The Clifford algebra of order 3

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Abstract

We define an algebra like the Clifford algebra but with relations of order 3

1 The Clifford algebra

The Clifford algebra for (E, g), E being a real vector space and g a symmetric bilinear form, is defined as the free algebra over E with the following relations:

$$ef + fe = -2g(e, f)$$

we have:

$$e^2 = -g(e, e)$$

The Clifford algebra is central simple.

2 The Clifford algebra of order 3

Let (E, h) be a real vector space E with a symmetric trilinear form h. Then the Clifford algebra of order 3 is defined by the following relations:

$$efg + fge + gef + feg + gfe + egf = 6h(e, f, g)$$

we have:

$$e^3 = h(e, e, e)$$

The proper value of e are so $0, 1, -1, j, -j, j^2, -j^2$ with a constant.

3 The Laplacian of order 3

We define a tensor with values in the differential operators of order 3 by mean of the connection nabla ∇:

$$\nabla(X, Y, Z) = \nabla_X \nabla_Y \nabla_Z - \nabla_X \nabla_{\nabla_Y Z} - \nabla_{\nabla_X Y} \nabla_Z +$$

$$+\nabla_{\nabla_{\nabla_X Y} Z} - \nabla_Y \nabla_{\nabla_X Z} + \nabla_{\nabla_Y \nabla_X Z}$$

The Laplacian of order 3 is then defined as:

$$\Delta_3 = \sum_{ijk} h(e_i, e_j, e_k) \nabla(e_i, e_j, e_k)$$

with (e_i) an orthonormal basis with respect of a metric. The Dirac operator is then:

$$\mathcal{D}_3 = \sum_i e_i.\nabla_{e_i}$$

with e_i in the Clifford algebra of order 3, such that:

$$\mathcal{D}_3^3 = \Delta_3 + \alpha$$

with α a lower term.

References

[BT] I.M.Benn & R.W.Tucker, "An Introduction to Spinors and Geometry with Applications in Physics", Adam Hilger, London, 1987.