+\frac{4+\sqrt{5}}{6-\sqrt{5}}}{2+\frac{-1+\frac{4+\sqrt{5}}{6-\sqrt{5}}}{3+\frac{4\left(-1+\frac{4+\sqrt{5}}{6-\sqrt{5}}\right)}{4+\frac{4\left(-1+\frac{4+\sqrt{5}}{6-\sqrt{5}}\right)}{5+\ldots}}}}\right)}$$

26*1/[1/20 log((1+2)^5/(1+2^5)) + log((1+2/2(sqrt(5) - 1) +4)/(1 - 2/2(sqrt(5)- 1) +4))/(4 sqrt(5))+1/20 sqrt(10 -2 sqrt(5)) tan^(-1)((2 sqrt(10 -2 sqrt(5)))/(4 -2 (sqrt(5) +1)))+0.164708638] +5

**Input interpretation:**

$$26\times 1\Bigg/\left(\frac{1}{20}\log\left(\frac{(1+2)^5}{1+2^5}\right)+\frac{\log\left(\frac{1+\frac{2}{2}\left(\sqrt{5}-1\right)+4}{1-\frac{2}{2}\left(\sqrt{5}-1\right)+4}\right)}{4\sqrt{5}}+\frac{1}{20}\sqrt{10-2\sqrt{5}}\tan^{-1}\left(\frac{2\sqrt{10-2\sqrt{5}}}{4-2\left(\sqrt{5}+1\right)}\right)+0.164708638\right)+5$$

log(x) is the natural logarithm

$\tan^{-1}(x)$ is the inverse tangent function

**Result:**

139.557430...

(result in radians)

139.55743… result practically equal to the rest mass of Pion meson 139.57 MeV

**Alternative representations:**

$$\frac{26}{\frac{1}{20}\log\left(\frac{(1+2)^5}{1+2^5}\right)+\frac{\log\left(\frac{1+\frac{2}{2}\left(\sqrt{5}-1\right)+4}{1-\frac{2}{2}\left(\sqrt{5}-1\right)+4}\right)}{4\sqrt{5}}+\frac{1}{20}\sqrt{10-2\sqrt{5}}\tan^{-1}\left(\frac{2\sqrt{10-2\sqrt{5}}}{4-2\left(\sqrt{5}+1\right)}\right)+0.164709}+5=$$

$$5+\frac{26}{0.164709+\frac{1}{20}\log\left(\frac{3^5}{1+2^5}\right)+\frac{\log\left(\frac{4+\sqrt{5}}{6-\sqrt{5}}\right)}{4\sqrt{5}}+\frac{1}{20}\tan^{-1}\left(1,\frac{2\sqrt{10-2\sqrt{5}}}{4-2\left(1+\sqrt{5}\right)}\right)\sqrt{10-2\sqrt{5}}}$$

$$\frac{26}{\frac{1}{20}\log\left(\frac{(1+2)^5}{1+2^5}\right)+\frac{\log\left(\frac{1+\frac{2}{2}\left(\sqrt{5}-1\right)+4}{1-\frac{2}{2}\left(\sqrt{5}-1\right)+4}\right)}{4\sqrt{5}}+\frac{1}{20}\sqrt{10-2\sqrt{5}}\tan^{-1}\left(\frac{2\sqrt{10-2\sqrt{5}}}{4-2\left(\sqrt{5}+1\right)}\right)+0.164709}+$$

$$5=5+\frac{26}{0.164709+\frac{1}{20}\log_e\left(\frac{3^5}{1+2^5}\right)+\frac{\log_e\left(\frac{4+\sqrt{5}}{6-\sqrt{5}}\right)}{4\sqrt{5}}+\frac{1}{20}\tan^{-1}\left(\frac{2\sqrt{10-2\sqrt{5}}}{4-2\left(1+\sqrt{5}\right)}\right)\sqrt{10-2\sqrt{5}}}$$

$$\frac{26}{\frac{1}{20}\log\left(\frac{(1+2)^5}{1+2^5}\right)+\frac{\log\left(\frac{1+\frac{2}{2}\left(\sqrt{5}-1\right)+4}{1-\frac{2}{2}\left(\sqrt{5}-1\right)+4}\right)}{4\sqrt{5}}+\frac{1}{20}\sqrt{10-2\sqrt{5}}\tan^{-1}\left(\frac{2\sqrt{10-2\sqrt{5}}}{4-2\left(\sqrt{5}+1\right)}\right)+0.164709}+$$

$$5=5+26\Big/\left(0.164709+\frac{1}{20}\log(a)\log_a\left(\frac{3^5}{1+2^5}\right)+\frac{\log(a)\log_a\left(\frac{4+\sqrt{5}}{6-\sqrt{5}}\right)}{4\sqrt{5}}+\frac{1}{20}\tan^{-1}\left(\frac{2\sqrt{10-2\sqrt{5}}}{4-2\left(1+\sqrt{5}\right)}\right)\sqrt{10-2\sqrt{5}}\right)$$

**Integral representations:**

$$\frac{26}{\frac{1}{20}\log\left(\frac{(1+2)^5}{1+2^5}\right)+\frac{\log\left(\frac{1+\frac{2}{2}\left(\sqrt{5}-1\right)+4}{1-\frac{2}{2}\left(\sqrt{5}-1\right)+4}\right)}{4\sqrt{5}}+\frac{1}{20}\sqrt{10-2\sqrt{5}}\tan^{-1}\left(\frac{2\sqrt{10-2\sqrt{5}}}{4-2\left(\sqrt{5}+1\right)}\right)+0.164709}+$$

$$5=5+\frac{26}{0.164709+\frac{1}{20}\log\left(\frac{81}{11}\right)+\frac{\log\left(\frac{4+\sqrt{5}}{6-\sqrt{5}}\right)}{4\sqrt{5}}+\frac{\sqrt{10-2\sqrt{5}}^{\,2}}{10\left(4-2\left(1+\sqrt{5}\right)\right)}\int_0^1\frac{1}{1+\frac{4t^2\sqrt{10-2\sqrt{5}}^{\,2}}{\left(4-2\left(1+\sqrt{5}\right)\right)^2}}\,dt}$$

$$\frac{26}{\frac{1}{20}\log\left(\frac{(1+2)^5}{1+2^5}\right)+\frac{\log\left(\frac{1+\frac{2}{2}\left(\sqrt{5}-1\right)+4}{1-\frac{2}{2}\left(\sqrt{5}-1\right)+4}\right)}{4\sqrt{5}}+\frac{1}{20}\sqrt{10-2\sqrt{5}}\tan^{-1}\left(\frac{2\sqrt{10-2\sqrt{5}}}{4-2\left(\sqrt{5}+1\right)}\right)+0.164709}+$$

$$5=5+26\Bigg/\left(0.164709+\int_1^{\frac{81}{11}}\left(\frac{1}{20t}-\frac{-1+\frac{4+\sqrt{5}}{6-\sqrt{5}}}{4\sqrt{5}\left(-\frac{81}{11}+t+\frac{4+\sqrt{5}}{6-\sqrt{5}}-\frac{t\left(4+\sqrt{5}\right)}{6-\sqrt{5}}\right)}+\frac{11\sqrt{10-2\sqrt{5}}^2}{700\left(4-2\left(1+\sqrt{5}\right)\right)\left(1+\frac{121(1-t)^2\sqrt{10-2\sqrt{5}}^2}{1225\left(4-2\left(1+\sqrt{5}\right)\right)^2}\right)}\right)dt\right)$$

$$\frac{26}{\frac{1}{20}\log\left(\frac{(1+2)^5}{1+2^5}\right)+\frac{\log\left(\frac{1+\frac{2}{2}\left(\sqrt{5}-1\right)+4}{1-\frac{2}{2}\left(\sqrt{5}-1\right)+4}\right)}{4\sqrt{5}}+\frac{1}{20}\sqrt{10-2\sqrt{5}}\tan^{-1}\left(\frac{2\sqrt{10-2\sqrt{5}}}{4-2\left(\sqrt{5}+1\right)}\right)+0.164709}+$$

$$5=5+26\Bigg/\left(0.164709+\frac{1}{20}\log\left(\frac{81}{11}\right)+\frac{\log\left(\frac{4+\sqrt{5}}{6-\sqrt{5}}\right)}{4\sqrt{5}}-\frac{i\sqrt{10-2\sqrt{5}}^2}{40\pi^{3/2}\left(4-2\left(1+\sqrt{5}\right)\right)}\int_{-i\infty+\gamma}^{i\infty+\gamma}\Gamma\left(\frac{1}{2}-s\right)\Gamma(1-s)\Gamma(s)^2\left(1+\frac{4\sqrt{10-2\sqrt{5}}^2}{\left(4-2\left(1+\sqrt{5}\right)\right)^2}\right)^{-s}ds\right)\quad\text{for } 0<\gamma<\frac{1}{2}$$

26*1/[1/20 log((1+2)^5/(1+2^5)) + log((1+2/2(sqrt(5) - 1) +4)/(1 - 2/2(sqrt(5)- 1) +4))/(4 sqrt(5))+1/20 sqrt(10 -2 sqrt(5)) tan^(-1)((2 sqrt(10 -2 sqrt(5)))/(4 -2 (sqrt(5) +1)))+0.164708638] -11+2

**Input interpretation:**

$$26\times1\Bigg/\left(\frac{1}{20}\log\left(\frac{(1+2)^5}{1+2^5}\right)+\frac{\log\left(\frac{1+\frac{2}{2}\left(\sqrt{5}-1\right)+4}{1-\frac{2}{2}\left(\sqrt{5}-1\right)+4}\right)}{4\sqrt{5}}+\frac{1}{20}\sqrt{10-2\sqrt{5}}\tan^{-1}\left(\frac{2\sqrt{10-2\sqrt{5}}}{4-2\left(\sqrt{5}+1\right)}\right)+0.164708638\right)-11+2$$

$\log(x)$ is the natural logarithm

$\tan^{-1}(x)$ is the inverse tangent function

**Result:**

125.557430...

(result in radians)

125.55743… result very near to the dilaton mass calculated as a type of Higgs boson: 125 GeV for T = 0 and to the Higgs boson mass 125.18 GeV

**Alternative representations:**

$$\frac{26}{\frac{1}{20}\log\left(\frac{(1+2)^5}{1+2^5}\right)+\frac{\log\left(\frac{1+\frac{2}{2}\left(\sqrt{5}-1\right)+4}{1-\frac{2}{2}\left(\sqrt{5}-1\right)+4}\right)}{4\sqrt{5}}+\frac{1}{20}\sqrt{10-2\sqrt{5}}\tan^{-1}\left(\frac{2\sqrt{10-2\sqrt{5}}}{4-2\left(\sqrt{5}+1\right)}\right)+0.164709}-11+2=-9+\frac{26}{0.164709+\frac{1}{20}\log\left(\frac{3^5}{1+2^5}\right)+\frac{\log\left(\frac{4+\sqrt{5}}{6-\sqrt{5}}\right)}{4\sqrt{5}}+\frac{1}{20}\tan^{-1}\left(1,\frac{2\sqrt{10-2\sqrt{5}}}{4-2\left(1+\sqrt{5}\right)}\right)\sqrt{10-2\sqrt{5}}}$$

$$\frac{26}{\frac{1}{20}\log\left(\frac{(1+2)^5}{1+2^5}\right)+\frac{\log\left(\frac{1+\frac{2}{2}\left(\sqrt{5}-1\right)+4}{1-\frac{2}{2}\left(\sqrt{5}-1\right)+4}\right)}{4\sqrt{5}}+\frac{1}{20}\sqrt{10-2\sqrt{5}}\tan^{-1}\left(\frac{2\sqrt{10-2\sqrt{5}}}{4-2\left(\sqrt{5}+1\right)}\right)+0.164709}-11+2=-9+\frac{26}{0.164709+\frac{1}{20}\log_e\left(\frac{3^5}{1+2^5}\right)+\frac{\log_e\left(\frac{4+\sqrt{5}}{6-\sqrt{5}}\right)}{4\sqrt{5}}+\frac{1}{20}\tan^{-1}\left(\frac{2\sqrt{10-2\sqrt{5}}}{4-2\left(1+\sqrt{5}\right)}\right)\sqrt{10-2\sqrt{5}}}$$

$$\frac{26}{\frac{1}{20}\log\left(\frac{(1+2)^5}{1+2^5}\right)+\frac{\log\left(\frac{1+\frac{2}{2}\left(\sqrt{5}-1\right)+4}{1-\frac{2}{2}\left(\sqrt{5}-1\right)+4}\right)}{4\sqrt{5}}+\frac{1}{20}\sqrt{10-2\sqrt{5}}\,\tan^{-1}\left(\frac{2\sqrt{10-2\sqrt{5}}}{4-2\left(\sqrt{5}+1\right)}\right)+0.164709}-$$

$$11+2=-9+26\Bigg/\left(0.164709+\frac{1}{20}\log(a)\log_a\left(\frac{3^5}{1+2^5}\right)+\frac{\log(a)\log_a\left(\frac{4+\sqrt{5}}{6-\sqrt{5}}\right)}{4\sqrt{5}}+\frac{1}{20}\tan^{-1}\left(\frac{2\sqrt{10-2\sqrt{5}}}{4-2\left(1+\sqrt{5}\right)}\right)\sqrt{10-2\sqrt{5}}\right)$$

## Integral representations:


$$\frac{26}{\frac{1}{20}\log\left(\frac{(1+2)^5}{1+2^5}\right)+\frac{\log\left(\frac{1+\frac{2}{2}\left(\sqrt{5}-1\right)+4}{1-\frac{2}{2}\left(\sqrt{5}-1\right)+4}\right)}{4\sqrt{5}}+\frac{1}{20}\sqrt{10-2\sqrt{5}}\,\tan^{-1}\left(\frac{2\sqrt{10-2\sqrt{5}}}{4-2\left(\sqrt{5}+1\right)}\right)+0.164709}-$$


$$11+2=-9+\frac{26}{0.164709+\frac{1}{20}\log\left(\frac{81}{11}\right)+\frac{\log\left(\frac{4+\sqrt{5}}{6-\sqrt{5}}\right)}{4\sqrt{5}}+\frac{\sqrt{10-2\sqrt{5}}^{\,2}}{10\left(4-2\left(1+\sqrt{5}\right)\right)}\int_0^1\frac{1}{1+\frac{4t^2\sqrt{10-2\sqrt{5}}^{\,2}}{\left(4-2\left(1+\sqrt{5}\right)\right)^2}}\,dt}$$


$$\frac{26}{\frac{1}{20}\log\left(\frac{(1+2)^5}{1+2^5}\right)+\frac{\log\left(\frac{1+\frac{2}{2}\left(\sqrt{5}-1\right)+4}{1-\frac{2}{2}\left(\sqrt{5}-1\right)+4}\right)}{4\sqrt{5}}+\frac{1}{20}\sqrt{10-2\sqrt{5}}\,\tan^{-1}\left(\frac{2\sqrt{10-2\sqrt{5}}}{4-2\left(\sqrt{5}+1\right)}\right)+0.164709}-$$

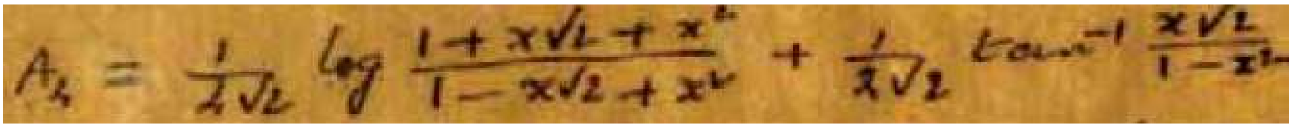

$$11+2=-9+26\Bigg/\left(0.164709+\int_1^{\frac{81}{11}}\left(\frac{1}{20\,t}-\frac{-1+\frac{4+\sqrt{5}}{6-\sqrt{5}}}{4\sqrt{5}\left(-\frac{81}{11}+t+\frac{4+\sqrt{5}}{6-\sqrt{5}}-\frac{t\left(4+\sqrt{5}\right)}{6-\sqrt{5}}\right)}+\frac{11\sqrt{10-2\sqrt{5}}^{\,2}}{700\left(4-2\left(1+\sqrt{5}\right)\right)\left(1+\frac{121(1-t)^2\sqrt{10-2\sqrt{5}}^{\,2}}{1225\left(4-2\left(1+\sqrt{5}\right)\right)^2}\right)}\right)dt\right)$$

$$\frac{26}{\frac{1}{20}\log\left(\frac{(1+2)^5}{1+2^5}\right)+\frac{\log\left(\frac{1+\frac{2}{2}\left(\sqrt{5}-1\right)+4}{1-\frac{2}{2}\left(\sqrt{5}-1\right)+4}\right)}{4\sqrt{5}}+\frac{1}{20}\sqrt{10-2\sqrt{5}}\tan^{-1}\left(\frac{2\sqrt{10-2\sqrt{5}}}{4-2\left(\sqrt{5}+1\right)}\right)+0.164709}-$$

$$11+2=$$

$$-9+26\Bigg/\left(0.164709+\frac{1}{20}\log\left(\frac{81}{11}\right)+\frac{\log\left(\frac{4+\sqrt{5}}{6-\sqrt{5}}\right)}{4\sqrt{5}}-\frac{i\sqrt{10-2\sqrt{5}}^{\,2}}{40\pi^{3/2}\left(4-2\left(1+\sqrt{5}\right)\right)}\int_{-i\infty+\gamma}^{i\infty+\gamma}\Gamma\left(\frac{1}{2}-s\right)\Gamma(1-s)\Gamma(s)^2\left(1+\frac{4\sqrt{10-2\sqrt{5}}^{\,2}}{\left(4-2\left(1+\sqrt{5}\right)\right)^2}\right)^{-s}ds\right) \text{ for } 0<\gamma<\frac{1}{2}$$

Now, we have that:



$$A_2 = \frac{1}{2\sqrt{2}}\log\frac{1+x\sqrt{2}+x^2}{1-x\sqrt{2}+x^2}+\frac{1}{2\sqrt{2}}\tan^{-1}\frac{x\sqrt{2}}{1-x^2}$$

1/(4sqrt2) ln (((1+2sqrt2+4)/(1-2sqrt2+4)))+1/(2sqrt2) tan^-1(((2sqrt2)/(1-4)))

**Input:**

$$\frac{1}{4\sqrt{2}}\log\left(\frac{1+2\sqrt{2}+4}{1-2\sqrt{2}+4}\right)+\frac{1}{2\sqrt{2}}\tan^{-1}\left(\frac{2\sqrt{2}}{1-4}\right)$$

log(x) is the natural logarithm

$\tan^{-1}(x)$ is the inverse tangent function

**Exact Result:**

$$\frac{\log\left(\frac{5+2\sqrt{2}}{5-2\sqrt{2}}\right)}{4\sqrt{2}}-\frac{\tan^{-1}\left(\frac{2\sqrt{2}}{3}\right)}{2\sqrt{2}}$$

(result in radians)

**Decimal approximation:**

−0.04059304540290341402684888493340270092590079222787614185...

(result in radians)

-0.0405930454029034…..

**Alternate forms:**

$$\frac{\log\left(\frac{1}{17}\left(33+20\sqrt{2}\right)\right)-2\tan^{-1}\left(\frac{2\sqrt{2}}{3}\right)}{4\sqrt{2}}$$

$$\frac{\log\left(\frac{1}{17}\left(33+20\sqrt{2}\right)\right)}{4\sqrt{2}}-\frac{\tan^{-1}\left(\frac{2\sqrt{2}}{3}\right)}{2\sqrt{2}}$$

$$\frac{\log\left(-\frac{1}{2\sqrt{2}-5}\right)+\log\left(5+2\sqrt{2}\right)-2\tan^{-1}\left(\frac{2\sqrt{2}}{3}\right)}{4\sqrt{2}}$$

**Alternative representations:**

$$\frac{\log\left(\frac{1+2\sqrt{2}+4}{1-2\sqrt{2}+4}\right)}{4\sqrt{2}}+\frac{\tan^{-1}\left(\frac{2\sqrt{2}}{1-4}\right)}{2\sqrt{2}}=\frac{\tan^{-1}\left(1,-\frac{2\sqrt{2}}{3}\right)}{2\sqrt{2}}+\frac{\log\left(\frac{5+2\sqrt{2}}{5-2\sqrt{2}}\right)}{4\sqrt{2}}$$

$$\frac{\log\left(\frac{1+2\sqrt{2}+4}{1-2\sqrt{2}+4}\right)}{4\sqrt{2}}+\frac{\tan^{-1}\left(\frac{2\sqrt{2}}{1-4}\right)}{2\sqrt{2}}=\frac{\tan^{-1}\left(-\frac{2\sqrt{2}}{3}\right)}{2\sqrt{2}}+\frac{\log_e\left(\frac{5+2\sqrt{2}}{5-2\sqrt{2}}\right)}{4\sqrt{2}}$$

$$\frac{\log\left(\frac{1+2\sqrt{2}+4}{1-2\sqrt{2}+4}\right)}{4\sqrt{2}}+\frac{\tan^{-1}\left(\frac{2\sqrt{2}}{1-4}\right)}{2\sqrt{2}}=\frac{\tan^{-1}\left(-\frac{2\sqrt{2}}{3}\right)}{2\sqrt{2}}+\frac{\log(a)\log_a\left(\frac{5+2\sqrt{2}}{5-2\sqrt{2}}\right)}{4\sqrt{2}}$$

**Series representations:**

$$\frac{\log\left(\frac{1+2\sqrt{2}+4}{1-2\sqrt{2}+4}\right)}{4\sqrt{2}}+\frac{\tan^{-1}\left(\frac{2\sqrt{2}}{1-4}\right)}{2\sqrt{2}}=$$
$$-\frac{\tan^{-1}\left(\frac{2\sqrt{2}}{3}\right)}{2\sqrt{2}}+\frac{\log\left(\frac{4}{17}\left(4+5\sqrt{2}\right)\right)}{4\sqrt{2}}-\frac{\sum_{k=1}^{\infty}\frac{\left(\frac{1}{8}\left(4-5\sqrt{2}\right)\right)^k}{k}}{4\sqrt{2}}$$

$$\frac{\log\left(\frac{1+2\sqrt{2}+4}{1-2\sqrt{2}+4}\right)}{4\sqrt{2}}+\frac{\tan^{-1}\left(\frac{2\sqrt{2}}{1-4}\right)}{2\sqrt{2}}=$$
$$\frac{\log\left(\frac{4}{17}\left(4+5\sqrt{2}\right)\right)-\sum_{k=1}^{\infty}\frac{\left(\frac{1}{8}\left(4-5\sqrt{2}\right)\right)^k}{k}-2\sum_{k=0}^{\infty}\frac{(-1)^k\,2^{3/2+3k}\times 3^{-1-2k}}{1+2k}}{4\sqrt{2}}$$

$$\frac{\log\left(\frac{1+2\sqrt{2}+4}{1-2\sqrt{2}+4}\right)}{4\sqrt{2}} + \frac{\tan^{-1}\left(\frac{2\sqrt{2}}{1-4}\right)}{2\sqrt{2}} = \frac{\log\left(-1+\frac{5+2\sqrt{2}}{5-2\sqrt{2}}\right) - \sum_{k=1}^{\infty} \frac{\left(\frac{1}{8}\left(4-5\sqrt{2}\right)\right)^k}{k} - 2\sum_{k=0}^{\infty} \frac{(-1)^k 2^{3/2+3k} \times 3^{-1-2k}}{1+2k}}{4\sqrt{2}}$$

$$\frac{\log\left(\frac{1+2\sqrt{2}+4}{1-2\sqrt{2}+4}\right)}{4\sqrt{2}} + \frac{\tan^{-1}\left(\frac{2\sqrt{2}}{1-4}\right)}{2\sqrt{2}} = -\frac{\tan^{-1}(z_0)}{2\sqrt{2}} + \frac{\log\left(-1+\frac{5+2\sqrt{2}}{5-2\sqrt{2}}\right)}{4\sqrt{2}} + \sum_{k=1}^{\infty}\left(\frac{(-1)^{-1+k}\left(-1+\frac{5+2\sqrt{2}}{5-2\sqrt{2}}\right)^{-k}}{4\sqrt{2}\,k} - \frac{i\left(-(-i-z_0)^{-k}+(i-z_0)^{-k}\right)\left(\frac{2\sqrt{2}}{3}-z_0\right)^k}{4\sqrt{2}\,k}\right)$$

for $(i z_0 \notin \mathbb{R}$ or ( not $(1 \leq i z_0 < \infty)$ and not $(-\infty < i z_0 \leq -1)))$

# Integral representations:

$$\frac{\log\left(\frac{1+2\sqrt{2}+4}{1-2\sqrt{2}+4}\right)}{4\sqrt{2}} + \frac{\tan^{-1}\left(\frac{2\sqrt{2}}{1-4}\right)}{2\sqrt{2}} = -3\int_0^1 \frac{1}{9+8t^2}\,dt + \frac{\log\left(\frac{1}{17}\left(33+20\sqrt{2}\right)\right)}{4\sqrt{2}}$$

$$\frac{\log\left(\frac{1+2\sqrt{2}+4}{1-2\sqrt{2}+4}\right)}{4\sqrt{2}} + \frac{\tan^{-1}\left(\frac{2\sqrt{2}}{1-4}\right)}{2\sqrt{2}} = \int_1^{\frac{1}{17}\left(33+20\sqrt{2}\right)} \left(-\frac{3}{\left(-1+\frac{1}{17}\left(33+20\sqrt{2}\right)\right)\left(9+\frac{8(1-t)^2}{\left(1+\frac{1}{17}\left(-33-20\sqrt{2}\right)\right)^2}\right)} + \frac{1}{4\sqrt{2}\,t}\right)dt$$

$$\frac{\log\left(\frac{1+2\sqrt{2}+4}{1-2\sqrt{2}+4}\right)}{4\sqrt{2}} + \frac{\tan^{-1}\left(\frac{2\sqrt{2}}{1-4}\right)}{2\sqrt{2}} = \frac{i}{12\pi^{3/2}} \int_{-i\infty+\gamma}^{i\infty+\gamma} \left(\frac{9}{17}\right)^s \Gamma\left(\frac{1}{2}-s\right)\Gamma(1-s)\,\Gamma(s)^2\,ds + \frac{\log\left(\frac{1}{17}\left(33+20\sqrt{2}\right)\right)}{4\sqrt{2}} \quad \text{for } 0<\gamma<\frac{1}{2}$$

## Continued fraction representations:

$$\frac{\log\left(\frac{1+2\sqrt{2}+4}{1-2\sqrt{2}+4}\right)}{4\sqrt{2}} + \frac{\tan^{-1}\left(\frac{2\sqrt{2}}{1-4}\right)}{2\sqrt{2}} = \frac{\log\left(\frac{1}{17}\left(33+20\sqrt{2}\right)\right)}{4\sqrt{2}} - \frac{1}{3\left(1+\underset{k=1}{\overset{\infty}{\mathrm{K}}}\frac{\frac{8k^2}{9}}{1+2k}\right)} =$$

$$\frac{\log\left(\frac{1}{17}\left(33+20\sqrt{2}\right)\right)}{4\sqrt{2}} - \frac{1}{3\left(1+\frac{8}{9\left(3+\frac{32}{9\left(5+\frac{8}{7+\frac{128}{9(9+\ldots)}}\right)}\right)}\right)}$$

$$\frac{\log\left(\frac{1+2\sqrt{2}+4}{1-2\sqrt{2}+4}\right)}{4\sqrt{2}} + \frac{\tan^{-1}\left(\frac{2\sqrt{2}}{1-4}\right)}{2\sqrt{2}} = \frac{\log\left(\frac{5+2\sqrt{2}}{5-2\sqrt{2}}\right)}{4\sqrt{2}} - \frac{1}{3\left(1+\underset{k=1}{\overset{\infty}{\mathrm{K}}}\frac{\frac{8k^2}{9}}{1+2k}\right)} =$$

$$\frac{\log\left(\frac{5+2\sqrt{2}}{5-2\sqrt{2}}\right)}{4\sqrt{2}} - \frac{1}{3\left(1+\frac{8}{9\left(3+\frac{32}{9\left(5+\frac{8}{7+\frac{128}{9(9+\ldots)}}\right)}\right)}\right)}$$

$$\frac{\log\left(\frac{1+2\sqrt{2}+4}{1-2\sqrt{2}+4}\right)}{4\sqrt{2}} + \frac{\tan^{-1}\left(\frac{2\sqrt{2}}{1-4}\right)}{2\sqrt{2}} = \frac{\log\left(\frac{1}{17}\left(33+20\sqrt{2}\right)\right)}{4\sqrt{2}} - \frac{1}{3\left(1+\underset{k=1}{\overset{\infty}{\mathrm{K}}}\frac{\frac{8}{9}(1-2k)^2}{\frac{1}{9}(17+2k)}\right)} =$$

$$\frac{\log\left(\frac{1}{17}\left(33+20\sqrt{2}\right)\right)}{4\sqrt{2}} - \frac{1}{3\left(1+\frac{8}{9\left(\frac{19}{9}+\frac{8}{\frac{7}{3}+\frac{200}{9\left(\frac{23}{9}+\frac{392}{9\left(\frac{25}{9}+\ldots\right)}\right)}}\right)}\right)}$$

$\underset{k=k_1}{\overset{k_2}{\mathrm{K}}} a_k / b_k$ is a continued fraction

(64+8)* -1/(((((1/(4sqrt2) ln (((1+2sqrt2+4)/(1-2sqrt2+4)))+1/(2sqrt2) tan^-1(((2sqrt2)/(1-4)))))))-47+Pi-(2-sqrt3+1/2)

**Input:**

$$\frac{(64+8)\times(-1)}{\frac{1}{4\sqrt{2}}\log\left(\frac{1+2\sqrt{2}+4}{1-2\sqrt{2}+4}\right)+\frac{1}{2\sqrt{2}}\tan^{-1}\left(\frac{2\sqrt{2}}{1-4}\right)} - 47 + \pi - \left(2-\sqrt{3}+\frac{1}{2}\right)$$

$\log(x)$ is the natural logarithm

$\tan^{-1}(x)$ is the inverse tangent function

**Exact Result:**

$$-\frac{99}{2}+\sqrt{3}+\pi-\frac{72}{\frac{\log\left(\frac{5+2\sqrt{2}}{5-2\sqrt{2}}\right)}{4\sqrt{2}}-\frac{\tan^{-1}\left(\frac{2\sqrt{2}}{3}\right)}{2\sqrt{2}}}$$

(result in radians)

**Decimal approximation:**

1729.0764855457834986270451992431707593020099622381767481 02...

(result in radians)

1729.076485545…

This result is very near to the mass of candidate glueball $f_0(1710)$ meson. Furthermore, 1728 occurs in the algebraic formula for the j-invariant of an elliptic curve. As a consequence, it is sometimes called a Zagier as a pun on the Gross–Zagier theorem. The number 1728 is one less than the Hardy–Ramanujan number 1729 (taxicab number)

**Alternate forms:**

$$-\frac{99}{2}+\sqrt{3}+\pi+\frac{144\sqrt{2}}{\tan^{-1}\left(\frac{2\sqrt{2}}{3}\right)-\tanh^{-1}\left(\frac{2\sqrt{2}}{5}\right)}$$

$$-\frac{99}{2}+\sqrt{3}+\pi+\frac{288\sqrt{2}}{\log\left(\frac{17}{33+20\sqrt{2}}\right)+2\tan^{-1}\left(\frac{2\sqrt{2}}{3}\right)}$$

$$-\frac{99}{2}+\sqrt{3}+\pi-\frac{288\sqrt{2}}{\log\left(-\frac{5+2\sqrt{2}}{2\sqrt{2}-5}\right)-2\tan^{-1}\left(\frac{2\sqrt{2}}{3}\right)}$$

$\tanh^{-1}(x)$ is the inverse hyperbolic tangent function

**Alternative representations:**

$$\frac{(64+8)(-1)}{\frac{\log\left(\frac{1+2\sqrt{2}+4}{1-2\sqrt{2}+4}\right)}{4\sqrt{2}}+\frac{\tan^{-1}\left(\frac{2\sqrt{2}}{1-4}\right)}{2\sqrt{2}}}-47+\pi-\left(2-\sqrt{3}+\frac{1}{2}\right)=$$
$$-\frac{99}{2}+\pi-\frac{72}{\frac{\tan^{-1}\left(1,-\frac{2\sqrt{2}}{3}\right)}{2\sqrt{2}}+\frac{\log\left(\frac{5+2\sqrt{2}}{5-2\sqrt{2}}\right)}{4\sqrt{2}}}+\sqrt{3}$$

$$\frac{(64+8)(-1)}{\frac{\log\left(\frac{1+2\sqrt{2}+4}{1-2\sqrt{2}+4}\right)}{4\sqrt{2}}+\frac{\tan^{-1}\left(\frac{2\sqrt{2}}{1-4}\right)}{2\sqrt{2}}}-47+\pi-\left(2-\sqrt{3}+\frac{1}{2}\right)=$$
$$-\frac{99}{2}+\pi-\frac{72}{\frac{\tan^{-1}\left(-\frac{2\sqrt{2}}{3}\right)}{2\sqrt{2}}+\frac{\log_e\left(\frac{5+2\sqrt{2}}{5-2\sqrt{2}}\right)}{4\sqrt{2}}}+\sqrt{3}$$

$$\frac{(64+8)(-1)}{\frac{\log\left(\frac{1+2\sqrt{2}+4}{1-2\sqrt{2}+4}\right)}{4\sqrt{2}}+\frac{\tan^{-1}\left(\frac{2\sqrt{2}}{1-4}\right)}{2\sqrt{2}}}-47+\pi-\left(2-\sqrt{3}+\frac{1}{2}\right)=$$
$$-\frac{99}{2}+\pi-\frac{72}{\frac{\tan^{-1}\left(1,-\frac{2\sqrt{2}}{3}\right)}{2\sqrt{2}}+\frac{\log_e\left(\frac{5+2\sqrt{2}}{5-2\sqrt{2}}\right)}{4\sqrt{2}}}+\sqrt{3}$$

## Series representations:

$$\frac{(64+8)(-1)}{\frac{\log\left(\frac{1+2\sqrt{2}+4}{1-2\sqrt{2}+4}\right)}{4\sqrt{2}}+\frac{\tan^{-1}\left(\frac{2\sqrt{2}}{1-4}\right)}{2\sqrt{2}}}-47+\pi-\left(2-\sqrt{3}+\frac{1}{2}\right)=$$
$$-\frac{99}{2}+\sqrt{3}+\pi+\frac{288\sqrt{2}}{2\tan^{-1}\left(\frac{2\sqrt{2}}{3}\right)+\log\left(\frac{1}{8}\left(-4+5\sqrt{2}\right)\right)+\sum_{k=1}^{\infty}\frac{\left(\frac{1}{8}\left(4-5\sqrt{2}\right)\right)^k}{k}}$$

$$\frac{(64+8)(-1)}{\frac{\log\left(\frac{1+2\sqrt{2}+4}{1-2\sqrt{2}+4}\right)}{4\sqrt{2}}+\frac{\tan^{-1}\left(\frac{2\sqrt{2}}{1-4}\right)}{2\sqrt{2}}}-47+\pi-\left(2-\sqrt{3}+\frac{1}{2}\right)=$$
$$-\frac{99}{2}+\sqrt{3}+\pi-\frac{72}{-\frac{\tan^{-1}\left(\frac{2\sqrt{2}}{3}\right)}{2\sqrt{2}}+\frac{\log\left(-1+\frac{5+2\sqrt{2}}{5-2\sqrt{2}}\right)-\sum_{k=1}^{\infty}\frac{\left(\frac{1}{8}\left(4-5\sqrt{2}\right)\right)^k}{k}}{4\sqrt{2}}}$$

$$\frac{(64+8)(-1)}{\frac{\log\left(\frac{1+2\sqrt{2}+4}{1-2\sqrt{2}+4}\right)}{4\sqrt{2}}+\frac{\tan^{-1}\left(\frac{2\sqrt{2}}{1-4}\right)}{2\sqrt{2}}}-47+\pi-\left(2-\sqrt{3}+\frac{1}{2}\right)=$$
$$-\frac{99}{2}+\sqrt{3}+\pi-\frac{72}{\frac{\log\left(-1+\frac{5+2\sqrt{2}}{5-2\sqrt{2}}\right)-\sum_{k=1}^{\infty}\frac{\left(\frac{1}{8}\left(4-5\sqrt{2}\right)\right)^k}{k}}{4\sqrt{2}}-\frac{\sum_{k=0}^{\infty}\frac{(-1)^k\,2^{3/2+3k}\times 3^{-1-2k}}{1+2k}}{2\sqrt{2}}}$$

$$\frac{(64+8)(-1)}{\frac{\log\left(\frac{1+2\sqrt{2}+4}{1-2\sqrt{2}+4}\right)}{4\sqrt{2}}+\frac{\tan^{-1}\left(\frac{2\sqrt{2}}{1-4}\right)}{2\sqrt{2}}}-47+\pi-\left(2-\sqrt{3}+\frac{1}{2}\right)=-\frac{99}{2}+\sqrt{3}+\pi-\frac{72}{\frac{\log\left(-1+\frac{5+2\sqrt{2}}{5-2\sqrt{2}}\right)-\sum_{k=1}^{\infty}\frac{\left(\frac{1}{8}\left(4-5\sqrt{2}\right)\right)^k}{k}}{4\sqrt{2}}-\frac{\tan^{-1}(z_0)+\frac{1}{2}i\sum_{k=1}^{\infty}\frac{\left(-(-i-z_0)^{-k}+(i-z_0)^{-k}\right)\left(\frac{2\sqrt{2}}{3}-z_0\right)^k}{k}}{2\sqrt{2}}}$$

for ($i z_0 \notin \mathbb{R}$ or ( not ($1 \le i z_0 < \infty$) and not ($-\infty < i z_0 \le -1$)))

## Integral representations:

$$\frac{(64+8)(-1)}{\frac{\log\left(\frac{1+2\sqrt{2}+4}{1-2\sqrt{2}+4}\right)}{4\sqrt{2}}+\frac{\tan^{-1}\left(\frac{2\sqrt{2}}{1-4}\right)}{2\sqrt{2}}}-47+\pi-\left(2-\sqrt{3}+\frac{1}{2}\right)=$$
$$-\frac{99}{2}+\sqrt{3}+\pi+\frac{1728}{72\int_0^1\frac{1}{9+8t^2}\,dt-3\sqrt{2}\log\left(\frac{1}{17}\left(33+20\sqrt{2}\right)\right)}$$

$$\frac{(64+8)(-1)}{\frac{\log\left(\frac{1+2\sqrt{2}+4}{1-2\sqrt{2}+4}\right)}{4\sqrt{2}}+\frac{\tan^{-1}\left(\frac{2\sqrt{2}}{1-4}\right)}{2\sqrt{2}}}-47+\pi-\left(2-\sqrt{3}+\frac{1}{2}\right)=-\frac{99}{2}+\sqrt{3}+\pi-\frac{72}{\frac{i}{12\pi^{3/2}}\int_{-i\infty+\gamma}^{i\infty+\gamma}\left(\frac{9}{17}\right)^s\Gamma\left(\frac{1}{2}-s\right)\Gamma(1-s)\Gamma(s)^2\,ds+\frac{\log\left(\frac{5+2\sqrt{2}}{5-2\sqrt{2}}\right)}{4\sqrt{2}}}\quad\text{for } 0<\gamma<\frac{1}{2}$$

$$\frac{(64+8)(-1)}{\frac{\log\left(\frac{1+2\sqrt{2}+4}{1-2\sqrt{2}+4}\right)}{4\sqrt{2}}+\frac{\tan^{-1}\left(\frac{2\sqrt{2}}{1-4}\right)}{2\sqrt{2}}}-47+\pi-\left(2-\sqrt{3}+\frac{1}{2}\right)=$$
$$-\frac{99}{2}+\sqrt{3}+\pi-\frac{72}{\int_1^{\frac{5+2\sqrt{2}}{5-2\sqrt{2}}}\left(-\frac{1}{3\left(-1+\frac{5+2\sqrt{2}}{5-2\sqrt{2}}\right)\left(1+\frac{8(1-t)^2}{9\left(1-\frac{5+2\sqrt{2}}{5-2\sqrt{2}}\right)^2}\right)}+\frac{1}{4\sqrt{2}\,t}\right)dt}$$

## Continued fraction representations:

$$\frac{(64+8)(-1)}{\frac{\log\left(\frac{1+2\sqrt{2}+4}{1-2\sqrt{2}+4}\right)}{4\sqrt{2}}+\frac{\tan^{-1}\left(\frac{2\sqrt{2}}{1-4}\right)}{2\sqrt{2}}}-47+\pi-\left(2-\sqrt{3}+\frac{1}{2}\right)=$$

$$-\frac{99}{2}+\sqrt{3}+\pi-\frac{72}{\frac{\log\left(\frac{1}{17}\left(33+20\sqrt{2}\right)\right)}{4\sqrt{2}}-\frac{1}{3\left(1+\underset{k=1}{\overset{\infty}{\mathrm{K}}}\frac{\frac{8k^2}{9}}{1+2k}\right)}}=$$

$$-\frac{99}{2}+\sqrt{3}+\pi-\frac{72}{\frac{\log\left(\frac{1}{17}\left(33+20\sqrt{2}\right)\right)}{4\sqrt{2}}-\frac{1}{3\left(1+\frac{8}{9\left(3+\frac{32}{9\left(5+\frac{8}{7+\frac{128}{9(9+\ldots)}}\right)}\right)}\right)}}$$

$$\frac{(64+8)(-1)}{\frac{\log\left(\frac{1+2\sqrt{2}+4}{1-2\sqrt{2}+4}\right)}{4\sqrt{2}}+\frac{\tan^{-1}\left(\frac{2\sqrt{2}}{1-4}\right)}{2\sqrt{2}}}-47+\pi-\left(2-\sqrt{3}+\frac{1}{2}\right)=$$

$$-\frac{99}{2}+\sqrt{3}+\pi-\frac{72}{\frac{\log\left(\frac{5+2\sqrt{2}}{5-2\sqrt{2}}\right)}{4\sqrt{2}}-\frac{1}{3\left(1+\underset{k=1}{\overset{\infty}{\mathrm{K}}}\frac{\frac{8k^2}{9}}{1+2k}\right)}}=$$

$$-\frac{99}{2}+\sqrt{3}+\pi-\frac{72}{\frac{\log\left(\frac{5+2\sqrt{2}}{5-2\sqrt{2}}\right)}{4\sqrt{2}}-\frac{1}{3\left(1+\frac{8}{9\left(3+\frac{32}{9\left(5+\frac{8}{7+\frac{128}{9(9+\ldots)}}\right)}\right)}\right)}}$$

$$\frac{(64+8)(-1)}{\frac{\log\left(\frac{1+2\sqrt{2}+4}{1-2\sqrt{2}+4}\right)}{4\sqrt{2}}+\frac{\tan^{-1}\left(\frac{2\sqrt{2}}{1-4}\right)}{2\sqrt{2}}}-47+\pi-\left(2-\sqrt{3}+\frac{1}{2}\right)=$$

$$-\frac{99}{2}+\sqrt{3}+\pi-\frac{72}{\frac{\log\left(\frac{1}{17}\left(33+20\sqrt{2}\right)\right)}{4\sqrt{2}}-\frac{1}{3\left(1+\underset{k=1}{\overset{\infty}{\mathrm{K}}}\frac{\frac{8}{9}(1-2k)^2}{\frac{1}{9}(17+2k)}\right)}}=$$

$$-\frac{99}{2}+\sqrt{3}+\pi-\frac{72}{\frac{\log\left(\frac{1}{17}\left(33+20\sqrt{2}\right)\right)}{4\sqrt{2}}-\frac{1}{3\left(1+\frac{8}{9\left(\frac{19}{9}+\frac{8}{\frac{7}{3}+\frac{200}{9\left(\frac{23}{9}+\frac{392}{9\left(\frac{25}{9}+\ldots\right)}\right)}}\right)}\right)}}$$

$\underset{k=k_1}{\overset{k_2}{\mathrm{K}}} a_k/b_k$ is a continued fraction

-5/((((1/(4sqrt2) ln (((1+2sqrt2+4)/(1-2sqrt2+4)))+1/(2sqrt2) tan^-1(((2sqrt2)/(1-4)))))))+18-golden ratio

**Input:**

$$-\frac{5}{\frac{1}{4\sqrt{2}}\log\left(\frac{1+2\sqrt{2}+4}{1-2\sqrt{2}+4}\right)+\frac{1}{2\sqrt{2}}\tan^{-1}\left(\frac{2\sqrt{2}}{1-4}\right)}+18-\phi$$

$\log(x)$ is the natural logarithm

$\tan^{-1}(x)$ is the inverse tangent function

$\phi$ is the golden ratio

**Exact Result:**

$$-\phi+18-\frac{5}{\frac{\log\left(\frac{5+2\sqrt{2}}{5-2\sqrt{2}}\right)}{4\sqrt{2}}-\frac{\tan^{-1}\left(\frac{2\sqrt{2}}{3}\right)}{2\sqrt{2}}}$$

(result in radians)

**Decimal approximation:**

139.5557744893490515472853513266333468858123288469297414193...

(result in radians)

139.555774489349… result practically equal to the rest mass of Pion meson 139.57 MeV

**Alternate forms:**

$$-\phi + 18 + \frac{20\sqrt{2}}{\log\left(\frac{1}{17}\left(33 - 20\sqrt{2}\right)\right) + 2\tan^{-1}\left(\frac{2\sqrt{2}}{3}\right)}$$

$$-\phi + 18 + \frac{20\sqrt{2}}{\log\left(\frac{17}{33+20\sqrt{2}}\right) + 2\tan^{-1}\left(\frac{2\sqrt{2}}{3}\right)}$$

$$-\phi + 18 - \frac{20\sqrt{2}}{\log\left(-\frac{5+2\sqrt{2}}{2\sqrt{2}-5}\right) - 2\tan^{-1}\left(\frac{2\sqrt{2}}{3}\right)}$$

**Alternative representations:**

$$-\frac{5}{\frac{\log\left(\frac{1+2\sqrt{2}+4}{1-2\sqrt{2}+4}\right)}{4\sqrt{2}} + \frac{\tan^{-1}\left(\frac{2\sqrt{2}}{1-4}\right)}{2\sqrt{2}}} + 18 - \phi = 18 - \phi - \frac{5}{\frac{\tan^{-1}\left(1, -\frac{2\sqrt{2}}{3}\right)}{2\sqrt{2}} + \frac{\log\left(\frac{5+2\sqrt{2}}{5-2\sqrt{2}}\right)}{4\sqrt{2}}}$$

$$-\frac{5}{\frac{\log\left(\frac{1+2\sqrt{2}+4}{1-2\sqrt{2}+4}\right)}{4\sqrt{2}} + \frac{\tan^{-1}\left(\frac{2\sqrt{2}}{1-4}\right)}{2\sqrt{2}}} + 18 - \phi = 18 - \phi - \frac{5}{\frac{\tan^{-1}\left(-\frac{2\sqrt{2}}{3}\right)}{2\sqrt{2}} + \frac{\log_e\left(\frac{5+2\sqrt{2}}{5-2\sqrt{2}}\right)}{4\sqrt{2}}}$$

$$-\frac{5}{\frac{\log\left(\frac{1+2\sqrt{2}+4}{1-2\sqrt{2}+4}\right)}{4\sqrt{2}} + \frac{\tan^{-1}\left(\frac{2\sqrt{2}}{1-4}\right)}{2\sqrt{2}}} + 18 - \phi = 18 - \phi - \frac{5}{\frac{\tan^{-1}\left(1, -\frac{2\sqrt{2}}{3}\right)}{2\sqrt{2}} + \frac{\log_e\left(\frac{5+2\sqrt{2}}{5-2\sqrt{2}}\right)}{4\sqrt{2}}}$$

**Series representations:**

$$-\frac{5}{\frac{\log\left(\frac{1+2\sqrt{2}+4}{1-2\sqrt{2}+4}\right)}{4\sqrt{2}} + \frac{\tan^{-1}\left(\frac{2\sqrt{2}}{1-4}\right)}{2\sqrt{2}}} + 18 - \phi =$$
$$\frac{35}{2} - \frac{\sqrt{5}}{2} + \frac{20\sqrt{2}}{2\tan^{-1}\left(\frac{2\sqrt{2}}{3}\right) + \log\left(\frac{1}{8}\left(-4 + 5\sqrt{2}\right)\right) + \sum_{k=1}^{\infty} \frac{\left(\frac{1}{8}\left(4 - 5\sqrt{2}\right)\right)^k}{k}}$$

$$-\frac{5}{\frac{\log\left(\frac{1+2\sqrt{2}+4}{1-2\sqrt{2}+4}\right)}{4\sqrt{2}}+\frac{\tan^{-1}\left(\frac{2\sqrt{2}}{1-4}\right)}{2\sqrt{2}}}+18-\phi=$$

$$-\left(\left(80-35\sqrt{2}\log\left(-1+\frac{5+2\sqrt{2}}{5-2\sqrt{2}}\right)+\sqrt{10}\log\left(-1+\frac{5+2\sqrt{2}}{5-2\sqrt{2}}\right)+35\sqrt{2}\sum_{k=1}^{\infty}\frac{\left(\frac{1}{8}\left(4-5\sqrt{2}\right)\right)^k}{k}-\sqrt{10}\sum_{k=1}^{\infty}\frac{\left(\frac{1}{8}\left(4-5\sqrt{2}\right)\right)^k}{k}+70\sqrt{2}\sum_{k=0}^{\infty}\frac{(-1)^k\, 2^{3/2+3k}\times 3^{-1-2k}}{1+2k}-2\sqrt{10}\sum_{k=0}^{\infty}\frac{(-1)^k\, 2^{3/2+3k}\times 3^{-1-2k}}{1+2k}\right)\Big/\left(2\sqrt{2}\left(\log\left(-1+\frac{5+2\sqrt{2}}{5-2\sqrt{2}}\right)-\sum_{k=1}^{\infty}\frac{\left(\frac{1}{8}\left(4-5\sqrt{2}\right)\right)^k}{k}-2\sum_{k=0}^{\infty}\frac{(-1)^k\, 2^{3/2+3k}\times 3^{-1-2k}}{1+2k}\right)\right)\right)$$

$$-\frac{5}{\frac{\log\left(\frac{1+2\sqrt{2}+4}{1-2\sqrt{2}+4}\right)}{4\sqrt{2}}+\frac{\tan^{-1}\left(\frac{2\sqrt{2}}{1-4}\right)}{2\sqrt{2}}}+18-\phi=$$

$$\left(80+70\sqrt{2}\tan^{-1}(z_0)-2\sqrt{10}\tan^{-1}(z_0)-35\sqrt{2}\log\left(-1+\frac{5+2\sqrt{2}}{5-2\sqrt{2}}\right)+\sqrt{10}\log\left(-1+\frac{5+2\sqrt{2}}{5-2\sqrt{2}}\right)+35\sqrt{2}\sum_{k=1}^{\infty}\frac{\left(\frac{1}{8}\left(4-5\sqrt{2}\right)\right)^k}{k}-\sqrt{10}\sum_{k=1}^{\infty}\frac{\left(\frac{1}{8}\left(4-5\sqrt{2}\right)\right)^k}{k}+35\,i\sqrt{2}\sum_{k=1}^{\infty}\frac{\left(-(-i-z_0)^{-k}+(i-z_0)^{-k}\right)\left(\frac{2\sqrt{2}}{3}-z_0\right)^k}{k}-i\sqrt{10}\sum_{k=1}^{\infty}\frac{\left(-(-i-z_0)^{-k}+(i-z_0)^{-k}\right)\left(\frac{2\sqrt{2}}{3}-z_0\right)^k}{k}\right)\Big/\left(2\sqrt{2}\left(2\tan^{-1}(z_0)-\log\left(-1+\frac{5+2\sqrt{2}}{5-2\sqrt{2}}\right)+\sum_{k=1}^{\infty}\frac{\left(\frac{1}{8}\left(4-5\sqrt{2}\right)\right)^k}{k}+i\sum_{k=1}^{\infty}\frac{\left(-(-i-z_0)^{-k}+(i-z_0)^{-k}\right)\left(\frac{2\sqrt{2}}{3}-z_0\right)^k}{k}\right)\right)$$

for $(i z_0 \notin \mathbb{R}$ or ( not $(1 \le i z_0 < \infty)$ and not $(-\infty < i z_0 \le -1)))$

## Integral representations:

$$-\frac{5}{\frac{\log\left(\frac{1+2\sqrt{2}+4}{1-2\sqrt{2}+4}\right)}{4\sqrt{2}}+\frac{\tan^{-1}\left(\frac{2\sqrt{2}}{1-4}\right)}{2\sqrt{2}}}+18-\phi = \frac{35}{2}-\frac{\sqrt{5}}{2}+\frac{120}{72\int_0^1 \frac{1}{9+8t^2}\,dt-3\sqrt{2}\log\left(\frac{1}{17}\left(33+20\sqrt{2}\right)\right)}$$

$$-\frac{5}{\frac{\log\left(\frac{1+2\sqrt{2}+4}{1-2\sqrt{2}+4}\right)}{4\sqrt{2}}+\frac{\tan^{-1}\left(\frac{2\sqrt{2}}{1-4}\right)}{2\sqrt{2}}}+18-\phi = \frac{35}{2}-\frac{\sqrt{5}}{2}-\frac{120}{\int_1^{\frac{1}{17}\left(33+20\sqrt{2}\right)}\left(-\frac{72}{\left(-1+\frac{1}{17}\left(33+20\sqrt{2}\right)\right)\left(9+\frac{8(1-t)^2}{\left(1+\frac{1}{17}\left(-33-20\sqrt{2}\right)\right)^2}\right)}+\frac{3\sqrt{2}}{t}\right)dt}$$

$$-\frac{5}{\frac{\log\left(\frac{1+2\sqrt{2}+4}{1-2\sqrt{2}+4}\right)}{4\sqrt{2}}+\frac{\tan^{-1}\left(\frac{2\sqrt{2}}{1-4}\right)}{2\sqrt{2}}}+18-\phi = \frac{35}{2}-\frac{\sqrt{5}}{2}-\frac{5}{\frac{i}{12\pi^{3/2}}\int_{-i\infty+\gamma}^{i\infty+\gamma}\left(\frac{9}{17}\right)^s\Gamma\left(\frac{1}{2}-s\right)\Gamma(1-s)\,\Gamma(s)^2\,ds+\frac{\log\left(\frac{5+2\sqrt{2}}{5-2\sqrt{2}}\right)}{4\sqrt{2}}} \quad \text{for } 0<\gamma<\frac{1}{2}$$

## Continued fraction representations:

$$-\frac{5}{\frac{\log\left(\frac{1+2\sqrt{2}+4}{1-2\sqrt{2}+4}\right)}{4\sqrt{2}}+\frac{\tan^{-1}\left(\frac{2\sqrt{2}}{1-4}\right)}{2\sqrt{2}}}+18-\phi = 18-\phi-\frac{5}{\frac{\log\left(\frac{1}{17}\left(33+20\sqrt{2}\right)\right)}{4\sqrt{2}}-\frac{1}{3\left(1+\underset{k=1}{\overset{\infty}{\mathrm{K}}}\frac{\frac{8k^2}{9}}{1+2k}\right)}} =$$

$$18-\phi-\frac{5}{\frac{\log\left(\frac{1}{17}\left(33+20\sqrt{2}\right)\right)}{4\sqrt{2}}-\frac{1}{3\left(1+\frac{8}{9\left(3+\frac{32}{9\left(5+\frac{8}{7+\frac{128}{9(9+\dots)}}\right)}\right)}\right)}}$$

$$-\frac{5}{\frac{\log\left(\frac{1+2\sqrt{2}+4}{1-2\sqrt{2}+4}\right)}{4\sqrt{2}}+\frac{\tan^{-1}\left(\frac{2\sqrt{2}}{1-4}\right)}{2\sqrt{2}}}+18-\phi=18-\phi-\frac{5}{\frac{\log\left(\frac{5+2\sqrt{2}}{5-2\sqrt{2}}\right)}{4\sqrt{2}}-\frac{1}{3\left(1+\underset{k=1}{\overset{\infty}{\mathrm{K}}}\frac{\frac{8k^2}{9}}{1+2k}\right)}}=$$

$$18-\phi-\frac{5}{\frac{\log\left(\frac{5+2\sqrt{2}}{5-2\sqrt{2}}\right)}{4\sqrt{2}}-\frac{1}{3\left(1+\frac{8}{9\left(3+\frac{32}{9\left(5+\frac{8}{7+\frac{128}{9(9+\ldots)}}\right)}\right)}\right)}}$$

$$-\frac{5}{\frac{\log\left(\frac{1+2\sqrt{2}+4}{1-2\sqrt{2}+4}\right)}{4\sqrt{2}}+\frac{\tan^{-1}\left(\frac{2\sqrt{2}}{1-4}\right)}{2\sqrt{2}}}+18-\phi=18-\phi-\frac{5}{\frac{\log\left(\frac{1}{17}\left(33+20\sqrt{2}\right)\right)}{4\sqrt{2}}-\frac{1}{3\left(1+\underset{k=1}{\overset{\infty}{\mathrm{K}}}\frac{\frac{8}{9}(1-2k)^2}{\frac{1}{9}(17+2k)}\right)}}=$$

$$18-\phi-\frac{5}{\frac{\log\left(\frac{1}{17}\left(33+20\sqrt{2}\right)\right)}{4\sqrt{2}}-\frac{1}{3\left(1+\frac{8}{9\left(\frac{19}{9}+\frac{8}{\frac{7}{3}+\frac{200}{9\left(\frac{23}{9}+\frac{392}{9\left(\frac{25}{9}+\ldots\right)}\right)}}\right)}\right)}}$$

$\underset{k=k_1}{\overset{k_2}{\mathrm{K}}} a_k / b_k$ is a continued fraction

-5/((((1/(4sqrt2) ln (((1+2sqrt2+4)/(1-2sqrt2+4)))+1/(2sqrt2) tan^-1(((2sqrt2)/(1-4)))))))+4-golden ratio

**Input:**

$$-\frac{5}{\frac{1}{4\sqrt{2}}\log\left(\frac{1+2\sqrt{2}+4}{1-2\sqrt{2}+4}\right)+\frac{1}{2\sqrt{2}}\tan^{-1}\left(\frac{2\sqrt{2}}{1-4}\right)}+4-\phi$$

log(x) is the natural logarithm

$\tan^{-1}(x)$ is the inverse tangent function

$\phi$ is the golden ratio

**Exact Result:**

$$-\phi+4-\frac{5}{\frac{\log\left(\frac{5+2\sqrt{2}}{5-2\sqrt{2}}\right)}{4\sqrt{2}}-\frac{\tan^{-1}\left(\frac{2\sqrt{2}}{3}\right)}{2\sqrt{2}}}$$

(result in radians)

**Decimal approximation:**

125.5557744893490515472853513266333468858123288469297414193...

(result in radians)

125.55577448934… result very near to the dilaton mass calculated as a type of Higgs boson: 125 GeV for T = 0 and to the Higgs boson mass 125.18 GeV

**Alternate forms:**

$$-\phi+4+\frac{20\sqrt{2}}{\log\left(\frac{1}{17}\left(33-20\sqrt{2}\right)\right)+2\tan^{-1}\left(\frac{2\sqrt{2}}{3}\right)}$$

$$-\phi+4+\frac{20\sqrt{2}}{\log\left(\frac{17}{33+20\sqrt{2}}\right)+2\tan^{-1}\left(\frac{2\sqrt{2}}{3}\right)}$$

$$-\phi+4-\frac{20\sqrt{2}}{\log\left(-\frac{5+2\sqrt{2}}{2\sqrt{2}-5}\right)-2\tan^{-1}\left(\frac{2\sqrt{2}}{3}\right)}$$

## Alternative representations:

$$-\frac{5}{\frac{\log\left(\frac{1+2\sqrt{2}+4}{1-2\sqrt{2}+4}\right)}{4\sqrt{2}}+\frac{\tan^{-1}\left(\frac{2\sqrt{2}}{1-4}\right)}{2\sqrt{2}}}+4-\phi=4-\phi-\frac{5}{\frac{\tan^{-1}\left(1,-\frac{2\sqrt{2}}{3}\right)}{2\sqrt{2}}+\frac{\log\left(\frac{5+2\sqrt{2}}{5-2\sqrt{2}}\right)}{4\sqrt{2}}}$$

$$-\frac{5}{\frac{\log\left(\frac{1+2\sqrt{2}+4}{1-2\sqrt{2}+4}\right)}{4\sqrt{2}}+\frac{\tan^{-1}\left(\frac{2\sqrt{2}}{1-4}\right)}{2\sqrt{2}}}+4-\phi=4-\phi-\frac{5}{\frac{\tan^{-1}\left(-\frac{2\sqrt{2}}{3}\right)}{2\sqrt{2}}+\frac{\log_e\left(\frac{5+2\sqrt{2}}{5-2\sqrt{2}}\right)}{4\sqrt{2}}}$$

$$-\frac{5}{\frac{\log\left(\frac{1+2\sqrt{2}+4}{1-2\sqrt{2}+4}\right)}{4\sqrt{2}}+\frac{\tan^{-1}\left(\frac{2\sqrt{2}}{1-4}\right)}{2\sqrt{2}}}+4-\phi=4-\phi-\frac{5}{\frac{\tan^{-1}\left(1,-\frac{2\sqrt{2}}{3}\right)}{2\sqrt{2}}+\frac{\log_e\left(\frac{5+2\sqrt{2}}{5-2\sqrt{2}}\right)}{4\sqrt{2}}}$$

## Series representations:

$$-\frac{5}{\frac{\log\left(\frac{1+2\sqrt{2}+4}{1-2\sqrt{2}+4}\right)}{4\sqrt{2}}+\frac{\tan^{-1}\left(\frac{2\sqrt{2}}{1-4}\right)}{2\sqrt{2}}}+4-\phi=$$
$$\frac{7}{2}-\frac{\sqrt{5}}{2}+\frac{20\sqrt{2}}{2\tan^{-1}\left(\frac{2\sqrt{2}}{3}\right)+\log\left(\frac{1}{8}\left(-4+5\sqrt{2}\right)\right)+\sum_{k=1}^{\infty}\frac{\left(\frac{1}{8}\left(4-5\sqrt{2}\right)\right)^k}{k}}$$

$$-\frac{5}{\frac{\log\left(\frac{1+2\sqrt{2}+4}{1-2\sqrt{2}+4}\right)}{4\sqrt{2}}+\frac{\tan^{-1}\left(\frac{2\sqrt{2}}{1-4}\right)}{2\sqrt{2}}}+4-\phi=$$
$$-\left(\left(80-7\sqrt{2}\log\left(-1+\frac{5+2\sqrt{2}}{5-2\sqrt{2}}\right)+\sqrt{10}\log\left(-1+\frac{5+2\sqrt{2}}{5-2\sqrt{2}}\right)+7\sqrt{2}\sum_{k=1}^{\infty}\frac{\left(\frac{1}{8}\left(4-5\sqrt{2}\right)\right)^k}{k}-\sqrt{10}\sum_{k=1}^{\infty}\frac{\left(\frac{1}{8}\left(4-5\sqrt{2}\right)\right)^k}{k}+14\sqrt{2}\sum_{k=0}^{\infty}\frac{(-1)^k\,2^{3/2+3k}\times 3^{-1-2k}}{1+2k}-2\sqrt{10}\sum_{k=0}^{\infty}\frac{(-1)^k\,2^{3/2+3k}\times 3^{-1-2k}}{1+2k}\right)\Big/\left(2\sqrt{2}\left(\log\left(-1+\frac{5+2\sqrt{2}}{5-2\sqrt{2}}\right)-\sum_{k=1}^{\infty}\frac{\left(\frac{1}{8}\left(4-5\sqrt{2}\right)\right)^k}{k}-2\sum_{k=0}^{\infty}\frac{(-1)^k\,2^{3/2+3k}\times 3^{-1-2k}}{1+2k}\right)\right)\right)$$

$$-\frac{5}{\frac{\log\left(\frac{1+2\sqrt{2}+4}{1-2\sqrt{2}+4}\right)}{4\sqrt{2}}+\frac{\tan^{-1}\left(\frac{2\sqrt{2}}{1-4}\right)}{2\sqrt{2}}}+4-\phi=$$

$$\left(80+14\sqrt{2}\tan^{-1}(z_0)-2\sqrt{10}\tan^{-1}(z_0)-7\sqrt{2}\log\left(-1+\frac{5+2\sqrt{2}}{5-2\sqrt{2}}\right)+\sqrt{10}\log\left(-1+\frac{5+2\sqrt{2}}{5-2\sqrt{2}}\right)+7\sqrt{2}\sum_{k=1}^{\infty}\frac{\left(\frac{1}{8}\left(4-5\sqrt{2}\right)\right)^k}{k}-\sqrt{10}\sum_{k=1}^{\infty}\frac{\left(\frac{1}{8}\left(4-5\sqrt{2}\right)\right)^k}{k}+7i\sqrt{2}\sum_{k=1}^{\infty}\frac{\left(-(-i-z_0)^{-k}+(i-z_0)^{-k}\right)\left(\frac{2\sqrt{2}}{3}-z_0\right)^k}{k}-i\sqrt{10}\sum_{k=1}^{\infty}\frac{\left(-(-i-z_0)^{-k}+(i-z_0)^{-k}\right)\left(\frac{2\sqrt{2}}{3}-z_0\right)^k}{k}\right)\Bigg/\left(2\sqrt{2}\left(2\tan^{-1}(z_0)-\log\left(-1+\frac{5+2\sqrt{2}}{5-2\sqrt{2}}\right)+\sum_{k=1}^{\infty}\frac{\left(\frac{1}{8}\left(4-5\sqrt{2}\right)\right)^k}{k}+i\sum_{k=1}^{\infty}\frac{\left(-(-i-z_0)^{-k}+(i-z_0)^{-k}\right)\left(\frac{2\sqrt{2}}{3}-z_0\right)^k}{k}\right)\right)$$

for ($i z_0 \notin \mathbb{R}$ or ( not ($1 \le i z_0 < \infty$) and not ($-\infty < i z_0 \le -1$)))

# Integral representations:

$$-\frac{5}{\frac{\log\left(\frac{1+2\sqrt{2}+4}{1-2\sqrt{2}+4}\right)}{4\sqrt{2}}+\frac{\tan^{-1}\left(\frac{2\sqrt{2}}{1-4}\right)}{2\sqrt{2}}}+4-\phi=\frac{7}{2}-\frac{\sqrt{5}}{2}+\frac{120}{72\int_0^1\frac{1}{9+8t^2}\,dt-3\sqrt{2}\log\left(\frac{1}{17}\left(33+20\sqrt{2}\right)\right)}$$

$$-\frac{5}{\frac{\log\left(\frac{1+2\sqrt{2}+4}{1-2\sqrt{2}+4}\right)}{4\sqrt{2}}+\frac{\tan^{-1}\left(\frac{2\sqrt{2}}{1-4}\right)}{2\sqrt{2}}}+4-\phi=\frac{7}{2}-\frac{\sqrt{5}}{2}-\frac{120}{\int_1^{\frac{1}{17}\left(33+20\sqrt{2}\right)}\left(-\frac{72}{\left(-1+\frac{1}{17}\left(33+20\sqrt{2}\right)\right)\left(9+\frac{8(1-t)^2}{\left(1+\frac{1}{17}\left(-33-20\sqrt{2}\right)\right)^2}\right)}+\frac{3\sqrt{2}}{t}\right)dt}$$

$$-\frac{5}{\frac{\log\left(\frac{1+2\sqrt{2}+4}{1-2\sqrt{2}+4}\right)}{4\sqrt{2}}+\frac{\tan^{-1}\left(\frac{2\sqrt{2}}{1-4}\right)}{2\sqrt{2}}}+4-\phi=$$

$$\frac{7}{2}-\frac{\sqrt{5}}{2}-\frac{5}{\frac{i}{12\pi^{3/2}}\int_{-i\infty+\gamma}^{i\infty+\gamma}\left(\frac{9}{17}\right)^s\Gamma\left(\frac{1}{2}-s\right)\Gamma(1-s)\Gamma(s)^2\,ds+\frac{\log\left(\frac{5+2\sqrt{2}}{5-2\sqrt{2}}\right)}{4\sqrt{2}}} \quad \text{for } 0<\gamma<\frac{1}{2}$$

## Continued fraction representations:

$$-\frac{5}{\frac{\log\left(\frac{1+2\sqrt{2}+4}{1-2\sqrt{2}+4}\right)}{4\sqrt{2}}+\frac{\tan^{-1}\left(\frac{2\sqrt{2}}{1-4}\right)}{2\sqrt{2}}}+4-\phi=4-\phi-\frac{5}{\frac{\log\left(\frac{1}{17}\left(33+20\sqrt{2}\right)\right)}{4\sqrt{2}}-\frac{1}{3\left(1+\underset{k=1}{\overset{\infty}{\mathrm{K}}}\frac{\frac{8k^2}{9}}{1+2k}\right)}}=$$

$$4-\phi-\frac{5}{\frac{\log\left(\frac{1}{17}\left(33+20\sqrt{2}\right)\right)}{4\sqrt{2}}-\frac{1}{3\left(1+\frac{8}{9\left(3+\frac{32}{9\left(5+\frac{8}{7+\frac{128}{9(9+\dots)}}\right)}\right)}\right)}}$$

$$-\frac{5}{\frac{\log\left(\frac{1+2\sqrt{2}+4}{1-2\sqrt{2}+4}\right)}{4\sqrt{2}}+\frac{\tan^{-1}\left(\frac{2\sqrt{2}}{1-4}\right)}{2\sqrt{2}}}+4-\phi=4-\phi-\frac{5}{\frac{\log\left(\frac{5+2\sqrt{2}}{5-2\sqrt{2}}\right)}{4\sqrt{2}}-\frac{1}{3\left(1+\underset{k=1}{\overset{\infty}{\mathrm{K}}}\frac{\frac{8k^2}{9}}{1+2k}\right)}}=$$

$$4-\phi-\frac{5}{\frac{\log\left(\frac{5+2\sqrt{2}}{5-2\sqrt{2}}\right)}{4\sqrt{2}}-\frac{1}{3\left(1+\frac{8}{9\left(3+\frac{32}{9\left(5+\frac{8}{7+\frac{128}{9(9+\dots)}}\right)}\right)}\right)}}$$

$$-\frac{5}{\frac{\log\left(\frac{1+2\sqrt{2}+4}{1-2\sqrt{2}+4}\right)}{4\sqrt{2}}+\frac{\tan^{-1}\left(\frac{2\sqrt{2}}{1-4}\right)}{2\sqrt{2}}}+4-\phi=4-\phi-\frac{5}{\frac{\log\left(\frac{1}{17}\left(33+20\sqrt{2}\right)\right)}{4\sqrt{2}}-\frac{1}{3\left(1+\underset{k=1}{\overset{\infty}{\mathrm{K}}}\frac{\frac{8}{9}(1-2k)^2}{\frac{1}{9}(17+2k)}\right)}}=$$

$$4-\phi-\frac{5}{\frac{\log\left(\frac{1}{17}\left(33+20\sqrt{2}\right)\right)}{4\sqrt{2}}-\frac{1}{3\left(1+\frac{8}{9\left(\frac{19}{9}+\frac{8}{\frac{7}{3}+\frac{200}{9\left(\frac{23}{9}+\frac{392}{9\left(\frac{25}{9}+\ldots\right)}\right)}}\right)}\right)}}$$

$\underset{k=k_1}{\overset{k_2}{\mathrm{K}}} a_k / b_k$ is a continued fraction

Now, we have that:

$$A_3=\frac{1}{6}\log\frac{(1+x)^3}{1+x^3}+\frac{1}{\sqrt{3}}\tan^{-1}\frac{x\sqrt{3}}{2-x}$$

For x = -2 and multiplying all the expression by -1, we obtain:

-((1/6 ln (((1-2)^3)/(1-8)) + 1/sqrt3 tan^-1 (-2sqrt3/(2+2))))

**Input:**

$$-\left(\frac{1}{6}\log\left(\frac{(1-2)^3}{1-8}\right)+\frac{1}{\sqrt{3}}\tan^{-1}\left(-2\times\frac{\sqrt{3}}{2+2}\right)\right)$$

$\log(x)$ is the natural logarithm

$\tan^{-1}(x)$ is the inverse tangent function

**Exact Result:**

$$\frac{\log(7)}{6} + \frac{\tan^{-1}\left(\frac{\sqrt{3}}{2}\right)}{\sqrt{3}}$$

(result in radians)

**Decimal approximation:**

0.736387320486844454951909129191439952702295682177676137042...

(result in radians)

0.7363873204…

**Alternate forms:**

$$\frac{\log(7)}{6} + \frac{\cot^{-1}\left(\frac{2}{\sqrt{3}}\right)}{\sqrt{3}}$$

$$\frac{1}{6}\left(\log(7) + 2\sqrt{3}\cot^{-1}\left(\frac{2}{\sqrt{3}}\right)\right)$$

$$\frac{1}{6}\left(\log(7) + 2\sqrt{3}\tan^{-1}\left(\frac{\sqrt{3}}{2}\right)\right)$$

$\cot^{-1}(x)$ is the inverse cotangent function

**Alternative representations:**

$$-\left(\frac{1}{6}\log\left(\frac{(1-2)^3}{1-8}\right) + \frac{\tan^{-1}\left(-\frac{2\sqrt{3}}{2+2}\right)}{\sqrt{3}}\right) = -\frac{1}{6}\log\left(\frac{-1}{-7}\right) - \frac{\tan^{-1}\left(1, -\frac{2\sqrt{3}}{4}\right)}{\sqrt{3}}$$

$$-\left(\frac{1}{6}\log\left(\frac{(1-2)^3}{1-8}\right) + \frac{\tan^{-1}\left(-\frac{2\sqrt{3}}{2+2}\right)}{\sqrt{3}}\right) = -\frac{1}{6}\log_e\left(\frac{-1}{-7}\right) - \frac{\tan^{-1}\left(-\frac{2\sqrt{3}}{4}\right)}{\sqrt{3}}$$

$$-\left(\frac{1}{6}\log\left(\frac{(1-2)^3}{1-8}\right) + \frac{\tan^{-1}\left(-\frac{2\sqrt{3}}{2+2}\right)}{\sqrt{3}}\right) = -\frac{1}{6}\log(a)\log_a\left(\frac{-1}{-7}\right) - \frac{\tan^{-1}\left(-\frac{2\sqrt{3}}{4}\right)}{\sqrt{3}}$$

**Series representations:**

$$-\left(\frac{1}{6}\log\left(\frac{(1-2)^3}{1-8}\right) + \frac{\tan^{-1}\left(-\frac{2\sqrt{3}}{2+2}\right)}{\sqrt{3}}\right) = \frac{\tan^{-1}\left(\frac{\sqrt{3}}{2}\right)}{\sqrt{3}} + \frac{\log(6)}{6} - \frac{1}{6}\sum_{k=1}^{\infty}\frac{\left(-\frac{1}{6}\right)^k}{k}$$

$$-\left(\frac{1}{6}\log\left(\frac{(1-2)^3}{1-8}\right)+\frac{\tan^{-1}\left(-\frac{2\sqrt{3}}{2+2}\right)}{\sqrt{3}}\right)=$$
$$\frac{1}{6}\left(\log(6)-\sum_{k=1}^{\infty}\frac{\left(-\frac{1}{6}\right)^k}{k}+2\sqrt{3}\sum_{k=0}^{\infty}\frac{(-1)^k\,2^{-1-2k}\times 3^{1/2+k}}{1+2k}\right)$$

$$-\left(\frac{1}{6}\log\left(\frac{(1-2)^3}{1-8}\right)+\frac{\tan^{-1}\left(-\frac{2\sqrt{3}}{2+2}\right)}{\sqrt{3}}\right)=$$
$$\frac{\tan^{-1}(z_0)}{\sqrt{3}}+\frac{\log(6)}{6}+\sum_{k=1}^{\infty}\left(\frac{(-1)^{-1+k}\,6^{-1-k}}{k}+\frac{i\left(-(-i-z_0)^{-k}+(i-z_0)^{-k}\right)\left(\frac{\sqrt{3}}{2}-z_0\right)^k}{2\sqrt{3}\,k}\right)$$
for $(i z_0 \notin \mathbb{R}$ or ( not $(1 \leq i z_0 < \infty)$ and not $(-\infty < i z_0 \leq -1)))$

$$-\left(\frac{1}{6}\log\left(\frac{(1-2)^3}{1-8}\right)+\frac{\tan^{-1}\left(-\frac{2\sqrt{3}}{2+2}\right)}{\sqrt{3}}\right)=$$
$$\frac{\tan^{-1}(z_0)}{\sqrt{3}}+\frac{\log(6)}{6}+\sum_{k=1}^{\infty}\left(\frac{\left(-\frac{1}{6}\right)^{1+k}}{k}+\frac{i\left(-(-i-z_0)^{-k}+(i-z_0)^{-k}\right)\left(\frac{\sqrt{3}}{2}-z_0\right)^k}{2\sqrt{3}\,k}\right)$$
for $(i z_0 \notin \mathbb{R}$ or ( not $(1 \leq i z_0 < \infty)$ and not $(-\infty < i z_0 \leq -1)))$

# Integral representations:

$$-\left(\frac{1}{6}\log\left(\frac{(1-2)^3}{1-8}\right)+\frac{\tan^{-1}\left(-\frac{2\sqrt{3}}{2+2}\right)}{\sqrt{3}}\right)=\int_1^7\left(\frac{1}{6t}+\frac{4}{49-2t+t^2}\right)dt$$

$$-\left(\frac{1}{6}\log\left(\frac{(1-2)^3}{1-8}\right)+\frac{\tan^{-1}\left(-\frac{2\sqrt{3}}{2+2}\right)}{\sqrt{3}}\right)=2\int_0^1\frac{1}{4+3t^2}\,dt+\frac{\log(7)}{6}$$

$$-\left(\frac{1}{6}\log\left(\frac{(1-2)^3}{1-8}\right)+\frac{\tan^{-1}\left(-\frac{2\sqrt{3}}{2+2}\right)}{\sqrt{3}}\right)=$$
$$-\frac{i}{8\pi^{3/2}}\int_{-i\infty+\gamma}^{i\infty+\gamma}\left(\frac{4}{7}\right)^s\Gamma\left(\frac{1}{2}-s\right)\Gamma(1-s)\,\Gamma(s)^2\,ds+\frac{\log(7)}{6}\quad\text{for } 0<\gamma<\frac{1}{2}$$

## Continued fraction representations:

$$-\left(\frac{1}{6}\log\left(\frac{(1-2)^3}{1-8}\right)+\frac{\tan^{-1}\left(-\frac{2\sqrt{3}}{2+2}\right)}{\sqrt{3}}\right)=$$

$$\frac{1}{6}\left(\log(7)+\frac{3}{1+\underset{k=1}{\overset{\infty}{\mathrm{K}}}\frac{\frac{3k^2}{4}}{1+2k}}\right)=\frac{1}{6}\left(\log(7)+\frac{3}{1+\frac{3}{4\left(3+\frac{3}{5+\frac{27}{4\left(7+\frac{12}{9+\dots}\right)}}\right)}}\right)$$

$$-\left(\frac{1}{6}\log\left(\frac{(1-2)^3}{1-8}\right)+\frac{\tan^{-1}\left(-\frac{2\sqrt{3}}{2+2}\right)}{\sqrt{3}}\right)=\frac{1}{6}\left(\log(7)+\frac{3}{1+\underset{k=1}{\overset{\infty}{\mathrm{K}}}\frac{\frac{3}{4}(1-2k)^2}{\frac{1}{4}(7+2k)}}\right)=$$

$$\frac{1}{6}\left(\log(7)+\frac{3}{1+\frac{3}{4\left(\frac{9}{4}+\frac{27}{4\left(\frac{11}{4}+\frac{75}{4\left(\frac{13}{4}+\frac{147}{4\left(\frac{15}{4}+\dots\right)}\right)}\right)}\right)}}\right)$$

$$-\left(\frac{1}{6}\log\left(\frac{(1-2)^3}{1-8}\right)+\frac{\tan^{-1}\left(-\frac{2\sqrt{3}}{2+2}\right)}{\sqrt{3}}\right)=\frac{1}{2}+\frac{\log(7)}{6}-\frac{3}{8\left(3+\underset{k=1}{\overset{\infty}{\mathrm{K}}}\frac{\frac{3}{4}\left(1+(-1)^{1+k}+k\right)^2}{3+2k}\right)}=$$

$$\frac{1}{2}+\frac{\log(7)}{6}-\frac{3}{8\left(3+\frac{27}{4\left(5+\frac{3}{7+\frac{75}{4\left(9+\frac{12}{11+\dots}\right)}}\right)}\right)}$$

$\underset{k=k_1}{\overset{k_2}{\mathrm{K}}} a_k / b_k$ is a continued fraction

48/(((-((1/6 ln (((1-2)^3)/(1-8)) + 1/sqrt3 tan^-1 (-2sqrt3/(2+2)))))))*2+11-2

**Input:**

$$-\frac{48}{\frac{1}{6}\log\left(\frac{(1-2)^3}{1-8}\right)+\frac{1}{\sqrt{3}}\tan^{-1}\left(-2\times\frac{\sqrt{3}}{2+2}\right)}\times 2+11-2$$

$\log(x)$ is the natural logarithm

$\tan^{-1}(x)$ is the inverse tangent function

**Exact Result:**

$$9+\frac{96}{\frac{\log(7)}{6}+\frac{\tan^{-1}\left(\frac{\sqrt{3}}{2}\right)}{\sqrt{3}}}$$

(result in radians)

**Decimal approximation:**

139.3661773216463714471334671833755532255719032123699111372...

(result in radians)

139.36617732… result practically equal to the rest mass of Pion meson 139.57 MeV

**Alternate forms:**

$$9+\frac{576}{\log(7)+2\sqrt{3}\,\cot^{-1}\left(\frac{2}{\sqrt{3}}\right)}$$

$$9+\frac{576}{\log(7)+2\sqrt{3}\,\tan^{-1}\left(\frac{\sqrt{3}}{2}\right)}$$

$$9+\frac{96}{\frac{\log(7)}{6}+\frac{\cot^{-1}\left(\frac{2}{\sqrt{3}}\right)}{\sqrt{3}}}$$

$\cot^{-1}(x)$ is the inverse cotangent function

**Alternative representations:**

$$-\frac{2\times 48}{\frac{1}{6}\log\left(\frac{(1-2)^3}{1-8}\right)+\frac{\tan^{-1}\left(-\frac{2\sqrt{3}}{2+2}\right)}{\sqrt{3}}}+11-2=9+\frac{96}{-\frac{1}{6}\log\left(\frac{-1}{-7}\right)-\frac{\tan^{-1}\left(1,-\frac{2\sqrt{3}}{4}\right)}{\sqrt{3}}}$$

$$-\frac{2\times 48}{\frac{1}{6}\log\left(\frac{(1-2)^3}{1-8}\right)+\frac{\tan^{-1}\left(-\frac{2\sqrt{3}}{2+2}\right)}{\sqrt{3}}}+11-2=9+\frac{96}{-\frac{1}{6}\log_e\left(\frac{-1}{-7}\right)-\frac{\tan^{-1}\left(-\frac{2\sqrt{3}}{4}\right)}{\sqrt{3}}}$$

$$-\frac{2\times 48}{\frac{1}{6}\log\left(\frac{(1-2)^3}{1-8}\right)+\frac{\tan^{-1}\left(-\frac{2\sqrt{3}}{2+2}\right)}{\sqrt{3}}}+11-2=9+\frac{96}{-\frac{1}{6}\log_e\left(\frac{-1}{-7}\right)-\frac{\tan^{-1}\left(1,-\frac{2\sqrt{3}}{4}\right)}{\sqrt{3}}}$$

**Series representations:**

$$-\frac{2\times 48}{\frac{1}{6}\log\left(\frac{(1-2)^3}{1-8}\right)+\frac{\tan^{-1}\left(-\frac{2\sqrt{3}}{2+2}\right)}{\sqrt{3}}}+11-2=9+\frac{576}{2\sqrt{3}\,\tan^{-1}\left(\frac{\sqrt{3}}{2}\right)+\log(6)-\sum_{k=1}^{\infty}\frac{\left(-\frac{1}{6}\right)^k}{k}}$$

$$-\frac{2\times 48}{\frac{1}{6}\log\left(\frac{(1-2)^3}{1-8}\right)+\frac{\tan^{-1}\left(-\frac{2\sqrt{3}}{2+2}\right)}{\sqrt{3}}}+11-2=$$
$$\frac{9\left(64+\log(6)-\sum_{k=1}^{\infty}\frac{\left(-\frac{1}{6}\right)^k}{k}+2\sqrt{3}\sum_{k=0}^{\infty}\frac{(-1)^k\,2^{-1-2k}\times 3^{1/2+k}}{1+2k}\right)}{\log(6)-\sum_{k=1}^{\infty}\frac{\left(-\frac{1}{6}\right)^k}{k}+2\sqrt{3}\sum_{k=0}^{\infty}\frac{(-1)^k\,2^{-1-2k}\times 3^{1/2+k}}{1+2k}}$$

$$-\frac{2\times 48}{\frac{1}{6}\log\left(\frac{(1-2)^3}{1-8}\right)+\frac{\tan^{-1}\left(-\frac{2\sqrt{3}}{2+2}\right)}{\sqrt{3}}}+11-2=$$

$$\left(9\left(64+2\sqrt{3}\tan^{-1}(z_0)+\log(6)-\sum_{k=1}^{\infty}\frac{\left(-\frac{1}{6}\right)^k}{k}+i\sqrt{3}\sum_{k=1}^{\infty}\frac{\left(-(-i-z_0)^{-k}+(i-z_0)^{-k}\right)\left(\frac{\sqrt{3}}{2}-z_0\right)^k}{k}\right)\right)\Big/\left(2\sqrt{3}\tan^{-1}(z_0)+\log(6)-\sum_{k=1}^{\infty}\frac{\left(-\frac{1}{6}\right)^k}{k}+i\sqrt{3}\sum_{k=1}^{\infty}\frac{\left(-(-i-z_0)^{-k}+(i-z_0)^{-k}\right)\left(\frac{\sqrt{3}}{2}-z_0\right)^k}{k}\right)$$

for $(i z_0 \notin \mathbb{R}$ or ( not $(1 \le i z_0 < \infty)$ and not $(-\infty < i z_0 \le -1)))$

**Integral representations:**

$$-\frac{2\times 48}{\frac{1}{6}\log\left(\frac{(1-2)^3}{1-8}\right)+\frac{\tan^{-1}\left(-\frac{2\sqrt{3}}{2+2}\right)}{\sqrt{3}}}+11-2=9+\frac{576}{\int_1^7\left(\frac{1}{t}+\frac{24}{49-2t+t^2}\right)dt}$$

$$-\frac{2\times 48}{\frac{1}{6}\log\left(\frac{(1-2)^3}{1-8}\right)+\frac{\tan^{-1}\left(-\frac{2\sqrt{3}}{2+2}\right)}{\sqrt{3}}}+11-2=9+\frac{576}{12\int_0^1\frac{1}{4+3t^2}\,dt+\log(7)}$$

$$-\frac{2\times 48}{\frac{1}{6}\log\left(\frac{(1-2)^3}{1-8}\right)+\frac{\tan^{-1}\left(-\frac{2\sqrt{3}}{2+2}\right)}{\sqrt{3}}}+11-2=$$

$$9+\frac{96}{-\frac{i}{8\pi^{3/2}}\int_{-i\infty+\gamma}^{i\infty+\gamma}\left(\frac{4}{7}\right)^s\Gamma\left(\frac{1}{2}-s\right)\Gamma(1-s)\Gamma(s)^2\,ds+\frac{\log(7)}{6}}\quad\text{for } 0<\gamma<\frac{1}{2}$$

**Continued fraction representations:**

$$-\frac{2\times 48}{\frac{1}{6}\log\left(\frac{(1-2)^3}{1-8}\right)+\frac{\tan^{-1}\left(-\frac{2\sqrt{3}}{2+2}\right)}{\sqrt{3}}}+11-2=$$

$$9+\frac{576}{\log(7)+\frac{3}{1+\underset{k=1}{\overset{\infty}{\mathrm{K}}}\frac{\frac{3k^2}{4}}{1+2k}}}=9+\frac{576}{\log(7)+\frac{3}{1+\frac{3}{4\left(3+\frac{3}{5+\frac{27}{4\left(7+\frac{12}{9+\dots}\right)}}\right)}}}$$

$$-\frac{2\times 48}{\frac{1}{6}\log\left(\frac{(1-2)^3}{1-8}\right)+\frac{\tan^{-1}\left(-\frac{2\sqrt{3}}{2+2}\right)}{\sqrt{3}}}+11-2=$$

$$9+\frac{576}{\log(7)+\frac{3}{1+\underset{k=1}{\overset{\infty}{\mathrm{K}}}\frac{\frac{3}{4}(1-2k)^2}{\frac{1}{4}(7+2k)}}}=9+\frac{576}{\log(7)+\frac{3}{1+\frac{3}{4\left(\frac{9}{4}+\frac{27}{4\left(\frac{11}{4}+\frac{75}{4\left(\frac{13}{4}+\frac{147}{4\left(\frac{15}{4}+\ldots\right)}\right)}\right)}\right)}}}$$

$$-\frac{2\times 48}{\frac{1}{6}\log\left(\frac{(1-2)^3}{1-8}\right)+\frac{\tan^{-1}\left(-\frac{2\sqrt{3}}{2+2}\right)}{\sqrt{3}}}+11-2=$$

$$9+\frac{96}{\frac{1}{2}+\frac{\log(7)}{6}-\frac{3}{8\left(3+\underset{k=1}{\overset{\infty}{\mathrm{K}}}\frac{\frac{3}{4}\left(1+(-1)^{1+k}+k\right)^2}{3+2k}\right)}}=9+\frac{96}{\frac{1}{2}+\frac{\log(7)}{6}-\frac{3}{8\left(3+\frac{27}{4\left(5+\frac{3}{7+\frac{75}{4\left(9+\frac{12}{11+\ldots}\right)}}\right)}\right)}}$$

$\underset{k=k_1}{\overset{k_2}{\mathrm{K}}} a_k/b_k$ is a continued fraction

48/(((-((1/6 ln (((1-2)^3)/(1-8)) + 1/sqrt3 tan^-1 (-2sqrt3/(2+2)))))))*2-5

**Input:**

$$-\frac{48}{\frac{1}{6}\log\left(\frac{(1-2)^3}{1-8}\right)+\frac{1}{\sqrt{3}}\tan^{-1}\left(-2\times\frac{\sqrt{3}}{2+2}\right)}\times 2-5$$

$\log(x)$ is the natural logarithm

$\tan^{-1}(x)$ is the inverse tangent function

**Exact Result:**

$$\frac{96}{\frac{\log(7)}{6}+\frac{\tan^{-1}\left(\frac{\sqrt{3}}{2}\right)}{\sqrt{3}}}-5$$

(result in radians)

**Decimal approximation:**

125.3661773216463714471334671833755532255719032123699111372...

(result in radians)

125.36617732… result very near to the dilaton mass calculated as a type of Higgs boson: 125 GeV for T = 0 and to the Higgs boson mass 125.18 GeV

**Alternate forms:**

$$\frac{576}{\log(7)+2\sqrt{3}\cot^{-1}\left(\frac{2}{\sqrt{3}}\right)}-5$$

$$\frac{576}{\log(7)+2\sqrt{3}\tan^{-1}\left(\frac{\sqrt{3}}{2}\right)}-5$$

$$\frac{96}{\frac{\log(7)}{6}+\frac{\cot^{-1}\left(\frac{2}{\sqrt{3}}\right)}{\sqrt{3}}}-5$$

$\cot^{-1}(x)$ is the inverse cotangent function

**Alternative representations:**

$$-\frac{2\times 48}{\frac{1}{6}\log\left(\frac{(1-2)^3}{1-8}\right)+\frac{\tan^{-1}\left(-\frac{2\sqrt{3}}{2+2}\right)}{\sqrt{3}}}-5=-5+\frac{96}{-\frac{1}{6}\log\left(\frac{-1}{-7}\right)-\frac{\tan^{-1}\left(1,-\frac{2\sqrt{3}}{4}\right)}{\sqrt{3}}}$$

$$-\frac{2\times 48}{\frac{1}{6}\log\left(\frac{(1-2)^3}{1-8}\right)+\frac{\tan^{-1}\left(-\frac{2\sqrt{3}}{2+2}\right)}{\sqrt{3}}}-5=-5+\frac{96}{-\frac{1}{6}\log_e\left(\frac{-1}{-7}\right)-\frac{\tan^{-1}\left(-\frac{2\sqrt{3}}{4}\right)}{\sqrt{3}}}$$

$$-\frac{2\times 48}{\frac{1}{6}\log\left(\frac{(1-2)^3}{1-8}\right)+\frac{\tan^{-1}\left(-\frac{2\sqrt{3}}{2+2}\right)}{\sqrt{3}}}-5=-5+\frac{96}{-\frac{1}{6}\log_e\left(\frac{-1}{-7}\right)-\frac{\tan^{-1}\left(1,-\frac{2\sqrt{3}}{4}\right)}{\sqrt{3}}}$$

## Series representations:

$$-\frac{2\times 48}{\frac{1}{6}\log\left(\frac{(1-2)^3}{1-8}\right)+\frac{\tan^{-1}\left(-\frac{2\sqrt{3}}{2+2}\right)}{\sqrt{3}}}-5=-5+\frac{576}{2\sqrt{3}\tan^{-1}\left(\frac{\sqrt{3}}{2}\right)+\log(6)-\sum_{k=1}^{\infty}\frac{\left(-\frac{1}{6}\right)^k}{k}}$$

$$-\frac{2\times 48}{\frac{1}{6}\log\left(\frac{(1-2)^3}{1-8}\right)+\frac{\tan^{-1}\left(-\frac{2\sqrt{3}}{2+2}\right)}{\sqrt{3}}}-5=$$
$$-\frac{-576+5\log(6)-5\sum_{k=1}^{\infty}\frac{\left(-\frac{1}{6}\right)^k}{k}+10\sqrt{3}\sum_{k=0}^{\infty}\frac{(-1)^k 2^{-1-2k}\times 3^{1/2+k}}{1+2k}}{\log(6)-\sum_{k=1}^{\infty}\frac{\left(-\frac{1}{6}\right)^k}{k}+2\sqrt{3}\sum_{k=0}^{\infty}\frac{(-1)^k 2^{-1-2k}\times 3^{1/2+k}}{1+2k}}$$

$$-\frac{2\times 48}{\frac{1}{6}\log\left(\frac{(1-2)^3}{1-8}\right)+\frac{\tan^{-1}\left(-\frac{2\sqrt{3}}{2+2}\right)}{\sqrt{3}}}-5=$$
$$-5+\frac{576}{2\sqrt{3}\tan^{-1}(z_0)+\log(6)+\sum_{k=1}^{\infty}\left(\frac{(-1)^{1+k}6^{-k}}{k}+\frac{i\sqrt{3}\left(-(-i-z_0)^{-k}+(i-z_0)^{-k}\right)\left(\frac{\sqrt{3}}{2}-z_0\right)^k}{k}\right)}$$

for ($i z_0 \notin \mathbb{R}$ or ( not ($1 \leq i z_0 < \infty$) and not ($-\infty < i z_0 \leq -1$)))

$$-\frac{2\times 48}{\frac{1}{6}\log\left(\frac{(1-2)^3}{1-8}\right)+\frac{\tan^{-1}\left(-\frac{2\sqrt{3}}{2+2}\right)}{\sqrt{3}}}-5=$$
$$-\left(\left(-576+10\sqrt{3}\tan^{-1}(z_0)+5\log(6)-5\sum_{k=1}^{\infty}\frac{\left(-\frac{1}{6}\right)^k}{k}+5i\sqrt{3}\sum_{k=1}^{\infty}\frac{\left(-(-i-z_0)^{-k}+(i-z_0)^{-k}\right)\left(\frac{\sqrt{3}}{2}-z_0\right)^k}{k}\right)\Bigg/\left(2\sqrt{3}\tan^{-1}(z_0)+\log(6)-\sum_{k=1}^{\infty}\frac{\left(-\frac{1}{6}\right)^k}{k}+i\sqrt{3}\sum_{k=1}^{\infty}\frac{\left(-(-i-z_0)^{-k}+(i-z_0)^{-k}\right)\left(\frac{\sqrt{3}}{2}-z_0\right)^k}{k}\right)\right)$$

for ($i z_0 \notin \mathbb{R}$ or ( not ($1 \leq i z_0 < \infty$) and not ($-\infty < i z_0 \leq -1$)))

## Integral representations:

$$-\frac{2\times 48}{\frac{1}{6}\log\left(\frac{(1-2)^3}{1-8}\right)+\frac{\tan^{-1}\left(-\frac{2\sqrt{3}}{2+2}\right)}{\sqrt{3}}}-5=-5+\frac{576}{\int_1^7\left(\frac{1}{t}+\frac{24}{49-2t+t^2}\right)dt}$$

$$-\frac{2\times 48}{\frac{1}{6}\log\left(\frac{(1-2)^3}{1-8}\right)+\frac{\tan^{-1}\left(-\frac{2\sqrt{3}}{2+2}\right)}{\sqrt{3}}}-5 = -5+\frac{576}{12\int_0^1 \frac{1}{4+3t^2}\,dt+\log(7)}$$

$$-\frac{2\times 48}{\frac{1}{6}\log\left(\frac{(1-2)^3}{1-8}\right)+\frac{\tan^{-1}\left(-\frac{2\sqrt{3}}{2+2}\right)}{\sqrt{3}}}-5 =$$
$$-5+\frac{96}{-\frac{i}{8\pi^{3/2}}\int_{-i\infty+\gamma}^{i\infty+\gamma}\left(\frac{4}{7}\right)^s\Gamma\left(\frac{1}{2}-s\right)\Gamma(1-s)\,\Gamma(s)^2\,ds+\frac{\log(7)}{6}} \quad \text{for } 0<\gamma<\frac{1}{2}$$

## Continued fraction representations:

$$-\frac{2\times 48}{\frac{1}{6}\log\left(\frac{(1-2)^3}{1-8}\right)+\frac{\tan^{-1}\left(-\frac{2\sqrt{3}}{2+2}\right)}{\sqrt{3}}}-5 =$$
$$-5+\frac{576}{\log(7)+\frac{3}{1+\underset{k=1}{\overset{\infty}{\mathrm{K}}}\frac{\frac{3k^2}{4}}{1+2k}}} = -5+\frac{576}{\log(7)+\frac{3}{1+\frac{3}{4\left(3+\frac{3}{5+\frac{27}{4\left(7+\frac{12}{9+\dots}\right)}}\right)}}}$$

$$-\frac{2\times 48}{\frac{1}{6}\log\left(\frac{(1-2)^3}{1-8}\right)+\frac{\tan^{-1}\left(-\frac{2\sqrt{3}}{2+2}\right)}{\sqrt{3}}}-5 =$$
$$-5+\frac{576}{\log(7)+\frac{3}{1+\underset{k=1}{\overset{\infty}{\mathrm{K}}}\frac{\frac{3}{4}(1-2k)^2}{\frac{1}{4}(7+2k)}}} = -5+\frac{576}{\log(7)+\frac{3}{1+\frac{3}{4\left(\frac{9}{4}+\frac{27}{4\left(\frac{11}{4}+\frac{75}{4\left(\frac{13}{4}+\frac{147}{4\left(\frac{15}{4}+\dots\right)}\right)}\right)}\right)}}}$$

$$-\frac{2\times 48}{\frac{1}{6}\log\left(\frac{(1-2)^3}{1-8}\right)+\frac{\tan^{-1}\left(-\frac{2\sqrt{3}}{2+2}\right)}{\sqrt{3}}}-5=$$

$$-5+\frac{96}{\frac{1}{2}+\frac{\log(7)}{6}-\frac{3}{8\left(3+\underset{k=1}{\overset{\infty}{\mathrm{K}}}\frac{\frac{3}{4}\left(1+(-1)^{1+k}+k\right)^2}{3+2k}\right)}}=-5+\frac{96}{\frac{1}{2}+\frac{\log(7)}{6}-\frac{3}{8\left(3+\frac{27}{4\left(5+\frac{3}{7+\frac{75}{4\left(9+\frac{12}{11+\dots}\right)}}\right)}\right)}}$$

27*1/2*((((((48/(((-((1/6 ln (((1-2)^3)/(1-8)) + 1/sqrt3 tan^-1 (-2sqrt3/(2+2)))))))*2-5)))+2)))+13-Pi-1/(2*golden ratio)

**Input:**

$$27\times\frac{1}{2}\left(\left(-\frac{48}{\frac{1}{6}\log\left(\frac{(1-2)^3}{1-8}\right)+\frac{1}{\sqrt{3}}\tan^{-1}\left(-2\times\frac{\sqrt{3}}{2+2}\right)}\times 2-5\right)+2\right)+13-\pi-\frac{1}{2\phi}$$

log(x) is the natural logarithm

$\tan^{-1}(x)$ is the inverse tangent function

$\phi$ is the golden ratio

**Exact Result:**

$$-\frac{1}{2\phi}+13-\pi+\frac{27}{2}\left(\frac{96}{\frac{\log(7)}{6}+\frac{\tan^{-1}\left(\frac{\sqrt{3}}{2}\right)}{\sqrt{3}}}-3\right)$$

(result in radians)

**Decimal approximation:**

1728.992784194261273873736870175107646602163369377715813100...

(result in radians)

1728.99278419… ≈ 1729

This result is very near to the mass of candidate glueball $f_0(1710)$ meson. Furthermore, 1728 occurs in the algebraic formula for the j-invariant of an elliptic curve. As a consequence, it is sometimes called a Zagier as a pun on the Gross–Zagier theorem. The number 1728 is one less than the Hardy–Ramanujan number 1729 (taxicab number)

**Alternate forms:**

$$-\frac{1}{2\phi}-\frac{55}{2}-\pi+\frac{7776}{\log(7)+2\sqrt{3}\tan^{-1}\left(\frac{\sqrt{3}}{2}\right)}$$

$$-\frac{1}{2\phi}-\frac{55}{2}-\pi+\frac{7776\sqrt{3}}{\sqrt{3}\log(7)+6\tan^{-1}\left(\frac{\sqrt{3}}{2}\right)}$$

$$-\frac{55}{2}-\frac{1}{1+\sqrt{5}}-\pi+\frac{1296}{\frac{\log(7)}{6}+\frac{\tan^{-1}\left(\frac{\sqrt{3}}{2}\right)}{\sqrt{3}}}$$

**Alternative representations:**

$$\frac{27}{2}\left(\left(-\frac{48\times 2}{\frac{1}{6}\log\left(\frac{(1-2)^3}{1-8}\right)+\frac{\tan^{-1}\left(-\frac{2\sqrt{3}}{2+2}\right)}{\sqrt{3}}}-5\right)+2\right)+13-\pi-\frac{1}{2\phi}=$$

$$13-\pi-\frac{1}{2\phi}+\frac{27}{2}\left(-3+\frac{96}{-\frac{1}{6}\log\left(\frac{-1}{-7}\right)-\frac{\tan^{-1}\left(1,-\frac{2\sqrt{3}}{4}\right)}{\sqrt{3}}}\right)$$

$$\frac{27}{2}\left(\left(-\frac{48\times 2}{\frac{1}{6}\log\left(\frac{(1-2)^3}{1-8}\right)+\frac{\tan^{-1}\left(-\frac{2\sqrt{3}}{2+2}\right)}{\sqrt{3}}}-5\right)+2\right)+13-\pi-\frac{1}{2\phi}=$$

$$13-\pi-\frac{1}{2\phi}+\frac{27}{2}\left(-3+\frac{96}{-\frac{1}{6}\log_e\left(\frac{-1}{-7}\right)-\frac{\tan^{-1}\left(-\frac{2\sqrt{3}}{4}\right)}{\sqrt{3}}}\right)$$

$$\frac{27}{2}\left(\left(-\frac{48\times 2}{\frac{1}{6}\log\left(\frac{(1-2)^3}{1-8}\right)+\frac{\tan^{-1}\left(-\frac{2\sqrt{3}}{2+2}\right)}{\sqrt{3}}}-5\right)+2\right)+13-\pi-\frac{1}{2\phi}=$$
$$13-\pi-\frac{1}{2\phi}+\frac{27}{2}\left(-3+\frac{96}{-\frac{1}{6}\log_e\left(\frac{-1}{-7}\right)-\frac{\tan^{-1}\left(1,-\frac{2\sqrt{3}}{4}\right)}{\sqrt{3}}}\right)$$

**Series representations:**

$$\frac{27}{2}\left(\left(-\frac{48\times 2}{\frac{1}{6}\log\left(\frac{(1-2)^3}{1-8}\right)+\frac{\tan^{-1}\left(-\frac{2\sqrt{3}}{2+2}\right)}{\sqrt{3}}}-5\right)+2\right)+13-\pi-\frac{1}{2\phi}=$$
$$-\frac{55}{2}-\frac{1}{1+\sqrt{5}}-\pi+\frac{1296}{\frac{\tan^{-1}\left(\frac{\sqrt{3}}{2}\right)}{\sqrt{3}}+\frac{1}{6}\left(\log(6)-\sum_{k=1}^{\infty}\frac{\left(-\frac{1}{6}\right)^k}{k}\right)}$$

$$\frac{27}{2}\left(\left(-\frac{48\times 2}{\frac{1}{6}\log\left(\frac{(1-2)^3}{1-8}\right)+\frac{\tan^{-1}\left(-\frac{2\sqrt{3}}{2+2}\right)}{\sqrt{3}}}-5\right)+2\right)+13-\pi-\frac{1}{2\phi}=$$
$$-\frac{55}{2}-\frac{1}{1+\sqrt{5}}-\pi+\frac{1296}{\frac{1}{6}\left(\log(6)-\sum_{k=1}^{\infty}\frac{\left(-\frac{1}{6}\right)^k}{k}\right)+\frac{\sum_{k=0}^{\infty}\frac{(-1)^k\,2^{-1-2k}\times 3^{1/2+k}}{1+2k}}{\sqrt{3}}}$$

$$\frac{27}{2}\left(\left(-\frac{48\times 2}{\frac{1}{6}\log\left(\frac{(1-2)^3}{1-8}\right)+\frac{\tan^{-1}\left(-\frac{2\sqrt{3}}{2+2}\right)}{\sqrt{3}}}-5\right)+2\right)+13-\pi-\frac{1}{2\phi}=-\frac{55}{2}-\frac{1}{1+\sqrt{5}}-\pi+$$
$$\frac{1296}{\frac{1}{6}\left(\log(6)-\sum_{k=1}^{\infty}\frac{\left(-\frac{1}{6}\right)^k}{k}\right)+\frac{\tan^{-1}(z_0)+\frac{1}{2}i\sum_{k=1}^{\infty}\frac{\left(-(-i-z_0)^{-k}+(i-z_0)^{-k}\right)\left(\frac{\sqrt{3}}{2}-z_0\right)^k}{k}}{\sqrt{3}}}$$

for $(i z_0 \notin \mathbb{R}$ or ( not $(1 \le i z_0 < \infty)$ and not $(-\infty < i z_0 \le -1)))$

## Integral representations:

$$\frac{27}{2}\left(\left(-\frac{48\times 2}{\frac{1}{6}\log\left(\frac{(1-2)^3}{1-8}\right)+\frac{\tan^{-1}\left(-\frac{2\sqrt{3}}{2+2}\right)}{\sqrt{3}}}-5\right)+2\right)+13-\pi-\frac{1}{2\phi} = -\frac{55}{2}-\frac{1}{2\phi}-\pi+\frac{7776}{12\int_0^1 \frac{1}{4+3t^2}\,dt+\log(7)}$$

$$\frac{27}{2}\left(\left(-\frac{48\times 2}{\frac{1}{6}\log\left(\frac{(1-2)^3}{1-8}\right)+\frac{\tan^{-1}\left(-\frac{2\sqrt{3}}{2+2}\right)}{\sqrt{3}}}-5\right)+2\right)+13-\pi-\frac{1}{2\phi} = -\frac{55}{2}-\frac{1}{2\phi}-\pi+\frac{1296}{\int_1^7\left(\frac{1}{6t}+\frac{4}{49-2t+t^2}\right)dt}$$

$$\frac{27}{2}\left(\left(-\frac{48\times 2}{\frac{1}{6}\log\left(\frac{(1-2)^3}{1-8}\right)+\frac{\tan^{-1}\left(-\frac{2\sqrt{3}}{2+2}\right)}{\sqrt{3}}}-5\right)+2\right)+13-\pi-\frac{1}{2\phi} = -\frac{55}{2}-\frac{1}{1+\sqrt{5}}-\pi+\frac{1296}{-\frac{i}{8\pi^{3/2}}\int_{-i\infty+\gamma}^{i\infty+\gamma}\left(\frac{4}{7}\right)^s\Gamma\left(\frac{1}{2}-s\right)\Gamma(1-s)\Gamma(s)^2\,ds+\frac{\log(7)}{6}} \quad \text{for } 0<\gamma<\frac{1}{2}$$

## Continued fraction representations:

$$\frac{27}{2}\left(\left(-\frac{48\times 2}{\frac{1}{6}\log\left(\frac{(1-2)^3}{1-8}\right)+\frac{\tan^{-1}\left(-\frac{2\sqrt{3}}{2+2}\right)}{\sqrt{3}}}-5\right)+2\right)+13-\pi-\frac{1}{2\phi} = -\frac{55}{2}-\frac{1}{2\phi}-\pi+\frac{7776}{\log(7)+\frac{3}{1+\mathop{K}_{k=1}^{\infty}\frac{\frac{3k^2}{4}}{1+2k}}} = -\frac{55}{2}-\frac{1}{2\phi}-\pi+\frac{7776}{\log(7)+\frac{3}{1+\frac{3}{4\left(3+\frac{3}{5+\frac{27}{4\left(7+\frac{12}{9+\dots}\right)}}\right)}}}$$

$$\frac{27}{2}\left(\left(-\frac{48\times 2}{\frac{1}{6}\log\left(\frac{(1-2)^3}{1-8}\right)+\frac{\tan^{-1}\left(-\frac{2\sqrt{3}}{2+2}\right)}{\sqrt{3}}}-5\right)+2\right)+13-\pi-\frac{1}{2\phi}=$$

$$13-\frac{1}{2\phi}-\pi+\frac{27}{2}\left(-3+\frac{96}{\frac{\log(7)}{6}+\frac{1}{2\left(1+\underset{k=1}{\overset{\infty}{\mathrm{K}}}\frac{\frac{3k^2}{4}}{1+2k}\right)}}\right)=$$

$$13-\frac{1}{2\phi}-\pi+\frac{27}{2}\left(-3+\frac{96}{\frac{\log(7)}{6}+\frac{1}{2\left(1+\frac{3}{4\left(3+\frac{3}{5+\frac{27}{4\left(7+\frac{12}{9+\ldots}\right)}}\right)}\right)}}\right)$$

$$\frac{27}{2}\left(\left(-\frac{48\times 2}{\frac{1}{6}\log\left(\frac{(1-2)^3}{1-8}\right)+\frac{\tan^{-1}\left(-\frac{2\sqrt{3}}{2+2}\right)}{\sqrt{3}}}-5\right)+2\right)+13-\pi-\frac{1}{2\phi}=$$

$$-\frac{55}{2}-\frac{1}{2\phi}-\pi+\frac{7776}{\log(7)+\frac{3}{1+\underset{k=1}{\overset{\infty}{\mathrm{K}}}\frac{\frac{3}{4}(1-2k)^2}{\frac{1}{4}(7+2k)}}}=$$

$$-\frac{55}{2}-\frac{1}{2\phi}-\pi+\frac{7776}{\log(7)+\frac{3}{1+\frac{3}{4\left(\frac{9}{4}+\frac{27}{4\left(\frac{11}{4}+\frac{75}{4\left(\frac{13}{4}+\frac{147}{4\left(\frac{15}{4}+\ldots\right)}\right)}\right)}\right)}}}$$

## EXAMPLE OF RAMANUJAN MATHEMATICS APPLIED TO THE COSMOLOGY

From:

**A Reissner-Nordstrom+Λ black hole in the Friedman-Robertson-Walker universe-** arXiv:1703.05119v1 [physics.gen-ph] 5 Mar 2017
*Safiqul Islam and Priti Mishra†*
Harish-Chandra Research Institute, Allahabad 211019, Uttar Pradesh, India
Homi Bhabha National Institute, Anushaktinagar, Mumbai 400094, India
*Farook Rahaman‡* - Department of Mathematics,Jadavpur University,Kolkata-700 032,West Bengal,India - (Dated: March 16, 2017)

From:

$$r_{\pm} = M \pm \sqrt{M^2 - Q^2},$$

For MBH87 data: mass = 13.12806e+39; radius = 1.94973e+13 , we obtain:

(1.94973e+13-13.12806e+39)^2 = ((13.12806e+39)^2-x^2)

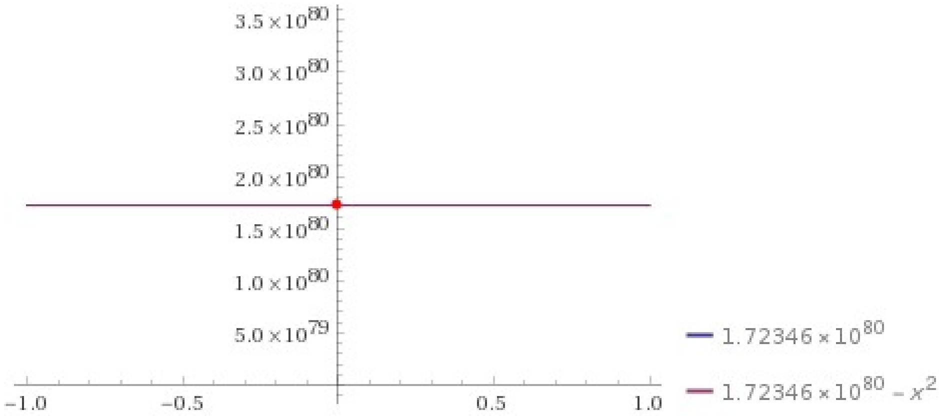
**Input interpretation:**

$$(1.94973 \times 10^{13} - 13.12806 \times 10^{39})^2 = (13.12806 \times 10^{39})^2 - x^2$$

**Result:**

$$1.72346 \times 10^{80} = 1.72346 \times 10^{80} - x^2$$

**Plot:**




**Alternate forms:**

$x^2 + 0 = 0$

$1.72346 \times 10^{80} = -(x - 1.31281 \times 10^{40})(x + 1.31281 \times 10^{40})$

**Solution:**

$x = 0$

Indeed:

(1.94973e+13-13.12806e+39)^2 = ((13.12806e+39)^2)

**Input interpretation:**

$(1.94973 \times 10^{13} - 13.12806 \times 10^{39})^2 = (13.12806 \times 10^{39})^2$

**Result:**

True

Thence Q = 0

Now, for

$a(v) > \frac{\sqrt{k}}{4}$. For the present universe, assuming $a(v) = 1$ and thus $k < 16$. Though constant k has an upper limit, it increases with the expansion of the universe and decreases with the contraction of the universe. We should observe a peculiar change when the constant k reaches this numerical value which is the limiting value for the expansion of the universe.

For $Q = 0$ in eqn.(64),

$$2\left(2 - \frac{\sqrt{1 + \frac{kx^2}{4}}}{ax}\right)\left[\frac{M^2}{\left(\frac{ax}{\sqrt{1+\frac{kx^2}{4}}}\right)^3} - \frac{Q^2}{\left(\frac{ax}{\sqrt{1+\frac{kx^2}{4}}}\right)^3} + \Lambda e^{-\frac{2ax}{\sqrt{1+\frac{kx^2}{4}}}}\right] + \frac{\sqrt{1 + \frac{kx^2}{4}}}{ax} = 0. \quad (64)$$

Hence at $x = R$ we get,

$$2\left(2 - \frac{\sqrt{1 + \frac{kR^2}{4}}}{aR}\right)\left[\frac{M^2}{\left(\frac{aR}{\sqrt{1+\frac{kR^2}{4}}}\right)^3} - \frac{Q^2}{\left(\frac{aR}{\sqrt{1+\frac{kR^2}{4}}}\right)^3} + \Lambda e^{-\frac{2aR}{\sqrt{1+\frac{kR^2}{4}}}}\right] + \frac{\sqrt{1 + \frac{kR^2}{4}}}{aR} = 0. \quad (65)$$

$$\Lambda = -e^{\frac{2aR}{\sqrt{1+\frac{kR^2}{4}}}} . \left[\frac{M^2}{\left(\frac{aR}{\sqrt{1+\frac{kR^2}{4}}}\right)^3} + \frac{1}{2\left(\frac{2aR}{\sqrt{1+\frac{kR^2}{4}}} - 1\right)}\right], \quad (67)$$

For k = 12, and a = 1, M = 13.12806e+39; R = 1.94973e+13, we obtain:

and:

(1+((12*(1.94973e+13)^2)/4))^1/2

**Input interpretation:**

$$\sqrt{1+\frac{1}{4}\left(12\left(1.94973\times 10^{13}\right)^2\right)}$$

**Result:**

$3.37703... \times 10^{13}$

3.37703e+13

Substituting in the eqs. (67), we obtain:

-exp(((2*1.94973e+13)/(3.37703e+13))) * [(((13.12806e+39)^2)) / (((1.94973e+13)/(3.37703e+13)))^3 + 1/((2((((2*1.94973e+13)/(3.37703e+13)-1)))))]

**Input interpretation:**

$$-\exp\left(\frac{2\times 1.94973\times 10^{13}}{3.37703\times 10^{13}}\right)\left(\frac{\left(13.12806\times 10^{39}\right)^2}{\left(\frac{1.94973\times 10^{13}}{3.37703\times 10^{13}}\right)^3}+\frac{1}{2\left(\frac{2\times 1.94973\times 10^{13}}{3.37703\times 10^{13}}-1\right)}\right)$$

**Result:**

$-2.84160... \times 10^{81}$

-2.84160…*$10^{81}$ which represents the Cosmological Constant inside the Schwarzschild black hole and also has a negative value.

Performing the following equation with the usual value of the Cosmological Constant 1.1056e-52, we obtain:

(1.1056e-52)x = -2.84160e+81

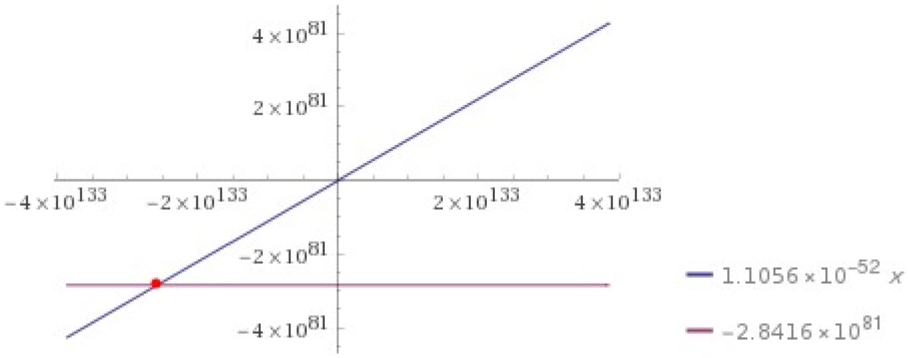
**Input interpretation:**

$1.1056\times 10^{-52}\, x = -2.84160\times 10^{81}$

**Result:**

$1.1056\times 10^{-52}\, x = -2.8416\times 10^{81}$

**Plot:**




**Alternate form:**

$1.1056\times10^{-52}\,x+2.8416\times10^{81}=0$

**Alternate form assuming x is real:**

$1.1056\times10^{-52}\,x+0=-2.8416\times10^{81}$

**Solution:**

$x =$

−25 701 881 331 403 766 886 664 569 715 710 133 147 602 520 011 173 198 993 507
564 120 861 732 475 370 738 202 865 312 319 616 245 712 374 922 255 343 303
805 210 672 526 000 128

**Integer solution:**

$x =$

−25 701 881 331 403 766 886 664 569 715 710 133 147 602 520 011 173 198 993 507
564 120 861 732 475 370 738 202 865 312 319 616 245 712 374 922 255 343 303
805 210 672 526 000 128

**Result:**

−2.5701881331403766886664569715710133147602520011173198993507564120
861732475370738202865312319616245712374922255343303805210672526
$000128\times10^{133}$

$-2.57018813314\ldots*10^{133}$

Value that multiplied by 1.1056e-52, give us $-2.84160 * 10^{81}$

Multiplying this result with the usual value of the Cosmological Constant, we obtain:

(1.1056e-52) * (-2.84160e+81)

**Input interpretation:**

$1.1056\times10^{-52}\,(-2.84160\times10^{81})$

**Result:**

−314 167 296 000 000 000 000 000 000 000

**Result:**

$-3.14167296\times10^{29}$

-3.14167296*10^29

We have also that, from the formula of coefficients of the '5th order' mock theta function $\psi_1$(q): (A053261 OEIS Sequence)

sqrt(golden ratio) * exp(Pi*sqrt(n/15)) / (2*5^(1/4)*sqrt(n))

for n = 230 and subtracting 47, that is a Lucas number, and π, we obtain:

sqrt(golden ratio) * exp(Pi*sqrt(230/15)) / (2*5^(1/4)*sqrt(230)) -47 – Pi

**Input:**

$$\sqrt{\phi}\times\frac{\exp\left(\pi\sqrt{\frac{230}{15}}\right)}{2\sqrt[4]{5}\sqrt{230}}-47-\pi$$

$\phi$ is the golden ratio

**Exact result:**

$$\frac{e^{\sqrt{46/3}\,\pi}\sqrt{\frac{\phi}{46}}}{2\times5^{3/4}}-47-\pi$$

**Decimal approximation:**

6122.273163239088047930830535468077939193046207568421910068...

6122.273163239…..

**Alternate forms:**

$$-47+\frac{1}{20}\sqrt{\frac{1}{23}\left(5+\sqrt{5}\right)}\,e^{\sqrt{46/3}\,\pi}-\pi$$

$$-47+\frac{\sqrt{\frac{1}{23}\left(1+\sqrt{5}\right)}\ e^{\sqrt{46/3}\ \pi}}{4\times 5^{3/4}}-\pi$$

$$\frac{1}{460}\left(-21620+\sqrt[4]{5}\ \sqrt{23\left(1+\sqrt{5}\right)}\ e^{\sqrt{46/3}\ \pi}-460\,\pi\right)$$

## Series representations:

$$\frac{\sqrt{\phi}\ \exp\left(\pi\sqrt{\frac{230}{15}}\right)}{2\sqrt[4]{5}\ \sqrt{230}}-47-\pi=$$
$$-\left(\left(470\sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k(230-z_0)^k z_0^{-k}}{k!}+10\,\pi\sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k(230-z_0)^k z_0^{-k}}{k!}-\right.\right.$$
$$5^{3/4}\exp\left(\pi\sqrt{z_0}\sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k\left(\frac{46}{3}-z_0\right)^k z_0^{-k}}{k!}\right)$$
$$\left.\left.\sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k(\phi-z_0)^k z_0^{-k}}{k!}\right)\Big/\left(10\sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k(230-z_0)^k z_0^{-k}}{k!}\right)\right)$$

for not $((z_0\in\mathbb{R}$ and $-\infty<z_0\leq 0))$

$$\frac{\sqrt{\phi}\ \exp\left(\pi\sqrt{\frac{230}{15}}\right)}{2\sqrt[4]{5}\ \sqrt{230}}-47-\pi=$$
$$-\left(\left(470\exp\left(i\,\pi\left\lfloor\frac{\arg(230-x)}{2\,\pi}\right\rfloor\right)\sum_{k=0}^{\infty}\frac{(-1)^k(230-x)^k x^{-k}\left(-\frac{1}{2}\right)_k}{k!}+\right.\right.$$
$$10\,\pi\exp\left(i\,\pi\left\lfloor\frac{\arg(230-x)}{2\,\pi}\right\rfloor\right)\sum_{k=0}^{\infty}\frac{(-1)^k(230-x)^k x^{-k}\left(-\frac{1}{2}\right)_k}{k!}-$$
$$5^{3/4}\exp\left(i\,\pi\left\lfloor\frac{\arg(\phi-x)}{2\,\pi}\right\rfloor\right)\exp\left(\pi\exp\left(i\,\pi\left\lfloor\frac{\arg\left(\frac{46}{3}-x\right)}{2\,\pi}\right\rfloor\right)\sqrt{x}\right.$$
$$\left.\left.\sum_{k=0}^{\infty}\frac{(-1)^k\left(\frac{46}{3}-x\right)^k x^{-k}\left(-\frac{1}{2}\right)_k}{k!}\right)\sum_{k=0}^{\infty}\frac{(-1)^k(\phi-x)^k x^{-k}\left(-\frac{1}{2}\right)_k}{k!}\right)\Big/$$
$$\left.\left(10\exp\left(i\,\pi\left\lfloor\frac{\arg(230-x)}{2\,\pi}\right\rfloor\right)\sum_{k=0}^{\infty}\frac{(-1)^k(230-x)^k x^{-k}\left(-\frac{1}{2}\right)_k}{k!}\right)\right)$$

for $(x\in\mathbb{R}$ and $x<0)$

From which:

(-(-2.84160e+81))^(5Pi/(((sqrt(golden ratio) * exp(Pi*sqrt(230/15)) / (2*5^(1/4)*sqrt(230)) -47 - Pi))))

**Input interpretation:**

$$(-(-2.84160\times10^{81}))^{5\times\pi/\left(\sqrt{\phi}\times\frac{\exp\left(\pi\sqrt{\frac{230}{15}}\right)}{2\sqrt[4]{5}\sqrt{230}}-47-\pi\right)}$$

$\phi$ is the golden ratio

**Result:**

1.6180279967015604382863892218765663179334071736938421506 42...

1.6180279967….. result that is a very good approximation to the value of the golden ratio 1,618033988749...

**Input interpretation:**

1.6180279967015604382863892218765663179334071736938421

**Possible closed forms:**

$$-\frac{8(45F_{FR}-1127)}{2047F_{FR}-800}\approx1.618027996701560429601$$

$$\frac{1}{3}\sqrt{\frac{1}{55}(-200+333e+162\pi+118\log(2))}\approx1.61802799670156043867372$$

$$-\frac{4(73-325\pi+39\pi^2)}{49-72\pi+159\pi^2}\approx1.61802799670156043858425$$

$$\pi\,\left[\text{root of }522x^4+580x^3-1362x^2+919x-228\text{ near }x=0.515034\right]\approx 1.61802799670156043816535$$

$$\frac{\sqrt[3]{\frac{2}{51}(984 - 89 e + 1000 \pi - 1707 \log(2))}}{5^{2/3}} \approx 1.61802799670156043826 5766$$

$$\frac{3\,709\,980\,781 \pi}{7\,203\,366\,314} \approx 1.61802799670156043829 6510$$

$$\text{root of } 647 x^4 - 350 x^3 - 4186 x^2 + 4220 x + 1179 \text{ near } x = 1.61803 \approx 1.61802799670156043829 0441$$

$$\frac{\sqrt[4]{\frac{31028619}{4409789} \pi}}{\sqrt{10}} \approx 1.6180279967015604 56743$$

$$\frac{1}{\text{root of } 1179 x^4 + 4220 x^3 - 4186 x^2 - 350 x + 647 \text{ near } x = 0.618036} \approx 1.61802799670156043829 0441$$

$$\text{root of } 5888 x^3 - 39\,087 x^2 + 37\,056 x + 17\,431 \text{ near } x = 1.61803 \approx 1.618027996701560438284 4533$$

$$\pi \, \text{root of } 29\,646 x^3 - 33\,474 x^2 - 52\,404 x + 31\,819 \text{ near } x = 0.515034 \approx 1.618027996701560438284 4495$$

$$\frac{1}{\text{root of } 17\,431 x^3 + 37\,056 x^2 - 39\,087 x + 5888 \text{ near } x = 0.618036} \approx 1.618027996701560438284 4533$$

$$\text{root of } 439 x^5 - 1047 x^4 + 217 x^3 + 924 x^2 - x - 1029 \text{ near } x = 1.61803 \approx 1.6180279967015604383 1097$$

$$\pi \, \text{root of } 657 x^5 + 621 x^4 + 647 x^3 - 1476 x^2 + 75 x + 197 \text{ near } x = 0.515034 \approx 1.61802799670156043826 3743$$

$$\frac{e^{\frac{3}{5} - \frac{9}{10 e} - \frac{3 e}{10} + \frac{2}{5 \pi} - \frac{3 \pi}{5}} \pi^{(19 e)/20 - 3/10}}{\sqrt[20]{\sin(e \pi)} \, (-\cos(e \pi))^{7/20}} \approx 1.6180279967015604386 2208$$

Now, we have that:

$$a = 3.2^{\frac{1}{3}}.(1 - 4Q^2\Lambda), \qquad (9)$$

$$b = [-54 + 972M^2\Lambda - 648Q^2\Lambda + [(-54 + 972M^2\Lambda - 648Q^2\Lambda)^2 - 4(9 - 36Q^2\Lambda)^3]^{\frac{1}{2}}]^{\frac{1}{3}}, \qquad (10)$$

$$c = 3.2^{\frac{1}{3}}\Lambda, \qquad (11)$$

For **Q = 0.00089**, Λ = 1.1056e-52 $m^{-2}$**:**

convert $1.1056 \times 10^{-52}$ $m^{-2}$ (reciprocal square meters) to per kilometers squared

$1.106 \times 10^{-46}$/$km^2$ (per kilometers squared) **Λ = -1.1056 * $10^{-46}$**

Mass = 3.8 solar masses**:**

$3.8 \times 1.9891 \times 10^{30} = 7558580000000000000000000000000 = 7.55858 \times 10^{30}$

**M = 7.55858e+30**

We obtain:

$$a = 3.2^{\frac{1}{3}}.(1 - 4Q^2\Lambda)$$

(3.2)^1/3 (1-((4*0.00089^2*(-1.1056e-46))))

**Input interpretation:**

$\sqrt[3]{3.2}\left(1 - 4 \times 0.00089^2 \left(-1.1056 \times 10^{-46}\right)\right)$

**Result:**

1.473612599456154642311929133431922888766903246975273583906...

1.4736125994561546.... = a

Now, we have that:

$$b = [-54 + 972M^2\Lambda - 648Q^2\Lambda + [(-54 + 972M^2\Lambda - 648Q^2\Lambda)^2 - 4(9 - 36Q^2\Lambda)^3]^{\frac{1}{2}}]^{\frac{1}{3}},$$

sqrt[(((((-54+972*((7.55858e+30)^2*(-1.1056e-46))-648*0.00089^2*(-1.1056e-46)+(((-54+972*((7.55858e+30)^2*(-1.1056e-46))-648*0.00089^2(-1.1056e-46)))^2-4(((9-36*0.00089^2*(-1.1056e-46)^3)))))))))]^1/3

**Input interpretation:**

$$\left(\sqrt{\left(-54+972\left((7.55858\times 10^{30})^2\left(-1.1056\times 10^{-46}\right)\right)-648\times 0.00089^2\left(-1.1056\times 10^{-46}\right)+\left(\left(-54+972\left((7.55858\times 10^{30})^2\left(-1.1056\times 10^{-46}\right)\right)-648\times 0.00089^2\left(-1.1056\times 10^{-46}\right)\right)^2-4\left(9-36\times 0.00089^2\left(-1.1056\times 10^{-46}\right)^3\right)\right)\right)}\right)\text{^}(1/3)$$

**Result:**

$1.831111995417529907080402771725336322228680076788385 40\ldots \times 10^6$

$1.8311119954175299\ldots * 10^6 = b$

And:

$$c = 3.2^{\frac{1}{3}}\Lambda,$$

(3.2)^(1/3) * (-1.1056e-46)

**Input interpretation:**

$$\sqrt[3]{3.2}\left(-1.1056\times 10^{-46}\right)$$

**Result:**

$-1.62923\ldots \times 10^{-46}$

$-1.62923\ldots * 10^{-46} = c$

From

$$r_4 = -\frac{1}{2}\cdot\left[\frac{2}{\Lambda}+\frac{a}{\Lambda b}+\frac{b}{c}\right]^{\frac{1}{2}} + \frac{1}{2}\cdot\left[\frac{4}{\Lambda}-\frac{a}{\Lambda b}-\frac{b}{c}+\frac{12M}{\Lambda(\frac{2}{\Lambda}+\frac{a}{\Lambda b}+\frac{b}{c})^{\frac{1}{2}}}\right]^{\frac{1}{2}}$$

We have that:

c = -1.62923e-46
b = 1.8311119954175299e+6
a = 1.4736125994561546
Λ = -1.1056e-46

-1/2((((2/(-1.1056e-46)+(1.4736125994561546) / (-1.1056e-46 * 1.8311119954175299e+6) + (1.8311119954175299e+6) / (-1.62923e-46)))))^1/2

**Input interpretation:**

$$-\frac{1}{2}\sqrt{\left(-\frac{2}{1.1056\times10^{-46}}+-\frac{1.4736125994561546}{1.1056\times10^{-46}\times1.8311119954175299\times10^{6}}+-\frac{1.8311119954175299\times10^{6}}{1.62923\times10^{-46}}\right)}$$

**Result:**

$-5.30074... \times 10^{25}\, i$

**Polar coordinates:**

$r = 5.30074\times10^{25}$ (radius), $\theta = -90°$ (angle)

$5.30074*10^{25}$

and:

$$+\frac{1}{2}\cdot\left[\frac{4}{\Lambda}-\frac{a}{\Lambda b}-\frac{b}{c}+\frac{12M}{\Lambda(\frac{2}{\Lambda}+\frac{a}{\Lambda b}+\frac{b}{c})^{\frac{1}{2}}}\right]^{\frac{1}{2}}:$$

1/2[(4/(-1.1056e-46)-(1.4736125994561546)/(-1.1056e-46 * 1.8311119954175299e+6)-(1.8311119954175299e+6)/(-1.62923e-46)+((((12* 7.55858e+30))))/((((-1.1056e-46)(2/(-1.1056e-46)+(1.4736125994561546)/(-1.1056e-46 * 1.8311119954175299e+6)+(1.8311119954175299e+6)/(-1.62923e-46))))]^(1/2)

**Input interpretation:**

$$-\frac{4}{1.1056\times10^{-46}}--\frac{1.4736125994561546}{1.1056\times10^{-46}\times1.8311119\times10^{6}}--\frac{1.8311119\times10^{6}}{1.62923\times10^{-46}}$$

**Result:**

$1.1239088437707639645816085733719240172831998373821284... \times 10^{52}$

1.1239088437707639645816085733719240172831998373821284 × 10^52

**Input interpretation:**

$$-\frac{12\times7.55858\times10^{30}}{1.1056\times10^{-46}\sqrt{-\frac{2}{1.1056\times10^{-46}}+-\frac{1.4736125994561546}{1.1056\times10^{-46}\times1.8311119\times10^{6}}+-\frac{1.8311119\times10^{6}}{1.62923\times10^{-46}}}}$$

**Result:**

$7.73850... \times 10^{51}\, i$

**Polar coordinates:**

$r = 7.7385\times10^{51}$ (radius), $\theta = 90°$ (angle)

7.7385e+51

1/2 (1.12390884377076396458l6e+52 + 7.7385e+51)^1/2

**Input interpretation:**

$$\frac{1}{2}\sqrt{1.12390884377076396458l6\times10^{52}+7.7385\times10^{51}}$$

**Result:**

$6.887958412640794909181674504887156505331221747079637 4\ldots\times10^{25}$

$6.88795841264\ldots*10^{25}$

$5.30074*10^{25} + 6.88795841264*10^{25}$

(5.30074*10^25 + 6.88795841264*10^25)

**Input interpretation:**

$5.30074\times10^{25}+6.88795841264\times10^{25}$

**Result:**

121 886 984 126 400 000 000 000 000

**Scientific notation:**

$1.218869841264\times10^{26}$

$r_4 = 1.218869841264 * 10^{26}$

(5.30074*10^25 - 6.88795841264*10^25)

**Result:**

$-1.58721841264\times10^{25}$

$r_3 = -1.58721841264 * 10^{25}$

**Input interpretation:**

$$\frac{1}{2}\sqrt{1.12390884377076396458l6\times10^{52}-7.7385\times10^{51}}$$

**Result:**

$2.95829\ldots\times10^{25}$

$2.95829\ldots*10^{25}$

(5.30074*10^25 + 2.9582885414153*10^25)

**Input interpretation:**

$5.30074\times 10^{25} + 2.9582885414153\times 10^{25}$

**Result:**

82590285414153000000000000

**Scientific notation:**

$8.2590285414153\times 10^{25}$

$r_2 = 8.2590285414153*10^{25}$

(5.30074*10^25 - 2.9582885414153*10^25)

**Input interpretation:**

$5.30074\times 10^{25} - 2.9582885414153\times 10^{25}$

**Result:**

23424514585847000000000000

**Scientific notation:**

$2.3424514585847\times 10^{25}$

$r_1 = 2.3424514585847*10^{25}$

From the four results (event horizons), we obtain:

$r_1 = 2.3424514585847*10^{25}$

$r_2 = 8.2590285414153*10^{25}$

$r_3 = -1.58721841264 * 10^{25}$

$r_4 = 1.218869841264 * 10^{26}$

(2.3424514585847*10^25 +8.2590285414153*10^25 -1.58721841264 * 10^25 +1.218869841264 * 10^26)

**Input interpretation:**

$2.3424514585847\times 10^{25} + 8.2590285414153\times 10^{25} + 10^{25}\times(-1.58721841264) + 1.218869841264\times 10^{26}$

**Result:**

212 029 600 000 000 000 000 000 000

**Scientific notation:**

$2.120296 \times 10^{26}$

$2.120296*10^{26}$

(2.3424514585847*10^25 +8.2590285414153*10^25 -1.58721841264 * 10^25 +1.218869841264 * 10^26)^1/126

**Input interpretation:**

$(2.3424514585847 \times 10^{25} + 8.2590285414153 \times 10^{25} + 10^{25} \times (-1.58721841264) + 1.218869841264 \times 10^{26})$ ^ $(1/126)$

**Result:**

1.61785522079119...

1.61785522079119... result that is a very good approximation to the value of the golden ratio 1,618033988749...

Now, we have:

$$\left(\frac{dr}{ds}\right)^2 = 2\left[-\frac{M}{r} + \frac{Q^2}{2r^2} - \frac{\Lambda r^2}{6} + k_1^2\left(-\frac{1}{2r^2} + \frac{M}{r^3} - \frac{Q^2}{2r^4}\right)\right], \quad (44)$$

For

r = 11225.7

Λ = -1.1056e-46

Q = 0.00089

M = 7.55858e+30

2[((((-7.55858e+30) / (11225.7) + (0.00089^2) / (2*11225.7^2) – (-1.1056e-46*11225.7^2)/6+x^2((-1/(2*11225.7^2)+(7.55858e+30)/(11225.7)^3-(0.00089)^2/(2*11225.7^4))))))] = 11225.7

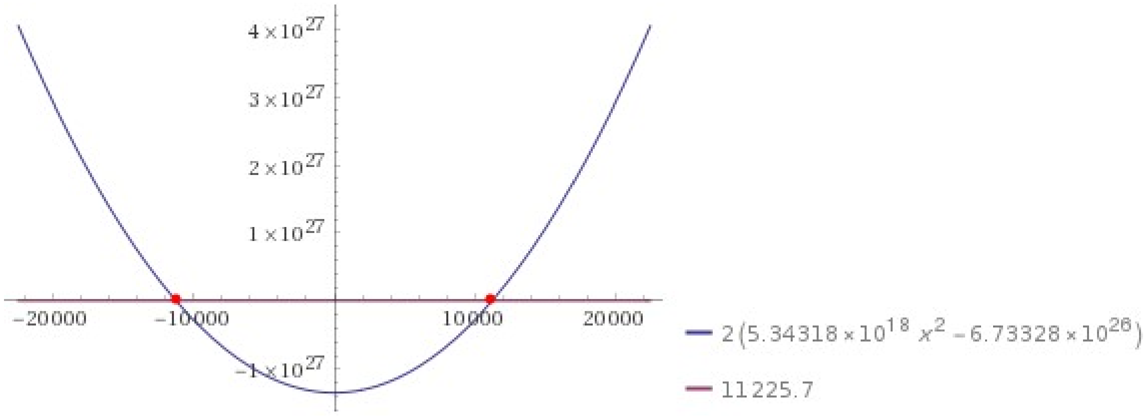
**Input interpretation:**

$$2\left(-\frac{7.55858\times10^{30}}{11\,225.7}+\frac{0.00089^2}{2\times11\,225.7^2}-\frac{1}{6}\left(-1.1056\times10^{-46}\times11\,225.7^2\right)+x^2\left(-\frac{1}{2\times11\,225.7^2}+\frac{7.55858\times10^{30}}{11\,225.7^3}-\frac{0.00089^2}{2\times11\,225.7^4}\right)\right)=11\,225.7$$

**Result:**

$$2\left(5.34318\times10^{18}\,x^2-6.73328\times10^{26}\right)=11\,225.7$$

**Plot:**




**Alternate forms:**

$$1.06864\times10^{19}\,x^2-1.34666\times10^{27}=0$$

$$1.06864\times10^{19}\,x^2-1.34666\times10^{27}=11\,225.7$$

$$1.06864\times10^{19}\,(x-11\,225.7)(x+11\,225.7)=11\,225.7$$

**Solutions:**

$x\approx-11\,225.7$

$x\approx11\,225.7$

11225.7

Thence, we have:

2[((((-7.55858e+30) / (11225.7) + (0.00089^2) / (2*11225.7^2) – (-1.1056e-46*11225.7^2)/6+11225.7^2((-1/(2*11225.7^2)+(7.55858e+30)/(11225.7)^3-(0.00089)^2/(2*11225.7^4))))))]-11225.7

**Input interpretation:**

$$2\left(-\frac{7.55858\times10^{30}}{11\,225.7}+\frac{0.00089^2}{2\times11\,225.7^2}-\frac{1}{6}\left(-1.1056\times10^{-46}\times11\,225.7^2\right)+11\,225.7^2\left(-\frac{1}{2\times11\,225.7^2}+\frac{7.55858\times10^{30}}{11\,225.7^3}-\frac{0.00089^2}{2\times11\,225.7^4}\right)\right)-11\,225.7$$

**Result:**

−11226.699999999999999999999999999999999999953558777984752...

-11226.6999….

We note that from the Ramanujan taxicab number:

$$11161^3+11468^3=14258^3+1$$

11161 + 64 + $\phi$ = 11226.61803398..... result, with positive sign, practically equal to the above value

Furthermore:

-(13+2)/10^3+(-(2[((((-7.55858e+30)/(11225.7)+(0.00089^2)/(2*11225.7^2)–(-1.1056e-46*11225.7^2)/6+11225.7^2((-1/(2*11225.7^2)+(7.55858e+30)/(11225.7)^3-(0.00089)^2/(2*11225.7^4))))))]-11225.7))^1/19

**Input interpretation:**

$$-\frac{13+2}{10^3}+\left(-\left(2\left(-\frac{7.55858\times10^{30}}{11\,225.7}+\frac{0.00089^2}{2\times11\,225.7^2}-\frac{1}{6}\left(-1.1056\times10^{-46}\times11\,225.7^2\right)+11\,225.7^2\left(-\frac{1}{2\times11\,225.7^2}+\frac{7.55858\times10^{30}}{11\,225.7^3}-\frac{0.00089^2}{2\times11\,225.7^4}\right)\right)-11\,225.7\right)\right)\verb|^|(1/19)$$

**Result:**

1.6186956929575781600816675562709037168219258081293574042 34...

1.6186956929575… result that is a very good approximation to the value of the golden ratio 1,618033988749...

Now, we have:

$$E(r)=\left(\frac{2Q^2}{r^4}-\frac{3M}{2r^3}\right)^{\frac{1}{2}}.$$

r = 11225.7

Λ = -1.1056e-46

Q = 0.00089

M = 7.55858e+30

sqrt((((2*0.00089^2)/(11225.7^4) – (3*7.55858e+30)/(2*11225.7^3))))

**Input interpretation:**

$$\sqrt{\frac{2\times0.00089^2}{11\,225.7^4}-\frac{3\times7.55858\times10^{30}}{2\times11\,225.7^3}}$$

**Result:**

$2.83104...\times10^9\, i$

$2.83104...*10^9$ i

(((sqrt((((2*0.00089^2)/(11225.7^4) – (3*7.55858e+30)/(2*11225.7^3)))))))^1/45

**Input interpretation:**

$$\sqrt[45]{\sqrt{\frac{2\times 0.00089^2}{11225.7^4}-\frac{3\times 7.55858\times 10^{30}}{2\times 11225.7^3}}}$$

**Result:**

1.620984... +
0.05660599... $i$

**Polar coordinates:**

$r = 1.62197$ (radius), $\theta = 2°$ (angle)

1.62197

(((sqrt((((2*0.00089^2)/(11225.7^4) –
(3*7.55858e+30)/(2*11225.7^3)))))))*1/4096^2 - 29i

**Input interpretation:**

$$\sqrt{\frac{2\times 0.00089^2}{11225.7^4}-\frac{3\times 7.55858\times 10^{30}}{2\times 11225.7^3}}\times\frac{1}{4096^2}-29\,i$$

$i$ is the imaginary unit

**Result:**

139.743... $i$

**Polar coordinates:**

$r = 139.743$ (radius), $\theta = 90°$ (angle)

139.743 result practically equal to the rest mass of Pion meson 139.57 MeV

(((sqrt((((2*0.00089^2)/(11225.7^4) –
(3*7.55858e+30)/(2*11225.7^3)))))))*1/4096^2 - (47-4)i

**Input interpretation:**

$$\sqrt{\frac{2\times 0.00089^2}{11225.7^4}-\frac{3\times 7.55858\times 10^{30}}{2\times 11225.7^3}}\times\frac{1}{4096^2}-(47-4)\,i$$

$i$ is the imaginary unit

**Result:**

125.743... $i$

**Polar coordinates:**

$r = 125.743$ (radius), $\theta = 90°$ (angle)

125.743 result very near to the dilaton mass calculated as a type of Higgs boson: 125 GeV for T = 0 and to the Higgs boson mass 125.18 GeV

From

$$\sigma(r) = \frac{(1 - \frac{2M}{r} + \frac{Q^2}{r^2} - \frac{\Lambda r^2}{3})^{\frac{1}{2}}(\frac{3M}{2r^3} - \frac{3Q^2}{r^4})}{2\pi r(\frac{2Q^2}{r^4} - \frac{3M}{2r^3})^{\frac{1}{2}}},$$

We obtain, for

r = 11225.7

Λ = -1.1056e-46

Q = 0.00089

M = 7.55858e+30

sqrt[((((1-(2*7.55858e+30)/11225.7 + (0.00089^2)/(11225.7^2) – (-1.1056e-46*11225.7^2)/3)))] * (3*7.55858e+30)/(2*11225.7^3) – (3*0.00089^2)/(11225.7^4) * 1/(2Pi*11225.7) * sqrt[((((2*0.00089^2)/(11225.7^4)-(3*7.55858e+30)/(2*11225.7^3))))]

[(((1-(2*7.55858e+30)/11225.7 + (0.00089^2)/(11225.7^2) – (-1.1056e-46*11225.7^2)/3)))]^0.5 * (3*7.55858e+30)/(2*11225.7^3) – (3*0.00089^2)/(11225.7^4)

**Input interpretation:**

$$\sqrt{1 - \frac{2 \times 7.55858 \times 10^{30}}{11\,225.7} + \frac{0.00089^2}{11\,225.7^2} - \frac{1}{3}\left(-1.1056 \times 10^{-46} \times 11\,225.7^2\right)} \times \frac{3 \times 7.55858 \times 10^{30}}{2 \times 11\,225.7^3} - \frac{3 \times 0.00089^2}{11\,225.7^4}$$

**Result:**

$-1.49640... \times 10^{-22} + 2.94117... \times 10^{32}\, i$

**Polar coordinates:**

$r = 2.94117\times10^{32}$ (radius), $\theta = 90°$ (angle)

2.94117*10$^{32}$

1/(2Pi*11225.7) * [((((2*0.00089^2)/(11225.7^4)-(3*7.55858e+30)/(2*11225.7^3))))]^0.5

**Input interpretation:**

$$\frac{1}{2\pi\times 11225.7}\sqrt{\frac{2\times0.00089^2}{11225.7^4}-\frac{3\times7.55858\times10^{30}}{2\times11225.7^3}}$$

**Result:**

$40137.7...\, i$

**Polar coordinates:**

$r = 40137.7$ (radius), $\theta = 90°$ (angle)

40137.7

40137.7i*((([((((1-(2*7.55858e+30)/11225.7 + (0.00089^2)/(11225.7^2) – (-1.1056e-46*11225.7^2)/3)))]^0.5 * (3*7.55858e+30)/(2*11225.7^3) – (3*0.00089^2)/(11225.7^4))))

**Input interpretation:**

$$40137.7\, i\left(\sqrt{1-\frac{2\times7.55858\times10^{30}}{11225.7}+\frac{0.00089^2}{11225.7^2}-\frac{1}{3}\left(-1.1056\times10^{-46}\times11225.7^2\right)}\times\frac{3\times7.55858\times10^{30}}{2\times11225.7^3}-\frac{3\times0.00089^2}{11225.7^4}\right)$$

$i$ is the imaginary unit

**Result:**

$-1.18052... \times 10^{37} - 6.00620... \times 10^{-18}\, i$

**Alternate form:**

$-1.18052\times10^{37}$

-1.18052*10$^{37}$

(((40137.7i*((([((((1-(2*7.55858e+30)/11225.7 + (0.00089^2)/(11225.7^2) – (-1.1056e-46*11225.7^2)/3)))]^0.5 * (3*7.55858e+30)/(2*11225.7^3) – (3*0.00089^2)/(11225.7^4)))))))^(1/(24e^2))

**Input interpretation:**

$$\left(40\,137.7\, i \left(\sqrt{1-\frac{2\times 7.55858\times 10^{30}}{11\,225.7}+\frac{0.00089^2}{11\,225.7^2}-\frac{1}{3}\left(-1.1056\times 10^{-46}\times 11\,225.7^2\right)}\times \frac{3\times 7.55858\times 10^{30}}{2\times 11\,225.7^3}-\frac{3\times 0.00089^2}{11\,225.7^4}\right)\right)^{\wedge}\left(\frac{1}{24\, e^2}\right)$$

$i$ is the imaginary unit

**Result:**

1.618006... +
0.02866654... $i$

**Polar coordinates:**

$r = 1.61826$ (radius), $\theta = -1.01501°$ (angle)

1.61826 result that is a very good approximation to the value of the golden ratio 1,618033988749...

(((40137.7i*((([((((1-(2*7.55858e+30)/11225.7 + (0.00089^2)/(11225.7^2) – (-1.1056e-46*11225.7^2)/3)))]^0.5 * (3*7.55858e+30)/(2*11225.7^3) – (3*0.00089^2)/(11225.7^4)))))))^1/17 -12

**Input interpretation:**

$$\left(40\,137.7\, i \left(\sqrt{1-\frac{2\times 7.55858\times 10^{30}}{11\,225.7}+\frac{0.00089^2}{11\,225.7^2}-\frac{1}{3}\left(-1.1056\times 10^{-46}\times 11\,225.7^2\right)}\times \frac{3\times 7.55858\times 10^{30}}{2\times 11\,225.7^3}-\frac{3\times 0.00089^2}{11\,225.7^4}\right)\right)^{\wedge}(1/17)-12$$

$i$ is the imaginary unit

**Result:**

137.022... +
27.8571... $i$

**Polar coordinates:**

$r = 139.826$ (radius), $\theta = -11.4918°$ (angle)

139.826 result practically equal to the rest mass of Pion meson 139.57 MeV

(((40137.7i*((([((((1-(2*7.55858e+30)/11225.7 + (0.00089^2)/(11225.7^2) – (-1.1056e-46*11225.7^2)/3)))]^0.5 * (3*7.55858e+30)/(2*11225.7^3) – (3*0.00089^2)/(11225.7^4)))))))^1/17 -27

**Input interpretation:**

$$\left(40\,137.7\,i\left(\sqrt{1-\frac{2\times 7.55858\times 10^{30}}{11\,225.7}+\frac{0.00089^2}{11\,225.7^2}-\frac{1}{3}\left(-1.1056\times 10^{-46}\times 11\,225.7^2\right)}\times \frac{3\times 7.55858\times 10^{30}}{2\times 11\,225.7^3}-\frac{3\times 0.00089^2}{11\,225.7^4}\right)\right)\hat{}\,(1/17)-27$$

$i$ is the imaginary unit

**Result:**

122.022... +
27.8571... $i$

**Polar coordinates:**

$r = 125.162$ (radius), $\theta = -12.86°$ (angle)

125.162 result very near to the dilaton mass calculated as a type of Higgs boson: 125 GeV for T = 0 and to the Higgs boson mass 125.18 GeV

From:

**An exact solution for a rotating black hole in modified gravity**
*Francesco Filippini, Gianmassimo Tasinato* - arXiv:1709.02147v2 [hep-th] 8 Apr 2018

We have that:

$$a_{max} = \frac{M}{2}\sqrt{2-q+2\sqrt{1-q}}$$

$$= \frac{M}{2}\sqrt{2+\frac{Q^2}{M^2}\left(\beta^2-1\right)+2\sqrt{1+\frac{Q^2}{M^2}\left(\beta^2-1\right)}}\,,$$

For M = 13.12806e+39 and $\beta^2 = 9$

1/2(13.12806e+39) * sqrt((((2+((x^2)(9-1))/((13.12806e+39)^2) + 2(((1+((x^2)(9-1))/((13.12806e+39)^2)))^1/2)))))

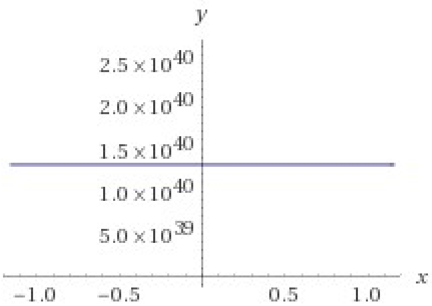
**Input interpretation:**

$$\frac{1}{2} \times 13.12806 \times 10^{39} \sqrt{2 + \frac{x^2 (9-1)}{(13.12806 \times 10^{39})^2} + 2 \sqrt{1 + \frac{x^2 (9-1)}{(13.12806 \times 10^{39})^2}}}$$

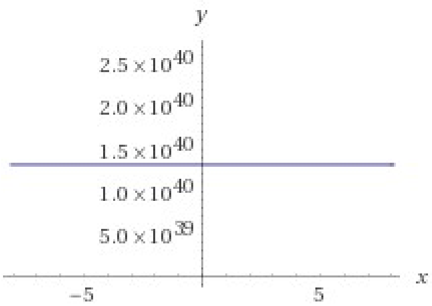
**Result:**

$$6.56403 \times 10^{39} \sqrt{4.64183 \times 10^{-80} x^2 + 2 \sqrt{4.64183 \times 10^{-80} x^2 + 1} + 2}$$

**Plots:**



(x from -1.2 to 1.2)



(x from -8.1 to 8.1)

**Alternate forms:**

$$9.28294 \times 10^{39} \sqrt{2.32091 \times 10^{-80} x^2 + \sqrt{4.64183 \times 10^{-80} x^2 + 1} + 1}$$

$$\begin{cases} 3.28202 \times 10^{39} \sqrt{1.85673 \times 10^{-79} x^2 + 4} + 6.56403 \times 10^{39} & -\frac{\pi}{2} < \arg\left(\sqrt{1.85673 \times 10^{-79} x^2 + 4} + 2\right) \leq \frac{\pi}{2} \\ -3.28202 \times 10^{39} \sqrt{1.85673 \times 10^{-79} x^2 + 4} - 6.56403 \times 10^{39} & \text{(otherwise)} \end{cases}$$

arg(z) is the complex argument

**Alternate form assuming x is real:**

$$6.56403\times 10^{39}\sqrt{4.64183\times 10^{-80}\,x^2+2\sqrt{4.64183\times 10^{-80}\,x^2+1}+2}+0$$

**Roots:**

(no roots exist)

**Properties as a real function:**

**Domain**

$\mathbb{R}$ (all real numbers)

**Range**

$\{y\in\mathbb{R} : y\geq 13\,128\,060\,000\,000\,000\,000\,000\,000\,000\,000\,000\,000\,000\}$

**Parity**

even

$\mathbb{R}$ is the set of real numbers

**Series expansion at x = 0:**

$$1.31281\times 10^{40}+1.52345\times 10^{-40}\,x^2-1.7679\times 10^{-120}\,x^4+O(x^5)$$

(Taylor series)

**Series expansion at x = -4.64147×10^39 i:**

$$6.56403\times10^{39}\sqrt{1+2\sqrt{2-(4.30898\times10^{-40}\,i)\,x}}+\frac{\left((0.353553\,i)\sqrt{2-(4.30898\times10^{-40}\,i)\,x}-1.41421\,i\right)(x+4.64147\times10^{39}\,i)}{\sqrt{1+2\sqrt{2-(4.30898\times10^{-40}\,i)\,x}}}+$$

$$\left(3.28202\times10^{39}\sqrt{1+2\sqrt{2-(4.30898\times10^{-40}\,i)\,x}}\left((-9.37572\times10^{-121}\,i)\,x+1.81321\times10^{-80}\sqrt{2-(4.30898\times10^{-40}\,i)\,x}+2.75608\times10^{-80}\right)(x+4.64147\times10^{39}\,i)^2\right)\Big/\left(0.5+\sqrt{2-(4.30898\times10^{-40}\,i)\,x}\right)^2+$$

$$\left(2.18801\times10^{39}\sqrt{1+2\sqrt{2-(4.30898\times10^{-40}\,i)\,x}}\left(\left(-2.14624\times10^{-160}-8.83745\times10^{-161}\sqrt{2-(4.30898\times10^{-40}\,i)\,x}\right)x+(1.55285\times10^{-120}\,i)\sqrt{2-(4.30898\times10^{-40}\,i)\,x}+2.75412\times10^{-120}\,i\right)(x+4.64147\times10^{39}\,i)^3\right)\Big/\left(0.5+\sqrt{2-(4.30898\times10^{-40}\,i)\,x}\right)^3+$$

$$\left(1.64101\times10^{39}\sqrt{1+2\sqrt{2-(4.30898\times10^{-40}\,i)\,x}}\left(3.76034\times10^{-240}\,x^2+\left((1.88135\times10^{-200}\,i)\sqrt{2-(4.30898\times10^{-40}\,i)\,x}+6.97007\times10^{-200}\,i\right)x-6.56759\times10^{-160}\sqrt{2-(4.30898\times10^{-40}\,i)\,x}-1.0505\times10^{-159}\right)(x+4.64147\times10^{39}\,i)^4\right)\Big/\left(0.5+\sqrt{2-(4.30898\times10^{-40}\,i)\,x}\right)^4+$$

$$O\left((x+4.64147\times10^{39}\,i)^5\right)$$

(Puiseux series)

**Series expansion at x = 4.64147×10^39 i:**

$$6.56403\times10^{39}\sqrt{1+2\sqrt{2+(4.30898\times10^{-40}\,i)\,x}}+\frac{\left(1.41421\,i-(0.353553\,i)\sqrt{2+(4.30898\times10^{-40}\,i)\,x}\right)(x-4.64147\times10^{39}\,i)}{\sqrt{1+2\sqrt{2+(4.30898\times10^{-40}\,i)\,x}}}+$$

$$\left(3.28202\times10^{39}\sqrt{1+2\sqrt{2+(4.30898\times10^{-40}\,i)\,x}}\left((9.37572\times10^{-121}\,i)\,x+1.81321\times10^{-80}\sqrt{2+(4.30898\times10^{-40}\,i)\,x}+2.75608\times10^{-80}\right)(x-4.64147\times10^{39}\,i)^2\right)\Big/\left(0.5+\sqrt{2+(4.30898\times10^{-40}\,i)\,x}\right)^2+$$

$$\left(2.18801\times10^{39}\sqrt{1+2\sqrt{2+(4.30898\times10^{-40}\,i)\,x}}\left(\left(-2.14624\times10^{-160}-8.83745\times10^{-161}\sqrt{2+(4.30898\times10^{-40}\,i)\,x}\right)x-(1.55285\times10^{-120}\,i)\sqrt{2+(4.30898\times10^{-40}\,i)\,x}-2.75412\times10^{-120}\,i\right)(x-4.64147\times10^{39}\,i)^3\right)\Big/\left(0.5+\sqrt{2+(4.30898\times10^{-40}\,i)\,x}\right)^3+$$

$$\left(1.64101\times10^{39}\sqrt{1+2\sqrt{2+(4.30898\times10^{-40}\,i)\,x}}\left(3.76034\times10^{-240}\,x^2+\left((-1.88135\times10^{-200}\,i)\sqrt{2+(4.30898\times10^{-40}\,i)\,x}-6.97007\times10^{-200}\,i\right)x-6.56759\times10^{-160}\sqrt{2+(4.30898\times10^{-40}\,i)\,x}-1.0505\times10^{-159}\right)(x-4.64147\times10^{39}\,i)^4\right)\Big/\left(0.5+\sqrt{2+(4.30898\times10^{-40}\,i)\,x}\right)^4+$$

$$O\left((x-4.64147\times10^{39}\,i)^5\right)$$

(Puiseux series)

**Series expansion at x = ∞:**

$$1.41421\,x + 6.56403\times10^{39} + \frac{1.52334\times10^{79}}{x} + O\left(\left(\frac{1}{x}\right)^3\right)$$

(Laurent series)

**Global minimum:**

$$\min\left\{6\,564\,029\,999\,999\,999\,699\,356\,864\,724\,551\,807\,795\,200\sqrt{4.64183\times10^{-80}\,x^2 + 2\sqrt{4.64183\times10^{-80}\,x^2+1} + 2}\right\} = 13\,128\,059\,999\,999\,999\,398\,713\,729\,449\,103\,615\,590\,400 \text{ at } x = 0$$

**Limit:**

$$\lim_{x\to\pm\infty} 6.56403\times10^{39}\sqrt{2 + 4.64183\times10^{-80}\,x^2 + 2\sqrt{1 + 4.64183\times10^{-80}\,x^2}} = 1.31281\times10^{40}$$

For Q = 4.64147e+39i, we obtain:

1/2(13.12806e+39) * sqrt((((2+(((((((4.64147e+39i)^2)(9-1))))/((((13.12806e+39)^2)))))) + 2(((1+((((((((4.64147e+39i)^2)(9-1)))))/(((((13.12806e+39)^2)))))))))^1/2)))

**Input interpretation:**

$$\frac{1}{2}\times13.12806\times10^{39}\sqrt{\left(2+\frac{(4.64147\times10^{39}\,i)^2\,(9-1)}{(13.12806\times10^{39})^2}\right)+2\sqrt{1+\frac{(4.64147\times10^{39}\,i)^2\,(9-1)}{(13.12806\times10^{39})^2}}}$$

$i$ is the imaginary unit

**Result:**

$$6.56555\ldots\times10^{39}$$

6.56555…*$10^{39}$

(((1/2(13.12806e+39) * sqrt((((2+((((((4.64147e+39i)^2)(9-1))))/((((13.12806e+39)^2)))))) + 2(((1+(((((((4.64147e+39i)^2)(9-1)))))/(((((13.12806e+39)^2)))))))))^1/2))))))^1/19+1/golden ratio

**Input interpretation:**

$$\left(\frac{1}{2} \times 13.12806 \times 10^{39} \sqrt{\left(2+\frac{(4.64147\times 10^{39}\, i)^2\,(9-1)}{(13.12806\times 10^{39})^2}\right)+2\sqrt{1+\frac{(4.64147\times 10^{39}\, i)^2\,(9-1)}{(13.12806\times 10^{39})^2}}}\right)\hat{}\,(1/19)+\frac{1}{\phi}$$

$i$ is the imaginary unit

$\phi$ is the golden ratio

**Result:**

125.255...

125.255… result very near to the dilaton mass calculated as a type of Higgs boson: 125 GeV for T = 0 and to the Higgs boson mass 125.18 GeV

(((1/2(13.12806e+39) * sqrt((((2+((((((4.64147e+39i)^2)(9-1))))/((((13.12806e+39)^2)))))) + 2(((1+(((((((4.64147e+39i)^2)(9-1)))))/(((((13.12806e+39)^2)))))))))^1/2))))))^1/19+13+golden ratio

**Input interpretation:**

$$\left(\frac{1}{2} \times 13.12806 \times 10^{39} \sqrt{\left(2+\frac{(4.64147\times 10^{39}\, i)^2\,(9-1)}{(13.12806\times 10^{39})^2}\right)+2\sqrt{1+\frac{(4.64147\times 10^{39}\, i)^2\,(9-1)}{(13.12806\times 10^{39})^2}}}\right)\hat{}\,(1/19)+13+\phi$$

$i$ is the imaginary unit

$\phi$ is the golden ratio

**Result:**

139.255...

139.255... result practically equal to the rest mass of Pion meson 139.57 MeV

For Q = 0.00089, we obtain:

1/2(13.12806e+39) * sqrt((((2+((0.00089^2)(9-1))/((13.12806e+39)^2) + 2(((1+((0.00089^2)(9-1))/((13.12806e+39)^2)))^1/2)))))

**Input interpretation:**

$$\frac{1}{2}\times 13.12806\times 10^{39}\sqrt{2+\frac{0.00089^2\,(9-1)}{(13.12806\times 10^{39})^2}+2\sqrt{1+\frac{0.00089^2\,(9-1)}{(13.12806\times 10^{39})^2}}}$$

**Result:**

$1.31281... \times 10^{40}$

$1.31281\times 10^{40} = 13.1281\times 10^{39}$

$13.1281 * 10^{39}$

(((1/2(13.12806e+39) * sqrt((((2+((0.00089^2)(9-1))/((13.12806e+39)^2) + 2(((1+((0.00089^2)(9-1))/((13.12806e+39)^2)))^1/2))))))))^1/19+7+Pi

**Input interpretation:**

$$\sqrt[19]{\frac{1}{2}\times 13.12806\times 10^{39}\sqrt{2+\frac{0.00089^2\,(9-1)}{(13.12806\times 10^{39})^2}+2\sqrt{1+\frac{0.00089^2\,(9-1)}{(13.12806\times 10^{39})^2}}}}+7+\pi$$

**Result:**

139.4076...

139.4076… result practically equal to the rest mass of Pion meson 139.57 MeV

(((1/2(13.12806e+39) * sqrt((((2+((0.00089^2)(9-1))/((13.12806e+39)^2) + 2(((1+((0.00089^2)(9-1))/((13.12806e+39)^2)))^1/2))))))))^1/19-4

**Input interpretation:**

$$\sqrt[19]{\frac{1}{2}\times 13.12806\times 10^{39}\sqrt{2+\frac{0.00089^2\,(9-1)}{(13.12806\times 10^{39})^2}+2\sqrt{1+\frac{0.00089^2\,(9-1)}{(13.12806\times 10^{39})^2}}}}-4$$

**Result:**

125.2660...

125.2660… result very near to the dilaton mass calculated as a type of Higgs boson: 125 GeV for T = 0 and to the Higgs boson mass 125.18 GeV

27*1/2*((((((1/2(13.12806e+39) * sqrt((((2+((0.00089^2)(9-1))/((13.12806e+39)^2) + 2(((1+((0.00089^2)(9-1))/((13.12806e+39)^2)))^1/2))))))))^1/19-4+e)))+sqrt2

**Input interpretation:**

$$27\times\frac{1}{2}\left(\sqrt[19]{\frac{1}{2}\times 13.12806\times 10^{39}\sqrt{2+\frac{0.00089^2\,(9-1)}{(13.12806\times 10^{39})^2}+2\sqrt{1+\frac{0.00089^2\,(9-1)}{(13.12806\times 10^{39})^2}}}}-4+e\right)+\sqrt{2}$$

**Result:**

1729.202...

1729.202…

Now, we have that:

$$\omega = 1 - \frac{r_h^{pol}}{r_h^{eq}} = 1 - \frac{1+\sqrt{1-q}}{2+\sqrt{2-3q-2\sqrt{1-q}}}.$$

For q = -0.2 , we obtain:

1-(((1+sqrt(1+0.2))))/(((2+sqrt(((2-3(-0.2)-2(1+0.2)^0.5))))))

**Input:**

$$1-\frac{1+\sqrt{1+0.2}}{2+\sqrt{2-3\times(-0.2)-2\sqrt{1+0.2}}}$$

**Result:**

0.206155590054464378289973211536470997616661831147095278081...

0.206155590054….

1/3*1/(((1-(((1+sqrt(1+0.2))))/(((2+sqrt(((2-3(-0.2)-2(1+0.2)^0.5)))))))))

**Input:**

$$\frac{1}{3}\times\frac{1}{1-\frac{1+\sqrt{1+0.2}}{2+\sqrt{2-3\times(-0.2)-2\sqrt{1+0.2}}}}$$

**Result:**

1.616901745158934497130440474700299294662044242674247828823...

1.6169017451589... result that is an approximation to the value of the golden ratio 1,618033988749...

26/(((1-(((1+sqrt(1+0.2))))/(((2+sqrt(((2-3(-0.2)-2(1+0.2)^0.5))))))))))-1/golden ratio

**Input:**

$$\frac{26}{1-\frac{1+\sqrt{1+0.2}}{2+\sqrt{2-3\times(-0.2)-2\sqrt{1+0.2}}}}-\frac{1}{\phi}$$

$\phi$ is the golden ratio

**Result:**

125.500...

125.500… result very near to the dilaton mass calculated as a type of Higgs boson: 125 GeV for T = 0 and to the Higgs boson mass 125.18 GeV

## Series representations:

$$\frac{26}{1-\frac{1+\sqrt{1+0.2}}{2+\sqrt{2-3(-0.2)-2\sqrt{1+0.2}}}}-\frac{1}{\phi}=$$

$$\frac{-1+52\phi-\sum_{k=0}^{\infty}\frac{(-1)^k(-0.59089)^k\left(-\frac{1}{2}\right)_k}{k!}+26\phi\sum_{k=0}^{\infty}\frac{(-1)^k(-0.59089)^k\left(-\frac{1}{2}\right)_k}{k!}+\sum_{k=0}^{\infty}\frac{(-0.2)^k\left(-\frac{1}{2}\right)_k}{k!}}{\phi\left(1+\sum_{k=0}^{\infty}\frac{(-1)^k(-0.59089)^k\left(-\frac{1}{2}\right)_k}{k!}-\sum_{k=0}^{\infty}\frac{(-0.2)^k\left(-\frac{1}{2}\right)_k}{k!}\right)}$$

$$\frac{26}{1-\frac{1+\sqrt{1+0.2}}{2+\sqrt{2-3(-0.2)-2\sqrt{1+0.2}}}}-\frac{1}{\phi}=$$

$$\left(-1+52\phi-\sqrt{z_0}\sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k(0.40911-z_0)^k z_0^{-k}}{k!}+26\phi\sqrt{z_0}\sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k(0.40911-z_0)^k z_0^{-k}}{k!}+\sqrt{z_0}\sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k(1.2-z_0)^k z_0^{-k}}{k!}\right)\Bigg/\left(\phi\left(1+\sqrt{z_0}\sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k(0.40911-z_0)^k z_0^{-k}}{k!}-\sqrt{z_0}\sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k(1.2-z_0)^k z_0^{-k}}{k!}\right)\right) \text{ for not } ((z_0\in\mathbb{R} \text{ and } -\infty<z_0\leq 0))$$

$$\frac{26}{1-\frac{1+\sqrt{1+0.2}}{2+\sqrt{2-3(-0.2)-2\sqrt{1+0.2}}}}-\frac{1}{\phi}=$$

$$\left(-2\sqrt{\pi}+104\phi\sqrt{\pi}+\sum_{j=0}^{\infty}\text{Res}_{s=-j}(-0.59089)^{-s}\,\Gamma\left(-\frac{1}{2}-s\right)\Gamma(s)-26\phi\sum_{j=0}^{\infty}\text{Res}_{s=-j}(-0.59089)^{-s}\,\Gamma\left(-\frac{1}{2}-s\right)\Gamma(s)-\sum_{j=0}^{\infty}\text{Res}_{s=-j}\,e^{1.60944s}\,\Gamma\left(-\frac{1}{2}-s\right)\Gamma(s)\right)\Bigg/\left(\phi\left(2\sqrt{\pi}-\sum_{j=0}^{\infty}\text{Res}_{s=-j}(-0.59089)^{-s}\,\Gamma\left(-\frac{1}{2}-s\right)\Gamma(s)+\sum_{j=0}^{\infty}\text{Res}_{s=-j}\,e^{1.60944s}\,\Gamma\left(-\frac{1}{2}-s\right)\Gamma(s)\right)\right)$$

26/(((1-(((1+sqrt(1+0.2))))/(((2+sqrt(((2-3(-0.2)-2(1+0.2)^0.5))))))))+13+1/golden ratio

**Input:**

$$\frac{26}{1-\frac{1+\sqrt{1+0.2}}{2+\sqrt{2-3\times(-0.2)-2\sqrt{1+0.2}}}}+13+\frac{1}{\phi}$$

$\phi$ is the golden ratio

**Result:**

139.736...

139.736… result practically equal to the rest mass of Pion meson 139.57 MeV

**Series representations:**

$$\frac{26}{1-\frac{1+\sqrt{1+0.2}}{2+\sqrt{2-3(-0.2)-2\sqrt{1+0.2}}}}+13+\frac{1}{\phi}=$$

$$\left(1+65\,\phi+\sum_{k=0}^{\infty}\frac{(-1)^k\,(-0.59089)^k\left(-\frac{1}{2}\right)_k}{k!}+39\,\phi\sum_{k=0}^{\infty}\frac{(-1)^k\,(-0.59089)^k\left(-\frac{1}{2}\right)_k}{k!}-\sum_{k=0}^{\infty}\frac{(-0.2)^k\left(-\frac{1}{2}\right)_k}{k!}-13\,\phi\sum_{k=0}^{\infty}\frac{(-0.2)^k\left(-\frac{1}{2}\right)_k}{k!}\right)\Big/\left(\phi\left(1+\sum_{k=0}^{\infty}\frac{(-1)^k\,(-0.59089)^k\left(-\frac{1}{2}\right)_k}{k!}-\sum_{k=0}^{\infty}\frac{(-0.2)^k\left(-\frac{1}{2}\right)_k}{k!}\right)\right)$$

$$\frac{26}{1-\frac{1+\sqrt{1+0.2}}{2+\sqrt{2-3(-0.2)-2\sqrt{1+0.2}}}}+13+\frac{1}{\phi}=$$

$$\left(1+65\phi+\sqrt{z_0}\sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k(0.40911-z_0)^k z_0^{-k}}{k!}+39\phi\sqrt{z_0}\sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k(0.40911-z_0)^k z_0^{-k}}{k!}-\sqrt{z_0}\sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k(1.2-z_0)^k z_0^{-k}}{k!}-13\phi\sqrt{z_0}\sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k(1.2-z_0)^k z_0^{-k}}{k!}\right)\Big/$$

$$\left(\phi\left(1+\sqrt{z_0}\sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k(0.40911-z_0)^k z_0^{-k}}{k!}-\sqrt{z_0}\sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k(1.2-z_0)^k z_0^{-k}}{k!}\right)\right)\quad \text{for not } ((z_0\in\mathbb{R} \text{ and } -\infty<z_0\le 0))$$

$$\frac{26}{1-\frac{1+\sqrt{1+0.2}}{2+\sqrt{2-3(-0.2)-2\sqrt{1+0.2}}}}+13+\frac{1}{\phi}=$$

$$\left(2\sqrt{\pi}+130\phi\sqrt{\pi}-\sum_{j=0}^{\infty}\text{Res}_{s=-j}(-0.59089)^{-s}\,\Gamma\left(-\frac{1}{2}-s\right)\Gamma(s)-39\phi\sum_{j=0}^{\infty}\text{Res}_{s=-j}(-0.59089)^{-s}\,\Gamma\left(-\frac{1}{2}-s\right)\Gamma(s)+\sum_{j=0}^{\infty}\text{Res}_{s=-j}\,e^{1.60944\,s}\,\Gamma\left(-\frac{1}{2}-s\right)\Gamma(s)+13\phi\sum_{j=0}^{\infty}\text{Res}_{s=-j}\,e^{1.60944\,s}\,\Gamma\left(-\frac{1}{2}-s\right)\Gamma(s)\right)\Big/$$

$$\left(\phi\left(2\sqrt{\pi}-\sum_{j=0}^{\infty}\text{Res}_{s=-j}(-0.59089)^{-s}\,\Gamma\left(-\frac{1}{2}-s\right)\Gamma(s)+\sum_{j=0}^{\infty}\text{Res}_{s=-j}\,e^{1.60944\,s}\,\Gamma\left(-\frac{1}{2}-s\right)\Gamma(s)\right)\right)$$

Now, we have that:

$$r_{erg} = M\left(1+\sqrt{1+\frac{Q^2}{M^2}(\beta^2-1)}\right),$$

(13.12806e+39) (((1+ (((1+((4.64147e+39i)^2)/((13.12806e+39)^2)*(9-1)))^1/2)))

**Input interpretation:**

$$13.12806\times 10^{39}\left(1+\sqrt{1+\frac{(4.64147\times 10^{39}\, i)^2}{(13.12806\times 10^{39})^2}\,(9-1)}\right)$$

$i$ is the imaginary unit

**Result:**

$1.3131105718306081506284250792549432118921377837108829... \times 10^{40}$

$1.313110571830\times 10^{40} = 13.13110571830\times 10^{39}$

$13.1311057183...*10^{39}$

(((((13.12806e+39) (((1+ (((1+((4.64147e+39i)^2)/((13.12806e+39)^2)*(9-1)))^1/2)))))))^1/19-4

**Input interpretation:**

$$\sqrt[19]{13.12806\times 10^{39}\left(1+\sqrt{1+\frac{(4.64147\times 10^{39}\, i)^2}{(13.12806\times 10^{39})^2}\,(9-1)}\right)}-4$$

$i$ is the imaginary unit

**Result:**

125.268...

125.268… result very near to the dilaton mass calculated as a type of Higgs boson: 125 GeV for T = 0 and to the Higgs boson mass 125.18 GeV

(((((13.12806e+39) (((1+ (((1+((4.64147e+39i)^2)/((13.12806e+39)^2)*(9-1)))^1/2)))))))^1/19+7+Pi

**Input interpretation:**

$$\sqrt[19]{13.12806\times 10^{39}\left(1+\sqrt{1+\frac{(4.64147\times 10^{39}\, i)^2}{(13.12806\times 10^{39})^2}\,(9-1)}\right)}+7+\pi$$

$i$ is the imaginary unit

**Result:**

139.409...

139.409... result practically equal to the rest mass of Pion meson 139.57 MeV

## Mathematical connections

Now, we describe some mathematical connections between the Ramanujan expressions and the equations concerning the black holes physics.

From pag.86, we have:

**Input interpretation:**

$$26\times 1\Bigg/\left(\frac{1}{20}\log\left(\frac{(1+2)^5}{1+2^5}\right)+\frac{\log\left(\frac{1+\frac{2}{2}\left(\sqrt{5}-1\right)+4}{1-\frac{2}{2}\left(\sqrt{5}-1\right)+4}\right)}{4\sqrt{5}}+\frac{1}{20}\sqrt{10-2\sqrt{5}}\tan^{-1}\left(\frac{2\sqrt{10-2\sqrt{5}}}{4-2\left(\sqrt{5}+1\right)}\right)+0.164708638\right)+5$$

log(x) is the natural logarithm

$\tan^{-1}(x)$ is the inverse tangent function

**Result:**

139.557430...

(result in radians)

139.55743...

From page 147, we have:

**Input interpretation:**

$$\left(40137.7\, i\left(\sqrt{1-\frac{2\times 7.55858\times 10^{30}}{11225.7}+\frac{0.00089^2}{11225.7^2}-\frac{1}{3}\left(-1.1056\times 10^{-46}\times 11225.7^2\right)\times}\right.\right.$$

$$\left.\left.\frac{3\times 7.55858\times 10^{30}}{2\times 11225.7^3}-\frac{3\times 0.00089^2}{11225.7^4}\right)\right)\text{^}(1/17)-12$$

$i$ is the imaginary unit

**Result:**

137.022... +
27.8571... $i$

**Polar coordinates:**

$r = 139.826$ (radius), $\theta = -11.4918°$ (angle)

139.826

Thence, we obtain:

$$\left(26\times 1\Bigg/\left(\frac{1}{20}\log\left(\frac{(1+2)^5}{1+2^5}\right)+\frac{\log\left(\frac{1+\frac{2}{2}\left(\sqrt{5}-1\right)+4}{1-\frac{2}{2}\left(\sqrt{5}-1\right)+4}\right)}{4\sqrt{5}}+\frac{1}{20}\sqrt{10-2\sqrt{5}}\tan^{-1}\left(\frac{2\sqrt{10-2\sqrt{5}}}{4-2\left(\sqrt{5}+1\right)}\right)+0.164708638\right)+5\right) = 139.55743 \approx$$

$$\approx \left( \left( 40\,137.7\,i \left( \sqrt{1 - \frac{2 \times 7.55858 \times 10^{30}}{11\,225.7} + \frac{0.00089^2}{11\,225.7^2} - \frac{1}{3} \left( -1.1056 \times 10^{-46} \times 11\,225.7^2 \right) \times \frac{3 \times 7.55858 \times 10^{30}}{2 \times 11\,225.7^3} - \frac{3 \times 0.00089^2}{11\,225.7^4}} \right) \right) \verb|^| (1/17) - 12 \right) = 139.826$$

Now, from page 89, we have:

**Input interpretation:**

$$26 \times 1 \Bigg/ \left( \frac{1}{20} \log\left( \frac{(1+2)^5}{1+2^5} \right) + \frac{\log\left( \frac{1+\frac{2}{2}(\sqrt{5}-1)+4}{1-\frac{2}{2}(\sqrt{5}-1)+4} \right)}{4\sqrt{5}} + \frac{1}{20}\sqrt{10-2\sqrt{5}}\,\tan^{-1}\left( \frac{2\sqrt{10-2\sqrt{5}}}{4-2(\sqrt{5}+1)} \right) + 0.164708638 \right) - 11 + 2$$

log(x) is the natural logarithm

$\tan^{-1}(x)$ is the inverse tangent function

**Result:**

125.557430...

(result in radians)

125.55743...

From page 148, we have:

**Input interpretation:**

$$\left(40137.7\,i\left(\sqrt{1-\frac{2\times 7.55858\times 10^{30}}{11225.7}+\frac{0.00089^2}{11225.7^2}-\frac{1}{3}\left(-1.1056\times 10^{-46}\times 11225.7^2\right)}\times\left(\frac{3\times 7.55858\times 10^{30}}{2\times 11225.7^3}-\frac{3\times 0.00089^2}{11225.7^4}\right)\right)\right)^{\wedge}(1/17)-27$$

$i$ is the imaginary unit

**Result:**

122.022... +
27.8571... $i$

**Polar coordinates:**

$r = 125.162$ (radius), $\theta = -12.86°$ (angle)

125.162

Thence, we obtain:

$$\left(26\times 1\Big/\left(\frac{1}{20}\log\left(\frac{(1+2)^5}{1+2^5}\right)+\frac{\log\left(\frac{1+\frac{2}{2}\left(\sqrt{5}-1\right)+4}{1-\frac{2}{2}\left(\sqrt{5}-1\right)+4}\right)}{4\sqrt{5}}+\frac{1}{20}\sqrt{10-2\sqrt{5}}\tan^{-1}\left(\frac{2\sqrt{10-2\sqrt{5}}}{4-2\left(\sqrt{5}+1\right)}\right)+0.164708638\right)-11+2\right) = 125.55743 \approx$$

$$\approx \left( \left( 40\,137.7\,i \left( \sqrt{1 - \frac{2 \times 7.55858 \times 10^{30}}{11\,225.7} + \frac{0.00089^2}{11\,225.7^2} - \frac{1}{3}\left(-1.1056 \times 10^{-46} \times 11\,225.7^2\right) \times \frac{3 \times 7.55858 \times 10^{30}}{2 \times 11\,225.7^3} - \frac{3 \times 0.00089^2}{11\,225.7^4}} \right) \right) \hat{}\,(1/17) - 27 \right) = 125.162$$

From page 53 and 58, we have:

**Input interpretation:**

$$42 \Big/ \left( \frac{1}{4} \tan^{-1}(2) - \frac{1}{20} \tan^{-1}(2)^5 + \frac{\tan^{-1}\left(\frac{(2-2^3)\sqrt{5}}{1-3\times 2^2+2^4}\right)}{4\sqrt{5}} + \frac{1}{40}\sqrt{10-2\sqrt{5}}\, \log\left(\frac{1+1\sqrt{10-2\sqrt{5}}+4}{1-1\sqrt{10-2\sqrt{5}}+4}\right) + 0.18987255794 \right) + \pi$$

$\tan^{-1}(x)$ is the inverse tangent function

$\log(x)$ is the natural logarithm

**Result:**

139.61873747...

(result in radians)

139.61873747…

**Input interpretation:**

$$42 \Big/ \left(\frac{1}{4}\tan^{-1}(2) - \frac{1}{20}\tan^{-1}(2)^5 + \frac{\tan^{-1}\left(\frac{(2-2^3)\sqrt{5}}{1-3\times 2^2+2^4}\right)}{4\sqrt{5}} + \frac{1}{40}\sqrt{10-2\sqrt{5}}\,\log\left(\frac{1+1\sqrt{10-2\sqrt{5}}+4}{1-1\sqrt{10-2\sqrt{5}}+4}\right) + 0.18987255794\right) - 11$$

$\tan^{-1}(x)$ is the inverse tangent function

$\log(x)$ is the natural logarithm

**Result:**

125.47714482...

(result in radians)

125.47714482...

From page 155 and 156, we have:

**Input interpretation:**

$$\sqrt[19]{\frac{1}{2}\times 13.12806\times 10^{39}\sqrt{2+\frac{0.00089^2\,(9-1)}{(13.12806\times 10^{39})^2}+2\sqrt{1+\frac{0.00089^2\,(9-1)}{(13.12806\times 10^{39})^2}}}+7+\pi}$$

**Result:**

139.4076...

139.4076...

(((1/2(13.12806e+39) * sqrt((((2+((0.00089^2)(9-1))/((13.12806e+39)^2) + 2(((1+((0.00089^2)(9-1))/((13.12806e+39)^2)))^1/2))))))))^1/19-4

**Input interpretation:**

$$\sqrt[19]{\frac{1}{2}\times 13.12806\times 10^{39}\sqrt{2+\frac{0.00089^2\,(9-1)}{(13.12806\times 10^{39})^2}+2\sqrt{1+\frac{0.00089^2\,(9-1)}{(13.12806\times 10^{39})^2}}}}-4$$

**Result:**

125.2660...

125.2660…

Thence, we obtain:

$$\left(42\Big/\left(\frac{1}{4}\tan^{-1}(2)-\frac{1}{20}\tan^{-1}(2)^5+\frac{\tan^{-1}\left(\frac{(2-2^3)\sqrt{5}}{1-3\times 2^2+2^4}\right)}{4\sqrt{5}}+\frac{1}{40}\sqrt{10-2\sqrt{5}}\,\log\left(\frac{1+1\sqrt{10-2\sqrt{5}}+4}{1-1\sqrt{10-2\sqrt{5}}+4}\right)+0.18987255794\right)+\pi\right) = 139.61873747 \approx$$

$$\approx \left( \sqrt[19]{\frac{1}{2} \times 13.12806 \times 10^{39} \sqrt{2 + \frac{0.00089^2 (9-1)}{(13.12806 \times 10^{39})^2} + 2\sqrt{1 + \frac{0.00089^2 (9-1)}{(13.12806 \times 10^{39})^2}}}} + 7 + \pi \right) = 139.4076$$

And:

$$\left( 42 \Bigg/ \left( \frac{1}{4} \tan^{-1}(2) - \frac{1}{20} \tan^{-1}(2)^5 + \frac{\tan^{-1}\left( \frac{(2-2^3)\sqrt{5}}{1-3\times 2^2+2^4} \right)}{4\sqrt{5}} + \frac{1}{40} \sqrt{10 - 2\sqrt{5}} \log\left( \frac{1 + 1\sqrt{10-2\sqrt{5}} + 4}{1 - 1\sqrt{10-2\sqrt{5}} + 4} \right) + 0.18987255794 \right) - 11 \right) = 125.47714482 \approx$$

$$\approx \left( \sqrt[19]{\frac{1}{2} \times 13.12806 \times 10^{39} \sqrt{2 + \frac{0.00089^2 (9-1)}{(13.12806 \times 10^{39})^2} + 2\sqrt{1 + \frac{0.00089^2 (9-1)}{(13.12806 \times 10^{39})^2}}}} - 4 \right) = 125.2660$$

## Appendix

## DILATON VALUE CALCULATIONS

from:

**Modular equations and approximations to $\pi$** - *Srinivasa Ramanujan*
Quarterly Journal of Mathematics, XLV, 1914, 350 – 372

We have that:

5. Since $G_n$ and $g_n$ can be expressed as roots of algebraical equations with rational coefficients, the same is true of $G_n^{24}$ or $g_n^{24}$. So let us suppose that

$$1 = ag_n^{-24} - bg_n^{-48} + \cdots,$$

or

$$g_n^{24} = a - bg_n^{-24} + \cdots.$$

But we know that

$$\begin{aligned} 64e^{-\pi\sqrt{n}}g_n^{24} &= 1 - 24e^{-\pi\sqrt{n}} + 276e^{-2\pi\sqrt{n}} - \cdots, \\ 64g_n^{24} &= e^{\pi\sqrt{n}} - 24 + 276e^{-\pi\sqrt{n}} - \cdots, \\ 64a - 64bg_n^{-24} + \cdots &= e^{\pi\sqrt{n}} - 24 + 276e^{-\pi\sqrt{n}} - \cdots, \\ 64a - 4096be^{-\pi\sqrt{n}} + \cdots &= e^{\pi\sqrt{n}} - 24 + 276e^{-\pi\sqrt{n}} - \cdots, \end{aligned}$$

that is

$$e^{\pi\sqrt{n}} = (64a + 24) - (4096b + 276)e^{-\pi\sqrt{n}} + \cdots \qquad (13)$$

Similarly, if

$$1 = aG_n^{-24} - bG_n^{-48} + \cdots,$$

then

$$e^{\pi\sqrt{n}} = (64a - 24) - (4096b + 276)e^{-\pi\sqrt{n}} + \cdots \qquad (14)$$

From (13) and (14) we can find whether $e^{\pi\sqrt{n}}$ is very nearly an integer for given values of $n$, and ascertain also the number of 9's or 0's in the decimal part. But if $G_n$ and $g_n$ be simple quadratic surds we may work independently as follows. We have, for example,

$$g_{22} = \sqrt{(1 + \sqrt{2})}.$$

Hence

$$
\begin{aligned}
64g_{22}^{24} &= e^{\pi\sqrt{22}} - 24 + 276e^{-\pi\sqrt{22}} - \cdots, \\
64g_{22}^{-24} &= \qquad\qquad 4096e^{-\pi\sqrt{22}} + \cdots,
\end{aligned}
$$

so that

$$64(g_{22}^{24} + g_{22}^{-24}) = e^{\pi\sqrt{22}} - 24 + 4372e^{-\pi\sqrt{22}} + \cdots = 64\{(1+\sqrt{2})^{12} + (1-\sqrt{2})^{12}\}.$$

Hence

$$e^{\pi\sqrt{22}} = 2508951.9982\ldots$$

Again

$$G_{37} = (6+\sqrt{37})^{\frac{1}{4}},$$

$$
\begin{aligned}
64G_{37}^{24} &= e^{\pi\sqrt{37}} + 24 + 276e^{-\pi\sqrt{37}} + \cdots, \\
64G_{37}^{-24} &= \qquad\qquad 4096e^{-\pi\sqrt{37}} - \cdots,
\end{aligned}
$$

so that

$$64(G_{37}^{24} + G_{37}^{-24}) = e^{\pi\sqrt{37}} + 24 + 4372e^{-\pi\sqrt{37}} - \cdots = 64\{(6+\sqrt{37})^{6} + (6-\sqrt{37})^{6}\}.$$

Hence

$$e^{\pi\sqrt{37}} = 199148647.999978\ldots.$$

Similarly, from

$$g_{58} = \sqrt{\left(\frac{5+\sqrt{29}}{2}\right)},$$

we obtain

$$64(g_{58}^{24} + g_{58}^{-24}) = e^{\pi\sqrt{58}} - 24 + 4372e^{-\pi\sqrt{58}} + \cdots = 64\left\{\left(\frac{5+\sqrt{29}}{2}\right)^{12} + \left(\frac{5-\sqrt{29}}{2}\right)^{12}\right\}.$$

Hence

$$e^{\pi\sqrt{58}} = 24591257751.99999982\ldots.$$

From:

**An Update on Brane Supersymmetry Breaking**
*J. Mourad and A. Sagnotti* - arXiv:1711.11494v1 [hep-th] 30 Nov 2017

Now, we have that:

From the following vacuum equations:

$$T\, e^{\gamma_E\, \phi} \;=\; -\, \frac{\beta_E^{(p)}\, h^2}{\gamma_E}\; e^{-2\,(8-p)\,C+2\,\beta_E^{(p)}\,\phi}$$

$$16\, k'\, e^{\;\;2C} \;=\; \frac{h^2 \left(p \,+\, 1 \,-\, \frac{2\,\beta_E^{(p)}}{\gamma_E}\right) e^{-2\,(8-p)\,C+2\,\beta_E^{(p)}\,\phi}}{(7\,-\,p)}$$

$$(A')^2 \;-\; k\, e^{-2A} \;+\; \frac{h^2}{16(p\,+\,1)} \left(7 \,-\, p \,+\, \frac{2\,\beta_E^{(p)}}{\gamma_E}\right) e^{-2\,(8-p)\,C+2\,\beta_E^{(p)}\,\phi}$$

We have obtained, from the results almost equals of the equations, putting

$4096\, e^{-\pi\sqrt{18}}$ instead of

$$e^{-2\,(8-p)\,C+2\,\beta_E^{(p)}\,\phi}$$

a new possible mathematical connection between the two exponentials. Thence, also the values concerning $p$, $C$, $\beta_E$ and $\phi$ correspond to the exponents of *e* (i.e. of exp). Thence we obtain for p = 5 and $\beta_E = 1/2$:

$$e^{-6C+\phi} = 4096e^{-\pi\sqrt{18}}$$

Therefore with respect to the exponential of the vacuum equation, the Ramanujan's exponential has a coefficient of 4096 which is equal to $64^2$, while -6C+$\phi$ is equal to -$\pi\sqrt{18}$. From this it follows that it is possible to establish mathematically, the dilaton value.

phi = -Pi*sqrt(18) + 6C, for C = 1, we obtain:

exp((-Pi*sqrt(18))

**Input:**

$\exp\left(-\pi\sqrt{18}\right)$

**Exact result:**

$e^{-3\sqrt{2}\,\pi}$

**Decimal approximation:**

$1.6272016226072509292942156739117979541838581136954016... \times 10^{-6}$

1.6272016… * $10^{-6}$

Now:

$e^{-6C+\phi} = 4096e^{-\pi\sqrt{18}}$

$e^{-\pi\sqrt{18}}$ = 1.6272016… * $10^{-6}$

$\frac{1}{4096} e^{-6C+\phi}$ = 1.6272016… * $10^{-6}$

0.000244140625 $e^{-6C+\phi} = e^{-\pi\sqrt{18}}$ = 1.6272016… * $10^{-6}$

$\ln\left(e^{-\pi\sqrt{18}}\right) = -13.328648814475 = -\pi\sqrt{18}$

(1.6272016* 10^-6) *1/ (0.000244140625)

**Input interpretation:**

$\frac{1.6272016}{10^6} \times \frac{1}{0.000244140625}$

**Result:**

0.0066650177536

0.006665017...

0.000244140625 $e^{-6C+\phi} = e^{-\pi\sqrt{18}}$

Dividing both sides by 0.000244140625, we obtain:

$\frac{0.000244140625}{0.000244140625} e^{-6C+\phi} = \frac{1}{0.000244140625} e^{-\pi\sqrt{18}}$

$e^{-6C+\phi}$ = 0.0066650177536

((((exp((-Pi*sqrt(18)))))))*1/0.000244140625

**Input interpretation:**

$\exp\left(-\pi\sqrt{18}\right) \times \frac{1}{0.000244140625}$

**Result:**

0.00666501785...

0.00666501785…

$e^{-6C+\phi} = 0.0066650177536$

$\exp\left(-\pi\sqrt{18}\right) \times \frac{1}{0.000244140625} =$

$e^{-\pi\sqrt{18}} \times \frac{1}{0.000244140625}$

= 0.00666501785…

ln(0.00666501784619)

**Input interpretation:**

log(0.00666501784619)

**Result:**

-5.010882647757...

-5.010882647757…

Now:

$-6C + \phi = -5.010882647757 \ldots$

For C = 1, we obtain:

$\phi = -5.010882647757 + 6 = \mathbf{0.989117352243} = \boldsymbol{\phi}$

## Observations

All the results of the most important connections are signed in blue throughout the drafting of the paper. We highlight as in the development of the various equations we use always the constants π, ϕ, 1/ϕ, the Fibonacci and Lucas numbers, linked to the golden ratio, that play a fundamental role in the development, and therefore, in the final results of the analyzed expressions.

In mathematics, the **Fibonacci numbers**, commonly denoted $F_n$, form a sequence, called the **Fibonacci sequence**, such that each number is the sum of the two preceding ones, starting from 0 and 1. Fibonacci numbers are strongly related to the golden ratio: Binet's formula expresses the $n$th Fibonacci number in terms of $n$ and the golden ratio, and implies that the ratio of two consecutive Fibonacci numbers tends to the golden ratio as $n$ increases.
Fibonacci numbers are also closely related to Lucas numbers ,in that the Fibonacci and Lucas numbers form a complementary pair of Lucas sequences

The beginning of the sequence is thus:

0, 1, 1, 2, 3, 5, 8, 13, 21, 34, 55, 89, 144, 233, 377, 610, 987, 1597, 2584, 4181, 6765, 10946, 17711, 28657, 46368, 75025, 121393, 196418, 317811, 514229, 832040, 1346269, 2178309, 3524578, 5702887, 9227465, 14930352, 24157817, 39088169, 63245986, 102334155...

The **Lucas numbers** or **Lucas series** are an integer sequence named after the mathematician François Édouard Anatole Lucas (1842–91), who studied both that sequence and the closely related Fibonacci numbers. Lucas numbers and Fibonacci numbers form complementary instances of Lucas sequences.

The Lucas sequence has the same recursive relationship as the Fibonacci sequence, where each term is the sum of the two previous terms, but with different starting values. This produces a sequence where the ratios of successive terms approach the golden ratio, and in fact the terms themselves are roundings of integer powers of the golden ratio.[1] The sequence also has a variety of relationships with the Fibonacci numbers, like the fact that adding any two Fibonacci numbers two terms apart in the Fibonacci sequence results in the Lucas number in between.

The sequence of Lucas numbers is:

2, 1, 3, 4, 7, 11, 18, 29, 47, 76, 123, 199, 322, 521, 843, 1364, 2207, 3571, 5778, 9349, 15127, 24476, 39603, 64079, 103682, 167761, 271443, 439204, 710647, 1149851, 1860498, 3010349, 4870847, 7881196, 12752043, 20633239, 33385282, 54018521, 87403803……

All Fibonacci-like integer sequences appear in shifted form as a row of the Wythoff array; the Fibonacci sequence itself is the first row and the Lucas sequence is the second row. Also like all Fibonacci-like integer sequences, the ratio between two consecutive Lucas numbers converges to the golden ratio.

A **Lucas prime** is a Lucas number that is prime. The first few Lucas primes are:

2, 3, 7, 11, 29, 47, 199, 521, 2207, 3571, 9349, 3010349, 54018521, 370248451, 6643838879, ... (sequence A005479 in the OEIS).

In geometry, a **golden spiral** is a logarithmic spiral whose growth factor is φ, the golden ratio.[1] That is, a golden spiral gets wider (or further from its origin) by a factor of φ for every quarter turn it makes. Approximate logarithmic spirals can occur in nature, for example the arms of spiral galaxies[3] - golden spirals are one special case of these logarithmic spirals

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