Sakata model revisited: Hadrons, nuclei, and scattering

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Modern theories of strong interactions suggest that baryon-antibaryon forces can be strongly attractive, and manifestations of "baryonium" states have been seen in experiments. In light of these new data, we attempt to revisit the Fermi-Yang-Sakata idea that mesons and baryons are bound states of few fundamental "sakatons" identified with p, n, Λ, and Λ_c particles. We optimized parameters of inter-sakaton potentials and calculated meson and baryon mass spectra in a fair agreement with experiment. Moreover, the same set of potentials allows us to reproduce approximately elastic scattering cross sections of baryons and binding energies of light nuclei and hypernuclei. This suggests that the Sakata model could be a promising organizing principle in particle and nuclear physics. This principle may also coexist with the modern quark model, where both valence and sea quark contributions to the hadron structure are allowed.

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I. INTRODUCTION

The quark model and its inspired quantum chromodynamics (QCD) are universally accepted as foundations of modern theory of strong interactions and integral parts of the Standard Model. Successes of this approach are well documented [1–3]. However, it is peculiar that this theory forbids observation of its fundamental constituents (quarks, gluons) and their properties (fractional charges, color). Theoretical manipulations (spontaneous symmetry breaking, confinement) lock these constituents inside hadrons, so that their direct measurements are deemed impossible. Low energy QCD calculations are extremely difficult [4, 5], and quark-gluon descriptions of hadrons and nuclei are still incomplete. For example, quark models predict too many excited baryon states, which do not show up in experiments [2, 6]. There are many puzzles in the meson spectrum as well [7, 8].

So, it would be of some interest to check different approaches, even if they lack the full rigor of QCD. One such idea was formulated by Fermi and Yang in 1949 [9]. They conjectured that if the nucleon-antinucleon attraction were strong enough, then the triplet of pions could be explained as a bound state $N\overline{N}$. This idea was developed into a full-fledged theory of hadrons by Sakata in 1956 [10], i.e., long before the advent of quarks.

The beauty of the Sakata model was that the number of arbitrary assumptions was reduced to a minimum. Elementary constituents (sakatons) were chosen to be the familiar proton (p), neutron (n), lambda-hyperon (Λ), and their antiparticles. (In Table I we also added the charmed Λ_c baryon to this list [11].) Moreover, it is now established (for some references see section IV) that baryons and antibaryons experience strong attraction and that at least some mesons can be explained as baryon-antibaryon bound states ("baryonia"). To these facts, the Sakata model adds a single hypothesis that *all* mesons and *all* non-sakaton baryons are multisakaton bound states.

In 1956 Matumoto [12] proposed a simple formula, which described masses of mesons and baryons in a surprisingly good agreement with experiment. This approach was based on the idea (borrowed from electrodynamics) that sakatons and antisakatons attract each other, while sakaton-sakaton and antisakaton-antisakaton interactions are repulsive. The energy $E_{Z\overline{Z}}$ of the $Z\overline{Z}$ attraction was assumed to be exactly equal to repulsion energies in corresponding ZZ and $\overline{Z}\overline{Z}$ pairs.[1] Then the mass M of any $\mathcal{N}$-sakaton group could be calculated as the sum of masses m_i of constituents (see Table I) plus the sum of interaction energies E_{ij} (divided by c^2) over all sakaton pairs:

TABLE I. Model properties of sakatons.

sakaton	Mass m MeV/c^2	Electric charge Q	Strangeness S	Charm C	Spin s
n	940	0	0	0	1/2
p	940	1	0	0	1/2
Λ	1116	0	-1	0	1/2
Λ_c	2285	1	0	1	1/2

$$M = \sum_{i=1}^{\mathcal{N}} m_i + \sum_{i<j}^{\mathcal{N}} E_{ij}/c^2. \tag{1}$$

In column 2 of Table II we listed Matumoto interaction energies

$$E_{Z\overline{Z}} = -E_{ZZ} = -E_{\overline{Z}\overline{Z}} \tag{2}$$

fitted to experimental masses of few reference particles shown in column 3. For example, the $\Lambda\Lambda$ interaction energy (1270 MeV) was calculated from the $\Xi(=\Lambda\Lambda\overline{N})$ baryon mass, where the $\Lambda\overline{N}$ attraction was taken from the $K(=\Lambda\overline{N})$ meson.

Without introducing any adjustable parameters, Matumoto's formula reproduced masses and stabilities of many particles such as $Z\overline{Z}$ mesons and $ZZ\overline{Z}$ baryons. Moreover, this approach described the famous Ω^- particle as a pentasakaton

[1] We use the symbol N to collectively denote nucleons n and p, while Z means all sakaton types considered in this work: $Z = (p, n, \Lambda, \Lambda_c)$. Addition of the bottom Λ_b baryon to this list should be also possible.

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($= \Lambda\Lambda\Lambda\overline{pn}$) and correctly predicted its stability with respect to dissociation into $K^-+\Xi^0 = (\Lambda\overline{p})+(\Lambda\Lambda\overline{n})$. The calculated binding energy of 404 MeV can be compared with the experimental value B.E.(Ω^-) = 136 MeV. Unfortunately, formula (1) predicted also non-existent strangeness $S = -3$ particles $\Lambda\Lambda\Lambda\overline{nn}$ and $\Lambda\Lambda\Lambda\overline{pp}$ with electric charges $Q = 0$ and -2, respectively, as well as many other non-existent multisakaton species [13]. So, it appears that Matumoto's approximation did capture important trends in the hadron structure, but overestimated the binding.

TABLE II. Interaction energies $E_{Z\overline{Z}}$ in the Matumoto formula and strengths of our potentials $V_{Z\overline{Z}}(r = 0.5$ fm) and $V_{ZZ}(r = 0.5$ fm) = $V_{\overline{ZZ}}(r = 0.5$ fm). For reference particles we show their approximate mass (in MeV/c^2) and sakaton composition.

Sakaton pair $Z\overline{Z}$	$E_{Z\overline{Z}}$ (MeV)	Reference particle	$V_{Z\overline{Z}}(0.5)$ (MeV)	$V_{ZZ}(0.5)$ (MeV)
$p\overline{p}$, $n\overline{n}$	-1742	$\pi^0(138)(= p\overline{p}, n\overline{n})$	-1776	4809
$p\overline{n}$	-1742	$\pi^+(138)(= p\overline{n})$	-1734	4785
$\Lambda\overline{N}$	-1562	$K(494)(= N\overline{\Lambda})$	-1590	1886
$\Lambda\overline{\Lambda}$	-1270	$\Xi(1318)(= \Lambda\Lambda\overline{N})$	-1035	1358
$\Lambda_c\overline{N}$	-1358	$D(1867)(= \Lambda_c\overline{N})$	-1301	1775
$\Lambda_c\overline{\Lambda}$	-1048	$\Xi_c(2469)(= \Lambda_c\Lambda\overline{N})$	-839	1456
$\Lambda_c\overline{\Lambda_c}$	-827	$\Xi_{cc}(3621)(= \Lambda_c\Lambda_c\overline{N})$	-850	1379

Another challenge is that the model forbids production of neutral kaons in $\overline{p}p$ collisions, whereas this reaction is just as probable as the production of charged kaons [14, 15]. Lipkin even called it *"The right experiment that killed the Sakata model"* [16]. In our opinion, this drawback is not sufficient to invalidate the Sakata-Matumoto approach. Indeed, the quark model also has to struggle to explain certain $\overline{p}p$ reaction channels that are supposed to be suppressed by the OZI rule. See [17] and references therein.

In 1991 Timmermans wrote in his Ph.D. thesis [18]: *"Although everyone nowadays believes in the existence of quarks and gluons, it can still be argued that the antinucleon-nucleon meson-exchange potential should have bound states at positions corresponding to the masses of the mesons. [...] If one takes this (maybe naive) Fermi-Yang picture seriously, it is of course a very heavy constraint on the dynamics to demand that the antinucleon-nucleon bound-state spectrum coincide with the experimental meson spectrum. In the nucleon-nucleon potential the only bound state, the deuteron, is already quite constraining. Nevertheless, the Fermi-Yang approach remains intriguing, and it will be interesting to pursue the issue sometime using a realistic antinucleon-nucleon force."* This is exactly the approach we decided to take in this work.

In our previous publication [11], we enhanced the Sakata-Matumoto theory by introducing realistic distance-dependent inter-sakaton potentials. We calculated multisakaton bound states by numerical solutions of the corresponding Schrödinger equations. A fair agreement was obtained with masses and stabilities of known mesons and baryons. However, our previous results turned out to be inadequate in two areas. First, scattering cross sections of baryons were overestimated by several orders of magnitude. Second, our NN potentials were completely repulsive, so they could not explain the binding of protons and neutrons in nuclei.

In section II we recalibrate sakaton interaction potentials for a better description of these two important aspects. In section III we demonstrate that our Hamiltonian yields a qualitatively reasonable description of strongly interacting systems. Some possible experimental manifestations of the Sakata model are discussed in section IV. Conclusions and directions for further studies are formulated in section V. The calculation tools used in this works are briefly described in Appendix.

II. MODEL

A. Selection of interaction

To proceed with calculations, we have to specify interaction potentials between sakatons. Despite identification of sakatons with real particles p, n, Λ, Λ_c, this information is not readily available. Indeed, many accurate nucleon-nucleon potentials were reported in the literature [19–22], but much less is known about interactions in other sectors ($N\overline{N}$, $N\Lambda$, etc.). Theoretically, $N\overline{N}$ interactions are usually derived by applying the G-parity transformation to NN potentials obtained from meson exchange models [23–25]. However, this method is applicable only at long and intermediate distances. The most interesting short-distance behavior usually remains ill-defined. Often inter-particle interactions are fitted to reproduce low-energy properties, like binding energies of nuclei and scattering data for collision energies below 1 GeV. However, in order to represent the deep sakaton bindings in mesons and baryons, we need potentials that describe a strong short-range attraction ($E < -1$ GeV) in $Z\overline{Z}$ pairs, which is difficult to measure.

We are not even sure about the basic functional form of sakaton interactions. Just to get a preliminary idea about the general shape of the potential, we selected four functions most popular in nuclear physics: Yukawa, exponential, Gaussian, and square well. All these potentials depend on two parameters A and α (see the first column in Table III). We decided to fit these parameters to the masses of π and $\pi(1300)$ mesons. In our interpretation, the π meson is the ground $1S$ state in the $N\overline{N}$ potential, and $\pi(1300)$ is the $2S$ excited state. The optimized parameters are shown in Table III and the resulting potentials are plotted in Fig. 1.

TABLE III. Attractive potentials reproducing experimental binding energies of two $p\overline{p}$ bound states: $B.E.[1S] = B.E.(\pi) = 1742$ MeV and $B.E.[2S] = B.E.(\pi(1300)) = 580$ MeV. Last column shows calculated $\overline{p}p$ elastic scattering cross sections at $P(lab) = 2$ GeV/c.

Potential type	α	A	$\sigma(2)$ (mbarn)
Yukawa*: $V = Ae^{-\alpha r}/r$	0.09 fm^{-1}	-551 MeV× fm	12900
exponent: $V = Ae^{-\alpha r}$	1.8 fm^{-1}	-4320 MeV	298
Gaussian: $V = Ae^{-\alpha r^2}$	1.45 fm^{-2}	-2880 MeV	120
square well: $V = A\theta(\alpha - r)$	0.85 fm	-2170 MeV	62
experiment [6]			≈ 32

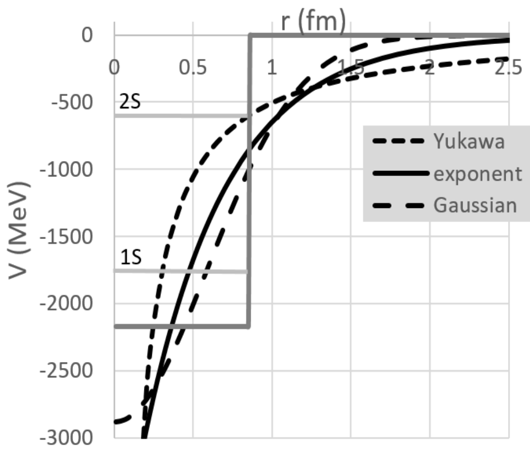

FIG. 1. Four kinds of model $N\overline{N}$ potentials: Yukawa, exponential, Gaussian, and square well. See Table III. Positions of $1S$ and $2S$ energy levels are indicated by grey lines. They are interpreted as π and $\pi(1300)$ mesons, respectively.

We should note, however, that the exact fit was not possible with the Yukawa potential even in the limit of very low exponent α. The potential shown in the table had the exponent of $\alpha = 0.09$ fm^{-1} taken from our previous study [11]. It overestimates the separation between the two levels: $E[1S] = 141$ MeV and $E[2S] = 1481$ MeV.

The interactions in the table are arranged from the long-range Yukawa to the extremely sort-range square well potential. The total elastic scattering cross section σ decreases predictably in this sequence. However, even the square well interaction cannot reproduce the low experimental value of $\sigma(2) = 32$ mbarn.

These observations have few important implications for the strategy of our studies. In particular, we are not expecting a perfect agreement with experimental data when using simple analytical forms of inter-sakaton potentials. Therefore, we will seek only a qualitative agreement with experiments, while trying to cover a broad spectrum of properties. In particular, when considering mesons, baryons, nuclei, and hypernuclei we will be interested in their binding energies more than in absolute values of their masses. It is important to formulate our model in such a way that particles, which are known to be stable with respect to strong decays, are not dissociating spontaneously into smaller fragments. On the other hand, sakaton combinations, which are unstable in nature, should not have positive calculated binding energies. Finally, we would like to make sure that our potentials can reproduce scattering cross sections at least within an order of magnitude.

As these studies are qualitative and approximate, we will use a non-relativistic description with simple central momentum-independent potentials and ignore all spin-dependent effects.

B. Hamiltonian

Taking into account all the above considerations, we decided to describe any $\mathcal{N}$-sakaton system by the Hamiltonian

$$H = \sum_{i=1}^{\mathcal{N}} m_i c^2 + \sum_{i=1}^{\mathcal{N}} \frac{p_i^2}{2m_i} + \sum_{i<j}^{\mathcal{N}} V_{ij}(r_{ij}),$$

where $\mathbf{p}_i$ are momenta of the sakatons and $r_{ij} \equiv |\mathbf{r}_i - \mathbf{r}_j|$ are their relative distances. Masses m_i of the sakatons are taken from Table I. The distance-dependent potentials $V(r)$ were selected as superpositions of three Gaussian-like terms

$$V_{ij}(r) = A_{ij} z_i z_j e^{-\alpha_{ij} r^2} + B_{ij} e^{-\beta_{ij} r^2} + C_{ij} z_i z_j r e^{-\gamma_{ij} r^2}, \quad (3)$$

where $z_i = +1$ for sakatons and $z_i = -1$ for antisakatons. The first term in (3) is of the Matumoto type as it describes attraction in sakaton-antisakaton pairs ($z_i z_j = -1$) and equal repulsion in sakaton-sakaton and antisakaton-antisakaton pairs ($z_i z_j = 1$). Our preliminary tests indicated that this interaction alone was inadequate as it systematically overestimated binding and thus predicted stability of many nonexistent species, similar to the original Matumoto formula (1). Some extra repulsion for all pairs is provided by the second term. The third term has the form "Gaussian × distance," which contributes only at intermediate distances. It was designed primarily to represent the nucleon-nucleon binding in nuclei.

TABLE IV. Parameters of sakaton-sakaton interaction potentials (3) optimized in this work.

ZZ	A MeV	α fm^{-2}	B MeV	β fm^{-2}	C MeV/fm	γ fm^{-2}
pp, nn	5232.231	1.432	2169.431	1.433	-880.000	0.745
pn	5282.496	1.410	2175.246	1.419	-1090.000	0.730
$N\Lambda$	2687.231	1.442	200.231	1.205	-318.000	0.637
$\Lambda\Lambda$	2052.000	1.633	209.231	1.031	-400.482	0.720
$\Lambda_c N$	2415.000	1.442	320.000	1.200	-342.000	0.637
$\Lambda_c\Lambda$	2072.000	1.613	410.000	1.140	-580.000	0.810
$\Lambda_c\Lambda_c$	1680.000	1.633	340.000	1.000	-5.000	0.110

Optimized parameters of sakaton interaction potentials are collected in Table IV. They were fitted by a combination of the Nelder-Mead "amoeba" algorithm [26] and manual optimization. Note that in our approximation, p and n sakatons have equal masses and the same interaction parameters with Λ and Λ_c. This implies that all properties calculated here (masses and scattering cross sections) are invariant with respect to replacements $p \leftrightarrow n$.

For illustration, in Fig. 2 we show the proton-proton interaction potential $V_{pp}(r)$ by the thick full line. The proton-neutron potential $V_{pn}(r)$ (thin full line) has a similar shape, but the attractive well at $r \approx 1.8$ fm is somewhat deeper, so that the bound state of the deuteron (pn) can be supported (see Table VI). It is interesting to compare our potential functions with well-known models of nucleon-nucleon interactions: the Malfliet-Tjon potential [27] and the scalar portion of the Reid potential [28]. In our case the attractive well is deeper and shifted to larger NN distances, thus implying a lower density

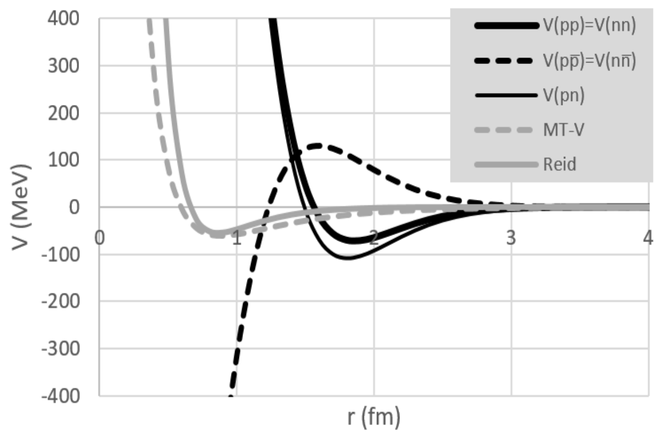

FIG. 2. Nucleon-nucleon interactions optimized in this work compared with the Malfliet-Tjon potential [27] and the Reid potential [28].

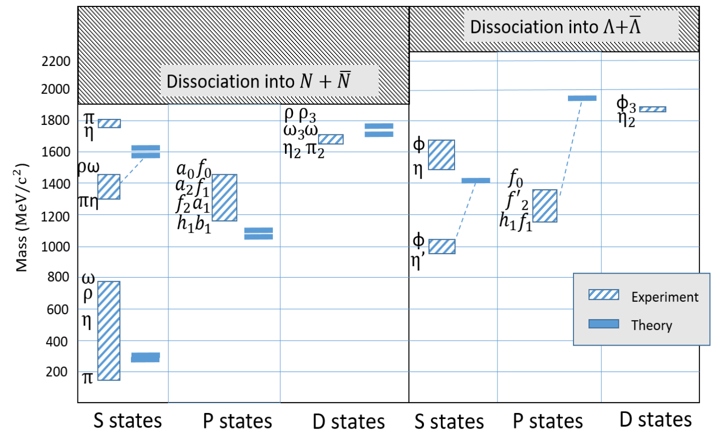

FIG. 3. Experimental [6] and calculated mass spectra of $N\overline{N}$ and $\Lambda\overline{\Lambda}$ mesons. The assignments of observed particles to $N\overline{N}$ and $\Lambda\overline{\Lambda}$ categories are tentative due to the strong $N\Lambda$ mixing.

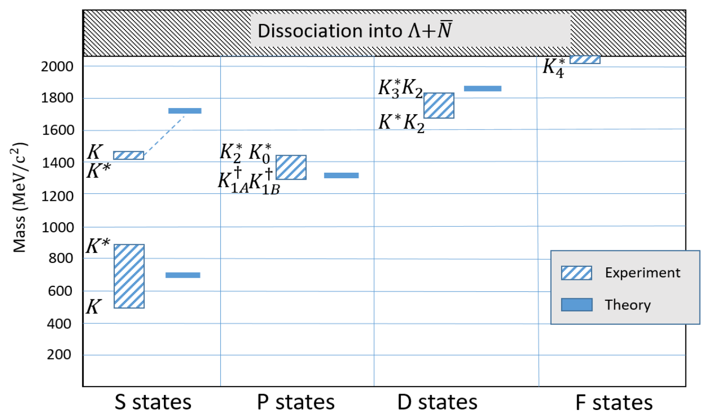

FIG. 4. Experimental [6] and calculated mass spectra of strange mesons $\Lambda\overline{N}$.

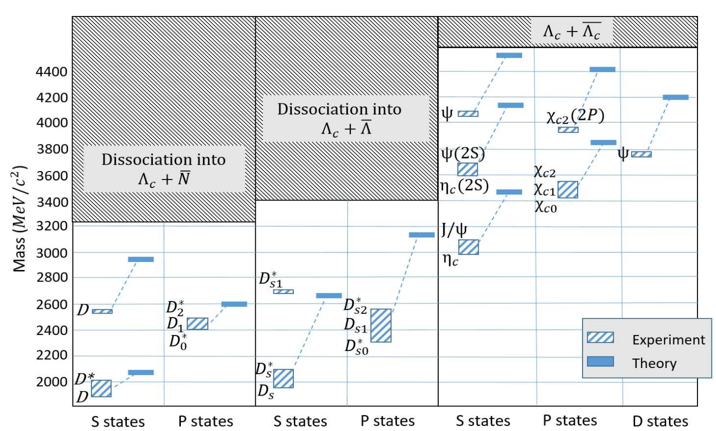

FIG. 5. Experimental [6] and calculated mass spectra of mesons with charmed sakatons $\Lambda_c\overline{N}$, $\Lambda_c\overline{\Lambda}$, and $\Lambda_c\overline{\Lambda_c}$.

of nuclear matter. The proton-antiproton interaction $V_{p\overline{p}}(r)$ (broken line) is roughly a mirror image of the $V_{pp}(r)$ potential. Potentials for other sakaton pairs have similar shapes.

Next we would like to compare our approach with the Matumoto mass formula (1). In column 4 of Table II we presented values of our optimized potentials $V_{Z\overline{Z}}(r)$ at the distance of r =0.5 fm. These values are rough indicators of interaction strengths and they correlate well with Matumoto interaction energies in column 2, thus confirming that our Hamiltonian inherits basic features of the Sakata-Matumoto theory. However, in contrast to the Matumoto approach, our potentials do not have the property (2). As shown in the last column of Table II, the repulsion in ZZ and $\overline{Z}\overline{Z}$ pairs is stronger than the $Z\overline{Z}$ attraction. This allows us to suppress the excessive binding, characteristic to the original Matumoto approach, and thus obtain stabilities of hadrons in qualitative agreement with experiments (see next section).

III. RESULTS

A. Mesons

Calculated mass spectra of $Z\overline{Z}$ mesons are compared with experimental data in Figs. 3 - 5. The obvious similarities support the Fermi-Yang-Sakata idea that mesons can be regarded as (ground and excited) energy levels in $Z\overline{Z}$ attractive potentials. Despite the similarities, the agreement is far from perfect: in many cases masses of particles are overestimated and some heavy meson states are missing in calculations. This indicates a certain repulsion bias in our fitted interactions.

We also explored properties of ground-state tetrasakatons $ZZ\overline{Z}\overline{Z}$, which can be interpreted as exotic spin-zero mesons. All 55 such compounds were found unstable with respect to dissociation into two mesons ($ZZ\overline{Z}\overline{Z} \to Z\overline{Z} + Z\overline{Z}$) or "baryon + antibaryon" ($ZZ\overline{Z}\overline{Z} \to Z + Z\overline{Z}\overline{Z}$).

B. Baryons

All 40 possible $ZZ\overline{Z}$ species and 170 $ZZZ\overline{ZZ}$ species were analyzed and only 11 of them were found stable with respect to dissociation into smaller fragments. All these stable combinations corresponded to the known baryons shown in Table V. As expected, these baryons have rather long lifetimes ($\tau > 10^{-20}s$) characteristic to electromagnetic or weak decays. Clearly, the calculated masses are overestimated and binding energies (in most cases) underestimated. This is just another indication that our optimized potentials are a bit less attractive than needed.

The remaining baryon-like combinations were found unstable with respect to dissociation into "baryon + meson(s)". So, even if such particles are observed, they have to be short-lived resonances. This is definitely true for $\Sigma_c (= \Lambda_c N\overline{N})$, which is known to dissociate into $\Lambda_c + \pi (= N\overline{N})$ with the decay width exceeding 2 MeV/c^2 [6]. The same may be true for yet unobserved $\Omega^+_{cc} (= \Lambda_c\Lambda_c\Lambda\overline{pn})$ and $\Omega^{++}_{ccc} (= \Lambda_c\Lambda_c\Lambda_c\overline{pn})$ particles, which turn out unstable in our approach. In the quark model, they correspond to the ccs and ccc combinations, respectively.

TABLE V. Masses (in MeV/c^2) and binding energies (in parentheses, in MeV) of stable compound baryons.

Baryon	sakaton composition	molecular composition	Calculated mass (B.E.)	Experimental mass (B.E.) [6]
Σ^0	$(\Lambda p\overline{p}, \Lambda n\overline{n})$	$\Lambda\pi^0$	1365.2 (13.8)	1192.6 (58.1)
Σ^+	$\Lambda p\overline{n}$	$\Lambda\pi^+$	1394.7 (5.2)	1189.4 (65.9)
Σ^-	$\Lambda n\overline{p}$	$\Lambda\pi^-$	1394.7 (5.2)	1197.4 (57.9)
Ξ^0	$\Lambda\Lambda\overline{n}$	$\Lambda\overline{K}^0$	1424.2 (378.3)	1314.9 (298.4)
Ξ^-	$\Lambda\Lambda\overline{p}$	ΛK^-	1424.2 (378.3)	1321.7 (287.7)
Ξ^+_c	$\Lambda_c\Lambda\overline{n}$	$\Lambda_c\overline{K}^0$	2808.2 (163.3)	2467.8 (316.3)
Ξ^0_c	$\Lambda_c\Lambda\overline{p}$	$\Lambda_c K^-$	2808.2 (163.3)	2470.9 (309.3)
Ξ^{++}_{cc}	$\Lambda_c\Lambda_c\overline{n}$	$\Lambda_c D^+$	4278.7 (69.2)	3621.4 (534.4)
Ξ^+_{cc}	$\Lambda_c\Lambda_c\overline{p}$	$\Lambda_c D^0$	4278.7 (69.2)	3518.9 (632.4)
Ω^-	$\Lambda\Lambda\Lambda\overline{pn}$	$\Lambda\overline{K}^0 K^-$	2100.8 (9.9)	1672.5 (136.1)
Ω^0_c	$\Lambda_c\Lambda\Lambda\overline{pn}$	$\Lambda\overline{K}^0 D^0$	3483.9 (3.3)	2695.2 (266.3)

Note that our studies suggest that compound baryons can be regarded as "molecules" consisting of one Λ or Λ_c baryon and one or two mesons (see column 3 in Table V).

C. Scattering

Calculated total elastic cross sections for pp, $p\overline{p}$, np, and Λp collisions are shown in Fig. 6. They deviate from experiment by no more than one order of magnitude in a broad range (0.1 – 2000 GeV/c) of lab momenta.

D. Nuclei and hypernuclei

Nuclei are bound states of protons and neutrons. So, any realistic theory of sakatons should describe the nuclear binding. Apparently, the Sakata-Matumoto theory was too approximate for this purpose. In the development of our approach, we made special efforts to make sure that NN potentials have the required attractive parts (see Fig. 2).

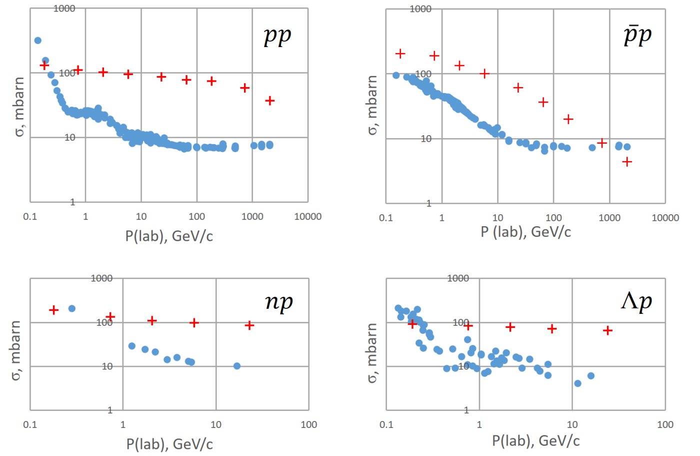

FIG. 6. Total elastic cross sections for pp, $\overline{p}p$, np, and Λp collisions. Dots "•" are experimental data [6], crosses "+" are our calculations.

For a nucleus composed of $\mathcal{N}_n$ neutrons and $\mathcal{N}_p$ protons, its experimental and calculated binding energies (in MeV) are defined as

$$B.E.(\text{exp.}) = \mathcal{N}_n \times 939.6 + \mathcal{N}_p \times 938.3 - M_{\text{exp}}c^2, \quad (4)$$

$$B.E.(\text{calc.}) = (\mathcal{N}_n + \mathcal{N}_p) \times 940.0 - M_{\text{calc}}c^2, \quad (5)$$

where $M_{\text{exp/calc}}$ is the mass of the nucleus (in MeV/c^2). These results are presented in Table VI. They demonstrate that our approach reproduces the nuclear stability pattern qualitatively correctly for systems containing up to 5 nucleons. In particular, the pn attraction is stronger than the attraction in nn/pp pairs (see Fig. 2). This explains the well-known fact that nuclei with roughly equal numbers of protons and neutrons tend to have a stronger binding.

We also calculated properties of hypernuclei, where one or more Λ-particle is bound to an $NN \cdots N$ core, and the total number of sakatons does not exceed 5. Only three such hypernuclei were found stable, as shown in Table VII. Unfortunately, our approach failed to reproduce the stability of ${}^5_\Lambda$He with respect to the dissociation $\to \Lambda + {}^4$He.

Finally, all 68 $ZZZ\overline{Z}$ tetrasakaton species (with baryon number $B = 2$) were found unstable with respect to dissociation channels into "two baryons" ($ZZZ\overline{Z} \to Z + ZZ\overline{Z}$) or "two baryons + meson" ($ZZZ\overline{Z} \to Z + Z + Z\overline{Z}$) or "nucleus + meson" ($ZZZ\overline{Z} \to ZZ + Z\overline{Z}$).

IV. DISCUSSION

We would like to stress that the Sakata and quark models need not to be antagonistic. The idea that basic sakatons are composed of three quarks (e.g., $p = uud$, $\Lambda = uds$) may co-exist with the idea that other hadrons are made of sakatons,

TABLE VI. Total binding energies (4) and (5) of light nuclei. For some species our calculations did not converge, possibly due to numerical instability of the FBS computer code [29].

sakaton composition	Nuclear symbol	Exp. binding energy (MeV) [30]	Calc. binding energy (MeV)
pp		unstable	unstable
pn	^{2}H	2.2	5.1
nn		unstable	unstable
ppp	^{3}Li	6.8	diverged
ppn	^{3}He	7.7	12.4
pnn	^{3}H	8.5	12.4
nnn		unstable	diverged
$pppp$		unstable	unstable
$pppn$	^{4}Li	4.6	12.0
$ppnn$	^{4}He	28.3	12.9
$pnnn$	^{4}H	5.6	12.0
$nnnn$		unstable	unstable
$ppppp$		unstable	diverged
$ppppn$		unstable	9.7
$pppnn$	^{5}Li	26.4	10.9
$ppnnn$	^{5}He	27.4	10.9
$pnnnn$	^{5}H	6.7	9.7
$nnnnn$		unstable	diverged

TABLE VII. Binding energies of light hypernuclei. B_Λ is the energy required to separate Λ from the non-strange core nucleus.

sakaton composition	Nuclear symbol	B_Λ (MeV) experiment [31]	B_Λ (MeV) this work
Λpn	$^3_\Lambda$H	0.13	0.21
Λpnn	$^4_\Lambda$H	2.04	0.30
Λppn	$^4_\Lambda$He	2.39	0.30
$\Lambda ppnn$	$^5_\Lambda$He	3.12	unstable

especially if the hadrons are allowed to have contributions from both valence quarks and sea quark-antiquark pairs [32, 33]. In the examples of the K^+ meson and the Ξ^- baryon, we obtain

$$K^+ = p\overline{\Lambda} = (uud)(\overline{u}\overline{d}\overline{s}) = [u\overline{s}](u\overline{u})(d\overline{d}),$$
$$\Xi^- = \Lambda\Lambda\overline{p} = (uds)(uds)(\overline{u}\overline{u}\overline{d}) = [ssd](u\overline{u})(u\overline{u})(d\overline{d}),$$

where the valence quark components are placed in square brackets.

There is a growing body of evidence in favor of the Sakata hypothesis. For example, now theorists agree that baryon-antibaryon (in particular, nucleon-antinucleon) potentials have to be much more attractive than baryon-baryon ones [23–25]. For many years this belief fueled searches for "baryonia" – hypothetical strongly bound $Z\overline{Z}$ states [34–37]. Exact shapes of $Z\overline{Z}$ potentials are difficult to establish, but some models support baryon-antibaryon states with binding energies of few hundred MeV [38, 39], or even above 1 GeV [24], i.e., in the range consistent with the Fermi-Yang-Sakata-Matumoto model. Interpretations of these theoretical results vary. One school of thought claims that the annihilation part of the potential is rather strong, so that only few broad $N\overline{N}$ states can survive [40, 41]. Shapiro and co-workers [23, 42] argued that the annihilation is less important and that many narrow bound or resonant $N\overline{N}$ states should exist near the $2m_N$ threshold.

From the point of view of the Sakata model, all mesons are, in fact, baryonium states. Then the importance of the nucleon-antinucleon annihilation can be deduced from the comparison of $\pi^0(= p\overline{p}, n\overline{n})$ and $\pi^\pm(= p\overline{n}/n\overline{p})$ decays. In the latter case the annihilation channel is closed, so the lifetime of charged pions is 9 orders of magnitude longer than for the neutral pion. A similar situation occurs with Σ baryons, which can be viewed as $\Lambda\pi$ molecules, according to Table V. The most unstable "atom" of the neutral $\Sigma^0(= \Lambda\pi^0)$ molecule is π^0, so the decay proceeds via π^0 annihilation: $\Sigma^0 \to \Lambda\gamma$. As the fast annihilation channel is not available for the charged $\Sigma^\pm(= \Lambda\pi^\pm)$ baryons, they decay primarily through decomposition of the Λ atom: $\Sigma^\pm \to N\pi$. This process is 9 orders of magnitude slower than the π^0 annihilation in Σ^0.

The Sakata model predicts a clustering of meson $Z\overline{Z'}$ states near the thresholds $m_Z + m_{\overline{Z'}}$, because this is the boundary between the discrete and continuous sakaton-antisakaton energy spectra. One can also expect a number of quasi-bound states above the threshold due to repulsive barriers at intermediate distances (see dark broken line in Fig. 2). Many experimental manifestations were attributed to near-threshold baryon-antibaryon states, and Sakata's ideas turned out to be helpful in organizing these data [43, 44].

Probably, the most convincing evidence of baryonium is the enhancement in the $p\overline{p}$ invariant mass spectrum in decays of the J/ψ particle [45–48]. Many groups attempted to fit these data by assuming a strongly bound protonium ($p\overline{p}$) state below the $2m_p$ threshold (see Table VIII and [49–51]). The results of different fits vary substantially, but they remain suspiciously close to the properties of the known mesons $\eta(1760)$ and $\pi(1800)$, which are described as 3S excited states of the $N\overline{N}$ system in our approach (see the leftmost panel in Fig. 3). If this identification is confirmed, then the Sakata-Matumoto model will get a solid experimental support.

TABLE VIII. Properties of the proposed $p\overline{p}$ bound state compared with the known mesons $\eta(1760)$ and $\pi(1800)$.

Particle	Mass (MeV/c^2)	Width (MeV/c^2)	J^{PC}	Fitted to decay	Ref.
$p\overline{p}$	1859^{+6}_{-27}	< 30	0^{-+}	$J/\psi \to \gamma p\overline{p}$	[45]
$X(1835)$	1833.7 ± 7	67.7 ± 21	0^{-+}	$J/\psi \to \gamma\pi^+\pi^-\eta'$	[46]
$p\overline{p}$	1831 ± 7	< 153	0^{-+}	$J/\psi \to \gamma p\overline{p}$	[46]
$p\overline{p}$	1871.6	53	0^{-+}	$J/\psi \to \gamma p\overline{p}$	[52]
$p\overline{p}$	1836-1846	"sizable"	0^{-+}	$J/\psi \to x p\overline{p}$	[53]
$X(1835)$	1836.5 ± 3.0	190.1 ± 9.0	0^{-+}	$J/\psi \to \gamma\pi^+\pi^-\eta'$	[47]
$p\overline{p}$	1832 ± 19	< 76	0^{-+}	$J/\psi \to x p\overline{p}$	[48]
$\eta(1760)$	1751 ± 15	240 ± 30	0^{-+}		[6]
$\pi(1800)$	1812 ± 12	208 ± 12	0^{-+}		[6]

One more evidence in support of the Sakata model comes from the observation [54] that K^+ and K^- mesons differ dramatically in their interactions with the nucleus: the K^- meson attracts strongly to nucleons and forms numerous quasi-bound states and resonances with them, while no such attraction is evident in the K^+N interaction. (This is in contrast to the π^+ and π^- mesons, whose strong interactions with nuclei are roughly the same.) From the point of view of Matumoto's mass formula (1), this difference is understandable, because the attractive $\overline{p} - N$ part of the interaction between $K^-(= \Lambda\overline{p})$ and N is stronger than the repulsive $\Lambda - N$ part (see Table II). So, in K^-N collisions, Σ resonances or $\Lambda\pi$ quasi-bound states form easily and decay promptly into $\Lambda + \pi$. Similarly, we expect a strong attraction of K^- (as well as $D^+(= \Lambda_c\overline{n})$ and $D_s^+(= \Lambda_c\overline{\Lambda})$) to other baryons and their repulsion from antibaryons. The situation is inverse for the repulsive $K^+ - N = (p\overline{\Lambda}) - N$ interaction. According to Matumoto's formula and our calculations, there are no bound states in the sakaton group $NN\overline{\Lambda}$, so no quasi-bound states or resonances are expected to be seen in experiments. The properties of π^+ and π^- are predicted to lie between these two extremes, because Matumoto interactions of their N and $\overline{N}$ constituents cancel out.

Possibly the most controversial part of the Sakata model is its interpretation of the Ω^- particle as a pentasakaton $\Lambda\Lambda\Lambda\overline{pn}$.[2] Our calculations describe Ω^- as a stable particle having the binding energy of 9.9 MeV with respect to dissociation into $\Xi^0 + K^-$ (see Table V). On a qualitative level, this agrees with the experimental binding energy B.E.$(\Omega^-) = 136.1$ MeV and with the presence of multiple $S = -3, Q = -1$ resonance states: $\Omega(2250)^-, \Omega(2380)^-$, $\Omega(2470)^-$ [6]. However, experimentally, there are no stable particles (and even resonances) with $S = -3$ and $Q = 0, -2$.[3] In the Sakata model, they are associated with $\Lambda\Lambda\Lambda\overline{nn}$ and $\Lambda\Lambda\Lambda\overline{pp}$ pentasakatons, which differ from Ω^- only by the replacement of the pn pair with nn and pp pairs, respectively. Our pn interaction potential is more attractive than the nn/pp potentials: the minimum of the $V_{pn}(r)$ curve in Fig. 2 is 36 MeV deeper than the minimum of $V_{pp}(r) = V_{nn}(r)$. This difference is sufficient to provide only 9.9 MeV of the binding energy in Ω^-, while rejecting bound states of $\Lambda\Lambda\Lambda\overline{nn}$ and $\Lambda\Lambda\Lambda\overline{pp}$. For a better agreement with experiment, we should make the pn and nn/pp potentials more distinct: add more attraction to the former and more repulsion to the latter.[4] Perhaps this can be achieved also by using spin-spin NN forces? If these forces favor the parallel orientation of pn spins (like in the triplet deuteron state), then they should also provide an additional repulsion in pp and nn pairs, where parallel spin orientations are forbidden by the Pauli exclusion principle.

[2] Similar considerations apply to the $\Omega_c^0(= \Lambda_c\Lambda\Lambda\overline{pn})$ particle.
[3] Although, the presence of such states was predicted by García-Recio and co-authors [55]. They suggested to look for Ω-like pentaquark ($sssu\overline{d}$ and $sssd\overline{u}$) resonances with electric charges 0 and -2 in $K^-\Xi^-$ and $\pi^\pm\Omega^-$ collisions. Observations of such resonances would give an indirect support to the Sakata-Matumoto model as well.
[4] Incidentally, this should help to improve the situation with nuclear masses in Table VI: the nuclei with balanced n and p numbers will get extra binding compared to n-dominated and p-dominated nuclei.

V. CONCLUSIONS

Our goal was to decide whether the sakaton-based approach is viable, and whether it is worth pursuing this line of thought. In spite of some deviations, meson, baryon, nuclear, and hypernuclear stability patterns are reproduced quite well in our studies. This is a strong indication that the Sakata model with simple interaction potentials does capture some important aspects of the physics of hadrons. There are theoretical approaches that are superior to ours in each individual sector (meson and baryon mass spectra, scattering of strongly-interacting particles, structure of light nuclei and hypernuclei). However, it is remarkable that such a simple model as ours is versatile enough to provide qualitatively correct answers in so different areas. This cannot be accidental, and we believe that our investigation has provided a convincing "proof of concept" for the Sakata model of hadrons. So, a continuing study into these matters is warranted. It would be interesting to broaden the scope of these investigations by addressing also excited states of nuclei, baryons, and baryonic resonances as well as inelastic collisions of mesons and baryons.

In addition to a more thorough optimization of the interaction parameters, we should add to our potentials also relativistic, momentum-dependent, spin-orbit, and spin-spin corrections. Furthermore, an important role should be given to mixing potentials like

$$V_{mix} \propto p^\dagger\overline{p}^\dagger n\overline{n} + \overline{n}^\dagger n^\dagger \overline{p}p,$$

where $p, n, \overline{p}, \overline{n}$ are annihilation operators for sakatons and antisakatons, while $p^\dagger$, $n^\dagger$, $\overline{p}^\dagger$, $\overline{n}^\dagger$ are their creation operators in the Fock space. In particular, this interaction may be responsible for the large mass difference (about 400 MeV/c^2) between π (isotriplet) and η (isosinglet) mesons.

Appendix: Computational tools

To calculate energy levels of $Z\overline{Z}$ mesons in Figs. 1 and 3 - 5, we used program RADIAL [56]. The same code was employed for calculating 2-body scattering cross sections in Fig. 6. From the phase shifts δ_l reported by the code we obtained partial cross sections as functions of the center-of-mass momentum [57]

$$\sigma_l(p) = 4\pi(2l + 1)\frac{\hbar^2 \sin^2 \delta_l}{p^2}$$

and then summed them up to obtain the total cross section

$$\sigma(p) = \sum_l \sigma_l(p). \tag{A.1}$$

To get converged results for high collision momenta, up to 200 partial waves had to be included in the expansion (A.1).

Bound state energies of multisakaton systems in Tables V - VII were calculated using the stochastic variational method of

Varga and Suzuki [29, 58, 59]. Only ground states with the lowest total spin ($s = 0$ for bosons and $s = 1/2$ for fermions) and zero orbital angular momentum were considered here. Within our approximations, multisakaton groups with calculated binding energies lower than 1 MeV were considered unbound. The basis set size (K) and parameters of the Gaussian exponents b_{min}, b_{max} depended on the type of the calculated system and on the number of sakatons there. For optimal balance between the accuracy and the speed of convergence, we used three different sets of these parameters (see Table IX). For other constants explained in [29] we chose the values $M_0 = 10$ and $K_0 = 50$.

The run times on a 2.4 GHz Intel computer varied from few seconds for 2-sakaton mesons and nuclei, up to 5 hours for some 5-sakaton species.

TABLE IX. Three sets of basis parameters [29]: (I) for hadrons without the heavy Λ_c sakaton; (II) for hadrons with Λ_c sakaton(s); (III) for nuclei and hypernuclei.

Number of sakatons	b_{min} I / II / III	b_{max} I / II / III	basis size (K) I / II / III
2	$10^{-5}/10^{-6}/5\times10^{-6}$	100/10/100	20/20/50
3	$10^{-5}/10^{-6}/5\times10^{-6}$	100/10/100	150/150/100
4	$10^{-5}/10^{-6}/5\times10^{-6}$	100/10/100	150/180/170
5	$10^{-5}/5\times10^{-7}/5\times10^{-6}$	100/5/100	400/400/200

Both FBS [29] and RADIAL [56] computer codes were downloaded from the CPC Program Library (Queen's University of Belfast, N. Ireland).

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