Title : Time & Relativity in Mathematics!! Godel Incompleteness Results, Cantor Diagonalization& Countability of Real Numbers, Famous paradoxes e.g. Richard, Russell, Skolem, Liar , Set Theory, ZFC, in Relativistic Mathematical Space-Time:

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Abstract :

In this paper, we try to revisit some of the most fundamental issues lying at the foundation of mathematics in space-time relativistic perspective ,rather than conventional absolute space. We are adding a new dimension "Time" to the mathematics and review it in Space-Time relativistic framework to resolve the major foundational issues and making it in line with physical world realities. We shall look at the famous Cantor's Diagonalization approach in to show the Countability of Real Numbers and explain Infiniteness in that perspective. We shall also look to resolve the famous paradoxes e.g. Richard, Russell,Liar, Skolem . We shall also look at the the foundation of Set theory historically in Space-time to restore the issues that had led to ZFC by elimination and restrictions. As a consequence, we shall also revisit Godel Incompleteness theorems for Real Numbers and also otherwise explain the "inconsistency" in the new framework. This could possibly lead to entirely new way of looking at conventional mathematics in broad sense.

Note: Space here means that mathematical space in which the Numbers are constructed physically ! Time is the dimension aligned with that mathematical Space in which Numbers are constructed ! The combined structure is Mathematical Space-Time or simply Space-time throughout this paper.

Moreover one key implications to mention is that Even Human Languages e.g. English, French, Hindi etc. are fundamentally structurally incomplete or inconsistent that would have huge implications in day to day life communications.

<u>Introduction to the idea :</u> There is an interesting paper by eminent Computer Scientist Gregory Chaitin [Reference 1]" How Real are the Real Numbers" in which he talks about the fundamental issues with Real Numbers, Borel's idea of definition of Real Numbers to the extremely powerful Richard's paradox. He points out how Real Numbers in Mathematics are at –a distance from the the measurement point of view in Physics where we have not been able to measure physically beyond few digits in the real world. Hence, a question is raised, how real are the Real Numbers really or Other mathematical entities in the physical real world. Infact, He talks interestingly that The Real Numbers are so Unreal if we compare them with Real world realities of the Physical Perspectives. In another Computable Universe Hypothesis by eminent physicist Max Tegmark,MIT, he talks about trying some ways to deal with Real Numbers issue as the fundamental issue to model Physical Realities.. Reference[2]. Recently Stephen Wolfram has launched Computer Project with Physics in Computational Aspects. All of them have some fundamental issues with Humanly Created Mathematics.

The fundamental issue is that Humans have created a Mathematical System which is at conflict with Nature's way of working. This is causing so many fundamental issues. Real Numbers being one of the prominent examples assuming infinite divisibility of Space/Particle . Mathematically, say a Real Number say Sqrt 2 and 1 are relatively defined where the former has unending decimal representation but Physically both have same finite Physical existence depicting a finite length. Evidently, the problem is the human defined system of Mathematics here Real Numbers. and Infact ,Physical theories have evolved from early age Galileo to Newton to Einstein to QM, but Mathematics still resides in old age where the ancient mathematicians came up with the system by deriving their at that time contemporary ideas about the physical universe.

Since Euclid to Einstein to Riemann all had raised doubt about the Continuous nature of Space at different times. . If there exits the most basic scale of the existence of physical object say the Planck Scale, then everything in this Universe is a multiple of that most basic unit and hence all are basically Natural Numbers fundamentally and naturally but Human defined system of mathematics doesn't seem to be coherent with all those.

In this paper, I would try to address and resolve these fundamental issues /problems in a completely new way of looking at the mathematical entities that could make it aligned with the physical world realities. So, here, I try to look at them in Physical world perspective of reality ! I'll try to expose a the hidden dimension to view the entire Mathematics in line with the Physical world reality that has often been ignored.

If we look at the evolution of terms in mathematics e.g. Set or Numbers etc. We would find that their construction have not been done without time dimension. They have been constructed at different stages in the time dimension. This is extremely vital when we go at the foundation of paradoxes, although we have been ignoring this otherwise in normal calculation. Traditionally, what we have been doing is to just presuming that the entire mathematical concepts exist at the same time absolutely, by overlooking their procedure of construction, that has actually occurred in time dimension. It's just like physics here.. Not to forget, one of the major related problem P vs NP in Theoretical Computer Science talks about the time involved to run an algorithm.

This paper looks into those vital hidden time dimension which is very much required along with space to construct the notion of mathematical basis e.g. Set

,Numbers. Infact, Unlike conventional approach, where mathematicians typically just talk about the Space aspects of Mathematics in Absolute sense, I would like to focus on the Time Aspects also leading to inclusive Space-Time view of these mathematical entities in Relativistic Sense. We shall see, this foundation change in view would try to resolve many important results e.g. Countability, Uncountability, Infinity , Paradoxes in Space- Time perspective that was derived using Cantor Diagonalization method, which is the backbone of so many important derived results and the Cantor based set theory. Historically many legendary mathematicians have spoken against the Cantor based set Theory! These traditional results at the foundation of arguably one of the the most important discoveries of mathematics and logic e.g. Godel Incompleteness Theorems. We can look at them in entirely new perspective in Space-Time to possibly address its limitations and expand them further in broader areas as well We would find that many fundamental limitations of Mathematics were long lasting historically so far because of the ignoring Relativistic and Time dimension that is coherent with the evolution of Physical realities.

Most importantly, The Newly Proposed Set theory ZFC was proposed as the foundation of Set theory by eliminating those paradoxes in erstwhile absoluteness of Set rather than resolving them foundationally. We would show that those paradoxes are no more existing when we include the Time dimension of mathematics rather say in Relativistic rather than the need to propose ZFC . So, this way, it also tries to solve the problem fundamentally rather than eliminating them.

Traditional Proof of Cantor Diagonalization Method to Prove the Uncountability of Real Numbers.

Set T of Infinite Sequence of all the binary digits exists

If s_1, s_2, … , s_n, … is any enumeration of elements from *T*, then there is always an element *s* of *T* which corresponds to no s_n in the enumeration.

The proof starts with an enumeration of elements from *T*.

Next, a sequence *s* is constructed by choosing the 1st digit as complementary to the 1st digit of s_1 (swapping 0s for 1s and vice versa), the 2nd digit as complementary to the 2nd digit of s_2, the 3rd digit as complementary to the 3rd digit of s_3, and generally for every *n*, the n^{th} digit as complementary to the n^{th} digit of s_n.

s_1=(**<u>0</u>**,0,0,0,0,0,0,...)

s_2=(1,**<u>1</u>**,1,1,1,1,1,...)

s_3=(0,1,**<u>0</u>**,1,0,1,0,...)

s_4=(1,0,1,**<u>0</u>**,1,0,1,...)

s_5=(1,1,0,1,**<u>0</u>**,1,1,...)

s_6=(0,0,1,1,0,**<u>1</u>**,1,...)

s_7=(1,0,0,0,1,0,**<u>0</u>**,...)

...

$s=(\underline{\mathbf{1}},\underline{\mathbf{0}},\underline{\mathbf{1}},\underline{\mathbf{1}},\underline{\mathbf{1}},\underline{\mathbf{0}},\underline{\mathbf{1}},\ldots)$

By construction, s differs from each s_n, since their n^{th} digits differ (highlighted in the example). Hence, s cannot occur in the enumeration.

Based on this theorem, Cantor then uses a proof of contradiction to show that:

The set T is uncountable.

The proof starts by assuming that T is countable. Then all its elements can be written as an enumeration s_1, s_2, ... , s_n, ... Applying the previous theorem to this enumeration produces a sequence s not belonging to the enumeration. However, this contradicts s being an element of T and therefore belonging to the enumeration. This contradiction implies that the original assumption is false. Therefore, T is uncountable.

Space-Time Proof of Countability of Real Numbers using Cantor Diagonalization:

The above proof seems to miss the more fundamental deep aspect while using the method of contradiction. It previously concluded that the assumption of "T is countable" is false but what could also be meant at the deeper level it's not actually about the UnCountability of the Set T rather that the assumption of the absolute existence of the set T is itself objectionable.

That means, There can't exist any such Absolute "Set T" statically in the dimension of Time nullifying the fundamental concept of "Set". Rather, here the Set is applicable not in absolute rather relative sense with respect to say time dimension in which numbers are constructed.

Numbers fundamentally come into existence in the dimension of time dynamically. They have been assumed to exist independent of time. That holds true for Natural, Real etc. numbers.

They have Relative Existence depending upon Observer like in Physics !

It's because of this assumption of "Absoluteness" of Set, this result of Uncountability comes into picture. But in our relativistic perspective, Such "absolute set T can't exist, they can exist in the dimension of time relativistically. That means, all the elements of the sets are created in different dimension of time rather their absolute existence at the same time for the observer. That also means that Uncountability and Countability have Relativistic existence.

Infact Construction of T is like the Set in Russell's paradox. We shall deal with the Russell's paradox in the next section in Relativistic Terms.

The foundational Concept of Set based on the Principle of Simultaneity is not aligned with Physics based realities. That Construction of a Set without Time dimension leads to all the Problems and its incompatibility with the Physics.

So, here goes on taking the traditional steps of Cantor Diagonalization of Binary digits

At time t= 0 say the following sequence exists i.e. T(0)

$s_1=(\underline{\mathbf{0}},0,0,0,0,0,0,\ldots)$

$s_2=(1,\underline{\mathbf{1}},1,1,1,1,1,\ldots)$

$s_3=(0,1,\underline{\mathbf{0}},1,0,1,0,\ldots)$

$s_4=(1,0,1,\underline{\mathbf{0}},1,0,1,\ldots)$

$s_5=(1,1,0,1,\underline{\mathbf{0}},1,1,\ldots)$

$s_6=(0,0,1,1,0,\underline{\mathbf{1}},1,\ldots)$

$s_7=(1,0,0,0,1,0,\underline{\mathbf{0}},\ldots)$

...

At t= 1 , T(1), the new s is added to T(0).

$s=(\underline{\mathbf{1}},\underline{\mathbf{0}},\underline{\mathbf{1}},\underline{\mathbf{1}},\underline{\mathbf{1}},\underline{\mathbf{0}},\underline{\mathbf{1}},\ldots)$

If the entire proof is viewed in this perspective, the end result of Cantor Diagonalization is Not the Absolute Uncountability rather a Relative Countability w.r.t. in the Time dimension. The absolute Uncountability arose because the absolute existence of the Set T was presumed to exist but that's not true and can be experimentally /practically realized because any time one tries to make an absolute confined existence, it will lead to the conclusion that after this there will be no Natural Number or any Number beyond that.

If we look at the process deeply, the contradiction arises because the definition of T varies in the dimension of time and the absolute confined existence of all the infinite possibilities at the same time i.e. independent of observer time doesn't hold true and possible.

In fact to start with, T at t= 0   and as a new s is created from the previous T at t=0, that becomes part of the new T at t= 1. and both the T at different times are not the same .They vary dynamically.

The new enumeration "s" is excluded/external to T(0) but included / internal to T(1). Hence, in Space-Time relativistic sense it has no contradiction but if time is removed and only Space is taken then, it leads to the contradiction as how s could be both external and internal at the same time!! This is the fundamental cause .

The earlier contradiction arose because the time evolution of T is ignored and the absolute existence of all the infinite elements of T is presumed independent of time in space. But fundamentally , it's in space -time as in physics. In other words, Infiniteness of Numbers or Numbers in general is a dynamic constructive process in space-time. not only space.

T (0)was Countable at t=0 but T(1) also countable at t=1 (after the new s enumeration comes up by taking earlier one as the absolute set of all infinite ones) . But if time dimension is hidden, it looks contradictory !

The Countability assumption appeared to be violated because earlier it was presumed to be absolute existence of T in space independent of time. But that's not possible regarding T. But the issue is more fundamental that lies in Relativistic Space-Time existence of Numbers and Infiniteness of elements for that matter.

T(at t=0) is assumed to be Countable ..but as the new enumeration s is appended, T(t) is also Countable. But Since time dimension is hidden/ignored, T overall becomes Uncountable because of Contradiction. But in Space-Time with time dimension included, both T exist at different time and hence co contradiction and hence no Uncountability.
Infact at any moment, they are finite also. This is because T(t) is never Complete list for any value of T ..ie T(0) or T(t). The assumption of Completeness of T is not valid and also the cause of Contradiction ! Hence, T (t) is actually Finite in Space-Time. Its when we remove Time dimension, it becomes Infinite !
In other words, Countability, Uncountability ,Set etc. are Relativistic in Space-Time rather Absolute . This time version would similarly hold true for any sequence including real numbers.

Infiniteness as Finiteness   in Mathematical Space-Time.

Infact also, Inifniteness of Numbers exist because of hidden dimension of time. As we view in the dimension of time, at any moment of time the mathematical infiniteness could also be finite in Space-time. Say for example Natural Numbers are assumed to be Infinite because they are presumed to be absolutely existing at the same time without time but in reality construction of Natural Number occurs in time dimension. At any particular moment which will always be finite, finite number of natural numbers would exist not all of them at the same time. All the natural numbers cant exist at the same time in the set. They need to be constructed by the observer relatvistically just like in physics and at any moment in time, there will be construction finite number of times only.

So, Infinite(at time time t)= Finite.

Cantor came up with different levels of Inifnity and reciprocally(just by dividing 1 by different levels of infinity could also prove to different levels of Inifinitesimals/0) These all issues arise because of lack of time dimension. But the construction of number requires time dimension.

Key Result **(A)

Hence the Conclusion is With Time dimension in Space-Time at any particular time t , any sequence(including Real Numbers) is Countable and Finite . It's when time is ignored, they appear to be Uncountable **& Infinity in Space. We need to look at them in Space-Time Reality as in Physics. Constructivism of Set,Numbers etc. can't happen without time.**
Infinitness (at time t) = Finiteness i.e. Inifniteness in Space becomes Finiteness in Space-Time.

The same issues arise in context of Russell's paradox .The paradox arises because the Master Set is assumed to exist in space independent of Time dimension. But the truth is they have hidden time dimension also I.e. space-time. The same set has time evolution also in space.Contradiction arises because the same Set varies in time and they appear contradictory in the absolute space. In reality, they exist in Space-Time not Space.

Henceforth, this implies that Numbers and Set etc should be seen as in Space-Time at fundamental levels. For example Set at T=0 and T= t . Real Number at T= 0 and at T= t . Then entire unending decimal parts of the Real Numbers cant exist at the same time T= 0. It can always be extened further and this can be viewed in Space-Time where at any particular T= t, the Real Number is actually Rational Number. Let's take an example sqrt(2)= 1.414.........

So, here at Sqrt2 (at T= 0) = 1.4
Sqrt2 (at T= 1) = 1.41
Sqrt2 (at T= 2) = 1.414
and so on....in the time dimension.

Sqrt2 cant exist completely in Space at any particular time T..It can always be extended further and that would be in the Space-Time.

Borel view of Real Numbers (Chaitin's paper) :A real number is really real only if it can be expressed, only if it can be defined using a finite number of words.It is only real if it can be named or specified as an individual mathematical object. And in order to do that we must necessarily employ some particular language e.g. English/French. Whatever the choice of language, there will only be countable infinity of possible texts.

Infact his view can be addressed in space-time perspective. Real Numbers will always be expressive in finite words at a particular T= t in Space-Time.

This space-time view will help resolve the issues of Countability and Uncountability of different type of Numbers. Even Real Number at a particular T= t will always be Countable. Even the Concept of Infinity will become Finite at T= t. So, many things will get more coherent practical view and compatible with Physics.
Hence, many fundamental issues regarding Real Numbers will get sorted out in Space-Time.

The previous assumption that the Set contains all the possible elements absolutely at the same time independent of time leads to contradiction..
The new element is not in the previous list Set at T= 0 but became the new element of the derived version of the Set which evolved at T= t. So, it's all about the evolution of the Set in the dimension of time.
Earlier the new element was not present at T=0 in the Set but it came in the list at T=t . But contradiction is because it is thought that how the new element outside the Set became the part of the previous Set. But it is happening at different Time as dimension.. not at the same time .This is the point of contradiction!!
This lack of time dimension leads to the contradiction that new element real number not in the previous Set is the part of the previous Set .This is because of assumption of Absoluteness of the Set in Space having all the elements at the same time and not taking into consideration the Time dimension aspect. The Set is not static rather it is changing over time. But the fact is Relativistic nature of the Set in Space-Time.

Skolem Paradox : Relativistic aspect in Space-Time.

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Skolem, the mathematician has pointed out the Relativistic aspect of Set theory, Counatability, Uncountability as a resolution to his Skolem Paradox.

A brief description of the problem :

Skolem (1922) pointed out the seeming contradiction between the Löwenheim–Skolem theorem on the one hand, which implies that there is a countable model of Zermelo's axioms, and Cantor's theorem on the other hand, which states that uncountable sets exist, and which is provable from Zermelo's axioms. "So far as I know," Skolem writes, "no one has called attention to this peculiar and apparently paradoxical state of affairs. By virtue of the axioms we can prove the existence of higher cardinalities... How can it be, then, that the entire domain *B* [a countable model of Zermelo's axioms] can already be enumerated by means of the finite positive integers?" (Skolem 1922, p. 295, translation by Bauer-Mengelberg)

More specifically, let *B* be a countable model of Zermelo's axioms. Then there is some set *u* in *B* such that *B* satisfies the first-order formula saying that *u* is uncountable. For example, *u* could be taken as the set of real numbers in *B*. Now, because *B* is countable, there are only countably many elements *c* such that $c \in u$ according to *B*, because there are only countably many elements *c* in *B* to begin with. Thus it appears that *u* should be countable. This is Skolem's paradox.

Skolem went on to explain why there was no contradiction. In the context of a specific model of set theory, the term "set" does not refer to an arbitrary set, but only to a set that is actually included in the model. The definition of countability requires that a certain one-to-one correspondence, which is itself a set, must exist. Thus it is possible to recognise that a particular set *u* is countable, but not countable in a particular model of set theory, because there is no set in the model that gives a one-to-one correspondence between *u* and the natural numbers in that model.
Skolem used the term "relative" to describe this state of affairs, where the same set is included in two models of set theory, is countable in one model, and is not countable in the other model. He described this as the "most important" result in his paper. Contemporary set theorists describe concepts that do not depend on the choice of a transitive model as absolute. From their point of view, Skolem's paradox simply shows that countability is not an absolute property in first order logic. (Kunen 1980 p. 141; Enderton 2001 p. 152; Burgess 1977 p. 406).

Skolem described his work as a critique of (first-order) set theory, intended to illustrate its weakness as a foundational system:

> "I believed that it was so clear that axiomatisation in terms of sets was not a satisfactory ultimate foundation of mathematics that mathematicians would, for the most part, not be very much concerned with it. But in recent times I have seen to my surprise that so many

mathematicians think that these axioms of set theory provide the ideal foundation for mathematics; therefore it seemed to me that the time had come for a critique." (Ebbinghaus and van Dalen, 2000, p. 147)

Space- Time Relativistic Resolution to Skolem Paradox.

The Set B is Countable at time t=0 in one frame with respect to the Set u . But at =1 , the Set u appears Uncountable at t=1 in another frame in which it is Real Number .

And as we have already shown using the Cantor Diagonalization in Space-Time that Real Numbers are Countable and Finite at any point in time t in Space-Time. So, in the Space-time perspective, the Set u is also Countable at time t and Set B is already Countable. Hence, there is no more contradiction in Space-Time.

Russell's Paradox Resolution in Space-Time Relativistic perspective

According to naive set theory, any definable collection is a set. Let *R* be the set of all sets that are not members of themselves. If *R* is not a member of itself, then its definition dictates that it must contain itself, and if it contains itself, then it contradicts its own definition as the set of all sets that are not members of themselves. This contradiction is Russell's paradox. Symbolically:

In 1908, two ways of avoiding the paradox were proposed, Russell's type theory and the Zermelo set theory. Zermelo's axioms went well beyond Gottlob Frege's axioms of extensionality and unlimited set abstraction; as the first constructed axiomatic set theory, it evolved into the now-standard Zermelo–Fraenkel set theory (ZFC). The essential difference between Russell's and Zermelo's solution to the paradox is that Zermelo altered the axioms of set theory while preserving the logical language in which they are expressed, while Russell altered the logical language itself. The language of ZFC, with the help of Thoralf Skolem, turned out to be first-order logic.

Example:

Most sets commonly encountered aren't members of themselves. For example, the set of all squares in the plane. This set is not itself a square in the plane, thus it is not a member of itself (of its own definition). Let us call a set "normal" if it is not a member of itself, and "abnormal" if it is a member of itself. The set of squares in the plane is normal. On the other hand, the complementary set that contains everything which is **not** a square in the plane is itself not a square in the plane, and so should be one of its own members and is therefore abnormal.

Now we consider the set of all normal sets, *R*, and try to determine whether *R* is normal or abnormal. If *R* were normal, it would be contained in the set of all normal sets (itself), and therefore be abnormal; on the other hand if *R* were abnormal, it would not be contained in the set of all normal sets (itself), and therefore be normal. This leads to the conclusion that *R* is neither normal nor abnormal: Russell's paradox.

Space-Time perspective :

We considered R as the set of all the normal sets to decide whether normal or abnormal.

Case 1 : If R is normal(i.e. not a member of itself as per first definition) , then it’s abnormal as per the second definition.

Case2 : If R is abnormal(i.e. member of itself as per the first definition, then it’s normal as per the second definition.

This leads to the conclusion R is neither Normal nor Abnormal.

The set of all squares is not a square itself making it abnormal.

If R is normal at t=0 , then R will be abnormal at t =1

If R is abnormal at t=0 then R will be normal at t= 1 as per the defintition

1st definition occurs at t=0 and 2nd definition occur at t= 1 i.e. different time not simultaneously.

R being Normal and Abnormal are occurring at different point in time not simultaneously. Infact , in Space-Time, this is possible and not contradictory.

That’s how it has been defined by us the definition of normal and abnormal. It has been hidden in time dimension, how they have been defined. that’s what the condition “If” & “Then” represent. If and then occur at different time dimension.

In other words, “Normal” and “Abnormal” terms are relativistically existing in Space-Time rather Absolute terms leading to pardox in Space.

In Space, this appears contradictory when time dimension is ignored.

Hence, if we look at Russell’s paradox in Space-Time perspective, it will not be a paradox any more. That’s how it can be resolved .

The Concept of Set is Relativistic in Space-Time . It has Time dimension included, which is often hidden.

As it has also been pointed out that the existence of Sequence T, (the set of all possible infinite binary digits sequence as explained before in the Cantor Diagonalization) is similar to Russell's Set.

Liar Paradox :

A says " I am lying"

Case 1 :If A is speaking Truth, then He is a Liar.

Case2 : If A is speaking Lie, then He is Not a Liar.

Lets again look at relativistic aspects hidden in Space-Time.

"I am lying" occurs at time t=0 and "I am saying that I am lying" occurs at time t =1 . Both the statements are occurring at different time in Space-Time. So, he is Liar at say time t=0 but Not a Liar at time t=1 or opposite.

In other words, He is lying for something else but He is Not lying overall or opposite. So, they are contextually and Relatively Liar or Not at different Times in the space-time

So, it's not a paradox in Space-Time once we add Time dimension . As earlier , it's because of ignoring Time dimension that it becomes a paradox !

Richard Paradox Resolution in Space-Time & Godel Incompleteness Theorems for Real Numbers:

Godel Incompleteness Results : Cantor-diagnolization to the list of all the possible mathematical definitions for individual real numbers in English !!

Conventional Description:

“Thus there is an infinite list of English phrases (such that each phrase is of finite length, but the list itself is of infinite length) that define real numbers unambiguously. We first arrange this list of phrases by increasing length, then order all phrases of equal lexicographically (in dictionary order, e.g. we can use the ASCII code, the phrases can only contain codes 32 to 126), so that the ordering is canonical . This yields an infinite list of the corresponding real numbers: r_1, r_2,...... Now define a new real number *r* as follows. The integer part of *r* is 0, the *n*th decimal place of *r* is 1 if the *n*th decimal place of r_n is not 1, and the *n*th decimal place of *r* is 2 if the *n*th decimal place of r_n is 1.

The preceding two paragraphs are an expression in English that unambiguously defines a real number *r*. Thus *r* must be one of the numbers r_n. However, *r* was constructed so that it cannot equal any of the r_n (thus, *r* is an undefinable This is the paradoxical contradiction.”

Space-Time Resolution :

If we apply here the previously explained view in Space-Time, Richard paradox gets resolved in Space-Time . The contradiction gets rooted out . The fundamental point is the absolute assumption of the Set of all the infinite list in dependent of Time dimension leads to the contradiction. As we view in Space-Time dimension, the Set of all English phrases encoded into Real Numbers is evolving in Time. In other words as Real Numbers would be Countable in Space-Time at T=t, it can be marched with the Set of English Phrases corresponding to them. If at new value of T =t’ if new r is constructed using the previous series, it can be defined using the new English phrase. The earlier issue was just because of hiding Time dimension that in Space, it seemed that Real Numbers are Uncountable but English Phrases are Countable and hence the issue. But in Space-Time there is no absolute existence and as shown in the Cantor Diagonalization that the issue regarding Real Number Uncountability was sorted out in Space-Time. Hence , Richard Paradox also gets resolved in Space-Time.

So, applying the Key Result A : That means, at any time t, the Real Numbers set is Countable and Finite in Space-Time and Hence,it can be matched with the English sequence one to one as both are Countable. The initial issue or contradiction was because of ignoring time dimension.

This entire Space-Time Relativistic concept can be applied to the Set of All Real Numbers say in the Semantic Language e.g. English in the Cantor Diagonalization Form . That’s what Richard paradox talks about. Hence, Similar Cantor Diagonalization result will be valid for Reals Numbers and their English equivalents . Kurt Godel considered Richard's paradox as the Semantic equivalent of Godel's Incomepleteness Results.

"Infact one major result to follow is that Humanly Created langauges e.g. English ,French, Hindi and others could also be fundamentally incomplete or inconsistent structurally ,which would have huge implications for day to day life communications"

Hence, in context of Richard paradox resultion in Space-time, Real Numbers can also be encoded in English Languages and then by applying Cantor Diagonlization, we can establish the Incompleteness and Inconsistency of Mathematics for Real Numbers systems also. We mean as earlier Godel Incompleteness results are confined to certain Natural Numbers, They can also be extended to Real Numbers as well.

Important Conclusion : The Resolution of Richard Paradox will also lead to the result that Godel Incompleteness Results which is basically proven for the set of Natural Numbers will also be valid for the set of Real Numbers!!

Secondly, What we call Inconsistency in context of Godel Incompleteness Theorems in general, is infact consistent in space-time framework as explained in the Cantor Diaonalization framework. Two contradictory results in absolute space could befree of contradictions in space-time at different points in time relativistically.

Space –Time Relativistic View of Set Theory rather than ZFC

In set theory , **Zermelo–Fraenkel set theory**, named after mathematicians Ernst Zermelo and Abraham Frenekel , is an axiomatic system that was proposed in the early twentieth century in order to formulate a theory of sets free of paradoxes such as Russell's paradox . Today, Zermelo–Fraenkel set theory, with the historically controversial axiom of choice (AC) included, is the standard form of axiomatic set theory and as such is the most common foundation of Mathematics Zermelo–Fraenkel set theory with the axiom of choice included is abbreviated **ZFC**, where C stands for "choice",[1] and **ZF** refers to the axioms of Zermelo–Fraenkel set theory with the axiom of choice excluded.

Zermelo–Fraenkel set theory is intended to formalize a single primitive notion, that of a hereditary well-founded set , so that all entities in the universe of discourse are such sets. Thus the axioms of Zermelo–Fraenkel set theory refer only to pure sets and prevent its models from containing urelements (elements of sets that are not themselves sets). Furthermore, proper classes (collections of mathematical objects defined by a property shared by their members where the collections are too big to be sets) can only be treated indirectly. Specifically, Zermelo–Fraenkel set theory does not allow for the existence of a universal sets (a set containing all sets) nor for unrestricted comprehensions , thereby avoiding Russell's paradox Von-Neumann-Berneys –Godel Set Theory(NBG) is a commonly used conservative extension of Zermelo–Fraenkel set theory that does allow explicit treatment of proper classes.

Some comments related to ZFC : As we have seen that ZFC was formulated to sort out the traditional issues e.g. paradoxes existing the erstwhile set theory . ZFC basically tries to just remove the weed out the problematic aspects of sets rather than providing the solution by going deep into the reasons behind those problems.

As in this paper, where I have been resolving those problems/paradoxes etc. in space-time, we can have entirely new aspects of “Set” in Space-time rathr than just disallowing the problematic parts of Set in ZFC. The problems arising was basically because of the lack of hidden time dimension and absoluteness of Set. The Space –time Relativistic Set doesn’t have those issues any more.

ZFC Problem for Real Numbers and Godel Incompleteness Theorems

A similar phenomenon occurs in formalized theories that are able to refer to their own syntax, in Zermelo-Frenkel-Set Theory (ZFC). Say that a formula φ(x) defines a real number if there is exactly one real number r such that φ(r) holds. Then it is not possible to define, by ZFC, the set of all Godel formulas that define real numbers. For, if it were possible to define this set, it would be possible to diagonalize over it to produce a new definition of a real number, following the outline of Richard's paradox above. Note that the set of formulas that define real numbers may exist, as a set F; the limitation of ZFC is that there is not any formula that defines F without reference to other sets. This is related Tarski indefinability theorem.

Space-Time Resolution :

In the Space-time view, when we have a new view about Real Numbers, Set ,the above constraint can also be resolved for real numbers as well. (which is not possible in ZFC). We have earlier demonstrated a new Space-time view to look at Set theory which resolves the fundamental constraints of paradoxes rather eliminating them to formulate ZFC.

The given we have shown the Countability and Finiteness of Real Numbers at any moment time and also Set in Space-Time, Godel Incompleteness Theorems would be applicable to Real Numbers as well in Space-TIme

I have just though of the following idea which I am just writing in Cantor’s theory

Higher Order of 0 or Infinitesimals Derived from in Cantor’s theory

In Cantor framework as there are different levels of Infinity, there are also different levels of 0.

By taking T as the Set of all possible binary digits. Put 0 before all so that all the sequences come after the decimal part. A New system, Now Just like Cantor constructed Cantor Diagonalization using the latest sequence, Similarly, the New sequence can be constructed after Decimal which is not in the list.. That shows even there are different levels of Infinitesimals(or 0) just like Infinity

In other words,just take the reciprocal of the different orders of Infinity.

Conclusion :

In this paper, we looked at the foundational mathematical entities e.g. Real Numbers,Set in Space-Time Relativistic perspective(making it in line with Physical world realities) to resolve major paradoxes e.g. Richard,Russell,Skolem , Liar Paradox,ZFC,The Concept of Infiniteness to Cantor Diagonalization to show the Countability snd Finiteness of Real Numbers .

The above results derived and analyzed would lead to the resolution of many existing issues lying the at the foundation of mathematics in future.

Moreover one key implications to mention is that Even Human Languages e.g. English, French, Hindi etc could be fundamentally incomplete or inconsistent structurally that would have huge day to day life implications to communicate with each other !!

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