

**On some Ramanujan equations: mathematical connections with various formulas concerning some topics of Cosmology and Black Holes/Wormholes Physics. VII**

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**Abstract**

*In this paper we have described several Ramanujan's formulas and obtained some mathematical connections with various equations concerning different arguments of Cosmology and Black Holes/Wormholes Physics.*

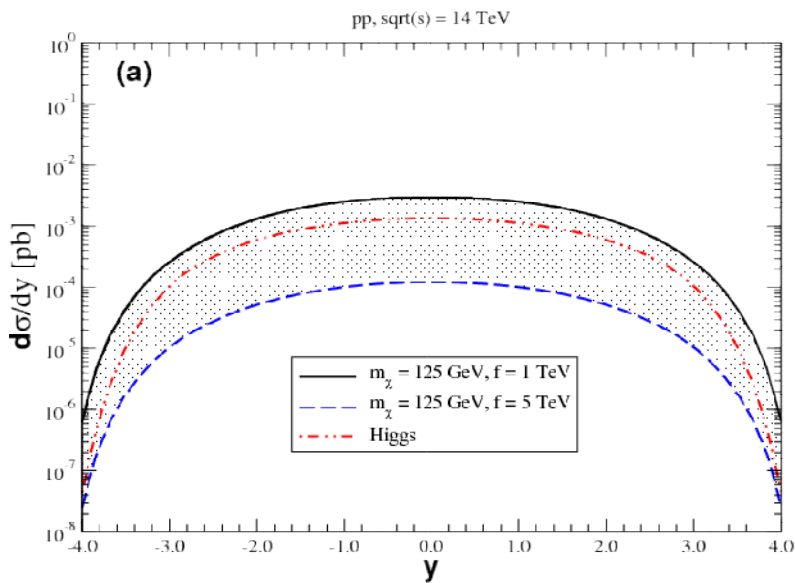
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<https://www.pinterest.it/pin/742319951051634216/?lp=true>



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(Color online)

Rapidity distribution for the dilaton production in  $\text{pom}\text{pom}$  interactions considering (a) pp and (b) PbPb collisions at LHC energies. The corresponding predictions for the SM Higgs production are also presented for comparison.

From:

**Can massless wormholes mimic a Schwarzschild black hole in the strong field lensing?** *Ramil N. Izmailov and Eduard R. Zhdanov† Amrita Bhattacharya,‡ Alexander A. Potapov, K.K. Nandi - arXiv:1909.13052v1 [gr-qc] 28 Sep 2019*

We have that:

For a correct comparison, the minimum impact parameter  $u_m$  of rays in the Schwarzschild black hole and EMD wormhole spacetime should be the same, which implies

$$u_m^{\text{Sch}} = \left( \frac{3\sqrt{3}}{2} \right) R_s = (3\sqrt{3}) M_s = u_m^{\text{EMD}} = 2q \quad (43)$$

$$\rightarrow \frac{q}{R_s} = \frac{3\sqrt{3}}{4}. \quad (44)$$

The last equation yields a formal identification of the Wheelerian mass  $q$  with the BH mass  $M_s$  as

$$q = \frac{(3\sqrt{3}) M_s}{2}. \quad (45)$$

The only variable in the Eqs.(38,39) now is the adimensionalized dilatonic charge  $\frac{\Sigma}{R_s}$ , and by varying it, we shall tabulate below the observables for massless EMD wormhole.

$$R_s = 1.67084e+37$$

$$q = 2.17049e+37$$

$$M_s = 8.35422e+36$$

$$\Sigma = 0.001$$

Indeed:

$$q = ((3\sqrt{3}) * (8.35422e+36)) / 2$$

**Input interpretation:**

$$\frac{1}{2} \left( (3\sqrt{3}) \times 8.35422 \times 10^{36} \right)$$

**Result:**

$$2.17049... \times 10^{37}$$

$$2.17049... * 10^{37}$$

With regard the dilatonic charge

dilatonic charge  $\frac{\Sigma}{R_s}$

We obtain

$$0.001 / (1.67084e+37)$$

**Input interpretation:**

$$\frac{0.001}{1.67084 \times 10^{37}}$$

**Result:**

$$5.9850135261305690550860644945057575830121376074309927... \times 10^{-41}$$

$$5.985013526... * 10^{-41}$$

From which:

$$(2e - 3) \left( \left( \frac{0.001}{1.67084e+37} \right) \right)^{1/6}$$

**Input interpretation:**

$$(2e - 3) \sqrt[6]{\frac{0.001}{1.67084 \times 10^{37}}}$$

**Result:**

$$4.81898... \times 10^{-7}$$

$4.81898... * 10^{-7}$  result very near to the value  $(4.81996... * 10^{-7} = \phi_1)$  of the scalar charge obtained from the following expression

$$\phi_1 = \frac{\alpha_0 Q_e^2}{M} \frac{1}{1 + \sqrt{1 + (\alpha_0^2 - 1) Q_e^2 / M^2}}$$

Now, we have that:

For EMD wormhole,  $u_m^{\text{Sch}}/R_s \rightarrow q/R_s = \frac{3\sqrt{3}}{4}$  [see Eq.(43,44)] so that

$$\alpha(\Delta, \Sigma/R_s) = -\bar{a} \log\left(\frac{\Delta}{q/R_s}\right) + \bar{b}(q/R_s, \Sigma/R_s), \quad (67)$$

From

$$q/R_s = \frac{3\sqrt{3}}{4}$$

$$(2.17049e+37) / (1.67084e+37)$$

**Input interpretation:**

$$\frac{2.17049 \times 10^{37}}{1.67084 \times 10^{37}}$$

**Result:**

1.299041200833113882837375212467980177635201455555289554954...

1.2990412008...

$$(3\sqrt{3})/4$$

**Input:**

$$\frac{1}{4}(3\sqrt{3})$$

**Exact result:**

$$\frac{3\sqrt{3}}{4}$$

**Decimal approximation:**

1.299038105676657970145584756129404275207103940357785471041...

1.299038105...

From

$$M_0 = \sqrt{3q^2 - \Sigma^2},$$

we have that:

$$\text{sqrt}(((3*(2.17049e+37)^2 - 0.001^2)))$$

**Input interpretation:**

$$\sqrt{3(2.17049 \times 10^{37})^2 - 0.001^2}$$

**Result:**

$$3.75940... \times 10^{37}$$

$$3.75940... * 10^{37} = M_0 \text{ (gravitating mass)}$$

**All 2nd roots of  $1.41331 \times 10^{75}$ :**

$$3.7594 \times 10^{37} e^0 \approx 3.7594 \times 10^{37} \text{ (real, principal root)}$$

$$3.7594 \times 10^{37} e^{i\pi} \approx -3.7594 \times 10^{37} \text{ (real root)}$$

(the leading order deflection by the massless EMD wormhole obtained in [24] using the Gauss-Bonnet method (75). which reveals, following Schwarzschild formula, that the effective gravitating mass is  $M_0$  and not merely  $q$ .)

$$2.17049 * 10^{37} = q$$

(when dilaton is switched off,  $\Sigma = 0$ , the metric (29-31) reduces to the famous Einstein-Rosen bridge [32] and in this case, the mass is proportional to just  $q$ ).

The ratio between  $M_0$  and  $q$

$$M_0 = \sqrt{3q^2 - \Sigma^2},$$

$$q = \frac{(3\sqrt{3}) M_s}{2}.$$

is equal to:

$$\sqrt{((3*(2.17049e+37)^2 - 0.001^2))} / ((3\sqrt{3})*(4.2*10^6 * 1.9891*10^30))/2$$

**Input interpretation:**

$$\frac{\sqrt{3(2.17049 \times 10^{37})^2 - 0.001^2}}{\frac{1}{2}((3\sqrt{3})(4.2 \times 10^6 \times 1.9891 \times 10^{30}))}$$

**Result:**

1.732050787905194420703947625671018160083566548802082460520...

**Input interpretation:**

1.7320507879

$1.7320507879 \approx \sqrt{3}$  that is the ratio between the gravitating mass  $M_0$  and the Wheelerian mass  $q$  of the wormhole

From

$$q/R_s = \frac{3\sqrt{3}}{4}$$

We have also:

$$4/3 * (((2.17049e+37) / (1.67084e+37) - (3.09516e-6) + (3.54428e-12)))$$

**Input interpretation:**

$$\frac{4}{3} \left( \frac{2.17049 \times 10^{37}}{1.67084 \times 10^{37}} - 3.09516 \times 10^{-6} + 3.54428 \times 10^{-12} \right)$$

**Result:**

1.732050807568877550449833616623973570180268607407052739939...

1.73205080756887755....

$1.7320508075688... = \sqrt{3}$  that is the ratio between the gravitating mass  $M_0$  and the Wheelerian mass  $q$

**Possible closed forms:**

$$\sqrt{3} \approx 1.732050807568877293$$

$$\frac{-572 + 361\pi + 7\pi^2}{116\pi} \approx 1.73205080756896800$$

$$\frac{1}{90} (-e^\pi + 25\pi + 85 \log(\pi) - 23 \log(2\pi) + 36 \tan^{-1}(\pi)) \approx 1.73205080756878279$$

$$\frac{-150\pi\pi! - 70 + 97\pi + 331\pi^2}{21\pi} \approx 1.7320508075691099$$

$$\frac{2703197\pi}{4903057} \approx 1.732050807568822535$$

$$\frac{4(V_{fe} + 125)}{146V_{fe} - 3} \approx 1.732050807572260$$

$$\frac{4(2\mathcal{G}_{Gi} + 125)}{292\mathcal{G}_{Gi} - 3} \approx 1.732050807572260$$

$$\frac{-38\bar{s}_{let} - 93}{5(21\bar{s}_{let} - 20)} \approx 1.732050807580252$$

Now, we have that:

and a much simplified expression

$$g(z, 0, 0) = \frac{\pi z - \sqrt{2}\sqrt{1 - \cos(\pi z)}}{z \sin\left(\frac{\pi z}{2}\right)},$$

leading to

$$b_R = \int_0^1 g(z, 0, 0) dz = \log(16) - 2 \log \pi.$$

$$(((\pi * z - \sqrt{2} * (1 - \cos(\pi * z))^{1/2}))/((z * \sin(\pi * z)/2))) = \ln(16) - 2 \ln(\pi)$$

**Input:**

$$\frac{\pi z - \sqrt{2} \sqrt{1 - \cos(\pi z)}}{z \left(\frac{1}{2} \sin(\pi z)\right)} = \log(16) - 2 \log(\pi)$$

$\log(x)$  is the natural logarithm

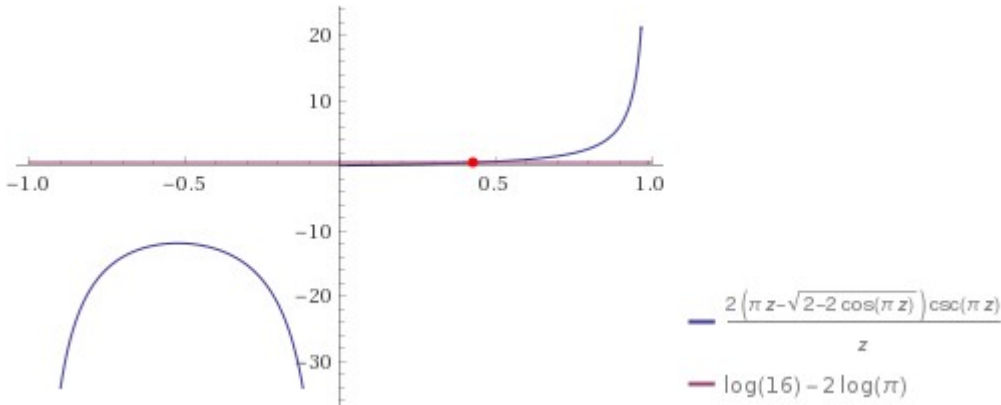
**Exact result:**

$$\frac{2 \left( \pi z - \sqrt{2} \sqrt{1 - \cos(\pi z)} \right) \csc(\pi z)}{z} = \log(16) - 2 \log(\pi)$$

$\csc(x)$  is the cosecant function



**Plot:**



**Alternate forms:**

$$\frac{2(\pi z - \sqrt{2 - 2 \cos(\pi z)}) \csc(\pi z)}{z} = \log\left(\frac{16}{\pi^2}\right)$$

$$\frac{(\pi z - \sqrt{2} \sqrt{1 - \cos(\pi z)}) \csc\left(\frac{\pi z}{2}\right) \sec\left(\frac{\pi z}{2}\right)}{z} = \log(16) - 2 \log(\pi)$$

$$\frac{4i\sqrt{2} \sqrt{1 + \frac{1}{2}(-e^{-i\pi z} - e^{i\pi z})}}{(e^{-i\pi z} - e^{i\pi z})z} - \frac{4i\pi}{e^{-i\pi z} - e^{i\pi z}} = \log(16) - 2 \log(\pi)$$

sec(x) is the secant function

**Alternate form assuming z>0:**

$$2\pi \csc(\pi z) - \frac{2\sqrt{2} \sqrt{1 - \cos(\pi z)} \csc(\pi z)}{z} = 4 \log(2) - 2 \log(\pi)$$

**Numerical solution:**

$z \approx 0.432506807719240\dots$

**0.432506807719240... = z**

We have, from

$$\frac{\pi z - \sqrt{2} \sqrt{1 - \cos(\pi z)}}{z \sin\left(\frac{\pi z}{2}\right)}$$

$$\frac{((\pi \cdot 0.4325068077) - \sqrt{2} \cdot \sqrt{1 - \cos(\pi \cdot 0.4325068077)})}{(0.4325068077 \cdot \sin(\frac{\pi \cdot 0.4325068077}{2}))}$$

**Input interpretation:**

$$\frac{\pi \times 0.4325068077 - \sqrt{2} \sqrt{1 - \cos(\pi \times 0.4325068077)}}{0.4325068077 \left(\frac{1}{2} \sin(\pi \times 0.4325068077)\right)}$$

**Result:**

0.483128951...

0.483128951...

**Alternative representations:**

$$\frac{\pi \cdot 0.432507 - \sqrt{2} \sqrt{1 - \cos(\pi \cdot 0.432507)}}{\frac{1}{2} \times 0.432507 \sin(\pi \cdot 0.432507)} = \frac{0.432507 \pi - \sqrt{1 - \cosh(0.432507 i \pi)} \sqrt{2}}{0.216253 \cos(0.0674932 \pi)}$$

$$\frac{\pi \cdot 0.432507 - \sqrt{2} \sqrt{1 - \cos(\pi \cdot 0.432507)}}{\frac{1}{2} \times 0.432507 \sin(\pi \cdot 0.432507)} = \frac{0.432507 \pi - \sqrt{1 - \cosh((-0.432507 i) \pi)} \sqrt{2}}{0.216253 \cos(0.0674932 \pi)}$$

$$\frac{\pi \cdot 0.432507 - \sqrt{2} \sqrt{1 - \cos(\pi \cdot 0.432507)}}{\frac{1}{2} \times 0.432507 \sin(\pi \cdot 0.432507)} = \frac{0.432507 \pi - \sqrt{1 - \cosh((-0.432507 i) \pi)} \sqrt{2}}{0.216253 \cos(0.932507 \pi)}$$

**Series representations:**

$$\frac{\pi \cdot 0.432507 - \sqrt{2} \sqrt{1 - \cos(\pi \cdot 0.432507)}}{\frac{1}{2} \times 0.432507 \sin(\pi \cdot 0.432507)} = \left( \pi - 2.3121 \exp\left(i \pi \left\lfloor \frac{\arg(2-x)}{2\pi} \right\rfloor \right) \sqrt{x} \sqrt{1 - \sum_{k=0}^{\infty} \frac{(-1)^k e^{-1.67631k} \pi^{2k}}{(2k)!}} \right) / \left( \sum_{k=0}^{\infty} \frac{(-1)^k (2-x)^k x^{-k} \left(-\frac{1}{2}\right)_k}{k!} \right) / \left( \sum_{k=0}^{\infty} (-1)^k J_{1+2k}(0.432507 \pi) \right) \text{ for } (x \in \mathbb{R} \text{ and } x < 0)$$

$$\frac{\pi \cdot 0.432507 - \sqrt{2} \sqrt{1 - \cos(\pi \cdot 0.432507)}}{\frac{1}{2} \times 0.432507 \sin(\pi \cdot 0.432507)} =$$

$$\left( 2 \left( \pi - 2.3121 \exp\left(i \pi \left[ \frac{\arg(2-x)}{2\pi} \right] \right) \sqrt{x} \sqrt{1 - \sum_{k=0}^{\infty} \frac{(-1)^k e^{-1.67631k} \pi^{2k}}{(2k)!}} \right. \right.$$

$$\left. \left. \sum_{k=0}^{\infty} \frac{(-1)^k (2-x)^k x^{-k} \left(-\frac{1}{2}\right)_k}{k!} \right) \right) /$$

$$\left( \sum_{k=0}^{\infty} \frac{(-1)^k 0.432507^{1+2k} \pi^{1+2k}}{(1+2k)!} \right) \text{ for } (x \in \mathbb{R} \text{ and } x < 0)$$

$$\frac{\pi \cdot 0.432507 - \sqrt{2} \sqrt{1 - \cos(\pi \cdot 0.432507)}}{\frac{1}{2} \times 0.432507 \sin(\pi \cdot 0.432507)} = \left( \pi - 2.3121 \exp\left(i \pi \left[ \frac{\arg(2-x)}{2\pi} \right] \right) \right.$$

$$\left. \sqrt{x} \sqrt{1 - J_0(0.432507 \pi) - 2 \sum_{k=1}^{\infty} (-1)^k J_{2k}(0.432507 \pi)} \right.$$

$$\left. \sum_{k=0}^{\infty} \frac{(-1)^k (2-x)^k x^{-k} \left(-\frac{1}{2}\right)_k}{k!} \right) /$$

$$\left( \sum_{k=0}^{\infty} (-1)^k J_{1+2k}(0.432507 \pi) \right) \text{ for } (x \in \mathbb{R} \text{ and } x < 0)$$

### Integral representations:

$$\frac{\pi \cdot 0.432507 - \sqrt{2} \sqrt{1 - \cos(\pi \cdot 0.432507)}}{\frac{1}{2} \times 0.432507 \sin(\pi \cdot 0.432507)} =$$

$$\frac{7.03138 \left( -0.657652 \pi + \sqrt{\pi \int_0^1 \sin(0.432507 \pi t) dt} \sqrt{2} \right)}{\pi \int_0^1 \cos(0.432507 \pi t) dt}$$

$$\frac{\pi \cdot 0.432507 - \sqrt{2} \sqrt{1 - \cos(\pi \cdot 0.432507)}}{\frac{1}{2} \times 0.432507 \sin(\pi \cdot 0.432507)} =$$

$$\frac{10.6916 \left( -0.432507 \pi + \sqrt{1 + \int_{\frac{\pi}{2}}^{0.432507 \pi} \sin(t) dt} \sqrt{2} \right)}{\pi \int_0^1 \cos(0.432507 \pi t) dt}$$

$$\frac{\pi 0.432507 - \sqrt{2} \sqrt{1 - \cos(\pi 0.432507)}}{\frac{1}{2} \times 0.432507 \sin(\pi 0.432507)} = \frac{18.4968 i \left( \pi - 1.52056 \sqrt{\pi \int_0^1 \sin(0.432507 \pi t) dt} \sqrt{2} \right)}{\sqrt{\pi} \int_{-i\infty+\gamma}^{i\infty+\gamma} \frac{e^{-(0.0467655 \pi^2)/s+s}}{s^{3/2}} ds} \quad \text{for } \gamma > 0$$

### Multiple-argument formulas:

$$\frac{\pi 0.432507 - \sqrt{2} \sqrt{1 - \cos(\pi 0.432507)}}{\frac{1}{2} \times 0.432507 \sin(\pi 0.432507)} = \frac{\pi - 3.26981 \sqrt{\sin^2(0.216253 \pi)} \sqrt{2}}{\cos(0.216253 \pi) \sin(0.216253 \pi)}$$

$$\frac{\pi 0.432507 - \sqrt{2} \sqrt{1 - \cos(\pi 0.432507)}}{\frac{1}{2} \times 0.432507 \sin(\pi 0.432507)} = \frac{\pi - 3.26981 \sqrt{1 - \cos^2(0.216253 \pi)} \sqrt{2}}{\cos(0.216253 \pi) \sin(0.216253 \pi)}$$

$$\frac{\pi 0.432507 - \sqrt{2} \sqrt{1 - \cos(\pi 0.432507)}}{\frac{1}{2} \times 0.432507 \sin(\pi 0.432507)} = \frac{0.5 \pi - 1.6349 \sqrt{\sin^2(0.216253 \pi)} \sqrt{2}}{0.75 \sin(0.144169 \pi) - \sin^3(0.144169 \pi)}$$

From which:

$$\left( \left( \sqrt{\frac{33}{5}} \cdot 1.395485972 \right) \cdot \left( \left( \left( \pi \cdot 0.4325068077 \right) - \sqrt{2} \cdot \left( 1 - \cos\left( \pi \cdot 0.4325068077 \right) \right)^{1/2} \right) \right) / \left( \left( 0.4325068077 \cdot \sin\left( \pi \cdot 0.4325068077 \right) \right) / 2 \right) \right)$$

Where 1.395485972 is the hard square hexagon constant that is given by

$$\begin{aligned} \kappa_6 &= \lim_{n \rightarrow \infty} [G(n)]^{1/n^2} \\ &= 1.395485972 \dots \end{aligned} \quad (2)$$

### Input interpretation:

$$\left( \sqrt{\frac{33}{5}} \times 1.395485972 \right) \times \frac{\pi \times 0.4325068077 - \sqrt{2} \sqrt{1 - \cos(\pi \times 0.4325068077)}}{0.4325068077 \left( \frac{1}{2} \sin(\pi \times 0.4325068077) \right)}$$

### Result:

1.73205032...

$1.73205032... \approx \sqrt{3}$  that is the ratio between the gravitating mass  $M_0$  and the Wheelerian mass  $q$

**Alternative representations:**

$$\frac{\left(\pi 0.432507 - \sqrt{2} \sqrt{1 - \cos(\pi 0.432507)}\right) \sqrt{\frac{33}{5}} 1.39549}{\frac{1}{2} \times 0.432507 \sin(\pi 0.432507)} =$$

$$\frac{1.39549 \left(0.432507 \pi - \sqrt{1 - \cosh(0.432507 i \pi)} \sqrt{2}\right) \sqrt{\frac{33}{5}}}{0.216253 \cos(0.0674932 \pi)}$$

$$\frac{\left(\pi 0.432507 - \sqrt{2} \sqrt{1 - \cos(\pi 0.432507)}\right) \sqrt{\frac{33}{5}} 1.39549}{\frac{1}{2} \times 0.432507 \sin(\pi 0.432507)} =$$

$$\frac{1.39549 \left(0.432507 \pi - \sqrt{1 - \cosh((-0.432507 i) \pi)} \sqrt{2}\right) \sqrt{\frac{33}{5}}}{0.216253 \cos(0.0674932 \pi)}$$

$$\frac{\left(\pi 0.432507 - \sqrt{2} \sqrt{1 - \cos(\pi 0.432507)}\right) \sqrt{\frac{33}{5}} 1.39549}{\frac{1}{2} \times 0.432507 \sin(\pi 0.432507)} =$$

$$\frac{1.39549 \left(0.432507 \pi - \sqrt{1 - \cosh((-0.432507 i) \pi)} \sqrt{2}\right) \sqrt{\frac{33}{5}}}{0.216253 \cos(0.932507 \pi)}$$

**Series representations:**

$$\frac{\left(\pi 0.432507 - \sqrt{2} \sqrt{1 - \cos(\pi 0.432507)}\right) \sqrt{\frac{33}{5}} 1.39549}{\frac{1}{2} \times 0.432507 \sin(\pi 0.432507)} =$$

$$\left(1.39549 \left(\frac{1}{z_0}\right)^{1/2} \left[\operatorname{arg}\left(\frac{33}{5} - z_0\right)/(2\pi)\right]_{z_0}^{1/2} \left[\operatorname{arg}\left(\frac{33}{5} - z_0\right)/(2\pi)\right]_{z_0}^{1/2}\right)$$

$$\left(\pi \sqrt{z_0} - 2.3121 \left(\frac{1}{z_0}\right)^{1/2} \left[\operatorname{arg}(2 - z_0)/(2\pi)\right]_{z_0}^{1+1/2} \left[\operatorname{arg}(2 - z_0)/(2\pi)\right]_{z_0}^{1+1/2}\right)$$

$$\sqrt{1 - \sum_{k=0}^{\infty} \frac{(-1)^k e^{-1.67631 k} \pi^{2k}}{(2k)!} \sum_{k=0}^{\infty} \frac{(-1)^k \left(-\frac{1}{2}\right)_k (2 - z_0)^k z_0^{-k}}{k!}}$$

$$\left. \sum_{k=0}^{\infty} \frac{(-1)^k \left(-\frac{1}{2}\right)_k \left(\frac{33}{5} - z_0\right)^k z_0^{-k}}{k!} \right) / \left( \sum_{k=0}^{\infty} (-1)^k J_{1+2k}(0.432507 \pi) \right)$$

$$\begin{aligned}
& \frac{(\pi 0.432507 - \sqrt{2} \sqrt{1 - \cos(\pi 0.432507)}) \sqrt{\frac{33}{5}} 1.39549}{\frac{1}{2} \times 0.432507 \sin(\pi 0.432507)} = \\
& \left( 2.79097 \left(\frac{1}{z_0}\right)^{1/2 [\arg(\frac{33}{5} - z_0)/(2\pi)]} z_0^{-1/2 [\arg(\frac{33}{5} - z_0)/(2\pi)]} \right. \\
& \quad \left( \pi \sqrt{z_0} - 2.3121 \left(\frac{1}{z_0}\right)^{1/2 [\arg(2-z_0)/(2\pi)]} z_0^{1+1/2 [\arg(2-z_0)/(2\pi)]} \right. \\
& \quad \left. \sqrt{1 - \sum_{k=0}^{\infty} \frac{(-1)^k e^{-1.67631k} \pi^{2k}}{(2k)!} \sum_{k=0}^{\infty} \frac{(-1)^k \left(-\frac{1}{2}\right)_k (2-z_0)^k z_0^{-k}}{k!}} \right) \\
& \quad \left. \sum_{k=0}^{\infty} \frac{(-1)^k \left(-\frac{1}{2}\right)_k \left(\frac{33}{5} - z_0\right)^k z_0^{-k}}{k!} \right) / \left( \sum_{k=0}^{\infty} \frac{(-1)^k 0.432507^{1+2k} \pi^{1+2k}}{(1+2k)!} \right)
\end{aligned}$$

$$\begin{aligned}
& \frac{(\pi 0.432507 - \sqrt{2} \sqrt{1 - \cos(\pi 0.432507)}) \sqrt{\frac{33}{5}} 1.39549}{\frac{1}{2} \times 0.432507 \sin(\pi 0.432507)} = \\
& \left( 1.39549 \left(\frac{1}{z_0}\right)^{1/2 [\arg(\frac{33}{5} - z_0)/(2\pi)]} z_0^{-1/2 [\arg(\frac{33}{5} - z_0)/(2\pi)]} \right. \\
& \quad \left( \pi \sqrt{z_0} - 2.3121 \left(\frac{1}{z_0}\right)^{1/2 [\arg(2-z_0)/(2\pi)]} z_0^{1+1/2 [\arg(2-z_0)/(2\pi)]} \right. \\
& \quad \left. \sqrt{1 - J_0(0.432507 \pi) - 2 \sum_{k=1}^{\infty} (-1)^k J_{2k}(0.432507 \pi)} \right. \\
& \quad \left. \sum_{k=0}^{\infty} \frac{(-1)^k \left(-\frac{1}{2}\right)_k (2-z_0)^k z_0^{-k}}{k!} \right) \\
& \quad \left. \sum_{k=0}^{\infty} \frac{(-1)^k \left(-\frac{1}{2}\right)_k \left(\frac{33}{5} - z_0\right)^k z_0^{-k}}{k!} \right) / \left( \sum_{k=0}^{\infty} (-1)^k J_{1+2k}(0.432507 \pi) \right)
\end{aligned}$$

### Integral representations:

$$\begin{aligned}
& \frac{(\pi 0.432507 - \sqrt{2} \sqrt{1 - \cos(\pi 0.432507)}) \sqrt{\frac{33}{5}} 1.39549}{\frac{1}{2} \times 0.432507 \sin(\pi 0.432507)} = \\
& \frac{9.81219 \left( -0.657652 \pi + \sqrt{\pi \int_0^1 \sin(0.432507 \pi t) dt} \sqrt{2} \right) \sqrt{\frac{33}{5}}}{\pi \int_0^1 \cos(0.432507 \pi t) dt}
\end{aligned}$$

$$\frac{\left(\pi 0.432507 - \sqrt{2} \sqrt{1 - \cos(\pi 0.432507)}\right) \sqrt{\frac{33}{5}} 1.39549}{\frac{1}{2} \times 0.432507 \sin(\pi 0.432507)} =$$

$$\frac{14.92 \left(-0.432507 \pi + \sqrt{1 + \int_{\frac{\pi}{2}}^{0.432507 \pi} \sin(t) dt} \sqrt{2}\right) \sqrt{\frac{33}{5}}}{\pi \int_0^1 \cos(0.432507 \pi t) dt}$$

$$\frac{\left(\pi 0.432507 - \sqrt{2} \sqrt{1 - \cos(\pi 0.432507)}\right) \sqrt{\frac{33}{5}} 1.39549}{\frac{1}{2} \times 0.432507 \sin(\pi 0.432507)} =$$

$$\frac{25.8121 i \left(\pi \sqrt{\frac{33}{5}} - 1.52056 \sqrt{\pi \int_0^1 \sin(0.432507 \pi t) dt} \sqrt{2} \sqrt{\frac{33}{5}}\right)}{\sqrt{\pi} \int_{-i \infty + \gamma}^{i \infty + \gamma} \frac{e^{-(0.0467655 \pi^2)/s+s}}{s^{3/2}} ds} \quad \text{for } \gamma > 0$$

### Multiple-argument formulas:

$$\frac{\left(\pi 0.432507 - \sqrt{2} \sqrt{1 - \cos(\pi 0.432507)}\right) \sqrt{\frac{33}{5}} 1.39549}{\frac{1}{2} \times 0.432507 \sin(\pi 0.432507)} =$$

$$\frac{1.39549 \left(\pi - 3.26981 \sqrt{\sin^2(0.216253 \pi)} \sqrt{2}\right) \sqrt{\frac{33}{5}}}{\cos(0.216253 \pi) \sin(0.216253 \pi)}$$

$$\frac{\left(\pi 0.432507 - \sqrt{2} \sqrt{1 - \cos(\pi 0.432507)}\right) \sqrt{\frac{33}{5}} 1.39549}{\frac{1}{2} \times 0.432507 \sin(\pi 0.432507)} =$$

$$\frac{1.39549 \left(\pi - 3.26981 \sqrt{1 - \cos^2(0.216253 \pi)} \sqrt{2}\right) \sqrt{\frac{33}{5}}}{\cos(0.216253 \pi) \sin(0.216253 \pi)}$$

$$\frac{\left(\pi 0.432507 - \sqrt{2} \sqrt{1 - \cos(\pi 0.432507)}\right) \sqrt{\frac{33}{5}} 1.39549}{\frac{1}{2} \times 0.432507 \sin(\pi 0.432507)} =$$

$$\frac{0.697743 \left(\pi - 3.26981 \sqrt{\sin^2(0.216253 \pi)} \sqrt{2}\right) \sqrt{\frac{33}{5}}}{\sin(0.144169 \pi) (-0.75 + \sin^2(0.144169 \pi))}$$

and:

$$((-e^{-1-3/e} \pi^e \tan(e \pi))) * (((((\pi * 0.4325068077) - \sqrt{2} * (1 - \cos(\pi * 0.4325068077)))^{1/2}))) / ((0.4325068077 * \sin(\pi * 0.4325068077) / 2)))$$

**Input interpretation:**

$$(-e^{-1-3/e} \pi^e \tan(e \pi)) \times \frac{\pi \times 0.4325068077 - \sqrt{2} \sqrt{1 - \cos(\pi \times 0.4325068077)}}{0.4325068077 \left(\frac{1}{2} \sin(\pi \times 0.4325068077)\right)}$$

**Result:**

1.61803435...

1.61803435... result that is a very good approximation to the value of the golden ratio 1,618033988749...

**Alternative representations:**

$$\frac{(\pi \cdot 0.432507 - \sqrt{2} \sqrt{1 - \cos(\pi \cdot 0.432507)}) (-e^{-1-3/e}) (\pi^e \tan(e \pi))}{\frac{1}{2} \times 0.432507 \sin(\pi \cdot 0.432507)} =$$

$$\frac{e^{-1-3/e} \pi^e \left(-i + \frac{2i}{1+e^{2ei\pi}}\right) \left(0.432507 \pi - \sqrt{1 + \frac{1}{2} (-e^{-0.432507i\pi} - e^{0.432507i\pi})} \sqrt{2}\right)}{0.216253 \cos(0.0674932 \pi)}$$

$$\frac{(\pi \cdot 0.432507 - \sqrt{2} \sqrt{1 - \cos(\pi \cdot 0.432507)}) (-e^{-1-3/e}) (\pi^e \tan(e \pi))}{\frac{1}{2} \times 0.432507 \sin(\pi \cdot 0.432507)} =$$

$$\frac{-e^{-1-3/e} \pi^e \left(-i + \frac{2i}{1+e^{2ei\pi}}\right) \left(0.432507 \pi - \sqrt{1 + \frac{1}{2} (-e^{-0.432507i\pi} - e^{0.432507i\pi})} \sqrt{2}\right)}{-0.216253 \cos(0.932507 \pi)}$$

$$\frac{(\pi \cdot 0.432507 - \sqrt{2} \sqrt{1 - \cos(\pi \cdot 0.432507)}) (-e^{-1-3/e}) (\pi^e \tan(e \pi))}{\frac{1}{2} \times 0.432507 \sin(\pi \cdot 0.432507)} =$$

$$\frac{e^{-1-3/e} \pi^e \left(-i + \frac{2i}{1+e^{2ei\pi}}\right) \left(0.432507 \pi - \sqrt{1 - \cosh((-0.432507i) \pi)} \sqrt{2}\right)}{\frac{0.216253 (-e^{-0.432507i\pi} + e^{0.432507i\pi})}{2i}}$$



## Series representations:

$$\frac{\left(\pi 0.432507 - \sqrt{2} \sqrt{1 - \cos(\pi 0.432507)}\right) (-e^{-1-3/e}) (\pi^e \tan(e \pi))}{\frac{1}{2} \times 0.432507 \sin(\pi 0.432507)} =$$

$$\left( 18.4968 e^{-3/e} \pi^{-1+e} \left( \sum_{k=1}^{\infty} \frac{1}{-4 e^2 + (1 - 2 k)^2} \right) \left( -0.432507 \pi + \exp\left(i \pi \left\lfloor \frac{\arg(2 - x)}{2 \pi} \right\rfloor\right) \right) \right.$$

$$\left. \sqrt{x} \sqrt{1 - J_0(0.432507 \pi) - 2 \sum_{k=1}^{\infty} (-1)^k J_{2k}(0.432507 \pi)} \right.$$

$$\left. \sum_{k=0}^{\infty} \frac{(-1)^k (2 - x)^k x^{-k} \left(-\frac{1}{2}\right)_k}{k!} \right) /$$

$$\left( \sum_{k=0}^{\infty} (-1)^k J_{1+2k}(0.432507 \pi) \right) \text{ for } (x \in \mathbb{R} \text{ and } x < 0)$$

$$\frac{\left(\pi 0.432507 - \sqrt{2} \sqrt{1 - \cos(\pi 0.432507)}\right) (-e^{-1-3/e}) (\pi^e \tan(e \pi))}{\frac{1}{2} \times 0.432507 \sin(\pi 0.432507)} =$$

$$- \left( \left( e^{-1-3/e} \left( i \pi^{1+e} \sum_{k=-\infty}^{\infty} (-1)^k \mathcal{A}^{2 e i k \pi} \operatorname{sgn}(k) - \right. \right. \right.$$

$$\left. \left. 2.3121 i \pi^e \exp\left(i \pi \left\lfloor \frac{\arg(2 - x \text{\$MFID})}{2 \pi} \right\rfloor\right) \sqrt{x \text{\$MFID}} \right. \right.$$

$$\left. \sqrt{1 - \sum_{k=0}^{\infty} \frac{(-1)^k e^{-1.67631 k} \pi^{2k}}{(2k)!} \sum_{k_1=0}^{\infty} \sum_{k_2=-\infty}^{\infty} \frac{1}{k_1!} (-1)^{k_1+k_2} \right. \right.$$

$$\left. \left. (2 - x \text{\$MFID})^{k_1} x \text{\$MFID}^{-k_1} \mathcal{A}^{2 e i \pi k_2} \left(-\frac{1}{2}\right)_{k_1} \operatorname{sgn}(k_2) \right) \right) /$$

$$\left( \sum_{k=0}^{\infty} (-1)^k J_{1+2k}(0.432507 \pi) \right) \text{ for } (x \text{\$MFID} \in \mathbb{R} \text{ and } x \text{\$MFID} < 0)$$

$$\begin{aligned}
& \frac{\left(\pi 0.432507 - \sqrt{2} \sqrt{1 - \cos(\pi 0.432507)}\right) (-e^{-1-3/e}) (\pi^e \tan(e \pi))}{\frac{1}{2} \times 0.432507 \sin(\pi 0.432507)} = \\
& - \left( \left( e^{-1-3/e} \left( i \pi^{1+e} \sum_{k=-\infty}^{\infty} (-1)^k \mathcal{A}^{2e i k \pi} \operatorname{sgn}(k) - 2.3121 i \pi^e \exp\left(i \pi \left[ \frac{\arg(2 - x\$MFID)}{2 \pi} \right] \right) \right) \right. \\
& \quad \sqrt{x\$MFID} \sqrt{1 - J_0(0.432507 \pi) - 2 \sum_{k=1}^{\infty} (-1)^k J_{2k}(0.432507 \pi)} \\
& \quad \sum_{k_1=0}^{\infty} \sum_{k_2=-\infty}^{\infty} \frac{1}{k_1!} (-1)^{k_1+k_2} (2 - x\$MFID)^{k_1} \\
& \quad \left. \left. x\$MFID^{-k_1} \mathcal{A}^{2e i \pi k_2} \left(-\frac{1}{2}\right)_{k_1} \operatorname{sgn}(k_2) \right) \right) / \\
& \left( \sum_{k=0}^{\infty} (-1)^k J_{1+2k}(0.432507 \pi) \right) \text{ for } (x\$MFID \in \mathbb{R} \text{ and } x\$MFID < 0)
\end{aligned}$$

### Multiple-argument formulas:

$$\begin{aligned}
& \frac{\left(\pi 0.432507 - \sqrt{2} \sqrt{1 - \cos(\pi 0.432507)}\right) (-e^{-1-3/e}) (\pi^e \tan(e \pi))}{\frac{1}{2} \times 0.432507 \sin(\pi 0.432507)} = \\
& \frac{2 e^{-(3+e)/e} \pi^e \left(\pi - 3.26981 \sqrt{\sin^2(0.216253 \pi) \sqrt{2}}\right) \tan\left(\frac{e \pi}{2}\right)}{\cos(0.216253 \pi) \sin(0.216253 \pi) \left(-1 + \tan^2\left(\frac{e \pi}{2}\right)\right)}
\end{aligned}$$

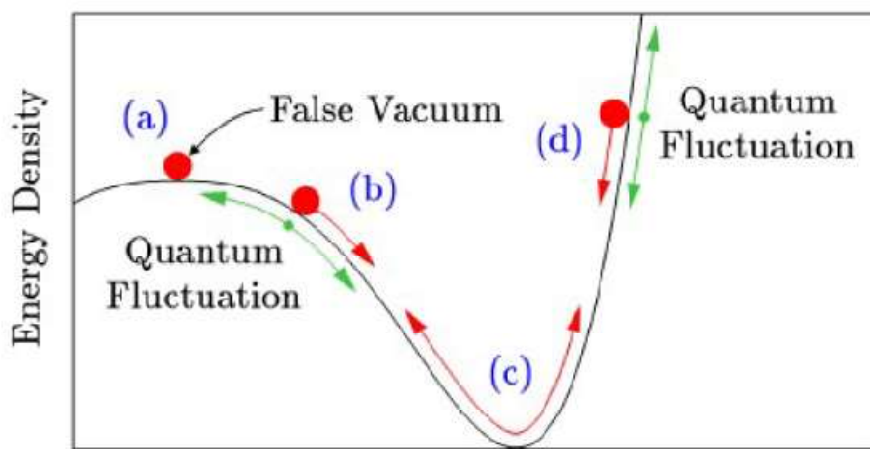
$$\begin{aligned}
& \frac{\left(\pi 0.432507 - \sqrt{2} \sqrt{1 - \cos(\pi 0.432507)}\right) (-e^{-1-3/e}) (\pi^e \tan(e \pi))}{\frac{1}{2} \times 0.432507 \sin(\pi 0.432507)} = \\
& \frac{2 e^{-(3+e)/e} \pi^e \left(\pi - 3.26981 \sqrt{1 - \cos^2(0.216253 \pi) \sqrt{2}}\right) \tan\left(\frac{e \pi}{2}\right)}{\cos(0.216253 \pi) \sin(0.216253 \pi) \left(-1 + \tan^2\left(\frac{e \pi}{2}\right)\right)}
\end{aligned}$$

$$\begin{aligned}
& \frac{\left(\pi 0.432507 - \sqrt{2} \sqrt{1 - \cos(\pi 0.432507)}\right) (-e^{-1-3/e}) (\pi^e \tan(e \pi))}{\frac{1}{2} \times 0.432507 \sin(\pi 0.432507)} = \\
& \frac{9.24841 e^{-(3+e)/e} \pi^e \left(0.432507 \pi - \sqrt{2 - 2 \cos^2(0.216253 \pi) \sqrt{2}}\right) \tan\left(\frac{e \pi}{2}\right)}{\sin(0.144169 \pi) \left(-3 + 4 \sin^2(0.144169 \pi)\right) \left(-1 + \tan^2\left(\frac{e \pi}{2}\right)\right)}
\end{aligned}$$

## On the Cubic Equations

Figure

**Inflationary Cosmology: Exploring the Universe from the Smallest to the Largest Scales**



$$\phi \quad \phi = 50 M_{\text{p}} = 1.2175 \times 10^{20} \text{ GeV}$$

$${}^{4096}\sqrt{\frac{1}{1.2175 \times 10^{20}}} = 0.98877237\dots$$

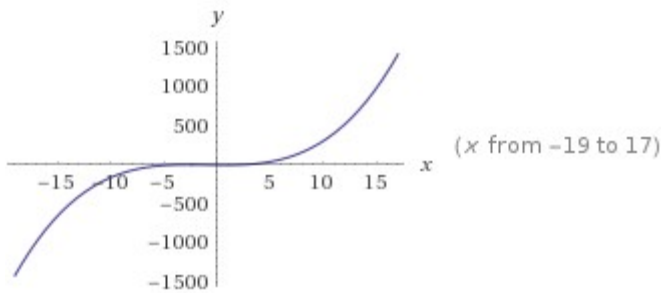
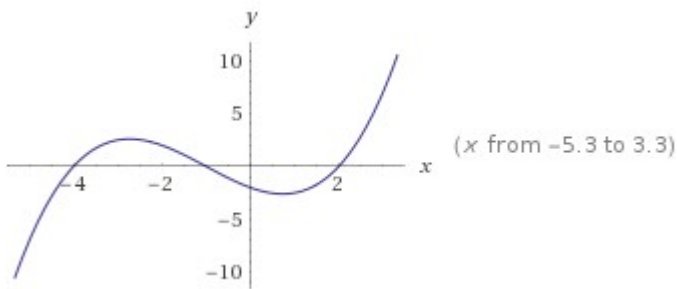
$$\sqrt{\log_{0.98877237}\left(\frac{1}{1.2175 \times 10^{20}}\right)} = 64.0000\dots$$

We have the following cubic function:

$$(x^3 + 3x^2 - 6x - 8)/4$$

**Input:**

$$\frac{1}{4}(x^3 + 3x^2 - 6x - 8)$$

**Plots:****Alternate forms:**

$$\frac{1}{4} (x + 1) (x - 2) (x + 4)$$

$$x \left( \left( \frac{x}{4} + \frac{3}{4} \right) x - \frac{3}{2} \right) - 2$$

$$\frac{1}{4} ((x + 1)^3 - 9(x + 1))$$

**Expanded form:**

$$\frac{x^3}{4} + \frac{3x^2}{4} - \frac{3x}{2} - 2$$

**Roots:**

$$x = -4$$

$$x = -1$$

$$x = 2$$

**Polynomial discriminant:**

$$\Delta = \frac{729}{64}$$

**Input:**

$$\frac{729}{64}$$

**Exact result:**

$$\frac{729}{64} \text{ (irreducible)}$$

**Decimal form:**

11.390625

11.390625

From which:

$$(729/64)^{1/5}$$

**Input:**

$$\sqrt[5]{\frac{729}{64}}$$

**Result:**

$$\frac{3 \sqrt[5]{\frac{3}{2}}}{2}$$

**Decimal approximation:**

1.626707656796547920618414883616769628316104511880939811006...

1.626707656...

**Properties as a real function:****Domain** $\mathbb{R}$  (all real numbers)**Range** $\mathbb{R}$  (all real numbers)**Surjectivity**surjective onto  $\mathbb{R}$  $\mathbb{R}$  is the set of real numbers

**Derivative:**

$$\frac{d}{dx} \left( \frac{1}{4} (x^3 + 3x^2 - 6x - 8) \right) = \frac{3}{4} (x^2 + 2x - 2)$$

**Indefinite integral:**

$$\int \frac{1}{4} (-8 - 6x + 3x^2 + x^3) dx = \frac{1}{4} \left( \frac{x^4}{4} + x^3 - 3x^2 - 8x \right) + \text{constant}$$

**Local maximum:**

$$\max \left\{ \frac{1}{4} (x^3 + 3x^2 - 6x - 8) \right\} = \frac{3\sqrt{3}}{2} \text{ at } x = -1 - \sqrt{3}$$

**Local minimum:**

$$\min \left\{ \frac{1}{4} (x^3 + 3x^2 - 6x - 8) \right\} = -\frac{3\sqrt{3}}{2} \text{ at } x = \sqrt{3} - 1$$

**Local maximum:**

$$\max \left\{ \frac{1}{4} (x^3 + 3x^2 - 6x - 8) \right\} \approx 2.5981 \text{ at } x \approx -2.7321$$

2.5981

**Local minimum:**

$$\min \left\{ \frac{1}{4} (x^3 + 3x^2 - 6x - 8) \right\} \approx -2.5981 \text{ at } x \approx 0.73205$$

-2.5981

**Definite integral area below the axis between the smallest and largest real roots:**

$$\int_{-4}^2 \frac{1}{4} (-8 - 6x + 3x^2 + x^3) \theta \left( \frac{1}{4} (8 + 6x - 3x^2 - x^3) \right) dx = -\frac{81}{16} = -5.0625$$

$\theta(x)$  is the Heaviside step function

**Definite integral area above the axis between the smallest and largest real roots:**

$$\int_{-4}^2 \frac{1}{4} (-8 - 6x + 3x^2 + x^3) \theta\left(\frac{1}{4} (-8 - 6x + 3x^2 + x^3)\right) dx = \frac{81}{16} = 5.0625$$

5.0625

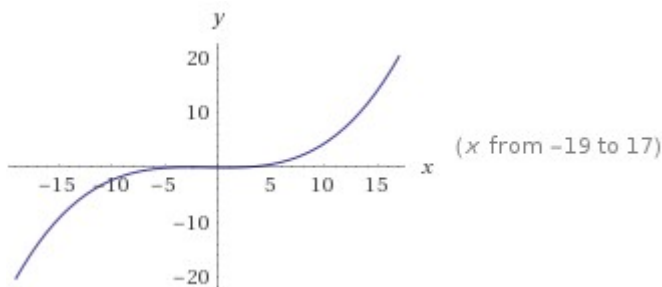
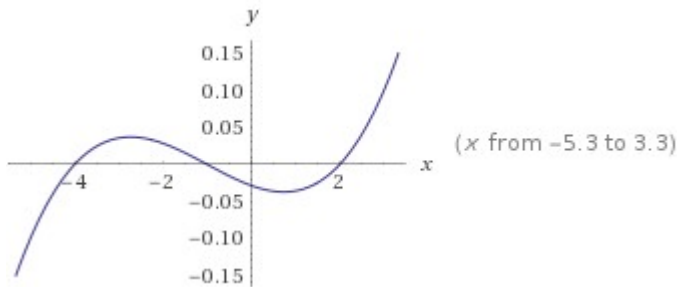
Now, we have the following cubic function:

$$(x^3 + 3x^2 - 6x - 8)/279$$

**Input:**

$$\frac{1}{279} (x^3 + 3x^2 - 6x - 8)$$

**Plots:**



**Alternate forms:**

$$\frac{1}{279} (x + 1) (x - 2) (x + 4)$$

$$x \left( \left( \frac{x}{279} + \frac{1}{93} \right) x - \frac{2}{93} \right) - \frac{8}{279}$$

$$\frac{1}{279} ((x + 1)^3 - 9(x + 1))$$

**Expanded form:**

$$\frac{x^3}{279} + \frac{x^2}{93} - \frac{2x}{93} - \frac{8}{279}$$

**Roots:**

$x = -4$

$x = -1$

$x = 2$

**Polynomial discriminant:**

$$\Delta = \frac{4}{8311689}$$

**Input:**

$$\frac{4}{8311689}$$

**Exact result:**

$$\frac{4}{8311689} \text{ (irreducible)}$$

**Decimal approximation:**

$$4.8124996014648767536898938350556667844525944125195252... \times 10^{-7}$$

4.812499601... \* 10<sup>-7</sup> result very near to the value (4.81996... \* 10<sup>-7</sup> =  $\phi_1$ ) of the scalar charge obtained from the following expression

$$\phi_1 = \frac{\alpha_0 Q_e^2}{M} \frac{1}{1 + \sqrt{1 + (\alpha_0^2 - 1) Q_e^2 / M^2}}$$

**Properties as a real function:****Domain**

$\mathbb{R}$  (all real numbers)

**Range**

$\mathbb{R}$  (all real numbers)

**Surjectivity**

surjective onto  $\mathbb{R}$



$\mathbb{R}$  is the set of real numbers

**Derivative:**

$$\frac{d}{dx} \left( \frac{1}{279} (x^3 + 3x^2 - 6x - 8) \right) = \frac{1}{93} (x^2 + 2x - 2)$$

**Indefinite integral:**

$$\int \frac{1}{279} (-8 - 6x + 3x^2 + x^3) dx = \frac{1}{279} \left( \frac{x^4}{4} + x^3 - 3x^2 - 8x \right) + \text{constant}$$

**Local maximum:**

$$\max \left\{ \frac{1}{279} (x^3 + 3x^2 - 6x - 8) \right\} = \frac{2}{31\sqrt{3}} \text{ at } x = -1 - \sqrt{3}$$

**Local minimum:**

$$\min \left\{ \frac{1}{279} (x^3 + 3x^2 - 6x - 8) \right\} = -\frac{2}{31\sqrt{3}} \text{ at } x = \sqrt{3} - 1$$

**Local maximum:**

$$\max \left\{ \frac{1}{279} (x^3 + 3x^2 - 6x - 8) \right\} \approx 0.037248 \text{ at } x \approx -2.7321$$

0.037248

**Local minimum:**

$$\min \left\{ \frac{1}{279} (x^3 + 3x^2 - 6x - 8) \right\} \approx -0.037248 \text{ at } x \approx 0.73205$$

-0.037248

**Definite integral area below the axis between the smallest and largest real roots:**

$$\int_{-4}^2 \frac{1}{279} (-8 - 6x + 3x^2 + x^3) \theta \left( \frac{1}{279} (8 + 6x - 3x^2 - x^3) \right) dx = -\frac{9}{124} \approx -0.0725806$$

$\theta(x)$  is the Heaviside step function

**Definite integral area above the axis between the smallest and largest real roots:**

$$\int_{-4}^2 \frac{1}{279} (-8 - 6x + 3x^2 + x^3) \theta \left( \frac{1}{279} (-8 - 6x + 3x^2 + x^3) \right) dx = \frac{9}{124} \approx 0.0725806$$

0.0725806

Now, we have the following cubic function:

$$(x^3 - 11x^2 + 18x - 1) \cdot \frac{1}{\left(\frac{1}{128\pi} + 1 + \sqrt{\pi} + \pi\right)}$$

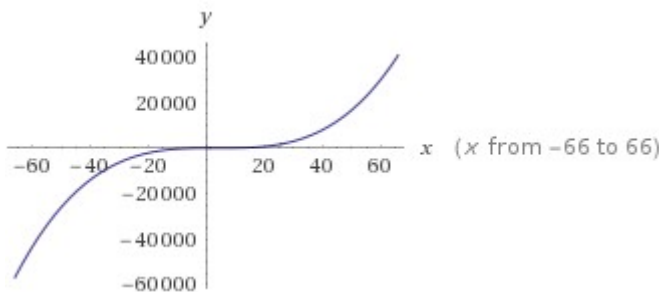
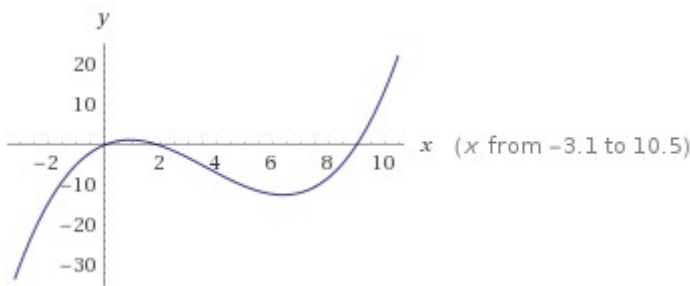
**Input:**

$$(x^3 - 11x^2 + 18x - 1) \times \frac{1}{\frac{1}{128\pi} + 1 + \sqrt{\pi} + \pi}$$

**Exact result:**

$$\frac{x^3 - 11x^2 + 18x - 1}{1 + \frac{1}{128\pi} + \sqrt{\pi} + \pi}$$

**Plots:**



**Alternate forms:**

Factor  $\left[ \frac{x^3 - 11x^2 + 18x - 1}{1 + \frac{1}{128\pi} + \sqrt{\pi} + \pi}, \text{Extension} \rightarrow \pi^{3/2} \right]$

$$x \left( x \left( \frac{128\pi x}{1 + 128\pi + 128\pi^{3/2} + 128\pi^2} - \frac{1408\pi}{1 + 128\pi + 128\pi^{3/2} + 128\pi^2} \right) + \frac{2304\pi}{1 + 128\pi + 128\pi^{3/2} + 128\pi^2} \right) - \frac{128\pi}{1 + 128\pi + 128\pi^{3/2} + 128\pi^2}$$

$$\frac{128\pi(x^3 - 11x^2 + 18x - 1)}{1 + 128\pi + 128\pi^{3/2} + 128\pi^2}$$

**Expanded form:**

$$\frac{x^3}{1 + \frac{1}{128\pi} + \sqrt{\pi} + \pi} - \frac{11x^2}{1 + \frac{1}{128\pi} + \sqrt{\pi} + \pi} + \frac{18x}{1 + \frac{1}{128\pi} + \sqrt{\pi} + \pi} - \frac{1}{1 + \frac{1}{128\pi} + \sqrt{\pi} + \pi}$$

**Roots:**

$$x \approx 0.057570$$

$$x \approx 1.9266$$

$$x \approx 9.0158$$

**Polynomial discriminant:**

$$\Delta_x = \frac{3781987139584\pi^4}{(1 + 128\pi + 128\pi^{3/2} + 128\pi^2)^4}$$

**Input:**

$$\frac{3781987139584\pi^4}{(1 + 128\pi + 128\pi^{3/2} + 128\pi^2)^4}$$

**Decimal approximation:**

11.49769850236354651302448002789869265539241404517386926706...

11.497698502...

**Property:**

$\frac{3781987139584\pi^4}{(1 + 128\pi + 128\pi^{3/2} + 128\pi^2)^4}$  is a transcendental number

**Alternate forms:**

$$\frac{3781987139584\pi^4}{(1 + 128\pi(1 + \sqrt{\pi} + \pi))^4}$$

$$\frac{230834176(129 - 128\sqrt{\pi} + 16512\pi + 256\pi^{3/2})}{(1 + 128\pi + 128\pi^{3/2} + 128\pi^2)^4} - \frac{461668352(65 - 64\sqrt{\pi} + 64\pi + 128\pi^{3/2})}{(1 + 128\pi + 128\pi^{3/2} + 128\pi^2)^3} + \frac{230834176}{(1 + 128\pi + 128\pi^{3/2} + 128\pi^2)^2}$$

**Alternative representations:**

$$\frac{3781987139584\pi^4}{(1 + 128\pi + 128\pi^{3/2} + 128\pi^2)^4} = \frac{3781987139584\pi^4}{(1 + 128\pi + 128\pi^{3/2} + 768\zeta(2))^4}$$

$$\frac{3781987139584\pi^4}{(1 + 128\pi + 128\pi^{3/2} + 128\pi^2)^4} = \frac{3781987139584 \cos^{-1}(-1)^4}{(1 + 128 \cos^{-1}(-1) + 128 \cos^{-1}(-1)^{3/2} + 128 \cos^{-1}(-1)^2)^4}$$

$$\frac{3781987139584\pi^4}{(1+128\pi+128\pi^{3/2}+128\pi^2)^4} = \frac{3781987139584(180^\circ)^4}{(1+23040^\circ+128(180^\circ)^{3/2}+128(180^\circ)^2)^4}$$

### Series representations:

$$\frac{3781987139584\pi^4}{(1+128\pi+128\pi^{3/2}+128\pi^2)^4} = \frac{340378842562560 \sum_{k=1}^{\infty} \frac{1}{k^4}}{(1+128\pi+128\pi^{3/2}+128\pi^2)^4}$$

$$\frac{3781987139584\pi^4}{(1+128\pi+128\pi^{3/2}+128\pi^2)^4} = \frac{3781987139584\pi^4}{(1+128\pi+128\pi^{3/2}+768 \sum_{k=1}^{\infty} \frac{1}{k^2})^4}$$

$$\frac{3781987139584\pi^4}{(1+128\pi+128\pi^{3/2}+128\pi^2)^4} = \frac{363070765400064 \sum_{k=0}^{\infty} \frac{1}{(1+2k)^4}}{(1+128\pi+128\pi^{3/2}+128\pi^2)^4}$$

### Integral representations:

$$\frac{3781987139584\pi^4}{(1+128\pi+128\pi^{3/2}+128\pi^2)^4} = \frac{60511794233344 \left( \int_0^{\infty} \frac{\sin(t)}{t} dt \right)^4}{\left( 1 + 256 \int_0^{\infty} \frac{\sin(t)}{t} dt + 256\sqrt{2} \left( \int_0^{\infty} \frac{\sin(t)}{t} dt \right)^{3/2} + 512 \left( \int_0^{\infty} \frac{\sin(t)}{t} dt \right)^2 \right)^4}$$

$$\frac{3781987139584\pi^4}{(1+128\pi+128\pi^{3/2}+128\pi^2)^4} = \frac{60511794233344 \left( \int_0^{\infty} \frac{1}{1+t^2} dt \right)^4}{\left( 1 + 256 \int_0^{\infty} \frac{1}{1+t^2} dt + 256\sqrt{2} \left( \int_0^{\infty} \frac{1}{1+t^2} dt \right)^{3/2} + 512 \left( \int_0^{\infty} \frac{1}{1+t^2} dt \right)^2 \right)^4}$$

$$\frac{3781987139584\pi^4}{(1+128\pi+128\pi^{3/2}+128\pi^2)^4} = \frac{968188707733504 \left( \int_0^1 \sqrt{1-t^2} dt \right)^4}{\left( 1 + 512 \int_0^1 \sqrt{1-t^2} dt + 1024 \left( \int_0^1 \sqrt{1-t^2} dt \right)^{3/2} + 2048 \left( \int_0^1 \sqrt{1-t^2} dt \right)^2 \right)^4}$$

### Properties as a real function:

#### Domain

$\mathbb{R}$  (all real numbers)

## Range

$\mathbb{R}$  (all real numbers)

## Surjectivity

surjective onto  $\mathbb{R}$

$\mathbb{R}$  is the set of real numbers

## Derivative:

$$\frac{d}{dx} \left( \frac{x^3 - 11x^2 + 18x - 1}{\frac{1}{128\pi} + 1 + \sqrt{\pi} + \pi} \right) = \frac{3x^2 - 22x + 18}{1 + \frac{1}{128\pi} + \sqrt{\pi} + \pi}$$

## Indefinite integral:

$$\int \frac{-1 + 18x - 11x^2 + x^3}{1 + \frac{1}{128\pi} + \sqrt{\pi} + \pi} dx = \frac{\frac{x^4}{4} - \frac{11x^3}{3} + 9x^2 - x}{1 + \frac{1}{128\pi} + \sqrt{\pi} + \pi} + \text{constant}$$

i.e.:

## Derivative:

$$\frac{d}{dx} \left( \frac{x^3 - 11x^2 + 18x - 1}{\frac{1}{128\pi} + 1 + \sqrt{\pi} + \pi} \right) \approx 0.169018 (3x^2 - 22x + 18)$$

## Indefinite integral:

$$\int \frac{-1 + 18x - 11x^2 + x^3}{1 + \frac{1}{128\pi} + \sqrt{\pi} + \pi} dx \approx \text{constant} + 0.169018 (0.25x^4 - 3.66667x^3 + 9x^2 - x)$$

## Local maximum:

$$\max \left\{ \frac{x^3 - 11x^2 + 18x - 1}{\frac{1}{128\pi} + 1 + \sqrt{\pi} + \pi} \right\} \approx 1.1884 \text{ at } x \approx 0.93822$$

1.1884

## Local minimum:

$$\min \left\{ \frac{x^3 - 11x^2 + 18x - 1}{\frac{1}{128\pi} + 1 + \sqrt{\pi} + \pi} \right\} \approx -12.544 \text{ at } x \approx 6.3951$$

-12.544 result very near to the Bekenstein-Hawking black hole entropy 12.5664 with minus sign

**Definite integral area below the axis between the smallest and largest real roots:**

$$\int_{\substack{\text{root of } -1+18x-11x^2+x^3 \text{ near } x=0.0575704 \\ \text{root of } -1+18x-11x^2+x^3 \text{ near } x=0.01581}}^{(-1+18x-11x^2+x^3)\theta\left(-\frac{-1+18x-11x^2+x^3}{1+\frac{1}{128\pi}+\sqrt{\pi}+\pi}\right)} dx =$$

$$\left( \left( \left( \left( -44 - \frac{134(1-i\sqrt{3})}{\sqrt[3]{\frac{1}{2}(-907+3i\sqrt{42267})}} - 2^{2/3}(1+i\sqrt{3})\sqrt[3]{-907+3i\sqrt{42267}} \right. \right. \right. \right.$$

$$\left. \left. \left. \left. \pi + 2 \left( 22 + \frac{134}{\sqrt[3]{\frac{1}{2}(907+3i\sqrt{42267})}} + 2^{2/3}\sqrt[3]{907+3i\sqrt{42267}} \right) \right. \right. \right. \right.$$

$$\left. \left. \left. \left. \pi + \left( -3060 - \frac{113742(1+i\sqrt{3})}{\sqrt[3]{\frac{1}{2}(-997247+591i\sqrt{42267})}} - \right. \right. \right. \right.$$

$$\left. \left. \left. \left. 9 \cdot 2^{2/3}(1-i\sqrt{3})\sqrt[3]{-997247+591i\sqrt{42267}} \right) \pi + \right. \right. \right. \right.$$

$$\left( 3060 - \frac{113742(1-i\sqrt{3})}{\sqrt[3]{\frac{1}{2}(997247+591i\sqrt{42267})}} - \right.$$

$$\left. \left. \left. \left. 9 \cdot 2^{2/3}(1+i\sqrt{3})\sqrt[3]{997247+591i\sqrt{42267}} \right) \pi + \right. \right. \right. \right.$$

$$\left( -6621 - \frac{75241 \cdot 579^{2/3}(1+i\sqrt{3})}{2^{2/3}\sqrt[3]{-993218337+26201i\sqrt{42267}}} - \right.$$

$$\left. \left. \left. \left. \frac{1}{2}(1-i\sqrt{3})\sqrt[3]{\frac{579}{2}(-993218337+26201i\sqrt{42267})} \right) \pi + \right. \right. \right. \right.$$

$$\left( 6621 - \frac{75241 \cdot 579^{2/3}(1-i\sqrt{3})}{2^{2/3}\sqrt[3]{993218337+26201i\sqrt{42267}}} - \right.$$

$$\left. \left. \left. \left. \frac{1}{2}(1+i\sqrt{3})\sqrt[3]{\frac{579}{2}(993218337+26201i\sqrt{42267})} \right) \pi + \right. \right. \right. \right.$$

$$\left( \frac{32560}{3} - \frac{11701294(1-i\sqrt{3})}{3\sqrt[3]{\frac{1}{2}(-775542967+96123i\sqrt{42267})}} - \right.$$

$$\left. \left. \left. \left. \frac{11}{3} \cdot 2^{2/3}(1+i\sqrt{3})\sqrt[3]{-775542967+96123i\sqrt{42267}} \right) \pi + \right. \right. \right. \right.$$

$$\left. \left. \left. \left. \frac{22}{3} \left( 1480 + \frac{1063754}{\sqrt[3]{\frac{1}{2}(775542967+96123i\sqrt{42267})}} + \right. \right. \right. \right.$$

$$\left. \left. \left. \left. 2^{2/3}\sqrt[3]{775542967+96123i\sqrt{42267}} \right) \pi \right) \right) /$$

$$\left( 3(1+128\pi+128\pi^{3/2}+128\pi^2) \right) \approx -54.3327$$

-54.3327

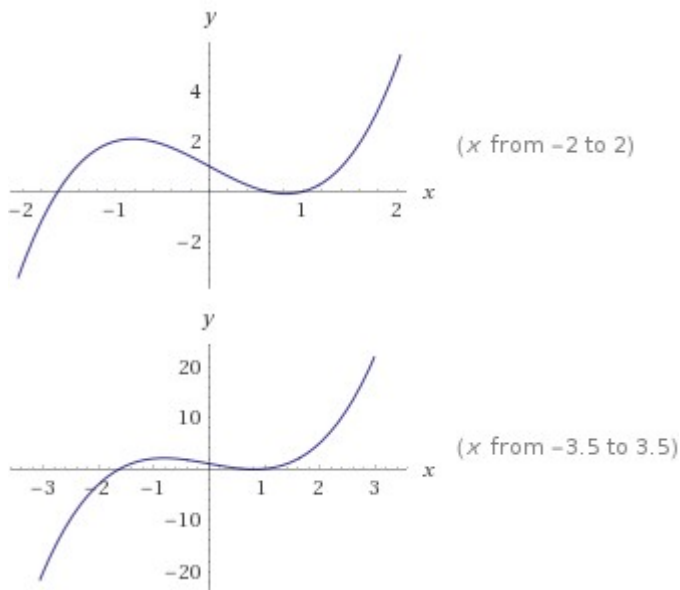
We have the following cubic function:

$$(x^3 - 2x + 1)$$

**Input:**

$$x^3 - 2x + 1$$

**Plots:**



**Alternate forms:**

$$(x - 1)(x^2 + x - 1)$$

$$x(x^2 - 2) + 1$$

$$-\frac{1}{4}(-2x + \sqrt{5} - 1)(x - 1)(2x + \sqrt{5} + 1)$$

**Roots:**

$$x = 1$$

$$x = -\frac{1}{2} - \frac{\sqrt{5}}{2}$$

$$x = \frac{\sqrt{5}}{2} - \frac{1}{2}$$

**Input:**

$$-\frac{1}{2} - \frac{\sqrt{5}}{2}$$



**Decimal approximation:**

-1.61803398874989484820458683436563811772030917980576286213...

-1.61803398874...

**Input:**

$$-\frac{1}{2} + \frac{\sqrt{5}}{2}$$

**Decimal approximation:**

0.618033988749894848204586834365638117720309179805762862135...

0.61803398874...

**Polynomial discriminant:**

$$\Delta = 5$$

**Properties as a real function:****Domain**

$\mathbb{R}$  (all real numbers)

**Range**

$\mathbb{R}$  (all real numbers)

**Surjectivity**

surjective onto  $\mathbb{R}$

$\mathbb{R}$  is the set of real numbers

**Derivative:**

$$\frac{d}{dx}(x^3 - 2x + 1) = 3x^2 - 2$$

**Indefinite integral:**

$$\int (1 - 2x + x^3) dx = \frac{x^4}{4} - x^2 + x + \text{constant}$$

**Local maximum:**

$$\max\{x^3 - 2x + 1\} = \frac{1}{9} (9 + 4\sqrt{6}) \text{ at } x = -\sqrt{\frac{2}{3}}$$

**Local minimum:**

$$\min\{x^3 - 2x + 1\} = \frac{1}{9} (9 - 4\sqrt{6}) \text{ at } x = \sqrt{\frac{2}{3}}$$

**Local maximum:**

$$\max\{x^3 - 2x + 1\} \approx 2.0887 \text{ at } x \approx -0.81650$$

2.0887

**Local minimum:**

$$\min\{x^3 - 2x + 1\} \approx -0.088662 \text{ at } x \approx 0.81650$$

-0.088662

**Definite integral area below the axis between the smallest and largest real roots:**

$$\int_{\frac{1}{2}(-1-\sqrt{5})}^1 (1 - 2x + x^3) \theta(-1 + 2x - x^3) dx = \frac{1}{8} (11 - 5\sqrt{5}) \approx -0.0225425$$

-0.0225425

$\theta(x)$  is the Heaviside step function

**Definite integral area above the axis between the smallest and largest real roots:**

$$\int_{\frac{1}{2}(-1-\sqrt{5})}^1 (1 - 2x + x^3) \theta(1 - 2x + x^3) dx = \frac{5\sqrt{5}}{4} \approx 2.79508$$

2.79508

$$(x^3 - 2x + 1) + (\sqrt{3})$$

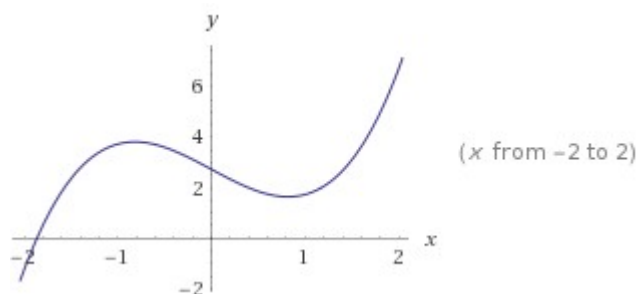
**Input:**

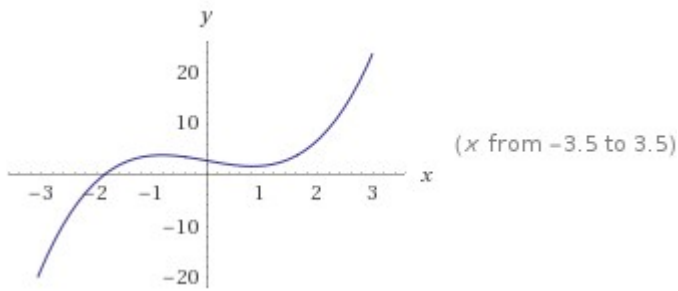
$$(x^3 - 2x + 1) + \sqrt{3}$$

**Exact result:**

$$x^3 - 2x + \sqrt{3} + 1$$

**Plots:**





**Alternate form:**

$$x(x^2 - 2) + \sqrt{3} + 1$$

**Real root:**

$$x \approx -1.8620$$

$$-1.8620$$

**Complex roots:**

$$x \approx 0.93102 - 0.77487i$$

$$x \approx 0.93102 + 0.77487i$$

**Polynomial discriminant:**

$$\Delta_x = -76 - 54\sqrt{3}$$

**Input:**

$$-76 - 54\sqrt{3}$$

**Decimal approximation:**

$$-169.530743608719373850482102441317107814911483705760553915\dots$$

$$-169.5307436\dots$$

**Properties as a real function:**

**Domain**

$\mathbb{R}$  (all real numbers)

**Range**

$\mathbb{R}$  (all real numbers)

**Surjectivity**

surjective onto  $\mathbb{R}$

$\mathbb{R}$  is the set of real numbers

**Derivative:**

$$\frac{d}{dx}((x^3 - 2x + 1) + \sqrt{3}) = 3x^2 - 2$$

**Indefinite integral:**

$$\int (1 + \sqrt{3} - 2x + x^3) dx = \frac{x^4}{4} - x^2 + \sqrt{3}x + x + \text{constant}$$

**Local maximum:**

$$\max\{(x^3 - 2x + 1) + \sqrt{3}\} = 1 + \frac{4\sqrt{\frac{2}{3}}}{3} + \sqrt{3} \text{ at } x = -\sqrt{\frac{2}{3}}$$

**Local minimum:**

$$\min\{(x^3 - 2x + 1) + \sqrt{3}\} = 1 - \frac{4\sqrt{\frac{2}{3}}}{3} + \sqrt{3} \text{ at } x = \sqrt{\frac{2}{3}}$$

**Local maximum:**

$$\max\{(x^3 - 2x + 1) + \sqrt{3}\} \approx 3.8207 \text{ at } x \approx -0.81650$$

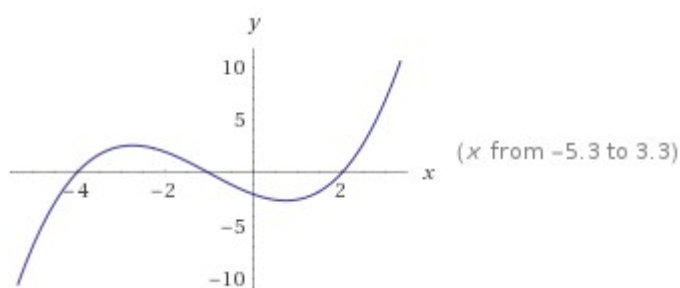
3.8207

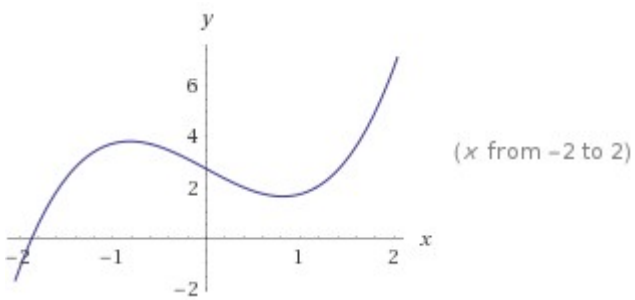
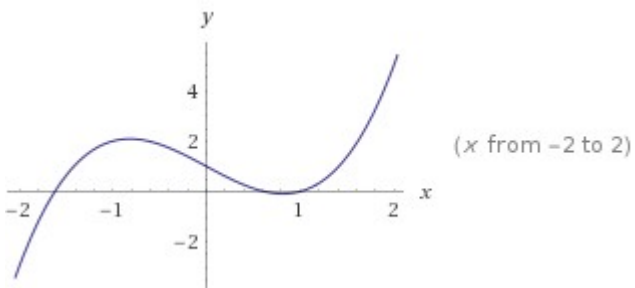
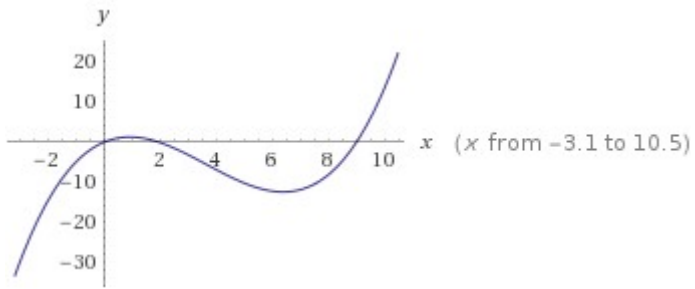
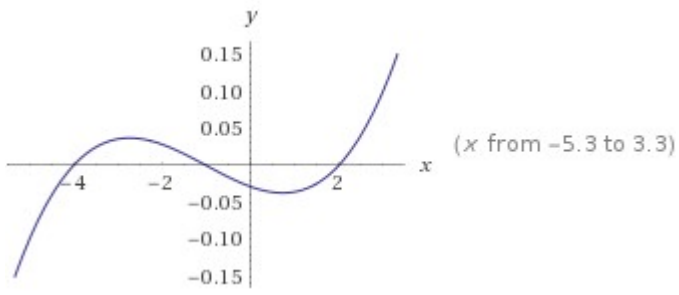
**Local minimum:**

$$\min\{(x^3 - 2x + 1) + \sqrt{3}\} \approx 1.6434 \text{ at } x \approx 0.81650$$

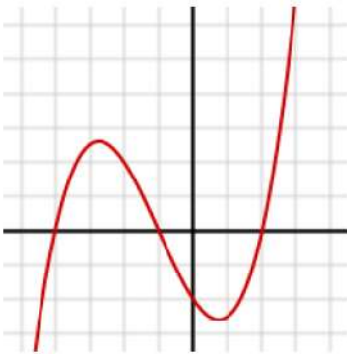
$$1.6434 \approx \zeta(2) = \frac{\pi^2}{6} = 1.644934 \dots$$

Now, we have that:





## From Wikipedia - Cubic Equation

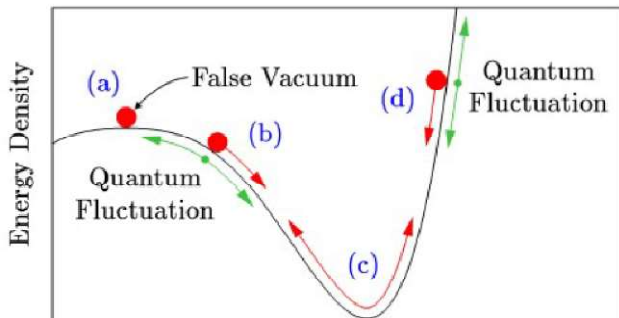


Graph of a cubic function with 3 real roots (where the curve crosses the horizontal axis at  $y = 0$ ). The case shown has two critical points.

Here the function is

$$f(x) = (x^3 + 3x^2 - 6x - 8)/4.$$

We observe how all the graphs above, concerning the cubic functions, are very similar to the following representation of the scalar field (in red). It is possible to hypothesize that cubic functions and the cubic equations, with their roots, are connected to the scalar field.



Now, from:

## LENSING OBSERVABLES: MASSLESS DYONIC vis-a-vis ELLIS WORMHOLE

*R.F. Lukmanova, G.Y. Tuleganova, R.N. Izmailov and K.K. Nandi*  
arXiv:1806.05441v1 [gr-qc] 14 Jun 2018

We have that:

The weak field deflection  $\hat{\alpha}$  in general has a major difference with strong field deflection. The strong field deflection suffered by light rays passing at an invariant impact parameter  $b$  closest to the photon sphere have a logarithmic divergence [32,33]. This fact prevents the *exact* deflection angle to be Taylor expanded to yield the same light deflection for the same  $b$ . For instance [33], for the Schwarzschild black hole of mass  $M$ ,

$$\tilde{\alpha}_{\text{strong}}(b') = -\pi + \log \left[ \frac{216(7 - 4\sqrt{3})}{b'} \right] + O(b'), \quad (27)$$

$$\hat{\alpha}_{\text{weak}}(b') = \frac{4}{3\sqrt{3}} (1 - b') + O(1 - b')^2, \quad (28)$$

where the redefined common impact parameter  $b'$  is  $1 - b' = \frac{3\sqrt{3}M}{b}$ . When  $b = 3\sqrt{3}M$ ,  $\tilde{\alpha}_{\text{strong}} \rightarrow \infty$ , but  $\hat{\alpha}_{\text{weak}} = \frac{4M}{b}$ , as expected. These facts indicate that the weak field lensing is expected to yield a set of lensing observables completely different from those of the strong field. We note that  $\tilde{\alpha}_{\text{strong}}$  is itself an approximation in the strong regime with  $O(b')$  neglected.<sup>4</sup>

From

$$\hat{\alpha}_{\text{strong}}(b') = -\pi + \log \left[ \frac{216(7 - 4\sqrt{3})}{b'} \right] + O(b'),$$

For  $b' = 3\sqrt{3} M = 6.82154e+40$

$3\sqrt{3} \times 13.12806 \times 10^{39} = 6.82154... \times 10^{40}$ , we obtain:







And:

$$\left( \frac{\left( \frac{4}{3\sqrt{3}} (1 - 6.82154e+40) + (1 - 6.82154e+40)^2 \right)}{\left( \frac{-\pi + \ln\left( \frac{216(7 - 4\sqrt{3})}{6.82154e+40} \right) + 6.82154e+40}{6.82154e+40} \right)} \right)^{1/172} + 5 \cdot \frac{1}{10^3}$$

**Input interpretation:**

$$\sqrt[172]{\frac{\frac{4}{3\sqrt{3}} (1 - 6.82154 \times 10^{40}) + (1 - 6.82154 \times 10^{40})^2}{-\pi + \log\left(\frac{216(7 - 4\sqrt{3})}{6.82154 \times 10^{40}}\right) + 6.82154 \times 10^{40}}} + 5 \times \frac{1}{10^3}$$

$\log(x)$  is the natural logarithm

**Result:**

1.732453211367567575444243530270325838135428137895883650539...

$1.73245\dots \approx \sqrt{3}$  that is the ratio between the gravitating mass  $M_0$  and the Wheelerian mass  $q$

**Possible closed forms:**

$$\sqrt{3} \approx 1.73205080$$

$$\sqrt{-5 + 5e - 2\pi + \log(2)} \approx 1.732446540$$

$$\frac{1}{\gamma} \approx 1.7324547146$$

From

$$\hat{\alpha}_{\text{weak}} = \frac{4M}{b}$$

we obtain:

$$(4 \cdot (13.12806e+39)) / (6.82154e+40)$$

**Input interpretation:**

$$\frac{4 \times 13.12806 \times 10^{39}}{6.82154 \times 10^{40}}$$

**Result:**

0.769800367658915728706421130712419776179572354629599767794...

**0.76980036765...**

**Possible closed forms:**

$$\frac{79564 W_{\text{Wad}}}{31007} \approx 0.76980036765891572870642113071241977617957235462959976779436901$$

$$\sin\left(\operatorname{csch}\left(\frac{3257021}{7598633}\right)\right) \approx 0.7698003676589157270135$$

$$6\pi \operatorname{sech}^2\left(\frac{6850447}{3002167}\right) \approx 0.7698003676589157273473$$

$$\frac{15\pi!}{2} + \frac{43}{6} + \frac{25}{66\pi} - \frac{2539\pi}{132} \approx 0.76980036765891572864855$$

$$-\frac{4(-278 + 40e + 5e^2)}{-1052 + 186e + 167e^2} \approx 0.76980036765891572869105$$

$$\frac{1}{88} (-150e^\pi + 326\pi - 857\log(\pi) + 1529\log(2\pi) + 543\tan^{-1}(\pi)) \approx 0.76980036765891572855746$$

$$\boxed{\text{root of } 1599x^3 - 61544x^2 + 55799x - 7213 \text{ near } x = 0.7698} \approx 0.76980036765891572869898$$

$$\pi \boxed{\text{root of } 61007x^3 + 79971x^2 - 42358x + 4680 \text{ near } x = 0.245035} \approx 0.769800367658915728783993$$

$$\frac{3217158089\pi}{13129378268} \approx 0.7698003676589157287091729$$

$$\pi \boxed{\text{root of } 1958x^4 - 3031x^3 + 2168x^2 - 3900x + 863 \text{ near } x = 0.245035} \approx 0.769800367658915728720420$$

$$\frac{\sqrt[3]{\frac{17525967}{3996211}} 10^{2/3}}{\pi^2} \approx 0.769800367658915735774$$

1

$$\boxed{\text{root of } 7213x^3 - 55799x^2 + 61544x - 1599 \text{ near } x = 1.29904} \approx 0.76980036765891572869898$$

$$\frac{1}{2} \sqrt{\frac{1}{393} (-5502 + 1973e - 178\pi + 2351\log(2))} \approx 0.769800367658915728780673$$

$$\log\left(\frac{1}{784} (-96 - 364\sqrt{2} - 226e + 128e^2 + 78\pi + 175\pi^2)\right) \approx 0.7698003676589157287081712$$

$$\boxed{\text{root of } 6986x^4 - 2072x^3 - 1689x^2 + 4588x - 4039 \text{ near } x = 0.7698} \approx 0.7698003676589157287039431$$

$W_{\text{Wad}}$  is the Wadsworth constant

$\text{csch}(x)$  is the hyperbolic cosecant function

$\text{sech}(x)$  is the hyperbolic secant function

$n!$  is the factorial function

$\tan^{-1}(x)$  is the inverse tangent function

$\log(x)$  is the natural logarithm

and:

### Input interpretation:

0.7698003676589

**0.7698003676589**

### Possible closed forms:

$$\sin\left(\cos\left(\frac{126}{253}\right)\right) \approx 0.769800367601826$$

$$-\frac{40(2\mathcal{K}_{\text{LR}} + 5)}{107\mathcal{K}_{\text{LR}} - 421} \approx 0.76980036765874674$$

$$\frac{536259\pi}{2188499} \approx 0.7698003676590251$$

$$\boxed{\text{root of } 120x^3 + 387x^2 + 274x - 495 \text{ near } x = 0.7698} \approx 0.769800367658963219$$

$$\frac{73(A^*)}{51} - \frac{\pi}{16} \approx 0.76980036765843485$$

$$\frac{87\Gamma(x_{\text{min}})}{34} - \frac{60e}{109} \approx 0.76980036765839321$$

$$-\frac{\pi!}{10} - \frac{23}{20} + \frac{63}{160\pi} + \frac{4\pi}{5} \approx 0.7698003676590988$$

$$\frac{1}{2} \sqrt{\frac{1}{19} (-30 - 31 e + 41 \pi + 44 \log(2))} \approx 0.76980036765853776$$

$$-e^{3+1/e-3e+7/\pi} \pi^{5-2e} \csc^2(e \pi) \sec^5(e \pi) \approx 0.7698003676598901$$

$$\frac{880 \cdot W_{Wy} - 3}{3(66 \cdot W_{Wy} + 1)} \approx 0.76980036750399$$

$\mathcal{K}_{LR}$  is the Landau-Ramanujan constant

$A^*$  is Graham's biggest little hexagon area

$\Gamma(x_{\min})$  is the minimal value of  $\Gamma$  function for positive argument

$n!$  is the factorial function

$\log(x)$  is the natural logarithm

$\csc(x)$  is the cosecant function

$\sec(x)$  is the secant function

$W_{Wy}$  is Wyler's constant

From the closed form

$$\sin\left(\cos\left(\frac{126}{253}\right)\right) \approx 0.769800367601826$$

we obtain:

**Input:**

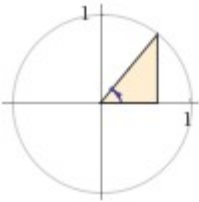
$$\sin\left(\cos\left(\frac{126}{253}\right)\right)$$

**Decimal approximation:**

0.769800367601826944551812208548234870647238145042843864199...

0.769800367601...

**Reference triangle for angle 0.8785 radians:**



width	$\cos\left(\cos\left(\frac{126}{253}\right)\right) \approx 0.638285$
height	$\sin\left(\cos\left(\frac{126}{253}\right)\right) \approx 0.7698$

**Alternate form:**

$$\frac{1}{2} i \exp\left(-\frac{1}{2} i \left(e^{-(126i)/253} + e^{(126i)/253}\right)\right) - \frac{1}{2} i \exp\left(\frac{1}{2} i \left(e^{-(126i)/253} + e^{(126i)/253}\right)\right)$$

**Alternative representations:**

$$\sin\left(\cos\left(\frac{126}{253}\right)\right) = \cos\left(\frac{\pi}{2} - \cos\left(\frac{126}{253}\right)\right)$$

$$\sin\left(\cos\left(\frac{126}{253}\right)\right) = -\cos\left(\frac{\pi}{2} + \cos\left(\frac{126}{253}\right)\right)$$

$$\sin\left(\cos\left(\frac{126}{253}\right)\right) = \frac{-e^{-i \cos(126/253)} + e^{i \cos(126/253)}}{2 i}$$

**Series representations:**

$$\sin\left(\cos\left(\frac{126}{253}\right)\right) = \sin\left(\sum_{k=0}^{\infty} \frac{\left(-\frac{15876}{64009}\right)^k}{(2k)!}\right)$$

$$\sin\left(\cos\left(\frac{126}{253}\right)\right) = \sum_{k=0}^{\infty} \frac{(-1)^k \cos^{1+2k}\left(\frac{126}{253}\right)}{(1+2k)!}$$

$$\sin\left(\cos\left(\frac{126}{253}\right)\right) = \sum_{k=0}^{\infty} \frac{(-1)^k \left(-\frac{\pi}{2} + \cos\left(\frac{126}{253}\right)\right)^{2k}}{(2k)!}$$

**Integral representations:**

$$\sin\left(\cos\left(\frac{126}{253}\right)\right) = \cos\left(\frac{126}{253}\right) \int_0^1 \cos\left(t \cos\left(\frac{126}{253}\right)\right) dt$$

$$\sin\left(\cos\left(\frac{126}{253}\right)\right) = -\frac{i \cos\left(\frac{126}{253}\right)}{4\sqrt{\pi}} \int_{-i\infty+\gamma}^{i\infty+\gamma} \frac{e^{s-\cos^2\left(\frac{126}{253}\right)/(4s)}}{s^{3/2}} ds \quad \text{for } \gamma > 0$$

$$\sin\left(\cos\left(\frac{126}{253}\right)\right) = -\frac{i}{2\sqrt{\pi}} \int_{-i\infty+\gamma}^{i\infty+\gamma} \frac{\Gamma(s) \left(2 \sec\left(\frac{126}{253}\right)\right)^{-1+2s}}{\Gamma\left(\frac{3}{2}-s\right)} ds \quad \text{for } 0 < \gamma < 1$$

### Multiple-argument formulas:

$$\sin\left(\cos\left(\frac{126}{253}\right)\right) = \sin\left(1 - 2 \sin^2\left(\frac{63}{253}\right)\right)$$

$$\sin\left(\cos\left(\frac{126}{253}\right)\right) = -\sin\left(1 - 2 \cos^2\left(\frac{63}{253}\right)\right)$$

$$\sin\left(\cos\left(\frac{126}{253}\right)\right) = -\sin\left(3 \cos\left(\frac{42}{253}\right) - 4 \cos^3\left(\frac{42}{253}\right)\right)$$

We have also:

$$89 \times \frac{1}{\left(\left(\left(\sin\left(\cos\left(\frac{126}{253}\right)\right)\right)\right)\right)} + 13 - \pi$$

**Input:**

$$89 \times \frac{1}{\sin\left(\cos\left(\frac{126}{253}\right)\right)} + 13 - \pi$$

**Exact result:**

$$13 - \pi + 89 \operatorname{csc}\left(\cos\left(\frac{126}{253}\right)\right)$$

$\operatorname{csc}(x)$  is the cosecant function

**Decimal approximation:**

125.4727974476559559197031752742542004173250207353189271013...

[125.47279744... result very near to the Higgs boson mass 125.18 GeV](#)

**Alternate forms:**

$$\left(-89 - 13 \sin\left(\cos\left(\frac{126}{253}\right)\right) + \pi \sin\left(\cos\left(\frac{126}{253}\right)\right)\right) \left(-\operatorname{csc}\left(\cos\left(\frac{126}{253}\right)\right)\right)$$

$$13 - \pi - \frac{178 \sin\left(\cos\left(\frac{126}{253}\right)\right)}{\cos\left(2 \cos\left(\frac{126}{253}\right)\right) - 1}$$

$$-\frac{1}{2} \left( -89 - 13 \sin\left(\cos\left(\frac{126}{253}\right)\right) + \pi \sin\left(\cos\left(\frac{126}{253}\right)\right) \right) \csc\left(\frac{1}{2} \cos\left(\frac{126}{253}\right)\right) \sec\left(\frac{1}{2} \cos\left(\frac{126}{253}\right)\right)$$

$\sec(x)$  is the secant function

### Alternative representations:

$$\frac{89}{\sin\left(\cos\left(\frac{126}{253}\right)\right)} + 13 - \pi = 13 - \pi + \frac{89}{\cos\left(\frac{\pi}{2} - \cos\left(\frac{126}{253}\right)\right)}$$

$$\frac{89}{\sin\left(\cos\left(\frac{126}{253}\right)\right)} + 13 - \pi = 13 - \pi + \frac{89}{\cos\left(\frac{\pi}{2} + \cos\left(\frac{126}{253}\right)\right)}$$

$$\frac{89}{\sin\left(\cos\left(\frac{126}{253}\right)\right)} + 13 - \pi = 13 - \pi + \frac{89}{\frac{-e^{-i \cos(126/253)} + e^{i \cos(126/253)}}{2i}}$$

### Series representations:

$$\frac{89}{\sin\left(\cos\left(\frac{126}{253}\right)\right)} + 13 - \pi = 13 - \pi - 178 i \sum_{k=1}^{\infty} q^{-1+2k} \text{ for } q = e^{i \cos(126/253)}$$

$$\frac{89}{\sin\left(\cos\left(\frac{126}{253}\right)\right)} + 13 - \pi = 13 - \pi + 89 \csc\left(\sum_{k=0}^{\infty} \frac{\left(-\frac{15876}{64009}\right)^k}{(2k)!}\right)$$

$$\frac{89}{\sin\left(\cos\left(\frac{126}{253}\right)\right)} + 13 - \pi = 13 - \pi + 89 \cos\left(\frac{126}{253}\right) \sum_{k=-\infty}^{\infty} \frac{(-1)^k}{-k^2 \pi^2 + \cos^2\left(\frac{126}{253}\right)}$$

### Integral representation:

$$\frac{89}{\sin\left(\cos\left(\frac{126}{253}\right)\right)} + 13 - \pi = 13 - \pi + \frac{89}{\pi} \int_0^{\infty} \frac{t^{\cos\left(\frac{126}{253}\right)/\pi}}{t+t^2} dt$$

### Multiple-argument formulas:

$$\frac{89}{\sin\left(\cos\left(\frac{126}{253}\right)\right)} + 13 - \pi = 13 - \pi + 89 \csc\left(1 - 2 \sin^2\left(\frac{63}{253}\right)\right)$$



$$\frac{89}{\sin\left(\cos\left(\frac{126}{253}\right)\right)} + 13 - \pi = 13 - \pi - 89 \csc\left(1 - 2 \cos^2\left(\frac{63}{253}\right)\right)$$

$$\frac{89}{\sin\left(\cos\left(\frac{126}{253}\right)\right)} + 13 - \pi = 13 - \pi - 89 \csc\left(3 \cos\left(\frac{42}{253}\right) - 4 \cos^3\left(\frac{42}{253}\right)\right)$$

$$89 * 1 / (((\sin(\cos(126/253)))))) + 13 + 11$$

**Input:**

$$89 \times \frac{1}{\sin\left(\cos\left(\frac{126}{253}\right)\right)} + 13 + 11$$

**Exact result:**

$$24 + 89 \csc\left(\cos\left(\frac{126}{253}\right)\right)$$

$\csc(x)$  is the cosecant function

**Decimal approximation:**

139.6143901012457491581658186575337033015221901346940329223...

139.61439... result practically equal to the rest mass of Pion meson 139.57 MeV

**Alternate forms:**

$$\left(89 + 24 \sin\left(\cos\left(\frac{126}{253}\right)\right)\right) \csc\left(\cos\left(\frac{126}{253}\right)\right)$$

$$24 - \frac{178 \sin\left(\cos\left(\frac{126}{253}\right)\right)}{\cos\left(2 \cos\left(\frac{126}{253}\right)\right) - 1}$$

$$\frac{1}{2} \left(89 + 24 \sin\left(\cos\left(\frac{126}{253}\right)\right)\right) \csc\left(\frac{1}{2} \cos\left(\frac{126}{253}\right)\right) \sec\left(\frac{1}{2} \cos\left(\frac{126}{253}\right)\right)$$

$\sec(x)$  is the secant function

**Alternative representations:**

$$\frac{89}{\sin\left(\cos\left(\frac{126}{253}\right)\right)} + 13 + 11 = 24 + \frac{89}{\cos\left(\frac{\pi}{2} - \cos\left(\frac{126}{253}\right)\right)}$$

$$\frac{89}{\sin\left(\cos\left(\frac{126}{253}\right)\right)} + 13 + 11 = 24 + -\frac{89}{\cos\left(\frac{\pi}{2} + \cos\left(\frac{126}{253}\right)\right)}$$

$$\frac{89}{\sin\left(\cos\left(\frac{126}{253}\right)\right)} + 13 + 11 = 24 + \frac{89}{\frac{-e^{-i \cos(126/253)} + e^{i \cos(126/253)}}{2i}}$$

### Series representations:

$$\frac{89}{\sin\left(\cos\left(\frac{126}{253}\right)\right)} + 13 + 11 = 24 - 178i \sum_{k=1}^{\infty} q^{-1+2k} \text{ for } q = e^{i \cos(126/253)}$$

$$\frac{89}{\sin\left(\cos\left(\frac{126}{253}\right)\right)} + 13 + 11 = 24 + 89 \operatorname{csc}\left(\sum_{k=0}^{\infty} \frac{\left(-\frac{15876}{64009}\right)^k}{(2k)!}\right)$$

$$\frac{89}{\sin\left(\cos\left(\frac{126}{253}\right)\right)} + 13 + 11 = 24 + 89 \cos\left(\frac{126}{253}\right) \sum_{k=-\infty}^{\infty} \frac{(-1)^k}{-k^2 \pi^2 + \cos^2\left(\frac{126}{253}\right)}$$

$n!$  is the factorial function

### Integral representation:

$$\frac{89}{\sin\left(\cos\left(\frac{126}{253}\right)\right)} + 13 + 11 = 24 + \frac{89}{\pi} \int_0^{\infty} \frac{t^{\cos\left(\frac{126}{253}\right)/\pi}}{t + t^2} dt$$

### Multiple-argument formulas:

$$\frac{89}{\sin\left(\cos\left(\frac{126}{253}\right)\right)} + 13 + 11 = 24 + 89 \operatorname{csc}\left(1 - 2 \sin^2\left(\frac{63}{253}\right)\right)$$

$$\frac{89}{\sin\left(\cos\left(\frac{126}{253}\right)\right)} + 13 + 11 = 24 - 89 \operatorname{csc}\left(1 - 2 \cos^2\left(\frac{63}{253}\right)\right)$$

$$\frac{89}{\sin\left(\cos\left(\frac{126}{253}\right)\right)} + 13 + 11 = 24 - 89 \operatorname{csc}\left(3 \cos\left(\frac{42}{253}\right) - 4 \cos^3\left(\frac{42}{253}\right)\right)$$

$$89 \times 1 / (((\sin(\cos(126/253)))))) + 2 \times 11$$

**Input:**

$$89 \times \frac{1}{\sin\left(\cos\left(\frac{126}{253}\right)\right)} + 2 \times 11$$

**Exact result:**

$$22 + 89 \operatorname{csc}\left(\cos\left(\frac{126}{253}\right)\right)$$

$\operatorname{csc}(x)$  is the cosecant function

**Decimal approximation:**

137.6143901012457491581658186575337033015221901346940329223...

137.61439... result practically equal to the golden angle value 137.5 and very near to the inverse of fine-structure constant 137.035

**Alternate forms:**

$$\left(89 + 22 \sin\left(\cos\left(\frac{126}{253}\right)\right)\right) \operatorname{csc}\left(\cos\left(\frac{126}{253}\right)\right)$$

$$22 - \frac{178 \sin\left(\cos\left(\frac{126}{253}\right)\right)}{\cos\left(2 \cos\left(\frac{126}{253}\right)\right) - 1}$$

$$\frac{1}{2} \left(89 + 22 \sin\left(\cos\left(\frac{126}{253}\right)\right)\right) \operatorname{csc}\left(\frac{1}{2} \cos\left(\frac{126}{253}\right)\right) \operatorname{sec}\left(\frac{1}{2} \cos\left(\frac{126}{253}\right)\right)$$

$\operatorname{sec}(x)$  is the secant function

**Alternative representations:**

$$\frac{89}{\sin\left(\cos\left(\frac{126}{253}\right)\right)} + 2 \times 11 = 22 + \frac{89}{\cos\left(\frac{\pi}{2} - \cos\left(\frac{126}{253}\right)\right)}$$

$$\frac{89}{\sin\left(\cos\left(\frac{126}{253}\right)\right)} + 2 \times 11 = 22 + - \frac{89}{\cos\left(\frac{\pi}{2} + \cos\left(\frac{126}{253}\right)\right)}$$

$$\frac{89}{\sin\left(\cos\left(\frac{126}{253}\right)\right)} + 2 \times 11 = 22 + \frac{89}{\frac{-e^{-i \cos(126/253)} + e^{i \cos(126/253)}}{2i}}$$

$i$  is the imaginary unit

### Series representations:

$$\frac{89}{\sin\left(\cos\left(\frac{126}{253}\right)\right)} + 2 \times 11 = 22 - 178i \sum_{k=1}^{\infty} q^{-1+2k} \text{ for } q = e^{i \cos(126/253)}$$

$$\frac{89}{\sin\left(\cos\left(\frac{126}{253}\right)\right)} + 2 \times 11 = 22 + 89 \operatorname{csc}\left(\sum_{k=0}^{\infty} \frac{\left(-\frac{15876}{64009}\right)^k}{(2k)!}\right)$$

$$\frac{89}{\sin\left(\cos\left(\frac{126}{253}\right)\right)} + 2 \times 11 = 22 + 89 \cos\left(\frac{126}{253}\right) \sum_{k=-\infty}^{\infty} \frac{(-1)^k}{-k^2 \pi^2 + \cos^2\left(\frac{126}{253}\right)}$$

$n!$  is the factorial function

### Integral representation:

$$\frac{89}{\sin\left(\cos\left(\frac{126}{253}\right)\right)} + 2 \times 11 = 22 + \frac{89}{\pi} \int_0^{\infty} \frac{t^{\cos\left(\frac{126}{253}\right)/\pi}}{t + t^2} dt$$

### Multiple-argument formulas:

$$\frac{89}{\sin\left(\cos\left(\frac{126}{253}\right)\right)} + 2 \times 11 = 22 + 89 \operatorname{csc}\left(1 - 2 \sin^2\left(\frac{63}{253}\right)\right)$$

$$\frac{89}{\sin\left(\cos\left(\frac{126}{253}\right)\right)} + 2 \times 11 = 22 - 89 \operatorname{csc}\left(1 - 2 \cos^2\left(\frac{63}{253}\right)\right)$$

$$\frac{89}{\sin\left(\cos\left(\frac{126}{253}\right)\right)} + 2 \times 11 = 22 - 89 \operatorname{csc}\left(3 \cos\left(\frac{42}{253}\right) - 4 \cos^3\left(\frac{42}{253}\right)\right)$$

$$27 \cdot \frac{1}{2} \cdot \left( \left( \left( 89 \cdot \frac{1}{\left( \sin \left( \cos \left( \frac{126}{253} \right) \right) \right) \right) \right) + 13 - \frac{1}{\phi} \right) + 1$$

**Input:**

$$27 \times \frac{1}{2} \left( 89 \times \frac{1}{\sin \left( \cos \left( \frac{126}{253} \right) \right)} + 13 - \frac{1}{\phi} \right) + 1$$

$\phi$  is the golden ratio

**Exact result:**

$$\frac{27}{2} \left( -\frac{1}{\phi} + 13 + 89 \operatorname{csc} \left( \cos \left( \frac{126}{253} \right) \right) \right) + 1$$

$\operatorname{csc}(x)$  is the cosecant function

**Decimal approximation:**

1728.950807518694033184476629612768879981325392890991645813...

$$1728.9508... \approx 1729$$

This result is very near to the mass of candidate glueball  **$f_0(1710)$  scalar meson**. Furthermore, 1728 occurs in the algebraic formula for the  $j$ -invariant of an elliptic curve. As a consequence, it is sometimes called a Zagier as a pun on the Gross–Zagier theorem. The number 1728 is one less than the Hardy–Ramanujan number 1729 (taxicab number)

**Alternate forms:**

$$\frac{353}{2} - \frac{27}{1 + \sqrt{5}} + \frac{2403}{2} \operatorname{csc} \left( \cos \left( \frac{126}{253} \right) \right)$$

$$1 + \frac{27}{2} \left( 13 - \frac{2}{1 + \sqrt{5}} + 89 \operatorname{csc} \left( \cos \left( \frac{126}{253} \right) \right) \right)$$

$$1 + \frac{27}{2} \left( \frac{1}{2} (27 - \sqrt{5}) + 89 \operatorname{csc} \left( \cos \left( \frac{126}{253} \right) \right) \right)$$

**Expanded form:**

$$-\frac{27}{2\phi} + \frac{353}{2} + \frac{2403}{2} \operatorname{csc} \left( \cos \left( \frac{126}{253} \right) \right)$$

### Alternative representations:

$$\frac{27}{2} \left( \frac{89}{\sin(\cos(\frac{126}{253}))} + 13 - \frac{1}{\phi} \right) + 1 = 1 + \frac{27}{2} \left( 13 - \frac{1}{\phi} + \frac{89}{\cos(\frac{\pi}{2} - \cos(\frac{126}{253}))} \right)$$

$$\frac{27}{2} \left( \frac{89}{\sin(\cos(\frac{126}{253}))} + 13 - \frac{1}{\phi} \right) + 1 = 1 + \frac{27}{2} \left( 13 - \frac{1}{\phi} + -\frac{89}{\cos(\frac{\pi}{2} + \cos(\frac{126}{253}))} \right)$$

$$\frac{27}{2} \left( \frac{89}{\sin(\cos(\frac{126}{253}))} + 13 - \frac{1}{\phi} \right) + 1 = 1 + \frac{27}{2} \left( 13 - \frac{1}{\phi} + \frac{89}{\frac{-e^{-i \cos(126/253)} + e^{i \cos(126/253)}}{2i}} \right)$$

$i$  is the imaginary unit

### Series representations:

$$\frac{27}{2} \left( \frac{89}{\sin(\cos(\frac{126}{253}))} + 13 - \frac{1}{\phi} \right) + 1 = \frac{353}{2} - \frac{27}{2\phi} - 2403 i \sum_{k=1}^{\infty} q^{-1+2k} \text{ for } q = e^{i \cos(126/253)}$$

$$\frac{27}{2} \left( \frac{89}{\sin(\cos(\frac{126}{253}))} + 13 - \frac{1}{\phi} \right) + 1 = \frac{353}{2} - \frac{27}{2\phi} + \frac{2403}{2} \operatorname{csc} \left( \sum_{k=0}^{\infty} \frac{\left(-\frac{15876}{64009}\right)^k}{(2k)!} \right)$$

$$\frac{27}{2} \left( \frac{89}{\sin(\cos(\frac{126}{253}))} + 13 - \frac{1}{\phi} \right) + 1 = \frac{353}{2} - \frac{27}{2\phi} + \frac{2403}{2} \cos\left(\frac{126}{253}\right) \sum_{k=-\infty}^{\infty} \frac{(-1)^k}{-k^2 \pi^2 + \cos^2\left(\frac{126}{253}\right)}$$

$n!$  is the factorial function

### Integral representation:

$$\frac{27}{2} \left( \frac{89}{\sin(\cos(\frac{126}{253}))} + 13 - \frac{1}{\phi} \right) + 1 = \frac{353}{2} - \frac{27}{2\phi} + \frac{2403}{2\pi} \int_0^{\infty} \frac{t^{\cos(\frac{126}{253})/\pi}}{t+t^2} dt$$

### Multiple-argument formulas:

$$\frac{27}{2} \left( \frac{89}{\sin(\cos(\frac{126}{253}))} + 13 - \frac{1}{\phi} \right) + 1 = 1 + \frac{27}{2} \left( 13 - \frac{1}{\phi} + 89 \operatorname{csc} \left( 1 - 2 \sin^2 \left( \frac{63}{253} \right) \right) \right)$$

$$\frac{27}{2} \left( \frac{89}{\sin\left(\cos\left(\frac{126}{253}\right)\right)} + 13 - \frac{1}{\phi} \right) + 1 = 1 + \frac{27}{2} \left( 13 - \frac{1}{\phi} - 89 \csc\left(1 - 2 \cos^2\left(\frac{63}{253}\right)\right) \right)$$

$$\frac{27}{2} \left( \frac{89}{\sin\left(\cos\left(\frac{126}{253}\right)\right)} + 13 - \frac{1}{\phi} \right) + 1 = 1 + \frac{27}{2} \left( 13 - \frac{1}{\phi} - 89 \csc\left(3 \cos\left(\frac{42}{253}\right) - 4 \cos^3\left(\frac{42}{253}\right)\right) \right)$$

From

$$\hat{\alpha}_{\text{weak}} = \frac{4M}{b}$$

we obtain:

$$(4 * (13.12806e+39)) / (6.82154e+40)$$

$$\frac{4 \times 13.12806 \times 10^{39}}{6.82154 \times 10^{40}}$$

0.769800367658915728706421130712419776179572354629599767794...

From which:

$$\sqrt{\left(\frac{2}{\left(\frac{4 * (13.12806e+39)}{6.82154e+40}\right)}\right)} + (4+2) * 1/10^3$$

**Input interpretation:**

$$\sqrt{\frac{2}{\frac{4 \times 13.12806 \times 10^{39}}{6.82154 \times 10^{40}}}} + (4 + 2) \times \frac{1}{10^3}$$

**Result:**

1.617854888585753438715512443679570132612090516880176433261...

1.61785488858... result that is a very good approximation to the value of the golden ratio 1,618033988749...

And also, as in the previous expression:

$$27 \times \frac{1}{2} \times \left( \left( \left( \left( 89 \times \frac{1}{\left( \left( \left( 4 \times (13.12806 \times 10^{39}) \right) / (6.82154 \times 10^{40}) \right) \right) \right) \right) + 13 - \frac{1}{\phi} \right) \right) + 1$$

**Input interpretation:**

$$27 \times \frac{1}{2} \left( 89 \times \frac{1}{\frac{4 \times 13.12806 \times 10^{39}}{6.82154 \times 10^{40}}} + 13 - \frac{1}{\phi} \right) + 1$$

$\phi$  is the golden ratio

**Result:**

1728.95...

$$1728.95... \approx 1729$$

This result is very near to the mass of candidate glueball  **$f_0(1710)$  scalar meson**. Furthermore, 1728 occurs in the algebraic formula for the  $j$ -invariant of an elliptic curve. As a consequence, it is sometimes called a Zagier as a pun on the Gross–Zagier theorem. The number 1728 is one less than the Hardy–Ramanujan number 1729 (taxicab number)

Now, from

$$2M = 2m^* - i; \text{ for } M_s = 13.12806 \times 10^{39}, \text{ we obtain:}$$

$$2 \times (13.12806 \times 10^{39}) = 2 \times x(-i)$$

**Input interpretation:**

$$2 \times 13.12806 \times 10^{39} = 2 \times x(-i)$$

$i$  is the imaginary unit

**Result:**

$$2.62561 \times 10^{40} = -2ix$$

**Alternate form:**

$$2.62561 \times 10^{40} + 2ix = 0$$

**Complex solution:**

$$x = 13\,128\,059\,999\,999\,999\,398\,713\,729\,449\,103\,615\,590\,400\,i$$

$$13.128059999... \times 10^{39} = m$$



We have that:

**(b) Ellis massless wormhole**

The action is

$$S_{EMS} = \int d^4x \sqrt{-g} [R + 2\partial_\mu \Psi \partial^\mu \Psi], \quad (51)$$

where the kinetic term  $+2\partial_\mu \Psi \partial^\mu \Psi$  is sign reversed here compared to that in action (1) meaning that the field  $\Psi$  represents exotic phantom matter. The Ellis massless solution is given by

$$d\tau^2 = -dt^2 + d\ell^2 + (\ell^2 + m^2) (d\theta^2 + \sin^2 \theta d\varphi^2), \quad (52)$$

$$\Psi = \frac{1}{\sqrt{2}} \left[ \frac{\pi}{2} - 2 \tan^{-1} \left( \frac{\ell}{m} \right) \right], \quad (53)$$

where  $m$  is a constant of integration that can be called the scalar charge proportional to the integrated total energy of the scalar field  $\Psi$ . Under the trans-

and from

$$\Psi = \frac{1}{\sqrt{2}} \left[ \frac{\pi}{2} - 2 \tan^{-1} \left( \frac{\ell}{m} \right) \right],$$

we obtain:

$$1/(\text{sqrt}2) (((\text{Pi}/2 - 2 \tan^{-1} (x/(13.128059999\text{e}+39)))))) = y$$

**Input interpretation:**

$$\frac{1}{\sqrt{2}} \left( \frac{\pi}{2} - 2 \tan^{-1} \left( \frac{x}{13.128059999 \times 10^{39}} \right) \right) = y$$

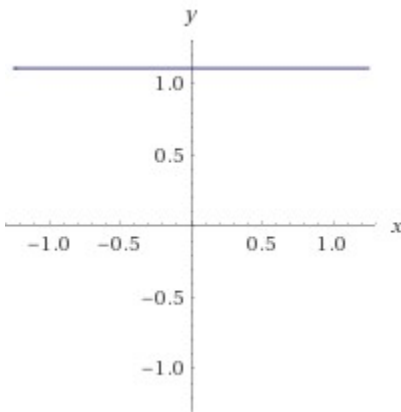
$\tan^{-1}(x)$  is the inverse tangent function

**Result:**

$$\frac{\frac{\pi}{2} - 2 \tan^{-1}(7.617271707 \times 10^{-41} x)}{\sqrt{2}} = y$$

**Geometric figure:**

line

**Implicit plot:****Alternate forms:**

$$\frac{\pi - 4 \tan^{-1}(7.617271707 \times 10^{-41} x)}{2\sqrt{2}} = y$$

$$y = \frac{\pi}{2\sqrt{2}} - \sqrt{2} \tan^{-1}(7.617271707 \times 10^{-41} x)$$

$$-\frac{i \log(1 - 7.617271707 \times 10^{-41} i x)}{\sqrt{2}} + \frac{i \log(1 + 7.617271707 \times 10^{-41} i x)}{\sqrt{2}} + \frac{\pi}{2\sqrt{2}} = y$$

$\log(x)$  is the natural logarithm

**Real solution:**

$$y \approx 0.250000000000000000 (4.442882938158366 - 5.656854249492380 \tan^{-1}(7.617271707138547 \times 10^{-41} x))$$

**Solution:**

$$y \approx 0.500000000000000000 (2.221441469079183 - 2.828427124746190 \tan^{-1}(7.617271707138547 \times 10^{-41} x))$$

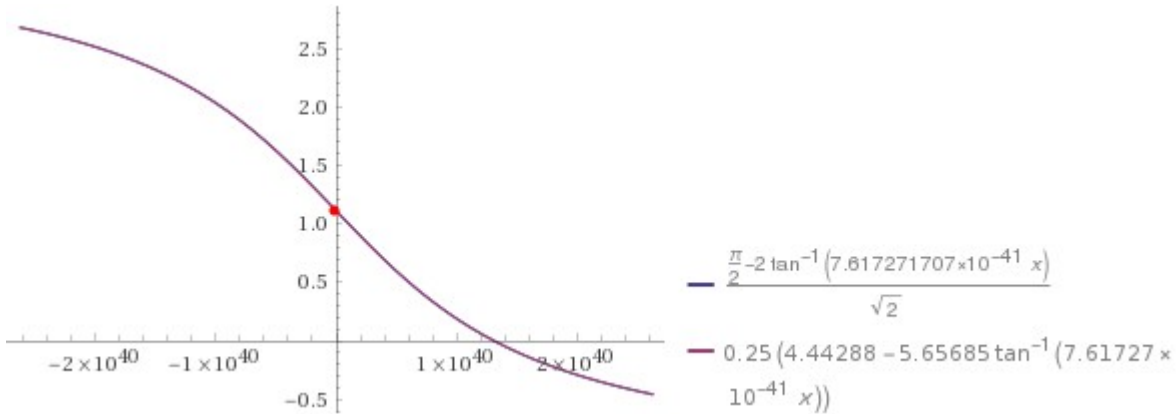
**Partial derivatives:**

$$\frac{\partial}{\partial x} \left( \frac{\frac{\pi}{2} - 2 \tan^{-1}(7.617271707 \times 10^{-41} x)}{\sqrt{2}} \right) = -\frac{1.0772448957 \times 10^{-40}}{5.802282826 \times 10^{-81} x^2 + 1}$$

$$\frac{\partial}{\partial y} \left( \frac{\frac{\pi}{2} - 2 \tan^{-1}(7.617271707 \times 10^{-41} x)}{\sqrt{2}} \right) = 0$$



**Plot:**



**Alternate forms:**

$$\tan^{-1}(7.61727 \times 10^{-41} x) - 1. \tan^{-1}(7.617271707 \times 10^{-41} x) = -5.19398 \times 10^{-7}$$

$$\frac{\pi - 4 \tan^{-1}(7.617271707 \times 10^{-41} x)}{2\sqrt{2}} = 1.11072 - 1.41421 \tan^{-1}(7.61727 \times 10^{-41} x)$$

$$\frac{\pi - 4 \tan^{-1}(7.617271707 \times 10^{-41} x)}{2\sqrt{2}} = -1.41421 (\tan^{-1}(7.61727 \times 10^{-41} x) - 0.785398)$$

**Alternate form assuming x is positive:**

$$\tan^{-1}(7.61727 \times 10^{-41} x) + 5.19398 \times 10^{-7} = 1. \tan^{-1}(7.617271707 \times 10^{-41} x)$$

**Expanded form:**

$$\frac{\pi}{2\sqrt{2}} - \sqrt{2} \tan^{-1}(7.617271707 \times 10^{-41} x) =$$

$$1.11072 - 1.41421 \tan^{-1}(7.61727 \times 10^{-41} x)$$

**Solution:**

$$x = 8\ 184\ 563\ 076\ 539\ 395\ 621\ 065\ 444\ 730\ 981\ 056\ 512\ 000$$

**Numerical solution:**

$$x \approx 8.18456307788921 \times 10^{39} \dots$$

$$8.18456307788921 * 10^{39} = \ell$$

Thence, we obtain:

$$1/(\text{sqrt}2) (((\text{Pi}/2 - 2 \tan^{-1} ((8.18456307788921\text{e}+39)/(13.128059999\text{e}+39))))))$$

**Input interpretation:**

$$\frac{1}{\sqrt{2}} \left( \frac{\pi}{2} - 2 \tan^{-1} \left( \frac{8.18456307788921 \times 10^{39}}{13.128059999 \times 10^{39}} \right) \right)$$

$\tan^{-1}(x)$  is the inverse tangent function

**Result:**

0.3223291657...

(result in radians)

0.3223291657...

from which:

$$5\left(\left(\frac{1}{\sqrt{2}}\right)\left(\left(\frac{\pi}{2} - 2 \tan^{-1}\left(\frac{8.18456307788921 \times 10^{39}}{13.128059999 \times 10^{39}}\right)\right)\right)\right) + (4+2) \times \frac{1}{10^3}$$

**Input interpretation:**

$$5\left(\frac{1}{\sqrt{2}}\left(\frac{\pi}{2} - 2 \tan^{-1}\left(\frac{8.18456307788921 \times 10^{39}}{13.128059999 \times 10^{39}}\right)\right)\right) + (4+2) \times \frac{1}{10^3}$$

$\tan^{-1}(x)$  is the inverse tangent function

**Result:**

1.617645828410589573688047898357437990090610503408837618266...

(result in radians)

1.61764582841..... result that is a very good approximation to the value of the golden ratio 1,618033988749...

and:

$$5\left(\left(\frac{1}{\sqrt{2}}\right)\left(\left(\frac{\pi}{2} - 2 \tan^{-1}\left(\frac{8.18456307788921 \times 10^{39}}{13.128059999 \times 10^{39}}\right)\right)\right)\right) + (4\pi)/10^2 - 5/10^3$$

**Input interpretation:**

$$5\left(\frac{1}{\sqrt{2}}\left(\frac{\pi}{2} - 2 \tan^{-1}\left(\frac{8.18456307788921 \times 10^{39}}{13.128059999 \times 10^{39}}\right)\right)\right) + \frac{4\pi}{10^2} - \frac{5}{10^3}$$

$\tan^{-1}(x)$  is the inverse tangent function

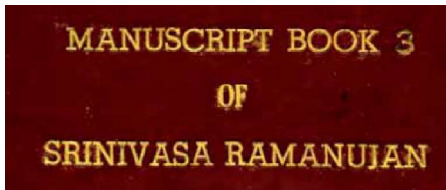
**Result:**

1.732309535...

(result in radians)

1.732309535...  $\approx \sqrt{3}$  that is the ratio between the gravitating mass  $M_0$  and the Wheelerian mass  $q$

From



Page 15

A photograph of a handwritten mathematical equation on aged, yellowish paper. The equation is  $e^{\frac{\pi}{4}\sqrt{78}} = 4\sqrt{3}(75 + 52\sqrt{2})$ . The handwriting is in dark ink and is slightly slanted.

$$4\sqrt{3}(75+52\sqrt{2})$$

**Input:**

$$4\sqrt{3}(75+52\sqrt{2})$$

**Decimal approximation:**

1029.109108769564232483268989990587119611758652119701575130...

1029.1091087695...

**Alternate forms:**

$$4(75\sqrt{3} + 52\sqrt{6})$$

$$300\sqrt{3} + 208\sqrt{6}$$

$$4\sqrt{3(11033 + 7800\sqrt{2})}$$

**Minimal polynomial:**

$$x^4 - 1059168x^2 + 108493056$$

$$\exp(\frac{\pi}{4}\sqrt{78})$$

**Input:**

$$\exp\left(\frac{\pi}{4}\sqrt{78}\right)$$

**Exact result:**

$$e^{1/2 \sqrt{39/2} \pi}$$

**Decimal approximation:**

1029.109108745708701845208873263603669484774707500189796766...

1029.1091087457...

**Property:**

$e^{1/2 \sqrt{39/2} \pi}$  is a transcendental number

**Series representations:**

$$e^{(\sqrt{78} \pi)/4} = e^{1/4 \pi \sqrt{77} \sum_{k=0}^{\infty} 77^{-k} \binom{1/2}{k}}$$

$$e^{(\sqrt{78} \pi)/4} = \exp\left(\frac{1}{4} \pi \sqrt{77} \sum_{k=0}^{\infty} \frac{\left(-\frac{1}{77}\right)^k \binom{-1/2}{k}}{k!}\right)$$

$$e^{(\sqrt{78} \pi)/4} = \exp\left(\frac{\pi \sum_{j=0}^{\infty} \operatorname{Res}_{s=-\frac{1}{2}+j} 77^{-s} \Gamma\left(-\frac{1}{2}-s\right) \Gamma(s)}{8 \sqrt{\pi}}\right)$$

**Integral representation:**

$$(1+z)^a = \frac{\int_{-i\infty+\gamma}^{i\infty+\gamma} \frac{\Gamma(s)\Gamma(-a-s)}{z^s} ds}{(2\pi i)\Gamma(-a)} \quad \text{for } (0 < \gamma < -\operatorname{Re}(a) \text{ and } |\arg(z)| < \pi)$$

From the expression, we obtain also:

$$4x(75+52\sqrt{2}) = \exp(\pi/4 \cdot \sqrt{78}) + 2.38549 \times 10^{-8}$$

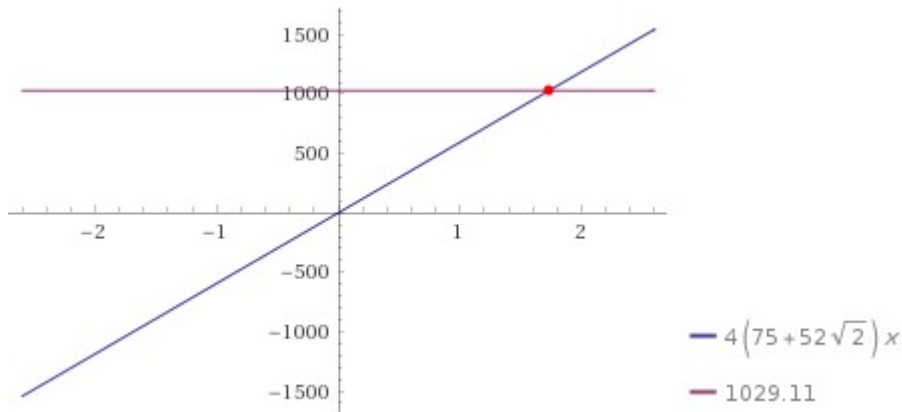
**Input interpretation:**

$$4x\left(75 + 52\sqrt{2}\right) = \exp\left(\frac{\pi}{4} \sqrt{78}\right) + 2.38549 \times 10^{-8}$$

**Result:**

$$4\left(75 + 52\sqrt{2}\right)x = 1029.11$$

**Plot:**



**Alternate forms:**

$$(300 + 208\sqrt{2})x = 1029.11$$

$$208\sqrt{2}x + 300x - 1029.11 = 0$$

**Expanded form:**

$$208\sqrt{2}x + 300x = 1029.11$$

**Solution:**

$$x \approx 1.73205$$

$1.73205 = \sqrt{3}$  that is the ratio between the gravitating mass  $M_0$  and the Wheelerian mass  $q$

Page 29

The image shows a handwritten mathematical expression on aged, yellowed paper. The expression is:  $\frac{5\sqrt{5}}{2\pi\sqrt{3}} = 1 + 12 \cdot \frac{1}{2} \cdot \frac{1 \cdot 5}{6^2} \cdot \left(\frac{4}{125}\right) + \dots$

$$(5\sqrt{5})/(2\pi\sqrt{3}) = 1 + 12/2 * (1*5)/6^2 * (4/125) + \dots$$



$$(5\sqrt{5})/(2\pi\sqrt{3})$$

**Input:**

$$\frac{5\sqrt{5}}{2\pi\sqrt{3}}$$

**Result:**

$$\frac{5\sqrt{\frac{5}{3}}}{2\pi}$$

**Decimal approximation:**

1.027340740102499675941615157239129241668605901250790303864...

1.02734074...

**Property:**

$\frac{5\sqrt{\frac{5}{3}}}{2\pi}$  is a transcendental number

**Alternate form:**

$$\frac{5\sqrt{15}}{6\pi}$$

**Series representations:**

$$\frac{5\sqrt{5}}{2\pi\sqrt{3}} = \frac{5\sqrt{4} \sum_{k=0}^{\infty} 4^{-k} \binom{\frac{1}{2}}{k}}{2\pi\sqrt{2} \sum_{k=0}^{\infty} 2^{-k} \binom{\frac{1}{2}}{k}}$$

$$\frac{5\sqrt{5}}{2\pi\sqrt{3}} = \frac{5\sqrt{4} \sum_{k=0}^{\infty} \frac{(-\frac{1}{4})^k (-\frac{1}{2})_k}{k!}}{2\pi\sqrt{2} \sum_{k=0}^{\infty} \frac{(-\frac{1}{2})^k (-\frac{1}{2})_k}{k!}}$$

$$\frac{5\sqrt{5}}{2\pi\sqrt{3}} = \frac{5 \sum_{k=0}^{\infty} \frac{(-1)^k (-\frac{1}{2})_k (5-z_0)^k z_0^{-k}}{k!}}{2\pi \sum_{k=0}^{\infty} \frac{(-1)^k (-\frac{1}{2})_k (3-z_0)^k z_0^{-k}}{k!}} \text{ for not } ((z_0 \in \mathbb{R} \text{ and } -\infty < z_0 \leq 0))$$



**Solution:**

$$x \approx 1.73205$$

$1.73205 = \sqrt{3}$  that is the ratio between the gravitating mass  $M_0$  and the Wheelerian mass  $q$

And:

$$\left(\frac{5\sqrt{5}}{2\pi\sqrt{3}}\right)^{18} - \frac{7}{10^3}$$

**Input:**

$$\left(\frac{5\sqrt{5}}{2\pi\sqrt{3}}\right)^{18} - \frac{7}{10^3}$$

**Result:**

$$\frac{7450580596923828125}{5159780352\pi^{18}} - \frac{7}{1000}$$

**Decimal approximation:**

1.618029293420459467453322017801847002267380068759590205781...

1.6180292934... result that is a very good approximation to the value of the golden ratio 1,618033988749...

**Alternate forms:**

$$\frac{4514807808\pi^{18} - 931322574615478515625}{644972544000\pi^{18}}$$

$$\frac{931322574615478515625 - 4514807808\pi^{18}}{644972544000\pi^{18}}$$

**Series representations:**

$$\left(\frac{5\sqrt{5}}{2\pi\sqrt{3}}\right)^{18} - \frac{7}{10^3} = -\frac{7}{1000} + \frac{7450580596923828125}{354577405862133891072 \left(\sum_{k=0}^{\infty} \frac{(-1)^k}{1+2k}\right)^{18}}$$

$$\left(\frac{5\sqrt{5}}{2\pi\sqrt{3}}\right)^{18} - \frac{7}{10^3} = -\frac{7}{1000} + \frac{7450580596923828125}{5159780352 \left(\sum_{k=0}^{\infty} \frac{4(-1)^k 1195^{-1-2k} (5^{1+2k} - 4 \times 239^{1+2k})}{1+2k}\right)^{18}}$$

$$\left(\frac{5\sqrt{5}}{2\pi\sqrt{3}}\right)^{18} - \frac{7}{10^3} = -\frac{7}{1000} + \frac{7450580596923828125}{5159780352 \left(\sum_{k=0}^{\infty} \left(-\frac{1}{4}\right)^k \left(\frac{1}{1+2k} + \frac{2}{1+4k} + \frac{1}{3+4k}\right)\right)^{18}}$$

**Integral representations:**

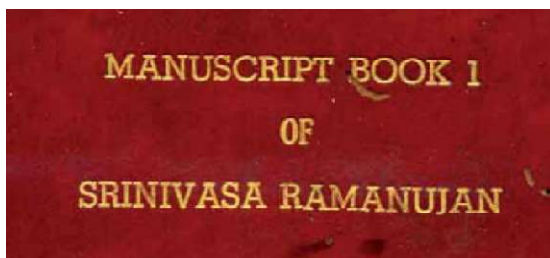
$$\left(\frac{5\sqrt{5}}{2\pi\sqrt{3}}\right)^{18} - \frac{7}{10^3} = -\frac{7}{1000} + \frac{7450580596923828125}{1352605460594688 \left(\int_0^{\infty} \frac{1}{1+t^2} dt\right)^{18}}$$

$$\left(\frac{5\sqrt{5}}{2\pi\sqrt{3}}\right)^{18} - \frac{7}{10^3} = -\frac{7}{1000} + \frac{7450580596923828125}{1352605460594688 \left(\int_0^1 \frac{1}{\sqrt{1-t^2}} dt\right)^{18}}$$

$$\left(\frac{5\sqrt{5}}{2\pi\sqrt{3}}\right)^{18} - \frac{7}{10^3} = -\frac{7}{1000} + \frac{7450580596923828125}{354577405862133891072 \left(\int_0^1 \sqrt{1-t^2} dt\right)^{18}}$$

Now, we have that:

From:



$$\left(\left(\frac{1}{2}(4+\sqrt{7})^{1/2}-(7)^{1/4}\right)\right)^{24} \left(\frac{1}{2}(\sqrt{7}-\sqrt{3})\right)^{12} (2-\sqrt{3})^4 \left(\frac{((3+\sqrt{7})^{1/2}-((6\sqrt{7}))^{1/4})}{((3+\sqrt{7})^{1/2}+((6\sqrt{7}))^{1/4})}\right)^{12}$$

**Input:**

$$\left(\frac{1}{2}\sqrt{4+\sqrt{7}}-\sqrt[4]{7}\right)^{24} \left(\frac{1}{2}(\sqrt{7}-\sqrt{3})\right)^{12} (2-\sqrt{3})^4 \left(\frac{\sqrt{3+\sqrt{7}}-\sqrt[4]{6\sqrt{7}}}{\sqrt{3+\sqrt{7}}+\sqrt[4]{6\sqrt{7}}}\right)^{12}$$

**Exact result:**

$$\frac{(2-\sqrt{3})^4 (\sqrt{7}-\sqrt{3})^{12} \left(\sqrt{3+\sqrt{7}}-\sqrt[4]{6\sqrt{7}}\right)^{12} \left(\frac{\sqrt{4+\sqrt{7}}}{2}-\sqrt[4]{7}\right)^{24}}{4096 \left(\sqrt[4]{6\sqrt{7}}+\sqrt{3+\sqrt{7}}\right)^{12}}$$

**Decimal approximation:**

$$3.8076936653286636541096070702737285701017658195906599... \times 10^{-31}$$

$$3.8076936653... \cdot 10^{-31}$$

From which:

$$\frac{1}{\left(\left(\left(\left(\left(\frac{1}{2}(4+\sqrt{7})^{1/2}-(7)^{1/4}\right)\right)^{24} \left(\frac{1}{2}(\sqrt{7}-\sqrt{3})\right)^{12} (2-\sqrt{3})^4 \left(\frac{((3+\sqrt{7})^{1/2}-((6\sqrt{7}))^{1/4})}{((3+\sqrt{7})^{1/2}+((6\sqrt{7}))^{1/4})}\right)^{12}\right)\right)\right)\right)^{1/128+4/10^3}}$$

**Input:**

$$\frac{1}{128 \sqrt{\left(\frac{1}{2} \sqrt{4+\sqrt{7}} - \sqrt[4]{7}\right)^{24} \left(\frac{1}{2}(\sqrt{7}-\sqrt{3})\right)^{12} (2-\sqrt{3})^4 \left(\frac{\sqrt{3+\sqrt{7}} - \sqrt[4]{6\sqrt{7}}}{\sqrt{3+\sqrt{7}} + \sqrt[4]{6\sqrt{7}}}\right)^{12}}} + \frac{4}{10^3}$$

**Exact result:**

$$\frac{1}{250} + \frac{\left(\frac{2\left(\sqrt[4]{6} \sqrt[8]{7} + \sqrt{3+\sqrt{7}}\right)}{(\sqrt{7}-\sqrt{3})\left(\sqrt{3+\sqrt{7}} - \sqrt[4]{6} \sqrt[8]{7}\right)}\right)^{3/32}}{\sqrt[32]{2-\sqrt{3}} \left(\sqrt[4]{7} - \frac{\sqrt{4+\sqrt{7}}}{2}\right)^{3/16}}$$

**Decimal approximation:**

1.732427144255371201882448621436228313907385480292631271656...

1.732427144...  $\approx \sqrt{3}$  that is the ratio between the gravitating mass  $M_0$  and the Wheelerian mass  $q$

**Alternate forms:**

$$\frac{2^{9/32}}{\sqrt[32]{2-\sqrt{3}} \left(2\sqrt[4]{7} - \sqrt{4+\sqrt{7}}\right)^{3/16}} \left(\frac{(\sqrt{7}-\sqrt{3})\left(\text{root of } x^4 - 6x^2 + 2 \text{ near } x = 2.37608 - \sqrt[4]{6} \sqrt[8]{7}\right)}{\text{root of } x^4 - 6x^2 + 2 \text{ near } x = 2.37608 + \sqrt[4]{6} \sqrt[8]{7}}\right)^{3/32} + \frac{1}{250}$$

$$\frac{250 \times 2^{9/32} \left(\frac{\sqrt[4]{6} \sqrt[8]{7} + \sqrt{3+\sqrt{7}}}{(\sqrt{7}-\sqrt{3})\left(\sqrt{3+\sqrt{7}} - \sqrt[4]{6} \sqrt[8]{7}\right)}\right)^{3/32} + \sqrt[32]{2-\sqrt{3}} \left(2\sqrt[4]{7} - \sqrt{4+\sqrt{7}}\right)^{3/16}}{250 \sqrt[32]{2-\sqrt{3}} \left(2\sqrt[4]{7} - \sqrt{4+\sqrt{7}}\right)^{3/16}}$$

$$\left(250 \left(\frac{2\left(\text{root of } x^4 - 6x^2 + 2 \text{ near } x = 2.37608 + \sqrt[4]{6} \sqrt[8]{7}\right)}{(\sqrt{7}-\sqrt{3})\left(\text{root of } x^4 - 6x^2 + 2 \text{ near } x = 2.37608 - \sqrt[4]{6} \sqrt[8]{7}\right)}\right)^{3/32} + \frac{\sqrt[32]{2-\sqrt{3}} \left(2\sqrt[4]{7} - \sqrt{4+\sqrt{7}}\right)^{3/16}}{2^{3/16}}\right) / \left(125 \times 2^{13/16} \sqrt[32]{2-\sqrt{3}} \left(2\sqrt[4]{7} - \sqrt{4+\sqrt{7}}\right)^{3/16}\right)$$



**Alternate forms:**

$$\begin{aligned}
 & \frac{\sqrt[4]{2}}{\frac{36\sqrt{2-\sqrt{3}} \sqrt[6]{2\sqrt[4]{7}-\sqrt{4+\sqrt{7}}} \sqrt[12]{\frac{(\sqrt{7}-\sqrt{3})\left(\sqrt[10]{x^4-6x^2+2} \text{ near } x=2.37608 - \sqrt[4]{6} \sqrt[8]{7}\right)}}{\sqrt[10]{x^4-6x^2+2} \text{ near } x=2.37608 + \sqrt[4]{6} \sqrt[8]{7}}}}}{\frac{1}{125}} \\
 & \frac{36\sqrt{2-\sqrt{3}} \sqrt[6]{2\sqrt[4]{7}-\sqrt{4+\sqrt{7}}} - 125 \sqrt[4]{2} \sqrt[12]{\frac{\sqrt[4]{6} \sqrt[8]{7} + \sqrt{3+\sqrt{7}}}{(\sqrt{7}-\sqrt{3})\left(\sqrt{3+\sqrt{7}} - \sqrt[4]{6} \sqrt[8]{7}\right)}}}{125 \sqrt[36]{2-\sqrt{3}} \sqrt[6]{2\sqrt[4]{7}-\sqrt{4+\sqrt{7}}}} \\
 & \frac{1}{125 \sqrt[36]{2-\sqrt{3}} \sqrt[6]{2\sqrt[4]{7}-\sqrt{4+\sqrt{7}}}} \sqrt{\frac{2}{2\left(\sqrt[10]{x^4-6x^2+2} \text{ near } x=2.37608 + \sqrt[4]{6} \sqrt[8]{7}\right)}} \\
 & \left( \frac{125 \sqrt[12]{\frac{2\left(\sqrt[10]{x^4-6x^2+2} \text{ near } x=2.37608 - \sqrt[4]{6} \sqrt[8]{7}\right)}{(\sqrt{7}-\sqrt{3})\left(\sqrt[10]{x^4-6x^2+2} \text{ near } x=2.37608 - \sqrt[4]{6} \sqrt[8]{7}\right)}}}{\frac{36\sqrt{2-\sqrt{3}} \sqrt[6]{2\sqrt[4]{7}-\sqrt{4+\sqrt{7}}}}{\sqrt[6]{2}}} \right)
 \end{aligned}$$



## Observations

*From:*

[https://www.scientificamerican.com/article/mathematics-ramanujan/?fbclid=IwAR2caRXrn\\_RpOSvJ1QxWsVLBcJ6KVgd\\_Af\\_hrmDYBNyU8mpSjRs1BDeremA](https://www.scientificamerican.com/article/mathematics-ramanujan/?fbclid=IwAR2caRXrn_RpOSvJ1QxWsVLBcJ6KVgd_Af_hrmDYBNyU8mpSjRs1BDeremA)

*Ramanujan's statement concerned the deceptively simple concept of partitions—the different ways in which a whole number can be subdivided into smaller numbers. Ramanujan's original statement, in fact, stemmed from the observation of patterns, such as the fact that  $p(9) = 30$ ,  $p(9 + 5) = 135$ ,  $p(9 + 10) = 490$ ,  $p(9 + 15) = 1,575$  and so on are all divisible by 5. Note that here the  $n$ 's come at intervals of five units.*

*Ramanujan posited that this pattern should go on forever, and that similar patterns exist when 5 is replaced by 7 or 11—there are infinite sequences of  $p(n)$  that are all divisible by 7 or 11, or, as mathematicians say, in which the "moduli" are 7 or 11.*

*Then, in nearly oracular tone Ramanujan went on: "There appear to be corresponding properties," he wrote in his 1919 paper, "in which the moduli are powers of 5, 7 or 11...and no simple properties for any moduli involving primes other than these three." (Primes are whole numbers that are only divisible by themselves or by 1.) Thus, for instance, there should be formulas for an infinity of  $n$ 's separated by  $5^3 = 125$  units, saying that the corresponding  $p(n)$ 's should all be divisible by 125. In the past methods developed to understand partitions have later been applied to physics problems such as the theory of the strong nuclear force or the entropy of black holes.*

*From Wikipedia*

*In particle physics, Yukawa's interaction or Yukawa coupling, named after Hideki Yukawa, is an interaction between a scalar field  $\phi$  and a Dirac field  $\psi$ . The Yukawa interaction can be used to describe the nuclear force between nucleons (which are fermions), mediated by pions (which are pseudoscalar mesons). The Yukawa interaction is also used in the Standard Model to describe the coupling between the Higgs field and massless quark and lepton fields (i.e., the fundamental fermion particles). Through spontaneous symmetry breaking, these fermions acquire a mass proportional to the vacuum expectation value of the Higgs field.*

Can be this the motivation that from the development of the Ramanujan's equations we obtain results very near to the dilaton mass calculated as a type of *Higgs boson*: *125 GeV* for  $T = 0$  and to the Higgs boson mass *125.18 GeV* and practically equal to the rest mass of *Pion meson* *139.57 MeV*

Note that:

$$g_{22} = \sqrt{(1 + \sqrt{2})}.$$

Hence

$$\begin{aligned} 64g_{22}^{24} &= e^{\pi\sqrt{22}} - 24 + 276e^{-\pi\sqrt{22}} - \dots, \\ 64g_{22}^{-24} &= 4096e^{-\pi\sqrt{22}} + \dots, \end{aligned}$$

so that

$$64(g_{22}^{24} + g_{22}^{-24}) = e^{\pi\sqrt{22}} - 24 + 4372e^{-\pi\sqrt{22}} + \dots = 64\{(1 + \sqrt{2})^{12} + (1 - \sqrt{2})^{12}\}.$$

Hence

$$e^{\pi\sqrt{22}} = 2508951.9982\dots$$

Thence:

$$64g_{22}^{-24} = 4096e^{-\pi\sqrt{22}} + \dots$$

And

$$64(g_{22}^{24} + g_{22}^{-24}) = e^{\pi\sqrt{22}} - 24 + 4372e^{-\pi\sqrt{22}} + \dots = 64\{(1 + \sqrt{2})^{12} + (1 - \sqrt{2})^{12}\}$$

That are connected with 64, 128, 256, 512, 1024 and  $4096 = 64^2$

(Modular equations and approximations to  $\pi$  - S. Ramanujan - Quarterly Journal of Mathematics, XLV, 1914, 350 – 372)

All the results of the most important connections are signed in blue throughout the drafting of the paper. We highlight as in the development of the various equations we use always the constants  $\pi$ ,  $\phi$ ,  $1/\phi$ , the Fibonacci and Lucas numbers, linked to the

golden ratio, that play a fundamental role in the development, and therefore, in the final results of the analyzed expressions.

In mathematics, the Fibonacci numbers, commonly denoted  $F_n$ , form a sequence, called the Fibonacci sequence, such that each number is the sum of the two preceding ones, starting from 0 and 1. Fibonacci numbers are strongly related to the golden ratio: Binet's formula expresses the  $n$ th Fibonacci number in terms of  $n$  and the golden ratio, and implies that the ratio of two consecutive Fibonacci numbers tends to the golden ratio as  $n$  increases.

Fibonacci numbers are also closely related to Lucas numbers, in that the Fibonacci and Lucas numbers form a complementary pair of Lucas sequences

The beginning of the sequence is thus:

0, 1, 1, 2, 3, 5, 8, 13, 21, 34, 55, 89, 144, 233, 377, 610, 987, 1597, 2584, 4181, 6765, 10946, 17711, 28657, 46368, 75025, 121393, 196418, 317811, 514229, 832040, 1346269, 2178309, 3524578, 5702887, 9227465, 14930352, 24157817, 39088169, 63245986, 102334155...

The Lucas numbers or Lucas series are an integer sequence named after the mathematician François Édouard Anatole Lucas (1842–91), who studied both that sequence and the closely related Fibonacci numbers. Lucas numbers and Fibonacci numbers form complementary instances of Lucas sequences.

The Lucas sequence has the same recursive relationship as the Fibonacci sequence, where each term is the sum of the two previous terms, but with different starting values. This produces a sequence where the ratios of successive terms approach the golden ratio, and in fact the terms themselves are roundings of integer powers of the golden ratio.<sup>[1]</sup> The sequence also has a variety of relationships with the Fibonacci numbers, like the fact that adding any two Fibonacci numbers two terms apart in the Fibonacci sequence results in the Lucas number in between.

The sequence of Lucas numbers is:

2, 1, 3, 4, 7, 11, 18, 29, 47, 76, 123, 199, 322, 521, 843, 1364, 2207, 3571, 5778, 9349, 15127, 24476, 39603, 64079, 103682, 167761, 271443, 439204, 710647, 1149851, 1860498, 3010349, 4870847, 7881196, 12752043, 20633239, 33385282, 54018521, 87403803.....

All Fibonacci-like integer sequences appear in shifted form as a row of the Wythoff array; the Fibonacci sequence itself is the first row and the Lucas sequence is the second row. Also like all Fibonacci-like integer sequences, the ratio between two consecutive Lucas numbers converges to the golden ratio.

A Lucas prime is a Lucas number that is prime. The first few Lucas primes are:

2, 3, 7, 11, 29, 47, 199, 521, 2207, 3571, 9349, 3010349, 54018521, 370248451, 6643838879, ...  
(sequence A005479 in the OEIS).

In geometry, a golden spiral is a logarithmic spiral whose growth factor is  $\phi$ , the golden ratio.<sup>[1]</sup> That is, a golden spiral gets wider (or further from its origin) by a factor of  $\phi$  for every

*quarter turn it makes. Approximate logarithmic spirals can occur in nature, for example the arms of spiral galaxies<sup>[3]</sup> - golden spirals are one special case of these logarithmic spirals*

**We note how the following three values: 137.508 (golden angle), 139.57 (mass of the Pion - meson Pi) and 125.18 (mass of the Higgs boson), are connected to each other. In fact, just add 2 to 137.508 to obtain a result very close to the mass of the Pion and subtract 12 to 137.508 to obtain a result that is also very close to the mass of the Higgs boson. We can therefore hypothesize that it is the golden angle (and the related golden ratio inherent in it) to be a fundamental ingredient both in the structures of the microcosm and in those of the macrocosm.**

## References

**Can massless wormholes mimic a Schwarzschild black hole in the strong field lensing?** *Ramil N. Izmailov and Eduard R. Zhdanov† Amrita Bhattacharya,‡ Alexander A. Potapov, K.K. Nandi* - arXiv:1909.13052v1 [gr-qc] 28 Sep 2019

## **LENSING OBSERVABLES: MASSLESS DYONIC vis-a-vis ELLIS WORMHOLE**

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arXiv:1806.05441v1 [gr-qc] 14 Jun 2018

