

# **On some Ramanujan equations: mathematical connections with various formulas concerning some topics of Cosmology and Black Holes/Wormholes Physics. VIII**

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## **Abstract**

*In this paper we have described several Ramanujan's formulas and obtained some mathematical connections with various equations concerning different arguments of Cosmology and Black Holes/Wormholes Physics.*

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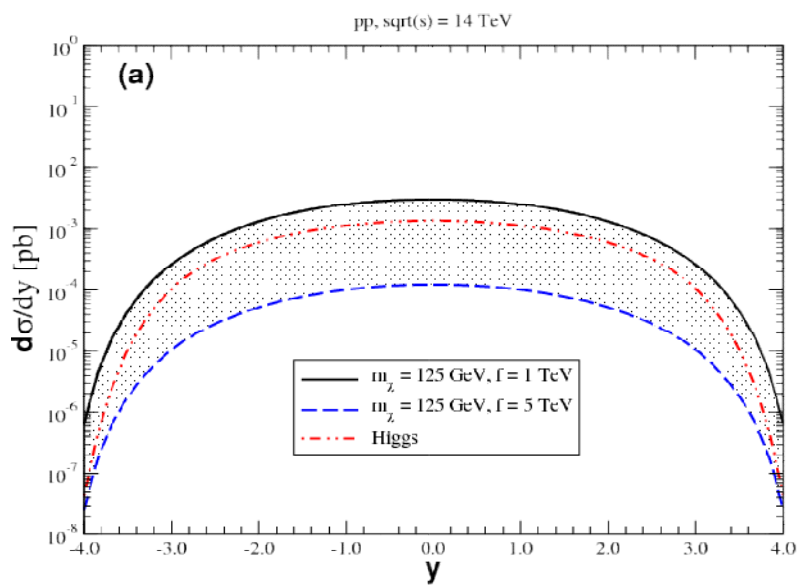
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<https://www.pinterest.it/pin/742319951051634216/?lp=true>



<http://inspirehep.net/record/1341042/plots>



(Color online)

Rapidity distribution for the dilaton production in  $\text{pom}\text{pom}$  interactions considering (a) pp and (b) PbPb collisions at LHC energies. The corresponding predictions for the SM Higgs production are also presented for comparison.

From:

**LENSING OBSERVABLES: MASSLESS DYONIC vis-a-vis ELLIS WORMHOLE**

*R.F. Lukmanova, G.Y. Tuleganova, R.N. Izmailov and K.K. Nandi*  
arXiv:1806.05441v1 [gr-qc] 14 Jun 2018

$$b = 3\sqrt{3}M$$

$$\hat{\alpha}(b) = A_2 \left(\frac{a}{b}\right)^2 = \frac{3\pi PQ}{2b^2} - \frac{\pi\Sigma^2}{4b^2}. \quad (26)$$

$$a^2 = 2PQ, k^2 = \Sigma^2 + a^2$$

$$\Sigma = 0.001$$

$$a = -0.1578947368421$$

$$\text{SMBH87 mass} = M = 13.12806e+39 ; \quad \text{charge} = Q = 1.12496e+15$$

so the invariant deflection angle is

$$\hat{\alpha}(b) = A_2 \left(\frac{a}{b}\right)^2 = \frac{3\pi PQ}{2b^2} - \frac{\pi\Sigma^2}{4b^2}. \quad (26)$$

From

$a^2 = 2PQ$ , we have that:

$$(-0.1578947368421)^2 = 2*(1.12496e+15)*x$$

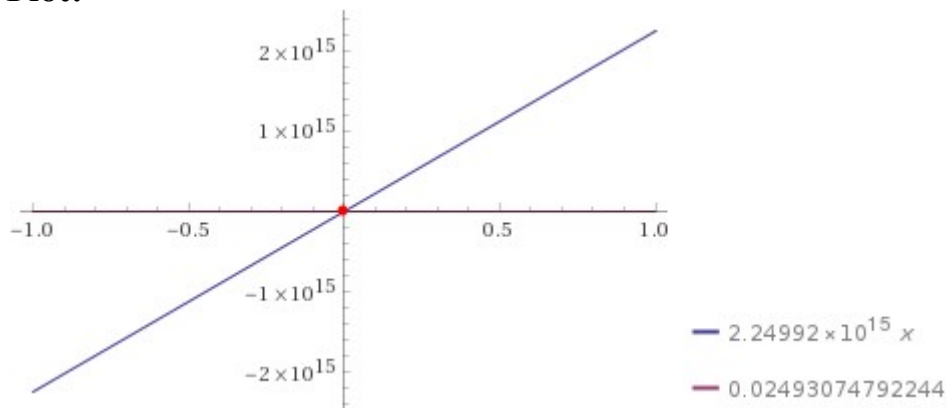
**Input interpretation:**

$$2 \times 1.12496 \times 10^{15} x = (-0.1578947368421)^2$$

**Result:**

$$2.24992 \times 10^{15} x = 0.02493074792244$$

**Plot:**



**Alternate form:**

$$2.24992 \times 10^{15} x - 0.02493074792244 = 0$$

**Alternate form assuming x is real:**

$$2.24992 \times 10^{15} x + 0 = 0.02493074792244$$

**Solution:**

$$x \approx 1.10807 \times 10^{-17}$$

$$P = 1.10807 * 10^{-17}$$

From

$$\hat{\alpha}(b) = A_2 \left( \frac{a}{b} \right)^2 = \frac{3\pi PQ}{2b^2} - \frac{\pi \Sigma^2}{4b^2}$$

we obtain:

$$\frac{(3\pi * 1.10807e-17 * 1.12496e+15) / (2 * (((3\sqrt{3} * 13.12806e+39)^2))) - ((\pi * (0.001)^2)) / (4 * (((3\sqrt{3} * 13.12806e+39)^2)))$$

**Input interpretation:**

$$\frac{3\pi \times 1.10807 \times 10^{-17} \times 1.12496 \times 10^{15}}{2(3\sqrt{3} \times 13.12806 \times 10^{39})^2} - \frac{\pi \times 0.001^2}{4(3\sqrt{3} \times 13.12806 \times 10^{39})^2}$$

**Result:**

$$1.26234... \times 10^{-83}$$

**1.26234 \* 10<sup>-83</sup> that is the invariant deflection angle**

Performing the 4096th root, where  $4096 = 64^2$  and adding

$$\frac{26 + 3\pi}{11(1 + \pi)} \approx 0.7775835601018$$

we obtain:

$$\left( \left( \left( \left( \frac{3\pi \times 1.10807 \times 10^{-17} \times 1.12496 \times 10^{15}}{2 \times ((3\sqrt{3} \times 13.12806 \times 10^{39})^2)} - \frac{\pi \times 0.001^2}{4 \times ((3\sqrt{3} \times 13.12806 \times 10^{39})^2)} \right) - \frac{\pi \times 0.001^2}{4 \times ((3\sqrt{3} \times 13.12806 \times 10^{39})^2)} \right) \right)^{1/4096} + \frac{26 + 3\pi}{11(1 + \pi)} \right)$$

**Input interpretation:**

$$\sqrt[4096]{\frac{3\pi \times 1.10807 \times 10^{-17} \times 1.12496 \times 10^{15}}{2(3\sqrt{3} \times 13.12806 \times 10^{39})^2} - \frac{\pi \times 0.001^2}{4(3\sqrt{3} \times 13.12806 \times 10^{39})^2} + \frac{26 + 3\pi}{11(1 + \pi)}}$$

**Result:**

$$1.73205080...$$

$$\sqrt{3} \approx 1.7320508075688$$

**1.7320508... ≈ √3 that is the ratio between the gravitating mass M<sub>0</sub> and the Wheelerian mass q**

$$M_0 = \sqrt{3q^2 - \Sigma^2},$$

$$q = \frac{(3\sqrt{3}) M_s}{2}.$$

(see: [Can massless wormholes mimic a Schwarzschild black hole in the strong field lensing? - arXiv:1909.13052v1 \[gr-qc\] 28 Sep 2019](#))

and performing the 397th root, where 397 is a prime number and is the sum of 377 + 13 + 7 (377 and 13 are Fibonacci numbers, while 7 is a Lucas number), we obtain:

$$1 + \left( \frac{(3\pi \cdot 1.10807 \times 10^{-17} \cdot 1.12496 \times 10^{15}) / (2 \cdot ((3\sqrt{3} \cdot 13.12806 \times 10^{39})^2)) - (\pi \cdot (0.001)^2)}{4 \cdot ((3\sqrt{3} \cdot 13.12806 \times 10^{39})^2)} \right)^{1/397}$$

**Input interpretation:**

$$1 + \sqrt[397]{\frac{3\pi \times 1.10807 \times 10^{-17} \times 1.12496 \times 10^{15}}{2(3\sqrt{3} \times 13.12806 \times 10^{39})^2} - \frac{\pi \times 0.001^2}{4(3\sqrt{3} \times 13.12806 \times 10^{39})^2}}$$

**Result:**

1.6182823...

1.6182823.... result that is a very good approximation to the value of the golden ratio 1,618033988749...

From:

**Ring-down gravitational waves and lensing observables: How far can a wormhole mimic those of a black hole?**

*Kamal K. Nandi, Ramil N. Izmailov, Almir A. Yanbekov and Azat A. Shayakhmetov - arXiv:1611.03479v2 [gr-qc] 19 May 2017*

We have that

Since  $\bar{a} = 1$ , we can *intuitively* insert the lensing observable  $u_m$  in the equation for  $\omega_{\text{QNM}}$  derived using the eikonal limit WKB approximation for Schwarzschild black hole (for details, see [27]), viz.,

$$\omega_{\text{QNM}} = \left(\frac{1}{u_m}\right) \left[ \left(l + \frac{1}{2}\right) - \frac{1}{3} \left(\frac{5\alpha^2}{12} - \beta + \frac{115}{144}\right) l^{-1} + \frac{1}{6} \left(\frac{5\alpha^2}{12} - \beta + \frac{115}{144}\right) l^{-2} \right] - i\alpha \left(\frac{1}{u_m}\right) \left[ 1 + \frac{1}{9} \left(\frac{235\alpha^2}{432} + \beta - \frac{1415}{1728}\right) l^{-2} \right], \quad (36)$$

where  $\alpha \equiv n + \frac{1}{2}$  and  $\beta = 0, 1, -3$  for scalar, electromagnetic and gravitational perturbations, respectively. We noted that the original expression for  $\omega_{\text{QNM}}$

$$l = 2 \quad (n = 0)$$

For  $u_m = 5.196$   $\alpha = 1/2$ ,  $l = 2$  and  $\beta = -3$ , we obtain:

$$\begin{aligned} & 1/(5.196) [((2+1/2)-1/3((((5*0.5^2)/12)+3+115/144)1/2))] \\ & + 1/6((((5*0.5^2)/12)+3+115/144)1/4] - i*0.5(1/(5.196)) * \\ & [1+1/9((((235*0.5^2)/432)-3-1415/1728)*2^{-1})] \end{aligned}$$

**Input:**

$$\begin{aligned} & \frac{1}{5.196} \\ & \left( \left( \left( 2 + \frac{1}{2} \right) - \frac{1}{3} \left( \left( \frac{1}{12} (5 \times 0.5^2) + 3 + \frac{115}{144} \right) \times \frac{1}{2} \right) \right) + \frac{1}{6} \left( \left( \frac{1}{12} (5 \times 0.5^2) + 3 + \frac{115}{144} \right) \times \frac{1}{4} \right) \right) - \\ & i \times 0.5 \times \frac{1}{5.196} \left( 1 + \frac{1}{9} \left( \frac{1}{2} \left( \frac{1}{432} (235 \times 0.5^2) - 3 - \frac{1415}{1728} \right) \right) \right) \end{aligned}$$

$i$  is the imaginary unit

**Result:**

$$\begin{aligned} & 0.387250... - \\ & 0.0765393... i \end{aligned}$$

**Polar coordinates:**

$$r = 0.394742 \text{ (radius)}, \quad \theta = -11.1803^\circ \text{ (angle)}$$

**0.394742 that is the value of quasinormal mode frequencies**

From:

$$u_m^{EB} = \sqrt{\frac{C(l_{ps})}{A(l_{ps})}} = M \sqrt{\left(4 + \frac{1}{\gamma^2}\right) \exp[2\pi\gamma - 4\gamma \tan^{-1}(2\gamma)]} = D_{OL}\theta_{\infty}. \quad (30)$$

$$13.12806e+39 * \text{sqrt}(\text{((((4+1/(-i)^2)*exp(2Pi*(-i)-4(-i) \tan^{-1}(-2i)))))))$$

**Input interpretation:**

$$13.12806 \times 10^{39} \sqrt{\left(4 + \frac{1}{(-i)^2}\right) \exp(2\pi(-i) - 4(-i) \tan^{-1}(-2i))}$$

$\tan^{-1}(x)$  is the inverse tangent function

$i$  is the imaginary unit

**Result:**

$$6.82154... \times 10^{40}$$

(result in radians)

$$6.82154... * 10^{40}$$

that is the value of minimum impact parameter, also called the radius of the shadow of the lens,

From

$$r_{ps}^{\pm} = \frac{M}{2} \left[ 2 \pm \sqrt{4 + \frac{1}{\gamma^2}} \right].$$

we obtain:

$$(13.12806e+39)/2 * (((2 + \text{sqrt}(4 + 1/(-i)^2))))$$

**Input interpretation:**

$$\frac{13.12806 \times 10^{39}}{2} \left( 2 + \sqrt{4 + \frac{1}{(-i)^2}} \right)$$

$i$  is the imaginary unit



**Result:**

$$2.44973... \times 10^{40}$$

2.44973... \* 10<sup>40</sup> that is radius of the photon sphere for the Ellis-Bronnikov wormhole

and:

$$(13.12806e+39)/2 * (((2 - \sqrt{4 + 1/(-i)^2})))$$

**Input interpretation:**

$$\frac{13.12806 \times 10^{39}}{2} \left( 2 - \sqrt{4 + \frac{1}{(-i)^2}} \right)$$

*i* is the imaginary unit

**Result:**

$$1.75883... \times 10^{39}$$

$$1.75883... * 10^{39}$$

From the ratio between the two previous following expressions, we obtain:

$$\frac{(((13.12806e+39 * \sqrt{((((4 + 1/(-i)^2) * \exp(2\pi * (-i) - 4(-i) \tan^{-1}(-2i)))))))))) / (((13.12806e+39)/2 * ((2 + \sqrt{4 + 1/(-i)^2}))))))$$

**Input interpretation:**

$$\frac{13.12806 \times 10^{39} \sqrt{\left( 4 + \frac{1}{(-i)^2} \right) \exp(2 \pi (-i) - 4 (-i) \tan^{-1}(-2 i))}}{\frac{13.12806 \times 10^{39}}{2} \left( 2 + \sqrt{4 + \frac{1}{(-i)^2}} \right)}$$

$\tan^{-1}(x)$  is the inverse tangent function

*i* is the imaginary unit

**Result:**

2.78461...

(result in radians)

2.78461...

From which:

$$\sqrt{\frac{13.12806 \times 10^{39} \sqrt{\left(4 + \frac{1}{(-i)^2}\right) \exp(2\pi(-i) - 4(-i) \tan^{-1}(-2i))}}{\frac{13.12806 \times 10^{39}}{2} \left(2 + \sqrt{4 + \frac{1}{(-i)^2}}\right)}} - \left(47 + \frac{18}{3+2}\right) \times \frac{1}{10^3}$$

**Input interpretation:**

$$\sqrt{\frac{13.12806 \times 10^{39} \sqrt{\left(4 + \frac{1}{(-i)^2}\right) \exp(2\pi(-i) - 4(-i) \tan^{-1}(-2i))}}{\frac{13.12806 \times 10^{39}}{2} \left(2 + \sqrt{4 + \frac{1}{(-i)^2}}\right)}} - \left(47 + \frac{18}{3+2}\right) \times \frac{1}{10^3}$$

 $\tan^{-1}(x)$  is the inverse tangent function $i$  is the imaginary unit**Result:**

1.61811498190269974046409345342736115928988542395553...

(result in radians)

1.6181149819.... result that is a very good approximation to the value of the golden ratio 1,618033988749...

And:

$$\sqrt{\frac{13.12806 \times 10^{39} \sqrt{\left(4 + \frac{1}{(-i)^2}\right) \exp(2\pi(-i) - 4(-i) \tan^{-1}(-2i))}}{\frac{13.12806 \times 10^{39}}{2} \left(2 + \sqrt{4 + \frac{1}{(-i)^2}}\right)}} + \frac{1}{\pi^2 \phi}$$

**Input interpretation:**

$$\sqrt{\frac{13.12806 \times 10^{39} \sqrt{\left(4 + \frac{1}{(-i)^2}\right) \exp(2\pi(-i) - 4(-i) \tan^{-1}(-2i))}}{\frac{13.12806 \times 10^{39}}{2} \left(2 + \sqrt{4 + \frac{1}{(-i)^2}}\right)}} + \frac{1}{\pi^2 \phi}$$

 $\tan^{-1}(x)$  is the inverse tangent function $i$  is the imaginary unit

**Result:**

1.73133491717403435261996674596330050284024759891737...

(result in radians)

$1.731334917\dots \approx \sqrt{3}$  that is the ratio between the gravitating mass  $M_0$  and the Wheelerian mass  $q$

$$M_0 = \sqrt{3q^2 - \Sigma^2},$$

$$q = \frac{(3\sqrt{3}) M_s}{2}.$$

(see: Can massless wormholes mimic a Schwarzschild black hole in the strong field lensing? - arXiv:1909.13052v1 [gr-qc] 28 Sep 2019)

**Possible closed forms:**

$$\sqrt{3} \approx 1.7320508$$

$$\frac{6}{\log(32)} \approx 1.73123404$$

$$\frac{116}{67} \approx 1.731343283$$

From

$$\tan^{-1}\left(\frac{r}{B}\right) \equiv \frac{\pi}{2} - \tan^{-1}\left(\frac{B}{r}\right)$$

with regard the post-post-Newtonian (PPN) Schwarzschild values,

$$\gamma = -i, \quad B = \frac{M}{2\gamma}$$

$r = 1.94973 \times 10^{13}$ , we obtain:

$$\frac{\pi}{2} - \tan^{-1}\left(\frac{13.12806 \times 10^{39}}{-2i} \times \frac{1}{1.94973 \times 10^{13}}\right)$$

**Input interpretation:**

$$\frac{\pi}{2} - \tan^{-1}\left(-\frac{13.12806 \times 10^{39}}{2i} \times \frac{1}{1.94973 \times 10^{13}}\right)$$

$\tan^{-1}(x)$  is the inverse tangent function

$i$  is the imaginary unit

**Result:**

$$-2.97032... \times 10^{-27} i$$

(result in radians)

**Polar coordinates:**

$$r = 2.97032 \times 10^{-27} \text{ (radius), } \theta = -90^\circ \text{ (angle)}$$

$$2.97032 * 10^{-27}$$

From which:

$$1/[\left(\left(\left(\left(\frac{\pi}{2} - \tan^{-1}\left(\frac{13.12806e+39}{-2i}\right) \times \frac{1}{1.94973e+13}\right)\right)\right)\right)^{1/128} + \left(\frac{7}{10^3}\right)]$$

**Input interpretation:**

$$\frac{1}{\sqrt[128]{\frac{\pi}{2} - \tan^{-1}\left(-\frac{13.12806 \times 10^{39}}{2i} \times \frac{1}{1.94973 \times 10^{13}}\right)}} + \frac{7}{10^3}$$

$\tan^{-1}(x)$  is the inverse tangent function

$i$  is the imaginary unit

**Result:**

$$1.618428... + 0.01977619... i$$

(result in radians)

**Polar coordinates:**

$$r = 1.61855 \text{ (radius), } \theta = 0.700084^\circ \text{ (angle)}$$

1.61855 result that is a very good approximation to the value of the golden ratio 1,618033988749...

And:

$$1/[\left(\left(\left(\left(\frac{\pi}{2} - \tan^{-1}\left(\frac{13.12806e+39}{-2i}\right) \times \frac{1}{1.94973e+13}\right)\right)\right)\right)^{1/128} + \left(\frac{123-2}{10^3}\right)]$$

**Input interpretation:**

$$\frac{1}{\sqrt[128]{\frac{\pi}{2} - \tan^{-1}\left(-\frac{13.12806 \times 10^{39}}{2i} \times \frac{1}{1.94973 \times 10^{13}}\right)}} + \frac{123-2}{10^3}$$

$\tan^{-1}(x)$  is the inverse tangent function

$i$  is the imaginary unit

**Result:**

$$1.732428... + 0.01977619... i$$

(result in radians)

**Polar coordinates:**

$$r = 1.73254 \text{ (radius)}, \quad \theta = 0.65402^\circ \text{ (angle)}$$

$1.73254 \approx \sqrt{3}$  that is the ratio between the gravitating mass  $M_0$  and the Wheelerian mass  $q$

$$M_0 = \sqrt{3q^2 - \Sigma^2},$$
$$q = \frac{(3\sqrt{3}) M_s}{2}.$$

(see: Can massless wormholes mimic a Schwarzschild black hole in the strong field lensing? - arXiv:1909.13052v1 [gr-qc] 28 Sep 2019)

**Possible closed forms:**

$$\sqrt{3} \approx 1.73205080$$

$$\frac{1}{\gamma} \approx 1.73245471$$

$$220 \Omega_U \approx 1.73249933$$

$\gamma$  is the Euler-Mascheroni constant

$\Omega_U$  is Chaitin's constant

From

**Modified gravity black hole lensing observables in weak and strong field of**

**gravity - R.N. Izmailov, R.Kh. Karimov, E.R. Zhdanov and K.K. Nandi**

arXiv:1905.01900v1 [gr-qc] 6 May 2019

$G = G_N(1 + \alpha)$ , where  $G_N$  is the Newtonian gravitational constant,  $\alpha$  is the MOG parameter representing a repulsive Yukawa-like force ( $\alpha > 0$ ). SMOG leads to SGR at  $\alpha = 0$ .

In this paper, it will be first argued that SMOG resembles a regular brane-world BH in the interval  $-1 < \alpha < 0$ , which joins the SMBH interval  $0 < \alpha < 2.47$  [13] across the SGR divide  $\alpha = 0$ . We shall then proceed beyond the

The departure of MOG from GR is characterized by Moffat's postulate [20] that the gravitational "source charge"  $q$  of the vector field  $\phi_\mu$  is proportional to the mass  $M$  of the gravitating source (since compact objects including black holes are generally electrically neutral) so that

$$q = \pm\sqrt{\alpha G_N} M, \quad (1)$$

where  $\alpha (= \frac{G-G_N}{G_N}) > 0$  determines the strength of gravitational vector forces [12]. The positive value ( $\alpha > 0$ ) produces a repulsive gravitational Yukawa-like force. The simplest case of spherical symmetry yields a solution

$$\phi_0 = \alpha C_N M \left( \frac{\exp(-\tilde{\mu}r)}{\tilde{\mu}r} \right), \quad (2)$$

where  $\tilde{\mu}$  is an independent mass parameter. Newtonian gravity is recovered in the weak field limit  $\tilde{\mu}r \ll 1$ . The SMOG metric is given by [9]

$$ds^2 = -A(r)c^2 dt^2 + B(r)dr^2 + C(r)(d\theta^2 + \sin^2\theta\phi^2), \quad (3)$$

$$A(r) = \frac{1}{B(r)} = -\frac{2G_N(1+\alpha)M}{c^2 r} + \frac{G_N^2 M^2(1+\alpha)\alpha}{c^4 r^2}, \quad (4)$$

$$C(r) = r^2. \quad (5)$$

## SMOG = Schwarzschild Modified Gravity

We have that:

$$q = \pm\sqrt{\alpha G_N} M,$$

For SMBH, the interval is  $0 < \alpha < 2.47$ ; we put  $\alpha = \sqrt{3}$ ;  
 $G_N = 6.67408\text{e-}11$ ;  $M = 13.12806\text{e}+39$

we obtain:

$$(((\text{sqrt}3 * 6.67408\text{e-}11)^{1/2})) * 13.12806\text{e}+39$$

### **Input interpretation:**

$$\sqrt{\sqrt{3} \times 6.67408 \times 10^{-11} \times 13.12806 \times 10^{39}}$$

### **Result:**

$$1.41149... \times 10^{35}$$

$1.41149... * 10^{35} = q$  is the gravitational "source charge"

From:

$$\phi_0 = -\alpha G_N M \left( \frac{\exp(-\tilde{\mu}r)}{\tilde{\mu}r} \right),$$

$$\tilde{\mu}r \ll 1 = 1/12$$

We obtain:

$$-\text{sqrt}3*(6.67408\text{e-}11)*(13.12806\text{e+}39)*(\exp(-1/12)/(1/12))$$

$$(-\text{sqrt}3)*(6.67408\text{e-}11)*(13.12806\text{e+}39)*((\exp(-1/12))*12)$$

**Input interpretation:**

$$-\sqrt{3} \times 6.67408 \times 10^{-11} \times 13.12806 \times 10^{39} \left( \exp\left(-\frac{1}{12}\right) \times 12 \right)$$

**Result:**

$$-1.67549... \times 10^{31}$$

**-1.67549... \* 10<sup>31</sup> =  $\phi_0$  is the repulsive gravitational Yukawa-like force.**

From the ratio between q and  $\phi_0$ , we obtain:

$$1/(\text{Pi}+\text{golden ratio}) * -((((\text{sqrt}3*6.67408\text{e-}11)^{1/2}) * 13.12806\text{e+}39)))/((((-\text{sqrt}3)*(6.67408\text{e-}11)*(13.12806\text{e+}39)*((\exp(-1/12))*12))))-47+2+4$$

**Input interpretation:**

$$\frac{1}{\pi + \phi} \times \frac{-\left(\sqrt{\sqrt{3} \times 6.67408 \times 10^{-11} \times 13.12806 \times 10^{39}}\right)}{-\sqrt{3} \times 6.67408 \times 10^{-11} \times 13.12806 \times 10^{39} \left(\exp\left(-\frac{1}{12}\right) \times 12\right)} - 47 + 2 + 4$$

$\phi$  is the golden ratio

**Result:**

$$1728.95...$$

$$1728.95.... \approx 1729$$

This result is very near to the mass of candidate glueball  **$f_0(1710)$  scalar meson**. Furthermore, 1728 occurs in the algebraic formula for the j-invariant of an elliptic curve. As a consequence, it is sometimes called a Zagier as a pun on the Gross–

Zagier theorem. The number 1728 is one less than the Hardy–Ramanujan number 1729 (taxicab number)

We have that:

$$\begin{aligned}
 r_m &= \left(\frac{3}{4}\right) \left[1 + \sqrt{1 - \frac{32Q}{9}}\right] \\
 &= \left(\frac{1}{4}\right) \left[3 + \sqrt{\frac{9 + \alpha}{1 + \alpha}}\right].
 \end{aligned}$$

For: SMBH87 mass = M = 13.12806e+39 ; charge = Q = 1.12496e+15

$(3/4) (1 + \sqrt{1 - (32 * 1.12496e+15)/9})$

**Input interpretation:**

$$\frac{3}{4} \left(1 + \sqrt{1 - \frac{1}{9} (32 \times 1.12496 \times 10^{15})}\right)$$

**Result:**

0.75 +  
4.74333... × 10<sup>7</sup> i

**Polar coordinates:**

$r = 4.74333 \times 10^7$  (radius),  $\theta = 90.^\circ$  (angle)

**4.74333\*10<sup>7</sup> is the scaled radius  $r_m$  of the photon spher**



And:

$$(1/4) (3+\sqrt{(9+3^{0.5})/(1+3^{0.5})})$$

**Input:**

$$\frac{1}{4} \left( 3 + \sqrt{\frac{9 + \sqrt{3}}{1 + \sqrt{3}}} \right)$$

**Decimal approximation:**

1.245492383283758786158113894411311144469126622074656532035...

**1.24549238...**

**Alternate forms:**

$$\frac{1}{4} \left( 3 + \sqrt{4\sqrt{3} - 3} \right)$$

$$\frac{3}{4} + \frac{1}{4} \sqrt{\frac{9 + \sqrt{3}}{1 + \sqrt{3}}}$$

**Minimal polynomial:**

$$8x^4 - 24x^3 + 30x^2 - 18x + 3$$

From which:

$$1 + \frac{1}{2} \left( \left( \left( \left( \frac{1}{4} \right) \left( 3 + \sqrt{\frac{9 + 3^{0.5}}{1 + 3^{0.5}}} \right) \right) \right) \right) - \frac{4}{10^3}$$

**Input:**

$$1 + \frac{1}{2} \left( \frac{1}{4} \left( 3 + \sqrt{\frac{9 + \sqrt{3}}{1 + \sqrt{3}}} \right) \right) - \frac{4}{10^3}$$

**Exact result:**

$$\frac{249}{250} + \frac{1}{8} \left( 3 + \sqrt{\frac{9 + \sqrt{3}}{1 + \sqrt{3}}} \right)$$

**Decimal approximation:**

1.618746191641879393079056947205655572234563311037328266017...

1.61874619.... result that is a very good approximation to the value of the golden ratio 1,618033988749...

**Alternate forms:**

$$\frac{1371}{1000} + \frac{1}{8} \sqrt{4\sqrt{3} - 3}$$

$$\frac{1371 + 125 \sqrt{4\sqrt{3} - 3}}{1000}$$

$$\frac{1371}{1000} + \frac{1}{8} \sqrt{\frac{9 + \sqrt{3}}{1 + \sqrt{3}}}$$

**Minimal polynomial:**

$31\,250\,000\,000\,x^4 - 171\,375\,000\,000\,x^3 + 355\,362\,375\,000\,x^2 - 330\,156\,679\,500\,x + 115\,617\,035\,883$

and:

$$1/2 + (((((1/4) (3 + \sqrt{(9 + 3^{0.5}) / (1 + 3^{0.5})})))) - 13/10^3$$

**Input:**

$$\frac{1}{2} + \frac{1}{4} \left( 3 + \sqrt{\frac{9 + \sqrt{3}}{1 + \sqrt{3}}} \right) - \frac{13}{10^3}$$

**Exact result:**

$$\frac{487}{1000} + \frac{1}{4} \left( 3 + \sqrt{\frac{9 + \sqrt{3}}{1 + \sqrt{3}}} \right)$$

**Decimal approximation:**

1.732492383283758786158113894411311144469126622074656532035...

1.73249238....  $\approx \sqrt{3}$  that is the ratio between the gravitating mass  $M_0$  and the Wheelerian mass  $q$

$$M_0 = \sqrt{3q^2 - \Sigma^2},$$

$$q = \frac{(3\sqrt{3}) M_s}{2}.$$

(see: [Can massless wormholes mimic a Schwarzschild black hole in the strong field lensing?](#) - arXiv:1909.13052v1 [gr-qc] 28 Sep 2019)

**Alternate forms:**

$$\frac{1237}{1000} + \frac{1}{4} \sqrt{4\sqrt{3} - 3}$$

$$\frac{1237 + 250 \sqrt{4\sqrt{3} - 3}}{1000}$$

$$\frac{1237}{1000} + \frac{1}{4} \sqrt{\frac{9 + \sqrt{3}}{1 + \sqrt{3}}}$$

**Minimal polynomial:**

$$1\,000\,000\,000\,000\,x^4 - 4\,948\,000\,000\,000\,x^3 + 9\,556\,014\,000\,000\,x^2 - 8\,499\,026\,212\,000\,x + 2\,762\,886\,793\,561$$

Now, we have the following equation:

$$(3/4) (1 + \sqrt{1 - (32 * 1.12496e+15)/9}) x = (1/4) (3 + \sqrt{(9 + 3^{0.5})/(1 + 3^{0.5})})$$

**Input interpretation:**

$$\frac{3}{4} \left( 1 + \sqrt{1 - \frac{1}{9} (32 \times 1.12496 \times 10^{15})} \right) x = \frac{1}{4} \left( 3 + \sqrt{\frac{9 + \sqrt{3}}{1 + \sqrt{3}}} \right)$$

**Result:**

$$(0.75 + 4.74333 \times 10^7 i) x = \frac{1}{4} \left( 3 + \sqrt{\frac{9 + \sqrt{3}}{1 + \sqrt{3}}} \right)$$

**Alternate forms:**

$$(0.75 + 4.74333 \times 10^7 i) x = \frac{1}{4} \left( 3 + \sqrt{4\sqrt{3} - 3} \right)$$

$$(0.75 + 4.74333 \times 10^7 i) x - \frac{1}{4} \sqrt{\frac{9 + \sqrt{3}}{1 + \sqrt{3}}} - \frac{3}{4} = 0$$

**Expanded form:**

$$(0.75 + 4.74333 \times 10^7 i)x = \frac{3}{4} + \frac{1}{4} \sqrt{\frac{9 + \sqrt{3}}{1 + \sqrt{3}}}$$

**Alternate form assuming x is real:**

$$0.75x + i(4.74333 \times 10^7 x + 0) + 0 = \frac{3}{4} + \frac{1}{4} \sqrt{\frac{9 + \sqrt{3}}{1 + \sqrt{3}}}$$

**Complex solution:**

$$x = 4.15179 \times 10^{-16} - 2.62578 \times 10^{-8} i$$

**Input interpretation:**

$$4.15179 \times 10^{-16} - 2.62578 \times 10^{-8} i$$

$i$  is the imaginary unit

**Result:**

$$4.15179... \times 10^{-16} - 2.62578... \times 10^{-8} i$$

**Polar coordinates:**

$$r = 2.62578 \times 10^{-8} \text{ (radius), } \theta = -90.^\circ \text{ (angle)}$$

$$2.62578 * 10^{-8}$$

from which we obtain:

$$(((3/4) (1 + \sqrt{1 - (32 * 1.12496e+15)/9})) 2.62578e-8$$

**Input interpretation:**

$$\frac{3}{4} \left( 1 + \sqrt{1 - \frac{1}{9} (32 \times 1.12496 \times 10^{15})} \right) \times 2.62578 \times 10^{-8}$$

**Result:**

$$1.96933... \times 10^{-8} + 1.24549... i$$

**Polar coordinates:**

$$r = 1.24549 \text{ (radius), } \theta = 90.^\circ \text{ (angle)}$$

$$1.24549$$

We have the following mathematical connection with the following Ramanujan mock theta function:

$$f(q) = 1 + \frac{q}{(1+q)^2} + \frac{q^4}{(1+q)^2(1+q^2)^2} + \dots$$

$$1 + (0.449329)/(1+0.449329)^2 + (0.449329)^4 / ((1+0.449329)^2(1+0.449329^2)^2)$$

**Input interpretation:**

$$1 + \frac{0.449329}{(1+0.449329)^2} + \frac{0.449329^4}{(1+0.449329)^2(1+0.449329^2)^2}$$

**Result:**

1.227343217712591575927923383010083014681378887610525818831...

**f(q) = 1.22734321771259...**

We note that the result 1.24549 is nearly to the value 1.2273432... and again, we observe:

$$1 + (0.449329)/(1+0.449329)^2 + (0.449329)^4 / ((1+0.449329)^2(1+0.449329^2)^2) + 18/10^3$$

**Input interpretation:**

$$1 + \frac{0.449329}{(1+0.449329)^2} + \frac{0.449329^4}{(1+0.449329)^2(1+0.449329^2)^2} + \frac{18}{10^3}$$

**Result:**

1.245343217712591575927923383010083014681378887610525818831...

**1.245343217...**

Thence, adding  $18/10^3$ , where 18 is a Lucas number, we obtain a result practically equal to the solution of the formula, i.e. **1.24549**

From:

The SMOG (Schwarzschild-Modified Gravity) metric is given by:

$$ds^2 = -A(r)c^2 dt^2 + B(r)dr^2 + C(r) (d\theta^2 + \sin^2 \theta \phi^2),$$

$$A(r) = \frac{1}{B(r)} = -\frac{2G_N(1+\alpha)M}{c^2 r} + \frac{G_N^2 M^2(1+\alpha)\alpha}{c^4 r^2},$$

$$C(r) = r^2.$$

$$A(r) = \frac{1}{B(r)} = -\frac{2G_N(1+\alpha)M}{c^2 r} + \frac{G_N^2 M^2(1+\alpha)\alpha}{c^4 r^2},$$

$$\tilde{\mu}r \ll 1.$$

$$(\alpha > 0)$$

For  $r = 1.94973 \times 10^{13}$ , we obtain:

$$\begin{aligned} &((-2 \times 6.67408 \times 10^{-11} (1 + \sqrt{3}) \times (13.12806 \times 10^{39}) \times 1 / ((9 \times 10^{16}) (1.94973 \times 10^{13})))) + \\ &(((6.67408 \times 10^{-11})^2 \times (13.12806 \times 10^{39})^2 \times (1 + \sqrt{3}) \times \sqrt{3} \times \\ &1 / (((3 \times 10^8)^4 \times (1.94973 \times 10^{13})^2)))) \end{aligned}$$

**Input interpretation:**

$$\begin{aligned} &-2 \times 6.67408 \times 10^{-11} \left( (1 + \sqrt{3}) \times 13.12806 \times 10^{39} \times \frac{1}{9 \times 10^{16} \times 1.94973 \times 10^{13}} \right) + \\ &\frac{(6.67408 \times 10^{-11})^2 (13.12806 \times 10^{39})^2 (1 + \sqrt{3}) \sqrt{3} \times 1}{(3 \times 10^8)^4 (1.94973 \times 10^{13})^2} \end{aligned}$$

**Result:**

$$-1.54853\dots$$

$$-1.54853\dots = A(r)$$

$$\begin{aligned} 1/x = &((((((-2 \times 6.67408 \times 10^{-11} (1 + \sqrt{3}) \times (13.12806 \times 10^{39}) \times 1 / ((9 \times 10^{16}) (1.94973 \times 10^{13})))) + \\ &(((6.67408 \times 10^{-11})^2 \times (13.12806 \times 10^{39})^2 \times (1 + \sqrt{3}) \times \sqrt{3} \times \\ &1 / (((3 \times 10^8)^4 \times (1.94973 \times 10^{13})^2))))))))) \end{aligned}$$

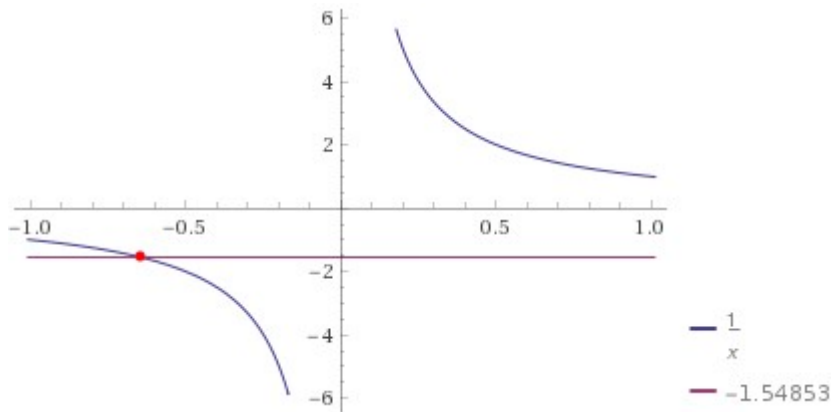
**Input interpretation:**

$$\begin{aligned} \frac{1}{x} = &-2 \times 6.67408 \times 10^{-11} \left( (1 + \sqrt{3}) \times 13.12806 \times 10^{39} \times \frac{1}{9 \times 10^{16} \times 1.94973 \times 10^{13}} \right) + \\ &\frac{(6.67408 \times 10^{-11})^2 (13.12806 \times 10^{39})^2 (1 + \sqrt{3}) \sqrt{3} \times 1}{(3 \times 10^8)^4 (1.94973 \times 10^{13})^2} \end{aligned}$$

**Result:**

$$\frac{1}{x} = -1.54853$$

**Plot:**



**Solution:**

$$x \approx -0.645772$$

$$-0.645772 = B(r)$$

$$(1.94973e+13)^2$$

**Input interpretation:**

$$(1.94973 \times 10^{13})^2$$

**Result:**

380 144 707 290 000 000 000 000 000

**Scientific notation:**

$$3.8014470729 \times 10^{26}$$

$$3.8014470729 * 10^{26} = C(r)$$

Thence, from

$$ds^2 = -A(r)c^2 dt^2 + B(r)dr^2 + C(r) (d\theta^2 + \sin^2 \theta \phi^2)$$

we perform the following calculations (our interpretation):

$$1.54853*(3e+8)^2 - 0.645772 + 3.8014470729e+26 (dx^2+\sin^2 xy^2)$$

for  $x = y = 1$ , we obtain:

$$1.54853*(3e+8)^2 - 0.645772 + 3.8014470729e+26 (\text{derivate}(1)+\sin^2 (1))$$

**Input interpretation:**

$$1.54853 (3 \times 10^8)^2 - 0.645772 + 3.8014470729 \times 10^{26} \times \frac{\partial(1 + \sin^2(1))}{\partial x}$$

**Result:**

$$1.3936769999999999999999999354228 \times 10^{17}$$

$$1.3936769999999999999999999354228 \times 10^{17} = ds^2$$

From which:

$$\text{sqrt}(((1.54853*(3e+8)^2 - 0.645772 + 3.8014470729e+26 (\text{derivate}(1)+\sin^2 (1))))))$$

**Input interpretation:**

$$\sqrt{1.54853 (3 \times 10^8)^2 - 0.645772 + 3.8014470729 \times 10^{26} \times \frac{\partial(1 + \sin^2(1))}{\partial x}}$$

**Result:**

$$3.73319836065537775696886355586935128955076890897718304... \times 10^8$$

$$3.7331983606... * 10^8 = ds$$

and:

$$7\ln(\text{sqrt}(((1.54853*(3e+8)^2 - 0.645772 + 3.8014470729e+26 (\text{derivate}(1)+\sin^2 (1)))))))+\text{golden ratio}$$

**Input interpretation:**

$$7\log\left(\sqrt{1.54853 (3 \times 10^8)^2 - 0.645772 + 3.8014470729 \times 10^{26} \times \frac{\partial(1 + \sin^2(1))}{\partial x}}\right) + \phi$$



log(x) is the natural logarithm

φ is the golden ratio

**Result:**

139.78366...

139.78366... result practically equal to the rest mass of Pion meson 139.57 MeV

And:

$$7\ln(\left(\sqrt{1.54853(3 \times 10^8)^2 - 0.645772 + 3.8014470729e+26} \frac{\partial(1 + \sin^2(1))}{\partial x}\right) - 13)$$

**Input interpretation:**

$$7 \log \left( \sqrt{1.54853 (3 \times 10^8)^2 - 0.645772 + 3.8014470729 \times 10^{26} \times \frac{\partial(1 + \sin^2(1))}{\partial x}} \right) - 13$$

log(x) is the natural logarithm

**Result:**

125.16562...

125.16562... result very near to the Higgs boson mass 125.18 GeV

$$27 \times \frac{1}{2} * \left( \left( 7 \ln \left( \sqrt{1.54853(3e+8)^2 - 0.645772 + 3.801447e+26} \frac{\partial(1 + \sin^2(1))}{\partial x} \right) - 7 - \pi \right) + \frac{1}{\phi} \right)$$

**Input interpretation:**

$$27 \times \frac{1}{2} \left( 7 \log \left( \sqrt{1.54853 (3 \times 10^8)^2 - 0.645772 + 3.801447 \times 10^{26} \times \frac{\partial(1 + \sin^2(1))}{\partial x}} \right) - 7 - \pi \right) + \frac{1}{\phi}$$

log(x) is the natural logarithm

φ is the golden ratio

**Result:**

1728.9424...

1728.9424... ≈ 1729

This result is very near to the mass of candidate glueball **f<sub>0</sub>(1710) scalar meson**. Furthermore, 1728 occurs in the algebraic formula for the j-invariant of an elliptic curve. As a consequence, it is sometimes called a Zagier as a pun on the Gross–Zagier theorem. The number 1728 is one less than the Hardy–Ramanujan number 1729 (taxicab number)

With regard 27 (From Wikipedia):

*“The fundamental group of the complex form, compact real form, or any algebraic version of E<sub>6</sub> is the cyclic group Z/3Z, and its outer automorphism group is the cyclic group Z/2Z. Its fundamental representation is 27-dimensional (complex), and a basis is given by the 27 lines on a cubic surface. The dual representation, which is inequivalent, is also 27-dimensional. In particle physics, E<sub>6</sub> plays a role in some grand unified theories”.*

(((27\*1/2\*(((7ln(((sqrt(((1.54853\*(3e+8)^2 - 0.645772 + 3.801447e+26  
(derivate(1)+sin^2 (1)))))))-7-Pi)))+1/golden ratio))))^1/15 - (21+5)\*1/10^3

**Input interpretation:**

$$\left( 27 \times \frac{1}{2} \left( 7 \log \left( \sqrt{1.54853 (3 \times 10^8)^2 - 0.645772 + 3.801447 \times 10^{26} \times \frac{\partial(1 + \sin^2(1))}{\partial x}} \right) - \left( 7 - \pi \right) + \frac{1}{\phi} \right)^{(1/15) - (21+5) \times \frac{1}{10^3}}$$

log(x) is the natural logarithm

φ is the golden ratio

**Result:**

1.61781158...

1.61781158... result that is a very good approximation to the value of the golden ratio 1,618033988749...

$(((((27 \times \frac{1}{2} \times (((7 \ln(\sqrt{(1.54853 \times (3e+8)^2 - 0.645772 + 3.801447e+26} \times \frac{\partial(1 + \sin^2(1))}{\partial x}})) - 7 - \pi))) + \frac{1}{\phi})))^{(1/13) - (21 \times 2) \times \frac{1}{10^3}}$

**Input interpretation:**

$$\left( 27 \times \frac{1}{2} \left( 7 \log \left[ \sqrt{1.54853 (3 \times 10^8)^2 - 0.645772 + 3.801447 \times 10^{26} \times \frac{\partial(1 + \sin^2(1))}{\partial x}} \right] - 7 - \pi \right) + \frac{1}{\phi} \right)^{(1/13) - (21 \times 2) \times \frac{1}{10^3}}$$

log(x) is the natural logarithm

φ is the golden ratio

**Result:**

1.73243478...

1.73243478... ≈ √3 that is the ratio between the gravitating mass M<sub>0</sub> and the Wheelerian mass q

$$M_0 = \sqrt{3q^2 - \Sigma^2},$$

$$q = \frac{(3\sqrt{3}) M_s}{2}.$$

(see: Can massless wormholes mimic a Schwarzschild black hole in the strong field lensing? - arXiv:1909.13052v1 [gr-qc] 28 Sep 2019)

From:

**Modular equations and approximations to π – Srinivasa Ramanujan**  
 Quarterly Journal of Mathematics, XLV, 1914, 350 – 372

We have:

$$G_{69}^2 = (3\sqrt{3} + \sqrt{23})^{\frac{1}{4}} \left( \frac{5 + \sqrt{23}}{4} \right)^{\frac{1}{6}} \left\{ \sqrt{\left( \frac{6 + 3\sqrt{3}}{4} \right)} + \sqrt{\left( \frac{2 + 3\sqrt{3}}{4} \right)} \right\},$$

$$(3\sqrt{3}+\sqrt{23})^{1/4} (1/4*5+\sqrt{23})^{(1/6)} [(1/4*6+3\sqrt{3})^{1/2} + (1/4*2+3\sqrt{3})^{1/2}]$$

**Input:**

$$\sqrt[4]{3\sqrt{3} + \sqrt{23}} \sqrt[6]{\frac{1}{4} \times 5 + \sqrt{23}} \left( \sqrt{\frac{1}{4} \times 6 + 3\sqrt{3}} + \sqrt{\frac{1}{4} \times 2 + 3\sqrt{3}} \right)$$

**Result:**

$$\sqrt[6]{\frac{5}{4} + \sqrt{23}} \sqrt[4]{3\sqrt{3} + \sqrt{23}} \left( \sqrt{\frac{1}{2} + 3\sqrt{3}} + \sqrt{\frac{3}{2} + 3\sqrt{3}} \right)$$

**Decimal approximation:**

11.93691992114149294736755866054151531884005922464082188394...

11.9369199.... result very near to the black hole entropy 11.8458 that is equal to  $\ln(139503)$

**Alternate forms:**

$$\frac{1}{2} \sqrt[4]{3\sqrt{3} + \sqrt{23}} \sqrt[6]{(5+4\sqrt{23})^2} \left( \sqrt{3(1+2\sqrt{3})} + \sqrt{1+6\sqrt{3}} \right)$$

$$\frac{\sqrt[4]{3\sqrt{3} + \sqrt{23}} \sqrt[6]{5+4\sqrt{23}} \left( \sqrt{3(1+2\sqrt{3})} + \sqrt{1+6\sqrt{3}} \right)}{2^{5/6}}$$

root of

$$256 x^8 - 2072390163636224 x^7 - 587898490908813667488288512 x^6 + 78728286687820879950394915658328064 x^5 + 558873562806062937828135138558332034276448 x^4 + 2315576050133859676321002858071659600384 x^3 - 508579491106461092118719849908808432 x^2 - 52729817541266994928596320384 x + 191581231380566414401 \text{ near } x = 8.36965 \times 10^{12}$$

**Minimal polynomial:**

$$256 x^{96} - 2072390163636224 x^{84} - 587898490908813667488288512 x^{72} + 78728286687820879950394915658328064 x^{60} + 558873562806062937828135138558332034276448 x^{48} + 2315576050133859676321002858071659600384 x^{36} - 508579491106461092118719849908808432 x^{24} - 52729817541266994928596320384 x^{12} + 191581231380566414401$$

and:

$$\left( \left( \left( \left( \left( 3\sqrt{3} + \sqrt{23} \right)^{1/4} \left( \frac{1}{4} \cdot 5 + \sqrt{23} \right)^{1/6} \left[ \left( \frac{1}{4} \cdot 6 + 3\sqrt{3} \right)^{1/2} + \left( \frac{1}{4} \cdot 2 + 3\sqrt{3} \right)^{1/2} \right] \right) \right)^{1/5} + 3^2 \cdot \frac{1}{10^2} \right)$$

**Input:**

$$\sqrt[5]{\sqrt[4]{3\sqrt{3} + \sqrt{23}} \sqrt[6]{\frac{1}{4} \times 5 + \sqrt{23}} \left( \sqrt{\frac{1}{4} \times 6 + 3\sqrt{3}} + \sqrt{\frac{1}{4} \times 2 + 3\sqrt{3}} \right) + 3^2 \times \frac{1}{10^2}}$$

**Exact result:**

$$\frac{9}{100} + \sqrt[30]{\frac{5}{4} + \sqrt{23}} \sqrt[20]{3\sqrt{3} + \sqrt{23}} \sqrt[5]{\sqrt{\frac{1}{2} + 3\sqrt{3}} + \sqrt{\frac{3}{2} + 3\sqrt{3}}}$$

**Decimal approximation:**

1.732020051070983847057986798759579546057133418314105638719...

1.732020051...  $\approx \sqrt{3}$  that is the ratio between the gravitating mass  $M_0$  and the Wheelerian mass  $q$

$$M_0 = \sqrt{3q^2 - \Sigma^2},$$

$$q = \frac{(3\sqrt{3}) M_s}{2}.$$

(see: [Can massless wormholes mimic a Schwarzschild black hole in the strong field lensing? - arXiv:1909.13052v1 \[gr-qc\] 28 Sep 2019](#))

**Alternate forms:**

$$\frac{9}{100} + \frac{\sqrt[20]{3\sqrt{3} + \sqrt{23}} \sqrt[30]{5 + 4\sqrt{23}} \sqrt[5]{\sqrt{3(1 + 2\sqrt{3})} + \sqrt{1 + 6\sqrt{3}}}}{\sqrt[6]{2}}$$

$$\frac{1}{100} \left( 9 + 50 \times 2^{5/6} \sqrt[20]{3\sqrt{3} + \sqrt{23}} \sqrt[30]{5 + 4\sqrt{23}} \sqrt[5]{\sqrt{3(1 + 2\sqrt{3})} + \sqrt{1 + 6\sqrt{3}}} \right)$$

$$\frac{1}{100 \times 2^{4/15}} \left( 9 \times 2^{4/15} + 100 \sqrt[20]{3\sqrt{3} + \sqrt{23}} \sqrt[30]{5 + 4\sqrt{23}} \left( \sqrt{3 - 3i\sqrt{11}} + \sqrt{1 - i\sqrt{107}} + 2 \left( \frac{1}{2} \sqrt{3i(\sqrt{11} + -i)} + \frac{1}{2} \sqrt{i(\sqrt{107} + -i)} \right) \right) \right)^{(1/5)}$$

**Possible closed forms:**

$$\sqrt{3} \approx 1.732050807$$

$$\sqrt[5]{e \pi \cos^2(e \pi)} \approx 1.732001437$$

$$\sqrt{4 + 4e - 4\pi + \log(2)} \approx 1.7320230599$$

$$\left( \left( \left( \left( 3\sqrt{3} + \sqrt{23} \right)^{1/4} \left( \frac{1}{4} \cdot 5 + \sqrt{23} \right)^{1/6} \left[ \left( \frac{1}{4} \cdot 6 + 3\sqrt{3} \right)^{1/2} + \left( \frac{1}{4} \cdot 2 + 3\sqrt{3} \right)^{1/2} \right] \right)^{1/5} - \frac{21+3}{10^3} \right) \right)$$

**Input:**

$$\sqrt[5]{\sqrt[4]{3\sqrt{3} + \sqrt{23}} \sqrt[6]{\frac{1}{4} \times 5 + \sqrt{23}} \left( \sqrt{\frac{1}{4} \times 6 + 3\sqrt{3}} + \sqrt{\frac{1}{4} \times 2 + 3\sqrt{3}} \right) - \frac{21+3}{10^3}}$$

**Exact result:**

$$\sqrt[30]{\frac{5}{4} + \sqrt{23}} \sqrt[20]{3\sqrt{3} + \sqrt{23}} \sqrt[5]{\sqrt{\frac{1}{2} + 3\sqrt{3}} + \sqrt{\frac{3}{2} + 3\sqrt{3}}} - \frac{3}{125}$$

**Decimal approximation:**

1.618020051070983847057986798759579546057133418314105638719...

1.61802005107.... result that is a very good approximation to the value of the golden ratio 1,618033988749...

**Alternate forms:**

$$\frac{\sqrt[20]{3\sqrt{3} + \sqrt{23}} \sqrt[30]{5 + 4\sqrt{23}} \sqrt[5]{\sqrt{3(1+2\sqrt{3})} + \sqrt{1+6\sqrt{3}}}}{\sqrt[6]{2}} - \frac{3}{125}$$

$$\frac{1}{250} \left( 125 \times 2^{5/6} \sqrt[20]{3\sqrt{3} + \sqrt{23}} \sqrt[30]{5 + 4\sqrt{23}} \sqrt[5]{\sqrt{3(1+2\sqrt{3})} + \sqrt{1+6\sqrt{3}}} - 6 \right)$$

$$\frac{1}{125 \times 2^{4/15}} \left( -3 \times 2^{4/15} + 125 \sqrt[20]{3\sqrt{3} + \sqrt{23}} \sqrt[30]{5 + 4\sqrt{23}} \left( \sqrt{3 - 3i\sqrt{11}} + \sqrt{1 - i\sqrt{107}} + 2 \left( \frac{1}{2} \sqrt{3i(\sqrt{11} + -i)} + \frac{1}{2} \sqrt{i(\sqrt{107} + -i)} \right) \right) \right)^{(1/5)}$$

$$\left( \left( \left( \left( 3\sqrt{3} + \sqrt{23} \right)^{1/4} \left( \frac{1}{4} \times 5 + \sqrt{23} \right)^{1/6} \left[ \left( \frac{1}{4} \times 6 + 3\sqrt{3} \right)^{1/2} + \left( \frac{1}{4} \times 2 + 3\sqrt{3} \right)^{1/2} \right] \right) \right)^{1/5} \right)$$

**Input:**

$$\sqrt[5]{\sqrt[4]{3\sqrt{3} + \sqrt{23}} \sqrt[6]{\frac{1}{4} \times 5 + \sqrt{23}} \left( \sqrt{\frac{1}{4} \times 6 + 3\sqrt{3}} + \sqrt{\frac{1}{4} \times 2 + 3\sqrt{3}} \right)}$$

**Exact result:**

$$\sqrt[30]{\frac{5}{4} + \sqrt{23}} \sqrt[20]{3\sqrt{3} + \sqrt{23}} \sqrt[5]{\sqrt{\frac{1}{2} + 3\sqrt{3}} + \sqrt{\frac{3}{2} + 3\sqrt{3}}}$$

**Decimal approximation:**

1.642020051070983847057986798759579546057133418314105638719...

$$1.64202005107\dots \approx \zeta(2) = \frac{\pi^2}{6} = 1.644934 \dots$$

**Alternate forms:**

$$\frac{\sqrt[20]{3\sqrt{3} + \sqrt{23}} \sqrt[30]{5 + 4\sqrt{23}} \sqrt[5]{\sqrt{3(1 + 2\sqrt{3})} + \sqrt{1 + 6\sqrt{3}}}}{\sqrt[6]{2}}$$

$$\frac{1}{2^{4/15}} \sqrt[20]{3\sqrt{3} + \sqrt{23}} \sqrt[30]{5 + 4\sqrt{23}} \sqrt[5]{\sqrt{3 - 3i\sqrt{11}} + \sqrt{1 - i\sqrt{107}} + 2 \left( \frac{1}{2} \sqrt{3i(\sqrt{11} + -i)} + \frac{1}{2} \sqrt{i(\sqrt{107} + -i)} \right)}$$

**All 5th roots of  $(\frac{5}{4} + \sqrt{23})^{1/6} (3\sqrt{3} + \sqrt{23})^{1/4} (\sqrt{1/2 + 3\sqrt{3}} + \sqrt{3/2 + 3\sqrt{3}})$ :**

$$\sqrt[30]{\frac{5}{4} + \sqrt{23}} \sqrt[20]{3\sqrt{3} + \sqrt{23}} \sqrt[5]{\sqrt{\frac{1}{2} + 3\sqrt{3}} + \sqrt{\frac{3}{2} + 3\sqrt{3}}} e^0 \approx 1.64202$$

(real, principal root)

$$\sqrt[30]{\frac{5}{4} + \sqrt{23}} \sqrt[20]{3\sqrt{3} + \sqrt{23}} \sqrt[5]{\sqrt{\frac{1}{2} + 3\sqrt{3}} + \sqrt{\frac{3}{2} + 3\sqrt{3}}} e^{(2i\pi)/5}$$

$$\approx 0.5074 + 1.5617i$$

$$\sqrt[30]{\frac{5}{4} + \sqrt{23}} \sqrt[20]{3\sqrt{3} + \sqrt{23}} \sqrt[5]{\sqrt{\frac{1}{2} + 3\sqrt{3}} + \sqrt{\frac{3}{2} + 3\sqrt{3}}} e^{(4i\pi)/5}$$

$$\approx -1.3284 + 0.9652i$$

$$\sqrt[30]{\frac{5}{4} + \sqrt{23}} \sqrt[20]{3\sqrt{3} + \sqrt{23}} \sqrt[5]{\sqrt{\frac{1}{2} + 3\sqrt{3}} + \sqrt{\frac{3}{2} + 3\sqrt{3}}} e^{-(4i\pi)/5}$$

$$\approx -1.3284 - 0.9652i$$

$$\sqrt[30]{\frac{5}{4} + \sqrt{23}} \sqrt[20]{3\sqrt{3} + \sqrt{23}} \sqrt[5]{\sqrt{\frac{1}{2} + 3\sqrt{3}} + \sqrt{\frac{3}{2} + 3\sqrt{3}}} e^{-(2i\pi)/5}$$

$$\approx 0.5074 - 1.5617i$$

Now, we have that:

$$G_{225} = \left( \frac{1 + \sqrt{5}}{4} \right) (2 + \sqrt{3})^{1/3} \{ \sqrt{(4 + \sqrt{15})} + 15^{1/4} \},$$

$$(1/4 * 1 + \sqrt{5}) (2 + \sqrt{3})^{1/3} (((4 + \sqrt{15})^{1/2} + 15^{1/4}))$$

**Input:**

$$\left( \frac{1}{4} \times 1 + \sqrt{5} \right) \sqrt[3]{2 + \sqrt{3}} \left( \sqrt{4 + \sqrt{15}} + \sqrt[4]{15} \right)$$

**Result:**

$$\sqrt[3]{2 + \sqrt{3}} \left( \frac{1}{4} + \sqrt{5} \right) \left( \sqrt[4]{15} + \sqrt{4 + \sqrt{15}} \right)$$



**Decimal approximation:**

18.40912205633178599323252650649463136656340256527495025167...

18.40912205633.... result very near to the black hole entropy 18.2773, that is equal to  $\ln(86645620)$

**Alternate forms:**

$$\frac{1}{24} (1+4\sqrt{5}) \sqrt[3]{2+\sqrt{3}} (3\sqrt{6} + 3\sqrt{10} + 6\sqrt[4]{15})$$

$$\frac{1}{4} \sqrt[4]{15} \sqrt[3]{2+\sqrt{3}} (1+4\sqrt{5}) + \frac{1}{4} \sqrt[3]{2+\sqrt{3}} (1+4\sqrt{5}) \sqrt{4+\sqrt{15}}$$

root of  $281474976710656x^8 -$   
 $10955709851389252337664x^7 + 2303992981906805814722560x^6 -$   
 $3086366580167815979587438903296x^5 +$   
 $1710410284704387675433353158143770624x^4 -$   
 $183168212592296105384197899984197124096x^3 +$   
 $8114959401324143640650624174862261698560x^2 -$   
 $2290067699804708236518788871661455323623179264x +$   
 $3491806676006468166913468087133050334443236481$

$\sqrt[6]{\text{near } x = 3.89223 \times 10^7}$

**Minimal polynomial:**

$$281474976710656x^{48} - 10955709851389252337664x^{42} +$$

$$2303992981906805814722560x^{36} -$$

$$3086366580167815979587438903296x^{30} +$$

$$1710410284704387675433353158143770624x^{24} -$$

$$183168212592296105384197899984197124096x^{18} +$$

$$8114959401324143640650624174862261698560x^{12} -$$

$$2290067699804708236518788871661455323623179264x^6 +$$

$$3491806676006468166913468087133050334443236481$$

$$\left( \left( \left( \left( \frac{1}{4} + \sqrt{5} \right) (2 + \sqrt{3}) \right)^{1/3} \left( \left( (4 + \sqrt{15})^{1/2} + 15^{1/4} \right) \right) \right) \right)^{1/6} - 7/10^3$$

**Input:**

$$\sqrt[6]{\left(\frac{1}{4} + \sqrt{5}\right) \sqrt[3]{2 + \sqrt{3}} \left(\sqrt{4 + \sqrt{15}} + \sqrt[4]{15}\right)} - \frac{7}{10^3}$$

**Exact result:**

$$\sqrt[18]{2 + \sqrt{3}} \sqrt[6]{\left(\frac{1}{4} + \sqrt{5}\right) \left(\sqrt[4]{15} + \sqrt{4 + \sqrt{15}}\right)} - \frac{7}{1000}$$

**Decimal approximation:**

1.617945674722075405155800494348777121511087719574135239711...

1.617945674722.... result that is a very good approximation to the value of the golden ratio 1,618033988749...

**Alternate forms:**

$$\frac{\sqrt[18]{2+\sqrt{3}} \sqrt[6]{(1+4\sqrt{5})\left(\sqrt[4]{15}+\sqrt{4+\sqrt{15}}\right)}}{\sqrt[3]{2}} - \frac{7}{1000}$$

$$\frac{500 \times 2^{2/3} \sqrt[18]{2+\sqrt{3}} \sqrt[6]{(1+4\sqrt{5})\left(\sqrt[4]{15}+\sqrt{4+\sqrt{15}}\right)} - 7}{1000}$$

$$\frac{1000 \sqrt[18]{2+\sqrt{3}} \sqrt[6]{(1+4\sqrt{5})\left(\sqrt[4]{15}+\sqrt{4+\sqrt{15}}\right)} - 7 \sqrt[3]{2}}{1000 \sqrt[3]{2}}$$

$$\left(\left(\left(\left(\frac{1}{4} \times 1 + \sqrt{5}\right) (2 + \sqrt{3})^{1/3} \left(\left(\left(4 + \sqrt{15}\right)^{1/2} + 15^{1/4}\right)\right)\right)\right)^{1/5} - (47 + 11) \frac{1}{10^3}\right)$$

**Input:**

$$\sqrt[5]{\left(\frac{1}{4} \times 1 + \sqrt{5}\right) \sqrt[3]{2 + \sqrt{3}} \left(\sqrt{4 + \sqrt{15}} + \sqrt[4]{15}\right) - (47 + 11) \times \frac{1}{10^3}}$$

**Result:**

$$\sqrt[15]{2 + \sqrt{3}} \sqrt[5]{\left(\frac{1}{4} + \sqrt{5}\right) \left(\sqrt[4]{15} + \sqrt{4 + \sqrt{15}}\right) - \frac{29}{500}}$$

**Decimal approximation:**

1.732633129846613565440747195665205400775614399847459811854...

1.73263....  $\approx \sqrt{3}$  that is the ratio between the gravitating mass  $M_0$  and the Wheelerian mass  $q$

$$M_0 = \sqrt{3q^2 - \Sigma^2},$$

$$q = \frac{(3\sqrt{3}) M_s}{2}.$$

(see: Can massless wormholes mimic a Schwarzschild black hole in the strong field lensing? - arXiv:1909.13052v1 [gr-qc] 28 Sep 2019)

**Alternate forms:**

$$\frac{250 \times 2^{2/5} \times 3^{4/5} \sqrt[15]{2 + \sqrt{3}} \sqrt[5]{(1 + 4\sqrt{5}) (3\sqrt{6} + 3\sqrt{10} + 6\sqrt[4]{15})} - 87}{1500}$$

$$\frac{\sqrt[15]{2 + \sqrt{3}} \sqrt[5]{(1 + 4\sqrt{5}) (\sqrt[4]{15} + \sqrt{4 + \sqrt{15}})}}{2^{2/5}} - \frac{29}{500}$$

$$\frac{500 \sqrt[15]{2 + \sqrt{3}} \sqrt[5]{(1 + 4\sqrt{5}) (\sqrt[4]{15} + \sqrt{4 + \sqrt{15}})}}{500 \times 2^{2/5}} - 29 \times 2^{2/5}$$

**Possible closed forms:**

$$\sqrt{3} \approx 1.73205080$$

$$\frac{4 \sqrt{\log(4)}}{e} \approx 1.73257976$$

$$\frac{-1 + e + \log(4)}{\log(6)} \approx 1.732696962$$

Now, we have that:

$$\frac{5\sqrt{5}}{2\pi\sqrt{3}} = 1 + 12 \frac{1}{2} \frac{1}{6} \frac{5}{6} \left(\frac{4}{125}\right) + 23 \frac{1 \cdot 3}{2 \cdot 4} \frac{1 \cdot 7}{6 \cdot 12} \frac{5 \cdot 11}{6 \cdot 12} \left(\frac{4}{125}\right)^2 + \dots,$$

$$(5\sqrt{5})/(2\pi\sqrt{3})$$

**Input:**

$$\frac{5\sqrt{5}}{2\pi\sqrt{3}}$$

**Result:**

$$\frac{5 \sqrt{\frac{5}{3}}}{2 \pi}$$

**Decimal approximation:**

1.027340740102499675941615157239129241668605901250790303864...

1.02734074010.....

**Property:**

$\frac{5 \sqrt{\frac{5}{3}}}{2 \pi}$  is a transcendental number

**Alternate form:**

$$\frac{5 \sqrt{15}}{6 \pi}$$

**Series representations:**

$$\frac{5 \sqrt{5}}{2 \pi \sqrt{3}} = \frac{5 \sqrt{4} \sum_{k=0}^{\infty} 4^{-k} \binom{\frac{1}{2}}{k}}{2 \pi \sqrt{2} \sum_{k=0}^{\infty} 2^{-k} \binom{\frac{1}{2}}{k}}$$

$$\frac{5 \sqrt{5}}{2 \pi \sqrt{3}} = \frac{5 \sqrt{4} \sum_{k=0}^{\infty} \frac{(-\frac{1}{4})^k (-\frac{1}{2})_k}{k!}}{2 \pi \sqrt{2} \sum_{k=0}^{\infty} \frac{(-\frac{1}{2})^k (-\frac{1}{2})_k}{k!}}$$

$$\frac{5 \sqrt{5}}{2 \pi \sqrt{3}} = \frac{5 \sum_{k=0}^{\infty} \frac{(-1)^k (-\frac{1}{2})_k (5-z_0)^k z_0^{-k}}{k!}}{2 \pi \sum_{k=0}^{\infty} \frac{(-1)^k (-\frac{1}{2})_k (3-z_0)^k z_0^{-k}}{k!}} \text{ for (not } (z_0 \in \mathbb{R} \text{ and } -\infty < z_0 \leq 0))$$

$$1+12*(1/2)*(1/6)*(5/6)*(4/125)+23*(1*3)/(2*4)*(1*7)/(6*12)*(5*11)/(6*12)*(4/125)^2$$

**Input:**

$$1+12 \times \frac{1}{2} \times \frac{1}{6} \times \frac{5}{6} \times \frac{4}{125} + 23 \times \frac{1 \times 3}{2 \times 4} \times \frac{1 \times 7}{6 \times 12} \times \frac{5 \times 11}{6 \times 12} \left(\frac{4}{125}\right)^2$$



$$\left(\frac{5\sqrt{5}}{2\pi\sqrt{3}}\right)^{18} - \frac{7}{10^3} = -\frac{7}{1000} + \frac{7450580596923828125}{5159780352 \left(\sum_{k=0}^{\infty} \frac{4(-1)^k 1195^{-1-2k} (5^{1+2k} - 4 \cdot 239^{1+2k})}{1+2k}\right)^{18}}$$

$$\left(\frac{5\sqrt{5}}{2\pi\sqrt{3}}\right)^{18} - \frac{7}{10^3} = -\frac{7}{1000} + \frac{7450580596923828125}{5159780352 \left(\sum_{k=0}^{\infty} \left(-\frac{1}{4}\right)^k \left(\frac{1}{1+2k} + \frac{2}{1+4k} + \frac{1}{3+4k}\right)\right)^{18}}$$

**Integral representations:**

$$\left(\frac{5\sqrt{5}}{2\pi\sqrt{3}}\right)^{18} - \frac{7}{10^3} = -\frac{7}{1000} + \frac{7450580596923828125}{1352605460594688 \left(\int_0^{\infty} \frac{1}{1+t^2} dt\right)^{18}}$$

$$\left(\frac{5\sqrt{5}}{2\pi\sqrt{3}}\right)^{18} - \frac{7}{10^3} = -\frac{7}{1000} + \frac{7450580596923828125}{1352605460594688 \left(\int_0^1 \frac{1}{\sqrt{1-t^2}} dt\right)^{18}}$$

$$\left(\frac{5\sqrt{5}}{2\pi\sqrt{3}}\right)^{18} - \frac{7}{10^3} = -\frac{7}{1000} + \frac{7450580596923828125}{354577405862133891072 \left(\int_0^1 \sqrt{1-t^2} dt\right)^{18}}$$

$$\left(\left(\frac{5\sqrt{5}}{2\pi\sqrt{3}}\right)^{21} - \frac{29}{10^3}\right)$$

**Input:**

$$\left(\frac{5\sqrt{5}}{2\pi\sqrt{3}}\right)^{21} - \frac{29}{10^3}$$

**Result:**

$$\frac{4656612873077392578125 \sqrt{\frac{5}{3}}}{123834728448 \pi^{21}} - \frac{29}{1000}$$

### Decimal approximation:

1.732995222429404742806504321335655854156167823519606105658...

$1.73299\dots \approx \sqrt{3}$  that is the ratio between the gravitating mass  $M_0$  and the Wheelerian mass  $q$

$$M_0 = \sqrt{3q^2 - \Sigma^2},$$

$$q = \frac{(3\sqrt{3}) M_s}{2}.$$

(see: Can massless wormholes mimic a Schwarzschild black hole in the strong field lensing? - arXiv:1909.13052v1 [gr-qc] 28 Sep 2019)

### Possible closed forms:

$$\sqrt{3} \approx 1.73205080$$

$$\pi - \log(2) - \frac{5 \log(\pi)}{8} \approx 1.732989294$$

$$9\lambda - 1 \approx 1.732967026$$

$\log(x)$  is the natural logarithm

$\lambda$  is the Gauss-Kuzmin-Wirsing constant

### Property:

$$-\frac{29}{1000} + \frac{4656612873077392578125 \sqrt{\frac{5}{3}}}{123834728448 \pi^{21}}$$

is a transcendental number

### Alternate forms:

$$\frac{1346702671872 \pi^{21} - 582076609134674072265625 \sqrt{15}}{46438023168000 \pi^{21}}$$

$$\frac{582076609134674072265625 \sqrt{15} - 1346702671872 \pi^{21}}{46438023168000 \pi^{21}}$$

$$\frac{582076609134674072265625 \sqrt{\frac{5}{3}} - 448900890624 \pi^{21}}{15479341056000 \pi^{21}}$$

**Series representations:**

$$\left(\frac{5\sqrt{5}}{2\pi\sqrt{3}}\right)^{21} - \frac{29}{10^3} = -\left(\left(7602176\pi^{21}\sqrt{2}^{21}\left(\sum_{k=0}^{\infty}2^{-k}\binom{\frac{1}{2}}{k}\right)^{21} - 59604644775390625\sqrt{4}^{21}\right)\left(\sum_{k=0}^{\infty}4^{-k}\binom{\frac{1}{2}}{k}\right)^{21}\right)/\left(262144000\pi^{21}\sqrt{2}^{21}\left(\sum_{k=0}^{\infty}2^{-k}\binom{\frac{1}{2}}{k}\right)^{21}\right)$$

$$\left(\frac{5\sqrt{5}}{2\pi\sqrt{3}}\right)^{21} - \frac{29}{10^3} = -\left(\left(7602176\pi^{21}\sqrt{2}^{21}\left(\sum_{k=0}^{\infty}\frac{\left(-\frac{1}{2}\right)^k\left(-\frac{1}{2}\right)_k}{k!}\right)^{21} - 59604644775390625\sqrt{4}^{21}\right)\left(\sum_{k=0}^{\infty}\frac{\left(-\frac{1}{4}\right)^k\left(-\frac{1}{2}\right)_k}{k!}\right)^{21}\right)/\left(262144000\pi^{21}\sqrt{2}^{21}\left(\sum_{k=0}^{\infty}\frac{\left(-\frac{1}{2}\right)^k\left(-\frac{1}{2}\right)_k}{k!}\right)^{21}\right)$$

$$\left(\frac{5\sqrt{5}}{2\pi\sqrt{3}}\right)^{21} - \frac{29}{10^3} = -\left(\left(7602176\pi^{21}\left(\sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k(3-z_0)^k z_0^{-k}}{k!}\right)^{21} - 59604644775390625\right)\left(\sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k(5-z_0)^k z_0^{-k}}{k!}\right)^{21}\right)/\left(262144000\pi^{21}\left(\sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k(3-z_0)^k z_0^{-k}}{k!}\right)^{21}\right)$$

for (not  $(z_0 \in \mathbb{R}$  and  $-\infty < z_0 \leq 0)$ )



## Observations

Figs.

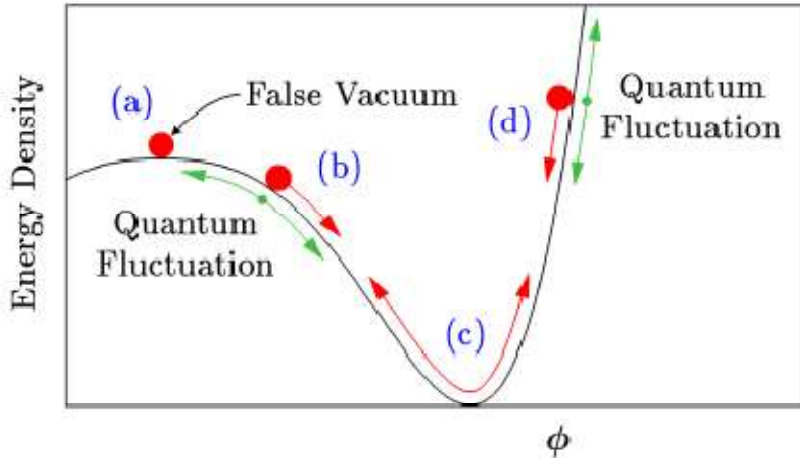
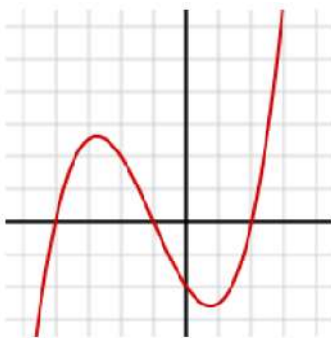


FIG. 1: In simple inflationary models, the universe at early times is dominated by the potential energy density of a scalar field,  $\phi$ . Red arrows show the classical motion of  $\phi$ . When  $\phi$  is near region (a), the energy density will remain nearly constant,  $\rho \cong \rho_f$ , even as the universe expands. Moreover, cosmic expansion acts like a frictional drag, slowing the motion of  $\phi$ : Even near regions (b) and (d),  $\phi$  behaves more like a marble moving in a bowl of molasses, slowly creeping down the side of its potential, rather than like a marble sliding down the inside of a polished bowl. During this period of “slow roll,”  $\rho$  remains nearly constant. Only after  $\phi$  has slid most of the way down its potential will it begin to oscillate around its minimum, as in region (c), ending inflation.



Graph of a cubic function with 3 real roots (where the curve crosses the horizontal axis at  $y = 0$ ). The case shown has two critical points. Here the function is  $f(x) = (x^3 + 3x^2 - 6x - 8)/4$ .

The ratio between  $M_0$  and  $q$

$$M_0 = \sqrt{3q^2 - \Sigma^2},$$

$$q = \frac{(3\sqrt{3}) M_s}{2}.$$

i.e. the gravitating mass  $M_0$  and the Wheelerian mass  $q$  of the wormhole, is equal to:

$$\text{sqrt}(((3*(2.17049\text{e}+37)^2 - 0.001^2))) / ((3\text{sqrt}3)*(4.2*10^6 * 1.9891*10^30))/2$$

**Input interpretation:**

$$\frac{\sqrt{3(2.17049 \times 10^{37})^2 - 0.001^2}}{\frac{1}{2}((3\sqrt{3})(4.2 \times 10^6 \times 1.9891 \times 10^{30}))}$$

**Result:**

1.732050787905194420703947625671018160083566548802082460520...

**Input interpretation:**

1.7320507879

$1.7320507879 \approx \sqrt{3}$  that is the ratio between the gravitating mass  $M_0$  and the Wheelerian mass  $q$  of the wormhole

We note that:

$$(-1/2+i/2(\text{sqrt}3))-(-1/2-i/2(\text{sqrt}3))$$

**Input:**

$$\left(-\frac{1}{2} + \frac{i}{2} \sqrt{3}\right) - \left(-\frac{1}{2} - \frac{i}{2} \sqrt{3}\right)$$

$i$  is the imaginary unit

**Result:**

$$i\sqrt{3}$$

**Decimal approximation:**

1.732050807568877293527446341505872366942805253810380628055...  $i$

**Polar coordinates:**

$r \approx 1.73205$  (radius),  $\theta = 90^\circ$  (angle)

1.73205

This result is very near to the ratio between  $M_0$  and  $q$ , that is equal to 1.7320507879  $\approx \sqrt{3}$

With regard  $\sqrt{3}$ , we note that is a fundamental value of the formula structure that we need to calculate a Cubic Equation

We have that the previous result

$$\left(-\frac{1}{2} + \frac{i}{2} \sqrt{3}\right) - \left(-\frac{1}{2} - \frac{i}{2} \sqrt{3}\right) = i\sqrt{3} =$$

$$= 1.732050807568877293527446341505872366942805253810380628055... i$$

$r \approx 1.73205$  (radius),  $\theta = 90^\circ$  (angle)

can be related with:

$$u^2(-u)\left(\frac{1}{2} \pm \frac{i\sqrt{3}}{2}\right) + v^2(-v)\left(\frac{1}{2} \pm \frac{i\sqrt{3}}{2}\right) = q$$

Considering:

$$(-1)\left(\frac{1}{2} - \frac{i\sqrt{3}}{2}\right) - (-1)\left(\frac{1}{2} + \frac{i\sqrt{3}}{2}\right) = q$$

$$= i\sqrt{3} = 1.732050807568877293527446341505872366942805253810380628055... i$$

$r \approx 1.73205$  (radius),  $\theta = 90^\circ$  (angle)

Thence:

$$\left(-\frac{1}{2} + \frac{i}{2} \sqrt{3}\right) - \left(-\frac{1}{2} - \frac{i}{2} \sqrt{3}\right) \Rightarrow$$

$$\Rightarrow (-1)\left(\frac{1}{2} - \frac{i\sqrt{3}}{2}\right) - (-1)\left(\frac{1}{2} + \frac{i\sqrt{3}}{2}\right) = q = 1.73205 \approx \sqrt{3}$$

**We observe how the graph above, concerning the cubic function, is very similar to the graph that represent the scalar field (in red). It is possible to hypothesize that cubic functions and cubic equations, with their roots, are connected to the scalar field.**

*From:*

[https://www.scientificamerican.com/article/mathematics-ramanujan/?fbclid=IwAR2caRXrn\\_RpOSvJ1QxWsVLBcJ6KVgd\\_Af\\_hrmDYBNyU8mpSjRs1BDeremA](https://www.scientificamerican.com/article/mathematics-ramanujan/?fbclid=IwAR2caRXrn_RpOSvJ1QxWsVLBcJ6KVgd_Af_hrmDYBNyU8mpSjRs1BDeremA)

*Ramanujan's statement concerned the deceptively simple concept of partitions—the different ways in which a whole number can be subdivided into smaller numbers. Ramanujan's original statement, in fact, stemmed from the observation of patterns, such as the fact that  $p(9) = 30$ ,  $p(9 + 5) = 135$ ,  $p(9 + 10) = 490$ ,  $p(9 + 15) = 1,575$  and so on are all divisible by 5. Note that here the  $n$ 's come at intervals of five units.*

*Ramanujan posited that this pattern should go on forever, and that similar patterns exist when 5 is replaced by 7 or 11—there are infinite sequences of  $p(n)$  that are all divisible by 7 or 11, or, as mathematicians say, in which the "moduli" are 7 or 11.*

*Then, in nearly oracular tone Ramanujan went on: "There appear to be corresponding properties," he wrote in his 1919 paper, "in which the moduli are powers of 5, 7 or 11...and no simple properties for any moduli involving primes other than these three." (Primes are whole numbers that are only divisible by themselves or by 1.) Thus, for instance, there should be formulas for an infinity of  $n$ 's separated by  $5^3 = 125$  units, saying that the corresponding  $p(n)$ 's should all be divisible by 125. In the past methods developed to understand partitions have later been applied to physics problems such as the theory of the strong nuclear force or the entropy of black holes.*

*From Wikipedia*

*In particle physics, Yukawa's interaction or Yukawa coupling, named after Hideki Yukawa, is an interaction between a scalar field  $\phi$  and a Dirac field  $\psi$ . The Yukawa interaction can be used to describe the nuclear force between nucleons (which are fermions), mediated by pions (which are pseudoscalar mesons). The Yukawa interaction is also used in the Standard Model to describe the coupling between the Higgs field and massless quark and lepton fields (i.e., the fundamental fermion particles). Through spontaneous symmetry breaking, these fermions acquire a mass proportional to the vacuum expectation value of the Higgs field.*

Can be this the motivation that from the development of the Ramanujan's equations we obtain results very near to the dilaton mass calculated as a type of *Higgs boson*: *125 GeV* for  $T = 0$  and to the Higgs boson mass *125.18 GeV* and practically equal to the rest mass of *Pion meson* *139.57 MeV*

Note that:

$$g_{22} = \sqrt{(1 + \sqrt{2})}.$$

Hence

$$\begin{aligned} 64g_{22}^{24} &= e^{\pi\sqrt{22}} - 24 + 276e^{-\pi\sqrt{22}} - \dots, \\ 64g_{22}^{-24} &= 4096e^{-\pi\sqrt{22}} + \dots, \end{aligned}$$

so that

$$64(g_{22}^{24} + g_{22}^{-24}) = e^{\pi\sqrt{22}} - 24 + 4372e^{-\pi\sqrt{22}} + \dots = 64\{(1 + \sqrt{2})^{12} + (1 - \sqrt{2})^{12}\}.$$

Hence

$$e^{\pi\sqrt{22}} = 2508951.9982\dots$$

Thence:

$$64g_{22}^{-24} = 4096e^{-\pi\sqrt{22}} + \dots$$

And

$$64(g_{22}^{24} + g_{22}^{-24}) = e^{\pi\sqrt{22}} - 24 + 4372e^{-\pi\sqrt{22}} + \dots = 64\{(1 + \sqrt{2})^{12} + (1 - \sqrt{2})^{12}\}$$

That are connected with 64, 128, 256, 512, 1024 and  $4096 = 64^2$

(Modular equations and approximations to  $\pi$  - S. Ramanujan - Quarterly Journal of Mathematics, XLV, 1914, 350 – 372)

All the results of the most important connections are signed in blue throughout the drafting of the paper. We highlight as in the development of the various equations we use always the constants  $\pi$ ,  $\phi$ ,  $1/\phi$ , the Fibonacci and Lucas numbers, linked to the

golden ratio, that play a fundamental role in the development, and therefore, in the final results of the analyzed expressions.

In mathematics, the Fibonacci numbers, commonly denoted  $F_n$ , form a sequence, called the Fibonacci sequence, such that each number is the sum of the two preceding ones, starting from 0 and 1. Fibonacci numbers are strongly related to the golden ratio: Binet's formula expresses the  $n$ th Fibonacci number in terms of  $n$  and the golden ratio, and implies that the ratio of two consecutive Fibonacci numbers tends to the golden ratio as  $n$  increases.

Fibonacci numbers are also closely related to Lucas numbers, in that the Fibonacci and Lucas numbers form a complementary pair of Lucas sequences

The beginning of the sequence is thus:

0, 1, 1, 2, 3, 5, 8, 13, 21, 34, 55, 89, 144, 233, 377, 610, 987, 1597, 2584, 4181, 6765, 10946, 17711, 28657, 46368, 75025, 121393, 196418, 317811, 514229, 832040, 1346269, 2178309, 3524578, 5702887, 9227465, 14930352, 24157817, 39088169, 63245986, 102334155...

The Lucas numbers or Lucas series are an integer sequence named after the mathematician François Édouard Anatole Lucas (1842–91), who studied both that sequence and the closely related Fibonacci numbers. Lucas numbers and Fibonacci numbers form complementary instances of Lucas sequences.

The Lucas sequence has the same recursive relationship as the Fibonacci sequence, where each term is the sum of the two previous terms, but with different starting values. This produces a sequence where the ratios of successive terms approach the golden ratio, and in fact the terms themselves are roundings of integer powers of the golden ratio.<sup>[1]</sup> The sequence also has a variety of relationships with the Fibonacci numbers, like the fact that adding any two Fibonacci numbers two terms apart in the Fibonacci sequence results in the Lucas number in between.

The sequence of Lucas numbers is:

2, 1, 3, 4, 7, 11, 18, 29, 47, 76, 123, 199, 322, 521, 843, 1364, 2207, 3571, 5778, 9349, 15127, 24476, 39603, 64079, 103682, 167761, 271443, 439204, 710647, 1149851, 1860498, 3010349, 4870847, 7881196, 12752043, 20633239, 33385282, 54018521, 87403803.....

All Fibonacci-like integer sequences appear in shifted form as a row of the Wythoff array; the Fibonacci sequence itself is the first row and the Lucas sequence is the second row. Also like all Fibonacci-like integer sequences, the ratio between two consecutive Lucas numbers converges to the golden ratio.

A Lucas prime is a Lucas number that is prime. The first few Lucas primes are:

2, 3, 7, 11, 29, 47, 199, 521, 2207, 3571, 9349, 3010349, 54018521, 370248451, 6643838879, ...  
(sequence A005479 in the OEIS).

In geometry, a golden spiral is a logarithmic spiral whose growth factor is  $\phi$ , the golden ratio.<sup>[1]</sup> That is, a golden spiral gets wider (or further from its origin) by a factor of  $\phi$  for every

*quarter turn it makes. Approximate logarithmic spirals can occur in nature, for example the arms of spiral galaxies<sup>[3]</sup> - golden spirals are one special case of these logarithmic spirals*

**We note how the following three values: 137.508 (golden angle), 139.57 (mass of the Pion - meson Pi) and 125.18 (mass of the Higgs boson), are connected to each other. In fact, just add 2 to 137.508 to obtain a result very close to the mass of the Pion and subtract 12 to 137.508 to obtain a result that is also very close to the mass of the Higgs boson. We can therefore hypothesize that it is the golden angle (and the related golden ratio inherent in it) to be a fundamental ingredient both in the structures of the microcosm and in those of the macrocosm.**

## References

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