Application of Markov Chain Model in Completion Rates

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Abstract

Completion Rate and Enrollment forecasting is an essential element in budgeting, resource allocation, and the overall planning for the growth of education sector. Our paper purposeful demonstrated the use of Markov chain techniques in studying progression of BSMST Programme Students from the time of entry/enrollment in each academic year to graduation after the expected year of study in MWECAU. The target population included all BSMST programme students in MWECAU from 2013 to 2015. The model used to determine the student's completion/dropout rate, retention rate and the expected duration of completing by sex. We established the completion rates for male students and that of female students and dropout rates. We saw how long Markov Transition Probability Matrices of BSMST students at MWECAU will be at a steady state. How the established completion and dropout rates will be in Absorbing rates/States. Also we saw female expectation of university education compared to male students in BSMST Programme students. The model was only suitable in make a short period projections.

Note: This is a BSc. (Mathematics and Statistics) Research Report, Mwenge Catholic University July, 2016.

CHAPTER ONE
1.0 INTRODUCTION

1.1 BACKGROUND OF THE PROBLEM

A graduation rate/completion rate follows the students who are first-time enrolled in a particular year and determines the percentage that graduate within a given year, such as one year in completing a year of study.

In the study transition model was used as an application of the more general Markov chain model in studying the continuous stream of the students from one stage to another stage. The students graduate from the system, transit from one grade year to the next higher grade, repeat the same grade or drop out of the system due to factors like lack of fees, sickness and poor academic performance. Thus students finally enter permanent states as graduates or drop outs. The student can either graduate at the final stage or drop out at any of its stages. (Burke, 1972)

Markov process can be defined as the stochastic process with the property that given value of X_t the probability of X_{t+k} where $k > 0$ is independent of $X_{t-1}, t - 1 < t$.

That is the condition distribution of the future X_{t+1} given the present X_t, X_t and the past X_{t-1} is independent of the past.

Mwenge Catholic University (Mwecau prospectus, 2012-2015) aims to produce graduates with skills, knowledge and confidence to work for Tanzanian society, able to use modern technology and improve where necessary when conducting their responsibilities. So as the university to make sure this aims are succeed, improved learning environments', tools and professionals. The achievements are measured through academic performance may be predicted from presents results to future results if and only if the system will remain unchanged for the certain period. This is by using Markov chain in stochastic process.

1.2 OBJECTIVES

1.2.1 General objective

To determine markov transitional matrix for students in the Bsc. Mathematics and Statistics Program in Mwecau.

1.2.2 Specific objectives

- To determine the dropout rate by sex in BSMST by use of Markov chain Model.
- To find the number of periods when transitional probabilities matrices of BSMST program will be at steady state.
- To demonstrate the use of Markov Chain model in enrollment projection in Mwenge Catholic University.

1.3 ASSUMPTIONS

- The study population (student's enrollment) was assumed to be closed i.e. there was no immigration and out migration of students with the neighboring Universities.
- Admissions took place only in first year.
- Dropouts were assumed to be uniformly distributed in the period $(x;\ x+1), x$ being the year of study.
- A student was not allowed to jump to higher level or demoted to lower grade of a system. So that ${P_{ij}}^{(n)} = 0$
- A student was not allowed to repeat a given year more than once

1.4 STATEMENT OF THE PROBLEM

The main problem in this study was the completion rates of BSMST students in MWECAU. Completion rate explain the number of the students who manage to graduate at the end of each academic year. Another name of the completion rate is graduation rate. The main goals concerned this problem was to identify and clarify factors that support students in the successful completion of the courses in which they were enroll and to suggest design and delivery factors that might remove barrier to successful student completion rate.

The factors which hinder students completion rates includes; specific personal characteristics commonly possessed by students to complete their courses and programs like laziness, drunkenness, and poor personal planning, Financial problems, family responsibilities, and weaknesses in institutional student support services like conduction of different seminars to address the problem. The effects of the level of the completion rate may be direct or indirect to the University and the students; we group them into three groups which are drop out, Overstaying and completing at a time.

According to (Mose J. Nyandwaki, 2014) holds that completion rate involves keeping learners in a programme until they achieve their learning goal.

Generally; when the students fail to complete studies within the expected time limit means that they have failed to reach their education and this frustrates them in their life.

1.5 SIGNIFICANCE OF THE STUDY

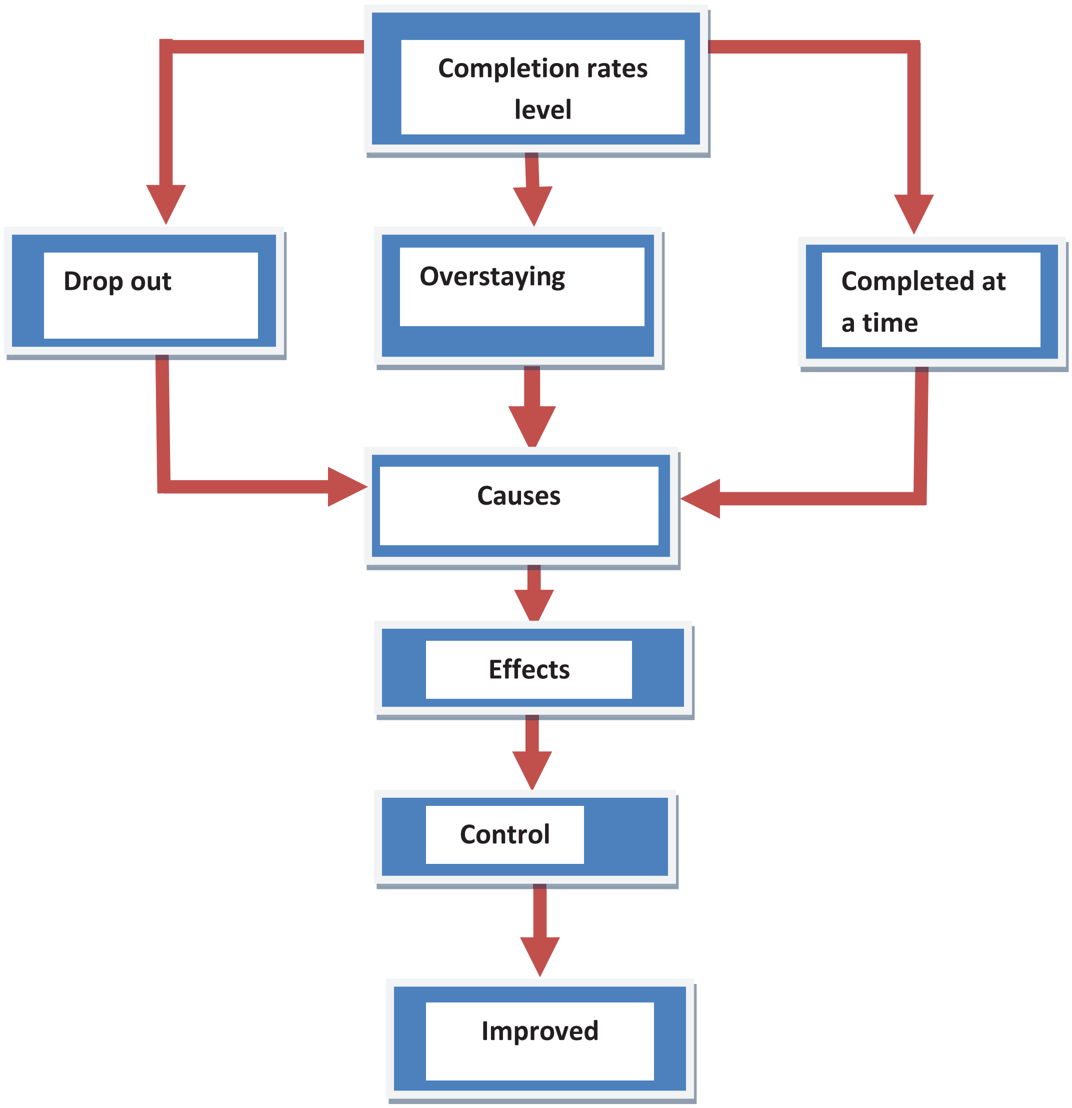
- The study will help the university to improve the learning environments and tools for the Bsc. Mathematics and Statistics program students.
- It will help mathematics and statistics department as well as University to know the completion rates of the BSMST program students.
- It will help in providing the means for projecting the number of students completing the program at the given time period.
- It will help to estimate mean time of a student to complete a study.
- The study also will establish opportunities to be harnessed to improve completion rate to be printed as a study material, science laboratories, TV and radio programs, forging partnerships, as well as investing more in Open Educational Resources (OERs).

1.6 CONCEPTUAL FRAMEWORK

Conceptual framework that guided this study was grounded on the realization that Mwenge Catholic University enrolls both loan and non-loan beneficial students. The MWECAU management has experienced that these students often faced a number of challenges that inhibited their completion of studies at MWECAU within the official duration, i.e., three years for the undergraduate students. These include financial difficulties, scarcity of study materials, isolation, limited supervision, and limited research skills among students. Our view that we had, factors affecting completion rates the university may uncover practical strategies for overcoming the challenges that inhibited completion rate at MWECAU.

Furthermore, the study set to investigate strategies that could be employed to help the Mwenge Catholic University to improve its completion rate. Effective orientation sessions, the university should be more opened in case of academic sessions to be conducted through the week. Investigating these variables was an important aspect in order to know which strategies work best within the university environment so as to adopt it in the process of improving completion rate at MWECAU. The conceptual framework is summarized below.

CONCEPTUAL FRAME WORK



CHAPTER TWO
2.0 LITERATURE REVIEW

2.1 INTODUCTION

This chapter introduces what others said about the project, what have being said about using of markov chain and transition probabilities and its practical application to real life data and forecasting of the future situation.

Over the years, many techniques have been suggested for forecasting enrollment and students flows at any level in education systems. (Wing, 1974)Classified them into curve fitting, causal models, attitude surveys and judgmental techniques. (Healy, 1978)Classified them into the judgmental, Markov process, trend analysis, regression, simulation and the ratio techniques.

A Markov model is a stochastic model (one which models random events) which has been used in diverse fields such as computer science, engineering, mathematics, genetics, agriculture economics, education, biology, etc. (Hillis, 1986)And (Stewart, 1994).

Markov chains have been widely used to model stochastic processes and to evaluate time to event data. Modeled a hierarchical system with a single absorbing state for an education system where dropouts and graduates were grouped together. Later, (Musiga.et.al, 2011), modeled a hierarchical system with double absorbing states for an education system, where graduates were separated from dropouts. These two papers form the basis for this study. In both, the education system under study is narrowed to one institution.

Forecasting Internal Labour Supply with a Use of Markov Chain Analysis. This paper was involving Manpower planning and forecasting can contribute to improving company's Performance. Implementation of certain straightforward planning techniques can result in higher effectiveness of human resource policy and increased competitiveness of the organization. In this paper were a number of methods of forecasting internal labour supply has been described. Markov chain model has been characterized in more detail and a numerical example of manpower planning in a retail store, based on this approach has been presented. The main limitations of implementation of Markov model to internal labour supply forecasting have been discussed and conclusions drawn in this paper (OCZKI, 2014)

Application of Markov Chain in Forecasting Demand of Trading Company. Markov chain is one of the techniques used in operations research with possibilities view that managers in organizational decision making (industrial and commercial) use it. Markov processes arise in probability and statistics in one of two ways. Markov process is a tool to predict that it can be make logical and accurate decisions about various aspects of management in the future. A stochastic process, defined via a separate argument, may be shown mathematically to have the Markov property, and as a consequence to have the properties that can be deduced from this for all Markov processes. Keywords: Markov Chain, Forecasting Demand, Trading Company. Management is defined decisions in a simple form and the most important factor for decision making is forecasting future. At present era those organizations have high complexity and much information is available, so their arrangement and refine can help to management in a logical and accurate decisions. The use of various aspects of operations research is easier to deal with complex issues for managers. Markov chain is one of the techniques used in operations research with possibilities view that managers in organizational decision making (industrial and commercial) use it. Successful decision is a picture of the future that this will not be achieved only from the prediction, based on scientific principles. Markov process is a chain of random events that by having information about the current location can be predicted next period and in fact Markov chain is a tool that used for forecasting of situation organization in future periods. (Hamed Alipoor Talemi, 2013)

Factors Affecting Enrollments Rates in Open University of Tanzania. This study explored factors affecting completion rates in open and distance learning institutions, using the Open University of Tanzania (OUT) as a case study. The study was done in Dar es Salaam, Kagera, Mwanza, Shinyanga, Singida, Dodoma, Kilimanjaro and Tanga regions. The study employed a mixed methods design, with emphasis on qualitative approach. It involved 224 respondents reached through questionnaires, focus group discussions, and interviews. The findings indicated that students failed to complete studies on time because of poor examination feedback, low students' commitment to studies, and students' low income, shortage of study materials, library materials, and supervision problems. The study established opportunities to be harnessed to improve completion rate to be, printed study materials, science laboratories, TV and radio programs, forging partnerships, as well as investing more in Open Educational Resources (OERs). The strategies to improve completion rate that were found included, use of e-counseling, mobile phones, group discussions with emphasis on interaction and networking, enhanced face to face sessions, enhanced orientation seminars and institutionalizing on demand examinations (ODEX). The recommendations made to improve completion rate at OUT were to adopt a study materials policy that would demand course tutors to ensure that within two years of their teaching they should have developed course materials for their respective courses. Information Communication Technologies (ICT) training programs should feature in every face to face and orientation programmers' that is organized. The Government of Tanzania was called upon to improve electrical power supply in rural areas to facilitate rural based students to learn using OERs ((Rwejuna, 2013).

Application of Markov Chain to the Assessment of Students' Admission and Academic Performance in Ekiti State University. This paper studies the pattern of students' enrolment and their academic performance in the Department of Mathematical Sciences (Mathematics Option) Ekiti State University, Ado – Ekiti, Nigeria. In this paper, a transition matrix was developed for ten consecutive academic sessions. The probabilities of absorption (Graduating and Withdrawal) were obtained. Also fundamental matrix was obtained to determine the expected length of students' stay before graduating. Prediction was made on the enrolment and academic performance of students (R.A. Adeleke, 2014)

An Enrollment Retention Study Using a Markov Model for a Regional State University Campus in Transition. This study shows that Enrollment retention rates are an accepted indicator of a university's success in providing quality degree programs and learning environments which will lead to a student's continued enrollment and timely graduation from the university campus

The university from which the data has been collected was a regional campus of a state university which has, over the past 40 years, transitioned from an off-campus site, to a two-year community college, and recently, to an autonomous four- year-degree granting institution with graduate programs in planning and student housing under construction. Retention figures tend to be used for comparison against national averages, and against rates of local, as well as similar peer institutions. This project was seeking to compare various retention rates within the schools of this campus for the purpose of gaining greater insight into areas where retention to be improved (Wainwright, 2007).

Generally, most of these studies are carried out in public organizations level, universities and single institutions of learning. Little has been done in modeling progress of a group of students in specific program within the University using Markov chains.

CHAPTER THREE

3.0 RESEARCH DESIGN AND METHODOLOGY

3.1 INTRODUCTION

This chapter describes the methods applied to get data for this study. The chapter is organized in different sub sections, which are selection of the study area, description of area under study, population understudy, sampling frame and procedure, data gathering, method of data collection, data analysis and presentation

3.2 Selection of the study area

We conducted our study in Mwenge Catholic University (MWECAU), specifically in Bsc. Mathematics and Statistics program students. We chose this group as our study area because were small enough to put in control and not wide in such a way that we cannot handle the data.

3.3 Description of the study area

Mwenge Catholic University (MWECAU) formally Mwenge University College of Education (MWUCE) was established in 2005 and is the successor to St. Joseph of Augustine's Teachers College, which opened to Diploma in Education courses in July 2001. The first students graduated in May 2003. Construction work started in 1995 with financial assistance from MISEREOR, an aid organization under the auspices of the Bishops‟ Conference of Germany. The College is owned by the Catholic Church but has an open access equal opportunities policy for both staff and students. The College was opened in response to Government requests for private organizations to become involved in secondary and higher education. When St. Joseph's Teachers College opened, it prepared students for the National Diploma in Education, a two year course, which qualifies graduates to teach in secondary Schools. The College specialized in teaching Science and Mathematics methods courses but in July 2003 added English methods. MWUCE continues to offer teacher training courses at degree level while also diversifying into other areas of study including Sociology and Social Work, Geography and Environmental Studies as well as Mathematics and Statistics. MWUCE also has programmes in Computer Science, Business Administration, Accounting and Laws at Certificate and Diploma levels. In 2013/2014 MWUCE launched a Postgraduate Diploma in Education. It is envisaged that MWUCE will continue to expand its programmes, the emphasis being Science (MWECAU_PROSPECTUS, 2015_2016).

3.4 Population

The population taken was the students taking Bsc. Mathematics and statistics program in Mwenge Catholic University in the specific period of time. The population of the students used in the study depended on available complete specific information for specific year.

3.5 Research design

The research design used was case study research design; a case study is the use of a single person in a research study. The deep information of our study were obtained from mathematics and statistics department at MWECAU about BSMST student's graduation rate from 2013/2014 to 2014/2015

3.6 Data collection procedures

Data collection procedures used by read the written documents from mathematics and statistics department concern BSMST students of the year2014/2015.

The source of data collection was only secondary data used from related literature on the topic done elsewhere and in particular on factors contributes to completion rate of BSMST students in MWECAU.

3.7 The sampling technique

The sampling technique used in our study was deliberate sampling to select BSMST students in MWECAU to represents the universe where; *Deliberate sampling* is also known as purposive or non-probability Sampling. This sampling method involves purposive or deliberate selection of particular units of the universe for constituting a sample which represents the universe. When population elements are selected for inclusion in the sample based on the ease of access; it can be called convenience sampling.

3.10 DATA ANALYSIS AND PRESENTATION

3.10.1 Model Development

The Markov model is based on an underlying stochastic process in which a system in one state, say $S_i$moves to a subsequent state, say S_j. The states are sometimes referred to as the Current state and the Nextstate. The act of moving from one state to the next is referred to as a step or transition.

In our study, a Markov chain model with t non-absorbing states; $1, 2 \dots., t$ corresponded to the classes of the education system and r absorbing states corresponded to the various final qualifications was considered. This implies that $N = t + r$, where N is the total number of possible states of the education system. Transition probabilities between absorbing states represented by one, hence the use of identity matrix. Transition from an absorbing state to a non-absorbing state which is impossible, was represented by zero, hence the matrix of zeroes. Transition from non-absorbing states to absorbing states are possible, likewise transitions between non-absorbing states.

3.10.2 Absorbing Markov Chain

A Markov Chain is absorbing if it has at least one absorbing state and if from every state it is possible to go to an absorbing state (not necessarily in one step).In an absorbing Markov Chain, a state which is not absorbing is called transient. If we have an absorbing Markov chain with t transient states and r absorbing states, the transition probability matrix $\boldsymbol{P}$, will take the following canonical form;

$$P = \begin{bmatrix} Q & R \\ 0 & I \end{bmatrix}$$

Where;

$\boldsymbol{Q}$ Is a $t \times t$ matrix, q_{ij} being the probability that a student who is in class i at time $(t-1)$ will be in class j at time t; $i, j = 1, 2, 3 \ldots, t$,

$\boldsymbol{R}$ Is a non-zero $t \times r$ matrix, r_{ij} being the probability that a student in class i at time $t-1$ will graduate with final education k at time t; $i = 1, 2, 3 \ldots, t$ & $k = 1, 2, 3 \ldots, r$;

$\boldsymbol{O}$ is an $r \times t$ zero matrix and,

$\boldsymbol{I}$ Is an $r \times r$ identity matrix.

The first t states are transient states and the last r states are absorbing states (Burke, 1972)

The ij^{th} entry, P_{ij}^{n} of the matrix P_n gives the probability that the Markov chain, starting in state S_i will be in state s_j after $n\ steps$ by Chapman-Kolmogorov theorem. The canonical form of the matrix P^n is given as;

$$P^n = \begin{bmatrix} Q^n & R^n \\ 0 & I \end{bmatrix}$$

Where;

$Q^n = t \times t$ Matrix which gives the probability that a student who is in class i will be in class j, n years later; i; $j = 1, 2, 3 \ldots, t$,

$R^n = (\boldsymbol{I} + \boldsymbol{Q} + \boldsymbol{Q}^2 + \cdots \ldots\ldots + \boldsymbol{Q}^{n-1})\boldsymbol{R}$.Is a $t \times r$ matrix which gives the probability that a student who is in class i will graduate with final education k within n years, $i = 1, 2 \ldots, t$; $k = 1, 2 \ldots, r$. It is also called the completion rate,

$\boldsymbol{O} = r \times t$ Matrix of zeros which gives transition probabilities from absorbing states to non absorbing states in n steps and,

$\boldsymbol{I} = r \times r$ Identity matrix which gives transition probabilities between absorbing states in n steps.

(al S. e., 1988) and (al U. e., 1986)concluded that while the probability matrix summarizes transition probability of the cohort, the transient states analysis allows prediction or prognosis for an individual subject, given their starting state, current state and cycle.

3.10.3 The Fundamental matrix

For an absorbing Markov chain, the matrix **N** is called the fundamental matrix where;

$$\boldsymbol{N} = (\boldsymbol{I} - \boldsymbol{Q})^{-1} = (\boldsymbol{I} + \boldsymbol{Q} + \boldsymbol{Q}^2 + \cdots \ldots \ldots \ldots. (3.1)$$

Where the ij^{th}entry $n_{ij}$of the matrix $\boldsymbol{N}$is the expected number of times the process is in the transient state $S_j$given that it started in the transient state S_i.

Hence$\boldsymbol{N}$gives the average number of cycles that a subject resides in transient states before absorption, given a specified starting state.

The states of the education system were denoted by integers $1, 2, 3, \ldots, n$at time $t = 0, 1, 2, \ldots$ While p_{ij}denoted the probability that a student in a year $i$at time $t-1$will be in year $j$at time t, then the transition matrix,

$$\boldsymbol{P} = ((P_{ij}));\ i, j = 1, 2, \ldots, N.$$

The non-absorbing states (transient states) were four and they were represented by values *1, 2* ,and *3*.This implies that the $\boldsymbol{Q}$ component of the transition matrix $\boldsymbol{P}$ is a 3×3matrix.The numbers of absorbing states will be two and represented by values *4* and *5*.The absorbing state 4 represents graduation from the system after attaining the maximum qualification and state 5 represents dropping out of the system before attaining the maximum qualification. Hence the $\boldsymbol{R}$component of matrix $\boldsymbol{P}$ will be a 3 × 2 matrix.

According to (Berk, 1983), the purpose of the transition matrix is to represent the probability of movement between states in a single time period. In this case, it was the probability that a student will reach a particular state by the end of the year of study.

3.10.4 Initial Transition Matrix

By letting $n_{ij}(t)$to represent the number of students in year $i$at time $(t-1)$who will be in year j at time $t$and $n_i(t-1)$,the number of students in a year$i$at time $t-1$, and by assumingthe multinomial distribution, the initial transition probabilities were estimated by;

$P_{ij} = n_{ij}(t)/n_i(t-1)$,Where; $i;\ j = 1, 2 \ldots, t$:

This was the proportion of students who were in a year$i$ at time $(t-1)$who ended up being in year$j$ at time t.

3.10.5 The n-step Transition Matrix

The $n - step$ transition probability matrix takes the canonical form below as per (Musiga.et.al, 2011) and (Berk, 1983) the Kolmogorov theorem;

$$P^n = \begin{bmatrix} Q^n & R^n \\ 0 & I \end{bmatrix}$$

The solution to this n-step transition matrix gives the state of a student $n - steps$(years later). The elements of the $n - step$ transition probability matrix represent the probabilities that an object in a given state will be in the next state $n - steps$ later.

3.10.6 Completion rates

(Musiga.et.al, 2011)Defined the dropout rate from year i, nyears later by

$$r_{ik}^{(n)} = \sum_{j=1}^{s} q_{ij}^{(n-1)} r_{ik} \quad i,j = 1,2 \dots, s$$

Where

q_{ij}^{n-1} is the probability that a student in class $i$will be in class $j, n-1$ years later and $r_{ik}$is the probability that a student in year i at time $t-1$ graduates with final education $k$at time t . It is the $(i,k)th$element of the product $Q^{n-1}\boldsymbol{R}$.

Therefore the cumulative dropout rate within y years from class i will be given by;

$r_{ij}^{(y)} = \sum_{n=1}^{y} r_{ik}^{(n)}$ $i = 1,2,\dots,t$And$k = 1,2,\dots,r$.

Where, r_{ik}^{y} is the $(i,k)th$element of $(\boldsymbol{I}+\boldsymbol{Q}+\boldsymbol{Q}^2+\cdots \dots\dots+\boldsymbol{Q}^{y-1})\boldsymbol{R}$.

3.10.7 Absorbing rates

Assuming that students will remain in the system indefinitely, then the absorbing rate is given by; $r_{i1}^{\infty} = \sum_{n=1}^{\infty} r_{i1}^{(n)}$

$$= (\boldsymbol{I}+\boldsymbol{Q}+\boldsymbol{Q}^2+\cdots \dots\dots)\boldsymbol{R}.$$
$$= (\boldsymbol{I}-\boldsymbol{Q})^{-1}\boldsymbol{R}$$

The solution to this gives the absorbing rate under double absorbing states system (Musiga.et.al, 2011) and (Uche, 1980)

3.10.8 Model Fitting

3.10.8.1 Initial transition Probabilities

By letting $n_{ij}(t)$represent the number of students in class $i$at time $(t-1)$who will be in class j at time t, and $n_i(t-1)$to represent the number of students in class $i$at time $(t-1)$, and by assuming the multinomial distribution, the transition probabilities can be estimated by;

$$P_{ij} = n_{ij}(t)/\, n_i(t-1), i,j = 1,2,..,N$$

P_{ij} is the proportion of students who are in class $i$at time $(t-1)$who ends up being in class $j$at time t.

3.10.8.2 Important Notations

Some of the notations used in the subsequent sections are defined below;

- Subscript d represents all BSMST program students
- Subscript m represents male students
- Subscript f represents female students

Such that $\boldsymbol{P}$ =BSMST program students transition probability matrix, $\boldsymbol{P_m}$ = male students transition probability matrix, and $\boldsymbol{P_f}$ =female students' transition probability matrix.

4.0 CHAPTER FOUR

RESULTS AND DISCUSSION

The table below shows the number of registered students in each level and number of students who successful completed their studies in the year

YEAR OF STUDY	2012			2013			2014			2015		
	MALE	FEMALE	TOTAL	MALE	FEMALE	TOTAL	MALE	FEMALE	TOTAL	MALE	FEMALE	TOTAL
1ST YEAR	99	14	113	76	19	95	71	23	94	67	14	81
2ND YEAR	27	2	29	94	14	108	75	18	93	69	20	79
3RD YEAR				27	2	29	93	12	105	73	18	91
NUMBER OF GRADUANTS							25	1	26			

In our research only two successive academic years were useful (2013/2014 and 2014/2015) so as to have complete number of students who were in first, second and third year within a year, because enrollment of BSMST students in MWECAU started in 2011/2012.

The table below shows the transient between two successive years (2013 and 2014) and total number of drop out / students who failed continue to the next year/level of their studies (carry over).

LEVEL	ENROLLED IN 2013/2014			PROCEEDED TO 2014/2015			DROPPED 2013/2014 TO 2014/2015		
	MALE	FEMALE	TOTAL	MALE	FEMALE	TOTAL	MALE	FEMALE	TOTAL
1STYEAR	76	19	95	75	18	93	1	1	2
2ND YEAR	94	14	108	93	12	105	1	2	3
3RD YEAR	27	2	29	25	1	26	1	1	2

4.1 The Initial Transition Probability matrix for BSMST Students

From the data obtained from Admission Office, students registered in first year, second year and third year for the year 2013/2014 and the registration for the same students in second, third year and who graduated in the following year 2014/2014 were as shown in the table 2. The dropout proportions before attaining maximum qualification for students who were in first, second and third year were $0.021052631, 0.0277777778$ and 0.068965517respectively. The proportion of students who graduated successfully after reaching third year is0.896551724. This gives rise to the Rcomponent of the matrix $\boldsymbol{P}$for the BSMST students. The $\boldsymbol{Q}$component of the matrix$\boldsymbol{P}$, whose states are transient states, its elements represent the proportions of students who proceeded to second year and third year in the following year, 2014/2015. The proportions of students who proceeded to second year and third year in the year of studies 2014/2015 0.978947368 and 0.97222222222 respectively.

Then, the transition probability for BSMST Students P is given as

$$P = \begin{bmatrix} 0 & 0.9789 & 0 & 0 & 0.0211 \\ 0 & 0 & 0.9722 & 0 & 0.0278 \\ 0 & 0 & 0 & 0.8966 & 0.1034 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix}$$

This means that $0.9789, 0.9722\ and\ 0.8966$ are the probabilities of completing first, second and third year respectively. While $0.0211, 0.0278\ and\ 0.1034$ are the probabilities of not completing first, second and third year respectively, and 1 is either a probability of completing or not completing at all.

Where Q and R components are;$R = \begin{bmatrix} 0 & 0.0211 \\ 0 & 0.0278 \\ 0.866 & 0.1034 \end{bmatrix}$ $Q = \begin{bmatrix} 0 & 0.9789 & 0 \\ 0 & 0 & 0.9722 \\ 0 & 0 & 0 \end{bmatrix}$

4.1.1The Initial Transition Probability matrix by gender

We now consider the same data in sex division, for the male students, the proportions of who dropout out before attaining maximum qualifications in first year, second year and third year $0.0132, 0.0106$ and 0.0741 respectively. The proportion of male students who graduated successfully after reaching third year 0.925. This gives rise to the R_m component of the matrix $\boldsymbol{P_m}$for the male students. The $\boldsymbol{Q_m}$ component for the transition matrix $\boldsymbol{P_m}$, for male students was given by the proportions of the male students who proceeded to second year and third in the year 2014/2015. They were given as 0.9868 and 0.9894 respectively.

Thus, the transition probability matrix $\boldsymbol{P_m}$for male students, with the double absorbing states, assuming time homogeneity is;

$$\boldsymbol{P_m} = \begin{bmatrix} 0 & 0.9868 & 0 & 0 & 0.0132 \\ 0 & 0 & 0.9894 & 0 & 0.0106 \\ 0 & 0 & 0 & 0.925 & 0.0741 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix}$$

Where the $\boldsymbol{Q_m}$ and $\boldsymbol{R_m}$are given as follows

$$\boldsymbol{Q_m} = \begin{bmatrix} 0 & 0.9868 & 0 \\ 0 & 0 & 0.9894 \\ 0 & 0 & 0 \end{bmatrix}$$

$$\boldsymbol{R_m} = \begin{bmatrix} 0 & 0.0132 \\ 0 & 0.0106 \\ 0.9259 & 0.0741 \end{bmatrix}$$

Similarly the transition probability for female students Pf is given as follows

$$P_f = \begin{bmatrix} 0 & 0.9474 & 0 & 0 & 0.0526 \\ 0 & 0 & 0.8571 & 0 & 0.1429 \\ 0 & 0 & 0 & 0.5 & 0.5 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix}$$

The R_f components for female students, is $R_f = \begin{bmatrix} 0 & 0.0526 \\ 0 & 0.1426 \\ 0.5 & 0.5 \end{bmatrix}$

The Q_f components for female students is $Q_f = \begin{bmatrix} 0 & 0.9474 & 0 \\ 0 & 0 & 0.8571 \\ 0 & 0 & 0 \end{bmatrix}$

4.2 Completion rates

Students in third year were grouped into those who dropped out of the system before attaining the maximum qualification and those who actually graduated from the system. The completion rate is the $(i, k)^{th}$ element of

$$(\boldsymbol{I} + \boldsymbol{Q} + \boldsymbol{Q}^2 + \dots + \boldsymbol{Q}^{n-1})\boldsymbol{R}$$

Where, I is an identity matrix correlated matrix to Q

For a one year, that is the coming year of studies (2015/2016), the completion rate BSMST students will be given as

$$(I + Q)R$$

By using MATLAB in matrix operation, the following output obtained,

The completion rate after one year is CR1= $\begin{bmatrix} 0 & 0.0483 \\ 0.8717 & 0.1283 \\ 0.8966 & 0.1034 \end{bmatrix}$

Within/ after two years will CR2=$(I + Q + Q^2)R$, then the output from MATLAB is

The completion rate within two years is given by CR2=$\begin{bmatrix} 0.8533 & 0.1467 \\ 0.8717 & 0.1283 \\ 0.8966 & 0.103 \end{bmatrix}$

4.2.1 Completion rates by sex

In addition to the above analysis, the results were further disaggregated by sex. The students were grouped into those who dropped out of the system before attaining the maximum qualification and those who actually graduated from the system by gender. The completion rate is the$(i, k)^{th}$element of$(\boldsymbol{I} + \boldsymbol{Q} + \boldsymbol{Q}^2 + \ldots + \boldsymbol{Q}^{n-1})\boldsymbol{R}$: in both cases. Within one year i.e. in the year 2015/2016,

The completion rate for female will be given by;$CR1_f = (I + Q_f)R_f$

Therefore the completion rate for female is $CR1_f = \begin{bmatrix} 0 & 0.1880 \\ 0.4285 & 0.5715 \\ 0.5 & 0.5 \end{bmatrix}$

For male students is $CR1_m = (I + Q_m)R_m$

Then the completion rate for male is $CR1_m = \begin{bmatrix} 0 & 0.0237 \\ 0.9161 & 0.0839 \\ 0.9259 & 0.0741 \end{bmatrix}$

Therefore from the above findings, After three years the completion rates for male students are higher compared to those for female students in all years of study, that is 0.9040, 0.9161 and 0.9259 for males and 0.4060, 0.4285 and 0.5 for females in first, second and third year respectively.

4.3 The expected duration of study

Here we consider the fundamental matrix **N**. The matrix gives the number of cycles that a subject resides in transient states before absorption, given a specified starting state. The fundamental matrix **N** is given as;

$$\boldsymbol{N} = \boldsymbol{I} + \boldsymbol{Q} + \boldsymbol{Q}^2 + \ldots = (\boldsymbol{I} - \boldsymbol{Q})^{-1}$$

To compute N for the BSMST, we first find $(\boldsymbol{I} - \boldsymbol{Q})$and it's inverse$ED = (I - Q)^{-1}(111)^T$

Therefore

$$(I - Q)^{-1} = \begin{bmatrix} 1 & 0.9789 & 0.9517 \\ 0 & 1 & 0.9722 \\ 0 & 0 & 1 \end{bmatrix}$$

The expected duration of study according to (Berk, 1983) is given by;

$$ED = (I - Q)^{-1}(111)^T$$

Therefore the Expected Duration is

$$ED = \begin{bmatrix} 2.9306 \\ 1.9722 \\ 1.0000 \end{bmatrix}$$

This implies that a student in first year has 2.9306 years of being in the system while that

In second year has 1.9722 years of schooling.

4.3.1 The expected duration of study by sex

Considering the fundamental matrix $\boldsymbol{N}$ for both cases, we can deduce the expected duration of studying for either sex. The matrix as stated earlier gives the number of cycles that a subject resides in transient states before absorption, given a specified starting state.

From the fundamental matrix $\boldsymbol{N}$;

$$\boldsymbol{N} = \boldsymbol{I} + \boldsymbol{Q} + \boldsymbol{Q}^{2} + \ldots = (\boldsymbol{I} - \boldsymbol{Q})^{-1}$$

Now for males will be as

$$ED_m = (I - Q_m)^{-1}(111)^T,$$

Therefore the expected duration for males is

$$ED_m = \begin{bmatrix} 2.9631 \\ 1.9894 \\ 1.0000 \end{bmatrix}$$

And that of females will be as

$ED_f = \left(I - Q_f\right)^{-1}(111)^T, N_f = \left(I - Q_f\right)^{-1}$

Then the expected duration for females is

$$ED_f = \begin{bmatrix} 2.7594 \\ 1.8571 \\ 1.0000 \end{bmatrix}$$

Therefore, from the above findings the expected duration for male students is generally higher compared to that of the female from the first year to third year students. This implies that male students have a higher chance of staying in the system compared to the female students.

4.4 Absorbing rates

When using this model a student has two ways of exiting the system; dropping out at any academic year or completing the three years successfully by graduating. Thus a student may graduate from the system or drop at any academic year. In the long run, the absorbing rate under double absorbing states is given by; $(\boldsymbol{I}-\boldsymbol{Q})^{-1}\boldsymbol{R}$

And in this study the BSMST absorbing rates were established to be;

$AR = (\boldsymbol{I}-\boldsymbol{Q})^{-1}\boldsymbol{R}$

The absorbing rate for BSMST is $AR = \begin{bmatrix} 0.8533 & 0.1467 \\ 0.8717 & 0.1283 \\ 0.8966 & 0.1034 \end{bmatrix}$

This implies that in Mwenge Catholic University a student in first, second and third year, has 0.8533, 0.8717and 0.8966 respectively chance of existing the system by completing successfully and 0.1467,0.1283 and 0.1034chance of not completing the system respectively. Means that out of 100 first, second and third year students registered in MWECAU 85, 87 and 90 complete the system while 15, 13 and 10 do not complete the system respectively.

4.4.1 Absorbing rates by sex

The absorbing rate was also computed with respect to gender. In the long run, the absorbing rate under double absorbing states is as discussed earlier. The absorbing rates for the male students are given as;

$$AB_m = (\mathrm{I} - \mathrm{Q}_m)^{-1} R_m$$

The absorbing rate for the male students in BSMST was established to be

$$AB_m = \begin{bmatrix} 0.9040 & 0.0960 \\ 0.9161 & 0.0839 \\ 0.9259 & 0.0741 \end{bmatrix}$$

And that for females student $AB_f = (\mathrm{I} - \mathrm{Q}_f)^{-1} R_f$

The absorbing rate for the female students in BSMST was established to be

$$AB_f = \begin{bmatrix} 0.4060 & 0.5940 \\ 0.4285 & 0.5715 \\ 0.5000 & 0.5000 \end{bmatrix}$$

From the above findings; Male students have higher absorbing state compared to female students, in first, second and third year, male student has 0.9040, $0.9161$and 0.9259 respectively chance of completing successfully from the system and $0.09600, 0.0839\ and\ 0.0741$ chance of not completing the system respectively.

For female student in first, second and third year, has 0.4060, 0.4285 and 0.5000 respectively chance of completing successfully from the system and $0.5940, 0.715\ and\ 0.5000$ chance of not completing the system respectively.

4.5 Retention rates

Retention rates can be obtained from the transition probabilities. In this study, the retention rates for first year, second year and third year was established to be 0.9789, 0.9722 and 0.8966 respectively. It can be observed that first year had the highest retention rate while third year had the lowest. The same was also determined by gender. Male student's retention rates in the district were 0.9868, 0.9894, and 0.9259 for first year, second year and third year respectively and that of female students were 0.9474, 0.8571, and 0.5, in the same order. It can be observed that the retention rate for male students in all years is than that of female students.

4.6 Enrollment Projections

From the transition matrix and the completion rate model, it is possible to project the number of students in future though in a short period of time. For example the number of students in the year 2013/2014, 2014/2015, 2015/2016 can be estimated using the transition matrix. By considering those were in 1styaer, the year 2013/2014, only 0.9789 proceeded to 2nd in the year 2014/2015, and $0.9517$are expected to proceed to 3rd year in the year 2015/2016, and finally only $0.8533$are expected to graduate from the system successfully in the year 2016.
This approach can be used to project enrollment for BSMST though in a short duration. To get the total number of students in BSMST in subsequent years, we need to know those who transit from High school to take Mwenge Catholic University to BSMST in the University. This will be treated as inflows into the system but cannot be estimated by this model as the model was specific to one university in specific course. Hence the short fall of the model.

Therefore, the total BSMST enrollment in the year 2014/2015 will be given by the sum of those who join 1st year the same year, 97.89% of those in 1styear, 97.22% of those in 2nd year, and 89.63640% of those in 3rdthe year 2013/2014.

4.7 Steady state

In this state, we project after how long n-period of time $(\boldsymbol{I} + \boldsymbol{Q} + \boldsymbol{Q}^2 + \ldots + \boldsymbol{Q}^{n-1})\boldsymbol{R}$

Will the transition matrix remain constant/unchanged. And the output gives us after three years$(\boldsymbol{I} + \boldsymbol{Q} + \boldsymbol{Q}^2)\boldsymbol{R}$ the transition probability matrix remained constant for this model. This was given as follows;

$$(\boldsymbol{I} + \boldsymbol{Q} + \boldsymbol{Q}^2)\boldsymbol{R} = \begin{bmatrix} 0 & 0 & 0 & 0.8533 & 0.1467 \\ 0 & 0 & 0 & 0.8717 & 0.1283 \\ 0 & 0 & 0 & 0.8966 & 0.1034 \\ 0 & 0 & 0 & 1.0000 & 0 \\ 0 & 0 & 0 & 0 & 1.0000 \end{bmatrix}$$

It can be concluded that within three years 85.33%, 0.8717% and 0.8966% will be the constant probabilities of completing/graduating from the system for first, second and third year student respectively.

5.0 CHAPTER FIVE
CONCLUSION AND RECOMMENDATION

5.1 Conclusions

From our database and analysis above the following conclusion were made:

- The completion rate for male students is 92% which is higher than 50% of female students for BSMST in MWECAU
- In every 100 BSMST students enrolled in MWECAU, only 90 students graduate from the system successfully
- The expected duration of schooling for male students is 2.96 which is higher compared to 2.76 for the female students of BSMST in MWECAU
- The dropout rates for female students is 50% higher than 8% that of male students in BSMST.
- Third year BSMST students in MWECAU has the least retention rate of 0.8966 compared to other academic year such as first year and second year.

5.2 Recommendations

From the above conclusions, the following recommendations were made;

- Research should be done to determine the causes of the gender disparity in expectation of schooling in universities especially science subjects.

- Research should be done to determine the root cause for high dropout rates in the third year students.

- This model/study can be extended to study the school and university enrollment trends in Country level of Government.

- The university admission office should keep clear data for student registration in each year
- Gender equality should be considered during enrollment of BSMST students in MWECAU in order to increase the completion rate for female students than now which is 0.5 compared to 0.95 completion rate of male
- In a closed hierarchical system, Markov chain model can be used to study how the system progresses with time.

- This model can only be used in a short period of time which is one of the weaknesses of the model.
- University should give more emphasize and efforts to female students like offering scholarship for further studies for females students who did better in a certain academic year so that to increase completion rate for female students in MWECAU especially BSMST students .

Appendix

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APPLICATION OF MARKOV CHAIN MODEL IN COMPLETION RATES IN MWENGE CATHOLIC UNIVERSITY.

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CERTIFICATION

The undersigned certify that they have read and here by recommend for acceptance by the Mwenge Catholic University a Research report entitled "**Application of Markov Chain Model in Completion Rates in Mwenge Catholic University",** in partial fulfillment of the requirements for the award of Bachelor degree of Science in Mathematics and Statistics.

Supervisor's name MISS MARY KAYANDA

Supervisor's Signature.....................................

Date July, 2016

DECLARATION AND COPYRIGHT

We Idd Sifael Omary, Jackson Ngong'homa and Timothy A. Peter declares that this Research report is our own original work and that it has not been presented and will not be presented to any other University for a similar or any other degree awarded

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ABREVIATIONS

MWECAU-Mwenge Catholic University

BSMST-Bachelor of Science in Mathematics and Statistics

HESLB-Higher Education Students Loan Board

OUT-Open Universities of Tanzania

ODEX-On Demand Examination

OERs-Open Educational Resource

MWUCE-Mwenge University College of Education

TABLE OF CONTENTS

APPLICATION OF MARKOV CHAIN MODEL IN COMPLETION RATES IN MWENGE CATHOLIC UNIVERSITY.

Abstract: Completion Rate and Enrollment forecasting is an essential element in budgeting, resource allocation, and the overall planning for the growth of education sector. Our paper purposeful demonstrated the use of Markov chain techniques in studying progression of BSMST Programme Students from the time of entry/enrollment in each academic year to graduation after the expected year of study in MWECAU. The target population included all BSMST programme students in MWECAU from 2013 to 2015. The model used to determine the student's completion/dropout rate, retention rate and the expected duration of completing by sex. We established the completion rates for male students and that of female students and dropout rates. We saw how long Markov Transition Probability Matrices of BSMST students at MWECAU will be at a steady state. How the established completion and dropout rates will be in Absorbing rates/States. Also we saw female expectation of university education compared to male students in BSMST Programme students. The model was only suitable in make a short period projections.

Keywords: Markov chain, transition matrix, steady state, absorption rate, Completion rate, retention rates, transient matrix.