# On the Belief Coulomb Force

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**Abstract**

Conflict management is a key issue in D-S evidence theory(DST) and has been the focus of many related researchers. However, there has been a lack of discussion about whether the evidence should be fused. In this paper, in the frame of DST, inspired by the belief universal gravitation[1], we proposed a concept of belief Coulomb force (BCF) to focus on whether or not the evidence should be fused. It aims to discuss the elimination of conflicts in the information fusion process from the perspective of electricity, which may provide us with a new idea to solve the problem of conflict evidence. An application is used to show that the conflict management is solved better than previous methods by using the proposed BCF.

*Keywords:* Dempster-Shafer evidence theory, Belief Coulomb force, BPA Inverse algorithm, Conflict management, Belief universal gravitation, Coulomb's law.

## 1. Introduction

D-S evidence theory, as a method of uncertain reasoning, based on the notion of belief function, has been widely used in information fusion and decision-making and many other fields (e.g. FMEA[2]). Dempster's combination rule

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plays an important role in DST. However, when the collected evidence is highly conflicting, the classical Dempster rule may result in illogical results. In order to solve the problem of highly conflicting evidence, many scholars have done a lot of research and put forward many solutions, which can be roughly divided into two categories. The first scheme is to modify the classical combination rules of DST, and the second scheme is to preprocess the evidence before fusion. Such methods are represented by the method of average support for propositions[3], average belief[4], modified average method[5] ,novel evidential correlation coefficient[6], etc.

Among them, the concept of belief universal gravitation (BUG)[1] is to combine evidence theory with gravity, and discuss the process of information fusion from the perspective of Newtonian mechanics. But this concept actually has some limitations. BUG only takes into account the gravitational force between evidence, without considering their repulsion force. In addition, none of the above approaches addresses a question that should be addressed: whether a piece of evidence should be fused when there are multiple sets of evidence. Therefore, the new belief Coulomb Force proposed in this paper fills in this gap, preprocessing the conflict evidence before fusion and sift out evidence that should not fuse so as to improve the composition of evidence. Using the combination rule of Dempster under the improved evidence will produce more reasonable conclusions.

The rest of this paper is organized as follows. First, some background knowledge is introduced in Section 2, including DST, belief universal gravitation[1] and Coulomb's law. In Section 3, belief Coulomb force are proposed, which introduces the basic theory and its calculation form. In Section 4, a numerical example is presented to test the improvement effect of belief Coulomb force on evidence fusion after preprocessing conflict evidence. In Section 5, we have a brief conclusion.

## 2. Preliminaries

### 2.1. *Dempster-Shafer evidence theory[7]*

Dempster-Shafer Evidence theory is used to deal with uncertain information, which satisfies weaker conditions than Bayes probability theory. It can integrate a variety of data and knowledge, and has the ability to directly express 'uncertainty' and 'don't know'. Here are some basic concepts of evidence theory that will be introduced.

#### 2.1.1. *Frame of discernment[7]*

If $\Theta$ is a set of mutually exclusive, exhaustive events defined by

$$\Theta = \{A_1, A_2, \cdots, A_n\}, \tag{1}$$

$\Theta$ is called the frame of discernment. If it has n elements, then its set of powers has $2^n$ elements. Each subset, including the empty set $\emptyset$ , corresponds to a proposition.

#### 2.1.2. *Mass function[7]*

Mass function, it's also called the basic probability assighment(BPA) in D-S evidence theory. For any of the propositions in the frame, it corresponds to an m,which satisfies:

$$m \in [0, 1] \tag{2}$$

If m meets the following two conditions:

$$m(\emptyset) = 0 \quad and \quad \sum_{T \subseteq \Theta} m(T) = 1 \tag{3}$$

it can be called a BPA.

*2.1.3. Belief function and plausibility function[8]*

For a BPA on $\Theta$, a belief function $Bel$ is defined by

$$Bel(T) = \sum_{E \subseteq T} m(E), \ \forall T \in 2^{\Theta} \tag{4}$$

where $Bel : 2^{\Theta} \in [0, 1]$. $Bel(T)$ can express total trust in hypothesis $T$.

Then, a plausibility function $Pl$ is defined as:

$$Pl(T) = 1 - Bel(\bar{T}) = \sum_{E \bigcap T = \emptyset} m(E), \ \forall T \in 2^{\Theta} \ (\bar{T} = \Theta - T) \tag{5}$$

where $Pl : 2^{\Theta} \in [0, 1]$. $Pl(T)$ expresses the degree of trust that does not negate the hypothesis $T$.

The above belief function and plausibility function have the following relationship:

$$Pl(T) \geq Bel(\bar{T}) \tag{6}$$

*2.2. Dempster's combination rule[7]*

$m_1$ and $m_2$ are two mass functions on the frame $\Theta$. The definition of Dempster's combination is:

$$m(T) = \begin{cases} \frac{\sum\limits_{A_1 \cap A_2 = T} m_1(A_1) m_2(A_2)}{1-K}, & T \neq \emptyset \\ 0, & T = \emptyset \end{cases} \tag{7}$$

with K (K¡1)

$$K = \sum_{A_1 \cap A_2 = \emptyset} m_1(A_1) m_2(A_2) \tag{8}$$

as the conflict coefficient.

Dempster's combination rule satisfies the associative and commutative laws when there are multiple evidence combinations.

*2.3. Evidence distance [9]*

Assumptions under frame $\Theta$, there are two evidence $T_1$ and $T_2$ and BPA $m_1$ and $m_2$, focus elements are $A_i$ and $A_j$ respectively. If $m1$ and $m_2$ both are row vectors, the evidence distance between $m_1$ and $m_2$ is shown as:

$$d\left(m_1, m_2\right) = \sqrt{\frac{1}{2}(m_1 - m_2)D(m_1 - m_2)^T} \tag{9}$$

while D is a $2^n \times 2^n$ symmetric matrix (n is the number of focus elements in the frame $\Theta$).The elements in the D matrix are expressed as:

$$e_{ij} = \frac{|A_i \cap A_j|}{|A_i \cup A_j|} \tag{10}$$

*2.4. Belief Universal Gravitation(BUG)*

Mi et al[1] proposed a belief universal graviation for belief functions, the evidence distance is defined as follows.

*2.4.1. EQC algorithm[1]*

In order to obtain evidence quality, Mi et al. proposed an EQC algorithm to generate evidence quality. The method is as follows:

Definition 1. *$m_1$ and $m_2$ are two mass functions on the frame $\Theta$={$A$,$B$,$C$},and their BPAs are following:*

$$m_1 : m_1(A) = \alpha_1, m_1(B) = \beta_1, m_1(C) = \theta_1$$

$$m_2 : m_2(A) = \alpha_2, m_2(B) = \beta_2, m_2(C) = \theta_2$$

*Note that $\alpha_i \neq 0$, $\beta_i \neq 0$, $\theta_i \neq 0$.*

*Then assign each BPA a binary code which based on the order of propositions in the discernment frame.The coding principle is expressed as: for each BPA, the position mark corresponding to the proposition in the recognition frame is 1, and the other position marks are 0. The binary codes of $m_1$ and $m_2$ are as* *follows:*

$$m_1 : m_1(A)- > 100, m_1(B)- > 010, m_1(C)- > 001$$

$$m_2 : m_2(A)- > 100, m_2(B)- > 010, m_2(C)- > 001$$

*After encoding, the binary encoding of each BPA is converted to decimal.*

$$m_1 : m_1(A)- > 4, m_1(B)- > 2, m_1(C)- > 1$$

$$m_2 : m_2(A)- > 4, m_2(B)- > 2, m_2(C)- > 1$$

*After the above transformation, the quality of evidence is generated:*

$$M_{m1} = \frac{4 \times \alpha_1 + 2 \times \beta_1 + 1 \times \theta_1}{n} = \frac{4 \times \alpha_1 + 2 \times \beta_1 + 1 \times \theta_1}{3} \tag{11}$$

$$M_{m2} = \frac{4 \times \alpha_2 + 2 \times \beta_2 + 1 \times \theta_2}{n} = \frac{4 \times \alpha_2 + 2 \times \beta_2 + 1 \times \theta_2}{3} \tag{12}$$

*The $n$ in this formula is the number of focus elements.*

*2.4.2. Belief Universal Gravitation*

Definition 2. *Let $m_1$ and $m_2$ be two separate and different evidences on the same discernment frame $\Theta$. The BUG formula is defined as*

$$F_{BPA} = G_{ET} \frac{M_{m1} M_{m2}}{d^2} \tag{13}$$

*where the $G_{ET}$ is defined as*

$$G_{ET} = 10^{-\delta|\Theta|} \tag{14}$$

*with*

$$0 \leq \delta \leq 1 \tag{15}$$

*In Eq (13), $G_{ET}$ is the evidence gravitation parameter used to distinguish different discernment frames. $\delta$ is an adjustable parameter that satisfies the equality constraint(15). $M_{m1}$ and $M_{m2}$ represent the quality of the evidence $m_1$*

*and $m_2$ by using the EQC algorithm [1].*

*As for the d in the formula, it is the evidence distance[9].*

### *2.5. Coulomb's law*

*Coulomb's law is the law of the interaction between charges at rest point. In 1785, the French scientist C, - A. de Coulomb obtained from the experiment that the interaction force between two static point charges in vacuum is directly proportional to the product of their charge quantity, and inversely proportional to the quadratic power of their distance. The direction of the force is on their connecting line, and the same charges repel each other and the opposite charges attract each other.*

*Its mathematical expression is following:*

$$F = k\frac{q_1 q_2}{r^2} \tag{16}$$

## 3. Proposed Belief Coulomb Force(BCF)

In this section,inspired by the BUG and the Coulomb's law, we proposed a new conception of the belief Coulomb force.

In order to make it easier to understand, here we use the method of analogy with Coulomb's law to illustrate.

### *3.1. Belief Coulomb gravitation*

We assume that the BPA with the greatest evidence electrification is the positive evidence, and that the remaining BPAs are the negative evidences, which have Coulomb gravitations with the positive evidence. Their belief Coulomb gravitation formula is defined as:

$$F_g = k_{gr}\frac{Q_{m_1} Q_{m_2}}{d^2} \tag{17}$$

in Eq (17), $Q_{m_1}$ and $Q_{m_2}$ represent the evidence electrification of each BPA. In the method we roposed, Q and M in the EQC algorithm proposed by Mi et al[1] are the same in terms of numerical value and calculation method, but the meaning represented by them is different, because Q here is the electrification in Coulomb force as evidence. According to the EQC algorithm proposed by Mi et al.[1], the evidence electrification of each BPA can be calculated, whick are shown in Eq (11) and Eq (12). The $k_{gr}$ is the evidence force parameter used to distinguish different recognition frameworks, which has the same numerical value and calculation form. d represents the evidence distance between $m_1$ and $m_2$.

### *3.2. BPA inverse algorithm*

By inversing the negative evidence of BPA, we will have some new positive BPA. In order to reverse the evidences, we propose a BI algorithm to inverse BPA.The introduction to the BI algorithm is shown below.

Assumption. *Assume $m_1$ ,$m_2$ and $m_3$ are three BPA on the same discernment frame$\Theta$=$\{A,B,C\}$,and their BPA are following:*

$$m_1 : m_1(A) = \alpha_1, m_1(B) = \beta_1, m_1(C) = \theta_1$$

$$m_2 : m_2(A) = \alpha_2, m_2(B) = \beta_2, m_2(C) = \theta_2$$

$$m_3 : m_3(A) = \alpha_3, m_3(B) = \beta_3, m_3(C) = \theta_3$$

*Notably, $\alpha_i \neq 0$, $\beta_i \neq 0$, $\theta_i \neq 0$.*

*Step1:*

*Assume that $m_2$ has the greatest evidence electrification, which we difine as the positive evidence. So $m_1$ and $m_3$ are the negative evidences. Then we inverse the $m_1$'s and $m_3$'s BPA, that's one minus the BPA of $m_1$ and $m_3$:*

$$\overline{m}_1(A) = 1 - m_1(A) = 1 - \alpha_1$$

$$\overline{m}_1(B) = 1 - m_1(B) = 1 - \beta_1$$

$$\overline{m}_1(C) = 1 - m_1(C) = 1 - \theta_1$$

$$\overline{m}_3(A) = 1 - m_3(A) = 1 - \alpha_3$$

$$\overline{m}_3(B) = 1 - m_3(B) = 1 - \beta_3$$

$$\overline{m}_3(C) = 1 - m_3(C) = 1 - \theta_3$$

*Step2:*

*After this inverse transpose, we will get the following result:*

$$\sum_{A \subseteq \Theta} \overline{m}(A) \neq 1$$

*In order to satisfy the normalization, we should normalize the inversed BPA:*

$$sum_1 = \sum_{A \subseteq \Theta} \overline{m}_1(A)$$

$$\overline{m}_1(A) = \frac{\overline{m}_1(A)}{sum_1}$$

$$\overline{m}_1(B) = \frac{\overline{m}_1(B)}{sum_1}$$

$$\overline{m}_1(C) = \frac{\overline{m}_1(C)}{sum_1}$$

*Similarly, the* $\overline{m_3}$ *also be normalized:*

$$sum_2 = \sum_{A \subseteq \Theta} \overline{m}_3(A)$$

$$\overline{m}_3(A) = \frac{\overline{m}_3(A)}{sum_2}$$

$$\overline{m}_3(B) = \frac{\overline{m}_3(B)}{sum_2}$$

$$\overline{m}_3(C) = \frac{\overline{m}_3(C)}{sum_2}$$

*Step3:*

*According to the EQC algorithm proposed by Mi et al.[1], the new evidence* *electrification of* $m_1$ *and* $m_3$ *can be calculated as following:*

$$\overline{m}_1 : \overline{m}_1(A) -> 100, \overline{m}_1(B) -> 010, \overline{m}_1(C) -> 001$$

$$\overline{m}_3 : \overline{m}_3(A) -> 100, \overline{m}_3(B) -> 010, \overline{m}_3(C) -> 001$$

*Convert binary to decimal:*

$$\overline{m}_1 : \overline{m}_1(A) -> 4, \overline{m}_1(B) -> 2, \overline{m}_1(C) -> 1$$

$$\overline{m}_3 : \overline{m}_3(A) -> 4, \overline{m}_3(B) -> 2, \overline{m}_3(C) -> 1$$

*Then the electrification of evidence is generated:*

$$\begin{aligned}\overline{Q_{m1}} &= \frac{4 \times (1-\alpha_1) + 2 \times (1-\beta_1) + 1 \times (1-\theta_1)}{n} \\ &= \frac{4 \times (1-\alpha_1) + 2 \times (1-\beta_1) + 1 \times (1-\theta_1)}{3}\end{aligned}$$

$$\begin{aligned}\overline{Q_{m3}} &= \frac{4 \times (1-\alpha_3) + 2 \times (1-\beta_3) + 1 \times (1-\theta_3)}{n} \\ &= \frac{4 \times (1-\alpha_3) + 2 \times (1-\beta_3) + 1 \times (1-\theta_3)}{3}\end{aligned}$$

*3.3. Belief Coulomb repulsion*

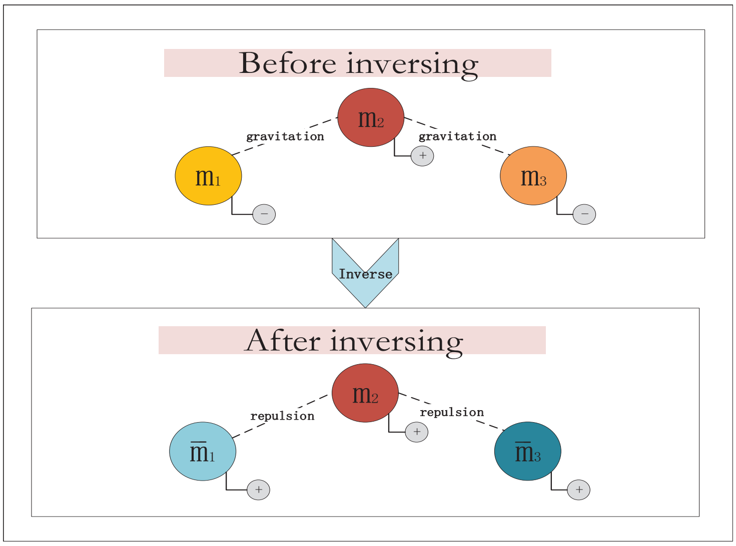


Figure 1: Show their forces in picture

Aftering inversing, as shown in Figure 1, $m_2$ is the positive evidence and the remaining BPA are also the positive evidences, which have Coulomb repulsion with the positive evidence. The formula for belief Coulomb repulsion is similar to the formula for belief gravitation:

$$F_r = k_{gr}\frac{\overline{Q_{m_1}}Q_{m_2}}{d^2} \tag{18}$$

In Eq (18), $k_{gr}$ is the same as the $k_{gr}$ in Eq (17).

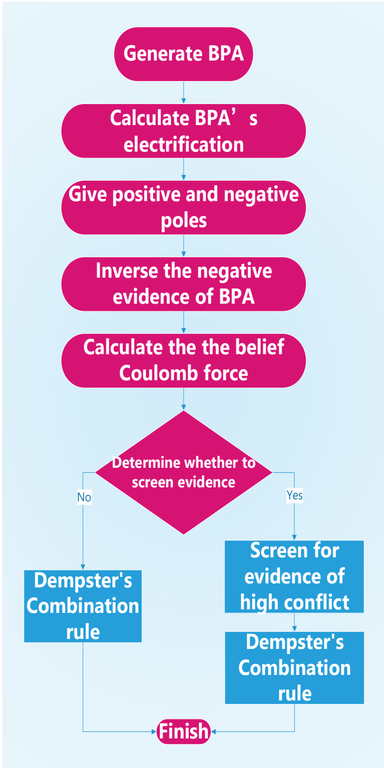


Figure 2: The procedure of the proposed method

## 4. Application of the proposed BCF in BPA fusion and discussions

In this section, we will briefly introduce a numerical example and apply belief Coulomb force in this example and discuss the effect of belief Coulomb force on conflict preprocessing.

Example 1. *Consider five BPA $m_1$, $m_2$, $m_3$, $m_4$ and $m_5$ on the same discernment frame*$\Theta$=$\{A,B,C\}$,*and their BPA are following[5]:*

$$\mathbf{m}_1 : m_1(\{A\}) = 0.5, m_1(\{B\}) = 0.2, m_1(\{C\}) = 0.3;$$
$$\mathbf{m}_2 : m_2(\{A\}) = 0, m_2(\{B\}) = 0.9, m_2(\{C\}) = 0.1;$$
$$\mathbf{m}_3 : m_3(\{A\}) = 0.55, m_3(\{B\}) = 0.1, m_3(\{A, C\}) = 0.35;$$
$$\mathbf{m}_4 : m_4(\{A\}) = 0.55, m_4(\{B\}) = 0.1, m_4(\{A, C\}) = 0.35;$$
$$\mathbf{m}_5 : m_5(\{A\}) = 0.60, m_5(\{B\}) = 0.1, m_5(\{A, C\}) = 0.3;$$

By simple analysis, the evidence $m_2$ is a conflict evidence because other evidence sopports the proposition A relatively, while evidence $m_2$ has no support for it.

Based on the theory of belief Coulomb force we proposed, the evidence quality of the five BPAs is obtained as

$$\begin{aligned}\mathbf{m}_1 &: Q_{m_1} = 0.9000\\ \mathbf{m}_2 &: Q_{m_2} = 0.9500\\ \mathbf{m}_3 &: Q_{m_3} = 1.3833\\ \mathbf{m}_4 &: Q_{m_4} = 1.3833\\ \mathbf{m}_5 &: Q_{m_5} = 1.3667\end{aligned}$$

And the size of their belief electrification is as follows:

$$Q_{m_3} = Q_{m_4} > Q_{m_5} > Q_{m_2} > Q_{m_1}$$

Obviously, $m_3$ and $m_4$ have the greatest evidence electrification. Under the proposed theory, we can choose either of them. Here choose $m_3$ as the positive evidence, so $m_1$, $m_2$, $m_4$ and $m_5$ are the negative evidences. The Coulomb grabitations and Coulomb repulsions of these five BPA are shown in Table 1 and Table 2.

Table 1: The belief Coulomb gravitaiton between remaining BPA and $m_3$ in Example 1.

| *Gravitation* | $m_1$ | $m_2$ | $m_3$* | $m_4$** | $m_5$ |
|---|---|---|---|---|---|
| $m_3$ | 18.1091 | 2.1325 | Inf | Inf | 1512.4444 |

* $m_3$ is the positive evidence itself, so it has infinite belief Coulomb gravitation with itself.

** $m_4$ is equal to $m_3$.

Table 2: The belief Coulomb repulsion between remaining BPA and $m_3$ in Example 1.

| *Repulsion* | $\overline{m}_1$ | $\overline{m}_2$ | $\overline{m}_3{}^*$ | $\overline{m}_4{}^{**}$ | $\overline{m}_5$ |
|---|---|---|---|---|---|
| $m_3$ | 11.3600 | 24.5242 | 9.3062 | 9.3062 | 9.2137 |

* $m_3$ is the evidence itself.

** $m_4$ is equal to $m_3$.

After analyzing the data, it can be seen from above tables that $m_2$ has the lowest Coulomb gravitation and the highest Coulomb repulsion with the $m_3$. More specifically, the $m_2$ is evidence of conflict and should be screened out.

The fusion results of these five pieces of evidence information after screening out the $m_2$ are shown below.

Table 3: The fusion result after screening

| | $m_1, m_2$ | $m_1, m_2, m_3$ | $m_1, m_2, m_3, m_4$ | $m_1, m_2, m_3, m_4, m_5$ |
|---|---|---|---|---|
| *Dempster − Shafer′s* | m(A)=0 | m(A)=0 | m(A)=0 | m(A)=0 |
| *combination rule* | m(B)=0.8571 | m(B)=0.6316 | m(B)=0.3288 | m(B)=0.1228 |
| | m(C)=0.1429 | m(C)=0.3684 | m(C)=0.6712 | m(C)=0.8772 |
| *Murphy′s average* | m(A)=0.1543 | m(A)=0.3500 | m(A)=0.6027 | m(A)=0.7958 |
| *combination rule* | m(B)=0.7469 | m(B)=0.5224 | m(B)=0.2627 | m(B)=0.0932 |
| | m(C)=0.0988 | m(C)=0.1276 | m(C)=0.1346 | m(C)=0.1110 |
| *Deng′s* | m(A)=0.1543 | m(A)=0.4861 | m(A)=0.7773 | m(A)=0.8909 |
| *modified average* | m(B)=0.7469 | m(B)=0.3481 | m(B)=0.0628 | m(B)=0.0086 |
| *combination rule* | m(C)=0.0988 | m(C)=0.1657 | m(C)=0.1600 | m(C)=0.1005 |
| | $m_1{}^*$ | $m_1, m_3{}^*$ | $m_1, m_3, m_4{}^*$ | $m_1, m_3, m_4, m_5{}^*$ |
| *Proposed* | m(A)=0.5000 | m(A)=0.7826 | m(A)=0.9137 | m(A)=0.9701 |
| *method* | m(B)=0.2000 | m(B)=0.0348 | m(B)=0.0045 | m(B)=0.0005 |
| | m(C)=0.3000 | m(C)=0.1826 | m(C)=0.0828 | m(C)=0.0293 |

* $m_2$ is screen out during evidence fusion in proposed method.

It can be easily seen from the Table 3 that the performance of the BCF method is better than the previous three methods, namely Dempster's Combination rule, Murphy's Average Combination rule and Deng's Modified Average Combination rule. The effect of evidence fusion is significantly increased and more reasonable after the application of The belief Coulomb force. The main reason for this phenomenon is that BCF makes use of the forces between the evidences to screen out the conflicting evidences, so that the influence of the conflicting evidences on the final combination results disappears. In this case, the original fusion is a simple combination of five pieces of evidence. However, the original fusion result do not accord with the reality, because of the conflict evidence $m_2$. Under the action of belief Coulomb force, evidence will be analyzed to find out whether there are some highly conflicting evidences and the conflicting evidences will be screened out. This treatment significantly improves the fusion effect. Rather than combining conflicting evidence in various ways, this is a new idea provided for information fusion.

## 5. Conclusion

In this paper, we proposed the belief Coulomb force in DST. In general, the contributions of this paper are as follows. First of all, to the best of our knowledge, this is the first time that evidence theory has discussed whether evidence should be fused and screened for conflicting evidence, rather than all evidence being fused in one way or another. Secondly, according to the relevant knowledge of DST, this theory can be used to show the interaction forces between evidence. Finally, the validity of the theory is illustrated by the evidence fusion of a numerical example.

In addition, there are still some shortcomings that need to be further overcome.For example, as a way to inverse BPA, is there a more reasonable alternative to the BI algorithm? How to consider more examples to reflact and increase the rationality and validity of belief repulsion and attraction, as well as consider its application to not numerical but more practical problems, etc.

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**Conflict of interest**

The authors declare that they have no conflict of interest.

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